Three dimensional analysis of a realistic trabecular bone structure, using a large-scale FE-model

THREE DIMENSIONAL ANALYSIS OF A REALISTIC TRABECULAR BONE STRUCTURE, USING A LARGE-SCALE FE-MODEL

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ABSTRACT

Mechanical variables at the microstructural level are believed to regulate biological processes in trabecular bone. In the present study it is attempted to determine these microstructural parameters, using a realistic 3-D finite element model of the trabecular structure. Three dimensional computer reconstruction techniques were used to obtain the microstructure of a 5mm cube of trabecular bone. A finite element model was created from this data by directly converting voxels to brick elements. The model thus obtained consisted of 296,676 brick elements and was solved using a special purpose finite element routine in combination with an iterative solving procedure.

The tissue strain-energy density distribution showed a wide range of values. With boundary elements not accounted for, a maximum tissue strain-energy density of 269 times the apparent strain-energy density was found. The tissue Young's modulus was estimated at 8245 MPa by comparing the calculated apparent density with literature values.

INTRODUCTION

The mechanical characteristics of trabecular bone depend on the morphology of its structure, which regulates the load transfer distribution from the articular surface through the bone matrix. Mechanical strains at the microstructural level of the individual trabeculae are believed to regulate the biological adaptive processes in trabecular bone. However, as yet not much is known about the micro-mechanical variables. Most of the stress analyses with the finite element method (FEM) reported use continuum assumption and apparent density and anisotropy variables to represent microstructural properties. Several attempts have been made to develop a model suitable to study stresses and strains at the microstructural level (Gibson, 1985; Williams and Lewis, 1982; Beaupré and Hayes, 1985). Recently, Hollister and Fyhrie (1991) introduced homogenization theory in combination with the FEM for modeling trabecular bone in more detail. This theory assumes trabecular bone to be a structure of unit cells, each having the same micro-structural morphology. Owing to periodicity in the FEM solution, the local stress distribution can be evaluated from a global model. The unit cells can either be simplified, regular structures (Hollister et al., 1991), or represent a bone detail of trabecular morphology (Hollister and Kikuchi, 1992a).

Two methods are presently available to measure and represent the trabecular structure on a micro level. One is a nondestructive method, using micro-CT-scanning, whereby a voxel size of 50 micron can be obtained (Feldkamp et al., 1989). The other technique involves 3-D serial reconstruction using microtome slices (Odgaard et al., 1990), which has a smallest possible voxel size of about 20 microns. Until recently, however, efforts to represent a substantial piece of bone in a FE model, for example in the size of a mechanical test specimen, have failed due to restrictions in computer memory and cpu-time.

The purpose of the present study was to develop a new FE strategy which does enable such a full, realistic 3-D analysis. To this end, use is made of the morphological characteristics of the voxel reconstruction and new algorithms which take advantage of the vectorizing capacity of computers. In this paper, the results of the first analyses are presented.

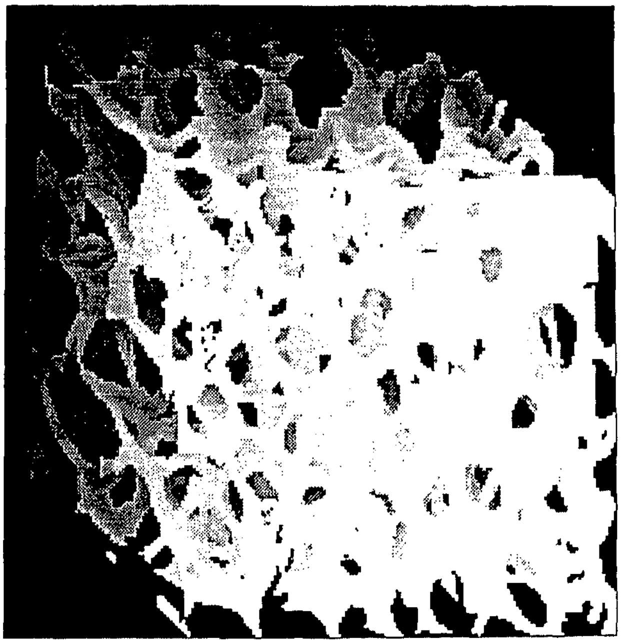

FIGURE 1. THE THREE DIMENSIONAL RECONSTRUCTION OF THE TRABECULAR BONE CUBE, WHICH ALSO REPRESENTS THE FE-MODEL.

