

## Effect of a tuned mass damper in footbridg design

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## EFFECT OF A TUNED MASS DAMPER IN FOOTBRIDGE DESIGN

**C. (Caspar) BREMAN**

MSc-student  
Eindhoven University of Technology  
Eindhoven, The Netherlands

**H.H. (Bert) SNIJDER**

Professor of Steel Structures  
Eindhoven University of Technology  
Eindhoven, The Netherlands

**H.G. (Herke) STUIT**

Senior Consultant  
Movares Nederland B.V.  
Utrecht, The Netherlands

**M.C.M. (Monique) BAKKER**

Associate professor of Numerical Mechanics  
Eindhoven University of Technology  
Eindhoven, The Netherlands

### Summary

Due to their length and slenderness, footbridges near train stations in The Netherlands form a challenge to meet dynamic criteria. A case study is performed to illustrate the effect of a tuned mass damper (TMD). The effect is studied by performing a dynamic analysis using a straightforward type of bridge subjected to a single and multiple pedestrians, modelled as moving harmonic point loads. The response has been determined for a bridge with and without a TMD. The stochastic properties of the load induced by pedestrians are taken into account using a Monte Carlo approach. The results are presented for different load cases which show the effect of a TMD for various group sizes. It is found that the effect of a TMD depends on the number of pedestrians involved.

**Keywords:** footbridge; tuned mass damper; vibration; response; dynamics; Monte Carlo simulation.

### 1. Introduction

Footbridges are a common feature at nearly all train stations in The Netherlands to facilitate safe access to different platforms. Figure 1 for example, shows a typical footbridge near one of the smaller train stations in The Netherlands. Vertical vibration can become a major point of concern when the bridge not only spans two or three main rail tracks but also an additional number of sidetracks. This means that the main span can easily exceed 50 meters, increasing the risk for vibration related problems. In this specific case, a solution by adding additional supports in between tracks is an undesired alternative because of collision hazards. An additional cause for potential vibration problems are common themes in current design such as *slenderness* and *transparency* which are becoming more important not only for aesthetic but also for social safety reasons.



Fig. 1 Typical footbridge near a train station (Boxtel, The Netherlands).



Fig. 2 Waiting area's (boxes) on a footbridge (Lage Zwaluwe, The Netherlands).

This characterisation of footbridges near train stations underlines the necessity for thorough assessment of their dynamic behaviour. This is particularly important for bridges which include waiting areas or other facilities (see figure 2). A more stringent vibration acceptance level applies because sitting people are more sensitive to vibrations than walking people.

In order to prevent dynamic related problems, the size of structural members can be increased to shift the fundamental frequency of the structure out of the critical range of 1 - 5 Hz. This approach often results in heavy and 'chunky' structures. Based on observations in other fields, it is believed that tuned mass dampers (TMD's) can be a valuable alternative. Although the effectiveness of TMD's has been shown in recent years, information on the initial design process is often very brief. This paper aims to illustrate the effect of a TMD by analysing a straightforward type of bridge.

First, the theoretical background is presented in section 2 followed by a case study in section 3 which describes the force and model used for the dynamic analyses of a footbridge. The type of dynamic analysis is discussed in more detail in section 4 followed by the results which are presented and analysed in section 5 and 6. Finally, conclusions and recommendations are given in section 7.

## 2. Theoretical background

The dynamic problems outlined in the introduction are related to the first vibration mode which will be further explained in section 3.1. In order to simulate this dynamic behaviour a simplified dynamic model with a single-degree-of-freedom (SDOF) is sufficient. The effect of a TMD can be assessed by adding an additional mass changing the model into a two-degrees-of-freedom (2DOF) system (see figure 3). The dynamic behaviour of these two models is illustrated by a *dynamic amplification factor* (DAF) in figure 4. This factor is defined as the quotient of the dynamic ( $\delta_{dyn}$ ) and static displacement ( $\delta_{st}$ ) for steady state response. The graphs show the response for different excitation frequencies ( $f_s$ ) relative to the fundamental frequency of the system ( $f_1$ ) for a harmonic force applied at a fixed point. The graph for a SDOF-system shows a clear peak when the excitation frequency equals the fundamental frequency of the system, also known as resonance. This peak can be reduced using a TMD as is shown in figure 4. However, the effect of a TMD is less significant for excitation frequencies away from the fundamental frequency. This shows that a TMD can be very effective however, to what extent this effect is utilised depends on the force characteristics. These characteristics become more complex when pedestrian group load is considered. This will be illustrated by a case study in section 3.

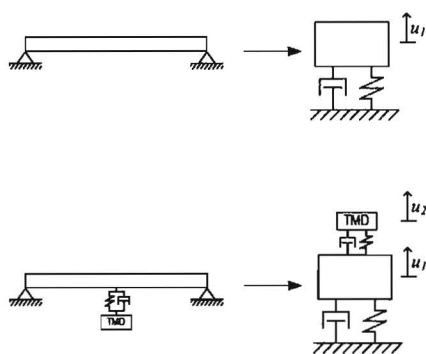


Fig. 3 A bridge modelled as single- (top) and two-degree-of-freedom system (bottom).

