Quasi-networks in social relational systems

TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Memorandum COSOR 91-27

Quasi-Networks in Social Relational Systems

Robert P. Gilles
Pieter H.M. Ruys
Shou Jilin

Eindhoven University of Technology
Department of Mathematics and Computing Science
P.O. Box 513
5600 MB Eindhoven
The Netherlands

Eindhoven, October 1991
The Netherlands

Quasi-Networks in Social Relational Systems*

Robert P. Gilles‡

Pieter H.M. Ruys§

Shou Jilin¶

October 1991

---

*The authors would like to thank Ton Storcken and Thijs ten Raa for their comments on previous drafts of this paper

‡Department of Economics, Virginia Polytechnic Institute & State University, Blacksburg, Virginia, USA. Financial support from the Netherlands Organization for Scientific Research (NWO), grant 450-228-009, is gratefully acknowledged.

§Department of Econometrics, Tilburg University, Tilburg, the Netherlands.

¶Department of Mathematics, Xi'an Jiaotong University, Xi'an, People's Republic of China. This author gratefully acknowledges financial support from the Department of Mathematics and Computing Sciences, Eindhoven University of Technology, Eindhoven, the Netherlands, and the CentER for Economic Research, Tilburg University, Tilburg, the Netherlands.

1 Introduction

In social sciences it is more or less generally acknowledged that (social) networks seem to be the natural tool in the organization of social activity. In the literature on the mathematical description of social phenomena the concept of a network is hardly used. In this paper we focus on the development of a possible mathematical formulation of the properties of a social network within the setting of an economic trade system.

In the literature on general economic equilibrium theory one uses the concept of a *social system* or an *abstract economy* to indicate a mathematical system that holds the principal features of a trade economy with a Walrasian market system. Essentially a social system consists of a set of agents and a mapping that assigns to every agent some tuple of individual economic attributes. Usually one takes a topological attribute space. In most cases we may therefore conclude that a social system is equivalent to a subspace of a topological attribute space. We specifically refer to the literature on attribute spaces, e.g., Hildenbrand (1974), Grodal (1974), and Mas-Colell (1985), as well as to the literature on abstract economies, e.g., Shafer and Sonnenschein (1975) and Vind (1983).

Within an abstract economy one describes the demand and supply of economic commodities, which are based on the individual attributes of the economic agents and certain prices that emerge on the markets. One of the major achievements of general equilibrium theory is that equilibrium prices, which assign zero net demands, have been shown to exist under conditions that allow for applying a fixed point argument. (For a complete treatment of this problem we refer to Debreu (1959).)

Agents in a standard social system that represent consumers or producers, are treated symmetrically. The same decision rule applies for all, and they differ only in their *individual* attributes. This symmetry is characteristic for the so called neoclassical general equilibrium models. Any asymmetry between agents, caused by e.g. monopolies or hierarchical industries, precludes general equilibrium analysis and requires the economic analysis to be of partial nature. In order to introduce asymmetry between agents with respect to either decisions, communication or productive capacities, we have to design and to analyse models in which there exist a relational structure between agents. This has also been done by Myerson (1977). In Gilles and Ruys (1990) and Gilles, Ruys and Shou (1991) the concept of a network has been introduced into a social system.

In order to capture asymmetric features of economic agents we propose to alter the fundamental notion of a social system by introducing a binary relation on the set of agents. This relation is describing the incomplete (asymmetric) possibilities of communication between the individual agents in the social system. Mathematically we thus introduce the notion of a ***social relational system*** consisting of a collection of agents, represented by their individual attributes within a topological attribute space and a binary relation on this set of agents. It may be clear that we require certain properties of the binary relation as well as the topology of the attribute space to hold.

Firstly we require that similar agents are socially related. Secondly, it is assumed that the total social system is connected, i.e., the binary relation connects all agent in the system with a finite number of links. Finally, we suppose that the topological attribute space consists of an at most countable number of (maximally) connected components.

