# Inventory replenishment in retail : the efficient full service strategy 

## Citation for published version (APA):

Donselaar, van, K. H., \& Broekmeulen, R. A. C. M. (2008). Inventory replenishment in retail : the efficient full service strategy. (BETA publicatie : working papers; Vol. 243). Technische Universiteit Eindhoven.

## Document status and date:

Published: 01/01/2008

## Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

## Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.
Link to publication


## General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25 fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

## Take down policy

If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.

# Inventory replenishment in retail: The Efficient Full Service strategy 

K.H. van Donselaar and R.A.C.M. Broekmeulen,<br>Eindhoven University of Technology, Dept of Technology Management, PO Box 513, 5600 MB Eindhoven, The Netherlands.<br>k.h.v.donselaar@tue.nl; r.a.c.m.broekmeulen@tue.nl


#### Abstract

In this paper we compare two inventory replenishment strategies for retailers and evaluate their effect on service levels, average inventory and the number of order lines. The first inventory replenishment strategy we consider is the Full Service strategy which is currently applied by many retailers and orders at a review period if either a case pack fits onto the shelves or the minimum reorder level is reached. This strategy is compared to an Efficient Full Service strategy where an order is placed only if at areview period the inventory position drops below the minimum reorder level; then as many case packs are ordered as possible taking into account the limited shelf space. This modified strategy will be compared with the current strategy. We will derive approximations for the key performance indicators and use simulation based on empirical data for thousands of SKU's at multiple stores from a European retailer to quantify the improvement potential of the new strategy and to evaluate our approximations. The results show that, on average, inventory can be reduced with $22 \%$ and the number of handled order lines can be reduced with $17 \%$ when applying the Efficient Full Service strategy, while guaranteeing the same target customer service level. The approximations for the average inventory and the number of order lines perform very well at the store level and perform well at the SKU level. We also show that these approximations can be used as good indicators for the improvement potential of the new replenishment strategy.


Keywords: Retail, Inventory, Replenishment, Handling, Order lines, Efficient Full Service

## 1. Introduction

Retailers are often dealing with an inventory replenishment environment in which deliveries are periodically (based on a delivery schedule per store), replenishment quantities are an integer multiple of a fixed case pack size, sales follow a weekly pattern with peak sales on Friday and Saturday and shelf space per SKU is limited. In such an environment many retailers apply a Full Service (FS) replenishment strategy. In this paper we will formalize the replenishment logic of this strategy and also introduce an alternative: the Efficient Full Service (EFS) strategy. We will determine the improvement potential of the new strategy and derive and test approximations for the key performance indicators for both strategies using simulation based on an extensive dataset of an European retailer.

The key performance indicators used to compare these two replenishment strategies, given the target customer service level they both aim for, are the average inventory and the number of order lines. Although historically the comparison of two replenishment strategies often focused on inventory levels, we explicitly also consider the number of order lines. This is based on two observations in the literature on Retail Operations. According to [1] handling costs and inventory costs at the operational level represent $66 \%$ and $12 \%$ of the retail supply chain costs, which shows that handling is a very important cost component. Furthermore [2] notes that the handling costs are mainly determined by the number of consumer units (CU) handled, the number of case packs and the number of order lines per year. Since the demand per year and the case pack size
are given per SKU, the number of order lines per year is the only component of the handling time which can be influenced by the inventory replenishment strategies.

In section 2 we will describe the two replenishment strategies in more detail. In section 3 we will derive approximations for the average inventory and the number of order lines for both strategies. The dataset and the simulation, which are used to determine the potential of the new strategy and to test the quality of the approximations, are described in section 4. In section 5 the results are presented and conclusions are given in section 6.

## 2. The two inventory replenishment strategies

Before describing the two replenishment strategies in more detail, we define the variables used in this paper in the following table:

| $S K U$ | Stock Keeping Unit | $I P$ | Inventory Position [CU] |
| :--- | :--- | :--- | :--- |
| $V$ | Adapted shelf capacity [CU] | $s$ | Reorder level [CU] |
| $Q$ | Case pack size [CU] | $s s$ | safety stock [CU] |
| $q$ | Order quantity [CU] | $\mu$ | Average demand per day [CU/day] |
| $P_{2}$ | Fill rate | $\hat{\mu}$ | Forecasted demand per day [CU/day] |
| $P_{2}^{*}$ | Target fill rate | $R$ | Review period [days] |

Table 1: Definitions of the variables used
To make sure the inventory does not get too high, companies often set a limit on the maximum inventory on hand, equal to $p$ days of the forecasted demand. To take this into account we will use the adapted shelf capacity $V$ in this paper, with $V=\min \{p h y s i c a l ~ s h e l f ~ c a p a c i t y, p \hat{\mu}\}$.

