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by

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Abstract In a practical multipurpose high frequency circuit, different oscillators are not completely isolated from each other. Instead, they interact with the environment, or with other oscillator. The interference between different oscillators may lead to generation of undesired signals. Therefore, the effect of oscillators on each other must be considered in the circuit design. As oscillators have nonlinear behavior, simulation of some of them which are coupled to each other needs more attention. In this report we present a mathematical model for three mutually coupled voltage controlled oscillators and solve it by a numerical method. The approach is illustrated by numerical experiments on realistic designs.

1 Introduction

The increasing speed of communication systems as well as the high-density packaging in those systems has focused renewed interest on the modeling and prediction of undesired electromagnetic interferences (EMI) such as crosstalk and mutual coupling. Oscillators are an important part of most circuits that influence each other by interference signals coming from other blocks or from other oscillators. These interferences may lead to locking in different frequency than desired, or maybe the pulling effect appears. In the latter case the oscillator is perturbed from its free running orbit without locking.

The relation between the locking range of an oscillator and the amplitude of the injected signal can be considered as a criterion in circuit design. Adler [1] shows that this relation is linear, but it is now well known that this is only the case for small injection levels and that the modeling fails for higher injection levels [2]. Also other linearization modeling techniques [3] cannot model nonlinear effects such as injection locking [2], [4]. Contrary to linear models, the nonlinear phase macromodel [5] is able to capture nonlinear effects such as injection locking. Harutyunyan [5] shows how such macromodel can be used in industrial practice to predict the behavior of inductively coupled oscillators. In this report we extend the mathematical model of two mutually coupled voltage controlled oscillators [5] to three and solve this model without additional approximation applied in macromodel simulations.

The report is build up as follows. In section 2, we briefly show the concept of a single oscillator modeling. Inductively coupled oscillators are discussed in detail in Section 3. Numerical results are presented in Section 4 and the conclusions are drawn in Section 5.

2 LC oscillator

Before we discuss the coupled oscillators, we will review a single oscillator. For many applications the oscillator can be modeled as an LC tank with a nonlinear resistor as shown in Fig. 1. This circuit is governed by the following differential equations for the unknowns (v, i):

$$\begin{aligned} &C\frac{dv(t)}{dt} + \frac{v(t)}{R} + i(t) + S\tanh\left(\frac{G_n v(t)}{S}\right) = b(t), \\ &L\frac{di(t)}{dt} - v(t) = 0, \\ &v(0) = v_0\,,\;\; i(0) = i_0\,. \end{aligned} \tag{1}$$

where C, L and R are the capacitance, inductance and resistance, respectively. The nodal voltage is denoted by *v* and the branch current of the inductor is denoted by *i*. The voltage controlled nonlinear resistor is defined by S and G_n parameters, where S has influence on the oscillation amplitude and G_n is the gain.

One of the most important methods of solution of the system of differential equations (1) is an explicit Runge-Kutta [6] which implemented as follow. Considering unknown *y* and coefficient *M* matrixes

$$y(t) = \begin{bmatrix} v(t) \\ i(t) \end{bmatrix},\; M = \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix},$$

The system of equations (1) can be written in a matrix form of

$$My' = f(t, y(t)), \quad y(t_0) = y_0\,. \tag{2}$$

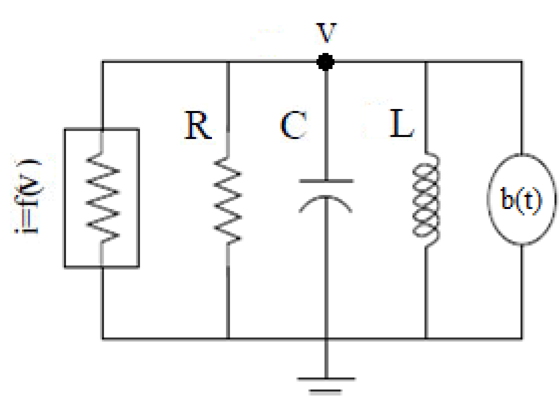


Figure 1: A common model for a voltage controlled oscillator, $f(v) = S\tanh\left(\frac{G_n v(t)}{S}\right)$