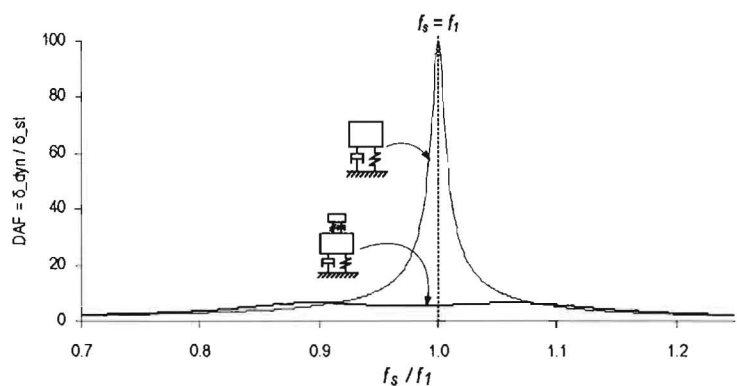


Fig. 4 Dynamic amplification factor (DAF) for a model with and without a TMD.

### 3. Case study

The case study involves the dynamic analysis of a straightforward type of bridge mentioned in the introduction. The bridge is analysed using an explicit finite element program known as *Beam Model* [1] which is a special purpose program developed to analyse beam / spring / damper compositions subjected to moving loads. In contrary to the SDOF and 2-DOF models shown in figure 3, this approach allows to apply moving forces over the full length of the structure. In this section the dynamic model of the bridge is described followed by the forces used to simulate pedestrian movement. The actual dynamic analysis will be further explained in section 4.

#### 3.1 Dynamic computer model of a footbridge

The computer model used for the analyses is based on a 50 meter long (L), simply supported beam as illustrated in figure 5. The bending stiffness (EI) is chosen based on static deflection criteria. For footbridges, the static deflection due to live load is in general limited to 1/250 of the span which equals to 200 mm for the analysed bridge. Assuming a four meter wide bridge, subjected to a maximum live load of 5.0 kN/m<sup>2</sup>, the required bending stiffness (EI) is found to be 8.16\*10<sup>9</sup> Nm<sup>2</sup>. A minimal amount of inherent damping is assumed expressed by a damping ratio ( $\zeta$ ) of 0.005. This means that the inherent damping is 0.5% of critical damping. The self weight (m) of the structure is relatively light: 1000 kg/m. Based on these values the natural frequency ( $f_n$ ), associated with the first vibration mode ( $n = 1$ ) can be determined as follows [2]:

$$f_n = \frac{(n\pi)^2}{2\pi L^2} \sqrt{\frac{EI}{m}} = \frac{(1 * \pi)^2}{2\pi * 50^2} \sqrt{\frac{8.15 * 10^9}{1000}} = 1.79\text{Hz} \quad (1)$$

The second natural frequency ( $n = 2$ ) equals 7.17 Hz which is outside the critical range of 0 - 5 Hz. Therefore, excessive vibrations are only expected at the centre of the bridge related to the first vibration mode. The bridge is analysed with *Beam Model* using a single beam divided into 0.5 m long segments connected by nodes. A separate model based on the same properties is analysed including a TMD modelled as an additional 1.0 m long beam. The TMD is attached to the centre of the beam by two springs and two dampers in stead of one due to limitations of the program (see figure 5). The specifications of both bridge and TMD are shown in figure 5. The mass of the TMD is arbitrarily chosen as 1/50 of the total mass of the bridge. The stiffness and damping of the TMD are based on optimum values according to [3]. Half of these values is applied to both springs and dampers to simulate the same behaviour as a single spring and damper. It should be noted that these optimum values are based on an *undamped* structure ( $\zeta = 0$ ). However, in [4] it is shown that the effect of inherent damping on the optimum values is minimal for moderately damped structures.

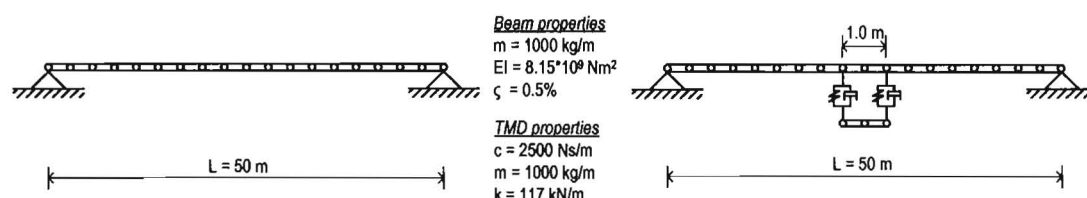


Fig. 5 Dynamic model of a bridge without and with a TMD.

A section of the model is shown in more detail in figure 6 to illustrate the application of a moving force. The force is applied to a single node if the force is situated directly above a node as illustrated with time step  $t_i$ . If the force is situated between two nodes at a following time step ( $t_{i+1}$ ), the force is divided between these two nodes proportional to the distance between the force and the nodes. Because the analysis is based on an explicit method, a short time step of  $1.5 * 10^{-5}$  seconds is required to guarantee accurate results.

