

Desynchronisability of (partial) closed loop systems

Citation for published version (APA):

Beohar, H., & Cuijpers, P. J. L. (2010). *Desynchronisability of (partial) closed loop systems*. (Computer science reports; Vol. 1019). Technische Universiteit Eindhoven.

Document status and date:

Published: 01/01/2010

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Desynchronisability of (partial) closed loop systems

Harsh Beohar

Pieter Cuijpers*

Abstract

The task of implementing a supervisory controller is non-trivial, even though there are different theories that allow automatic synthesis of these controllers in the form of automata. One of the reasons for this discord is due to the asynchronous interaction between a plant and its controller in implementations, whereas the existing supervisory control theories assume synchronous interaction. As a consequence the implementation suffers from the so-called inexact synchronisation problem. In this paper we address the issue of inexact synchronisation in a process algebraic setting, by solving a more general problem of refinement. We construct an asynchronous closed loop system by introducing a communication medium in a given synchronous closed loop system. Our goal is to find sufficient conditions under which a synchronous closed loop system is branching bisimilar to its corresponding asynchronous closed loop system. Furthermore, we extend our results to a class of synchronous closed loop systems called partial synchronous closed loop systems, whose alphabet contains external actions of both the plant and its controller that do not result in interaction.

1 Introduction

The task of implementing a supervisory controller is non-trivial, even though there are different theories that allow automatic synthesis of these controllers in the form of automata. One of the reasons for this discord is due to the asynchronous interaction between a plant and its controller in implementations, whereas the existing supervisory control theories assume synchronous interaction. We elaborate on this mismatch by first introducing some terminology that is often used in supervisory control theory [15].

Supervisory control theory provides an automatic synthesis of a supervisor that controls a plant in such a way that a corresponding requirement (legal behaviour) is achieved. In supervisory control theory terminology,

- the model that is to be controlled is known as the *plant*,
- the model that specifies the requirement is known as the *specification*,
- the model that forces the plant to meet the specification by interacting with it is known as the *supervisor* or the *controller*.
- the interaction between a plant and its supervisor is known as *closed-loop behavior*.

*Department of Mathematics and Computer Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, email: `x@tue.nl`, `x` \in {H.Beohar, P.J.L.Cuijpers}.

The closed loop behaviour in supervisory control theory is realized by synchronous parallel composition. Informally, it allows a plant and a supervisor to synchronise on common events while other events can happen independently.

One of the main drawbacks while implementing the interaction between a plant and its supervisor, synthesised by supervisory control theory, is inexact synchronization [8]. In practical industrial applications, the interaction between a plant and its supervisor is not synchronous but rather asynchronous. Due to the synchronous parallel composition used in supervisory control theory, the interaction between a plant and its supervisor is strict. By strict, we mean that either the plant or the supervisor has to wait for the other party while synchronising. To overcome this problem we study asynchronous communication between a plant and its supervisor where communications are delayed in buffers.

Balemi was the first to consider the inexact synchronisation problem, and the solutions given in his PhD thesis [3] were in the domain of automata theory. In [3], an *input-output* interpretation was given between a plant and its supervisor and a special delay operator was introduced to model the delay in communication between the plant and the supervisor. Moreover, for this setup the existence of a supervisor in the presence of delays was also shown in [3]. It was required that the output actions from a plant can occur asynchronously, while the output actions from a supervisor must occur synchronously [20]. In [20] this requirement was relaxed. Furthermore, necessary and sufficient conditions were also provided for the existence of a controller under bounded delay between a plant and its supervisor.

The solutions provided in [3, 20] construct a new supervisor under the presence of bounded delay, which is a computationally expensive procedure. To circumvent this, we present sufficient conditions on a synchronous closed loop system under which the asynchronous closed loop system constructed from it is a refinement of the given synchronous closed loop system. Moreover, the technique developed in this paper is independent of the size of the buffers used. However, we do not analyse the computational complexities associated with the sufficient conditions presented in this paper.

In this paper, we reformulate the inexact synchronisation problem as a problem of refinement in the process algebra TCP [2]. The synchronous closed loop system can be considered to be a specification with the asynchronous closed loop system as its implementation. If the given synchronous closed loop system and its corresponding asynchronous closed loop system are branching bisimilar [17], then the asynchronous closed loop system is said to be a refinement of its corresponding synchronous closed loop system. Note that we do not compute an additional supervisor under the presence of delays, but we assume a given plant and its supervisor. Thus, we solve a refinement problem instead of solving a control synthesis problem.

In the past, the idea of solving a refinement problem was studied [9, 11, 12], but different setups compared to the current paper were used in these studies. These studies were motivated by the so-called “Foam-rubber wrapper” principle [16], borrowed from the field of delay insensitive circuits. Mathematically, it states that “a process and the same process connected with buffers are equivalent”. In [9], the foam-rubber wrapper principle was also studied in the context of the parallel composition and it was shown that an extra condition is required to preserve this principle. In brief, we have a different architecture for the asynchronous closed loop system in this paper and we study the components in the asynchronous closed loop system conjointly, in order to capture desynchronisability.

We extend our previous result [6] to a class of synchronous closed loop systems called partial synchronous closed loop systems, whose alphabet contains external actions of both

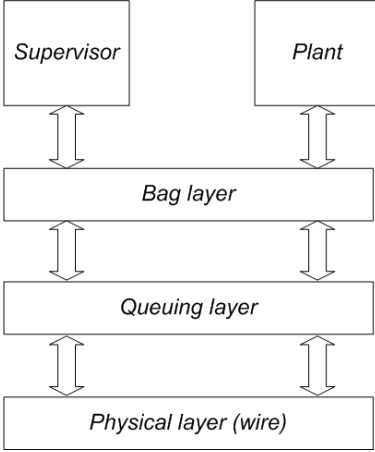


Figure 1: Asynchronous closed loop system in practice.

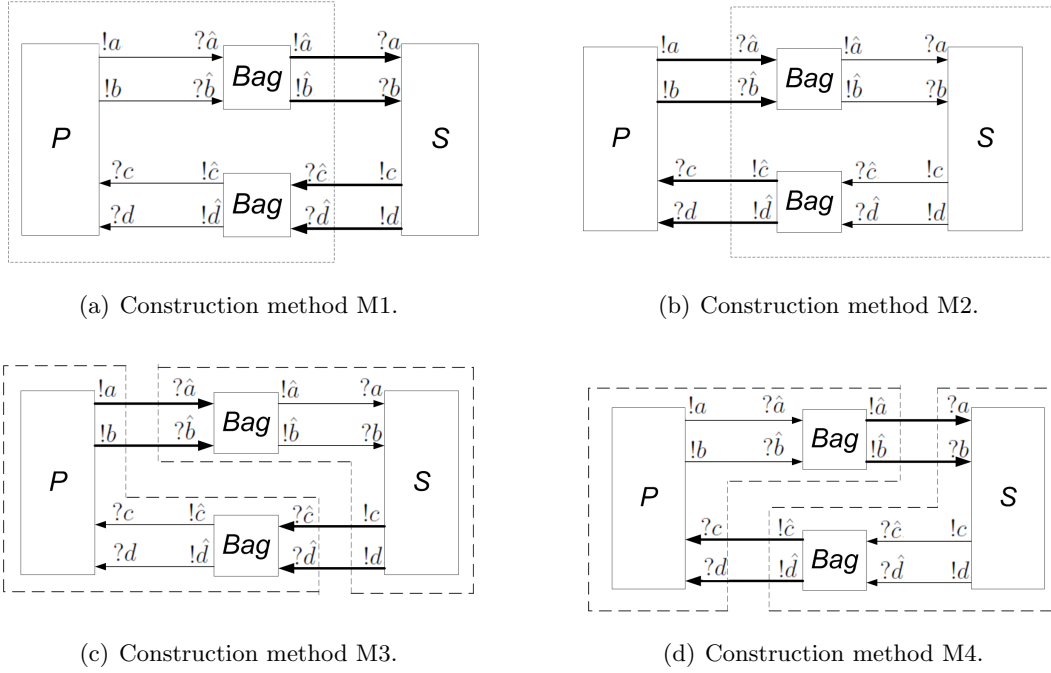
the plant and its supervisor that do not result in interaction. This makes it possible to desynchronise a synchronous closed loop system present in a decentralised or hierarchical architecture [19] by decomposing the given synchronous closed loop system into a number of partial synchronous closed loop systems and by verifying each of the individual partial closed loop systems. However, more research is required to achieve this compositional result for a desynchronisable closed loop system.

1.1 Architecture

This paper and the companion paper [6] are the results of the pre-study carried out in [7], where four construction methods are proposed to construct an asynchronous closed loop system from its corresponding synchronous one. In this subsection, we introduce the architecture of an asynchronous closed loop system, discuss the reasonability of using a bag as a buffer and describe one of the abstraction schemes that will be used throughout this paper. We elucidate on these points in the following paragraphs.

An asynchronous closed loop system can be constructed by introducing a buffer between a plant and its supervisor in order to decouple the synchronisation of events between the two. In practice, the buffering mechanism is realised by the interactions of different layers (also known as protocol stack) as shown in Figure 1. In theory, various authors [9, 11, 13] have abstracted from the interaction of different layers by using data structures based on a particular level of abstraction. For example, to model delay insensitive (DI) circuits, which are at a lower level of abstraction (physical layer), wires are used as a buffering mechanism [12]. On the other hand, to model data flow networks which are at a higher level of abstraction (in comparison to DI circuits), queues are used as a buffering mechanism [11]. In this paper, we are interested in studying the asynchronous interaction in a closed loop system at an even higher level of abstraction by having a unique queue for every message. Thus, a queue stores only one type of unique message and all queues are allowed to run concurrently without interacting with one another. Such interleaving queues are equivalent to a bag modulo strong bisimulation. Hence, we use a bag as the buffering mechanism in this paper.

It is obvious that when introducing the bag as a buffer, the asynchronous closed loop system contains interactions that are not present in the synchronous closed loop system.



(a) Construction method M1.

(b) Construction method M2.

(c) Construction method M3.

(d) Construction method M4.

Figure 2: Different ways to construct an asynchronous closed loop system.

However, to compare these two closed loop systems by a branching bisimulation relation [17], it is necessary to hide some interactions or to define a suitable abstraction scheme. In principle, a synchronous closed loop system can be converted into an asynchronous closed loop system by introducing bags with the following abstraction schemes:

- M1. by introducing bags between a plant and its supervisor such that *the interaction between plant and bag is hidden* (see Figure 2(a)).
- M2. by introducing bags between a plant and its supervisor such that *the interaction between supervisor and bag is hidden* (see Figure 2(b)).
- M3. by introducing bags between a plant and its supervisor such that *the communication among the input actions of both plant and supervisor with bags are hidden* (see Figure 2(c)).
- M4. by introducing bags between a plant and its supervisor such that *the communication among the output actions of both plant and supervisor with bags are hidden* (see Figure 2(d)).

In Figure 2, thick lines are used to show the visible interaction and thin lines are used to show the invisible interaction. The notation $!a$ means ‘send action a ’ and $?a$ means ‘receive action a ’. In this paper, we develop the theory for the construction method M1 (see Section 4 for the rationale behind this choice) and leave other construction methods as open for future study. Moreover, the techniques presented in this paper are restricted to reactive systems (so, we do not consider termination).

1.2 Outline

The remainder of this paper is organized as follows. In Section 2, we start our exposition by defining the overall background required for this paper. Section 3 provides a brief introduction to supervisory control theory with respect to our setup. In Section 4, the construction method M1 is defined formally with its abstraction scheme. In Section 5, we give the formal definition of a desynchronisable closed loop system with the conditions that are sufficient for desynchronisability. In Section 6, we extend our results for partial synchronous closed loop system. In Section 7 we discuss in the context of synchronous closed loop systems whether some of the sufficient conditions can be any further weakened. Finally, the conclusions are presented in Section 8 with some directions for future research.

2 Background

In this section, we define the basic notations and definitions that will be used throughout this paper. Let Act be a set of action names. We use the symbols a, b, c, \dots to range over the set Act . Then we define the following actions for an action label $a \in Act$,

- $!a$: send action labeled a .
- $?a$: receive action labeled a .
- $\sharp a$: communication of action labeled a .

Let A denote the set of all actions label defined in the following way

$$A = \{!a, ?a, \sharp a\}_{a \in Act}.$$

The variables x, y, z, \dots are used to denote elements from set A when the information about the type of action is irrelevant. The set of all process terms (denoted by \mathbb{P}) is then defined by the grammar in Table 1. The constant $\mathbf{0}$ is a process term that cannot perform any action is called an *inaction* process term. A unary operator $x._$ for each action $x \in A$ is introduced in the TCP syntax, denoting an *action prefix*. Intuitively, the process term $x.p$ performs the action x and then behaves as the process term p . The binary operator $+$ denotes the *alternative composition or choice* between any two process terms. The encapsulation operator $\partial_H(-)$, *blocks execution of actions* from H while enduring the execution of other actions from the set $A \setminus H$. The abstraction operator $\tau_I(-)$ *renames* the actions in the set I to the silent step τ , and leaves other actions unchanged. Note that the silent step $\tau \notin A$.

In the remainder of this paper, we assume that the symbols $P, R, S, p, p', s, s' \dots$ range over the set \mathbb{P} . We fix the capital letters P, R, S for process terms associated with supervisory control theory. Note that we also use the renaming operator ρ from TCP algebra for technical reasons, but not for modeling purposes. The empty process $\mathbf{1}$ is not defined because we are only interested in modeling reactive systems. The notation \mathcal{R} denotes a recursion definition by a set of pairs $\{X_0 = t_0, \dots, X_m = t_m\}$ where X_i denotes a recursion variable and t_i the process term defining it. The parallel composition operator is parameterized with a partial communication function $\gamma : A \times A \rightarrow A$ such that

$$\forall a \in Act. [\gamma(?a, !a) = \gamma(!a, ?a) = \sharp a] .$$

The semantic domain of the process terms is a transition system (see [2] for details), which is achieved by the so-called SOS rules [14].

\mathbb{P}	::=	$\mathbf{0}$	inaction
		$x.\mathbb{P}$	action prefix, where $x \in A \cup \{\tau\}$
		$\mathbb{P} + \mathbb{P}$	alternative composition
		$\mathbb{P} \parallel_\gamma \mathbb{P}$	parallel composition
		$\partial_H(\mathbb{P})$	action encapsulation, where $H \subseteq A$
		$\tau_I(\mathbb{P})$	abstraction (hiding of actions), where $I \subseteq A$
		\mathcal{R}	recursive definition

Table 1: Syntax of TCP [2].

$$\begin{array}{c}
\frac{}{x.p \xrightarrow{x} p} \quad (1) \quad \frac{p \xrightarrow{x} p'}{p + q \xrightarrow{x} p'} \quad (2) \\
\frac{p \xrightarrow{x} p'}{p \parallel_\gamma q \xrightarrow{x} p' \parallel_\gamma q} \quad (3) \quad \frac{p \xrightarrow{x} p', q \xrightarrow{x'} q', \gamma(x, x') = x''}{p \parallel_\gamma q \xrightarrow{x''} p' \parallel_\gamma q'} \quad (4) \\
\frac{p \xrightarrow{x} p', x \notin H}{\partial_H(p) \xrightarrow{x} \partial_H(p')} \quad (5) \quad \frac{p \xrightarrow{x} p', x \notin I}{\tau_I(p) \xrightarrow{x} \tau_I(p)} \quad (6) \quad \frac{p \xrightarrow{x} p', x \in I}{\tau_I(p) \xrightarrow{\tau} \tau_I(p)} \quad (7) \\
\frac{t_0 \xrightarrow{x} p, X_0 = t_0}{X_0 \xrightarrow{x} p} \quad (8) \quad \frac{p \xrightarrow{x} p', f : A \rightarrow A}{\rho_f(p) \xrightarrow{f(x)} \rho_f(p')} \quad (9)
\end{array}$$

Table 2: Operational semantics for a fragment of TCP, where $x \in A \cup \{\tau\}$.

Definition 1. A transition system over a set of actions A is a set Q of states, equipped with a transition relation $\rightarrow \subseteq Q \times (A \cup \{\tau\}) \times Q$. The action $\tau \notin A$ denotes the invisible action. In the semantics of TCP, Q is usually taken to be the set of process terms, i.e., $Q = \mathbb{P}$, and the initial state of a process is defined as the process term itself. \square

For the sake of completeness, we give the SOS rules of the operators used here in the Table 2.

Definition 2. The alphabet of a process term p , written as $\alpha(p)$, is the set of atomic actions that it can perform. It is defined in the following way

$$\begin{aligned}
\alpha(p) &= \{x \mid p \xrightarrow{x} p'\} \cup \bigcup_{p \xrightarrow{x} p'} \alpha(p'), \\
\alpha(p) &= \emptyset \quad \text{if } \nexists p', x. [p \xrightarrow{x} p'].
\end{aligned}$$

\square

As mentioned in the introduction, we use branching bisimulation to relate a synchronous closed loop system with its corresponding asynchronous closed loop system in which τ actions

are present. The presence of τ actions in an asynchronous closed loop system will become evident in Section 4. We write the transitive closure of the transition relation \rightarrow as \rightarrow^* . The notation $q \xrightarrow{w} q'$ for some $w \in A^*$ is inductively defined in the following way,

$$\begin{aligned} q &\xrightarrow{w} q' \triangleq q \xrightarrow{\epsilon} q' && \text{if } w = \epsilon, \\ q &\xrightarrow{x.w} q' \triangleq \exists q''. [q \xrightarrow{x} q'' \wedge q'' \xrightarrow{w} q']. \end{aligned}$$

The symbol \equiv is used to denote syntactical equivalence between process terms. The shorthand notation $q \xrightarrow{\tau^*} q'$ is defined as iff there is $n \geq 0$ and $q_0, \dots, q_n \in Q$ such that $q \equiv q_0 \xrightarrow{\tau} \dots \xrightarrow{\tau} q_n \equiv q'$.

Definition 3. A binary relation $\Phi \subseteq Q \times Q$ is called a branching bisimulation relation [2, 17] iff:

- $\forall q, q_1, q', x \in A. [(q, q') \in \Phi \wedge q \xrightarrow{x} q_1 \Rightarrow \exists q'_1, q'_2. [q' \xrightarrow{\tau^*} q'_1 \xrightarrow{x} q'_2 \wedge (q, q'_1) \in \Phi \wedge (q_1, q'_2) \in \Phi]]$.
- $\forall q, q_1, q'. [(q, q') \in \Phi \wedge q \xrightarrow{\tau} q_1 \Rightarrow (q_1, q') \in \Phi \vee \exists q'_1, q'_2. [q' \xrightarrow{\tau^*} q'_1 \xrightarrow{\tau} q'_2 \wedge (q, q'_1) \in \Phi \wedge (q_1, q'_2) \in \Phi]]$.
- $\forall q, q', q'_1, x \in A. [(q, q') \in \Phi \wedge q' \xrightarrow{x} q'_1 \Rightarrow \exists q_1, q_2. [q \xrightarrow{\tau^*} q_1 \xrightarrow{x} q_2 \wedge (q_1, q') \in \Phi \wedge (q_2, q'_1) \in \Phi]]$.
- $\forall q, q', q'_1. [(q, q') \in \Phi \wedge q' \xrightarrow{\tau} q'_1 \Rightarrow (q, q'_1) \in \Phi \vee \exists q_1, q_2. [q \xrightarrow{\tau^*} q_1 \xrightarrow{\tau} q_2 \wedge (q_1, q') \in \Phi \wedge (q_2, q'_1) \in \Phi]]$.

Let $q, q' \in Q$ be the initial states of processes $p, p' \in \mathbb{P}$, respectively. Two processes p and p' are said to be branching bisimilar (denoted as $p \dot{\leftrightarrow}_b p'$) iff there exists a branching bisimulation relation Φ such that their initial states q, q' are related, i.e. $(q, q') \in \Phi$. \square

Note that in the absence of τ actions, branching bisimulation coincides with strong bisimulation. The phenomena of the occurrences of redundant silent steps can be formulated by the following notion of τ -inertness [10].

Definition 4. Let $p \in \mathbb{P}$ be an arbitrary process term. A process p is said to be τ -inert with respect to $\dot{\leftrightarrow}_b$ iff for all states q of the transition system (generated by operational rules) of p it holds that $q \xrightarrow{\tau} q' \Rightarrow q \dot{\leftrightarrow}_b q'$ where, $q' \in Q$. \square

The essence of the above definition is that an inert τ action does not affect the future choices of a process modulo branching bisimulation. In Section 5, we also show that an asynchronous closed loop system constructed from a synchronous closed loop system satisfying Definitions 10, 11, and 12 is always τ -inert with respect to $\dot{\leftrightarrow}_b$.

3 Supervisory control theory

In this section, we give a brief introduction to supervisory control theory and define its fundamental entities in our setup. The basic entity (a plant, or a supervisor, or a requirement) in the supervisory control theory is deterministic. Furthermore, the proofs of main Theorems 1 and 2 requires the fact that a given synchronous closed loop system is also deterministic. Therefore, we now introduce the term deterministic process.

Definition 5. A process $p \in \mathbb{P}$ is called a deterministic process iff for all states q of the transition system (generated by the operational rules) of p and for all $x \in A \cup \{\tau\}$ it holds that $q \xrightarrow{x} q_1 \wedge q \xrightarrow{x} q_2 \Rightarrow q_1 \equiv q_2$ where, $q_1, q_2 \in Q$. \square

In supervisory control theory, plants and supervisors are allowed to perform events that are divided into two disjoint subsets: *controllable* and *uncontrollable* events. The idea behind this partition is that the supervisor can enable or disable controllable events so that the closed loop behavior is equivalent to the requirement. The supervisor can observe but cannot influence uncontrollable events. In this paper, we follow the input-output interpretation [3] between a plant and its supervisor, wherein the uncontrollable events are outputs from a plant to a supervisor and the controllable events are outputs from a supervisor to a plant. Thus, processes that model plants or supervisors must have distinct (because of the above partition) input and output actions in its alphabet. Such processes are called input-output processes.

Definition 6. The set of input actions for an arbitrary process $p \in \mathbb{P}$ is denoted by $\alpha^?(p)$ and is defined as $\alpha^?(p) \triangleq \{a \mid ?a \in \alpha(p)\}$. Similarly, the set of output actions (denoted by $\alpha^!(p)$) is defined as $\alpha^!(p) \triangleq \{a \mid !a \in \alpha(p)\}$. A process p is called an input-output process iff

$$\alpha^?(p) \cap \alpha^!(p) = \emptyset \wedge \alpha(p) \cap I = \emptyset \wedge \tau \notin \alpha(p)$$

where, $I = \{?a \mid a \in Act\}$. \square

The condition $\alpha(p) \cap I = \emptyset$ ensures that an input-output process does not contain communicated actions in its alphabet. This is because bags are introduced to buffer both input and output events of an input-output process $p \in \mathbb{P}$. So if communicated actions are allowed in the specification of the process p then, the information whether the action $?a$ is an input or an output action of the process p is unknown.

We now define the three basic entities in the supervisory control theory in our setup. A *plant* $P \in \mathbb{P}$ is a deterministic input-output process. Similarly, a *supervisor* is a deterministic input-output process. A requirement is a process specifying the legal interaction that should occur while the plant and its supervisor are interacting such that a required task (for which the supervisor is synthesised) is completed. Thus, a *requirement* is a deterministic process $R \in \mathbb{P}$ such that

$$\alpha(R) \cap H = \emptyset \wedge \tau \notin \alpha(R)$$

where, $H = \{!a, ?a \mid a \in Act\}$. This condition ensures that a requirement process only contains communicated actions in its alphabet.

Now, we can state the *control problem* as follows: given a plant P and a requirement R , find a supervisor S such that

$$\partial_H(P \parallel_\gamma S) \stackrel{\text{b}}{\Leftarrow} R.$$

In this paper, we are not interested in how this supervisor is computed and rather assume that we are provided with a solution to the above equation. The goal of this paper is then to find certain conditions on the given synchronous closed loop system such that it is desynchronisable. Note that in supervisory control theory the control problem is based on language equivalence, but branching bisimilarity coincides with language equivalence in the presence of determinism and in the absence of τ actions. However, we use branching bisimulation because the asynchronous closed loop systems as constructed in the next section are always nondeterministic. In brief, this cause of nondeterminism is due to the abstraction of interactions between a plant and the buffer.

4 From synchrony to asynchrony

The aim of this section is to extend our setup in accordance with the architecture of Subsection 1.1, to model asynchronous communication by introducing two bags between a given plant and its supervisor; one bag that contains input actions of P and another one that contains output actions of P . But, to define a bag we need to define a multiset and some operations over multisets.

A multiset ξ over the set of communicated actions $I' \subset I$ is a tuple (I', κ) where $\kappa : I' \rightarrow \mathbb{N}$ is the corresponding multiplicity function. We write the empty multiset as ε , which is defined as (\emptyset, κ_0) , where $\kappa_0 : \emptyset \rightarrow 0$ is the *zero function*.

Definition 7. Let $\xi = (I', \kappa)$ be a multiset over the set I .

- The predicate \in' is used to denote an element that belongs to a multiset. It is defined as $\?a \in' \xi \triangleq \?a \in I' \wedge \kappa(\?a) > 0$.
- The operator \oplus is used to denote an addition of an element to a multiset. It is defined as $\xi \oplus \?a \triangleq (I_1, \kappa_1)$ where,

$$I_1 = \begin{cases} I', & \text{if } \?a \in I' \\ I' \cup \{\?a\}, & \text{if } \?a \notin I' \end{cases}, \quad \kappa_1(x) = \begin{cases} \kappa(\?a) + 1, & \text{if } x = \?a \wedge \?a \in I' \\ 1, & \text{if } x = \?a \wedge \?a \notin I' \\ \kappa(x) & \text{if } x \neq \?a \wedge x \in I' \end{cases}.$$

- The operator \ominus is used to denote a removal of an element from a multiset. It is defined as $\xi \ominus \?a \triangleq (I_1, \kappa_1)$ where,

$$I_1 = \begin{cases} I', & \kappa(\?a) > 1 \\ I' \setminus \{\?a\}, & \kappa(\?a) = 1 \end{cases}, \quad \kappa_1(x) = \begin{cases} \kappa(\?a) - 1, & \text{if } x = \?a \wedge \kappa(\?a) > 0 \\ \kappa(x), & \text{if } x \neq \?a \wedge x \in I' \end{cases}. \quad \square$$

For each $x \in A$ we define an auxiliary element \hat{x} . Let \hat{A} denote the set of new elements of the form \hat{x} . Similarly we assume that there exists auxiliary hidden and blocking sets \hat{I} and \hat{H} , respectively.