A solution to equation (2) can be written as

$$My(t_{k+1}) = My(t_k) + \int_{t_k}^{t_{k+1}} f(t, y(t))dt\,, \tag{3}$$

which can be calculated by quadrature approximation of the integral:

$$\begin{aligned} My_{n+1} &= My_n + h\sum_{i=1}^{s} b_i k_i\,, \\ k_i &= f(t_n + c_i h, y_n + \sum_{j=1}^{i-1} a_{ij} h k_j)\,. \end{aligned} \tag{4}$$

A particular method is specified by the number of stages, *s*, and the coefficients a_{ij}, b_i, and c_i [7]. We apply Runge-Kutta (4,5) formula using 'ode45' function in our MATLAB programming. For one oscillator the program is very simple as *y* is a 2×1 matrix and *dy* is defined as

$$\begin{aligned} dy(1) &= -\frac{y(1)}{R} - S\tanh(\frac{G_n y(1)}{S}) - y(2) + b(t)\,, \\ dy(2) &= y(1)\,. \end{aligned} \tag{5}$$

For example, consider an oscillator designed for a frequency of 6 GHz with following parameters

$L = 0.53$ nH, $C = 1.33$ pF, $R = 250$ ohm, $S = 1/R$, and $G_n = -1.1/R$.

The power spectral density (PSD) of output voltage, *y(1)*, is shown in Fig. 2. The maximum of output is appeared at 6 GHz, as expected and there is not any valuable side band as *b(t)=0*.

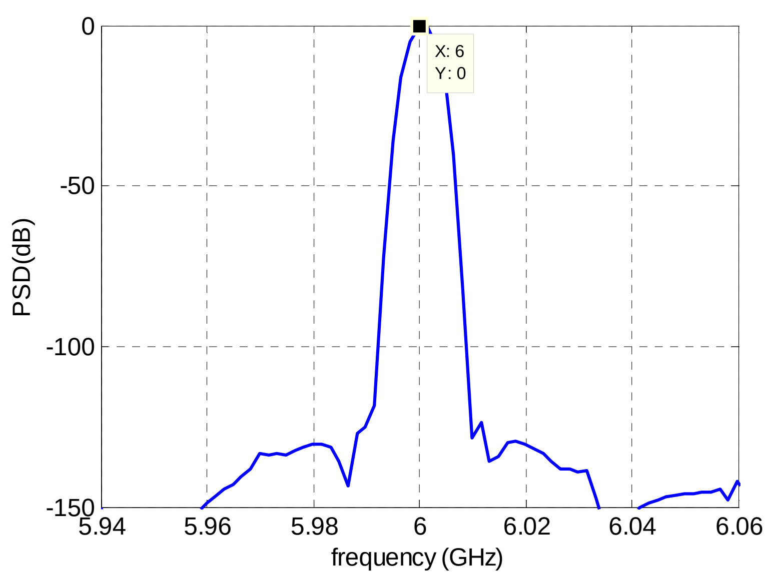


Figure 2: Output voltage of an example oscillator designed for 6 GHz.

3 Mutual inductive coupling

Next we consider three mutually coupled LC oscillators as shown in Fig. 3. The inductive coupling between these oscillators can be modeled as

$$
\begin{aligned}
L_1 \frac{di_1(t)}{dt} + M_{12} \frac{di_2(t)}{dt} + M_{13} \frac{di_3(t)}{dt} &= v_1(t), \\
M_{12} \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} + M_{23} \frac{di_3(t)}{dt} &= v_2(t), \\
M_{13} \frac{di_1(t)}{dt} + M_{23} \frac{di_2(t)}{dt} + L_3 \frac{di_3(t)}{dt} &= v_3(t).
\end{aligned}
\tag{6}
$$

where $M_{ij} = k_{ij}\sqrt{L_i L_j}$ is the mutual inductance and $|k_{ij}| < 1$ is the coupling factor. This makes the matrix

$$
\begin{bmatrix} L_1 & M_{12} & M_{13} \\ M_{12} & L_2 & M_{23} \\ M_{13} & M_{23} & L_3 \end{bmatrix}.
$$