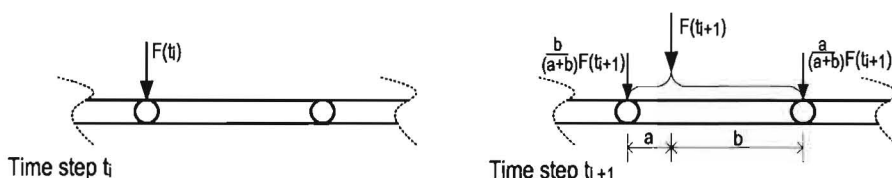


Fig. 6 Moving force at two locations at different time steps.

Inherent damping as well as the fundamental frequency are verified by applying a 100 kN pulse load at the centre of the bridge without a TMD. The fundamental frequency ( $f_1$ ) and damping ratio ( $\zeta$ ) are derived from figure 7 as follows:

$$f_1 = \frac{\text{number of cycles}}{\Delta T} = \frac{6}{3.41} = 1.76 \text{ Hz} \quad (2) \text{ (Close to the natural frequency of eq.(1): 1.79 Hz)}$$

$$\frac{u_2}{u_1} = e^{-2\pi\zeta} \Rightarrow \frac{1.71}{1.77} = e^{-2\pi\zeta} \Rightarrow \zeta = 0.005 = 0.5\% \quad (3) \text{ (In accordance with expected value)}$$

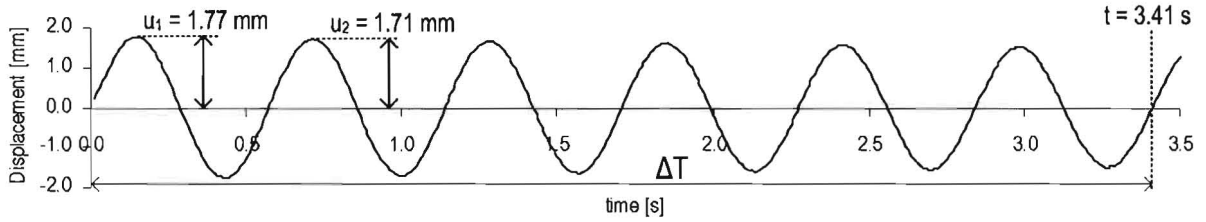


Fig. 7 Response at the centre of the bridge without a TMD after applying a 100 kN pulse load at the same location.

### 3.2 Force imposed by pedestrians

The dynamic analyses are performed using forces defined in the time-domain. This offers the ability to incorporate transient characteristics of the force induced by pedestrians. This is relevant because pedestrians represent a moving load which excites a bridge for only a short period of time. Also, pedestrians close to the abutments of a bridge have far less effect on the response of the structure than pedestrians walking at the centre of the bridge. The forces applied in *Beam Model* are represented by a harmonic load  $F(t)$  moving at a constant, average speed  $v$ :

$$F(t) = G\alpha \sin(2\pi f_s t + \phi) \text{ [N]} \quad (4)$$

$$v = \sum_1^n \frac{f_{s,i} * L_{s,i}}{n} \text{ [m/s]} \quad (5)$$

where ( $G$ ) represents the pedestrians weight in [N], ( $\alpha$ ) the dynamic load factor, ( $f_s$ ) the step frequency in [Hz], ( $\phi$ ) the phase of the force in [rad] and ( $L_s$ ) the step length in [m]. The weight, step frequency and step length are derived from normal distributions found in literature (see figure 8). The phase is defined by a uniform distribution between 0 and  $2\pi$ .

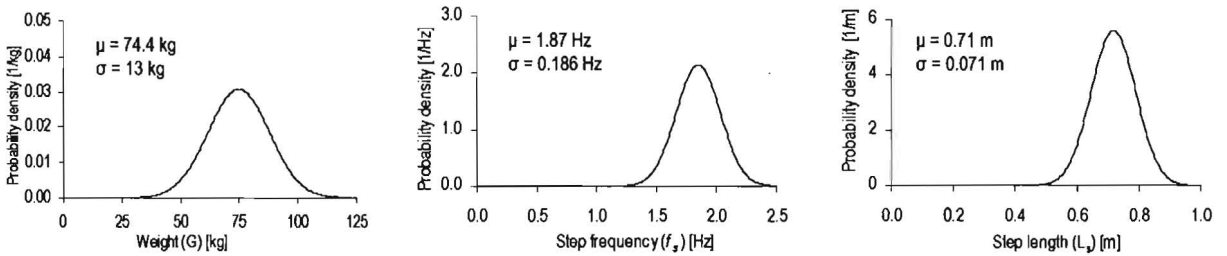


Fig. 8 Force variables: weight ( $G$ ) [5], step frequency ( $f_s$ ) [6] and step length ( $L_s$ ) [6].