Definition 8. (*Bag*). Let $n > 0$ be a natural number representing the size of a bag process. Let ε denote the empty multiset and ξ denote a multiset of communicated actions (i.e. the actions that are decorated with the symbol $\?$). Then a bounded bag process over a set of

action labels $A_1 \subseteq \text{Act}$ of size n is defined in the following way.

$$\begin{aligned}
B_{A_1}^n(\varepsilon) &= \sum_{?a \in A_1} ?\hat{a}.B_{A_1}^n(\varepsilon \oplus ?a) \quad \text{for every } n > 0, \\
B_{A_1}^n(\xi) &= \sum_{!a \in \xi} !\hat{a}.B_{A_1}^n(\xi \ominus ?a) + \sum_{b \in A_1} ?\hat{b}.B_{A_1}^n(\xi \oplus ?b) \\
&\quad \text{if } |\xi| < n, \\
B_{A_1}^n(\xi) &= \sum_{!a \in \xi} !\hat{a}.B_{A_1}^n(\xi \ominus ?a) \quad \text{if } |\xi| = n.
\end{aligned}$$

□

The above definition is bounded with variable n which not only helps in modeling a realistic asynchronous implementation (as they contain buffers with finite memory). In contrast, it also helps in modeling an asynchronous implementation that have buffers of infinite size, i.e., when $n = \infty$.

Next we formally define an abstraction scheme that implements the construction method M1. Informally, it decorates the interaction between a plant and the two interleaving bags with the symbol $\hat{\cdot}$, indicating such interactions are to be made hidden. We write the asynchronous closed loop system as $\tau_{\hat{I}}(\partial_{H \cup \hat{H}}(P \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} S))$ (for some $m, n > 0$) constructed from its corresponding synchronous closed loop system $\partial_H(P \parallel_{\gamma} S)$ where,

- The notation $B^{m,n}[\mu, \nu]$ represents two empty interleaving bags and is defined in the following way,

$$B^{m,n}[\mu, \nu] \triangleq B_{A_1}^m(\mu) \parallel B_{A_2}^n(\nu)$$

where, $A_1 = \alpha^?(P)$, $A_2 = \alpha^!(P)$ and $m > 0$ ($n > 0$) denotes the size of bag associated with input (output) actions of the plant P . Furthermore, the sets A_1 and A_2 denote the set of input and output action labels of the plant P , respectively.

- $\gamma' : (A \cup \hat{A}) \times (A \cup \hat{A}) \rightarrow (A \cup \hat{A})$ is the modified communication function (or the abstraction scheme for method M1) defined in following way,

$$\gamma'(!a, ?\hat{a}) = \begin{cases} ?\hat{a} & \text{if } !a \in \alpha^!(P) \\ ?a & \text{if } !a \in \alpha^!(S) \end{cases} \quad \gamma'(!\hat{a}, ?a) = \begin{cases} ?\hat{a} & \text{if } ?a \in \alpha^?(P) \\ ?a & \text{if } ?a \in \alpha^?(S) \end{cases}.$$

Intuitively, the communication function γ' with the operators $\tau_{\hat{I}}, \partial_{H \cup \hat{H}}$ ensures the interactions between the plant and the bag are invisible while the interactions between the supervisor and the bag are visible.

The rationale behind the choice of M1 is based on the observation that a transition system generated by a supervisor S is isomorphic to the corresponding synchronous closed loop system $\partial_H(P \parallel_{\gamma} S)$, modulo the difference in the type of action labels [7]. This is because in the synthesis of supervisors no transitions are introduced that a plant cannot execute. Moreover, the action labels in S will be decorated as either an input action (?) or an output action (!) while in $\partial_H(P \parallel_{\gamma} S)$ the same label will be decorated as a communicated action (?). Formally, this fact is equivalent to

$$\rho_f(S) \Leftrightarrow \partial_H(P \parallel_{\gamma} S)$$

where, ρ is the renaming operator from TCP [2] and $f : A \rightarrow A$ is a function that renames an input/output action to a communicated action, i.e., $\forall ?a, !a \in A. [f(!a) = f(?a) = ?a]$. As a consequence, when one introduces bags and abstracts from the interaction between plant and bags, the supervisor model remains unaffected. While in other abstraction schemes (M2, M3 and M4) the above fact does not hold. Thus, it is easier to study abstraction scheme M1 than other schemes.

5 Desynchronisable closed loop system

In the previous section, we have shown how to construct an asynchronous closed loop system from a given synchronous closed loop system. In general, the newly constructed asynchronous closed loop system will not be branching bisimilar to the given synchronous closed loop system. To this end, we introduce a special class of synchronous closed loop systems called desynchronisable closed loop systems that are always branching bisimilar to their corresponding asynchronous closed loop system. We then present sufficient conditions for desynchronisability.

Definition 9. *Let $\partial_H(P \parallel_\gamma S)$ be a synchronous closed loop system and let m, n be any two nonzero natural numbers. Then, $\partial_H(P \parallel_\gamma S)$ is said to be desynchronisable with input and output buffers of size n and m (or in short desynchronisable closed loop system), respectively, if*

$$\partial_H(P \parallel_\gamma S) \stackrel{b}{\leftrightarrow} \tau_I(\partial_{H \cup \hat{H}}(P \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} S)) .$$

□

We now present three sufficient conditions for desynchronisability with buffers of arbitrary size. The objective of these conditions is the following. The conditions given in Definition 10 and Definition 11 prevent an asynchronous closed loop system from getting deadlocked. The condition in Definition 12 ensures that the silent steps introduced by the abstraction scheme are inert.

Definition 10. *Let $\partial_H(P \parallel_\gamma S)$ be a synchronous closed loop system. Then, $\partial_H(P \parallel_\gamma S)$ is called well posed if there exists a binary relation $W \subseteq \mathbb{P} \times \mathbb{P}$ such that $(P, S) \in W$ and the following conditions are satisfied:*

- $\forall !a, p, p', s. [(p, s) \in W \wedge p \xrightarrow{!a} p' \Rightarrow \exists s'. [s \xrightarrow{?a} s' \wedge (p', s') \in W]]$, and
- $\forall !a, p, s, s'. [(p, s) \in W \wedge s \xrightarrow{!a} s' \Rightarrow \exists p'. [p \xrightarrow{?a} p' \wedge (p', s') \in W]]$.

In other words, if a plant (supervisor) is able to send an output label $!a$ then the supervisor (plant) is able to receive the input label $?a$. □

We now partition the set I into two disjoint non-empty subsets $I_P^?, I_P^!$ with respect to a plant process P as:

- $I_P^? \triangleq \{?a \mid ?a \in I \wedge a \in \alpha^?(P)\}$.
- $I_P^! \triangleq \{?a \mid ?a \in I \wedge a \in \alpha^!(P)\}$.

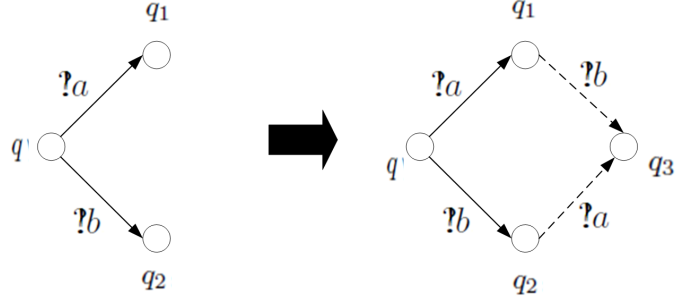


Figure 3: Diamond property.

Definition 11. Let $\mu \in I_P^{? *}$ and $\nu \in I_P^! *$ be sequences in $I_P^?$ and $I_P^!$, respectively. Let $p \in \mathbb{P}$ be an arbitrary process. We define the set of reachable states of p in the following way,

$$\text{Reach}(p) = \{p' \mid \exists w \in A^*. [p \xrightarrow{w} p']\} .$$

By the semantics of TCP we know that if the initial state of a process has the form $\partial_H(- \parallel_\gamma -)$ then, all the reachable states will also be of the same structure. A synchronous closed loop system $\partial_H(P \parallel_\gamma S)$ is said to satisfy the reordering property iff both the following conditions are satisfied,

- $\forall p', p_2, s', \partial_H(p_1 \parallel_\gamma s_1) \in \text{Reach}(\partial_H(P \parallel_\gamma S)), ?a \in I_P^?$
 $[\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\mu. ?a} \partial_H(p' \parallel_\gamma s') \wedge p_1 \xrightarrow{?a} p_2 \Rightarrow$
 $\exists s_2. [\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{?a} \partial_H(p_2 \parallel_\gamma s_2)]]$
- $\forall p', s', s_2, \partial_H(p_1 \parallel_\gamma s_1) \in \text{Reach}(\partial_H(P \parallel_\gamma S)), ?a \in I_P^!$
 $[\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\nu. ?a} \partial_H(p' \parallel_\gamma s') \wedge s_1 \xrightarrow{?a} s_2 \Rightarrow$
 $\exists p_2. [\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{?a} \partial_H(p_2 \parallel_\gamma s_2)]] .$

□

Definition 12. Let $q \in \mathbb{P}$ be an arbitrary process. Then, q is said to satisfy the diamond property iff the following condition hold (see Figure 3)

- $\forall x, y, q_1, q_2. [q \xrightarrow{x} q_1 \wedge q \xrightarrow{y} q_2 \wedge x \neq y \Rightarrow \exists q_3. [q_1 \xrightarrow{y} q_3 \wedge q_2 \xrightarrow{x} q_3]] .$

A process p is said to satisfy the diamond property iff for all reachable states p' from p satisfy the diamond property. □

For a reader familiar with the concepts of *true concurrency* [18], the conditions given in Definition 5, 11 and 12 are similar to the axioms of asynchronous transition system. The formulation of these axioms is based on the definition of an independence relation, which is an irreflexive and symmetric relation on the set of actions A . However, the techniques for desynchronisability for such models are not investigated here, although it will be worthwhile to examine this research direction in the future. Note that in our approach we do not need an additional notion of the independence relation.

Next, we present the following main results of this paper.

- If an arbitrary synchronous closed loop system satisfies the conditions in Definitions 10, 11 and 12 then, it is a desynchronisable with buffers of arbitrary size.

- The transition system generated by an asynchronous closed loop system constructed from a synchronous closed loop system satisfying the conditions in Definitions 10, 11 and 12 is always τ -inert with respect to \Leftrightarrow_b .

To prove the above statements, we first fix some notations and then prove some lemmas, which are necessary for the proof of main theorem.

We denote the contents of an arbitrary bag by the symbols ξ, ξ' , i.e., ξ, ξ' are of the form (I_0, κ_0) and (I_1, κ_1) respectively, where $I_0, I_1 \subseteq I$. The contents of the bag attached to input actions of P is denoted by μ , i.e., μ is of the form (I_μ, κ_μ) where $I_\mu \subseteq I_P^?$. Similarly the contents of the bag attached to output actions of P is denoted by ν , i.e., ν is of the form (I_ν, κ_ν) where $I_\nu \subseteq I_P^!$. For an arbitrary multiset ξ , we define a sequence (denoted as $\vec{\xi}$) over ξ as,

$$\vec{\xi} \triangleq \langle x_1.x_2.\dots \rangle$$

such that $\#(x_i, \vec{\xi}) = \kappa(x_i)$, where $\#$ is a function that returns the maximum number of occurrences of x_i in $\vec{\xi}$ for some $i > 0$. For example consider a multiset $\xi = \{\?a, \?a, \?b, \?b\}$. Then a possible sequence $\vec{\xi}$ over the given ξ can be of the form $\langle \?a. \?b. \?a. \?b \rangle$. Let $f_i : I^* \rightarrow H^*$ be the function defined as $f_i(\?a.\vec{\xi}) = \?a.f_i(\vec{\xi})$. Similarly, let $f_o : I^* \rightarrow H^*$ be the function defined as $f_o(\!a.\vec{\xi}) = \!a.f_o(\vec{\xi})$.

Proposition 1. *Given a trace $\partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\mu}} \partial_H(p_1 \parallel_\gamma s_1)$, we find using the above function f_i and semantics of \parallel_γ that $P \xrightarrow{f_i(\vec{\mu})} p_1 \wedge S \xrightarrow{f_o(\vec{\mu})} s_1$.*

Proposition 2. *Similarly, given a trace $\partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\nu}} \partial_H(p_1 \parallel_\gamma s_1)$, we conclude that $P \xrightarrow{f_o(\vec{\nu})} p_1 \wedge S \xrightarrow{f_i(\vec{\nu})} s_1$.*

The following lemma is a generalisation of Definition 12. It states that if two different states q_1, q_2 are reachable from a state q_0 , then there exists a state q_3 reachable from q_1 and q_2 such that, the trace between q_0, q_1 and the trace between q_0, q_2 commute.

Lemma 1 (Generalised diamond property). *Let $\partial_H(P \parallel_\gamma S)$ be an arbitrary synchronous closed loop system satisfying the conditions in Definitions 10, 11 and 12. If $\partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\xi}} \partial_H(p_1 \parallel_\gamma s_1) \wedge \partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\xi}'} \partial_H(p_2 \parallel_\gamma s_2)$ then,*

$$\exists p_3, s_3. [\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\xi}'} \partial_H(p_3 \parallel_\gamma s_3) \wedge \partial_H(p_2 \parallel_\gamma s_2) \xrightarrow{\vec{\xi}} \partial_H(p_3 \parallel_\gamma s_3)].$$

The following Lemmas 2, 3 are the results (See [5] for the proofs) obtained by direct instantiation of reordering property (Definition 11) and generalised diamond property (Lemma 1).

Lemma 2. *Let $\partial_H(P \parallel_\gamma S)$ be a synchronous closed loop system satisfying the conditions in Definitions 10, 11 and 12. Suppose $\?a \in I_P^? \wedge \partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\mu}.\?a} \partial_H(p_2 \parallel_\gamma s_2) \wedge P \xrightarrow{\?a} p_1$ then,*

$$\exists s_1. \left[\partial_H(P \parallel_\gamma S) \xrightarrow{\?a} \partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_\gamma s_2) \right].$$

Proof. It is given that $\partial_H(P \parallel_\gamma S)$ satisfies the conditions in Definitions 10, 11 and 12 with $\dagger a \in I_P^? \wedge \partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\mu}.\dagger a} \partial_H(p_2 \parallel_\gamma s_2) \wedge P \xrightarrow{?a} p_1$. Then by reordering property (Definition 11) we get,

$$\exists s_1. [\partial_H(P \parallel_\gamma S) \xrightarrow{?a} \partial_H(p_1 \parallel_\gamma s_1)] .$$

By the given transitions $\partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\mu}.\dagger a} \partial_H(p_2 \parallel_\gamma s_2)$ we infer that,

$$\exists p'_1, s'_1. [\partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\mu}} \partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{?a} \partial_H(p_2 \parallel_\gamma s_2)] .$$

Applying generalised diamond property (Lemma 1) at the state $\partial_H(P \parallel_\gamma S)$ we get,

$$\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_\gamma s_2) .$$

Hence, the desired result is achieved. \square

Lemma 3. Let $\partial_H(P \parallel_\gamma S)$ be a synchronous closed loop system satisfying the conditions in Definitions 10, 11 and 12. Suppose $\dagger a \in I_P^! \wedge \partial_H(P \parallel_\gamma S) \xrightarrow{\vec{v}.\dagger a} \partial_H(p_3 \parallel_\gamma s_3) \wedge S \xrightarrow{?a} s_1$ then,

$$\exists p_1. \left[\partial_H(P \parallel_\gamma S) \xrightarrow{?a} \partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}} \partial_H(p_3 \parallel_\gamma s_3) \right] .$$

Proof. Similar as the proof of Lemma 2. \square

We now pose the main theorem of this paper which proves the following statement. If the given synchronous closed system satisfies the conditions in Definition 10, 11 and 12, then it is desynchronisable independent of the size of the buffers introduced between the given plant and its supervisor.

Theorem 1. Let $\partial_H(P \parallel_\gamma S)$ be an arbitrary synchronous closed loop system satisfying the conditions in Definitions 10, 11 and 12. Then for any $m, n > 0$ we have,

$$\partial_H(P \parallel_\gamma S) \xleftrightarrow{b} \tau_{\hat{I}}(\partial_{H \cup \hat{H}}(P \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} S)) .$$

Proof. Appendix A \square

In hindsight, what we have actually proven is that all τ actions generated by the abstraction scheme are τ -inert with respect to \xleftrightarrow{b} . The following corollary states this fact.

Corollary 1. Let q_c be a process of the form $\partial_H(P \parallel_\gamma S)$. And let q_a be an asynchronous closed loop system of the form $\tau_{\hat{I}}(\partial_{H \cup \hat{H}}(P \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} S))$ such that $(q_c, q_a) \in \Phi$. Then,

$$\forall q_c, q_a, q'_a. [(q_c, q_a) \in \Phi \wedge q_a \xrightarrow{\tau} q'_a \Rightarrow q_a \xleftrightarrow{b} q'_a] .$$

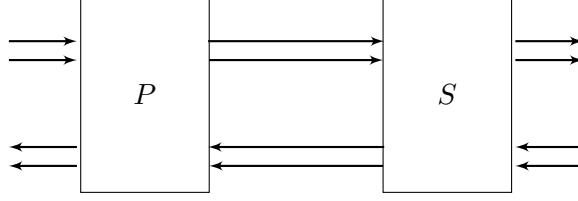


Figure 4: A partial closed loop system.

6 Desynchronisation of partial closed loop systems

In the previous section, we showed that the well posedness, reordering and diamond properties are sufficient conditions under which a synchronous closed loop system is desynchronisable. In this section, we extend the desynchronisability result to a class of synchronous closed loop systems whose alphabet contains communicated actions with certain send/recieve actions from a plant and its supervisor. Such closed loop systems are called *partial* closed loop systems and Figure 4 shows the context diagram of a partial closed loop system.

To carry out similar work as in Section 5, we first modify the definitions of a plant, a supervisor, and a requirement in order to construct a partial closed loop system. Secondly, we extend the definitions of aforementioned sufficient conditions in a conservative way. We assume that for an input-output process $p \in \mathbb{P}$ we are given with a set of external actions $\bar{\alpha}(p) \subseteq H$.

Definition 13. A plant (supervisor) $P \in \mathbb{P}$ ($S \in \mathbb{P}$) is an input-output and deterministic process such that the set of external actions are nonempty, i.e., $\bar{\alpha}(P) \neq \emptyset$. Define the modified blocking set,

$$H(P, S) = \{\square a \mid \square a \notin \bar{\alpha}(P) \wedge \square a \notin \bar{\alpha}(S) \wedge \square \in \{!, ?\}\}.$$

A requirement $R \in \mathbb{P}$ for a partial closed loop system $\partial_{H(P,S)}(P \parallel_{\gamma} S)$ is a deterministic process such that,

$$\alpha(R) \cap H(P, S) = \emptyset \wedge \tau \notin \alpha(R).$$

Furthermore, the control equation in this new setting is the following.

$$\partial_{H(P,S)}(P \parallel_{\gamma} S) \triangleleft_b R.$$

□

The above modifications results in the following change of the definition of the communication function γ' which implements the abstraction scheme M1. We write the asynchronous closed loop system as

$$\tau_i(\partial_{H(P,S) \cup \hat{H}(P,S)}(P \parallel_{\gamma'} B^{m,n}(\varepsilon, \varepsilon) \parallel_{\gamma'} S))$$

(for some $m, n > 0$), constructed from a partial synchronous closed loop system $\partial_{H(P,S)}(P \parallel_{\gamma} S)$ with,

$$\gamma'(!a, ?\hat{a}) = \begin{cases} \sharp \hat{a} & \text{if } !a \in \alpha^!(P) \wedge !a \in H(P, S) \\ \sharp a & \text{if } !a \in \alpha^!(S) \wedge !a \in H(P, S) \end{cases},$$

$$\gamma'(!\hat{a}, ?a) = \begin{cases} \sharp \hat{a} & \text{if } ?a \in \alpha^?(P) \wedge ?a \in H(P, S) \\ \sharp a & \text{if } ?a \in \alpha^?(S) \wedge ?a \in H(P, S) \end{cases}.$$

Informally, the above definition of communication function γ' not only implements the abstraction scheme M1, but it also *restricts* the communication of external actions of both the plant and the supervisor with the bag.

6.1 Modified sufficient conditions

In this section, we extend the well posedness, the reordering and the diamond property in a conservative way. Furthermore, we introduce an additional property, called fair-noise property, that ensures the presence of external actions of plant is safe. The intuition of these conditions will be explain alongside with their respective formal definitions.

Notation. We write a partial synchronous closed loop system $\partial_{H(P,S)}(P \parallel_{\gamma} S)$ as $\partial_{\circ}(P \parallel_{\gamma} S)$ and the corresponding partial asynchronous closed loop system as $\widehat{\nabla}(P \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} S)$.

Definition 14. Let $\partial_{\circ}(P \parallel_{\gamma} S)$ be a partial synchronous closed loop system. Then, $\partial_{\circ}(P \parallel_{\gamma} S)$ is called well posed if there exists a binary relation $W \subseteq \mathbb{P} \times \mathbb{P}$ such that $(P, S) \in W$ and the following conditions are satisfied:

1. $\forall_{p,p',s}. [(p, s) \in W \wedge p \xrightarrow{!a} p' \wedge !a \notin \bar{\alpha}(p) \Rightarrow \exists s'. [s \xrightarrow{?a} s' \wedge (p', s') \in W]]$,
2. $\forall_{p,p',s}. [(p, s) \in W \wedge p \xrightarrow{x} p' \wedge x \in \bar{\alpha}(p) \Rightarrow (p', s) \in W]$,
3. $\forall_{p,s,s'}. [(p, s) \in W \wedge s \xrightarrow{!a} s' \wedge !a \notin \bar{\alpha}(s) \Rightarrow \exists p'. [p \xrightarrow{?a} p' \wedge (p', s') \in W]]$,
4. $\forall_{p,s,s'}. [(p, s) \in W \wedge s \xrightarrow{x} s' \wedge x \in \bar{\alpha}(s) \Rightarrow (p, s') \in W]$.

□

The conditions 1,3 in the above definition are the usual conditions (See Definition 10) of well posedness property. However, the conditions 2,4 ensures that an external step $\square \xrightarrow{x} \square'$ performed at a receiver's state (either plant's state or supervisor's state, i.e., $\square \in \{p, s\}$) do not alter the set of input actions enabled at the state \square and thus remain well-posed.

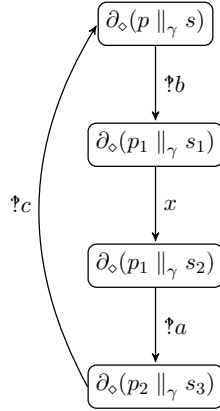
As already mentioned, the above well posedness property was designed to prevent a partial asynchronous closed loop system from getting deadlocked. Unfortunately, the well posedness property *alone* is not sufficient for this purpose. The following example illustrates this fact.

Example 1. Let $x \in \bar{\alpha}(S)$ and consider the following equations describing incomplete behaviour of a plant and a supervisor.

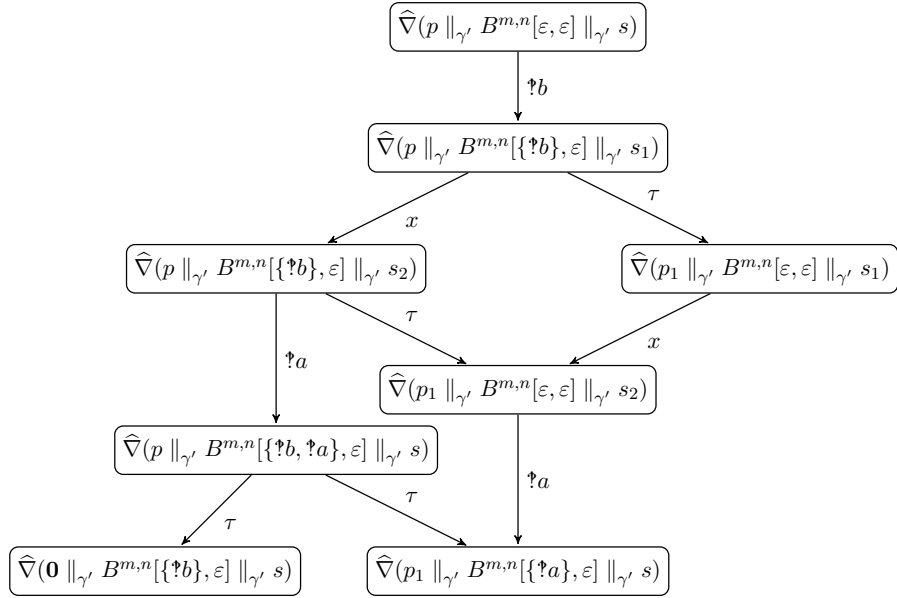
$$\begin{array}{ll} p = ?a.0 + ?b.p_1 & s = !b.s_1, s_1 = x.s_2 \\ p_1 = ?a.p_2, p_2 = !c.p & s_2 = !a.s_3, s_3 = ?c.s \end{array}$$

The Figures 5(a) and 5(b) shows the transition system of $\partial_{\circ}(p \parallel_{\gamma} s)$ and $\widehat{\nabla}(p \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$, respectively. Note, that the partial synchronous closed loop system $\partial_{\circ}(p \parallel_{\gamma} s)$ is deadlock free; however, the asynchronous closed loop system $\widehat{\nabla}(p \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$ contains the following deadlock trace $\langle ?b.x. ?a.\tau \rangle$.

