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Iterative Solution of Field Problems with a Varying Physical Parameter

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Abstract In this paper, linear field problems with a varying physical parameter are solved with the conjugate gradient method and a dedicated extrapolation procedure for generating the initial estimate. The scheme is formulated in detail, and its application to three-dimensional scattering problems for a rectangular conducting plate and an inhomogeneous, dispersive dielectric body are discussed.

1 Introduction

In modern society different trends are recognized in the usage of the available electromagnetic spectrum. One can think of wireless communication or transport of (digital) information. The density of such applications is increasing rapidly. Obtaining electromagnetic compatibility and/or reducing electromagnetic interference sometimes seems to be an impossible task. Another trend is found in electromagnetic inverse scattering and profiling. For example, this development is used in the detection and classification of land mines and other unexploded ordnance. Regarding electromagnetic inversion, one can also think of medical applications such as tomography or the detection of defects in metallic heart valves. Finally, we would like to mention the problem of electromagnetic coupling into humans in the area of clinical hyperthermia or non-ionizing radiation hazards analysis. In these applications, a rigorous electromagnetic analysis is indispensable.

The focus of this chapter is found in computational tools in the field of electromagnetic analysis and design. One can identify the roadmap "going from engineering electromagnetics to electromagnetic engineering". One of the extensively used and most versatile methods is the integral equation technique. It takes into account that the irradiated object is present in free space and that it manifests itself through the presence of secondary sources or contrast sources. Integral equations can be solved by the method of moments [1]. This leads to a system of linear algebraic equations.

To solve this system, one can use direct discrete numerical solution methods, such as Gaussian Elimination or Singular Value Decomposition, or suitable iterative techniques such as a conjugate gradient (CG) method. An overview of numerical solution methods for linear systems of equations can be found in the book by Golub and Van Loan [2]. In electromagnetic scattering and coupling problems, integral equations are often solved by using a Fast Fourier Transform to compute the spatial convolution of the integral operator and a conjugate gradient iterative scheme. This so-called CGFFT method has been used successfully for many electromagnetic scattering and coupling problems [3, 4, 5, 6, 7, 8, 9].

In analysis or design procedures, the engineer has the freedom to change one or more parameters to obtain an optimal design with respect to performance and costs. This means that he or she will need to consider the determination of electromagnetic fields for a (large) number of values of a physical parameter. In this chapter we present a strong approach for this type of problem, which utilizes the iterative schemes mentioned above. We restrict ourselves to the case where the linear system originates from one or more integral equations. We apply an iterative procedure based on the minimization of an integrated squared error, and start this procedure from an initial estimate that is a linear combination of the last few "final" results. When the coefficients in this extrapolation are determined by minimizing the integrated squared error for the actual value of the parameter, the built-in orthogonality in this type of scheme ensures that only a few iteration steps are required to obtain the solution. The success of this strategy has been demonstrated before [10, 11, 12]. However, it has not been applied to 3-D problems.

The outline of the chapter is as follows. In Sect. 2, the method of solution is discussed. Special attention is given to the iterative procedure and the implementation of a relevant initial estimate based on the previous solutions. Explicit examples are discussed in Sect. 4 and Sect. 5 presents the conclusions.

2 Method of Solution

In the computational modeling of electromagnetic fields for practical applications, typically a large system of linear equations must be solved. This system originates from spatially discretizing Maxwell's differential equations (in "finite" or "local" techniques) or equivalent integral equations (in "global" techniques). In formal notation, such a system can be written as

$$L(p)\,u(p) = f(p), \tag{1}$$

where

$$\begin{aligned} L(p) &= \text{a linear operator,} \\ u(p) &= \text{the unknown field,} \\ f(p) &= \text{the forcing function,} \\ p &= \text{a physical parameter.} \end{aligned}$$

The operator $L(p)$ originates from discretizing its counterpart in the continuous equation, $u(p)$ is a discretized field and $f(p)$ corresponds to an impressed source or an incident field. We are interested in the situation where this problem must be solved for a large number of sampled values of the parameter p, e.g., $p_m = p_0 + m\Delta p$, with $m = 0, 1, \ldots, M$.