The dynamic load factor in eq. (4) is given by the following equation [7]:

$$\alpha = -0.2649f_s^3 + 1.3206f_s^2 - 1.7597f_s + 0.7613 \quad (6)$$

#### 4. Monte Carlo simulation

In footbridge engineering, dynamic forces induced by pedestrians are often modelled as deterministic periodic forces. Based on research conducted in recent years [5, 6], it is now widely accepted that pedestrian induced forces vary strongly among pedestrians. These variations are expressed by normal distributions for several force variables in figure 8. A realistic assessment of the dynamic behaviour of a footbridge can be obtained using a probabilistic type of analysis such as a Monte Carlo simulation. This simulation can be used to estimate the probability of failure ( $P_f$ ) which corresponds to the number of simulations in which an acceleration limit ( $a_{limit}$ ) is exceeded ( $N_f$ ), divided by the total number of simulations ( $N$ ):

$$P_f = \frac{N_f}{N} \quad (7)$$

The standard deviation of the error which is made in the estimated probability  $P_f$  decreases with larger  $N$  and larger  $P_f$ . If  $N$  approaches infinity then  $P_f$  approaches the true probability of failure. The Monte Carlo simulations have been performed using five different load cases as shown in figure 9. They involve a single pedestrian and groups of multiples of three pedestrians walking next to each other. The groups consist of 3, 6, 12 and 24 pedestrians walking from one side of the bridge to the other. The forces applied in *Beam Model* consist of harmonic point loads as illustrated in figure 6. Each point load represents either a single pedestrian in load case 1 or the sum of three pedestrians walking next to each other for the other load cases. The analyses involve 500 different load samples for each load case. This number is large enough to give an accurate prediction of the 50% response limit, but is still too low to result in an accurate prediction of the 95% response limit as discussed in section 5. The same load samples are used for the model with and without TMD. As a result, any change in response can be attributed solely to the TMD.

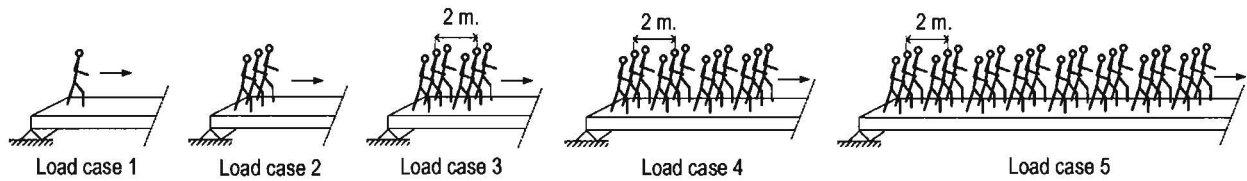


Fig. 9 Load cases.

The force characteristics of all pedestrians are chosen randomly according to the distributions in figure 8. This means that the harmonic force induced by each individual pedestrian has a different frequency and amplitude. The walking speed of a single pedestrian is given by equation (5). For groups, all pedestrians walk at the same speed which is derived from the average speed among all pedestrians. Although the walking speed for all pedestrians in a group is identical for all pedestrians within a load sample, the step frequencies of the individual pedestrians are not affected by this assumption. In other words: the group loads are unsynchronised. This assumption is correct for low density crowds. However, if the density of pedestrians increases their dynamic behaviour becomes more complex and synchronisation among pedestrians may occur, which got great attention after vibration problems mainly related to lateral excitation [8]. On the other hand, the walking speed and step frequency are in general lower for high density of pedestrians which results in lower values for the dynamic load factor (eq. (6)) and therefore a lower dynamic force.

#### 5. Results

The peak acceleration observed for each simulation is calculated after applying a band-pass filter (0 - 5 Hz) in order to remove any high-frequency numerical noise. Two examples are shown in figure 10 for a single pedestrian and a group of 24 pedestrians. Both graphs show the response at the centre of the bridge without as well as with a TMD. The first graph represents the response caused by a single pedestrian walking at 1.80 steps per second and a walking speed of 1.27 m/s. The peak acceleration is obtained after approximately 20 seconds when the pedestrian reaches the centre of the bridge ( $20 \text{ s} * 1.27 \text{ m/s} \approx 25 \text{ m}$ ). For multiple pedestrians, the response is less straightforward as shown in figure 10 (bottom). The graph shows a group of 24 pedestrians walking with different step frequencies and an average walking speed of 1.12 m/s. In this case the peak acceleration is obtained after 20 seconds, well before the whole group reaches the centre of the bridge ( $25 \text{ m} / 1.12 \text{ m/s} = 22.4 \text{ s}$ ).