Thus, the following reordering property is designed to eliminate such scenarios. □



(a) Transition system of $\partial_\circ(p \parallel_\gamma s)$.



(b) Partial transition system of $\widehat{\nabla}(p \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$ showing the deadlock at the state $\widehat{\nabla}(\mathbf{0} \parallel_{\gamma'} B^{m,n}[\{?\?b\}, \varepsilon] \parallel_{\gamma'} s)$.

Figure 5: Example 1.

Definition 15. Let $\vec{\mu} \in (I_P^? \cup \bar{\alpha}(S))^*$ and $\vec{\nu} \in (I_P^! \cup \bar{\alpha}(P))^*$ be sequences in $I_P^?$ and $I_P^!$, respectively. Let $p \in \mathbb{P}$ be an arbitrary process and let $q \in Q$ be its initial state. A partial synchronous closed loop system $\partial_\circ(P \parallel_\gamma S)$ is said to satisfy the reordering property iff both the following conditions are satisfied,

- $\forall p', p_2, s', \partial_\circ(p_1 \parallel_\gamma s_1) \in \text{Reach}(\partial_\circ(P \parallel_\gamma S)), \dagger a \in I_P^?$

$$\left[\partial_\circ(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\mu}.\dagger a} \partial_\circ(p' \parallel_\gamma s') \wedge p_1 \xrightarrow{?a} p_2 \Rightarrow \right.$$

$$\left. \exists s_2. [\partial_\circ(p_1 \parallel_\gamma s_1) \xrightarrow{\dagger a} \partial_\circ(p_2 \parallel_\gamma s_2)] \right]$$
- $\forall p', s', s_2, \partial_\circ(p_1 \parallel_\gamma s_1) \in \text{Reach}(\partial_\circ(P \parallel_\gamma S)), \dagger a \in I_P^!$

$$\left[\partial_\circ(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\nu}.\dagger a} \partial_\circ(p' \parallel_\gamma s') \wedge s_1 \xrightarrow{?a} s_2 \Rightarrow \right.$$

$$\left. \exists p_2. [\partial_\circ(p_1 \parallel_\gamma s_1) \xrightarrow{\dagger a} \partial_\circ(p_2 \parallel_\gamma s_2)] \right].$$

□

In the new setting, the diamond property (Definition 16) has the same objective that the silent steps generated by the abstraction scheme are τ -inert.

Definition 16. Let $q \in \mathbb{P}$ be an arbitrary process. Then, q is said to satisfy the diamond property iff the following condition holds

- $\forall x, y, q_1, q_2. \left[q \xrightarrow{x} q_1 \wedge q \xrightarrow{y} q_2 \wedge x \neq y \Rightarrow \exists q_3. [q_1 \xrightarrow{y} q_3 \wedge q_2 \xrightarrow{x} q_3] \right].$

A process p is said to satisfy the diamond property iff for all reachable states p' from p satisfy the diamond property. □

We have extended the old sufficient conditions in the setting of partial closed loop systems and expect the desynchronisability result to hold via a branching bisimulation relation $\widehat{\Phi}$ similar to the one used in the proof of Theorem 1. The idea behind the design of the relation $\widehat{\Phi}$ should be to relate a state $\partial_\circ(p \parallel_\gamma s)$ in a partial synchronous closed loop system to those states $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_\gamma s)$ in an asynchronous closed loop system that contain the same supervisor state s . This is due to the abstraction scheme used to construct an partial asynchronous closed loop system from a given partial synchronous closed loop system. Unfortunately, there are certain scenarios (explained in the following paragraph) due to the external step made by plant, which causes more behaviour in an asynchronous closed loop system that will not be present in the corresponding synchronous closed loop system. However, if a partial closed loop system contains only the external actions of the supervisor (i.e. no external actions of the plant) in its alphabet then the above modified conditions are sufficient for desynchronisability.

Next we explore the scenarios in which the external step made by a plant in a partial closed loop system obstructs its desynchronisability.

Example 2. Consider the behaviour of a synchronous closed loop system specified by the following equations

$$\partial_\circ(p \parallel_\gamma s) = \dagger a. \partial_\circ(p_1 \parallel_\gamma s_1), \quad \partial_\circ(p_1 \parallel_\gamma s_1) = x. \partial_\circ(p_2 \parallel_\gamma s_1)$$

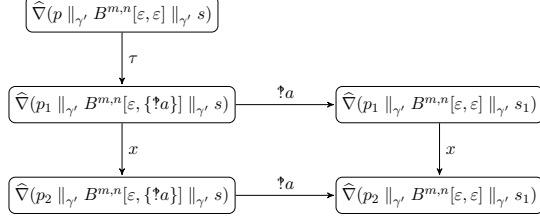


Figure 6: Transition system of $\widehat{\nabla}(p \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$ in the Example 2.

where $!a \in I_P^!$, $x \in \bar{\alpha}(P)$. The transition system generated by the asynchronous closed loop system is shown in Figure 6. Immediately, we observe the trace $\langle \tau.x. !a \rangle$ from the state $\widehat{\nabla}(p \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$ and thus disallowing the states $\partial_{\circ}(p \parallel_{\gamma} s)$, $\widehat{\nabla}(p_1 \parallel_{\gamma'} B^{m,n}[\varepsilon, \{!a\}] \parallel_{\gamma'} s)$ to be related by a branching bisimulation relation $\widehat{\Phi}$. Moreover, it contradicts our intuition about $\widehat{\Phi}$ that “a state $\partial_{\circ}(p \parallel_{\gamma} s)$ in a partial synchronous closed loop system to those states in an asynchronous closed loop system that contain the same supervisor state s ”. To rectify this, we require that the state $\partial_{\circ}(p \parallel_{\gamma} s)$ must contain the trace $\langle x. !a \rangle$ reachable to the state $\partial_{\circ}(p_2 \parallel_{\gamma} s_1)$.

Intuitively, we require that a plant must treat its output action $!a \in I_P^!$ to supervisor and a command $x \in \bar{\alpha}(P)$ by or to an external process in a ‘fair’ way. This is formalised by “fair-noise” property (See Definition 17); note an execution of external step by a plant can be considered as a noise from the viewpoint of the supervisor and hence the name fair-noise.

Definition 17. Let $!a \in I_P^!$ and $x \in \bar{\alpha}(P)$. A plant P is said to satisfy the fair-noise property in a partial synchronous closed loop system $\partial_{\circ}(P \parallel_{\gamma} S)$ iff the following condition holds.

$$\begin{aligned} \forall \partial_{\circ}(p_1 \parallel_{\gamma} s_1) \in \text{Reach}(\partial_{\circ}(P \parallel_{\gamma} S)). \left[\partial_{\circ}(p_1 \parallel_{\gamma} s_1) \xrightarrow{!a.x} \partial_{\circ}(p_2 \parallel_{\gamma} s_2) \right. \\ \left. \Rightarrow \exists p'_1. [\partial_{\circ}(p_1 \parallel_{\gamma} s_1) \xrightarrow{x} \partial_{\circ}(p'_1 \parallel_{\gamma} s_1)] \right]. \end{aligned}$$

□

Lemma 4. Let $\vec{v} \in I_P^{!*}$, $x \in \bar{\alpha}(P)$ and $\partial_{\circ}(p_1 \parallel_{\gamma} s_1)$ satisfies the fair-noise property (Definition 17) and the diamond property (Definition 16). If $\partial_{\circ}(p_1 \parallel_{\gamma} s_1) \xrightarrow{\vec{v}.x} \partial_{\circ}(p_2 \parallel_{\gamma} s_2)$ then,

$$\exists p'_1. [\partial_{\circ}(p'_1 \parallel_{\gamma} s_1) \xrightarrow{x.\vec{v}} \partial_{\circ}(p_2 \parallel_{\gamma} s_2)].$$

Proof. Straightforward, by induction on \vec{v} . □

Proposition 3. Let $\vec{v} \in I_P^{!*}$, $\vec{\sigma} \in \bar{\alpha}(P)^*$ and suppose $\partial_{\circ}(p \parallel_{\gamma} s)$ satisfies the fair-noise property (Definition 17) and the diamond property (Definition 16). If $\partial_{\circ}(p \parallel_{\gamma} s) \xrightarrow{\vec{v}.\vec{\sigma}} \partial_{\circ}(p_1 \parallel_{\gamma} s_1)$ then,

$$\exists p', s'. \left[\partial_{\circ}(p \parallel_{\gamma} s) \xrightarrow{\vec{\sigma}} \partial_{\circ}(p' \parallel_{\gamma} s') \xrightarrow{\vec{v}} \partial_{\circ}(p_1 \parallel_{\gamma} s_1) \right].$$

Lemma 5. Let $\partial_H(P \parallel_\gamma S)$ be an arbitrary synchronous closed loop system satisfying the conditions in Definitions 14, 15, 16 and 17. If $\partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\xi}} \partial_H(p_1 \parallel_\gamma s_1) \wedge \partial_H(P \parallel_\gamma S) \xrightarrow{\vec{\xi}} \partial_H(p_2 \parallel_\gamma s_2)$ then,

$$\exists p_3, s_3. [\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\xi}} \partial_H(p_3 \parallel_\gamma s_3) \wedge \partial_H(p_2 \parallel_\gamma s_2) \xrightarrow{\vec{\xi}} \partial_H(p_3 \parallel_\gamma s_3)].$$

Lemma 6. Let $\partial_\diamond(p_1 \parallel_\gamma s_1)$ be a partial synchronous closed loop system satisfying the well posedness (Definition 14), reordering property (Definition 15) and the diamond property (Definition 16). Suppose $\dagger a \in I_P^?$, $\vec{\sigma} \in (\bar{\alpha}(P) \cup \bar{\alpha}(S))^*$ such that $\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\dagger a} \partial_\diamond(p_3 \parallel_\gamma s_3)$. Then,

$$\exists p'_3, s'_3. [\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{\dagger a} \partial_\diamond(p'_3 \parallel_\gamma s'_3) \xrightarrow{\vec{\sigma}} \partial_\diamond(p_3 \parallel_\gamma s_3)].$$

Proof. We prove it by structural induction on $\vec{\sigma}$.

- Base case. In this case, $\vec{\sigma} = \varepsilon$, or $\vec{\sigma} = x, x \in \bar{\alpha}(P)$, or $\vec{\sigma} = x, x \in \bar{\alpha}(S)$. When $\vec{\sigma} = \varepsilon$ the proof is trivial.

1. When $\vec{\sigma} = x, x \in \bar{\alpha}(P)$. Then we have,

$$\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{x} \partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\dagger a} \partial_\diamond(p_3 \parallel_\gamma s_3)$$

with $s_1 = s_2$ due to semantics of \parallel_γ and $x \in \bar{\alpha}(P)$. Furthermore, from the semantics of \parallel_γ and the fact $\dagger a \in I_P^?$ we get $p_2 \xrightarrow{\dagger a} p_3 \wedge s_1 \xrightarrow{!a} s_3$. So by using the extended well posedness (Definition 14) we get,

$$\exists p'_3. [\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{\dagger a} \partial_\diamond(p'_3 \parallel_\gamma s_3)].$$

And by the Lemma 5 we get the desired result,

$$\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{\dagger a} \partial_\diamond(p'_3 \parallel_\gamma s_3) \xrightarrow{x} \partial_\diamond(p_3 \parallel_\gamma s_3).$$

2. When $\vec{\sigma} = x, x \in \bar{\alpha}(S)$. Then we have,

$$\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{x} \partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\dagger a} \partial_\diamond(p_3 \parallel_\gamma s_3)$$

with $p_1 = p_2$ due to semantics of \parallel_γ and $x \in \bar{\alpha}(S)$. Furthermore, from the semantics of \parallel_γ and the fact $\dagger a \in I_P^?$ we get $p_1 \xrightarrow{\dagger a} p_3 \wedge s_1 \xrightarrow{!a} s_3$. But from the Definition 15 we get,

$$\exists p'_3, s'_3. [\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{\dagger a} \partial_\diamond(p'_3 \parallel_\gamma s'_3)].$$

Again by Lemma 5 we get,

$$\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{\dagger a} \partial_\diamond(p'_3 \parallel_\gamma s'_3) \xrightarrow{x} \partial_\diamond(p_3 \parallel_\gamma s_3).$$

- Inductive case. Let $\vec{\sigma} = x.\vec{\sigma}'$ and assume the transition $\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{x} \partial_\diamond(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\sigma}'} \partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{!a} \partial_\diamond(p_3 \parallel_\gamma s_3)$. By induction hypothesis we have,

$$\exists p'_3, s'_3. [\partial_\diamond(p'_1 \parallel_\gamma s'_1) \xrightarrow{!a} \partial_\diamond(p'_3 \parallel_\gamma s'_3) \xrightarrow{\vec{\sigma}'} \partial_\diamond(p_3 \parallel_\gamma s_3)].$$

We identify two cases based on the external action x performed either by the plant (P) or the supervisor (S).

1. $x \in \bar{\alpha}(P)$. Then by semantics of \parallel_γ we know that $s_1 = s'_1$. Furthermore, from the induction hypothesis, $!a \in I_P^?$ and the semantics of \parallel_γ we get, $p'_1 \xrightarrow{?a} p'_3 \wedge s'_1 \xrightarrow{!a} s'_3$. By applying well posedness (Definition 14) at the state $\partial_\diamond(p_1 \parallel_\gamma s'_1)$ we get,

$$\exists p''_1. [\partial_\diamond(p_1 \parallel_\gamma s'_1) \xrightarrow{!a} \partial_\diamond(p''_1 \parallel_\gamma s'_3)].$$

Applying the Lemma 5 we get the desired result,

$$\partial_\diamond(p_1 \parallel_\gamma s'_1) \xrightarrow{!a} \partial_\diamond(p''_1 \parallel_\gamma s'_3) \xrightarrow{x} \partial_\diamond(p'_3 \parallel_\gamma s'_3).$$

2. $x \in \bar{\alpha}(P)$. Then by semantics of \parallel_γ we know that $p_1 = p'_1$. Furthermore, from the induction hypothesis, $!a \in I_P^?$ and the semantics of \parallel_γ we get, $p'_1 \xrightarrow{?a} p'_3 \wedge s'_1 \xrightarrow{!a} s'_3$. Using Definition 15 with the transitions $\partial_\diamond(p'_1 \parallel_\gamma s_1) \xrightarrow{x.!a} \partial_\diamond(p'_3 \parallel_\gamma s'_3)$ and $p'_1 \xrightarrow{?a} p'_3$ we get,

$$\exists p''_1, s''_1. [\partial_\diamond(p'_1 \parallel_\gamma s_1) \xrightarrow{!a} \partial_\diamond(p''_1 \parallel_\gamma s''_1)].$$

Applying the Lemma 5 we get the desired result,

$$\partial_\diamond(p'_1 \parallel_\gamma s_1) \xrightarrow{!a} \partial_\diamond(p''_1 \parallel_\gamma s''_1) \xrightarrow{x} \partial_\diamond(p'_3 \parallel_\gamma s'_3).$$

□

Lemma 7. Let $\partial_\diamond(p_1 \parallel_\gamma s_1)$ be a partial synchronous closed loop system satisfying the well posedness (Definition 14), reordering property (Definition 15) and the diamond property (Definition 16). Suppose $!a \in I_P^!$, $\vec{\sigma} \in (\bar{\alpha}(P) \cup \bar{\alpha}(S))^*$ such that $\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{!a} \partial_\diamond(p_3 \parallel_\gamma s_3)$. Then,

$$\exists p'_3, s'_3. [\partial_\diamond(p_1 \parallel_\gamma s_1) \xrightarrow{!a} \partial_\diamond(p'_3 \parallel_\gamma s'_3) \xrightarrow{\vec{\sigma}} \partial_\diamond(p_3 \parallel_\gamma s_3)].$$

Proof. Similar to the proof of Lemma 6. □

Next we pose the following main result of this section: “If a partial synchronous closed loop system satisfies the condition of Definition 14, 15, 16 and 17 then it is desynchronisable”.

Theorem 2. Let $\partial_\diamond(P \parallel_\gamma S)$ be an arbitrary partial synchronous closed loop system satisfying the conditions in Definitions 14, 15, 16 and 17. Then for any $m, n > 0$ we have,

$$\partial_\diamond(P \parallel_\gamma S) \Leftrightarrow_b \widehat{\nabla}(P \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} S).$$

Proof. Appendix B. □

7 Discussion

In this section, we discuss in the context of desynchronisable closed loop system (Section 5) whether the reordering property (Definition 11) and the diamond property (Definition 12) presented here can be further weakened while desynchronising. The choice of these properties is motivated by the fact that these conditions seems more restrictive than well posedness property.

Informally, the reordering property states that every reachable state from an initial state in a synchronous closed loop system satisfy the following condition. If a receiver (either plant or supervisor) is able to receive an input $?a$ and a sender is able to send a sequence of outputs $f_o(\mu)!.a$ ($\mu \in I_P^{?*}$) then, the sender must send the output $!a$ before the sequences of outputs $f_o(\mu)^1$. To circumvent such a stringent condition, one may design a synchronous closed loop system that allows a sender to progress with an output action if the sender has received the acknowledgements of previously send output actions and thus, implicitly creating a *locking* mechanism.

Example 3. Consider the behaviour of a plant p and a supervisor s specified by the following set of equations.

$$\begin{aligned} p &=?a.p_1, \quad p_1 =!b.p_2, \quad p_2 =?c.p \\ s &=!a.s_1, \quad s_1 =?b.s_2, \quad s_2 =!c.s \end{aligned}$$

To observe the effect of locking mechanism lets consider the initial state of the asynchronous closed loop system, $\tau_{\hat{I}}(\partial_{H \cup \hat{H}}(p \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} s))$. The supervisor performs the output $!a$ and transforms to the state s_1 . Thus, we infer the following transition by the asynchronous closed loop system.

$$\tau_{\hat{I}}(\partial_{H \cup \hat{H}}(p \parallel_{\gamma'} B^{m,n}[\varepsilon, \varepsilon] \parallel_{\gamma'} s)) \xrightarrow{!a} \tau_{\hat{I}}(\partial_{H \cup \hat{H}}(p \parallel_{\gamma'} B^{m,n}[\{!a\}, \varepsilon] \parallel_{\gamma'} s_1))$$

Note the supervisor in the state s_1 can only wait for the input action $?b$ and the plant in the state p can only receive the input action $?a$. Thus, the plant in the state $\tau_{\hat{I}}(\partial_{H \cup \hat{H}}(p \parallel_{\gamma'} B^{m,n}[\{!a\}, \varepsilon] \parallel_{\gamma'} s_1))$ can only remove the content from its input bag. A similar phenomenon can be observed in the other states of the asynchronous closed loop system. Moreover, it can be verified that the synchronous closed loop system $\partial_H(p \parallel_{\gamma} s)$ is desynchronisable.

Note that from the above example it is clear that a locking mechanism can be implicitly built-in a synchronous closed loop system in order to avoid the reordering property. However, upon inspection it can be concluded that the reordering property is vacuously satisfied in the above example. This also suggests that the reordering property can be a suitable candidate for the necessary conditions for desynchronisability.

In comparison to the reordering property, the diamond property property can be further weakened. We give an example where a synchronous closed loop system satisfying well posedness and reordering properties is still desynchronisable.

Example 4. Consider the behaviour of a plant and a supervisor specified by the following set of equations:

$$\begin{aligned} p &=?a.p_1 + ?c.p_2, & s &=!a.s_1 + !c.s_2, \\ p_1 &=!b.p, \quad p_2 =!d.p & s_1 &=?b.s, \quad s_2 =?d.s \end{aligned}$$

The transition system of the synchronous and asynchronous closed loop system is depicted in Figure 7. Clearly, the two transition system are branching bisimilar.

¹Recall the definition of the function f_o from Page 13.

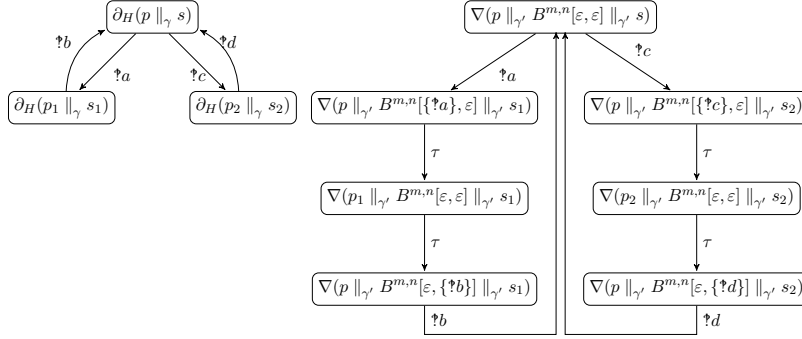


Figure 7: Example 4, $\nabla(-) = \tau_{I \cup \hat{I}}(\partial_{H \cup \hat{H}}(-))$.

Thus, we anticipate that the diamond property (Definition 12) can be further weakened. In particular, if the actions $\dagger a, \dagger b \in I_P^?$ are enabled at a state q then it may not be necessary for the traces $\dagger a. \dagger b$ and $\dagger b. \dagger a$ to commute. Furthermore, we conjecture that if a synchronous closed loop system satisfies the well posedness property (Definition 10), the reordering property (Definition 11), and the weaker form of diamond property, then it is desynchronisable.

8 Conclusions and future work

The goal of this paper was to check for desynchronisability of a synchronous closed loop system without building the corresponding asynchronous system. We presented sufficient conditions for desynchronisability in a process algebraic setting and showed that an asynchronous implementation using bags (of arbitrary size) is a refinement of the synchronous closed loop system satisfying these conditions. Moreover, we generalise this result for partial closed loop systems whose alphabets may contain the input/output actions from the plant and its supervisor in addition to the communicated actions.

The prominent features of our work can be summarised in the following main points:

- We solve a refinement problem instead of a supervisory control problem, and do not compute a new supervisor in the presence of buffers, as done in [3, 20]. Our approach is *intended* to be computationally cheaper than the one developed in [3, 20], however this conjecture needs to be verified by analysing the complexities associated with the conditions presented here. In particular, we conjecture that supervisory control theory always results in synchronous closed loop systems, which are well-posed (Definition 10), but the other conditions, (Definition 11 and Definition 12), are not likely to be attained so easily.
- We present our conditions for desynchronisability over the components of a synchronous closed loop system conjointly, in contrast with [9], where the check for the foam rubber wrapper principle on the two components was applied separately. Note the sender domination property from [9] is equivalent to the well posed condition (Definition 10). However, the two approaches are incomparable because in [9] the construction method M3 was studied while in this paper the construction method M1 is studied.
- We use branching bisimulation equivalence instead of the failure equivalence that was adopted in [9]. As a consequence our techniques are applicable to all the weak equiva-

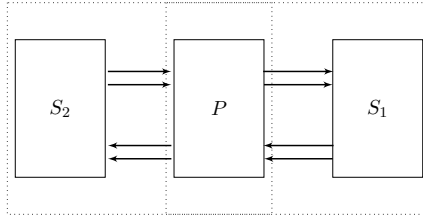


Figure 8: A synchronous closed loop system in a decentralised architecture [19].

lences in the ‘van Glabbeek spectrum’ [17] (including failure equivalence). The branching bisimulation is the preferred equivalence in TCP process algebra under the presence of τ action [2]. Furthermore, the conditions (well posedness and diamond property) given here are similar to the ones mentioned in [9], where sufficient conditions for desynchronisability was given modulo failure equivalence. Thus, we conjecture that achieving desynchronisability for weaker equivalences will not lead to weaker sufficient conditions.

A question that was not treated in this paper, is whether the conditions we posed are in fact reasonable for industrial applications. This may become clear in the near future, when we study the case studies involved with supervisory control theory in the context of the MULTIFORM project [1] with the language CIF [4]. The authors of CIF are currently developing techniques that will incorporate supervisory control theory and model based engineering into a single framework, thus making it suitable for the design of industrial applications. In particular, the elevator case study and the toy example, which were desynchronisable in [7] using the construction method M1, satisfy our conditions.

The issue of compositionality for desynchronisable closed loop system is also not answered completely, although initial results in this directions are presented here, i.e, the desynchronisability of partial synchronous closed loop systems. Consider the synchronous closed loop system $\partial_H(P \parallel_\gamma S_1 \parallel_\gamma S_2)$ in a decentralised architecture, which can be further decomposed into two partial synchronous closed loop systems as shown in Figure 8. Next, these individual partial synchronous closed loop systems can be verified for desynchronisability by inspecting the sufficient conditions on them. However, to conclude the overall desynchronisable closed loop system (i.e. $\partial_H(P \parallel_\gamma S_1 \parallel_\gamma S_2)$) is desynchronisable, more research is required.

Lastly, the research performed in this paper can of course be repeated for different architectures. One might study whether wires or queues can be used instead of bags, or study different abstraction schemes, or try to study the conditions for desynchronisability by focusing on other notions of weak equivalences.

9 Acknowledgements

The authors would like to thank the reviewers of the ICE Workshop, 2010 for their valuable critical remarks on the earlier draft of this paper and pointing out that the sender domination property and the well posed property are equivalent. The authors would also like to thank Jos Baeten, Koos Rooda, Bert van Beek, Rong Su, Jasen Markowski, Damian Nadales, and Mihaly Petreczky for various discussions and clarification about this work.

This work has been performed as part of the ‘‘Integrated Multi-formalism Tool Support for the Design of Networked Embedded Control Systems’’ (MULTIFORM) project, supported by

the Seventh Research Framework Programme of the European Commission (Grant agreement number: INFSO-ICT-224249).