### 2.1 Full Service strategy

In the Full Service (FS) strategy, the order quantity at a review period is equal to:

$$
\begin{equation*}
q=\max \left\{\left\lfloor\frac{(V-I P)}{Q}\right\rfloor \cdot Q,\left\lceil\frac{(s-I P)}{Q}\right\rceil \cdot Q, 0\right\} \tag{1}
\end{equation*}
$$

with $\lfloor x\rfloor$ resp. $\lceil x\rceil$ representing the nearest integer less or equal to $x$ resp. greater or equal to $x$. The first term on the right-hand side reflects the basic idea behind the FS strategy: at a review period we order as many case packs that fit on the shelf, given the current inventory position $I P$. The second term on the right-hand side is needed in order to satisfy the requirement that the fill rate $P_{2}$ is at least equal to the target fill rate $P_{2}^{*}$ for a $S K U$. So if the inventory position $I P$ is less than the reorder level $s$, which is based on the target fill rate, we need to order the minimum number of case packs which is needed to raise $I P$ back to or above the reorder level. Finally the third term reflects the notion that the order quantity should always be non-negative (due to the dynamic $s$ the $I P$ may sometimes be larger than $s+Q$ ).

The replenishment strategy described above is a generalization of the Full Service strategy as described by [3] by incorporating the possibilities that the reorder level may be larger than the shelf space minus one case pack size, $s$ may be dynamic and/or case pack sizes may be larger than one consumer unit.

### 2.2 Efficient Full Service strategy

The Efficient Full Service strategy (EFS) is similar to the FS strategy, but aims to minimize the number of order lines per year, while still guaranteeing the target service level. If at a review period the inventory position is strictly below the reorder level $s$, we order the maximum number of case packs such that the inventory position (IP) after ordering is less than or equal to the shelf capacity $V$. Unless this $I P$ is still below $s$, i.e., the shelf is not large enough to accommodate all units, then we order as many case packs as needed to bring the inventory position after reordering to (or just above) $s$. In summary: if at a review period $I P$ is strictly less than $s$, the order quantity becomes:

$$
\begin{equation*}
q=\max \left\{\left\lfloor\frac{(V-I P)}{Q}\right\rfloor \cdot Q,\left\lceil\frac{(s-I P)}{Q}\right\rceil \cdot Q, 0\right\} \text { if } I P<s \tag{2}
\end{equation*}
$$

Note that this is the same formula as in Full Service, but now the order is only triggered when $I P<s$. This EFS strategy extends and generalizes the (s, S, Q)-strategy proposed by [4] by including situations in which the shelf space is smaller than the reorder level, demand is not stationary or in which the shelf space and reorder level are not a strict multiple of $Q$.

### 2.3 Comparison of the Full Service and the Efficient Full Service strategy: an example

The EFS strategy typically differs from the FS strategy for SKU's with ample shelf capacity. The example in Figure 1 shows the typical pattern of the inventory position for both strategies for such a SKU (canned mushroom slices). This SKU has the following characteristics: $V=48$; $Q=12 ; \mu=3.6$. The review period is equal to 2 days (the inventory is reviewed at odd days) and the lead-time is equal to 1 day. For this SKU, the FS strategy keeps the shelf as full as possible


Figure 1 The inventory position pattern for the FS and EFS strategy; an example.
and orders as soon as the inventory position drops below $37(=V-Q+1)$; then a new case pack fits on the shelf. The EFS strategy only orders when the inventory position drops below the EFS reorder level. The EFS reorder level depends on a dynamic forecast and for this SKU it varies around 12. Note that in this example the number of case packs per order line in the FS strategy is equal to one, while in the EFS strategy this is equal to two or three case packs. As a result the EFS strategy has fewer order lines with larger order quantities. Since the reorder level in the EFS strategy is much lower than in the FS strategy, the average inventory in the EFS strategy is also lower than in the FS strategy.