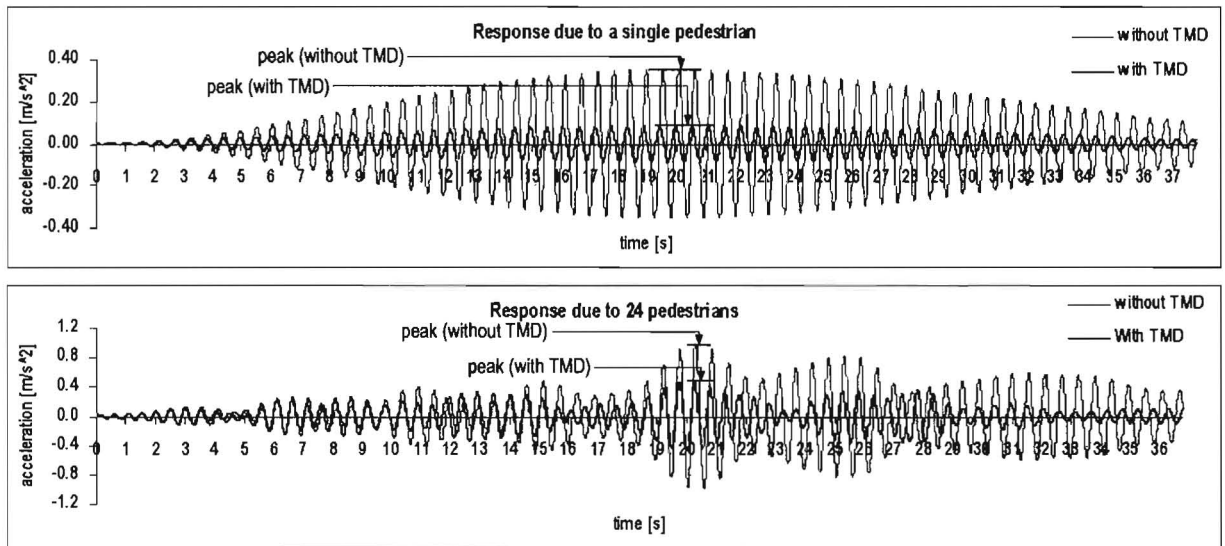


Fig. 10 Examples of response observed at the centre of the bridge produced by a single pedestrian (top,  $f_s = 1.80$  Hz,  $v = 1.27$  m/s) and 24 pedestrians (bottom,  $v = 1.12$  m/s).

The peak accelerations at the centre of the bridge for all load samples are presented in figure 11 by separate histograms for each load case. These graphs are used to assess the dynamic performance of the footbridge and the effectiveness of the TMD. Dynamic serviceability criteria are normally based on accelerations. The BS5400 for instance, prescribes an peak acceleration limit for footbridges of  $0.5\sqrt{f_1}$  where ( $f_1$ ) represents the fundamental frequency of the bridge (here,  $0.5\sqrt{1.79} = 0.67$  m/s<sup>2</sup>). Design criteria based on the peak accelerations are safe but often result in highly conservative structures. Therefore, a 95% response limit is added to ignore the relatively few high peaks. This 95% response limit can then be compared to the serviceability criterion.

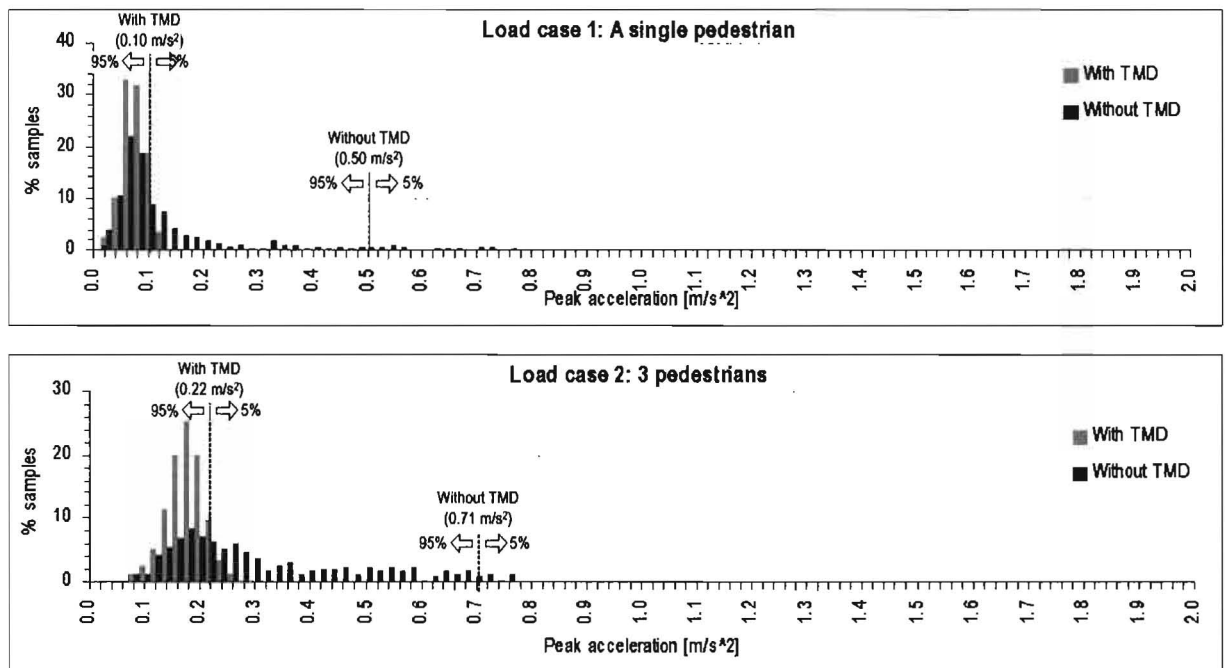


Fig. 11 Response per load case (note that the scale on the vertical axis is adjusted for each load case).