References

- [1] Integrated multi-formalism tool support for the design of networked embedded control systems : Multiform. <http://cms.multiform.bci.tu-dortmund.de/>.
- [2] J. C. M. Baeten, T. Basten, and M. Reniers. *Process Algebra: Equational Theories of Communicating Processes*. (Cambridge Tracts in Theoretical Computer Science; 50). Cambridge University Press, 2009.
- [3] S. Balemi. *Control of Discrete Event Systems: Theory And Application*. PhD thesis, Swiss Federal Institute of Technology, Automatic Control Laboratory, ETH Zurich, May 1992.
- [4] D. A. van Beek, P. Collins, D. E. Ndales, J.E. Rooda, and R. R. H. Schiffelers. New concepts in the abstract format of the compositional interchange format. In A. Giua, C. Mahuela, M. Silva, and J. Zaytoon, editors, *3rd IFAC Conference on Analysis and Design of Hybrid Systems*, pages 250–255, Zaragoza, Spain, 2009.
- [5] H. Beohar and P. J. L. Cuijpers. Desynchronisability of (partial) synchronous closed loop systems. CS Technical report: 10-18, Eindhoven university of technology, 2010. <http://www.win.tue.nl/~pcuijper/pages/publications.html>.
- [6] H. Beohar and P.J.L. Cuijpers. A theory of desynchronisable closed loop systems. In S. Bliudze, R. Bruni, D. Grohmann, and A. Silva, editors, *Proceedings of Interaction and Concurrency Experience (ICE'10)*, volume 38 of *EPTCS*, 2010.
- [7] H. Beohar, P. J. L. Cuijpers, and J. C. M. Baeten. Design of asynchronous supervisors. [abs/0910.0868](http://arxiv.org/abs/0910.0868), 2009. <http://arxiv.org/abs/0910.0868>.
- [8] M. Fabian and A. Hellgren. PLC-based implementation of supervisory control for discrete event systems. *Proceedings of the 37th IEEE Conference on Decision and Control, 1998*, 3:3305–3310, 1998.
- [9] C. Fischer and W. Janssen. Synchronous development of asynchronous systems. In Ugo Montanari and Vladimiro Sassone, editors, *Proceedings of CONCUR'96*, volume 1119 of *Lecture Notes in Computer Science*, pages 735–750. Springer-Verlag, 1996.
- [10] J. F. Groote and M. P. A. Sellink. Confluence for process verification. *Theor. Comput. Sci.*, 170(1-2):47–81, 1996.
- [11] He Jifeng, M. B. Josephs, and C. A. R. Hoare. A theory of synchrony and asynchrony. In M. Broy and C. B. Jones, editors, *Programming Concepts and Methods*, pages 459–479, 1990.
- [12] H. K. Kapoor and M. B. Josephs. Modelling and verification of delay-insensitive circuits using CCS and the concurrency workbench. *Inf. Process. Lett.*, 89(6):293–296, 2004.

- [13] M. Mousavi, P. Le Guernic, J.-P. Talpin, S. K. Shukla, and T. Basten. Modeling and validating globally asynchronous design in synchronous frameworks. In *Proceedings of the Conference on Design Automation and Test in Europe*, pages 384–389. IEEE Computer Society Press, 2003.
- [14] G. D. Plotkin. A Structural Approach to Operational Semantics. Technical Report DAIMI FN-19, University of Aarhus, 1981.
- [15] P. J. Ramadge and W. M. Wonham. Supervisory control of a class of discrete event processes. *SIAM Journal on Control and Optimization*, 25(1):206–230, 1987.
- [16] J.T. Udding. *Classification and Composition of Delay-Insensitive Circuits*. PhD thesis, Eindhoven University of Technology, Eindhoven, 1984.
- [17] R. J. v. Glabbeek. *Comparative concurrency semantics and refinement of actions*. Ph.D. thesis, CWI, Amsterdam, 1990.
- [18] G. Winskel and M. Nielsen. Models for concurrency. In *Handbook of Logic in Computer Science (vol. 4)*, pages 1–148. Oxford University Press, 1995.
- [19] W. M. Wonham. Supervisory control of discrete-event systems. Monograph ECE 1636F/1637S, University of Toronto, Dept. of Electrical & Computer Engineering, 2008.
- [20] S. Xu and R. Kumar. Asynchronous implementation of synchronous discrete event control. In *9th International Workshop on Discrete Event Systems, WODES 2008*, pages 181–186, Goteborg, Sweden, May 2008.

A Proof of Theorem 1

Proof. Let p, p', s be free process variables. Let μ, ν be two free variables representing the contents of an input and an output bag of P , respectively. For brevity we write $\tau_{\hat{I}}(\partial_{H \cup \hat{H}}(-))$ as $\nabla(-)$. Then, define a relation Φ as follows.

$$\Phi \triangleq \{(\partial_H(p \parallel_\gamma s), \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)) \mid (p' = p \wedge \mu = \varepsilon \wedge \nu = \varepsilon) \bigvee \quad \text{[C1]}$$

$$(\mu = \varepsilon \wedge \exists s'. [\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_H(p' \parallel_\gamma s')]) \bigvee \quad \text{[C2]}$$

$$(\nu = \varepsilon \wedge \exists s'. [\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_\gamma s)]) \bigvee \quad \text{[C3]}$$

$$(\exists p'', s', s''. [\partial_H(p' \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_H(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_\gamma s)]) \bigvee \quad \text{[C4]}$$

$$(\exists p'', s', s''. [\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s')]) \}. \quad \text{[C5]}$$

The proof of the theorem is based on showing that Φ is a witnessing branching bisimulation relation. The intuition behind the definition of Φ is following. A state $\partial_H(p \parallel_\gamma s)$ in a synchronous closed loop system is related to those states in an asynchronous closed loop

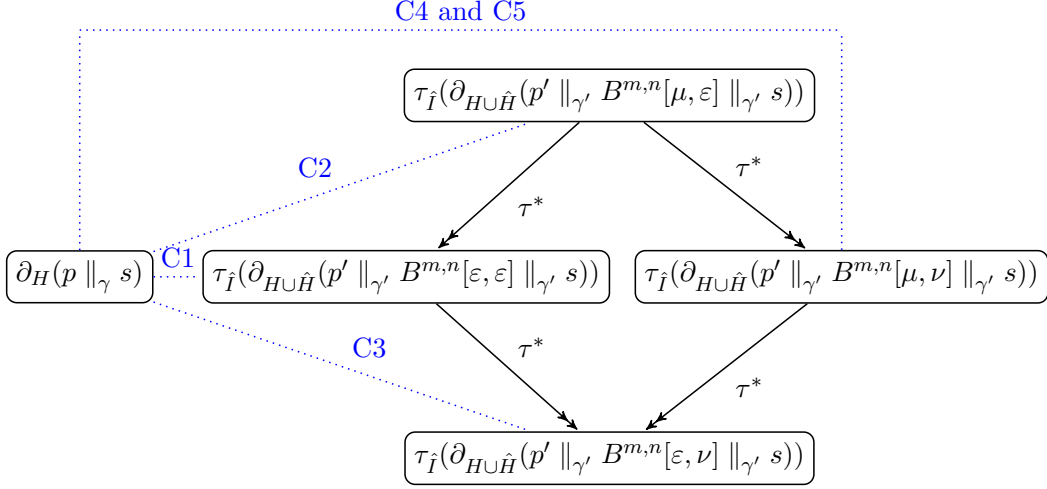


Figure 9: Illustration of relation Φ .

system which contain the same supervisor state s . The Φ relation between two states is indicated by dotted lines in Figure 9.

Let q_c, q_a be the initial states of the processes $\partial_H(p \parallel_{\gamma} s)$ and $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$, respectively. From the definition of branching bisimilarity we need to show the following four transfer conditions:

1. $\forall ?a, q_c, q'_c, q_a. [q_c \xrightarrow{?a} q'_c \wedge (q_c, q_a) \in \Phi \Rightarrow \exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{?a} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi]]$.
2. $\forall q_c, q'_c, q_a. [q_c \xrightarrow{\tau} q'_c \wedge (q_c, q_a) \in \Phi \Rightarrow (q'_c, q_a) \in \Phi \vee \exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{\tau} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi]]$.
3. $\forall q_c, q_a, q'_a. [q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \Rightarrow (q'_a, q_c) \in \Phi \vee \exists q'_c, q''_c. [q_c \xrightarrow{\tau^*} q'_c \xrightarrow{\tau} q''_c \wedge (q'_c, q_a), (q''_c, q'_a) \in \Phi]]$.
4. $\forall ?a, q_c, q_a, q'_a. [q_a \xrightarrow{?a} q'_a \wedge (q_c, q_a) \in \Phi \Rightarrow \exists q'_c, q''_c. [q_c \xrightarrow{\tau^*} q'_c \xrightarrow{?a} q''_c \wedge (q'_c, q_a), (q''_c, q'_a) \in \Phi]]$.

Since the synchronous closed loop system does not contain τ actions in its alphabet, there are following three effects on the above transfer conditions.

- The above condition 2 will be vacuously satisfied.
- The condition 3 will be reduced to the simpler form,

$$\forall q_c, q_a, q'_a. [q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \Rightarrow (q_c, q'_a) \in \Phi].$$

- And similarly condition 4 will be reduced to:

$$\forall ?a, q_c, q_a, q'_a. [q_a \xrightarrow{?a} q'_a \wedge (q_c, q_a) \in \Phi \Rightarrow \exists q'_c. [q_c \xrightarrow{?a} q'_c \wedge (q'_c, q'_a) \in \Phi]].$$

But to show that these conditions hold, we need to know whether an action label $?a$ occurring in each condition is either an input or output action with respect to P , i.e. $?a \in I_P^?$ or $?a \in O_P^?$ (see the partition of the set I in Page 11).

Thus, we get six transfer conditions in total which are shown in Table 3. Furthermore, for each case we apply case distinction based on the structure of μ and ν . In each subcase we use C1,C2,C3,C4, and C5 (the conditions from the definition of Φ) to determine the relation between free process variable p, p' and then prove the conclusion as shown in Table 3. The notation $\tau = \tau_{\sharp a}(\sharp a)$ is used to denote that the τ action is a result of abstraction of the communicated action $\sharp a$.

Table 3: Proof structure of Theorem 1.

Case No.	Hypothesis	Conclusion
T1	$q_c \xrightarrow{\sharp a} q'_c \wedge (q_c, q_a) \in \Phi \wedge \sharp a \in I_P^?$	$\exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{\sharp a} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi.]$
T2	$q_c \xrightarrow{\sharp a} q'_c \wedge (q_c, q_a) \in \Phi \wedge \sharp a \in I_P^!$	$\exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{\sharp a} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi.]$
T3	$q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \wedge \tau = \tau_{\sharp a}(\sharp a) \wedge \sharp a \in I_P^?$	$(q_c, q'_a) \in \Phi.$
T4	$q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \wedge \tau = \tau_{\sharp a}(\sharp a) \wedge \sharp a \in I_P^!$	$(q_c, q'_a) \in \Phi.$
T5	$q_a \xrightarrow{\sharp a} q'_a \wedge (q_c, q_a) \in \Phi \wedge \sharp a \in I_P^?$	$\exists q'_c. [q_c \xrightarrow{\sharp a} q'_c \wedge (q'_c, q'_a) \in \Phi.]$
T6	$q_a \xrightarrow{\sharp a} q'_a \wedge (q_c, q_a) \in \Phi \wedge \sharp a \in I_P^!$	$\exists q'_c. [q_c \xrightarrow{\sharp a} q'_c \wedge (q'_c, q'_a) \in \Phi.]$

Using the above ideas, we now show that the relation Φ is a branching bisimulation relation.

T1. Let $q_c \xrightarrow{\sharp a} q'_c \wedge (q_c, q_a) \in \Phi \wedge \sharp a \in I_P^?$. Then from the construction of Φ we know that,

$$\begin{aligned} q_c &\equiv \partial_H(p \parallel_\gamma s), \\ q_a &\equiv \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of \parallel_γ we know that,

$$q'_c \equiv \partial_H(p_1 \parallel_\gamma s_1).$$

Thus, the given transition $q_c \xrightarrow{\sharp a} q'_c$ will be of the form $\partial_H(p \parallel_\gamma s) \xrightarrow{\sharp a} \partial_H(p_1 \parallel_\gamma s_1)$. From the SOS rule of \parallel_γ , and from the assumption $\sharp a \in I_P^?$ we infer that,

$$p \xrightarrow{\sharp a} p_1 \wedge s \xrightarrow{!a} s_1. \quad (1)$$

Before showing that the state q_a can perform action $\sharp a$, we need to retrieve the relation between process terms $\partial_H(p \parallel_\gamma s)$ and $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ from the conditions C1,C2,C3,C4, and C5. We apply case distinction based on the structure of μ and ν .

- (a) when $\mu = \varepsilon \wedge \nu = \varepsilon$. Then the state q_a is of the form $\nabla(p' \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$. Note that the states q_c, q_a can be Φ -related only by C1 because of the assumption $\mu = \varepsilon \wedge \nu = \varepsilon$. Thus, $p' = p$. Moreover from the transition $s \xrightarrow{!a} s_1$ in Equation 1, we infer that

$$\nabla(p \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s) \xrightarrow{\sharp a} \nabla(p \parallel_{\gamma'} B[\varepsilon \oplus \sharp a, \varepsilon] \parallel_{\gamma'} s_1).$$

Substituting $p = p_1, s = s_1, s' = s, p' = p, \nu = \varepsilon$ and $\mu = \{\sharp a\}$ in C3 and from the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\sharp a} \partial_H(p_1 \parallel_\gamma s_1)$ we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p \parallel_{\gamma'} B[\varepsilon \oplus \sharp a, \varepsilon] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

- (b) when $\mu \neq \varepsilon \wedge \nu = \varepsilon$. Similar as Subcase T1.d.
- (c) when $\mu = \varepsilon \wedge \nu \neq \varepsilon$. Similar as Subcase T1.d.
- (d) when $\mu \neq \varepsilon \wedge \nu \neq \varepsilon$. Then the state q_a is of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$. Note that the states q_c and q_a can be Φ -related either by C4 or C5 because of the assumption that $\mu \neq \varepsilon \wedge \nu \neq \varepsilon$. Thus we get following two cases.
- i. Either $(q_c, q_a) \in \Phi$ by C4. Then using the definition of Φ we get,

$$\exists p'', s', s''. [\partial_H(p' \parallel_{\gamma'} s') \xleftarrow{\vec{v}} \partial_H(p'' \parallel_{\gamma'} s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_{\gamma} s)].$$

Moreover from the transition $s \xrightarrow{!a} s_1$ in Equation 1 (Page 28) we infer that,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{!a} \nabla(p' \parallel_{\gamma'} B[\mu \oplus ?a, \nu] \parallel_{\gamma'} s_1).$$

Recall the given transitions $\partial_H(p'' \parallel_{\gamma'} s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_{\gamma} s)$ and $\partial_H(p \parallel_{\gamma} s) \xrightarrow{!a} \partial_H(p_1 \parallel_{\gamma} s_1)$. This implies that,

$$\partial_H(p'' \parallel_{\gamma'} s'') \xrightarrow{\vec{\mu}.!a} \partial_H(p_1 \parallel_{\gamma} s_1).$$

Note that the sequence $\vec{\mu}.!a$ is a sequence over the multiset $\mu \oplus ?a$. Now substitute $p'' = p'', s'' = s'', p' = p', s' = s', p = p_1, s = s_1, \nu = \nu$, and $\mu = \mu \oplus ?a$ in C4 and from the transitions $\partial_H(p' \parallel_{\gamma'} s') \xleftarrow{\vec{v}} \partial_H(p'' \parallel_{\gamma'} s'') \xrightarrow{\vec{\mu}.!a} \partial_H(p_1 \parallel_{\gamma} s_1)$ we infer that,

$$\left(\partial_H(p_1 \parallel_{\gamma} s_1), \nabla(p' \parallel_{\gamma'} B[\mu \oplus ?a, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

- ii. Or $(q_c, q_a) \in \Phi$ by C5. Then using the definition of Φ we get,

$$\exists p'', s', s''. [\partial_H(p \parallel_{\gamma} s) \xrightarrow{\vec{v}} \partial_H(p'' \parallel_{\gamma'} s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_{\gamma'} s')].$$

Using the transition $s \xrightarrow{!a} s_1$ from Equation 1 we get,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{!a} \nabla(p' \parallel_{\gamma'} B[\mu \oplus ?a, \nu] \parallel_{\gamma'} s_1).$$

Now we need to show that the state $\nabla(p' \parallel_{\gamma'} B[\mu \oplus ?a, \nu] \parallel_{\gamma'} s_1)$ is Φ -related to the state $\partial_H(p_1 \parallel_{\gamma} s_1)$. From the hypothesis we have $\partial_H(p'' \parallel_{\gamma'} s'') \xleftarrow{\vec{v}} \partial_H(p \parallel_{\gamma} s) \xrightarrow{!a} \partial_H(p_1 \parallel_{\gamma} s_1)$. Then, applying generalised diamond property (Lemma 1) we get,

$$\exists p'_1, s'_1. [\partial_H(p'' \parallel_{\gamma'} s'') \xrightarrow{!a} \partial_H(p'_1 \parallel_{\gamma'} s'_1) \wedge \partial_H(p_1 \parallel_{\gamma} s_1) \xrightarrow{\vec{v}} \partial_H(p'_1 \parallel_{\gamma'} s'_1)].$$

Recall the given transition $\partial_H(p' \parallel_{\gamma'} s') \xrightarrow{\vec{\mu}} \partial_H(p'' \parallel_{\gamma'} s'')$. Then from transitivity of transition relation \rightarrow we get,

$$\partial_H(p' \parallel_{\gamma'} s') \xrightarrow{\vec{\mu}.!a} \partial_H(p'_1 \parallel_{\gamma'} s'_1).$$

Substituting $\mu = \mu \oplus \mathfrak{!}a, \nu = \nu, p' = p', s' = s', p'' = p'_1, s'' = s'_1, p = p_1, s = s_1$ in C5 and from the transitions $\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\nu}} \partial_H(p'_1 \parallel_\gamma s'_1) \xleftarrow{\vec{\mu}.\mathfrak{!}a} \partial_H(p' \parallel_\gamma s')$ we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p' \parallel_{\gamma'} B[\mu \oplus \mathfrak{!}a, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

T2. Let $q_c \xrightarrow{\mathfrak{!}a} q'_c \wedge (q_c, q_a) \in \Phi \wedge \mathfrak{!}a \in I_P^1$. Then from the construction of Φ we know that,

$$\begin{aligned} q_c &\equiv \partial_H(p \parallel_\gamma s), \\ q_a &\equiv \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of \parallel_γ we know that,

$$q'_c \equiv \partial_H(p_1 \parallel_\gamma s_1).$$

Thus the given transition $q_c \xrightarrow{\mathfrak{!}a} q'_c$ will be of the form $\partial_H(p \parallel_\gamma s) \xrightarrow{\mathfrak{!}a} \partial_H(p_1 \parallel_\gamma s_1)$. From the SOS rule of \parallel_γ and from the assumption that $\mathfrak{!}a \in I_P^1$ we infer that,

$$p \xrightarrow{\mathfrak{!}a} p_1 \wedge s \xrightarrow{\mathfrak{!}a} s_1. \quad (2)$$

Before showing that the state q_a can perform action $\mathfrak{!}a$, we need to retrieve the relation between process terms $\partial_H(p \parallel_\gamma s)$ and $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ using conditions C1,C2,C3,C4, and C5. So apply case distinction based on the structure of μ and ν .

- (a) when $\mu = \varepsilon$ and $\nu = \varepsilon$. Then the state q_a is of the form $\nabla(p' \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$. Note the states q_c , and q_a can be Φ -related only by C1 because of the assumption $\mu = \varepsilon$ and $\nu = \varepsilon$. Thus, $p' = p$. From the transition $p \xrightarrow{\mathfrak{!}a} p_1$ in Equation 2 we infer that,

$$\nabla(p \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p_1 \parallel_{\gamma'} B[\varepsilon, \{\mathfrak{!}a\}] \parallel_{\gamma'} s).$$

Using C2 and from the given transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\mathfrak{!}a} \partial_H(p_1 \parallel_\gamma s_1)$ we get,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p_1 \parallel_{\gamma'} B[\varepsilon, \{\mathfrak{!}a\}] \parallel_{\gamma'} s) \right) \in \Phi.$$

Moreover from the transition $s \xrightarrow{\mathfrak{!}a} s_1$ (Equation 2,Page 30) we know that,

$$\nabla(p_1 \parallel_{\gamma'} B[\varepsilon, \{\mathfrak{!}a\}] \parallel_{\gamma'} s) \xrightarrow{\mathfrak{!}a} \nabla(p_1 \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s_1).$$

Substituting $p = p_1, \mu = \varepsilon$, and $\nu = \varepsilon$ in C1 we get,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p_1 \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

- (b) when $\mu = \varepsilon$ and $\nu \neq \varepsilon$. Similar as Subcase T2.d.
(c) when $\mu \neq \varepsilon$ and $\nu = \varepsilon$. Similar as Subcase T2.d.
(d) when $\mu \neq \varepsilon$ and $\nu \neq \varepsilon$. Then the state q_a is of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$. Note that the states q_c and q_a can be Φ -related either by C4 or C5 because of the assumption that $\mu \neq \varepsilon$ and $\nu \neq \varepsilon$. Thus, we get following two cases:

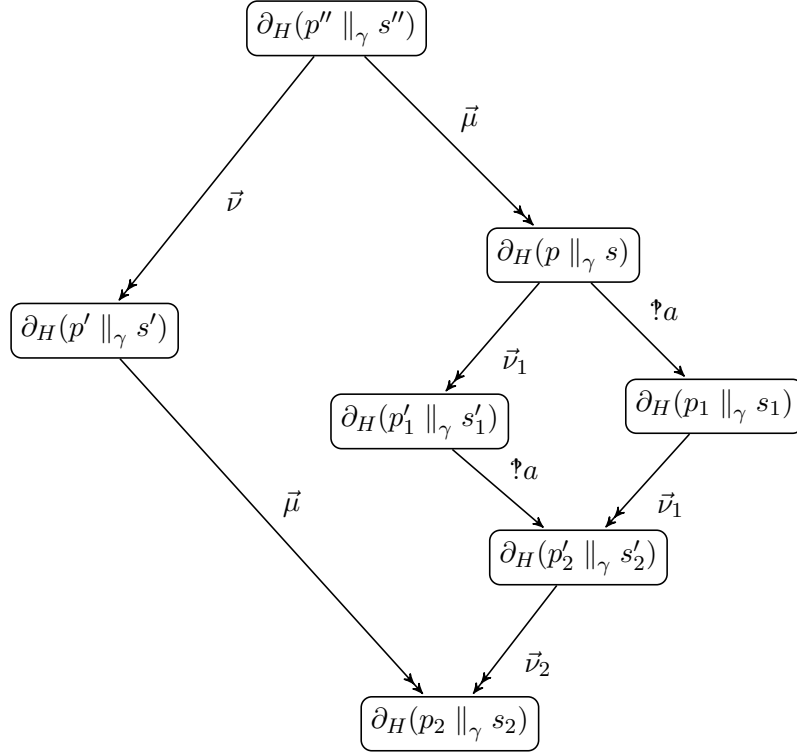


Figure 10: Proof in Subcase C2.d.A.

- i. Either $(q_c, q_a) \in \Phi$ by C4. Then, $\exists p'', s', s''. [\partial_H(p' \parallel_\gamma s') \xleftarrow{\vec{v}} \partial_H(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_\gamma s)]$. By applying generalised diamond property (Lemma 1) at state $\partial_H(p'' \parallel_\gamma s'')$ we get,

$$\exists p_2, s_2. [\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{v}} \partial_H(p \parallel_\gamma s)]. \quad (3)$$

The state q_a can perform action $?a$ only if $?a \in' \nu$. Otherwise the plant component p' in state q_a will have to increase the content ν output bag $?a$. Thus, we have following two possibilities:

- A. Either $?a \in' \nu$. Then there exists p'_1, s'_1, p'_2, s'_2 such that the transition

$\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_H(p_2 \parallel_\gamma s_2)$ derived in the Equation 3 will be of the following form (see Figure 10),

$$\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}_1} \partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{?a} \partial_H(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{v}_2} \partial_H(p_2 \parallel_\gamma s_2) \quad (4)$$

with $\vec{v}_1. ?a. \vec{v}_2$ being a sequence over the multiset \vec{v} . But, the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{?a} \partial_H(p_1 \parallel_\gamma s_1)$ is given. So, applying generalised diamond property (Lemma 1) at the state $\partial_H(p \parallel_\gamma s)$ we get ,

$$\exists p''_2, s''_2. [\partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{?a} \partial_H(p''_2 \parallel_\gamma s''_2) \xleftarrow{\vec{v}_1} \partial_H(p_1 \parallel_\gamma s_1)].$$

But we know that there exists the following transition,

$$\partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{?a} \partial_H(p'_2 \parallel_\gamma s'_2).$$

And since synchronous closed loop systems are deterministic we can conclude that,

$$p'_2 \xleftrightarrow{\text{b}} p''_2 \wedge p'_2 \xleftrightarrow{\text{b}} s''_2.$$

Using the transition $\partial_H(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{v}_2} \partial_H(p_2 \parallel_\gamma s_2)$ derived in Equation 4 and from the above transition we get,

$$\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1.\vec{v}_2} \partial_H(p_2 \parallel_\gamma s_2). \quad (5)$$

Now using the above derived facts in the state $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$. From the transition $s \xrightarrow{?a} s_1$ (Equation 2, Page 30) and the assumption $?a \in' \nu$ we get,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{?a} \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus ?a] \parallel_{\gamma'} s_1).$$

Note that $\vec{v}_1.\vec{v}_2$ is a sequence over the multiset $\nu \ominus ?a$. Substituting $p = p_1, s = s_1, p'' = p_2, s'' = s_2, p' = p', s' = s', \mu = \mu, \nu = \nu \ominus ?a$ in C5 and from the following transitions,

$$\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1.\vec{v}_2} \partial_H(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s') \quad (\text{Equations 3 and 5}).$$

we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus ?a] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

B. Or $?a \notin' \nu$. From Equation 3 (Page 31) we have the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_H(p_2 \parallel_\gamma s_2)$ and the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{?a} \partial_H(p_1 \parallel_\gamma s_1)$ is given. So applying the generalised diamond property (Lemma 1) at the state $\partial_H(p \parallel_\gamma s)$ we get (See Figure 11),

$$\exists p'_2, s'_2. [\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}} \partial_H(p'_2 \parallel_\gamma s'_2) \wedge \partial_H(p_2 \parallel_\gamma s_2) \xrightarrow{?a} \partial_H(p'_2 \parallel_\gamma s'_2)].$$

Using the above derived transition $\partial_H(p_2 \parallel_\gamma s_2) \xrightarrow{?a} \partial_H(p'_2 \parallel_\gamma s'_2)$ and the fact that $?a \in I_P^!$ we get,

$$p_2 \xrightarrow{!a} p'_2 \quad (\text{SOS rule of } \parallel_\gamma.) \quad (6)$$

From the Equation 3 (Page 31) we have the transition $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_\gamma s_2)$ and from hypothesis we have

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

By applying result of Proposition 1 in the transition $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_\gamma s_2)$ we infer that,

$$p' \xrightarrow{f_i(\vec{\mu})} p_2 \quad (\text{SOS rule of } \parallel_\gamma).$$

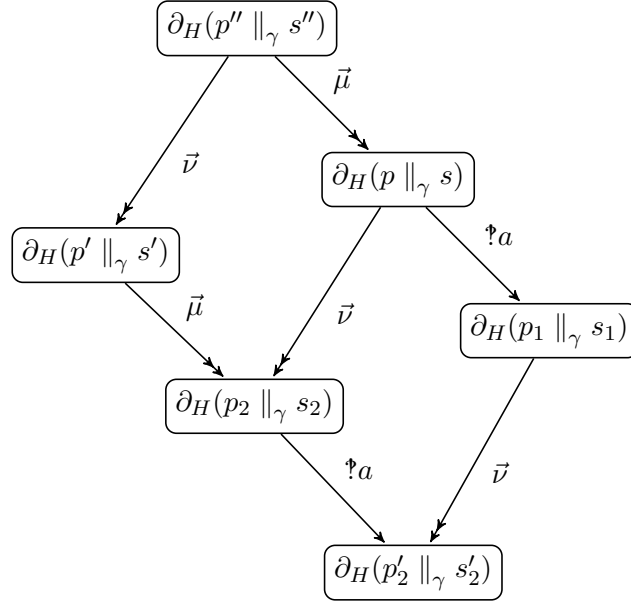


Figure 11: Proof in Subcase C2.d.B.