## 3. Approximations

In this paragraph, we establish closed-form expressions that approximately describe the behavior of the percentual difference in inventory $\Delta\left(I_{F S}, I_{E F S}\right)$ and in order lines $\Delta\left(O L_{F S}, O L_{E F S}\right)$, when comparing the FS and EFS replenishment strategies. We define a percentual difference as follows:

$$
\begin{equation*}
\Delta\left(X_{B A S E}, X_{A L T}\right)=\frac{\left(X_{B A S E}-X_{A L T}\right)}{X_{\text {BASE }}} \cdot 100 \tag{3}
\end{equation*}
$$

In this section we derive simple approximations for the average inventory and the average number of order lines, denoted by $\hat{I}$ and $\hat{O} L$, for both strategies. These can be used to approximate the reduction in inventory and number of order lines by calculating $\Delta\left(\hat{I}_{F S}, \hat{I}_{E F S}\right)$ and $\Delta\left(\hat{O} L_{F S}, \hat{O} L_{E F S}\right)$

Before these approximations are derived, we make two main assumptions: delivery moments are equidistant and demand is stationary. Both in reality and in our simulation experiments delivery moments are not always equidistant and demand follows a weekly pattern with peak demand on Friday and Saturday.

To derive an approximation for the average inventory, we start with the notion that the FS strategy is actually a discrete ( $\mathrm{R}, \mathrm{s}, \mathrm{nQ}$ )-strategy with $s_{F S}=\max \{V-Q+1,\lceil(L+R) \mu+s s\rceil\}$, with ss an integer safety stock. This safety stock is usually set to meet a pre-determined target service level. For a discrete ( $\mathrm{R}, \mathrm{s}, \mathrm{nQ}$ )-strategy with stationary demand, the inventory position just after a review moment is known to be uniformly distributed between $s-1$ and $s-1+Q$ with expectation $s+(Q-1) / 2$ and its expected net inventory $L$ days after a review moment is $s+(Q-1) / 2-L \mu$ (see [5]). In the next $R$ days, the average net inventory decreases with $\mu$ each day, leading to an average net inventory measured at each day equal to $s+(Q-1) / 2-L \mu-(R-1) \cdot \mu / 2$. When we approximate the average physical inventory by the average net inventory, this results in the following approximation for the average physical inventory in the FS replenishment strategy:

$$
\begin{align*}
\hat{I}_{F S} & =s_{F S}+\frac{Q-1}{2}-\left(L+\frac{R-1}{2}\right) \cdot \mu \\
& =\max \left\{V-Q+1,\lceil(L+R) \mu\rceil+s s_{F S}\right\}+\frac{Q-1}{2}-\left(L+\frac{R-1}{2}\right) \cdot \mu \tag{4}
\end{align*}
$$

Similar, we approximate the average physical inventory for the Efficient Full Service (EFS) strategy by using the same approximation but now replacing $s_{F S}$ with $s_{E F S}=\lceil(L+R) \mu\rceil+s s_{E F S}$ and replacing $Q$ with $\hat{N} \cdot Q$, where $\hat{N}$ is an approximation of the number of case packs ordered. Summarizing, this leads to the following expressions:

$$
\begin{equation*}
\hat{I}_{E F S}=s_{E F S}+\frac{\hat{N} \cdot Q-1}{2}-\left(L+\frac{R-1}{2}\right) \cdot \mu=\lceil(L+R) \mu\rceil+s S_{E F S}+\frac{\hat{N} \cdot Q-1}{2}-\left(L+\frac{R-1}{2}\right) \cdot \mu \tag{5}
\end{equation*}
$$