Within the setting of a relational social system as described above we are able to analyse several subsystems with additional properties. In this paper we discuss subsystems that are relationally complete in the sense that it is connected and that it can be reached by all agents in the system. In Gilles, Ruys and Shou (1991) it is shown that only under several restricting properties there exists a minimal subsystem that satisfies these requirements of completeness. Here we show that generically there exist minimal subsystems, which are nearly complete in the sense that within the hyperspace endowed with the topology of closed convergence it is the limit of a sequence of complete subsystems. We refer to such a minimal and nearly complete social relational subsystem as a quasi-network. We argue that the notion of a quasi-network is giving a proper description of a crudely efficient organization that is able to handle all communication within a social relational system.

2 Semi-networks in social relational systems

The main mathematical setting in which we develop our notion of a quasi-network, is that of a social relational system. As mentioned in the introduction it is a modification of the well known concept of a social system or an abstract economy.

Definition 2.1 *A triple (A, τ, R) is a* **social relational system** *if A is a set, $\tau \subset 2^A$ is a topology on A, and $R \subset A \times A$ is a reflexive and symmetric relation on A such* *that the following properties are satisfied:*

(i) *For every $a \in A$ it holds that $\mathcal{N}_a \neq \emptyset$, where*

$$\mathcal{N}_a = \{U \in \tau \mid a \in U \text{ and for every } b \in U\colon (a,b) \in R\}.$$

(ii) *For every $a, b \in A$ there exists a finite sequence $c_1, \ldots, c_n$ in A such that $c_1 = a$, $c_n = b$, and $(c_i, c_{i+1}) \in R$ for every $i \in \{1, \ldots, n-1\}$.*

(iii) *There is a countable covering $(C_n)_{n \in \mathbb{N}}$ of A with each C_n $(n \in \mathbb{N})$ a connected subspace of the topological space (A, τ).*

As indicated above a social relational system represents the bare mathematical structure of a collection of economic agents endowed with a social relation. In a social relational system (A, τ, R) the set A represents a collection of *types*, i.e., a class of agents described with the use of tuples of individual attributes. The topology $\tau \subset 2^A$ represents a generalized notion of distance between the various tuples of individual attributes or types. The topological space (A, τ) thus describes the attribute space that is relevant with respect to the description of economic trading processes. (See for the properties of some well known topological attribute spaces Grodal (1974) and Hildenbrand (1974).) Finally the relation $R \subset A \times A$ describes binary social relations between economic agents of various types.

From Condition (i) as stated in the definition of a social relational system it is clear that similar types are socially related. Hence, economic agents with similar individual attributes are socially related, and thus are able to communicate with each other. Moreover from this condition it is clear that the topology τ on A is precisely the one generated by the neighbourhood system $\{\mathcal{N}_a \mid a \in A\}$ as defined in (i).

Condition (ii) states that the system is socially connected. Condition (iii) of Definition 2.1 of a social relational system (A, τ, R) implies that there exists an at most countable sequence $(A_n)_{n \in \mathbb{N}}$ of maximally connected components of (A, τ), i.e., $(A_n)_{n \in \mathbb{N}}$ is a partition of A and every A_n $(n \in \mathbb{N})$ is a connected subspace of (A, τ). We define $S := \{A_n \mid n \in \mathbb{N}\}$ as the *subdivision* of (A, τ, R). The mapping $p\colon A \to S$, which assigns to every type $a \in A$ the unique component $p(a) = A_n$ such that $a \in A_n$, is referred to as the *projection* of (A, τ, R). Finally we define the relation $P \subset S \times S$ with for every $A_n, A_m \in S$, $(A_n, A_m) \in P$ if there are types $a, b \in A$ with $p(a) = A_n$

and $p(b) = A_m$ such that $(a,b) \in R$. The pair (S,P) as defined above is denoted as the *condensation* of (A,τ,R).