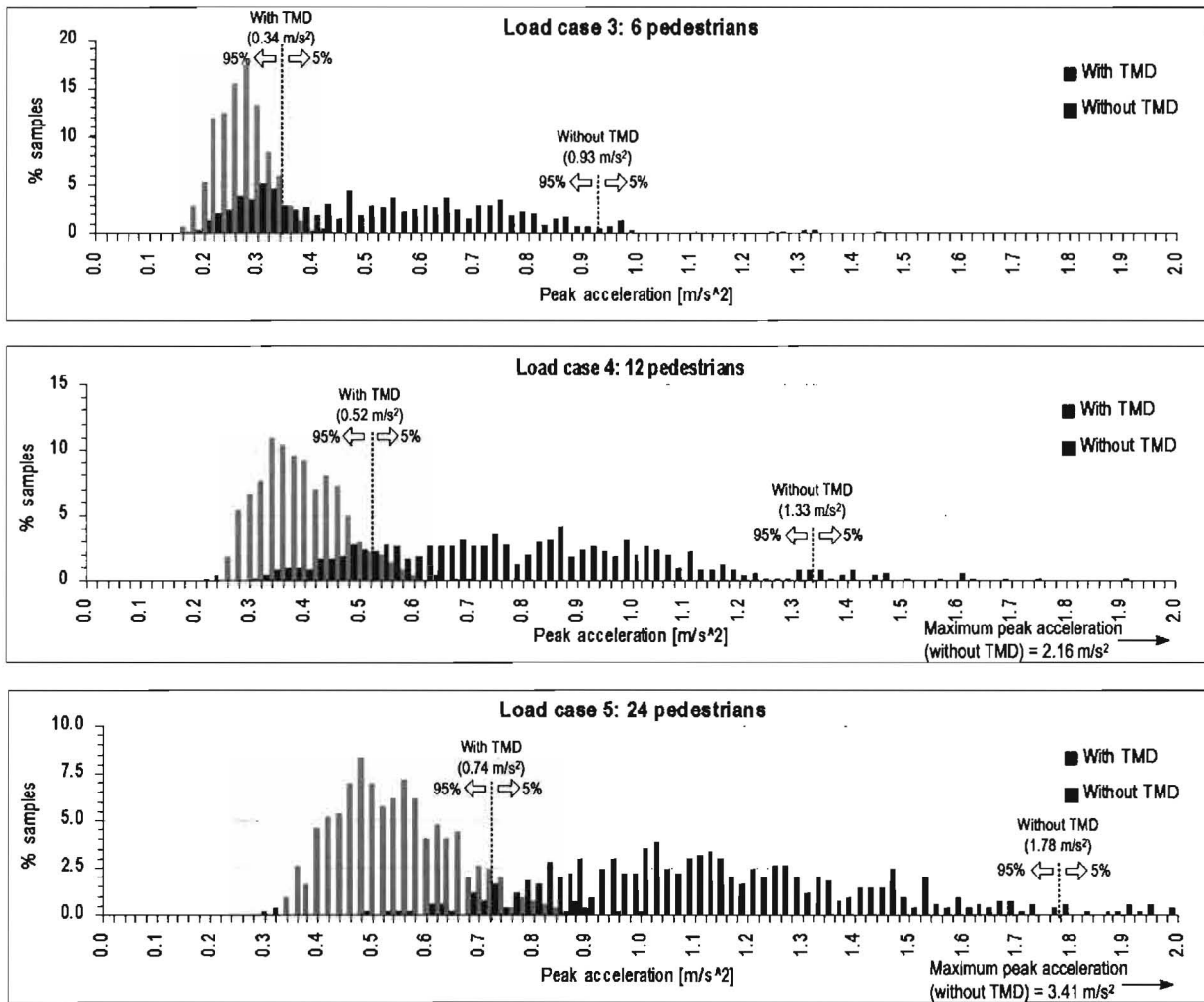


Fig. 11 (continued).

Two parameters ( $p$  and  $q$ ), illustrated in figure 12, are used to analyse the results. The value  $p$  divides the sample distribution into two equal parts which both comprise 50% of the samples. For large numbers of pedestrians, the response samples approach a normal distribution and thus  $p$  approaches the mean value of the response samples. For small numbers of pedestrians,  $p$  is close to, but not necessarily the same as the top of the response distribution. The value  $q$  divides the sample distribution into two parts of 95% and 5% of the total number of samples: the 95% response limit. The values  $p$  and  $q$  are presented in table 1 for both bridges with and without a TMD. In addition, the relation between the response without and with a TMD is given to illustrate the effect of the TMD. This effect for  $p$  and  $q$  is defined as  $p_{without} / p_{with TMD}$  and  $q_{without} / q_{with TMD}$  respectively. The factors  $\beta$  and  $\gamma$  in table 1 are explained in section 6.

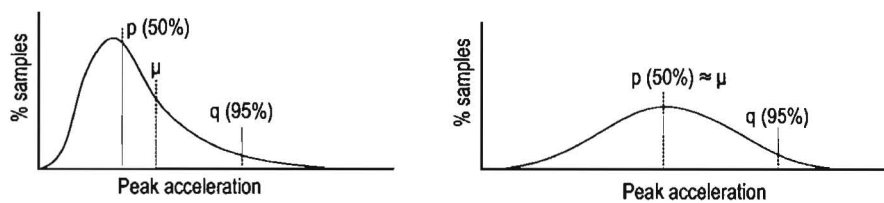


Fig. 12 Characterisation of response distribution for small number of pedestrians (left) and large number of pedestrians (right).



Table 1 Summary of response data obtained from Monte Carlo simulations (all values represent accelerations in m/s<sup>2</sup>).

