

# Analytical model of the interaction force between a rectangular coil and a cuboidal permanent magnet

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### Analytical model of the interaction force between a rectangular coil and a cuboidal permanent magnet.

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#### 1. Introduction

Single-stage magnetically levitated platforms (planar actuators) (e.g. [1]) are being researched for use in the lithographic industry (e.g. wafer stages), which requires a positioning accuracy in the nanometer (nm) range. Such actuators consist of a stator with coils and a plate with a Halbach array of permanent magnets (PMs) mounted on it (translator). To achieve such nm accuracies, the deformation of the translator plate has to be limited. Therefore, we are interested in the force acting on each PM in the array, since these forces cause the deformation. In previous research, the static forces among the PMs in a Halbach array were determined [2]. A next step is to determine the force due to the coils in the stator. For real-time implementation, this model has to be accurate and fast. This paper presents a new analytical method to determine the force between a cuboidal PM and a rectangular coil using analytical equations.

#### 2. Lorentz force on a cuboidal current carrying volume due to a cuboidal permanent magnet

To determine the Lorentz force acting on a cuboidal volume with a uniform current distribution as shown in Fig. 1 (right), the magnetic flux density due to a PM (Fig. 1 (left)) is integrated over the current carrying volume. This flux density is determined using the magnetic nodes method as presented in [2], which is derived from the magnetic surface charge method and assumes a relative permeability ( $\mu_r$ ) of the PM material equal to unity. Analytical solutions for the Lorentz integrals over the cuboidal volume, which are fast and accurate, are found. The analytical equations and a verification of the results by FEM will be presented in the final paper.

#### 3. Modeling a rectangular coil as four cuboidal volumes

A rectangular coil is shown in Fig. 2. Since the analytical equations only offer solutions for the force acting on straight segments, the rectangular coil is modeled as four cuboidal volumes ( $V_{1-4}$ ) shown in Fig. 2 and the Lorentz force acting on the coil is calculated as the sum of the Lorentz forces acting on these volumes. The force acting on the coil can also be obtained from numerical integration, in which case the corner segments can be taken into account. The dimensions of the four cuboidal volumes are optimized to minimize the error due to neglecting the corner segments.

#### 4. Results

A top and side view of a coil and a PM and the dimensions are shown in Fig. 2. The magnetization vector of the PM is in the positive z-direction and indicated by  $M$ , the remanence  $B_r = 1.23\text{T}$  and  $\mu_r = 1.03$ . The orthocyclically wound coil has 319 turns and a current of 2.00A. The dimensions of the four cuboidal volumes to represent the coil are optimized such that  $c_{1,V1}=73.2\text{mm}$ ,  $c_{1,V3}=10.3\text{mm}$ ,  $c_{w,V1}=c_{w,V3}=b_w$ . The force acting on the coil for variation of  $x_c$  and  $y_0=y_c-y_m=0$ ,  $z_0=z_c-z_m=12.2\text{mm}$  is then determined using the analytical models (where the coil is modeled using four cuboidal volumes, neglecting corner segments) and compared with results obtained from measurements (using a 6DOF load cell) and numerical integration (including the corner segments). The results are shown in Fig. 3 and show good agreement with a maximum error between the results obtained from the analytical equations and numerical integration of approximately 4%. This difference is due to the fact that the analytical models neglect the corner segments of the coil. The results obtained from analytical equations and measurements show good agreement, with an error of approximately 5%, which is partly due to the assumption that  $\mu_r = 1$ .

#### 5. Conclusions

The paper presents a new method to calculate the force between a magnet and a rectangular coil using analytical equations. The results show good agreement compared to results obtained from measurements and numerical models. The advantage of such analytical models over numerical models is that the models are fast to solve. A disadvantage is that the corners of the coils cannot be taken into account. The result can be extended for use in the design and control of a magnetically levitated planar actuator such as presented in [1].

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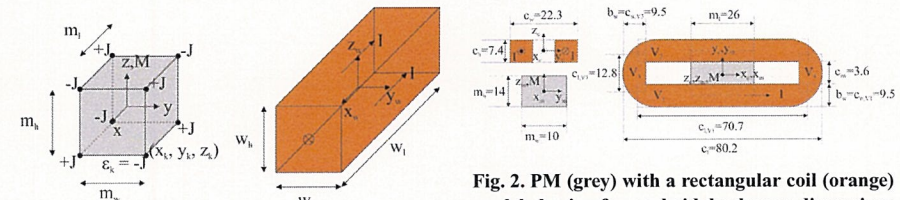


Fig. 1. PM modeled using magnetic nodes method (left) cuboidal current carrying volume (right).

Fig. 2. PM (grey) with a rectangular coil (orange) modeled using four cuboidal volumes, dimensions in mm.

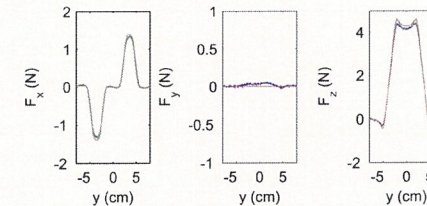


Fig. 3. Forces ( $F_x$ ,  $F_y$ ,  $F_z$ ) acting on the coil due to the PM determined from measurements (in blue), analytical models (in green) and numerical models (in red)