Furthermore using the above fact at state $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ we get,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau^*} \nabla(p_2 \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s).$$

In addition from the transition $p_2 \xrightarrow{!a} p'_2$ (Equation 6, page 32) we infer,

$$\nabla(p_2 \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_2 \parallel_{\gamma'} B[\varepsilon, \nu \oplus \vec{a}] \parallel_{\gamma'} s).$$

Then from C2 and the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu} \cdot \vec{a}} \partial_H(p_2 \parallel_\gamma s_2)$ we get,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p'_2 \parallel_{\gamma'} B[\varepsilon, \nu \oplus \vec{a}] \parallel_{\gamma'} s) \right) \in \Phi.$$

And from the transition $s \xrightarrow{?a} s_1$ derived in Equation 2 (Page 30) we get,

$$\nabla(p'_2 \parallel_{\gamma'} B[\varepsilon, \nu \oplus \vec{a}] \parallel_{\gamma'} s) \xrightarrow{!a} \nabla(p'_2 \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s_1).$$

Substituting $p = p_1, s = s_1, p' = p'_2, s' = s'_2, \mu = \varepsilon, \nu = \nu$ in C2 and from the transition $\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\nu}} \partial_H(p'_2 \parallel_\gamma s'_2)$ we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p'_2 \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

- ii. Or $(q_c, q_a) \in \Phi$ by C5. Then $\exists p'', s', s''. [\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s')]$. Again there are two possibilities:

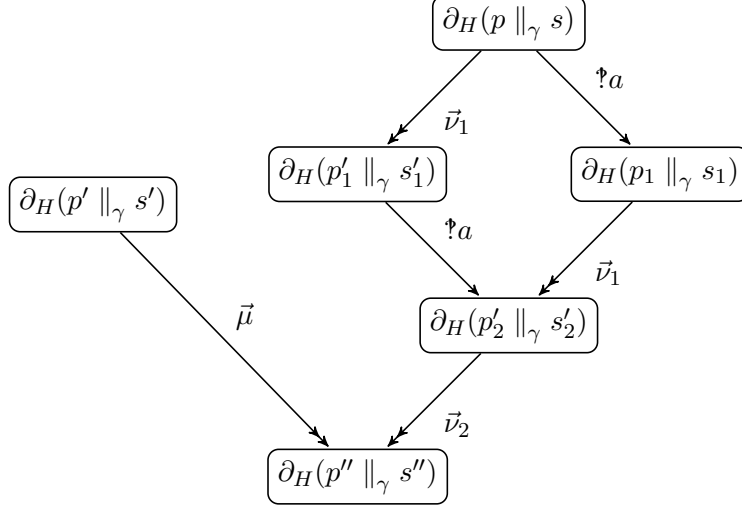


Figure 12: Proof in Subcase C2.e.ii.

- A. Either $\dagger a \in' \nu$. Then there exists p'_1, s'_1, p'_2, s'_2 such that the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_H(p'' \parallel_\gamma s'')$ will be of the following form (see Figure 12),

$$\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}_1} \partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\dagger a} \partial_H(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{v}_2} \partial_H(p'' \parallel_\gamma s'')$$

with $\vec{v}_1. \dagger a. \vec{v}_2$ being a sequence over the multiset \vec{v} . But the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\dagger a} \partial_H(p_1 \parallel_\gamma s_1)$ is given. So applying generalised diamond property (See Figure 12) at state $\partial_H(p \parallel_\gamma s)$ we get,

$$\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1} \partial_H(p'_2 \parallel_\gamma s'_2).$$

Recall that $\partial_H(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{v}_2} \partial_H(p'' \parallel_\gamma s'')$, so we have

$$\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1. \vec{v}_2} \partial_H(p'' \parallel_\gamma s'').$$

Using the above derived facts in asynchronous transition system at state $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$. From the transition $s \xrightarrow{\dagger a} s_1$ derived in Equation 2 (Page 30) we get,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\dagger a} \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus \dagger a] \parallel_{\gamma'} s_1).$$

Note that $\vec{v}_1. \vec{v}_2$ is a sequence over the multiset $\nu \ominus \dagger a$. By substituting $p = p_1, s = s_1, p' = p', s' = s', p'' = p'', s'' = s'', \nu = \nu \ominus \dagger a, \mu = \mu$ in C5 and from the transitions

$$\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1. \vec{v}_2} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s')$$

we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus \dagger a] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

B. Or $\sharp a \notin \nu$. Then from hypothesis we have,

$$\partial_H(p_1 \parallel_\gamma s_1) \xleftarrow{\sharp a} \partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_H(p'' \parallel_\gamma s'')$$

So by applying generalised diamond property (Lemma 1) at state $\partial_H(p \parallel_\gamma s)$ we get,

$$\exists p'_1, s'_1. [\partial_H(p'' \parallel_\gamma s'') \xrightarrow{\sharp a} \partial_H(p'_1 \parallel_\gamma s'_1) \xleftarrow{\vec{\nu}} \partial_H(p_1 \parallel_\gamma s_1)]$$

which further implies that,

$$p'' \xrightarrow{!a} p'_1 \quad (\text{from SOS rule of } \parallel_\gamma \text{ and } \sharp a \in I_P^1). \quad (7)$$

Furthermore, from the hypothesis we have the transition $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p'' \parallel_\gamma s'')$ and $(\partial_H(p \parallel_\gamma s), \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)) \in \Phi$. By applying result of Proposition 1 in the above transition $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p'' \parallel_\gamma s'')$ we get $p' \xrightarrow{f_i(\mu)} p''$. Thus,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau^*} \nabla(p'' \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s).$$

From Equation 7 we get,

$$\nabla(p'' \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \nu \oplus \sharp a] \parallel_{\gamma'} s).$$

Using C2 and the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}, \sharp a} \partial_H(p'_1 \parallel_\gamma s'_1)$ we infer that,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \nu \oplus \sharp a] \parallel_{\gamma'} s) \right) \in \Phi.$$

And from the transition $s \xrightarrow{?a} s_1$ derived in Equation 2 (Page 30) we get,

$$\nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \nu \oplus \sharp a] \parallel_{\gamma'} s) \xrightarrow{?a} \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s_1).$$

By substituting $p = p_1, s = s_1, p' = p'_1, s' = s'_1, \nu = \nu, \mu = \varepsilon$ in C2 and from the transition $\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{\nu}} \partial_H(p'_1 \parallel_\gamma s'_1)$ we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

T3. Let $q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \wedge \tau = \tau_{\sharp a}(\sharp a) \wedge \sharp a \in I_P^2$. Then from the construction of Φ relation we know that,

$$\begin{aligned} q_a &\equiv \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s), \\ q_c &\equiv \partial_H(p \parallel_\gamma s). \end{aligned}$$

From the transition $q_a \xrightarrow{\tau} q'_a$ and the abstraction of action $\sharp a \in I_P^2$, i.e. $\tau = \tau_{\sharp a}(\sharp a)$ we know that $\sharp a \in' \mu$. Furthermore from the semantics of $\parallel_{\gamma'}$ and consumption of action $\sharp a$ by plant we infer that,

$$q'_a \equiv \nabla(p'_1 \parallel_{\gamma'} B[\mu \ominus \sharp a, \nu] \parallel_{\gamma'} s).$$

Then the transition $q_a \xrightarrow{\tau} q'_a$ will be of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s)$. And from the SOS rule of $\parallel_{\gamma'}$ and the assumption that $\dagger a \in I_P^?$, we get,

$$p' \xrightarrow{\dagger a} p'_1. \quad (8)$$

Before showing that the state q_c is Φ -related to q'_a , we need to retrieve the relation between free process variables $\partial_H(p \parallel_{\gamma} s)$ and $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ using conditions from the definition of Φ . So applying case distinction on the structure of μ and ν we get following cases.

- (a) when $\mu = \{\dagger a\}$ and $\nu = \varepsilon$. Then the transition $q_a \xrightarrow{\tau} q'_a$ will be of the form $\nabla(p' \parallel_{\gamma'} B[\{\dagger a\}, \varepsilon] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s)$. Since the input bag contains a single element $\dagger a$, so we know that,

$$\exists s'. [\nabla(p' \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s') \xrightarrow{\dagger a} \nabla(p' \parallel_{\gamma'} B[\{\dagger a\}, \varepsilon] \parallel_{\gamma'} s)]$$

which further implies that,

$$s' \xrightarrow{!a} s \quad (\text{SOS rule of } \parallel_{\gamma'}.)$$

Moreover from the transition $p' \xrightarrow{\dagger a} p'_1$ (Equation 8) and above transition we get,

$$\partial_H(p' \parallel_{\gamma'} s') \xrightarrow{\dagger a} \partial_H(p'_1 \parallel_{\gamma} s).$$

Thus from the condition C3 and the above transition we infer that,

$$\left(\partial_H(p'_1 \parallel_{\gamma} s), \nabla(p' \parallel_{\gamma'} B[\{\dagger a\}, \varepsilon] \parallel_{\gamma'} s) \right) \in \Phi.$$

And finally from C1 we conclude that,

$$\left(\partial_H(p'_1 \parallel_{\gamma} s), \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s) \right) \in \Phi.$$

- (b) when $\mu = \{\dagger a\}$ and $\nu \neq \varepsilon$. Similar as case T3.d.

- (c) when $\dagger a \in' \mu$ and $\nu = \varepsilon$. Similar as case T3.d.

- (d) when $\dagger a \in' \mu$ and $\nu \neq \varepsilon$. Then the transition $q_a \xrightarrow{\tau} q'_a$ will be of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s)$. Note that the states q_c and q_a can be Φ -related either by C4 or C5.

- i. Either $(q_c, q_a) \in \Phi$ by C4. Then, $\exists p'', s', s''. [\partial_H(p' \parallel_{\gamma'} s') \xleftarrow{\vec{v}} \partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_{\gamma} s)]$. Applying generalised diamond property (Lemma 1) at state $\partial_H(p'' \parallel_{\gamma} s'')$ we get,

$$\exists p_2, s_2. [\partial_H(p' \parallel_{\gamma'} s') \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_{\gamma} s_2) \xleftarrow{\vec{v}} \partial_H(p \parallel_{\gamma} s)].$$

Then there exists $p''_1, s''_1, p''_2, s''_2$ such that the above transition $\partial_H(p' \parallel_{\gamma'} s') \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_{\gamma} s_2)$ will be of the following form,

$$\partial_H(p' \parallel_{\gamma'} s') \xrightarrow{\vec{\mu}_1} \partial_H(p''_1 \parallel_{\gamma} s''_1) \xrightarrow{\dagger a} \partial_H(p''_2 \parallel_{\gamma} s''_2) \xrightarrow{\vec{\mu}_2} \partial_H(p_2 \parallel_{\gamma} s_2)$$

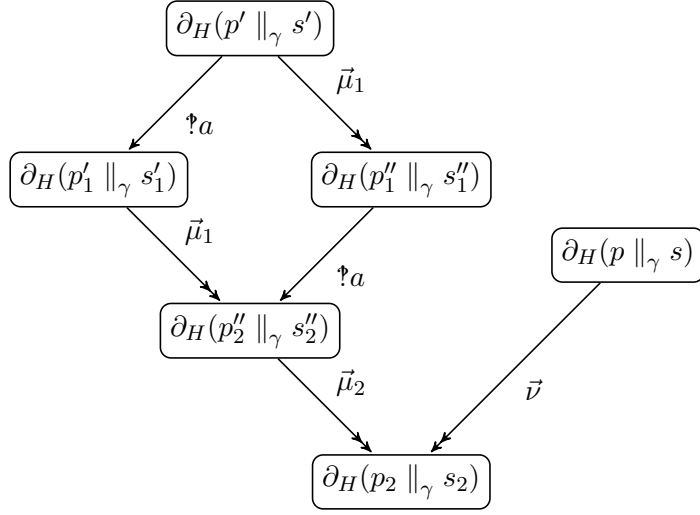


Figure 13: Proof in Subcase C3.d.i.

with $\vec{\mu}_1. \dagger a. \vec{\mu}_2$ being a sequence over the multiset μ . Thus we have,

$$\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}_1. \dagger a} \partial_H(p''_2 \parallel_\gamma s''_2).$$

But, the transition $p' \xrightarrow{\dagger a} p'_1$ (Equation 8, Page 36) is given, and from the result of Lemma 2 we know that (see Figure 13),

$$\exists s'_1. [\partial_H(p' \parallel_\gamma s') \xrightarrow{\dagger a} \partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}_1} \partial_H(p''_2 \parallel_\gamma s''_2)].$$

Hence,

$$\partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}_1. \vec{\mu}_2} \partial_H(p_2 \parallel_\gamma s_2).$$

Applying the above derived facts in the asynchronous closed loop system at state $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ such that,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

Moreover from the transition $p' \xrightarrow{\dagger a} p'_1$ (Equation 8, Page 36) we get,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s).$$

Note that $\vec{\mu}_1. \vec{\mu}_2$ is a sequence over the multiset $\mu \ominus \dagger a$. By substituting $p = p, s = s, p'' = p_2, s'' = s_2, p' = p'_1, s' = s'_1, \mu = \mu \ominus \dagger a, \nu = \nu$ in C5 and from the transitions $\partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}_1. \vec{\mu}_2} \partial_H(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_H(p \parallel_\gamma s)$ we infer that,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p'_1 \parallel_{\gamma'} B[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

ii. Or $(q_c, q_a) \in \Phi$ by C5. Then

$$\exists p'', s', s''. [\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s')].$$

Moreover there exists p'_1, s'_1, p''_2, s''_2 such that the above given transition $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p'' \parallel_\gamma s'')$ is of the following form,

$$\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}_1} \partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\dagger a} \partial_H(p''_2 \parallel_\gamma s''_2) \xrightarrow{\vec{\mu}_2} \partial_H(p'' \parallel_\gamma s'')$$

with $\vec{\mu}_1. \dagger a. \vec{\mu}_2$ being a sequence over the multiset μ . Thus we have,

$$\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}_1. \dagger a} \partial_H(p''_2 \parallel_\gamma s''_2).$$

Using the transition $p' \xrightarrow{?a} p'_1$ (Equation 8 in Page 36), and from the result of Lemma 2 we know that,

$$\exists s'_1. [\partial_H(p' \parallel_\gamma s') \xrightarrow{?a} \partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}_1} \partial_H(p''_2 \parallel_\gamma s''_2)].$$

Hence,

$$\partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}_1. \vec{\mu}_2} \partial_H(p'' \parallel_\gamma s'').$$

Applying the above derived facts in the asynchronous closed loop system at state $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ such that,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \right) \in \Phi \quad (\text{hypothesis.})$$

Again from the transition $p' \xrightarrow{?a} p'_1$ (Equation 8, Page 36) we infer that,

$$\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s).$$

Note that $\vec{\mu}_1. \vec{\mu}_2$ is a sequence over the multiset $\mu \ominus \dagger a$. By substituting $p = p, s = s, p'' = p'', s'' = s'', p' = p'_1, s' = s'_1, \mu = \mu \ominus \dagger a, \nu = \nu$ in C5 and from the transitions $\partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}_1. \vec{\mu}_2} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\nu}} \partial_H(p \parallel_\gamma s)$ we infer that,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p'_1 \parallel_{\gamma'} B[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

T4. Let $q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \wedge \tau = \tau_{\dagger a}(\dagger a) \wedge \dagger a \in I_P^!$. Then from the construction of Φ relation we know that,

$$\begin{aligned} q_a &\equiv \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s), \\ q_c &\equiv \partial_H(p \parallel_\gamma s). \end{aligned}$$

The transition $q_a \xrightarrow{\tau} q'_a$ with $\dagger a \in I_P^!$ implies that plant is increasing the contents of its output bag content ν by $\dagger a$. Thus from the semantics of $\parallel_{\gamma'}$ we know that,

$$q'_a \equiv \nabla(p'_1 \parallel_{\gamma'} B[\mu, \nu \oplus \dagger a] \parallel_{\gamma'} s).$$

Then the transition $q_a \xrightarrow{\tau} q'_a$ will be of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\mu, \nu \oplus \dagger a] \parallel_{\gamma'} s)$. Furthermore from the SOS rule of $\parallel_{\gamma'}$ and the assumption that $\dagger a \in I_P^!$ we know that,

$$p' \xrightarrow{\dagger a} p'_1. \quad (9)$$

Before showing that the state q_c is Φ -related to q'_a , we need to retrieve the relation between free process variables $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ and $\partial_H(p \parallel_{\gamma} s)$ using conditions given in the definition of Φ . So applying case distinction on the structure of μ and ν we get following cases.

- (a) when $\mu = \varepsilon$ and $\nu = \varepsilon$. Then the states q_c, q_a can be Φ -related only by C1, because of the assumption that $\mu = \varepsilon$ and $\nu = \varepsilon$. Thus, $p' = p$ and by hypothesis we have,

$$\left(\partial_H(p \parallel_{\gamma} s), \nabla(p \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s) \right) \in \Phi.$$

Moreover from the transition $p \xrightarrow{!a} p'_1$ (in Equation 9) we get,

$$\nabla(p \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \{!a\}] \parallel_{\gamma'} s).$$

Also from the condition of well posedness we know that,

$$\exists s_1. [s \xrightarrow{?a} s_1].$$

And from SOS rule of \parallel_{γ} , Equation 9, and the above equation we infer that,

$$\partial_H(p \parallel_{\gamma} s) \xrightarrow{?a} \partial_H(p'_1 \parallel_{\gamma} s_1).$$

Substituting $p' = p'_1, s' = s_1, p = p, s = s, \nu = \{!a\}, \mu = \varepsilon$ in C2 and from the above transition $\partial_H(p \parallel_{\gamma} s) \xrightarrow{?a} \partial_H(p'_1 \parallel_{\gamma} s_1)$ we infer that,

$$\left(\partial_H(p \parallel_{\gamma} s), \nabla(p'_1 \parallel_{\gamma'} B[\varepsilon, \{!a\}] \parallel_{\gamma'} s) \right) \in \Phi.$$

- (b) when $\mu = \varepsilon$ and $\nu \neq \varepsilon$. Similar as Case T4.d.
(c) when $\mu \neq \varepsilon$ and $\nu = \varepsilon$. Similar as Case T4.d.
(d) when $\mu \neq \varepsilon$ and $\nu \neq \varepsilon$. Then the transition $q_a \xrightarrow{\tau} q'_a$ is of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \nabla(p'_1 \parallel_{\gamma'} B[\mu, \nu \oplus !a] \parallel_{\gamma'} s)$. Note that the states q_c, q_a can be Φ -related either by C4 or C5.

- i. Either $(q_c, q_a) \in \Phi$ by C4. Then, $\exists p'', s', s''. [\partial_H(p' \parallel_{\gamma} s') \xleftarrow{\vec{v}} \partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_{\gamma} s)]$. From the transition $p' \xrightarrow{!a} p'_1$ (Equation 9) and applying well posedness condition at state $\partial_H(p' \parallel_{\gamma} s')$ we get,

$$\exists s'_1. [s' \xrightarrow{?a} s'_1] \quad (\text{well posedness condition.})$$

And using SOS rule of \parallel_{γ} we get,

$$\partial_H(p' \parallel_{\gamma} s') \xrightarrow{?a} \partial_H(p'_1 \parallel_{\gamma} s'_1). \quad (10)$$

Recall the given transition $\partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{v}} \partial_H(p' \parallel_{\gamma} s')$, and from the above transition we infer that,

$$\partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{v}.?a} \partial_H(p'_1 \parallel_{\gamma} s'_1).$$

Note that $\vec{\nu}. \mathfrak{!}a$ is a sequence over the multiset $\nu \oplus \mathfrak{!}a$. Substituting $p'' = p''$, $s'' = s''$, $p' = p'_1$, $s' = s'_1$, $p = p$, $s = s$, $\mu = \mu$, $\nu = \nu \oplus \mathfrak{!}a$ in C4 and from the transitions $\partial_H(p'_1 \parallel_\gamma s'_1) \xleftarrow{\vec{\nu}. \mathfrak{!}a} \partial_H(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_\gamma s)$ we infer that,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p'_1 \parallel_{\gamma'} B[\mu, \nu \oplus \mathfrak{!}a] \parallel_{\gamma'} s) \right) \in \Phi.$$

ii. Or $(q_c, q_a) \in \Phi$ by C5. Then,

$$\exists p'', s', s''. [\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s')].$$

Using the same reasoning as in the previous subcase we can derive the Equation 10 i.e.,

$$\partial_H(p' \parallel_\gamma s') \xrightarrow{\mathfrak{!}a} \partial_H(p'_1 \parallel_\gamma s'_1).$$

Recall from the hypothesis we have the transition $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p'' \parallel_\gamma s'')$. So applying generalised diamond property (Lemma 1) at state $\partial_H(p' \parallel_\gamma s')$ we get,

$$\exists p_2, s_2. [\partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_\gamma s_2) \xleftarrow{\mathfrak{!}a} \partial_H(p'' \parallel_\gamma s'')].$$

Also from the hypothesis we have the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_H(p'' \parallel_\gamma s'')$, and from transitivity of the transition relation we get,

$$\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}. \mathfrak{!}a} \partial_H(p_2 \parallel_\gamma s_2).$$

Note that $\vec{\nu}. \mathfrak{!}a$ is a sequence over the multiset $\nu \oplus \mathfrak{!}a$. Now substituting $p = p$, $s = s$, $p' = p'_1$, $s' = s'_1$, $p'' = p''$, $s'' = s''$, $\mu = \mu$, $\nu = \nu \oplus \mathfrak{!}a$ in C5 and from the transitions $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{\nu}. \mathfrak{!}a} \partial_H(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\mu}} \partial_H(p'_1 \parallel_\gamma s'_1)$ we infer that,

$$\left(\partial_H(p \parallel_\gamma s), \nabla(p'_1 \parallel_{\gamma'} B[\mu, \nu \oplus \mathfrak{!}a] \parallel_{\gamma'} s) \right) \in \Phi.$$

T5. Let $q_a \xrightarrow{\mathfrak{!}a} q'_a \wedge (q_c, q_a) \in \Phi \wedge \mathfrak{!}a \in I_P^?$. Then from the construction of Φ relation we know that,

$$\begin{aligned} q_a &\equiv \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s), \\ q_c &\equiv \partial_H(p \parallel_\gamma s). \end{aligned}$$

From the transition $q_a \xrightarrow{\mathfrak{!}a} q'_a$ we know that the supervisor component s in the state q_a will increase the queue content μ by performing action $\mathfrak{!}a$. Furthermore, from the semantics of $\parallel_{\gamma'}$ we know that,

$$q'_a \equiv \nabla(p' \parallel_{\gamma'} B[\mu \oplus \mathfrak{!}a, \nu] \parallel_{\gamma'} s_1).$$

Then the transition $q_a \xrightarrow{\mathfrak{!}a} q'_a$ will be of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\mathfrak{!}a} \nabla(p' \parallel_{\gamma'} B[\mu \oplus \mathfrak{!}a, \nu] \parallel_{\gamma'} s_1)$. Thus, from the semantics of $\parallel_{\gamma'}$ and the assumption that $\mathfrak{!}a \in I_P^?$ we know that,

$$s \xrightarrow{\mathfrak{!}a} s_1. \tag{11}$$

Before we show that the state q_c perform the action $\dagger a$ we need to retrieve the relation between free process variables $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ and $\partial_H(p \parallel_{\gamma} s)$. So applying case distinction on the structure of μ and ν .