It is our purpose to describe specific collections of types in a social relational system, who are jointly able to take care of all communication in the system. This implies we introduce two properties of such a class of types, namely ***direct reachability*** of that class by all outside types and ***connectedness*** of that class within the social relation R of the social relational system (A,τ,R).

Before introducing such a collection formally we define a mapping $R\colon A \to 2 \diamond A$, where for every type $a \in A$ we define

$$R(a) := \{b \in A \mid (a,b) \in R\}.$$

The mapping R is representing the relation R in the social relational system (A,τ,R), and therefore is also reflexive and symmetric, i.e., $a \in R(a)$ for every $a \in A$ and $a \in R(b)$ implies $b \in R(a)$ for every $a,b \in A$. Finally we introduce for every subset $E \subset A$

$$R(E) := \bigcup_{a \in E} R(a).$$

In the formal definition of a collection of types as described above we additionally require that this collection is a closed subset of (A,τ).

Definition 2.2 *Let (A,τ,R) be a social relational system. A set $N \subset A$ is a* **semi-network** *in (A,τ,R) if N is a closed subset of (A,τ) and it satisfies the following properties:*

Reachability

It holds that $R(N) = A$.

Connectivity

For every $a,b \in N$ there exists a finite sequence $c_1,\ldots,c_n$ in N with $c_1 = a$, $c_n = b$, and $(c_i,c_{i+1}) \in R$ for every $i \in \{1,\ldots,n-1\}$.

It is clear that the set of all types A itself is a semi-network in the system (A,τ,R).

3 Quasi-networks

A semi-network N in a social relational system (A, τ, R) is, because of its crude inefficiency, insufficient to describe an organization that is able to take care of all communication within the system. With the purpose to develop such an organization, we first extend the class of semi-networks and then take the minimal elements in this extended class as the proper description of such an organization.

Let (A, τ, R) be a social relational system. We define

$$\mathcal{F}(A) := \{F \subset A \mid F \text{ is a closed subset of } (A, \tau)\}$$

as the collection of all closed subsets of the topological space (A, τ). We endow this collection $\mathcal{F}(A)$ with a topology as follows: Take a *compact* set $K \in \mathcal{F}(A)$ and take $\mathcal{G} \subset \tau$ to be a *finite* family of non-empty open subsets. Now we define

$$\mathcal{U}(K, \mathcal{G}) := \{E \in \mathcal{F}(A) \mid E \cap K = \emptyset \text{ and } E \cap G \neq \emptyset,\ G \in \mathcal{G}\}.$$

Now the topology $\mathcal{T}_c$ on $\mathcal{F}(A)$ is taken to be the topology generated by the collection

$$\{\mathcal{U}(K, \mathcal{G}) \mid K \in \mathcal{F}(A) \text{ compact and } \mathcal{G} \subset \tau \text{ finite}\}.$$

The space $(\mathcal{F}(A), \mathcal{T}_c)$ is denoted as the hyper-space with the topology of closed convergence generated by (A, τ).*

Definition 3.1 *Let (A, τ, R) be a social relational system and let $(\mathcal{F}(A), \mathcal{T}_c)$ be the generated hyper-space with the topology of closed convergence.*

(a) *A set $M \subset A$ is an* **asymptotic semi-network** *in (A, τ, R) if $M \in \mathcal{F}(A)$ and for every $\mathcal{T}_c$-neighbourhood $\mathcal{V}_M$ of M there exists a semi-network $N \subset A$ such that $N \in \mathcal{V}_M$.*

(b) *A set $N \subset A$ is a* **quasi-network** *in (A, τ, R) if N is an asymptotic semi-network in (A, τ, R) and there is no proper subset $M \subsetneq N$ which is also an asymptotic semi-network in (A, τ, R).*

From Definition 3.1 we deduce that the collection of asymptotic semi-networks is the closure of the set of semi-networks within the generated hyper-space with the topology

*For properties of this topological space we refer to Hildenbrand (1974) and Klein-Thompson (1984).

of closed convergence $(\mathcal{F}(A), \mathcal{T}_c)$. Similar we may conclude that the collection of quasi-networks in a social relational system (A, τ, R) is exactly the set of minimal elements of this extension of the class of semi-networks within $(\mathcal{F}(A), \mathcal{T}_c)$ with respect to set inclusion. Therefore the notion of a quasi-network is giving a proper description of a crudely efficient organization that is able to handle all communication within a social relational system.