To find an approximation for $\hat{N}$, we note that the number of case packs ordered increases with the undershoot. The undershoot is defined as the difference between $s_{E F S}-1$ and the inventory position just before ordering and is denoted by the variable $U$. If e.g. for an SKU with $V=48$, $Q=12$ and $s_{E F S}=15$ the inventory position at a review period is equal to $I P=14$, then the undershoot is equal to zero and $\lfloor(48-14) / 12\rfloor=2$ case packs are ordered, while if the inventory position is equal to $I P=12$, then the undershoot is equal to 2 and $\lfloor(48-12) / 12\rfloor=3$ case packs are ordered. In [6] the following simple estimators for the mean and variance of the undershoot in a periodic review system are derived:

$$
\begin{align*}
& E[U]=\frac{1}{2}\left(1+c_{R}^{2}\right) \mu_{R},  \tag{6}\\
& \operatorname{var}[U]=\frac{1}{3}\left(1+c_{R}^{2}\right)\left(1+2 c_{R}^{2}\right) \mu_{R}^{2}-\frac{1}{4}\left(1+c_{R}^{2}\right)^{2} \mu_{R}^{2} \tag{7}
\end{align*}
$$

where $\mu_{R}$ and $c_{R}$ are the mean and coefficient of variation of the demand during R periods. Next, we distinguish two situations: $V-s_{E F S}+1<Q$ and $V-s_{E F S}+1 \geq Q$. In the first case, i.e., if $V-s_{E F S}+1<Q$, we simply assume we will always order one case pack, so $\hat{N}=1$. In the second case, i.e. if $V-s_{E F S}+1 \geq Q$, the number of case packs ordered depends on the undershoot. If the undershoot would be zero, then the inventory position at the time of ordering would be $s_{E F S}-1$ and so we order $n_{0}$ case packs of size Q , with $n_{0}=\left\lfloor\left(V-s_{E F S}+1\right) / Q\right\rfloor$. If the undershoot is sufficiently large, we may order $n_{0}+1$ case packs. For sake of simplicity we restrict ourselves in our approximation to the possibilities that either $n_{0}$ or $n_{0}+1$ case packs are ordered. The probability that $n_{0}$ case packs are ordered, denoted by $P\left(N=n_{0}\right)$, is equal to the probability that $V-s_{E F S}+1+U$ is strictly less than $\left(n_{0}+1\right) Q$, so:

$$
\begin{equation*}
P\left(N=n_{0}\right)=P\left(U<\left(n_{0}+1\right) Q+s_{E F S}-1-V\right) . \tag{8}
\end{equation*}
$$

This probability can be calculated easily, using equations (3) and (4) above and assuming that the undershoot is gamma distributed. Our estimator for the expected number of case packs per order line is then equal to:

$$
\hat{N}= \begin{cases}1 & \text { if } n_{0} \leq 0  \tag{9}\\ n_{0} P\left(N=n_{0}\right)+\left(n_{0}+1\right)\left(1-P\left(N=n_{0}\right)\right) & \text { otherwise }\end{cases}
$$

As the last step, we now derive approximations for the number of order lines: $\hat{O} L_{F S}$ and $\hat{O} L_{E F S}$. First we note that the number of order lines per day is equal to the average sales per day divided by the average order size. If we approximate the average sales by the target fill rate $P_{2}^{*}$ times the average demand, we get: $\hat{O} L=P_{2}^{*} \mu / \hat{q}$. To approximate $\hat{q}$ for the FS-strategy we define $n_{U}$ as the number of case packs ordered if the inventory position just before ordering is at $s_{F S}-1-E[U]$, so $n_{U}=\lceil(1+E[U]) / Q\rceil$ and we define $p_{U}$ as $P\left(U \leq n_{U} Q\right)$. Next, we approximate $\hat{q}_{F S}$ as follows:

$$
\begin{equation*}
\hat{q}_{F S}=\max \left\{R \mu, n_{U} Q p_{U}+\left(n_{U}+1\right) Q\left(1-p_{U}\right)\right\} \tag{10}
\end{equation*}
$$

This gives:

$$
\begin{equation*}
\hat{O} L_{F S}=\frac{P_{2}^{*} \mu}{\max \left\{R \mu, n_{U} Q p_{U}+\left(n_{U}+1\right) Q\left(1-p_{U}\right)\right\}} \tag{11}
\end{equation*}
$$

The approximation for $\hat{O} L_{E F S}$ is derived in a similar fashion, after $\hat{q}_{E F S}$ is approximated by : $\hat{q}_{E F S}=\max \left\{R \mu, n_{U} Q p_{U}+\left(n_{U}+1\right) Q\left(1-p_{U}\right), \hat{N Q} Q\right.$, where $\hat{N}$ is determined by equation (9).