Load case	No. of ped. (n)	Values related to p (50%)			Values related to q (95%)			$\beta (= p / \sqrt{n})$		$\gamma (= q / \sqrt{n})$	
		p_without TMD	p_with TMD	p_effect TMD	q_without TMD	q_with TMD	q_effect TMD	Without TMD	With TMD	Without TMD	With TMD
1	1	0.075	0.062	1.210	0.499	0.096	5.198	0.075	0.062	0.499	0.096
2	3	0.256	0.170	1.506	0.711	0.221	3.217	0.147	0.098	0.410	0.128
3	6	0.518	0.262	1.977	0.925	0.343	2.697	0.211	0.107	0.378	0.140
4	12	0.782	0.372	2.102	1.329	0.520	2.556	0.226	0.107	0.384	0.150
5	24	1.100	0.522	2.107	1.777	0.739	2.405	0.225	0.107	0.363	0.151

6. Interpretation of results

The results in table 1 are interpreted using the graphs shown in figure 13. The graphs show that, in all cases, the response is significantly higher for groups than for a single pedestrian. The response can be approximated by  $p = \beta\sqrt{n}$  and  $q = \gamma\sqrt{n}$  where  $n$  represents the number of pedestrians involved and  $\beta$  and  $\gamma$  represent additional factors related to  $p$  and  $q$  respectively. For large numbers of pedestrians,  $\beta$  approaches 0.23 for the bridge without a TMD and 0.11 for the bridge with a TMD. These values for  $\gamma$  are 0.36 and 0.15 respectively. It is expected that  $\beta$  and  $\gamma$  are bridge dependent. The factor  $\sqrt{n}$  is in accordance with [9] where the same factor is derived based on random vibration theory. It should be noted that due to the limited number of load samples, the standard deviation of the error in the value of  $q$  is quite large.

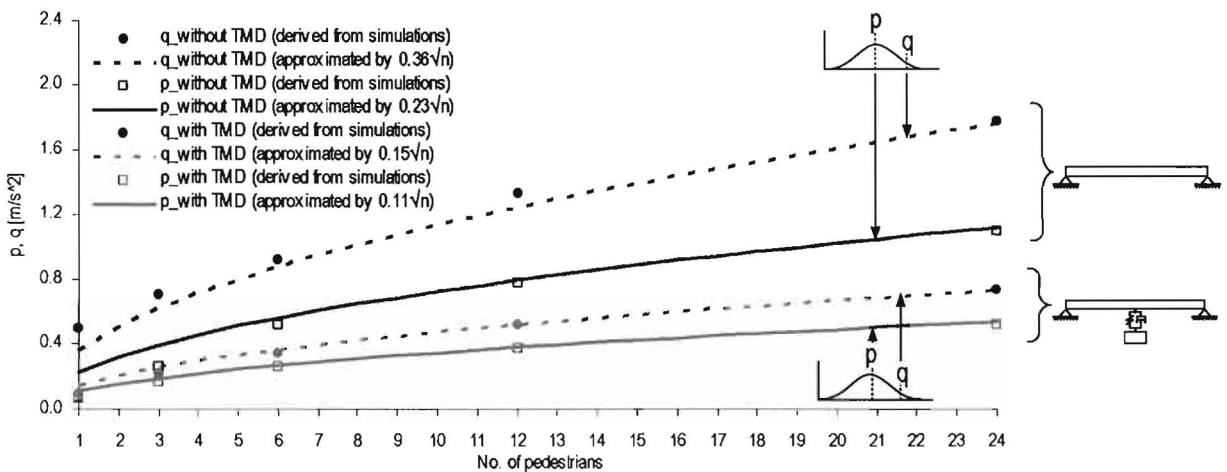


Fig. 13 Relation between the number of pedestrians and response.

Table 1 shows that the response with TMD is significantly less compared to the response without TMD. This effect can be illustrated by the ratio between the response without and with a TMD for  $p$  and  $q$ . This ratio is presented in figure 14 for each load case. The graph shows that for large groups the ratio for  $p$  approaches a constant of 2.1. The ratio for  $q$  converges more slowly, and may converge to a value lower than 2.4.

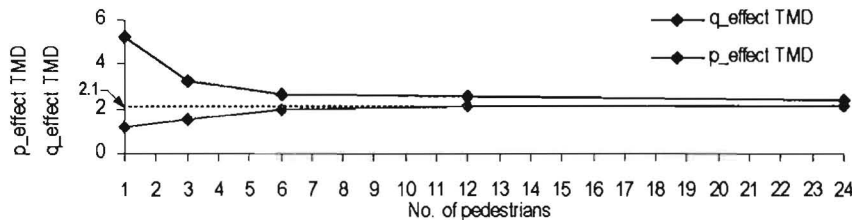


Fig. 14 Effect of a TMD for increasing number of pedestrians for  $p$  and  $q$ .

The graphs in figure 14 show deviations from these constant values 2.1 and 2.4 for small numbers of pedestrians. The same phenomenon can be seen in figure 13 where the approximation curve is slightly off for small numbers of pedestrians. This can be explained by the fact that the response distribution changes from an asymmetric shape for a single pedestrian to a symmetric shape for multiple pedestrians. This phenomenon can be observed in figure 11 for the response without a TMD and is schematically reproduced in figure 15.