To solve the system of equations (1), we use the conjugate gradient method. This algorithm is described in detail by Van den Berg [3, 4]. Further, we organize the space discretization such that the convolution structure of the continous equation is preserved. In that case, the matrix-vector products in the CG algorithm can be evaluated by FFT operations, which considerably improves the speed of the so-called CGFFT algorithm [3, 4, 5, 6, 7, 8, 9].

In many applications of the conjugate gradient method, a simple initial estimate is used. Typically, the scheme is started from an initial vector $u^{(0)} = 0$. Depending on the nature of the problem at hand, we can also start from an incident field or from the Kirchhoff approximation to an unknown surface current.

Our choice of the initial estimate is inspired by the fact that $u(p)$ depends in a well-behaved manner on the parameter p. Therefore, it should be possible to extrapolate, by choosing

$$u^{(0)}(p_m) = \sum_{k=1}^{K} \gamma_k\, u(p_{m-k}). \tag{2}$$

The interpretation of the conjugate gradient scheme given above suggested that the values $\{\gamma_k \mid k = 1, \ldots, K\}$ should be found by minimizing the squared error

$$< L(p_m)u^{(0)}(p_m) - f(p_m) \mid L(p_m)u^{(0)}(p_m) - f(p_m) > . \tag{3}$$

where we have defined the inner product as

$$< g \mid h >= \sum_j g_j^* h_j, \tag{4}$$

where g_j and h_j denote the components of g and h, and where the asterisk denotes complex conjugation.

Because of the built-in orthogonality of the conjugate gradient method, we are then certain that this procedure must start its search for components of $f(p_m)$ outside the space spanned by the "previous" functions $\{Lu(p_{m-k}) \mid k = 1, \ldots, K\}$.

The coefficients γ_k that minimize the squared error in (3) can be found from the system of linear equations

$$\sum_{k'=1}^{K} < L(p_m)u(p_{m-k}) \mid L(p_m)u(p_{m-k'}) > \gamma_{k'} \\ =< L(p_m)u(p_{m-k}) \mid f(p_m) >, \tag{5}$$

with $k = 1, \ldots, K$. Typically, we choose $K = 2$ (linear extrapolation) or $K = 3$ (quadratic extrapolation). For larger values of K, the basis vectors $L(p_m)u(p_{m-n})$ with $n = 1, \ldots, K$ become almost linearly dependent, and therefore the coefficients $\{\gamma_k\}$ can no longer be resolved from (5).

3 Scattering by Three-Dimensional Objects

To illustrate our approach, we have extended existing implementations of the CGFFT procedure for two three-dimensional objects that have become standards in the literature. In both cases, no special precautions were taken to enhance the discretization, which is first-order accurate as a function of the mesh size.

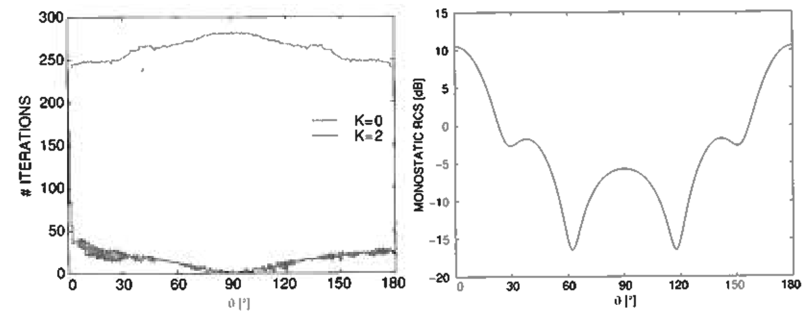


Fig. 1. Marching-on-in-angle version of the CGFFT method for a flat plate. (a) Number of iterations required to reach a relative error of 10^{-3} versus angle of incidence using zero (gray line) and two previous results (black line) as an initial estimate. (b) Monostatic radar cross section versus angle of incidence