This gives:

$$
\begin{equation*}
\hat{O} L_{E F S}=\frac{P_{2}^{*} \mu}{\max \left\{R \mu, n_{U} Q p_{U}+\left(n_{U}+1\right) Q\left(1-p_{U}\right), \hat{N} Q\right\}} \tag{12}
\end{equation*}
$$

Note that $\hat{O} L_{E F S} \leq \hat{O} L_{E F S}$, since the denominator in (12) is always larger than or equal to the denominator in (11).

## 4. Datasets and Simulation

To determine the improvement potential of the new replenishment strategy and to evaluate our approximations we used empirical data on daily sales, product attributes and available shelf space from an European supermarket-chain. We focused on dry groceries which are delivered from the retailers' DC. The lead-time from the DC to the stores is one day. The delivery frequency differs per store; it varies from 2 to 5 deliveries per week. The stores differ substantially in size and in sales volume. For more details on the characteristics of the stores and the products we refer to the descriptive statistics in section 5.1. Table 1 lists all merchandising categories which were included in the experiment. The experiment contained 184,901 SKU's
from 44 stores. Sales in these stores typically follow a seasonality pattern during the week with high sales on Friday and Saturday ( $>50 \%$ of weekly sales) and no sales on Sunday. Median sales per SKU are 3.7 CU/week.

| Baby food | Cosmetics | Pet food |
| :--- | :--- | :--- |
| Beer | Crackers / Cereals | Paper ware |
| Bread spreads | Dessert components | Sauces / Acids / Oils |
| Canned fruits | Diapers | Soft drinks / Juices |
| Canned meat / fish | Dairy (ambient storage) | Soups |
| Canned vegetables | Baking ingredients | Spices |
| Chips / Nuts | Frying fat \& oil | Sweets |
| Cleaning detergents | Health food | Washing detergents |
| Coffee creamers | Household | Wines |
| Coffee / Tea | Margarine / Butter |  |
| Cookies | Meal components |  |

Table 1: List of merchandising categories included in our dataset
We carried out a simulation in which weekly demand is gamma distributed (as suggested by [7]) with mean and standard deviation based on the empirical data. Promotions are eliminated when we calculate the mean and the standard deviation of the sales per week. Demand for SKU's which are out-of-stock is lost. Since daily demand varies strongly within the week (due to the seasonality pattern), we used dynamic reorder levels which are equal to the forecasted demand during the lead-time plus review period plus the safety stock. We then optimized the safety stock for each SKU and each target service level and each replenishment strategy. Finally, the maximum inventory level expressed in weeks of average demand is set equal to $p=72$ selling days (equal to 12 weeks).

The timing of events in the simulation is: during opening hours inventory decreases due to customers' demand and after closing the store the service level is calculated, goods arrive in the backroom and are stacked on the shelves, inventory is counted and finally orders are placed before opening the store. SKU's for which the daily demand is larger than the available shelf space are also replenished from the backroom during the day to prevent out-of-stocks.

Following [8], the reported values for the simulation are the averages from at least 10 replications. In each replication, the first 50 weeks were the warming-up periods and statistics are recorded for the last 1000 weeks. We replicated until a $95 \%$ confidence interval was reached for the fill rate $P_{2} \pm 0.005$.

## 5. Empirical results

In this section, we show the results obtained in the analysis of the FS and the EFS strategy. The analysis is split up into two parts: first we analyse the performance improvement when changing from a FS strategy to an EFS strategy (Section 5.1) and then we analyze the quality of our approximations (Section 5.2). Both parts are done for the scenario where the target fill rate $P_{2}^{*}$ equals $98 \%$, which is the target set by our retailer. The last section (Section 5.3) analyses
the sensitivity of the results, for example by evaluating the effects of changing the target fill rate $P_{2}^{*}$.