Although the collection of asymptotic semi-networks in a system (A, τ, R) is not empty we do however not know whether the collection of quasi-networks in a system (A, τ, R) is non-empty. Under some additional restrictions we can show the following.

Theorem 3.2 *Let (A, τ, R) be a social relational system. If (A, τ) is a locally compact Hausdorff space, then there exists a quasi-network in (A, τ, R).*

PROOF

Take a fixed type $d \in A$. Next define

$$\mathcal{S}_d := \left\{ N \subset A \;\middle|\; \begin{array}{c} d \in N \text{ and} \\ N \text{ is an asymptotic semi-network} \\ \text{in } (A, \tau, R) \end{array} \right\}$$

We note that by the connectedness of (A, τ, R) (2.1 (ii)) the set of all types A is a semi-network and so $A \in \mathcal{S}_d \neq \emptyset$.

In order to use Zorn's lemma on the class $\mathcal{S}_d$, we now take a totally ordered subcollection $\mathcal{B}_d \subset \mathcal{S}_d$, where $\mathcal{S}_d$ is ordered with respect to inclusion. Since for every asymptotic semi-network $N \in \mathcal{B}_d$ by definition $d \in N$ it is obvious that

$$d \in N_0 := \cap \mathcal{B}_d \neq \emptyset.$$

We now show that the set N_0 is a lower bound for the totally ordered subcollection $\mathcal{B}_d \subset \mathcal{S}_d$, i.e., we will prove that $N_0 \in \mathcal{S}_d$. In order to do so, we note that we only have to check whether N_0 is an asymptotic semi-network in (A, τ, R).

In fact we know that the collection $\mathcal{B}_d$ is a decreasing net with respect to inclusion, and so $N_0 := \mathrm{Li}(\mathcal{B}_d) \equiv \mathrm{Ls}(\mathcal{B}_d)$. So by Theorem 4.5.4 of Klein-Thompson (1984), we establish that $N_0 = \lim_{N \in \mathcal{B}_d} N$ in the topology of closed convergence $\mathcal{T}_c$ on the class of closed sets $\mathcal{F}(A)$.

By definition any $\mathcal{T}_c$-neighbourhood can be written as the collection

$$\mathcal{U}(K,\mathcal{G}) := \{F \in \mathcal{F}(A) \mid F \cap K = \emptyset \text{ and } F \cap G \neq \emptyset,\ G \in \mathcal{G}\},$$

where $K \subset A$ is a compact subset of (A,τ) and $\mathcal{G} \subset \tau$ is some finite collection of non-empty open subsets.

Hence, for each $\mathcal{T}_c$-neighbourhood $\mathcal{U}(K,\mathcal{G})$ of N_0, there is an asymptotic closed pre-network $N_1 \in \mathcal{B}_d$ such that $N_1 \in \mathcal{U}(K,\mathcal{G})$. But $\mathcal{U}(K,\mathcal{G})$ is then also a $\mathcal{T}_c$-neighbourhood of N_1, and hence by the definition of an asymptotic closed pre-network, there exists a closed pre-network, denoted by N, such that $N \in \mathcal{U}(K,\mathcal{G})$.

So we conclude that for every $\mathcal{T}_c$-neighbourhood $\mathcal{U}(K,\mathcal{G})$ of N_0, there is a semi-network $N \in \mathcal{B}_d$ such that $N \in \mathcal{U}(K,\mathcal{G})$. With the use of the Definition 3.1 we establish that N_0 is also an asymptotic semi-network in (A,τ,R), i.e., $N_0 \in \mathcal{S}_d$.