- (a) when $\mu = \varepsilon$ and $\nu = \varepsilon$. Then from C1 we have $p' = p$ and the transition $q_a \xrightarrow{\dagger a} q'_a$ is of the form $\nabla(p \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s) \xrightarrow{\dagger a} \nabla(p \parallel_{\gamma'} B[\{\dagger a\}, \varepsilon] \parallel_{\gamma'} s_1)$. Moreover from the transition $s \xrightarrow{!a} s_1$ (Equation 11) and by the condition of well posedness we know that,

$$\exists p_1. [p \xrightarrow{?a} p_1] \quad (\text{well posedness condition.})$$

And using SOS rule of \parallel_{γ} we get,

$$\partial_H(p \parallel_{\gamma} s) \xrightarrow{\dagger a} \partial_H(p_1 \parallel_{\gamma} s_1).$$

From the above transition and condition C3 we infer that,

$$\left(\partial_H(p_1 \parallel_{\gamma} s_1), \nabla(p \parallel_{\gamma'} B[\{\dagger a\}, \varepsilon] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

- (b) when $\mu = \varepsilon$ and $\nu \neq \varepsilon$. Similar as Case T5.d.
(c) when $\mu \neq \varepsilon$ and $\nu = \varepsilon$. Similar as Case T5.d.
(d) when $\mu \neq \varepsilon$ and $\nu \neq \varepsilon$. Note that the states q_c, q_a can be Φ -related either by C4 or C5.

- i. Either $(q_c, q_a) \in \Phi$ by C4. Then, $\exists p'', s', s''. [\partial_H(p' \parallel_{\gamma} s') \xleftarrow{\vec{v}} \partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_{\gamma} s)]$. And from the transition $s \xrightarrow{!a} s_1$ (Equation 11, Page 40),

$$\exists p_1. [p \xrightarrow{?a} p_1] \quad (\text{Well posedness condition.})$$

Using SOS rule of \parallel_{γ} we get,

$$\partial_H(p \parallel_{\gamma} s) \xrightarrow{\dagger a} \partial_H(p_1 \parallel_{\gamma} s_1). \quad (12)$$

Recall that the transition $\partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_{\gamma} s)$ is given. Thus,

$$\partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{\mu}. \dagger a} \partial_H(p_1 \parallel_{\gamma} s_1).$$

Note that $\vec{\mu}. \dagger a$ is a sequence over the multiset $\mu \oplus \dagger a$. Substituting $p'' = p, s'' = s'', s' = s', s' = s', p = p_1, s = s_1, \mu = \mu \oplus \dagger a, \nu = \nu$ in C4 and from the transitions $\partial_H(p' \parallel_{\gamma} s') \xleftarrow{\vec{v}} \partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{\mu}. \dagger a} \partial_H(p_1 \parallel_{\gamma} s_1)$ we infer that,

$$\left(\partial_H(p_1 \parallel_{\gamma} s_1), \nabla(p' \parallel_{\gamma'} B[\mu \oplus \dagger a, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

ii. Or $(q_c, q_a) \in \Phi$ by C5. Then,

$$\exists p'', s', s''. [\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s')].$$

Note that using the same reasoning as in the above subcase we can derive Equation 12, i.e.

$$\exists p_1. [\partial_H(p \parallel_\gamma s) \xrightarrow{?a} \partial_H(p_1 \parallel_\gamma s_1)].$$

Now recall the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_H(p'' \parallel_\gamma s'')$ from the hypothesis and by applying generalised diamond property (Lemma 1) at state $\partial_H(p \parallel_\gamma s)$ we get,

$$\exists p_2, s_2. [\partial_H(p'' \parallel_\gamma s'') \xrightarrow{?a} \partial_H(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{v}} \partial_H(p_1 \parallel_\gamma s_1)].$$

Furthermore, from the hypothesis we have the transition $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p'' \parallel_\gamma s'')$ and by combining with the transition $\partial_H(p'' \parallel_\gamma s'') \xrightarrow{?a} \partial_H(p_2 \parallel_\gamma s_2)$ we get,

$$\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}.?a} \partial_H(p_2 \parallel_\gamma s_2).$$

Note that $\vec{\mu}.?a$ is a sequence over the multiset $\mu \oplus ?a$. Substituting $p = p_1, s = s_1, p'' = p_2, s'' = s_2, p' = p', s' = s', \mu = \mu \oplus ?a, \nu = \nu$ in C5 and from the transitions $\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}} \partial_H(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\mu}.?a} \partial_H(p' \parallel_\gamma s')$ we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p' \parallel_{\gamma'} B[\mu \oplus ?a, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

T6. Let $q_a \xrightarrow{?a} q'_a \wedge (q_c, q_a) \in \Phi \wedge ?a \in I_P^!$. Then from the construction of Φ relation we know that,

$$\begin{aligned} q_a &\equiv \nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s), \\ q_c &\equiv \partial_H(p \parallel_\gamma s). \end{aligned}$$

From the transition $q_a \xrightarrow{?a} q'_a$ we know that the supervisor component s in the state q_a will decrease the bag content ν by performing action $?a$. So assume that $?a \in' \nu$. Moreover from the semantics of $\parallel_{\gamma'}$ we know that,

$$q'_a \equiv \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus ?a] \parallel_{\gamma'} s_1).$$

Then transition $q_a \xrightarrow{?a} q'_a$ is of the form $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{?a} \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus ?a] \parallel_{\gamma'} s_1)$. Using the assumption that $?a \in I_P^!$, the above transition implies that,

$$s \xrightarrow{?a} s_1. \tag{13}$$

Before we show that the state q_c perform the action $?a$ we need to retrieve the relation between free process variables $\nabla(p' \parallel_{\gamma'} B[\mu, \nu] \parallel_{\gamma'} s)$ and $\partial_H(p \parallel_\gamma s)$. So by applying case distinction on the structure of μ and ν we get following cases.

- (a) when $\mu = \varepsilon$ and $\nu = \{\!|a\!\}$. Then the state q_a is of the form $\nabla(p' \parallel_{\gamma'} B[\varepsilon, \{\!|a\!\}] \parallel_{\gamma'} s)$ and the transition $q_a \xrightarrow{\tau} q'_a$ will be of the form

$$\nabla(p' \parallel_{\gamma'} B[\varepsilon, \{\!|a\!\}] \parallel_{\gamma'} s) \xrightarrow{\!|a\!\} \nabla(p' \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s_1).$$

Moreover (q_c, q_a) can be Φ -related only by condition C2 because of the assumption $\mu = \varepsilon$ and $\nu = \{\!|a\!\}$. Thus we have the following transition,

$$\partial_H(p \parallel_{\gamma} s) \xrightarrow{\!|a\!\} \partial_H(p' \parallel_{\gamma} s_1).$$

And from condition C1 we know that,

$$\left(\partial_H(p' \parallel_{\gamma} s_1), \nabla(p' \parallel_{\gamma'} B[\varepsilon, \varepsilon] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

- (b) when $\mu = \varepsilon$ and $\!|a \in' \nu$. Similar as Case T6.d.
(c) when $\mu \neq \varepsilon$ and $\nu = \{\!|a\!\}$. Similar as Case T6.d.
(d) when $\mu \neq \varepsilon$ and $\!|a \in' \nu$. Note that the states q_c, q_a can be Φ -related either by C4 or C5. So we get following two cases.

- i. Either $(q_c, q_a) \in \Phi$ by C4. Then, $\exists p'', s', s''. [\partial_H(p' \parallel_{\gamma} s') \xleftarrow{\vec{\nu}} \partial_H(p'' \parallel_{\gamma} s'') \xrightarrow{\vec{\mu}} \partial_H(p \parallel_{\gamma} s)]$. By applying generalised diamond property (Lemma 1) at state $\partial_H(p'' \parallel_{\gamma} s'')$ we get,

$$\exists p_2, s_2. [\partial_H(p' \parallel_{\gamma} s') \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_{\gamma} s_2) \xleftarrow{\vec{\nu}} \partial_H(p \parallel_{\gamma} s)].$$

Then there exists p'_1, s'_1, p'_2, s'_2 such that the transition $\partial_H(p \parallel_{\gamma} s) \xrightarrow{\vec{\nu}} \partial_H(p_2 \parallel_{\gamma} s_2)$ will be of the following form,

$$\partial_H(p \parallel_{\gamma} s) \xrightarrow{\vec{\nu}_1} \partial_H(p'_1 \parallel_{\gamma} s'_1) \xrightarrow{\!|a\!\} \partial_H(p'_2 \parallel_{\gamma} s'_2) \xrightarrow{\vec{\nu}_2} \partial_H(p_2 \parallel_{\gamma} s_2)$$

such that $\vec{\nu}_1. \!|a. \vec{\nu}_2$ is a sequence over the multiset ν . By applying result of Lemma 3 on the transitions $\partial_H(p \parallel_{\gamma} s) \xrightarrow{\nu_1. \!|a} \partial_H(p'_2 \parallel_{\gamma} s'_2)$ and $s \xrightarrow{\!|a} s_1$ (Equation 13) we know that,

$$\exists p_1. [\partial_H(p \parallel_{\gamma} s) \xrightarrow{\!|a\!\} \partial_H(p_1 \parallel_{\gamma} s_1) \xrightarrow{\vec{\nu}_1} \partial_H(p'_2 \parallel_{\gamma} s'_2)].$$

Thus,

$$\partial_H(p \parallel_{\gamma} s) \xrightarrow{\!|a\!\} \partial_H(p_1 \parallel_{\gamma} s_1).$$

Now we need to show that state $\partial_H(p_1 \parallel_{\gamma} s_1)$ is Φ -related to q'_a . Recall that

$\partial_H(p'_2 \parallel_{\gamma} s'_2) \xrightarrow{\vec{\nu}_2} \partial_H(p_2 \parallel_{\gamma} s_2)$, and from above derived transition $\partial_H(p_1 \parallel_{\gamma} s_1) \xrightarrow{\vec{\nu}_1} \partial_H(p'_2 \parallel_{\gamma} s'_2)$ we have,

$$\partial_H(p_1 \parallel_{\gamma} s_1) \xrightarrow{\vec{\nu}_1. \vec{\nu}_2} \partial_H(p_2 \parallel_{\gamma} s_2).$$

Note that $\vec{\nu}_1. \vec{\nu}_2$ is a sequence over the multiset $\nu \ominus \!|a$. Substituting $p'' = p_2, s'' = s_2, p' = p'_1, s' = s'_1, p = p_1, s = s_1, \mu = \mu, \nu = \nu \ominus \!|a$ in C5 and from the transitions $\partial_H(p' \parallel_{\gamma} s') \xrightarrow{\vec{\mu}} \partial_H(p_2 \parallel_{\gamma} s_2) \xleftarrow{\vec{\nu}_1. \vec{\nu}_2} \partial_H(p_1 \parallel_{\gamma} s_1)$ we infer that,

$$\left(\partial_H(p_1 \parallel_{\gamma} s_1), \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus \!|a] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

ii. Or $(q_c, q_a) \in \Phi$ by C5. Then,

$$\exists p'', s', s''. [\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_H(p' \parallel_\gamma s')].$$

Then there exists p'_1, s'_1, p'_2, s'_2 such that the transition $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_H(p'' \parallel_\gamma s'')$ will be of the following form,

$$\partial_H(p'' \parallel_\gamma s) \xrightarrow{\vec{v}_1} \partial_H(p'_1 \parallel_\gamma s'_1) \xrightarrow{?a} \partial_H(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{v}_2} \partial_H(p'' \parallel_\gamma s'')$$

such that $\vec{v}_1. ?a. \vec{v}_2$ is a sequence over the multiset ν . By result of Lemma 3 and the transitions $\partial_H(p \parallel_\gamma s) \xrightarrow{\vec{v}_1. ?a} \partial_H(p'_2 \parallel_\gamma s'_2)$ and $s \xrightarrow{?a} s_1$ (Equation 13) we know that,

$$\exists p_1. [\partial_H(p \parallel_\gamma Xs) \xrightarrow{?a} \partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1} \partial_H(p'_2 \parallel_\gamma s'_2)]$$

Now we need to show that the state $\partial_H(p_1 \parallel_\gamma s_1)$ is Φ -related to q'_a . Recall that $\partial_H(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{v}_2} \partial_H(p'' \parallel_\gamma s'')$, and from above derived transition $\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1} \partial_H(p'_2 \parallel_\gamma s'_2)$ we have,

$$\partial_H(p_1 \parallel_\gamma s_1) \xrightarrow{\vec{v}_1. \vec{v}_2} \partial_H(p' \parallel_\gamma s').$$

Note that $\vec{v}_1. \vec{v}_2$ is a sequence over the multiset $\nu \ominus ?a$. Substituting $p'' = p', s'' = s', p' = p', s' = s', p = p_1, s = s_1, \mu = \mu, \nu = \nu \ominus ?a$ in C5 and from the transitions $\partial_H(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_H(p'' \parallel_\gamma s'') \xleftarrow{\vec{v}_1. \vec{v}_2} \partial_H(p_1 \parallel_\gamma s_1)$ we infer that,

$$\left(\partial_H(p_1 \parallel_\gamma s_1), \nabla(p' \parallel_{\gamma'} B[\mu, \nu \ominus ?a] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

□

B Proof Of Theorem 2

Proof. Let p, p', s be free process variables. Let μ, ν be two free variables representing the contents of an input and output bag of P , respectively. Recall the relation Φ defined in the proof of Theorem 1 and define a relation $\widehat{\Phi}$ as follows,

$$\begin{aligned} \widehat{\Phi} \triangleq \Phi \cup \left\{ (\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)) \mid \right. \\ \left. \exists s_1, \vec{\sigma} \in \bar{\alpha}(S)^*. \left[(\partial_\diamond(p \parallel_\gamma s_1), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1)) \in \Phi \wedge \right. \right. \\ \left. \left. \partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s) \right] \vee \right\} \quad (C6) \end{aligned}$$

$$\begin{aligned} \exists p_1, p'_1, s_1, s', \vec{\sigma} \in \bar{\alpha}(P)^*. \left[(\partial_\diamond(p_1 \parallel_\gamma s), \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)) \in \Phi \wedge \right. \\ \left. \partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s') \wedge \partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s) \right] \}. \quad (C7) \end{aligned}$$

Next we need to show that Φ is a witnessing branching bisimulation relation. Let q_c, q_a denote the following processes $\partial_\diamond(p \parallel_\gamma s)$ and $\widehat{\nabla}(p \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$, respectively. We get 10 transfer conditions in total (shown in Table 4) based on similar reasoning used to get the Table 3 (Page 28) in the proof of Theorem 1.

Table 4: Proof structure of Theorem 2.

Case No.	Hypothesis
S1	$q_c \xrightarrow{\dagger a} \bar{q}_c \wedge (q_c, q_a) \in \Phi \wedge \dagger a \in I_P^? \Rightarrow$ $\exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{\dagger a} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi.]$
S2	$q_c \xrightarrow{\dagger a} q'_c \wedge (q_c, q_a) \in \Phi \wedge \dagger a \in I_P^! \Rightarrow$ $\exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{\dagger a} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi.]$
S3	$q_c \xrightarrow{x} q'_c \wedge (q_c, q_a) \in \Phi \wedge x \in \bar{\alpha}(P) \Rightarrow$ $\exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{x} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi.]$
S4	$q_c \xrightarrow{x} q'_c \wedge (q_c, q_a) \in \Phi \wedge x \in \bar{\alpha}(S) \Rightarrow$ $\exists q'_a, q''_a. [q_a \xrightarrow{\tau^*} q'_a \xrightarrow{x} q''_a \wedge (q_c, q'_a), (q'_c, q''_a) \in \Phi.]$
S5	$q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \wedge \tau = \tau_{\dagger a}(\dagger a) \wedge \dagger a \in I_P^? \Rightarrow (q_c, q'_a) \in \Phi.$
S6	$q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \Phi \wedge \tau = \tau_{\dagger a}(\dagger a) \wedge \dagger a \in I_P^! \Rightarrow (q_c, q'_a) \in \Phi.$
S7	$q_a \xrightarrow{\dagger a} q'_a \wedge (q_c, q_a) \in \Phi \wedge \dagger a \in I_P^? \Rightarrow \exists q'_c. [q_c \xrightarrow{\dagger a} q'_c \wedge (q'_c, q'_a) \in \Phi.]$
S8	$q_a \xrightarrow{\dagger a} q'_a \wedge (q_c, q_a) \in \Phi \wedge \dagger a \in I_P^! \Rightarrow \exists q'_c. [q_c \xrightarrow{\dagger a} q'_c \wedge (q'_c, q'_a) \in \Phi.]$
S9	$q_a \xrightarrow{x} q'_a \wedge (q_c, q_a) \in \Phi \wedge x \in \bar{\alpha}(P) \Rightarrow \exists q'_c. [q_c \xrightarrow{x} q'_c \wedge (q'_c, q'_a) \in \Phi.]$
S10	$q_a \xrightarrow{x} q'_a \wedge (q_c, q_a) \in \Phi \wedge x \in \bar{\alpha}(S) \Rightarrow \exists q'_c. [q_c \xrightarrow{x} q'_c \wedge (q'_c, q'_a) \in \Phi.]$

S1 Let $q_c \xrightarrow{\dagger a} q'_c \wedge (q_c, q_a) \in \widehat{\Phi} \wedge \dagger a \in I_P^?$. From construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned}
q_c &\equiv \partial_\diamond(p \parallel_\gamma s), \\
q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} S).
\end{aligned}$$

Furthermore, from the semantics of \parallel_γ and $\dagger a \in I_P^?$ we know that,

$$q'_c \equiv \partial_\diamond(\bar{p} \parallel_\gamma \bar{s}) \wedge p \xrightarrow{?a} \bar{p} \wedge s \xrightarrow{!a} \bar{s}. \quad (14)$$

Before showing that the state q_a can perform action $\dagger a$, we need to retrieve the relation between process terms $\partial_\diamond(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} S)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. when $\mu, \nu = \varepsilon$.
2. when $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 is already proven in [5]. Here

we give the proof of the remaining cases. Note that from Equation 14 we have,

$$\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\dagger a} \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu \oplus \dagger a, \nu] \parallel_{\gamma'} \bar{s}).$$

In the upcoming cases we will show how to relate the states $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu \oplus \dagger a, \nu] \parallel_{\gamma'} \bar{s})$ and $\partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s})$.

(a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Then,

$$\exists s_1, \sigma \in \bar{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\bar{\sigma}} q_c \right] \quad (15)$$

where,

$$q_c^1 = \partial_{\diamond}(p \parallel_{\gamma} s_1), \quad q_a^1 = \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s'', s'. [\partial_{\diamond}(p' \parallel_{\gamma} s') \xleftarrow{\bar{v}} \partial_{\diamond}(p'' \parallel_{\gamma} s'') \xrightarrow{\bar{\mu}} \partial_{\diamond}(p \parallel_{\gamma} s_1)].$$

From Equation (14),(15) we know that, $\partial_{\diamond}(p \parallel_{\gamma} s) \xrightarrow{\dagger a} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s})$ and $\partial_{\diamond}(p \parallel_{\gamma} s_1) \xrightarrow{\bar{\sigma}} \partial_{\diamond}(p \parallel_{\gamma} s)$, respectively. And by Lemma 6 we get,

$$\exists \bar{s}_1. [\partial_{\diamond}(p \parallel_{\gamma} s_1) \xrightarrow{\dagger a} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1) \xrightarrow{\bar{\sigma}} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s})].$$

Thus, $\partial_{\diamond}(p'' \parallel_{\gamma} s'') \xrightarrow{\bar{\mu}, \dagger a} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1)$. And from C4 we get,

$$\left(\partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu \oplus \dagger a, \nu] \parallel_{\gamma'} \bar{s}_1) \right) \in \Phi. \quad (16)$$

Finally, using the Equation 16, the transition $\partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1) \xrightarrow{\sigma} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s})$ and the Condition C6 we conclude that,

$$\left(\partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu \oplus \dagger a, \nu] \parallel_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Then,

$$\exists p'', s'', s'. [\partial_{\diamond}(p \parallel_{\gamma} s_1) \xrightarrow{\bar{v}} \partial_{\diamond}(p'' \parallel_{\gamma} s'') \xleftarrow{\bar{\mu}} \partial_{\diamond}(p' \parallel_{\gamma} s')].$$

Applying the result of Lemma 6 on the transitions in $\partial_{\diamond}(p \parallel_{\gamma} s) \xrightarrow{\dagger a} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s})$ (hypothesis), $\partial_{\diamond}(p \parallel_{\gamma} s_1) \xrightarrow{\bar{\sigma}} \partial_{\diamond}(p \parallel_{\gamma} s)$ (Equation (15)) we get,

$$\exists \bar{s}_1. [\partial_{\diamond}(p \parallel_{\gamma} s_1) \xrightarrow{\dagger a} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1) \xrightarrow{\bar{\sigma}} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s})].$$

Applying the result of Lemma 5 on the transitions $\partial_{\diamond}(p \parallel_{\gamma} s_1) \xrightarrow{\bar{v}} \partial_{\diamond}(p'' \parallel_{\gamma} s'')$ and $\partial_{\diamond}(p \parallel_{\gamma} s_1) \xrightarrow{\dagger a} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1)$ we get,

$$\exists p_2, s_2. [\partial_{\diamond}(p'' \parallel_{\gamma} s'') \xrightarrow{\dagger a} \partial_{\diamond}(p_2 \parallel_{\gamma} s_2) \xleftarrow{\bar{v}} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1)].$$

Thus, from C5 we get,

$$\left(\partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu \oplus \mathfrak{?}a, \nu] \parallel_{\gamma'} \bar{s}_1) \right) \in \Phi. \quad (17)$$

Finally, from the Equation (17), the transition $\partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}_1) \xrightarrow{\bar{\sigma}} \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s})$ and by the Condition C6 we get,

$$\left(\partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu \oplus \mathfrak{?}a, \nu] \parallel_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

(b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Similar to the above case.

S2 Let $q_c \xrightarrow{\mathfrak{?}a} q'_c \wedge (q_c, q_a) \in \Phi \wedge \mathfrak{?}a \in I_P^1$. From construction of Φ we know that,

$$\begin{aligned} q_c &\equiv \partial_{\diamond}(p \parallel_{\gamma} s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} S). \end{aligned}$$

Furthermore, from the semantics of \parallel_{γ} and $\mathfrak{?}a \in I_P^1$ we know that,

$$q'_c \equiv \partial_{\diamond}(\bar{p} \parallel_{\gamma} \bar{s}) \wedge p \xrightarrow{\mathfrak{!}a} \bar{p} \wedge s \xrightarrow{\mathfrak{?}a} \bar{s}. \quad (18)$$

Before showing that the state q_a can perform action $\mathfrak{?}a$, we need to retrieve the relation between process terms $\partial_{\diamond}(p \parallel_{\gamma} s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 is already proven in [5]. Here we give the proof of the remaining cases. Note that the state q_a can perform action $\mathfrak{?}a$ only if $\mathfrak{?}a \in' \nu$. Otherwise the plant component p' in state q_a will have to increase the content ν of output bag by $\mathfrak{?}a$. In the following we assume that $\mathfrak{?}a \notin' \nu$; the proof for the other case when $\mathfrak{?}a \in' \nu$ can be constructed similarly.

(a) Either $(q_c, q_a) \in \Phi$ due to C6. Then,

$$\exists s_1, \sigma \in \bar{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\bar{\sigma}} q_c \right] \quad (19)$$

where,

$$q_c^1 = \partial_{\diamond}(p \parallel_{\gamma} s_1), \quad q_a^1 = \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s'', s'. [\partial_{\diamond}(p' \parallel_{\gamma'} s') \xleftarrow{\bar{\nu}} \partial_{\diamond}(p'' \parallel_{\gamma'} s'') \xrightarrow{\bar{\mu}} \partial_{\diamond}(p \parallel_{\gamma} s_1)].$$

And by Lemma 5 we get,

$$\exists p_2, s_2. [\partial_{\diamond}(p' \parallel_{\gamma'} s') \xrightarrow{\bar{\mu}} \partial_{\diamond}(p_2 \parallel_{\gamma} s_2) \xleftarrow{\bar{\nu}} \partial_{\diamond}(p \parallel_{\gamma} s_1)].$$

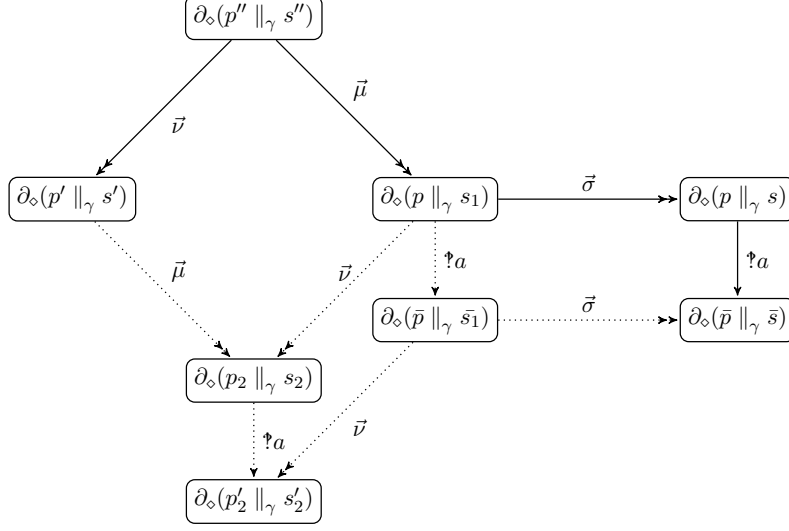


Figure 14: Proof in Subcase S2-2-a-i

And by applying Lemma 7 on the transitions $q_c \xrightarrow{\uparrow a} q'_c$ and $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s)$ (Equation (19)) we get,

$$\exists \bar{p}, \bar{s}_1. [\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\uparrow a} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})].$$

Again applying Lemma 5 on the transitions $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{v}} \partial_\diamond(p_2 \parallel_\gamma s_2)$ and $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\uparrow a} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1)$ we get (See Figure 14),

$$\exists p'_2, s'_2. [\partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\uparrow a} \partial_\diamond(p'_2 \parallel_\gamma s'_2) \xleftarrow{\vec{v}} \partial_\diamond(\bar{s} \parallel_\gamma \bar{s}_1)].$$

From the transition $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2)$ and the semantics of \parallel_γ we infer that, $p' \xrightarrow{f_i(\vec{\mu})} p_2$. Thus,

$$\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau^*} \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s).$$

From the Condition C2 and the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{v}} \partial_\diamond(p_2 \parallel_\gamma s_2)$ we get,

$$\left(\partial_\diamond(p \parallel_\gamma s_1), \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi. \quad (20)$$

From the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s)$, the Equation (20) and the Condition C6 with $\mu = \varepsilon$ we get,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

Furthermore, from the transition $\partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\uparrow a} \partial_\diamond(p'_2 \parallel_\gamma s'_2)$ and the semantics of \parallel_γ we get,

$$\widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau} \widehat{\nabla}(p'_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu \oplus \uparrow a] \parallel_{\gamma'} s).$$

From the Condition C2 and the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\nu}.\dagger a} \partial_\diamond(p'_2 \parallel_\gamma s'_2)$ we get,

$$\left(\partial_\diamond(p \parallel_\gamma s_1), \widehat{\nabla}(p'_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu \oplus \dagger a] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

Using the above fact, the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s)$ and the Condition C6 with $\mu = \varepsilon$ we have,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(p'_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu \oplus \dagger a] \parallel_{\gamma'} s) \right) \in \Phi.$$

Using the transition $s \xrightarrow{?a} \bar{s}$ (Equation (18)) and the semantics of $\parallel_{\gamma'}$ we get,

$$\widehat{\nabla}(p'_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu \oplus \dagger a] \parallel_{\gamma'} s) \xrightarrow{\dagger a} \widehat{\nabla}(p'_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} \bar{s}).$$

Applying the condition C2 with the transition $\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1) \xrightarrow{\vec{\nu}} \partial_\diamond(p'_2 \parallel_\gamma s'_2)$ we get,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1), \widehat{\nabla}(p'_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} \bar{s}_1) \right) \in \Phi.$$

Finally, by the above equation, the transition $\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})$ and the Condition C6 with $\mu = \varepsilon$ we conclude that,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}), \widehat{\nabla}(p'_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

ii. Or $(q_c, q_a) \in \Phi$ due to C5. Similar to the previous case.