### 5.1 Performance improvement

This section describes the performance improvement in terms of reduction of inventories and number of order lines, when changing from a FS strategy to an EFS strategy. The performance improvement is first analyzed at the store level. All data is aggregated over all SKUs for each of the 44 stores in our dataset: e.g. total adapted shelf space ( $V^{s}$ ), the total daily sales $\left(\mu^{s}\right)$ are obtained by summing up all SKU level data within each store. Table 2 shows the descriptive statistics for each of the variables.

|  | Mean | Median | Std Deviation | Min | Max |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mu^{s}$ | 5706 | 5239 | 2088 | 2801 | 11000 |
| $V^{s}$ | 94626 | 96684 | 15828 | 64165 | 121110 |
| $R$ | 2.14 | 2 | 0.48 | 1.20 | 3 |
| $I_{F S}^{s}$ | 75285 | 76082 | 13607 | 49228 | 97622 |
| $I_{E F S}^{s}$ | 58507 | 57075 | 9735 | 39882 | 78568 |
| $O L_{F S}^{s}$ | 380.57 | 350.23 | 124.63 | 202.86 | 685.46 |
| $O L_{E F S}^{s}$ |  | 315.69 | 283.44 | 100.92 | 168.65 |
| $\Delta\left(I_{F S}^{s}, I_{E F S}^{s}\right)$ | $[\%]$ | 22.00 | 22.25 | 3.29 | 15.03 |
| $\Delta\left(O L_{F S}^{s}, O L_{E F S}^{s}\right)[\%]$ | 16.68 | 17.29 | 4.53 | 8.60 | 28.92 |

Table 2: Descriptive statistics (44 store observations)
In Table 2, $I_{F S}^{s}$ and $I_{E F S}^{s}$ represent the total average inventory for store s, summed over all SKU's of store s, for the FS and the EFS replenishment strategy; $\Delta\left(I_{F S}^{s}, I_{E F S}^{s}\right)$ represents the percentual reduction in total inventory for store $s$, when changing from the FS strategy to the EFS strategy.

Table 2 shows a very large performance improvement when changing from a FS strategy to an EFS strategy: inventories per store are reduced by $22.0 \%$ on average, while the number of order lines per store are reduced by $16.7 \%$ on average. The performance improvement varies per store, particularly for the reduction in number of order lines. This variation is no surprise, since the stores are quite different; e.g. the maximum daily demand per store is almost four times as large as the minimum daily demand and some stores are delivered only twice per week $(R=3)$, while others are delivered five times per week $(R=1.2)$. Despite this variation, all stores clearly benefit from the EFS strategy.

Table 3 shows the descriptive statistics of the same variables as in Table 2, but now at the SKU level: Table 3 confirms our store level insights: the EFS strategy results in less inventory and less order lines with a reduction of $16.80 \%$ in inventory and $13.78 \%$ in order lines respectively. These numbers are slightly different than the ones reported above, but this is due to the different level of analysis (stores versus SKU). Table 3 also shows that the performance improvement varies strongly between SKU's, as is evident from the large standard deviation (17.86 resp.
23.47) relative to the mean performance improvement ( 16.80 resp. 13.78) for the reduction in inventories resp. the number of order lines. This variation is again explained by the large differences between SKU's. The highest average daily demand per SKU e.g. is more than 60,000 times the lowest average daily demand. Also the available shelf capacity and the case pack size differ greatly per SKU. The reason for a relatively high percentage for the inventory reduction is due to the fact that a substantial reduction in number of order lines can only be achieved if the difference between the shelf capacity and the EFS reorder level is at least equal to two case packs, while the inventory can already be reduced with the EFS strategy when this difference is approximately equal to one case pack.

|  | Mean | Median | Std | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Deviation |  |  |  |  |
| $\mu$ | 1.358 | 0.623 | 3.534 | 0.004 | 255.333 |
| V | 22.52 | 18 | 19.85 | 1 | 522 |
| $R$ | 2.13 | 2 | 0.47 | 1.2 | 3 |
| $Q$ | 11.66 | 12 | 6.63 | 1 | 100 |
| $I_{F S}$ | 17.915 | 12.759 | 20.187 | 2.378 | 1032.009 |
| $I_{\text {EFS }}$ | 13.923 | 10.210 | 16.777 | 2.378 | 1032.009 |
| $O L_{\text {FS }}$ | 0.0906 | 0.0582 | 0.0934 | 0.0001 | 0.8256 |
| $O L_{\text {EFS }}$ | 0.0751 | 0.0444 | 0.0845 | 0.0001 | 0.8127 |
| $\Delta\left(I_{F S}, I_{E F S}\right) \quad[\%]$ | 16.80 | 12.08 | 17.86 | -34.35 | 61.93 |
| $\Delta\left(O L_{F S}, O L_{E F S}\right)[\%]$ | 13.78 | 0.89 | 23.47 | -17.25 | 96.77 |