This implies that we are able to apply Zorn's lemma on the collection $\mathcal{S}_d$ to establish the existence of a minimal element, say $\widetilde{N}$, in $\mathcal{S}_d$. (Note that $d \in \widetilde{N}$.)

Next we define the following collections:

$$\mathcal{S} := \{N \subset A \mid N \text{ is a semi-network in } (A,\tau,R)\}.$$

$$\mathcal{S}' := \{N \subset A \mid N \text{ is an asymptotic semi-network in } (A,\tau,R)\}.$$

Obviously $\mathcal{S} \subset \mathcal{S} \diamond /$. In order to complete the proof of the theorem we first prove the following claim:

Claim. *There is no asymptotic semi-network $\bar{N} \in \mathcal{S}'$ such that $\bar{N} \subset \widetilde{N} \setminus \{d\}$, $\bar{N} \neq \widetilde{N} \setminus \{d\}$.*

Proof of the claim

Suppose that there is an asymptotic semi-network $\bar{N} \in \mathcal{S}'$ such that $\bar{N} \subset \widetilde{N} \setminus \{d\}$, $\bar{N} \neq \widetilde{N} \setminus \{d\}$. Then $\bar{N} \cup \{d\} \subset \widetilde{N}$ and $\bar{N} \cup \{d\} \neq \widetilde{N}$.

First we note that $\bar{N} \cup \{d\}$ is a closed subset in (A,τ). (Use the T_2-separation property of (A,τ).) Next take a $\mathcal{T}_c$-neighbourhood $\mathcal{U}(K,\mathcal{G})$ of $\bar{N} \cup \{d\}$, where $K \subset A$ is a compact set, and $\mathcal{G} = \{G_1,\ldots,G_k\}$ is a finite collection of open subsets of (A,τ). We now prove that there exists a closed pre-network in this $\mathcal{T}_c$-neighbourhood $\mathcal{U}(K,\mathcal{G})$ of $\bar{N} \cup \{d\}$. First define

$$\mathcal{G}' := \{G \in \mathcal{G} \mid d \notin G\} \subset \mathcal{G}$$

If $\mathcal{G}' \neq \emptyset$, then $\mathcal{U}(K, \mathcal{G}')$ is a neighbourhood of $\bar{N}$. Since $\bar{N} \in \mathcal{S}'$ we know that there is a semi-network $N \in \mathcal{S}$ such that $N \in \mathcal{U}(K, \mathcal{G}')$.

If $\mathcal{G}\prime = \emptyset$, then $\mathcal{U}(K, \{A\})$ is a neighbourhood of $\bar{N}$. By the same reasoning as above, there exists a semi-network $N \in \mathcal{S}$ such that $N \in \mathcal{U}(K, \{A\})$.

In both cases above it is obvious that $N \cup \{d\}$ belongs to $\mathcal{S}$, i.e., is a semi-network, and moreover $(N \cup \{d\}) \in \mathcal{U}(K, \mathcal{G})$.
Hence we may conclude that $\bar{N} \cup \{d\}$ is an asymptotic semi-network, and thus $\bar{N} \cup \{d\} \in \mathcal{S}_d$. This contradicts the minimality assumption on $\widetilde{N}$ in the collection $\mathcal{S}_d$.

THIS COMPLETES THE PROOF OF THE CLAIM.

We can distinguish two cases:

(i) $\widetilde{N} \setminus \{d\}$ is an asymptotic semi-network, i.e., $\widetilde{N} \setminus \{d\} \in \mathcal{S}'$.
Then by the claim, the set $\widetilde{N} \setminus \{d\}$ has to be a minimal element of the collection $\mathcal{S} \diamond \prime$, and so $\widetilde{N} \setminus \{d\}$ is the required quasi-network in (A, τ, R).