3.1 Scattering by a Flat Plate

The first example is a flat, rectangular plate in free space located at $0 < x < a$, $0 < y < b$ and $z = 0$. For this problem, we solve the well-known electric-field integral equation

$$\left[\nabla_{\mathrm{T}}\nabla_{\mathrm{T}}\cdot - \frac{s^2}{c_0^2}\right] \int_0^a dx' \int_0^b dy'\, \frac{\exp(-sR/c_0)}{4\pi R}\, \boldsymbol{J}_{\mathrm{S}}(\boldsymbol{r}'_{\mathrm{T}}, s) \\ = -s\varepsilon_0\, \boldsymbol{E}^{\mathrm{i}}_{\mathrm{T}}(\boldsymbol{r}_{\mathrm{T}}, s), \tag{6}$$

where s is a complex frequency, $R = |\boldsymbol{r}_{\mathrm{T}} - \boldsymbol{r}'_{\mathrm{T}}|$, and where the subscript T stands for a transverse component. The unknown surface current $\boldsymbol{J}_{\mathrm{S}}(\boldsymbol{r}_{\mathrm{T}}, s)$ is approximated by rooftop functions, and we use a weak formulation of (6), weighted by the same rooftop functions [13]. In the resulting discretized form, the convolution symmetry is preserved, so that the matrix-vector products in the conjugate gradient procedure can be evaluated with the aid of two-dimensional FFT operations.

In particular, we have computed the monostatic radar cross section of a $\lambda \times \lambda$ plate for the special case $s = j\omega$. A plane wave is incident on the plate at an angle ϑ with respect to the z-axis and an angle $\phi = 90°$ with respect to the x-axis. The incident plane wave is x-polarized. The discretized plate has a mesh of 31×31 points. Figure 1(a) shows the number of iterations for increasing ϑ. In the generic formulation of Section 2, this means that $p = \vartheta$. The gray line represents starting from a zero initial estimate, and the black line is for two previous results in the initial estimate, i.e. $K = 2$. Figure 1 presents the monostatic radar cross section of the $\lambda \times \lambda$ plate in the plane $\phi = 90°$.

Another result for the plate concerns marching on in length. Now, the parameter p represents the length of the plate in the x-direction. The idea was inspired by the shape sensitivity analysis in [14, 15]. Here, we start from a $\lambda \times \lambda$ plate and we increase the length of the plate in 100 steps to a $2\lambda \times \lambda$ plate. We used a fixed spatial discretization of 62×31 mesh points. The number of iterations required to reach a relative error of 10^{-3} versus the length of the plate is shown in Fig. 2. In the computations leading to Figs. 1 and 2, it turned out that extrapolation with $K = 2$ was in fact more efficient than extrapolation with $K = 3$.

3.2 Scattering by an Inhomogeneous Dielectric Cube

The second example is an inhomogeneous dielectric cube, again in free space. We formulate the scattering problem as a domain integral equation over the object domain $\mathcal{D}$ as

$$\boldsymbol{E}^{i}(\boldsymbol{r}, s) = \frac{\boldsymbol{D}(\boldsymbol{r}, s)}{\varepsilon(\boldsymbol{r}, s)} + \left(\frac{s^2}{c_0} - \nabla\nabla\cdot\right) \boldsymbol{A}(\boldsymbol{r}, s), \tag{7}$$

where s is a complex frequency and where the vector potential $\boldsymbol{A}(\boldsymbol{r}, s)$ is given by

$$\boldsymbol{A}(\boldsymbol{r}, s) = \frac{1}{\varepsilon_0} \iiint_{\mathcal{D}} \frac{\exp(-sR/c_0)}{4\pi R}\, \frac{\varepsilon(\boldsymbol{r}, s) - \varepsilon_0}{\varepsilon(\boldsymbol{r}, s)}\, \boldsymbol{D}(\boldsymbol{r}, s)\, dV', \tag{8}$$

where $R = |\boldsymbol{r} - \boldsymbol{r}'|$. We take the contrast function in (8) constant in each rectangular subdomain in the space discretization. Like the current in the plate problem, the dielectric displacement $\boldsymbol{D}(\boldsymbol{r}, s)$ is approximated by an expansion