### 5.3 Quality of the approximations

To test the quality of our approximations we start at the SKU level, since this is the original level from which we derived our approximations. The first four rows in Table 4 show the relative approximation error for the average inventory and the average number of order lines per SKU. It shows that the mean error is close to zero, while the standard deviation of the error is low for the inventories and moderate for the number of order lines. The latter is due to some extreme values. If we look in more detail at the frequency distribution of the relative errors (see Figure 2), we note that $94 \%$ of the SKU's have a relative error for the number of order lines for the FS strategy which is between $-5 \%$ and $+5 \%$. In conclusion, the approximations at SKU level perform well for the vast majority of SKU's.

The approximations at SKU level can be used very well to get an approximation of the total inventory and number of order lines per store. The last four rows in Table 4 show the approximation errors at store level for both strategies. Both the mean and the standard deviation of the errors are close to zero.

| $\Delta(\cdot \cdot)[\%]$ | Mean | Median | Std Deviation | Min | Max |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\Delta\left(I_{F S}, \hat{I}_{F S}\right)$ | -0.42 | -0.46 | 2.62 | -24.78 | 21.19 |
| $\Delta\left(I_{E F S}, \hat{I}_{E F S}\right)$ | 1.07 | 0.91 | 3.36 | -16.63 | 25.29 |
| $\Delta\left(O L_{F S}, \hat{O} L_{F S}\right)$ | 0.21 | 0.22 | 5.63 | $-46,42$ | 65.76 |
| $\Delta\left(O L_{E F S}, \hat{O} L_{E F S}\right)$ | -0.17 | -0.41 | 5.31 | $-46,42$ | 65.76 |
| $\Delta\left(I_{F S}^{s}, \hat{I}_{F S}^{s}\right)$ | -0.62 | -0.79 | 0.87 | -2.23 | 1.27 |
| $\Delta\left(I_{E F S}^{s}, \hat{I}_{E F S}^{s}\right)$ | 0.81 | 0.74 | 1.02 | -1.11 | 2.93 |
| $\Delta\left(O L_{F S}^{s}, \hat{O} L_{F S}^{s}\right)$ | -0.60 | -0.31 | 1.29 | -5.00 | 1.19 |
| $\Delta\left(O L_{E F S}^{s}, \hat{O} L_{E F S}^{s}\right)$ | -0.94 | -0.72 | 0.87 | -3.68 | 0.30 |
| Table 4: Descriptive statistics for approximation errors |  |  |  |  |  |

Table 4: Descriptive statistics for approximation errors


Figure 2: The frequency distribution for the relative approximation error for the inventories and the number of order lines for the FS and the EFS strategy.

The approximations for the inventories and the number of order lines perform well when they are used to approximate the performance improvement when changing from a FS strategy to an EFS strategy. Table 5 reports the difference between the variables $\Delta\left(I_{F S}, I_{E F S}\right)$ and $\Delta\left(\hat{I}_{F S}, \hat{I}_{E F S}\right)$, where $\Delta\left(I_{F S}, I_{E F S}\right)$ represents the reduction in total inventory for store s, while $\Delta\left(\hat{I}_{F S}, \hat{I}_{E F S}\right)$ represents the approximated reduction in total inventory for store s. Table 5 shows that both at the SKU level (the first two rows) and at the store level (the last two rows) the mean difference between the actual reduction and the approximated reduction is close to zero.

| Difference [\%] | Mean | Median | Std Deviation | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta\left(I_{F S}, I_{E F S}\right)-\Delta\left(\hat{I}_{F S}, \hat{I}_{E F S}\right)$ | -1.03 | 0 | 2.44 | -20.68 | 12.98 |
| $\Delta\left(O L_{F S}, O L_{E F S}\right)-\Delta\left(\hat{O} L_{F S}, \hat{O} L_{E F S}\right)$ | 0.35 | 0 | 2.57 | -28.19 | 54.43 |
| $\Delta\left(I_{F S}^{s}, I_{E F S}^{s}\right)-\Delta\left(\hat{I}_{F S}^{s}, \hat{I}_{E F S}^{s}\right)$ | -1.11 | -0.98 | 0.64 | -3.50 | -0.08 |
| $\Delta\left(O L_{F S}^{s}, O L_{E F S}^{s}\right)-\Delta\left(\hat{O} L_{F S}^{s}, \hat{O} L_{E F S}^{s}\right)$ | 0.29 | 0.35 | 0.60 | -1.58 | 1.42 |