(ii) $\widetilde{N} \setminus \{d\}$ is not an asymptotic semi-network, i.e., $\widetilde{N} \setminus \{d\} \notin \mathcal{S}'$.
Then by applying the claim we arrive at the conclusion that $\widetilde{N}$ is a minimal element in $\mathcal{S}'$, and so it is the required quasi-network in (A, τ, R).

Q.E.D.

Although in Theorem 3.2 we have established that under mild restrictions there exists a quasi-network, we do not know whether the size of such a quasi-network is acceptable. Next we address the question in which cases there exist "small" quasi-networks.

For that purpose we call a social relational system (A, τ, R) *strongly connected* if its condensation (S, P) satisfies the condition that for every component $A_n \in S$ it holds that

$$|\{A_m \in S \mid (A_n, A_m) \in P\}| < \infty.$$

As a direct consequence of this additional property of a social relational system we deduce the following lemma.

Reordering Lemma. *Let (A, τ, R) be strongly connected. There exists an ordering of the set $S = \{A_n \mid n \in \mathbb{N}\}$ such that*

(i) for every $k \in \mathbb{N}$ the graph $(\bigcup_{n=1}^{k} A_n, R \cap [\bigcup_{n=1}^{k} A_n \times \bigcup_{n=1}^{k} A_n])$ is finitely connected and

(ii) S can be partitioned into a countable collection of finite sets $(B_i)_{i \in \mathbb{N}}$ with

$$B_1 = \{A_1\},$$

$$B_2 = \{A_2, \ldots, A_{n_1}\} \text{ with } n_1 > 1,$$

$$B_r = \{A_{n_{r-1}}, \ldots, A_{n_r}\} \text{ with } n_r > n_{r-1}, \text{ for } r \geq 2,$$

where $n_r \in \mathbb{N}$ for every $r \in \mathbb{N}$.
If $|r_1 - r_2| = 1$, then there exist components $A_{k_1} \in B_{r_1}$ and $A_{k_2} \in B_{r_2}$ such that $(A_{k_1}, A_{k_2}) \in P$.
If $|r_1 - r_2| > 1$, then for all components $A_{k_1} \in B_{r_1}$ and $A_{k_2} \in B_{r_2}$ it holds that $(A_{k_1}, A_{k_2}) \notin P$.

For a proof of this Reordering Lemma we refer to Gilles, Ruys and Shou (1991). We are now in a position to prove the following result.

Theorem 3.3 *Let (A, τ, R) be a social relational system and (S, P) its condensation. Suppose that the following requirements are satisfied:*

(i) *(A, τ, R) is strongly connected.*

(ii) *(A, τ) is a Hausdorff space.*

(iii) *Every component $A_n \in S$ is a compact subspace of (A, τ).*

Then there exists an at most countable quasi-network in (A, τ, R).

PROOF
First we note that from the assumptions (ii) and (iii) in the assertion it follows that (A, τ) is a locally compact topological space. Using the Reordering Lemma we can order $S = \{A_n \mid n \in \mathbb{N}\}$ such that for every $k \in \mathbb{N}$ the triple $(B_k, \tau|B_k, R \cap [B_k \times B_k])$ is a social relational system, where $B_k = \bigcup_{m=1}^{k} A_m$.
By Theorem 3.5 in Gilles et al. (1991) we conclude that for every $n \in \mathbb{N}$ there exists a finite network N_n in the system $(A_n, \tau|A_n, R \cap [A_n \times A_n])$. Now we construct the following sequence $(F_n)_{n \in \mathbb{N}}$ of finite subsets of A:

- $F_1 := N_1$;
- Given the set F_n ($n \in \mathbb{N}$) we define $F_{n+1} := F_n \cup N_{n+1} \cup \{a, b\}$, where taking a number $1 \leq k \leq n$ such that $(A_k, A_{n+1}) \in P$, we choose $a \in A_k$ and $b \in A_{n+1}$ such that $(a, b) \in R$.