(b) Or $(q_c, q_a) \in \Phi$ due to C7. Similar to the previous case.

S3 Let $q_c \xrightarrow{x} q'_c \wedge (q_c, q_a) \in \widehat{\Phi} \wedge x \in \bar{\alpha}(P)$. From construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\diamond(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} S). \end{aligned}$$

Furthermore, from the semantics of \parallel_γ and $x \in \bar{\alpha}(P)$ we know that,

$$q'_c \equiv \partial_\diamond(\bar{p} \parallel_\gamma s) \wedge p \xrightarrow{x} \bar{p}. \quad (21)$$

Before showing that the state q_a can perform action x , we need to retrieve the relation between process terms $\partial_\diamond(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 are special cases of C6 and C7. Here we prove the cases C6 and C7 only.

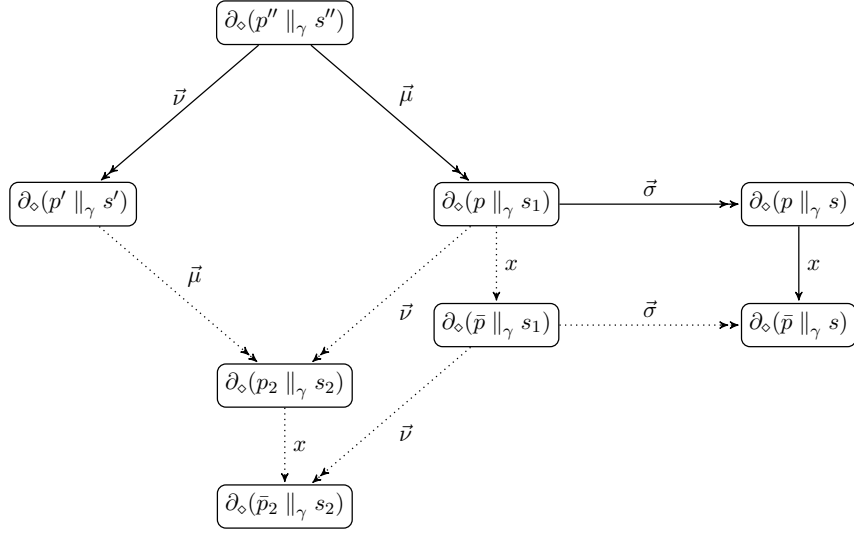


Figure 15: Proof in Subcase S3-2-a-i.

(a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Then,

$$\exists s_1, \sigma \in \bar{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\vec{\sigma}} q_c \right]$$

where,

$$q_c^1 = \partial_\circ(p \parallel_\gamma s_1), \quad q_a^1 = \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s'', s'. [\partial_\circ(p' \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\circ(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\circ(p \parallel_\gamma s_1)].$$

From Equation (21) and semantics of \parallel_γ we get, $\partial_\circ(p \parallel_\gamma s_1) \xrightarrow{x} \partial_\circ(\bar{p} \parallel_\gamma s_1)$. Applying Lemma 5 at the state $\partial_\circ(p \parallel_\gamma s_1)$ we get (See Figure 15),

$$\partial_\circ(\bar{p} \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\circ(\bar{p} \parallel_\gamma s).$$

Again applying Lemma 5 but at the state $\partial_\circ(p'' \parallel_\gamma s'')$ we get (See Figure 15),

$$\exists p_2, s_2. [\partial_\circ(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\circ(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_\circ(p \parallel_\gamma s_1)].$$

Again applying Lemma 5 but at the state $\partial_\circ(p \parallel_\gamma s_1)$ we get (See Figure 15),

$$\exists \bar{p}_2. [\partial_\circ(p_2 \parallel_\gamma s_2) \xrightarrow{x} \partial_\circ(\bar{p}_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_\circ(\bar{p} \parallel_\gamma s_1)].$$

From the transition $\partial_\circ(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\circ(p_2 \parallel_\gamma s_2)$ and the semantics of \parallel_γ we get $p' \xrightarrow{f_i(\vec{\mu})} p_2$. Now applying this fact at the state q_a we get,

$$\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau^*} \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s).$$

And from the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\nu}} \partial_\diamond(p_2 \parallel_\gamma s_2)$ and the Condition C2 we get,

$$\left(\partial_\diamond(p \parallel_\gamma s_1), \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

Using the above fact, the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s)$ and the Condition 6 with $\mu = \varepsilon$ we get,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

Moreover, from the transition $\partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{x} \partial_\diamond(\bar{p}_2 \parallel_\gamma s_2)$ and the semantics of \parallel_γ we get, $p_2 \xrightarrow{x} \bar{p}_2$. Thus, applying this fact at the state $\widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s_1)$ we get,

$$\widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s_1) \xrightarrow{x} \widehat{\nabla}(\bar{p}_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s_1).$$

And from C2 with the transition $\partial_\diamond(\bar{p} \parallel_\gamma s_1) \xrightarrow{\vec{\nu}} \partial_\diamond(\bar{p}_2 \parallel_\gamma s_2)$ we get,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma s_1), \widehat{\nabla}(\bar{p}_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

Finally from the above fact, the transition $\partial_\diamond(\bar{p} \parallel_\gamma s_1) \xrightarrow{\sigma} \partial_\diamond(\bar{p} \parallel_\gamma s)$ and the Condition C6 we get,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma s), \widehat{\nabla}(\bar{p}_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

- ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar as previous subcase.
- (b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Then,

$$\begin{aligned} \exists p_1, p'_1, s_1, s', \vec{\sigma} \in \bar{\alpha}(P)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge \partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s') \right. \\ \left. \wedge \partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s) \right] \quad (22) \end{aligned}$$

where,

$$q_c^1 = \partial_\diamond(p_1 \parallel_\gamma s), \quad q_a^1 = \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s''. [\partial_\diamond(p'_1 \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

Applying Lemma 5 at the state $\partial_\diamond(p'' \parallel_\gamma s'')$ we get (See Figure 16),

$$\exists p_2, s_2. [\partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

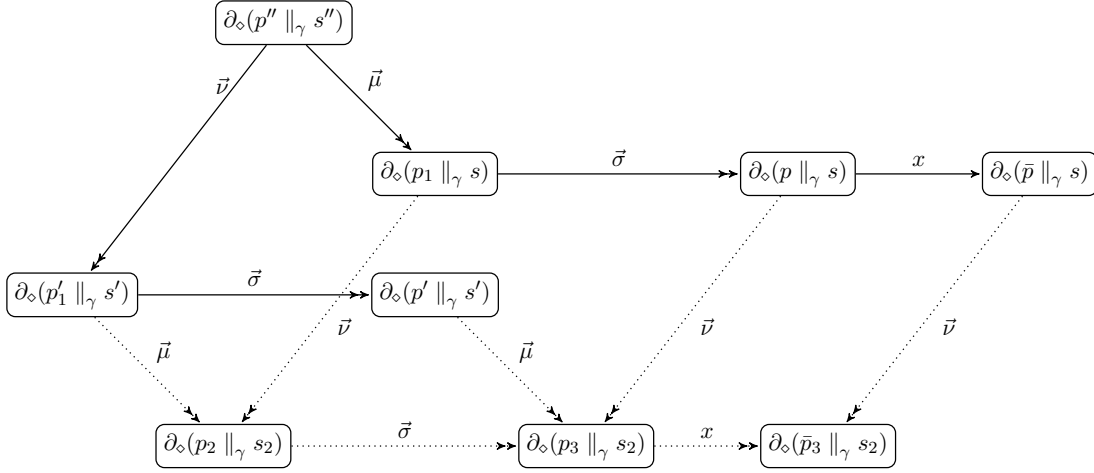


Figure 16: Proof in Subcase S3-2-b-i.

Now applying Lemma 5 at the state $\partial_\diamond(p'_1 \parallel_\gamma s')$ we get,

$$\exists p_3. [\partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\vec{\sigma}} \partial_\diamond(p_3 \parallel_\gamma s_2) \xleftarrow{\vec{\mu}} \partial_\diamond(p' \parallel_\gamma s)].$$

Again applying Lemma 5, but at the state $\partial_\diamond(p_1 \parallel_\gamma s)$ we get, $\partial_\diamond(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_\diamond(p_3 \parallel_\gamma s_2)$. Again applying Lemma 5, but at the state $\partial_\diamond(p \parallel_\gamma s)$ we get,

$$\exists \bar{p}_3. [\partial_\diamond(p_3 \parallel_\gamma s_2) \xrightarrow{x} \partial_\diamond(\bar{p}_3 \parallel_\gamma s_2) \xleftarrow{\vec{v}} \partial_\diamond(\bar{p} \parallel_\gamma s)].$$

Using the derived transition $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_3 \parallel_\gamma s_2)$ and the semantics of \parallel_γ we get, $p' \xrightarrow{f_i(\vec{\mu})} p_3$. Using this fact at the state q_a we get,

$$\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau^*} \widehat{\nabla}(p_3 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s).$$

And from the Condition C2 with the transition $\partial_\diamond(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_\diamond(p_3 \parallel_\gamma s_2)$ (See Figure 16) we get,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(p_3 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

From the derived transition $\partial_\diamond(p_3 \parallel_\gamma s_2) \xrightarrow{x} \partial_\diamond(\bar{p}_3 \parallel_\gamma s_2)$ (See Figure 16) and the semantics of \parallel_γ we get, $p_3 \xrightarrow{x} \bar{p}_3$. Thus,

$$\widehat{\nabla}(p_3 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \xrightarrow{x} \widehat{\nabla}(\bar{p}_3 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s).$$

Finally, by the transition $\partial_\diamond(\bar{p} \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_\diamond(\bar{p}_3 \parallel_\gamma s_2)$, and the Condition C2 we get,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma s), \widehat{\nabla}(\bar{p}_3 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

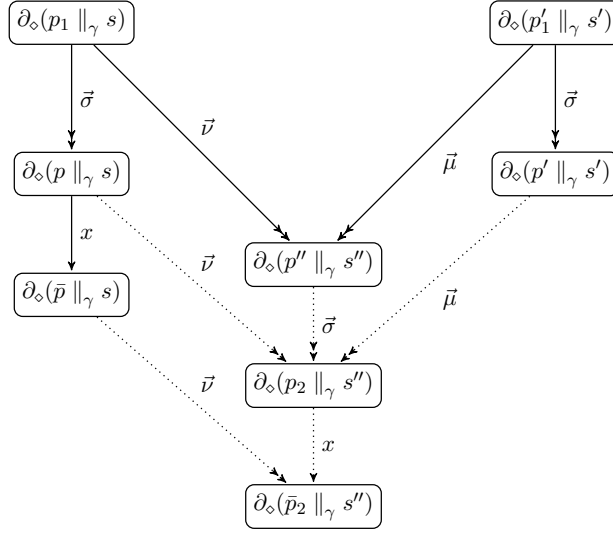


Figure 17: Proof in Subcase S3-2-b-ii.

ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Then,

$$\exists p'', s''. [\partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_\diamond(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_\diamond(p'_1 \parallel_\gamma s')].$$

Applying Lemma 5 at the state $\partial_\diamond(p_1 \parallel_\gamma s)$ we get (See Figure 17),

$$\exists p_2. [\partial_\diamond(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_\diamond(p_2 \parallel_\gamma s'') \xleftarrow{\vec{\sigma}} \partial_\diamond(p'' \parallel_\gamma s')].$$

Again applying Lemma 5, but at the state $\partial_\diamond(p \parallel_\gamma s)$ we get (See Figure 17),

$$\exists \bar{p}_2. [\partial_\diamond(p_2 \parallel_\gamma s'') \xrightarrow{x} \partial_\diamond(\bar{p}_2 \parallel_\gamma s'') \xleftarrow{\vec{v}} \partial_\diamond(\bar{p} \parallel_\gamma s)].$$

And applying Lemma 5 at the state $\partial_\diamond(p'_1 \parallel_\gamma s')$ we get (See Figure 17), $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s'')$. Thus, from the semantics of \parallel_γ we have $p' \xrightarrow{f_i(\vec{\mu})} p_2$. Hence,

$$\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{\tau^*} \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s).$$

From the transition $\partial_\diamond(p \parallel_\gamma s) \xrightarrow{\vec{v}} \partial_\diamond(p_2 \parallel_\gamma s'')$ and the Condition C2 we get,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

Now from the transition $\partial_\diamond(p_2 \parallel_\gamma s'') \xrightarrow{x} \partial_\diamond(\bar{p}_2 \parallel_\gamma s'')$ and the semantics of \parallel_γ we have $p_2 \xrightarrow{x} \bar{p}_2$. Thus,

$$\widehat{\nabla}(p_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \xrightarrow{x} \widehat{\nabla}(\bar{p}_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s).$$

Finally, from the transition $\partial_\diamond(\bar{p} \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_\diamond(\bar{p}_2 \parallel_\gamma s)$ and the Condition C2 we get,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma s), \widehat{\nabla}(\bar{p}_2 \parallel_{\gamma'} B^{m,n}[\varepsilon, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

S4 Let $q_c \xrightarrow{x} q'_c \wedge (q_c, q_a) \in \widehat{\Phi} \wedge x \in \bar{\alpha}(S)$. From construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\diamond(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of \parallel_γ and $x \in \bar{\alpha}(S)$ we know that,

$$q'_c \equiv \partial_\diamond(p \parallel_\gamma \bar{s}) \wedge s \xrightarrow{x} \bar{s}. \quad (23)$$

Before showing that the state q_a can perform action x , we need to retrieve the relation between process terms $\partial_\diamond(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 are special cases under conditions C6 and C7, respectively. Here we give the proof of the remaining cases.
 - (a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Trivial.
 - (b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Then,

$$\begin{aligned} \exists p_1, p'_1, s_1, s', \vec{\sigma} \in \bar{\alpha}(P)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge \partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s') \right. \\ \left. \wedge \partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s) \right] \quad (24) \end{aligned}$$

where,

$$q_c^1 = \partial_\diamond(p_1 \parallel_\gamma s), \quad q_a^1 = \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s''. [\partial_\diamond(p'_1 \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

Applying Lemma 5 at the state $\partial_\diamond(p'' \parallel_\gamma s'')$ we get (See Figure 18),

$$\exists p_2, s_2. [\partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\mu}} \partial_\diamond(p'_1 \parallel_\gamma s')].$$

Again applying Lemma 5, but at the state $\partial_\diamond(p'_1 \parallel_\gamma s')$ we get (See Figure 18),

$$\exists p_3. [\partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\vec{\sigma}} \partial_\diamond(p_3 \parallel_\gamma s_2) \xleftarrow{\vec{\mu}} \partial_\diamond(p' \parallel_\gamma s')].$$

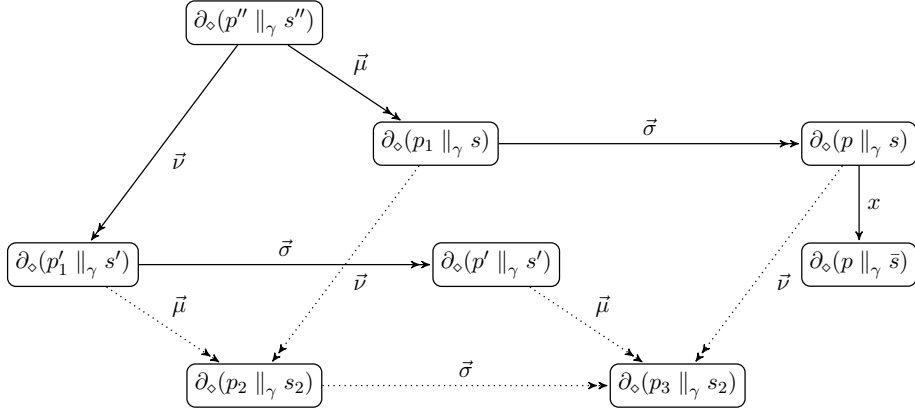


Figure 18: Proof in Subcase S4-2-b-i.

Applying Lemma 5 at the state $\partial_\circ(p_1 \parallel_\gamma s)$ we get (See Figure 18), $\partial_\circ(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_\circ(p_3 \parallel_\gamma s_2)$. Recall from the Equation (23) the transition $s \xrightarrow{x} \bar{s}$ and thus we get,

$$\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \xrightarrow{x} \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} \bar{s}).$$

Using the transitions $\partial_\circ(p' \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\circ(p_3 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_\circ(p \parallel_\gamma s)$ and the Condition C5 we get,

$$\left(\partial_\circ(p \parallel_\gamma s), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

Finally, from the transition $\partial_\circ(p \parallel_\gamma s) \xrightarrow{x} \partial_\circ(p \parallel_\gamma \bar{s})$ and the Condition C6 we get,

$$\left(\partial_\circ(p \parallel_\gamma \bar{s}), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar to the above case.

S5 Let $q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \widehat{\Phi} \wedge \tau = \tau_{\mathfrak{?}a}(\mathfrak{?}a) \wedge \mathfrak{?}a \in I_P^?$. From construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\circ(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of $\parallel_{\gamma'}$ and $\mathfrak{?}a \in I_P^?$ we know that,

$$\mathfrak{?}a \in \mu \wedge q'_a \equiv \widehat{\nabla}(\bar{p}_1 \parallel_{\gamma'} B^{m,n}[\mu \ominus \mathfrak{?}a, \nu] \parallel_{\gamma'} s) \wedge p' \xrightarrow{\mathfrak{?}a} \bar{p}_1. \quad (25)$$

Now, we retrieve the relation between process terms $\partial_\circ(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.

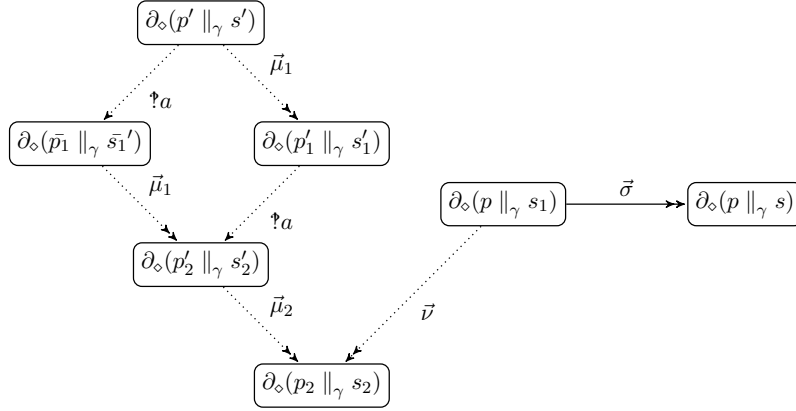


Figure 19: Proof in Subcase S5-2-a-i.

2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 is already proven in [5]. Here we give the proof of the remaining cases.

(a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Then,

$$\exists s_1, \sigma \in \bar{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\vec{\sigma}} q_c \right]$$

where,

$$q_c^1 = \partial_\diamond(p \parallel_\gamma s_1), \quad q_a^1 = \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s'', s'. [\partial_\diamond(p' \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p \parallel_\gamma s_1)].$$

Applying Lemma 5 at the state $\partial_\diamond(p'' \parallel_\gamma s'')$ we get,

$$\exists p_2, s_2. [\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_\diamond(p \parallel_\gamma s_1)].$$

From Equation (25) we know that $\uparrow a \in \mu$ which further implies that $\uparrow a \in \vec{\mu}$. Suppose $\vec{\mu} = \vec{\mu}_1 \cdot \uparrow a \cdot \vec{\mu}_2$ for some $\vec{\mu}_1, \vec{\mu}_2 \in I_P^*$. Thus the transition $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2)$ will be of the following form (See Figure 19),

$$\exists p'_1, s'_1, p'_2, s'_2. [\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}_1} \partial_\diamond(p'_1 \parallel_\gamma s'_1) \xrightarrow{\uparrow a} \partial_\diamond(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{\mu}_2} \partial_\diamond(p_2 \parallel_\gamma s_2)].$$

Furthermore, we have $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}_1 \cdot \uparrow a} \partial_\diamond(p'_2 \parallel_\gamma s'_2)$ and $p' \xrightarrow{\uparrow a} \bar{p}_1$. But from Definition 15 we know that,

$$\exists \bar{s}_1'. [\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\uparrow a} \partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}_1')].$$

Using the Lemma 5 and the transition $\partial_\diamond(p'_2 \parallel_\gamma s'_2) \xrightarrow{\vec{\mu}_2} \partial_\diamond(p_2 \parallel_\gamma s_2)$ we get (See Figure 19),

$$\partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}'_1) \xrightarrow{\vec{\mu}_1 \cdot \vec{\mu}_2} \partial_\diamond(p_2 \parallel_\gamma s_2).$$

From the transitions $\partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}'_1) \xrightarrow{\vec{\mu}_1 \cdot \vec{\mu}_2} \partial_\diamond(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_\diamond(p \parallel_\gamma s_1)$ and the Condition C5 we have,

$$\left(\partial_\diamond(p \parallel_\gamma s_1), \widehat{\nabla}(\bar{p}_1 \parallel_{\gamma'} B^{m,n}[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

Finally from the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s)$ and the Condition C6 we conclude that,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(\bar{p}_1 \parallel_{\gamma'} B^{m,n}[\mu \ominus \dagger a, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

- ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar to the previous case.
- (b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Then,

$$\begin{aligned} \exists p_1, p'_1, s_1, s', \vec{\sigma} \in \bar{\alpha}(P)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge \partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s') \right. \\ \left. \wedge \partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s) \right] \quad (26) \end{aligned}$$

where,

$$q_c^1 = \partial_\diamond(p_1 \parallel_\gamma s), \quad q_a^1 = \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s''. [\partial_\diamond(p'_1 \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

Applying Lemma 5 at the state $\partial_\diamond(p'' \parallel_\gamma s'')$ we get (See Figure 20),

$$\exists p_2, s_2. [\partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

Now applying Lemma 5 at the state $\partial_\diamond(p'_1 \parallel_\gamma s')$ we get (See Figure 20),

$$\exists p'_2. [\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p'_2 \parallel_\gamma s_2) \xleftarrow{\vec{\sigma}} \partial_\diamond(p_2 \parallel_\gamma s_2)].$$

Again applying Lemma 5 but at the state $\partial_\diamond(p_1 \parallel_\gamma s)$ we get (See Figure 20),

$$\partial_\diamond(p \parallel_\gamma s) \xrightarrow{\vec{\nu}} \partial_\diamond(p'_2 \parallel_\gamma s_2). \quad (27)$$

Consider the transition $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p'_2 \parallel_\gamma s_2)$, the fact $\dagger a \in \vec{\mu}$ (Equation (25)) we get (See Figure 20),

$$\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}_1} \partial_\diamond(p''_1 \parallel_\gamma s''_1) \xrightarrow{\dagger a} \partial_\diamond(p''_2 \parallel_\gamma s''_2) \xrightarrow{\vec{\mu}_2} \partial_\diamond(p'_2 \parallel_\gamma s_2).$$

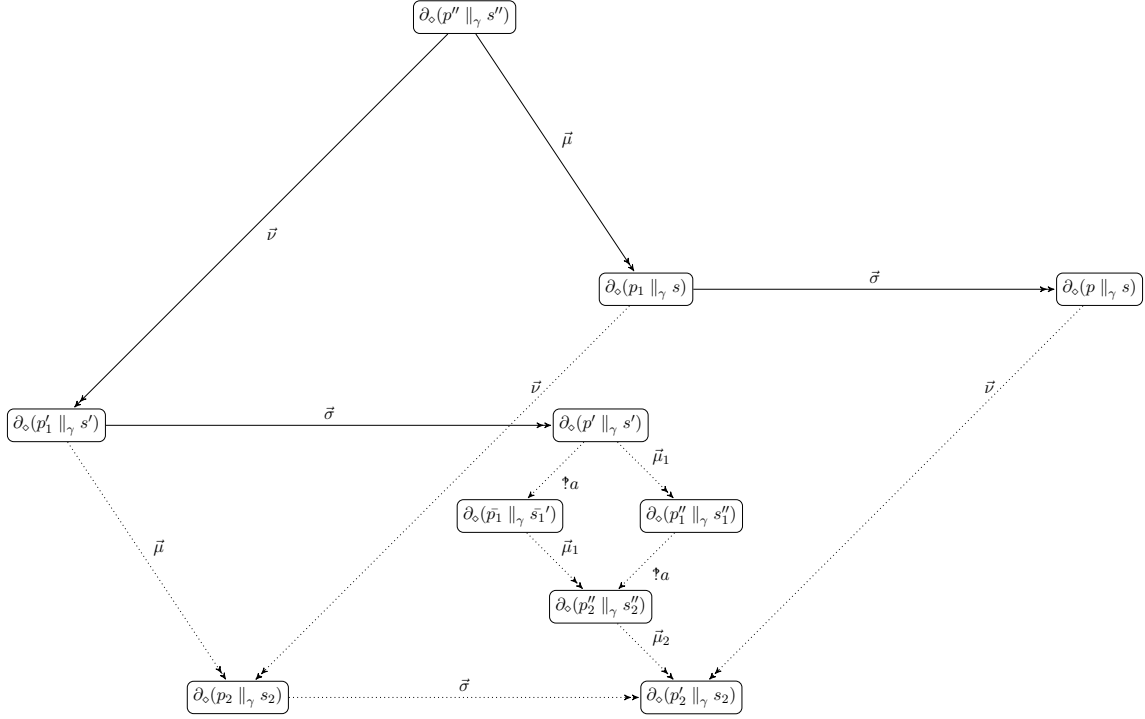


Figure 20: Proof in Subcase S5-2-b-i.