### 5.4 Sensitivity analysis

In the above analysis, the target fill rate $P_{2}^{*}$ is set to $98 \%$ which is what the retailer was using at that time. To evaluate and quantify the effect of changing this target fill rate, we performed the same analysis but for a range of $P_{2}^{*} \in\{97 \%, 97.5 \%, 98 \%, 98.5 \%, 99 \%\}$. Figure 3 shows the results for the inventory and order gains achieved when comparing the FS versus the EFS strategy.


Figure 3: The reduction in inventory and number of order lines versus the target fill rate.
Table 6 shows the percentage error made in approximating the performance improvement when changing from an FS strategy to an EFS strategy but now for the different target fill rates. As observed from the table, the error made is declining in the target fill rate.

| $P_{2}^{*}$ | Inventory | Order lines |
| ---: | ---: | ---: |
| 0.970 | -1.26 | 0.58 |
| 0.975 | -1.19 | 0.42 |
| 0.980 | -1.11 | 0.29 |
| 0.985 | -1.01 | 0.15 |
| 0.990 | -0.90 | 0.03 |

Table 6: Error in approximation of the performance improvement versus the target fill rate $P_{2}^{*}$
The Full Service strategy would be greatly simplified if we would only have to consider the shelf space capacity. Then personnel in the store who are responsible for replenishment only would need to check whether one or more case packs fit on the shelf and they do not have to rely on demand forecasting. To evaluate the implications of such a strategy, we studied the system in which the order quantity at a review period is determined by:

$$
q=\max \left\{\left(\frac{(V-I P)}{Q}\right\rfloor * Q, 0\right\}
$$

The weighted fill rate for all SKU's using this simplified strategy is equal to $95.6 \%$. With this simplified strategy $35 \%$ of the SKU's did not reach a fill rate equal to $98 \%$ and $3 \%$ had a fill rate below $85 \%$. This shows that such a simplification has severe consequences for the overall customer service.

## 6. Conclusions and future research

In this paper the Efficient Full Service (EFS) inventory replenishment strategy is introduced and compared with the Full Service (FS) strategy. A simulation study using empirical data has shown that on average inventory can be reduced with $22 \%$ and the number of order lines can be reduced with $17 \%$ guaranteeing the same target service level. The approximations we derived for the average inventory and the average number of order lines performed well, even though they were based on the assumptions that delivery moments were equidistant and demand was stationary, while in the test environment these assumptions were often not valid.

To further support the implementation of the EFS strategy, formulas for the safety stock norms could be derived. Another opportunity for future research is the investigation of the impact of a safety stock norm which is dynamic throughout the week, based on the weekpattern.

## References

1. Broekmeulen, R. A. C. M., J. C. Fransoo, K. H. van Donselaar, T. van Woensel. 2007. Shelf space excesses and shortages in grocery retail stores. Working Paper, Eindhoven University of Technology, The Netherlands.
2. van Zelst, S., K. van Donselaar, T. Van Woensel, R. Broekmeulen and J. Fransoo. 2007. A Model for Shelf Stacking in Grocery Retail Stores: Potential for Efficiency Improvement, Working Paper, Eindhoven University of Technology, The Netherlands, to be published in International Journal of Production Economics.
3. Cachon, G. 2001. Managing a Retailer's Shelf Space, Inventory, and Transportation. Manufacturing \& Service Operations Management, 3(3), 211-229.
4. Hill, R.M. 2006. Inventory control with indivisible units of stock transfer. European Journal of Operational Research. 175 (11) 593-601.
5. Axsater, S. 2006. Inventory Control (International Series in Operations Research \& Management Science), Springer, New York.
6. Tijms, H.C., 1986, Stochastic modelling and analysis: a computational approach, Wiley, Chichester.
7. Burgin, T.A. 1975. The Gamma Distribution and Inventory Control. Operational Research Quarterly, 26, 507-508.
8. Law, A.M. and Kelton, W.D. 2000. Simulation modeling and analysis (third ed.), McGrawHill, Boston.