Now for each $n \in \mathbb{N}$ the set F_n is finite, and thus closed in (A, τ). Obviously it satisfies reachability and connectivity with respect to the social relational sub-system $(B_n, \tau|B_n, R \cap [B_n \times B_n])$, where $B_n := \bigcup_{m=1}^{n} A_m$. Moreover, the sequence $(F_n)_{n\in\mathbb{N}}$ is increasing, i.e., $F_j \subset F_{j+1}$ for all $j \in \mathbb{N}$.

Define $\widetilde{N} := \mathrm{Ls}(F_n) \equiv \mathrm{Li}(F_n)$. It is easy to check that $\widetilde{N} \in \mathcal{F}(A)$ and thus satisfies all properties of a semi-network. Hence, $\widetilde{N}$ is a countable semi-network.

This means that there exists a countable asymptotic semi-network in (A, τ, R). Take $d \in A$, and define

$$\mathcal{S}_d := \left\{ N \subset A \;\middle|\; \begin{array}{c} d \in N \text{ and} \\ N \text{ is an at most countable} \\ \text{asymptotic semi-network in } (A, R) \end{array} \right\}.$$

Clearly $\mathcal{S}_d \neq \emptyset$. (Take $\widetilde{N} \cup \{d\}$ as an example of an element in the collection $\mathcal{S}_d$.) Similarly as is done in the proof of Theorem 3.2 on general existence of quasi-networks, we are able to establish that:

1. By Zorn's lemma there exists a minimal element in the collection $\mathcal{S}_d$.
2. Now we define

$$\mathcal{S} := \left\{ N \subset A \;\middle|\; \begin{array}{c} N \text{ is an at most} \\ \text{countable semi-network} \\ \text{in } (A, \tau, R) \end{array} \right\} \text{ and}$$

$$\mathcal{S}' := \left\{ N \subset A \;\middle|\; \begin{array}{c} N \text{ is an at most countable} \\ \text{asymptotic semi-network} \\ \text{in } (A, \tau, R) \end{array} \right\}.$$

   By repeating a course of reasoning as followed in the proof of Theorem 3.2, we arrive at the conclusion that there exists a minimal element in the collection $\mathcal{S}'$. This is the desired countable quasi-network in (A, τ, R).

Q.E.D.

References

CSÁSZÀR, A. (1978), *General Topology*, Adam Hilger Ltd, Bristol.

DEBREU, G. (1959), *Theory of Value*, Wiley, New York.

ENGELKING, R. (1989), *General Topology*, Heldermann Verlag, Berlin.

GILLES, R.P., AND P.H.M. RUYS (1990), "Characterization of Economic Agents in Arbitrary Communication Structures", *Nieuw Archief voor Wiskunde*, 8, 325–345.

GILLES, R.P., P.H.M. RUYS, AND J. SHOU (1991), *Existence of Networks in Social Relational Systems*, Working Paper, Department of Economics, VPI&SU, Blacksburg.

GRODAL, B. (1974), "A Note on the Space of Preference Relations", *Journal of Mathematical Economics*, 1, 279-294.

HILDENBRAND, W. (1974), *Core and Equilibria of a Large Economy*, Princeton UP, Princeton.

KLEIN, E., AND A.C. THOMPSON (1984), *Theory of Correspondences, Including Applications to Mathematical Economics*, Wiley, New York.

KOPPERMAN, R. (1988), "All Topologies Come From Generalized Metrics", *American Mathematical Monthly*, 95, 89–97.

MAS-COLELL, A. (1985), *The Theory of General Economic Equilibrium. A differentiable approach*, Cambridge UP, Cambridge.

MYERSON, R.B. (1977), "Graphs and Cooperation in Games", *Mathematics of Operations Research*, 2, 225-229.

SHAFER, W., AND H. SONNENSCHEIN (1975), "Some Theorems on the Existence of Competitive Equilibrium", *Journal of Economic Theory*, 11, 83–93.

VIND, K. (1983), "Equilibrium with Coordination", *Journal of Mathematical Economics*, 12, 275–285.