METHODS

The three dimensional microstructure of a characteristic piece of trabecular bone was obtained by digitizing 128 slices from a 5mm cube of bone taken from the proximal human tibia. A three dimensional reconstruction of this bony cube is shown in Figure 1. The rectangular voxels in the digitized cross sections were converted to equally sized elements using a three dimensional FE pre-processor developed in our laboratory. The connectivity of the resulting element mesh was checked for elements not connected to the main structure. All elements in the model are rectangular and equally sized: 40.26 by 28.44 by 40 microns in x-, y- and z-direction respectively. The cube has $N_{grid}=128$ voxels at each side (Fig. 2), hence, $(128)^3$ voxels in total of which $N_{el}=296{,}676$ represent bone. Accordingly, the volume fraction of the bone thus modeled equals $\nu=N_{el}/(N_{grid})^3= 0.14$. The total number of degrees of freedom equals 1,381,602.

A uniformly distributed unit load in the z-direction was applied at the top of the model at $z=z_{max}$ (Fig. 2). Boundary conditions were chosen such that the displacements in the x-direction were constrained at the $x=x_{min}$ and the $x=x_{max}$ boundaries, in the y-direction at the $y=y_{min}$ and the $y=y_{max}$ boundaries, and in the z-direction at the $z=z_{min}$ boundary.

All elements in the model were given isotropic material properties with a Young's modulus of 1000 MPa, and a Poisson's ratio of 0.3.

A special purpose FE-code was developed to solve the

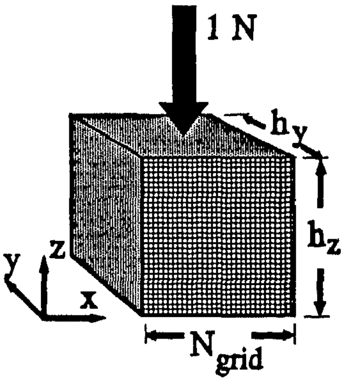

FIGURE 2. DIMENSIONS AND LOADING OF THE FE-MODEL

set of equations that results from the finite element approach. In this code the Conjugate Gradient Iterative Solving technique, with diagonal scaling as a preconditioner was used (Strang, 1986; Hughes, 1987). Each iteration requires a multiplication of the global stiffness matrix with a vector. The memory required to solve this problem was reduced by three separate strategies. First, the entire global stiffness matrix is never actually computed. Instead, an Element By Element (EBE) approach (Hughes et al.; 1987) is applied which allows for the multiplication at element level. Second, because all elements are identical in size, the global structure can be described by the element connectivity alone. Hence, no nodal coordinate data has to be stored. Finally, all elements have identical material properties which allows for an identical element stiffness matrix for all elements. This reduction even applies when each element would have a different Young's modulus, because the isotropic element stiffness matrix is a linear function of this modulus. Compared to traditional EBE-solving techniques which must store all elements stiffness matrices, the required core size is drastically reduced. The matrix-vector multiplication was performed in a highly vectorized manner (Hughes et al.; 1987, Hayes and Devloo; 1986), taking full advantage of the CRAY YMP computer used for the calculations.

The solution was considered sufficiently accurate if the error in the calculated strain field was less than 25 microstrain. For the element size used in the present study (40 micron), this criterium implies a maximum allowable error in the displacement field of 1E-6mm (1 micron). Residual forces were calculated to check for the accuracy.