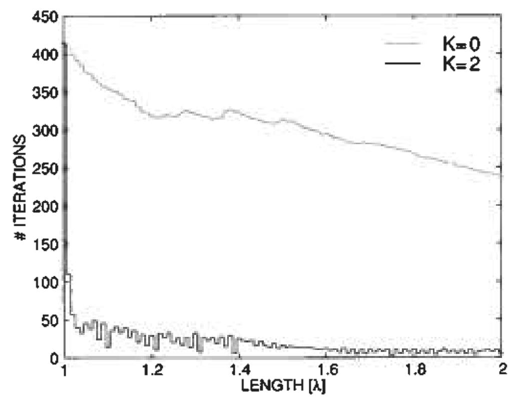
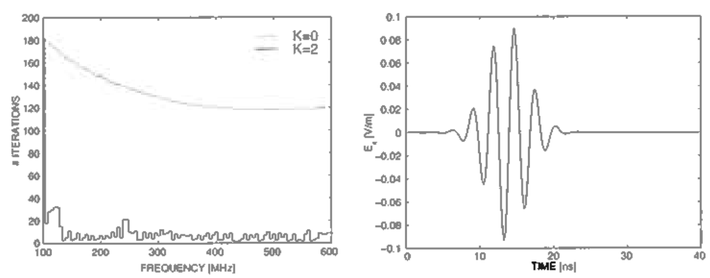


Fig. 2. Number of iterations required to reach a relative error of 10^{-3} versus length of the plate for the marching-on-in-length version of the CGFFT method for a flat plate using zero (gray line) and two previous results (black line) as an initial estimate



(a) (b)

Fig. 3. Marching-on-in-frequency version of the CGFFT method for an inhomogeneous dielectric cube. (a) Number of iterations required to reach a relative error of 10^{-3} versus frequency for the using zero (gray line) and two previous results (black line) as an initial estimate. (b) Time domain signal at the center of the muscle cube for an incident x-polarized wave of 1 V/m

that is piecewise linear in the longitudinal direction and constant in the transverse directions. The Green's function is replaced by a weak form, and the result is weighted by testing functions that are identical to the expansion functions. Again, the space discretization preserves the convolution symmetry of the continuous form of the integral equation given in (7) and (8). More details can be found in the papers by Zwamborn and Van den Berg [16, 17].

As an illustration, we have modeled a cube of muscle tissue centered inside a cube of fat tissue. The incident field is x-polarized with propagation vector parallel to the z-axis and a strength of 1 V/m. The dispersive tissues are modeled using a Debye model [18] and the dimensions of the inner and outer cubes are 0.14 m and 0.30 m, respectively. The discretized object has $30 \times 30 \times 30$ mesh points. The field is computed in the middle of the muscle cube for real-valued frequencies $f = \omega/2\pi = -js/2\pi$ of 100 to 600 MHz and then converted to a time domain signal. In this case, we vary $p = \omega$. The number of iterations needed is shown in Fig. 3(a), where the gray line is for a zero initial estimate, and the black line for minimization using two previous results. Again, using $K = 2$ in the extrapolation procedure led to the most rapid convergence. The time signal, shown in Fig. 3(b), is computed by an FFT using the waveform $\exp\left[-(t-\tau)^2/(2T^2)\right]\sin(\omega_0 t)$, where $\tau = 14$ ns, $T = 2.75$ ns and $\omega_0/2\pi = 450$ MHz.

4 Conclusions

In this chapter, we have extended the conventional conjugate gradient method with a dedicated extrapolation procedure that considerably enhances the speed of convergence. Although the procedure has already been demonstrated successfully for a range of applications, including transient scattering, radar cross section computations and inverse profiling,

until now no applications to three-dimensional configurations have been reported. In the present chapter, this gap has been filled.

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