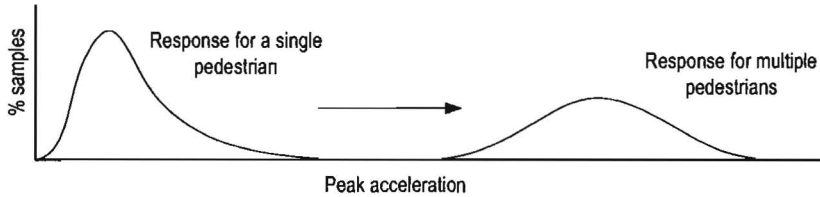


Fig. 15 Change in response distribution from a asymmetric shape for a single pedestrian to a symmetric shape for multiple pedestrians.

The shape of the response distribution is dominated by the probability that the step frequency of a pedestrian coincides with the fundamental frequency of the bridge. This phenomenon is illustrated in figures 16 and 17. These figures show the response related to all step frequencies applied in load case 1 (figure 16) and load case 2 (figure 17). For a single pedestrian walking over a bridge without TMD (figure 16, left), high accelerations are only observed if the pedestrian walks close to the fundamental frequency of the bridge. Therefore, the majority of pedestrians cause minimal vibration which is expressed by a large, asymmetric peak in the response pattern shown in figure 16 (left). However, when multiple pedestrians are concerned, the probability that one of the pedestrians walks close the fundamental frequency increases. The pedestrians involved in the same load sample also experiences this response. In other words: A pedestrian walking with a step frequency away from the fundamental frequency also experience large vibrations if one of his fellow pedestrians walks close to the fundamental frequency of the bridge which explains the large scatter in data in figure 17 and the normal distribution in figure 17 (left).

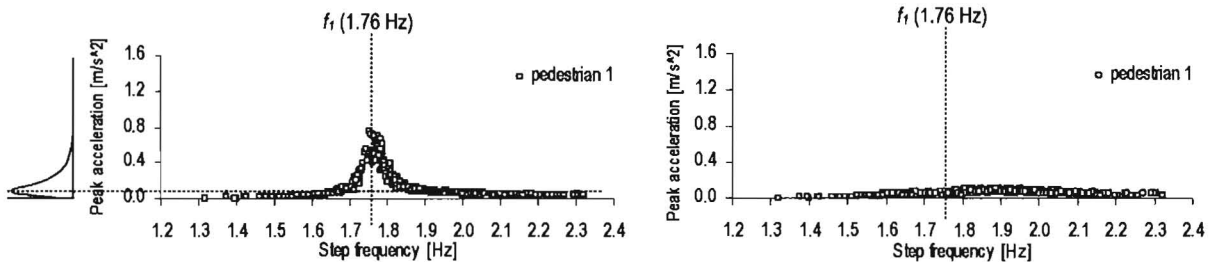


Fig. 16 Step frequencies applied in load case 1 (single pedestrian) without TMD (left) and with TMD (right).

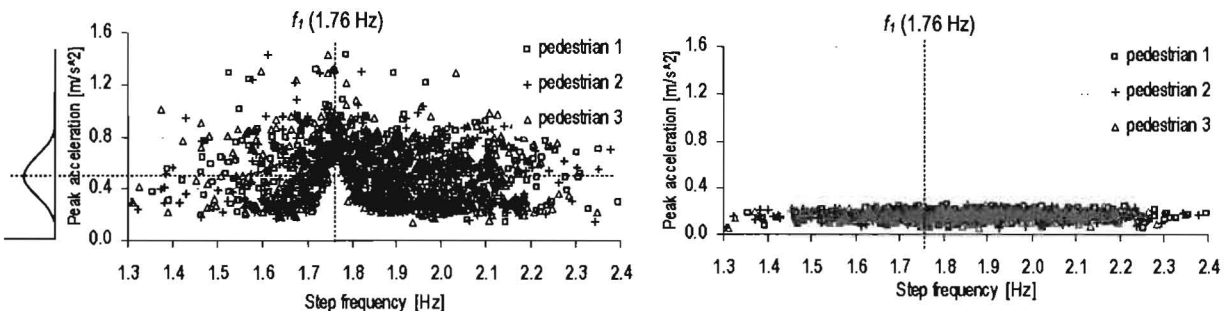


Fig. 17 Step frequencies applied in load case 2 (three pedestrians) without TMD (left) and with TMD (right).

The 95% response limit including a TMD for a group of 24 pedestrians is approximately 0.74 (table 1) which is slightly higher than the criterion stated in BS5400. A solution can be found in installing a heavier TMD.

## 7. Conclusions & Recommendations

The effect of a TMD is studied based on a straightforward type of bridge subjected to a single pedestrian and several groups of pedestrians. It is shown that the effect of a TMD depends on the number of pedestrians. For large groups of pedestrians, the reduction in the 50% response limit of peak acceleration approaches a factor of 2.1. For the accurate determination of the 95% response limit of peak acceleration, more load samples are needed.

A clear increase in response is found when the number of pedestrians increases. This group effect is described by the square root of the number of pedestrians multiplied by an additional factor. This implies that even higher accelerations are expected for groups exceeding 24 pedestrians. However, the dynamic load is becoming more complex for high density crowds. First of all, the average walking speed of pedestrians in a dense crowd reduces which results in a lower dynamic force. On the other hand, some degree of synchronisation is expected which results in an increase in response. These phenomena require further research.

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