At tissue level, results will be presented for the tissue strain energy density distribution, calculated directly from the element nodal displacements and forces, and the element volume. These values are compared to the apparent strain energy density, which was calculated from the external nodal forces and the corresponding

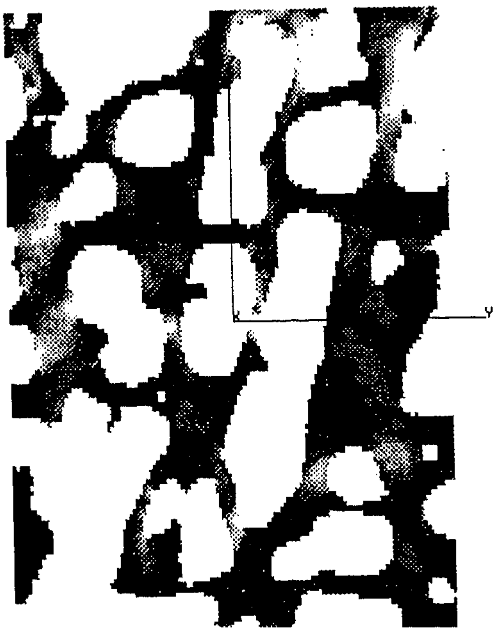

FIGURE 3. STRAIN-ENERGY DENSITY DISTRIBUTION IN A TEN ELEMENTS THICK SLICE TAKEN FROM THE MIDDLE OF THE MODEL. WHITE AREAS ARE HIGHLY LOADED, BLACK AREAS ARE UNLOADED.

displacements, and the apparent volume.

To compare the results with those of earlier continuum models, the apparent Young's modulus for the specimen as a whole was calculated from the formula $E_a=\sigma_a/\varepsilon_a$, where $\sigma_a=1/(h_x.h_y)$ is the apparent stress due to the applied unit load, and $\varepsilon_a=u_m/h_z$ is apparent strain; u_m represents the mean displacement of the loaded area, and h_x, h_y and h_z represent the external dimensions of the cube in x-, y-, z-directions, respectively.

RESULTS

The Conjugate Gradient Iterative solver used a total of 5074 iterations and 2.5 hours cpu time to obtain the solution within the accuracy interval. Residual forces were found to be less than 1E-5 N.

The calculated tissue strain-energy density distribution is shown in Fig. 3 for a 10 elements thick slice taken from the middle of the cube. Throughout the cross section, the SED distribution shows relatively high loaded regions inside trabeculae oriented in the overall load direction. To quantify the results for the whole structure, the ratio between the apparent SED U_a and the microstructural tissue SED U_t in each element was calculated. Results are presented in the histogram of Figure 4. The mean ratio of tissue and apparent SED was $(U_t/U_a)_{mean}$= 7.07. The maximum value for this ratio was $(U_t/U_a)_{max}$=16,343. To investigate whether this high maximum value was due to boundary effects, the

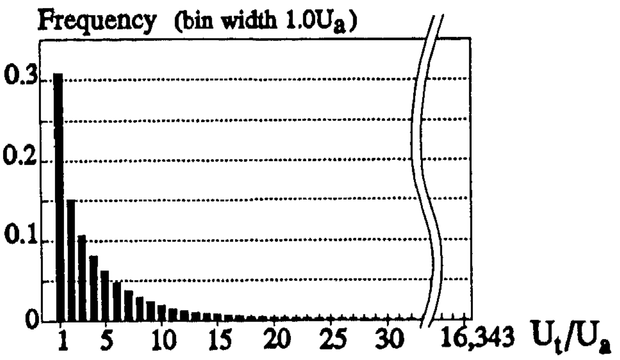

FIGURE 4. DISTRIBUTION OF THE TISSUE STRAIN-ENERGY DENSITY U_t OVER THE APPARENT STRAIN-ENERGY DENSITY U_a. THE BIN WIDTH IS SET TO $1.0U_a$.

maximum ratio was calculated again, now excluding the elements close to the boundary i.e. within 20 grid units. The maximum U_t/U_a ratio in the remaining structure was 268.7.

The average displacement of the externally loaded nodal points was u_m=9.68E-3mm. The apparent Young's modulus calculated from this value was E_a= 28.2MPa.