EINDHOVEN UNIVERSITY OF TECHNOLOGY
Department of Mathematics and Computing Science
PROBABILITY THEORY, STATISTICS, OPERATIONS RESEARCH
AND SYSTEMS THEORY
P.O. Box 513
5600 MB Eindhoven, The Netherlands

Secretariate: Dommelbuilding 0.03
Telephone : 040-473130

-List of COSOR-memoranda - 1991

Number	Month	Author	Title
91-01	January	M.W.I. van Kraaij W.Z. Venema J. Wessels	The construction of a strategy for manpower planning problems.
91-02	January	M.W.I. van Kraaij W.Z. Venema J. Wessels	Support for problem formulation and evaluation in manpower planning problems.
91-03	January	M.W.P. Savelsbergh	The vehicle routing problem with time windows: minimizing route duration.
91-04	January	M.W.I. van Kraaij	Some considerations concerning the problem interpreter of the new manpower planning system formasy.
91-05	February	G.L. Nemhauser M.W.P. Savelsbergh	A cutting plane algorithm for the single machine scheduling problem with release times.
91-06	March	R.J.G. Wilms	Properties of Fourier-Stieltjes sequences of distribution with support in [0,1).
91-07	March	F. Coolen R. Dekker A. Smit	Analysis of a two-phase inspection model with competing risks.
91-08	April	P.J. Zwietering E.H.L. Aarts J. Wessels	The Design and Complexity of Exact Multi-Layered Perceptrons.
91-09	May	P.J. Zwietering E.H.L. Aarts J. Wessels	The Classification Capabilities of Exact Two-Layered Peceptrons.
91-10	May	P.J. Zwietering E.H.L. Aarts J. Wessels	Sorting With A Neural Net.
91-11	May	F. Coolen	On some misconceptions about subjective probability and Bayesian inference.

91-12	May	P. van der Laan	Two-stage selection procedures with attention to screening.
91-13	May	I.J.B.F. Adan G.J. van Houtum J. Wessels W.H.M. Zijm	A compensation procedure for multiprogramming queues.
91-14	June	J. Korst E. Aarts J.K. Lenstra J. Wessels	Periodic assignment and graph colouring.
91-15	July	P.J. Zwietering M.J.A.L. van Kraaij E.H.L. Aarts J. Wessels	Neural Networks and Production Planning.
91-16	July	P. Deheuvels J.H.J. Einmahl	Approximations and Two-Sample Tests Based on P - P and Q - Q Plots of the Kaplan-Meier Estimators of Lifetime Distributions.
91-17	August	M.W.P. Savelsbergh G.C. Sigismondi G.L. Nemhauser	Functional description of MINTO, a Mixed INTeger Optimizer.
91-18	August	M.W.P. Savelsbergh G.C. Sigismondi G.L. Nemhauser	MINTO, a Mixed INTeger Optimizer.
91-19	August	P. van der Laan	The efficiency of subset selection of an almost best treatment.
91-20	September	P. van der Laan	Subset selection for an -best population: efficiency results.
91-21	September	E. Levner A.S. Nemirovsky	A network flow algorithm for just-in-time project scheduling.
91-22	September	R.J.M. Vaessens E.H.L. Aarts J.H. van Lint	Genetic Algorithms in Coding Theory - A Table for $A_3(n,d)$.
91-23	September	P. van der Laan	Distribution theory for selection from logistic populations.

COSOR-MEMORANDA (3)

91-24	October	I.J.B.F. Adan J. Wessels W.H.M. Zijm	Matrix-geometric analysis of the shortest queue problem with threshold jockeying.
91-25	October	I.J.B.F. Adan J. Wessels W.H.M. Zijm	Analysing Multiprogramming Queues by Generating Functions.
91-26	October	E.E.M. van Berkum P.M. Upperman	D-optimal designs for an incomplete quadratic model.
91-27	October	R.P. Gilles P.H.M. Ruys S. Jilin	Quasi-Networks in Social Relational Systems.