Thus we have $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\bar{\mu}_1.\text{?}a} \partial_\diamond(p_2'' \parallel_\gamma s_2'')$. But by Equation (25) we know that $p' \xrightarrow{\text{?}a} \bar{p}_1$ and further applying the Definition 15 we get,

$$\exists \bar{s}_1'. [\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\text{?}a} \partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}_1')].$$

Furthermore, applying Lemma 5 at the state $\partial_\diamond(p' \parallel_\gamma s')$ we get,

$$\partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}_1') \xrightarrow{\bar{\mu}_1.\bar{\mu}_2} \partial_\diamond(p'_2 \parallel_\gamma s_2).$$

Finally from the above transition, and the transition $\partial_\diamond(p'_2 \parallel_\gamma s_2) \xleftarrow{\bar{\nu}} \partial_\diamond(p \parallel_\gamma s)$ and the Condition C5 we get,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(\bar{p}_1 \parallel_{\gamma'} B^{m,n}[\mu \ominus \text{?}a, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar as the previous case.

S6 Let $q_a \xrightarrow{\tau} q'_a \wedge (q_c, q_a) \in \widehat{\Phi} \wedge \tau = \tau_{\text{?}a}(\text{?}a) \wedge \text{?}a \in I_P^1$. From the construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\diamond(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of $\parallel_{\gamma'}$ and $\text{?}a \in I_P^1$ we know that,

$$q'_a \equiv \widehat{\nabla}(\bar{p}_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu \oplus \text{?}a] \parallel_{\gamma'} s) \wedge p' \xrightarrow{\text{!}a} \bar{p}_1. \quad (28)$$

Now, we retrieve the relation between process terms $\partial_\circ(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 is already proven in [5]. Here we give the proof of the remaining cases.

(a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Then,

$$\exists s_1, \sigma \in \overline{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\vec{\sigma}} q_c \right]$$

where,

$$q_c^1 = \partial_\circ(p \parallel_\gamma s_1), \quad q_a^1 = \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Trivial.
- ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Then,

$$\exists p'', s'', s'. [\partial_\circ(p \parallel_\gamma s_1) \xrightarrow{\vec{v}} \partial_\circ(p'' \parallel_\gamma s'') \xleftarrow{\vec{\mu}} \partial_\circ(p' \parallel_\gamma s')].$$

From Equation 28 we know that $p' \xrightarrow{!a} \bar{p}_1$ and by Definition 14 we get,

$$\exists \bar{s}'. [\partial_\circ(p' \parallel_\gamma s') \xrightarrow{!a} \partial_\circ(\bar{p}_1 \parallel_\gamma \bar{s}')].$$

Now applying Lemma 5 at the state $\partial_\circ(p' \parallel_\gamma s')$ we get,

$$\exists p_2, s_2. [\partial_\circ(\bar{p}_1 \parallel_\gamma \bar{s}') \xrightarrow{\vec{\mu}} \partial_\circ(p_2 \parallel_\gamma s_2) \xleftarrow{!a} \partial_\circ(p'' \parallel_\gamma s'')].$$

Thus, from the transitions $\partial_\circ(\bar{p}_1 \parallel_\gamma \bar{s}') \xrightarrow{\vec{\mu}} \partial_\circ(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{v}.!a} \partial_\circ(p \parallel_\gamma s_1)$ and the Condition C5 we get,

$$\left(\partial_\circ(p \parallel_\gamma s_1), \widehat{\nabla}(\bar{p}_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu \oplus !a] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

Finally, from the transition $\partial_\circ(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\circ(p \parallel_\gamma s)$ and the Condition C6 we conclude that,

$$\left(\partial_\circ(p \parallel_\gamma s), \widehat{\nabla}(\bar{p}_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu \oplus !a] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

(b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Similar as previous subcase.

S7 Let $q_a \xrightarrow{!a} q'_a \wedge (q_c, q_a) \in \widehat{\Phi} \wedge !a \in I_P^?$. From the construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\circ(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of $\|_{\gamma'}$ and $\!?\!a \in I_P^?$ we know that,

$$q'_a \equiv \widehat{\nabla}(p' \|_{\gamma'} B^{m,n}[\mu \oplus \!?\!a, \nu] \|_{\gamma'} \bar{s}) \wedge s \xrightarrow{!a} \bar{s}. \quad (29)$$

Before showing that the state q_a can perform action $\!?\!a$, we need to retrieve the relation between process terms $\partial_\diamond(p \|_\gamma s)$, $\widehat{\nabla}(p' \|_{\gamma'} B^{m,n}[\mu, \nu] \|_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 is already proven in [5]. Here we give the proof of the remaining cases.
 - (a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Then,

$$\exists s_1, \sigma \in \bar{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\bar{\sigma}} q_c \right]$$

where,

$$q_c^1 = \partial_\diamond(p \|_\gamma s_1), \quad q_a^1 = \widehat{\nabla}(p' \|_{\gamma'} B^{m,n}[\mu, \nu] \|_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s'', s'. [\partial_\diamond(p' \|_\gamma s') \xleftarrow{\bar{\nu}} \partial_\diamond(p'' \|_\gamma s'') \xrightarrow{\bar{\mu}} \partial_\diamond(p \|_\gamma s_1)].$$

From Equation (29) we know that $s \xrightarrow{!a} \bar{s}$ and by well posedness (Definition 14) we infer that,

$$\exists \bar{p}. [\partial_\diamond(p \|_\gamma s) \xrightarrow{!a} \partial_\diamond(\bar{p} \|_\gamma \bar{s})].$$

Applying Lemma 6 on the transition $\partial_\diamond(p \|_\gamma s_1) \xrightarrow{\bar{\sigma}, !a} \partial_\diamond(\bar{p} \|_\gamma \bar{s})$ we get,

$$\exists \bar{s}_1. [\partial_\diamond(p \|_\gamma s_1) \xrightarrow{!a} \partial_\diamond(\bar{p} \|_\gamma \bar{s}_1)].$$

And by Lemma 5 we get, $\partial_\diamond(\bar{p} \|_\gamma \bar{s}_1) \xrightarrow{\bar{\sigma}} \partial_\diamond(\bar{p} \|_\gamma \bar{s})$. Thus we have $\partial_\diamond(p'' \|_\gamma s'') \xrightarrow{\bar{\mu}, !a} \partial_\diamond(\bar{p} \|_\gamma \bar{s}_1)$. From the above transition, the transition $\partial_\diamond(p'' \|_\gamma s'') \xrightarrow{\bar{\nu}} \partial_\diamond(p' \|_\gamma s')$ and by Condition C4 we have,

$$\left(\partial_\diamond(\bar{p} \|_\gamma \bar{s}_1), \widehat{\nabla}(p' \|_{\gamma'} B^{m,n}[\mu \oplus \!?\!a, \nu] \|_{\gamma'} \bar{s}_1) \right) \in \Phi.$$

Finally, from the transition $\partial_\diamond(\bar{p} \|_\gamma \bar{s}_1) \xrightarrow{\bar{\sigma}} \partial_\diamond(\bar{p} \|_\gamma \bar{s})$ and the Condition C6 we conclude that,

$$\left(\partial_\diamond(\bar{p} \|_\gamma \bar{s}), \widehat{\nabla}(p' \|_{\gamma'} B^{m,n}[\mu \oplus \!?\!a, \nu] \|_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

- ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar to the above case.
- (b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Similar to the above case.

S8 Let $q_a \xrightarrow{?a} q'_a \wedge (q_c, q_a) \in \widehat{\Phi} \wedge ?a \in I_P^!$. From the construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\circ(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of $\parallel_{\gamma'}$ and $?a \in I_P^!$ we know that,

$$q'_a \equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu \ominus ?a] \parallel_{\gamma'} \bar{s}) \wedge s \xrightarrow{?a} \bar{s} \wedge ?a \in' \nu. \quad (30)$$

Before showing that the state q_a can perform action $?a$, we need to retrieve the relation between process terms $\partial_\circ(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 is already proven in [5]. Here we give the proof of the remaining cases.
 - (a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Then,

$$\exists s_1, \sigma \in \bar{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\bar{\sigma}} q_c \right]$$

where,

$$q_c^1 = \partial_\circ(p \parallel_\gamma s_1), \quad q_a^1 = \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s'', s'. [\partial_\circ(p' \parallel_{\gamma'} s') \xleftarrow{\bar{v}} \partial_\circ(p'' \parallel_{\gamma'} s'') \xrightarrow{\bar{\mu}} \partial_\circ(p \parallel_\gamma s_1)].$$

By Lemma 5 we get,

$$\exists p_2, s_2. [\partial_\circ(p' \parallel_{\gamma'} s') \xrightarrow{\bar{\mu}} \partial_\circ(p_2 \parallel_\gamma s_2) \xleftarrow{\bar{v}} \partial_\circ(p \parallel_\gamma s_1)].$$

Again applying Lemma 5, but at the state $\partial_\circ(p \parallel_\gamma s_1)$ we get (See Figure 21),

$$\exists s'_2. [\partial_\circ(p_2 \parallel_\gamma s_2) \xrightarrow{\bar{\sigma}} \partial_\circ(p_2 \parallel_\gamma s'_2) \xleftarrow{\bar{v}} \partial_\circ(p \parallel_\gamma s)].$$

From Equation 30 we know that $?a \in' \nu$ and thus assume $\bar{v} = \bar{v}_1. ?a. \bar{v}_2$ for some $\bar{v}_1, \bar{v}_2 \in I_P^!*$. Hence, we know that the transition $\partial_\circ(p \parallel_\gamma s) \xrightarrow{\bar{v}} \partial_\circ(p_2 \parallel_\gamma s'_2)$ is of the following form,

$$\exists p''_1, s''_1, p''_2, s''_2. [\partial_\circ(p \parallel_\gamma s) \xrightarrow{\bar{v}_1} \partial_\circ(p''_1 \parallel_\gamma s''_1) \xrightarrow{?a} \partial_\circ(p''_2 \parallel_\gamma s''_2) \xrightarrow{\bar{v}_2} \partial_\circ(p_2 \parallel_\gamma s'_2)].$$

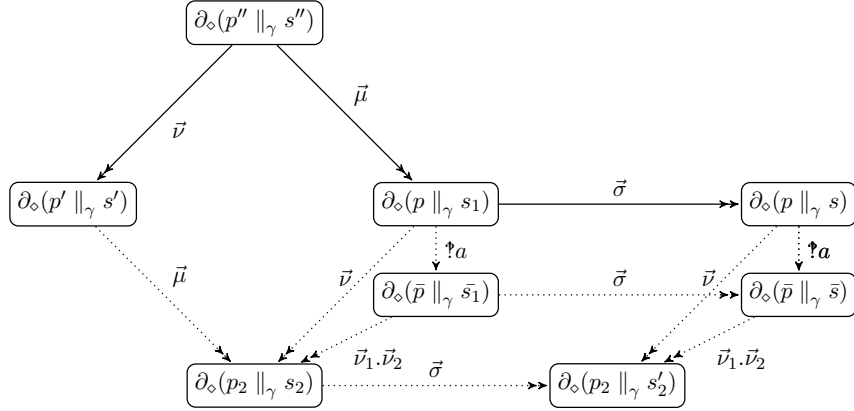


Figure 21: Proof in Subcase S8-2-a-i.

But, from the Definition 15 (Reordering property) and the transition $s \xrightarrow{?a} \bar{s}$ (Equation (30)) we get (See Figure 21),

$$\exists \bar{p}. [\partial_\diamond(p \parallel_\gamma s) \xrightarrow{?a} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})].$$

Furthermore, from the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})$ and the Lemma 7 we get (See Figure 21),

$$\exists \bar{s}_1. [\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{?a} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})].$$

Using the above derived facts we know that $\vec{\nu} = \vec{\nu}_1 \cdot ?a \cdot \vec{\nu}_2$ and there exists the transition $\partial_\diamond(p \parallel_\gamma s_1) \xrightarrow{?a} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1)$. And by Lemma 5 we know that,

$$\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1) \xrightarrow{\vec{\nu}_1 \cdot \vec{\nu}_2} \partial_\diamond(p_2 \parallel_\gamma s_2).$$

And from the above transition, and the transition $\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2)$ and the condition C5 we get,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu \ominus ?a] \parallel_{\gamma'} \bar{s}_1) \right) \in \Phi.$$

Finally, from the transition $\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}_1) \xrightarrow{\vec{\sigma}} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})$ and the Condition C6 we conclude that,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu \ominus ?a] \parallel_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

- ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar to the above case.
- (b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Then,

$$\begin{aligned} \exists p_1, p'_1, s_1, s', \vec{\sigma} \in \bar{\alpha}(P)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge \partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s') \right. \\ \left. \wedge \partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s) \right] \end{aligned}$$

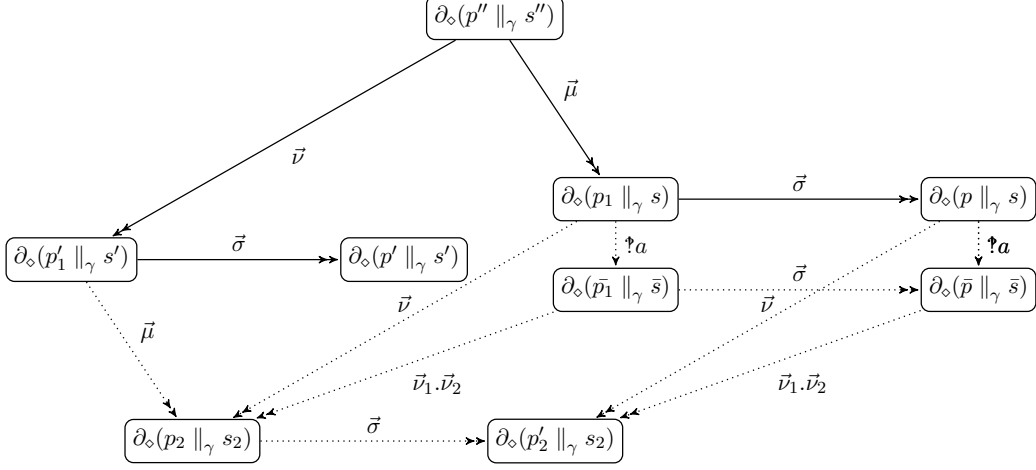


Figure 22: Proof in Subcase S8-2-b-i.

where,

$$q_c^1 = \partial_\diamond(p_1 \parallel_\gamma s), \quad q_a^1 = \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s''. [\partial_\diamond(p'_1 \parallel_\gamma s'_1) \xleftarrow{\vec{v}} \partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

Now applying Lemma 5 we get,

$$\exists p_2, s_2. [\partial_\diamond(p'_1 \parallel_\gamma s'_1) \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{v}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

Again applying Lemma 5, but at the state $\partial_\diamond(p_1 \parallel_\gamma s)$ we get (See Figure 22),

$$\exists p'_2. [\partial_\diamond(p_2 \parallel_\gamma s_2) \xrightarrow{\vec{\sigma}} \partial_\diamond(p'_2 \parallel_\gamma s_2) \xleftarrow{\vec{v}} \partial_\diamond(p \parallel_\gamma s)].$$

Using the fact $\dagger a \in' \nu$ and the transition $s \xrightarrow{\dagger a} \bar{s}$ (Equation (30)) in the Definition 15 we get (See Figure 22),

$$\exists \bar{p}. [\partial_\diamond(p \parallel_\gamma s) \xrightarrow{\dagger a} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})].$$

Thus we have the transitions $\partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\sigma} \cdot \dagger a} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})$ and by applying Lemma 7 we get (See Figure 22),

$$\exists \bar{p}_1. [\partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\dagger a} \partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}) \xrightarrow{\vec{\sigma}} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})].$$

Since $\dagger a \in' \nu \Rightarrow \dagger a \in \vec{v}$ (Equation 30), let $\vec{v} = \vec{v}_1 \cdot \dagger a \cdot \vec{v}_2$ for some $\vec{v}_1, \vec{v}_2 \in I_P^!$. Then, applying Lemma 5 at the state $\partial_\diamond(p_1 \parallel_\gamma s)$ we get (See Figure 22),

$$\partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}) \xrightarrow{\vec{v}_1 \cdot \vec{v}_2} \partial_\diamond(p_2 \parallel_\gamma s_2).$$

And from the transitions $\partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\mu}} \partial_\diamond(p_2 \parallel_\gamma s_2) \xleftarrow{\vec{\nu}_1, \vec{\nu}_2} \partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s})$ and the Condition C5 we get,

$$\left(\partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}), \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu \ominus \dagger a] \parallel_{\gamma'} \bar{s}) \right) \in \Phi.$$

Finally, by the above fact, the transitions $\partial_\diamond(\bar{p}_1 \parallel_\gamma \bar{s}) \xrightarrow{\vec{\sigma}} \partial_\diamond(\bar{p} \parallel_\gamma \bar{s})$, $\partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s')$ and the Condition C7 we get,

$$\left(\partial_\diamond(\bar{p} \parallel_\gamma \bar{s}), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu \ominus \dagger a] \parallel_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Same as the above case.

S9 Let $q_a \xrightarrow{x} q'_a \wedge (q_c, q_a) \in \widehat{\Phi} \wedge x \in \bar{\alpha}(P)$. From the construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\diamond(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of $\parallel_{\gamma'}$ and $x \in \bar{\alpha}(P)$ we know that,

$$q'_a \equiv \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \wedge p' \xrightarrow{x} p'_1. \quad (31)$$

Before showing that the state q_a can perform action x , we need to retrieve the relation between process terms $\partial_\diamond(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 are special cases under conditions C6 and C7, respectively. Here we give the proof of the remaining cases.

(a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Then,

$$\exists s_1, \sigma \in \bar{\alpha}(S)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge q_c^1 \xrightarrow{\vec{\sigma}} q_c \right]$$

where,

$$q_c^1 = \partial_\diamond(p \parallel_\gamma s_1), \quad q_a^1 = \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s'', s'. [\partial_\diamond(p' \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p \parallel_\gamma s_1)].$$

But from Equation 31 we have that $p' \xrightarrow{x} p'_1$ and thus by well posedness (Definition 14) and the semantics of \parallel_γ we get,

$$\partial_\diamond(p' \parallel_\gamma s') \xrightarrow{x} \partial_\diamond(p'_1 \parallel_\gamma s').$$

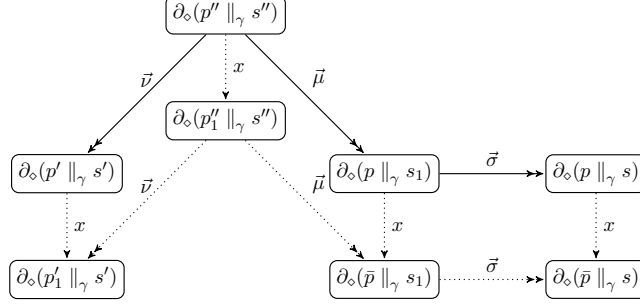


Figure 23: Proof in Subcase S9-2-a-i.

Now applying the result of Lemma 4 on the transition $\partial_\circ(p'' \parallel_\gamma s'') \xrightarrow{\vec{\nu}.x} \partial_\circ(p'_1 \parallel_\gamma s')$ we get (See Figure 23),

$$\exists p''_1. [\partial_\circ(p'' \parallel_\gamma s'') \xrightarrow{x} \partial_\circ(p''_1 \parallel_\gamma s'') \xrightarrow{\vec{\nu}} \partial_\circ(p'_1 \parallel_\gamma s')].$$

Now applying Lemma 5 at the state $\partial_\circ(p'' \parallel_\gamma s'')$ we get (See Figure 23),

$$\exists \bar{p}. [\partial_\circ(p \parallel_\gamma s_1) \xrightarrow{x} \partial_\circ(\bar{p} \parallel_\gamma s_1) \xleftarrow{\vec{\mu}} \partial_\circ(p'' \parallel_\gamma s'')].$$

Again applying Lemma 5, but at the state $\partial_\circ(p \parallel_\gamma s_1)$ we get (See Figure 23),

$$\partial_\circ(p \parallel_\gamma s) \xrightarrow{x} \partial_\circ(\bar{p} \parallel_\gamma s) \xleftarrow{\vec{\sigma}} \partial_\circ(\bar{p} \parallel_\gamma s_1).$$

And from the transitions $\partial_\circ(p'_1 \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\circ(p''_1 \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\circ(\bar{p} \parallel_\gamma s_1)$ and the Condition C4 we get,

$$\left(\partial_\circ(\bar{p} \parallel_\gamma s_1), \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s_1) \right) \in \Phi.$$

Using the above fact, the transitions $\partial_\circ(\bar{p} \parallel_\gamma s_1) \xrightarrow{\vec{\sigma}} \partial_\circ(\bar{p} \parallel_\gamma s)$ and the Condition C6 we conclude that,

$$\left(\partial_\circ(\bar{p} \parallel_\gamma s), \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \right) \in \widehat{\Phi}.$$

- ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar to the previous case.
- (b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Trivial.

S10 Let $q_a \xrightarrow{x} q'_a \wedge (q_c, q_a) \in \widehat{\Phi} \wedge x \in \bar{\alpha}(S)$. From the construction of $\widehat{\Phi}$ we know that,

$$\begin{aligned} q_c &\equiv \partial_\circ(p \parallel_\gamma s), \\ q_a &\equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s). \end{aligned}$$

Furthermore, from the semantics of $\parallel_{\gamma'}$ and $x \in \bar{\alpha}(S)$ we know that,

$$q'_a \equiv \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} \bar{s}) \wedge s \xrightarrow{x} \bar{s}. \quad (32)$$

Before showing that the state q_a can perform action x , we need to retrieve the relation between process terms $\partial_\circ(p \parallel_\gamma s)$, $\widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s)$ from the Conditions Ci where, $i \in [1, 7]$. We apply case distinction based on the structure of μ and ν .

1. When $\mu, \nu = \varepsilon$.
2. When $\mu, \nu \neq \varepsilon$. Then the states q_c, q_a can be related by the condition C4, or C5, or C6, or C7. The case under conditions C4 and C5 is already proven in [5]. Here we give the proof of the remaining cases.
 - (a) Either $(q_c, q_a) \in \widehat{\Phi}$ due to C6. Trivial.
 - (b) Or $(q_c, q_a) \in \widehat{\Phi}$ due to C7. Then,

$$\begin{aligned} \exists p_1, p'_1, s_1, s', \vec{\sigma} \in \overline{\alpha}(P)^*. \left[(q_c^1, q_a^1) \in \Phi \wedge \partial_\diamond(p'_1 \parallel_\gamma s') \xrightarrow{\vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s') \right. \\ \left. \wedge \partial_\diamond(p_1 \parallel_\gamma s) \xrightarrow{\vec{\sigma}} \partial_\diamond(p \parallel_\gamma s) \right] \end{aligned}$$

where,

$$q_c^1 = \partial_\diamond(p_1 \parallel_\gamma s), \quad q_a^1 = \widehat{\nabla}(p'_1 \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s).$$

Note $(q_c^1, q_a^1) \in \Phi$ and thus we get the following two cases:

- i. Either $(q_c^1, q_a^1) \in \Phi$ due to C4. Then,

$$\exists p'', s''. [\partial_\diamond(p'_1 \parallel_\gamma s') \xleftarrow{\vec{\nu}} \partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p_1 \parallel_\gamma s)].$$

Applying Proposition 3 on the transitions $\partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\nu}, \vec{\sigma}} \partial_\diamond(p' \parallel_\gamma s')$ we get,

$$\exists p_2. [\partial_\diamond(p'' \parallel_\gamma s'') \xrightarrow{\vec{\sigma}} \partial_\diamond(p_2 \parallel_\gamma s'') \xrightarrow{\vec{\nu}} \partial_\diamond(p' \parallel_\gamma s')].$$

Now applying Lemma 5 at the state $\partial_\diamond(p'' \parallel_\gamma s'')$ we get,

$$\partial_\diamond(p_2 \parallel_\gamma s'') \xrightarrow{\vec{\mu}} \partial_\diamond(p \parallel_\gamma s).$$

Thus from the above transition, and the transition $\partial_\diamond(p_2 \parallel_\gamma s'') \xrightarrow{\vec{\nu}} \partial_\diamond(p' \parallel_\gamma s')$ and the Condition C4 we get,

$$\left(\partial_\diamond(p \parallel_\gamma s), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} s) \right) \in \Phi.$$

Finally, from the above fact and the transition $\partial_\diamond(p \parallel_\gamma s) \xrightarrow{x} \partial_\diamond(p \parallel_\gamma \bar{s})$ and the Condition C6 we conclude that,

$$\left(\partial_\diamond(p \parallel_\gamma \bar{s}), \widehat{\nabla}(p' \parallel_{\gamma'} B^{m,n}[\mu, \nu] \parallel_{\gamma'} \bar{s}) \right) \in \widehat{\Phi}.$$

- ii. Or $(q_c^1, q_a^1) \in \Phi$ due to C5. Similar to the above case.

□