DISCUSSION

In the present paper it is shown that the model developed can be used to obtain relations between local tissue quantities and apparent quantities, for a reasonably large piece of trabecular bone.

The mean ratio of tissue and apparent SED determined in the present model equals exactly the estimation for this value U_a/ν=7.07, as introduced by Carter et al. (1987). This result confirms the validity of the local strain-energy density calculation as the total apparent energy V_aU_a must equal the total internal energy in the load carrying structure $V_t(U_t)_{mean}$, where V_a is the apparent volume and $V_t=\nu V_a$ the total tissue volume. However, the tissue SED distribution shows a wide range of values, with a maximum U_t/U_a ratio of about 2312 times the average value although the number of elements with excessive SED values is relatively small (Fig. 4). If the elements close to the boundary are excluded, this maximum ratio drops to 38 times the average value. It is possible that a more physiological loading condition (for instance obtained from multiaxial load cases) will further reduce the maximal SED value.

Assuming that the bone tissue is a homogeneous, isotropic and linear elastic material, the actual value of its Young's modulus can be obtained by scaling the calculated apparent modulus to an experimentally determined value. For a human proximal tibia, the mean apparent Young's modulus can be estimated from the apparent density using for instance the formula of

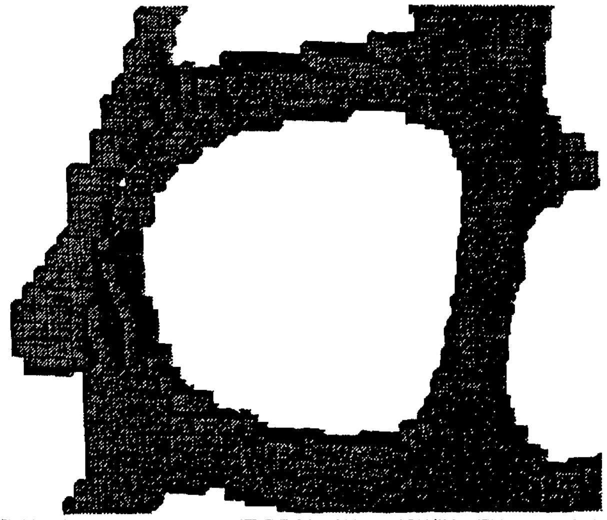

FIGURE 5. DETAIL OF THE FE-MODEL SHOWING THE ELEMENT SIZE RELATIVE TO THE TRABECULAR WIDTH.

Hodgkinson and Currey (1992): $E_a = 10^{(-2.43+1.96\log\rho)}$. In the present example an approximated value for the apparent density is 280 kgm^{-3} (approximated from the volume fraction of 0.14). In the above formula this gives an apparent Young's modulus of 232.5 MPa . We have calculated an apparent Young's modulus of 28.2 MPa with a tissue modulus of 1000 MPa in the present model. Since the model is linear, a tissue modulus of 8245 MPa would give exactly the apparent modulus value of 232.5 MPa. The tissue value of 8245 MPa is in agreement with results found by Hollister (1992a) who suggested that the tissue modulus should be greater than 5000 MPa. A better determination of the tissue modulus can be obtained by experimentally testing the same specimens that are actually modeled. These experiments can also be used to determine material properties that are a non-linear function of the deformations, such as the Poisson's ratio. Optimization procedures are required to fit experimental and FE model results, from which the unknown parameters can be obtained.

The geometry of the FE-model was chosen such that the average trabecular cross section as seen in the model was covered with at least four elements. A typical trabecula in the model is shown in Fig. 5. The jagged boundary that can be seen in the digitized mesh may introduce inaccuracies in the calculations. However, Hollister (1992b) has shown that the resulting errors in the stiffness determination resulting from the digitization will be less than 10% for images with 50 micron cubic voxels where the major structures are 100-200 microns in thickness. For the 40 micron voxels used in the present study, the resulting error is expected to be less than the error found by Hollister.

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