positive definite, which ensures that the problem is well posed. In this section all the parameters with a subindex refer to the parameters of the oscillator with the same subindex. If we combine the mathematical model (1) of each oscillator with (6), then the two inductively coupled oscillators can be described by the following differential equations

$$C_1 \frac{dv_1(t)}{dt} + \frac{v_1(t)}{R_1} + i_1(t) + S_1 \tanh(\frac{G_n v_1(t)}{S_1}) = 0 \,, \tag{7a}$$

$$L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt} - v_1(t) = 0 \,, \tag{7b}$$

$$C_2 \frac{dv_2(t)}{dt} + \frac{v_2(t)}{R_2} + i_2(t) + S_2 \tanh(\frac{G_n v_2(t)}{S_2}) = 0 \,, \tag{7c}$$

$$L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} + M_{23} \frac{di_3}{dt} - v_2(t) = 0 \,, \tag{7d}$$

$$C_3 \frac{dv_3(t)}{dt} + \frac{v_3(t)}{R_3} + i_3(t) + S_3 \tanh(\frac{G_n v_3(t)}{S_3}) = 0 \,, \tag{7e}$$

$$L_3 \frac{di_3}{dt} + M_{13} \frac{di_1}{dt} + M_{23} \frac{di_2}{dt} - v_3(t) = 0 \,. \tag{7f}$$

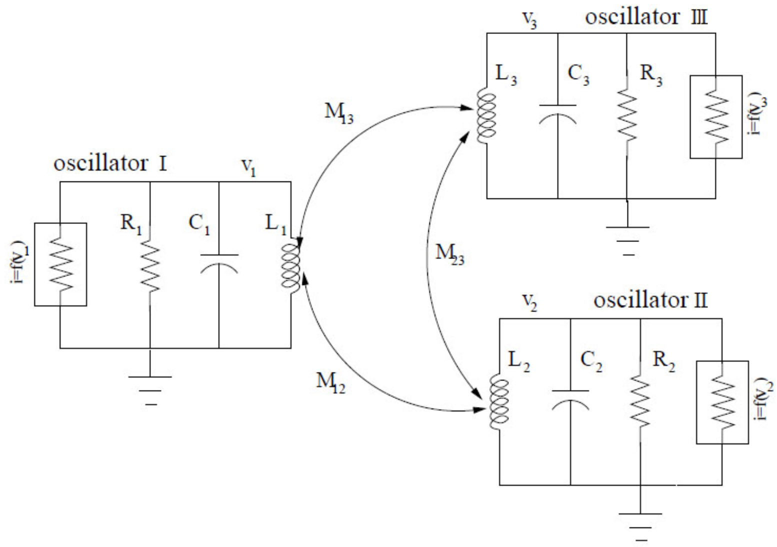


Figure 3: Three mutually coupled LC oscillators.

To solve the system of ODE (7) we define unknown matrix *y* and coefficient matrix *M* as follow

$$y=\begin{bmatrix} v_1 \\ i_1 \\ v_2 \\ i_2 \\ v_3 \\ i_3 \end{bmatrix}, M=\begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_1 & 0 & M_{12} & 0 & M_{13} \\ 0 & 0 & C_2 & 0 & 0 & 0 \\ 0 & M_{12} & 0 & L_2 & 0 & M_{23} \\ 0 & 0 & 0 & 0 & C_3 & 0 \\ 0 & M_{13} & 0 & M_{23} & 0 & L_3 \end{bmatrix}. \tag{8}$$

Therefore differential equations can be written as

$$\begin{aligned} dy(1) &= -\frac{y(1)}{R_1} - y(2) - S_1 \tanh(\frac{G_n y(1)}{S_1}), \\ dy(2) &= y(1), \\ dy(3) &= -\frac{y(3)}{R_2} - y(4) - S_2 \tanh(\frac{G_n y(3)}{S_2}), \\ dy(4) &= y(3) \\ dy(5) &= -\frac{y(5)}{R_3} - y(6) - S_3 \tanh(\frac{G_n y(5)}{S_3}) \\ dy(6) &= y(5). \end{aligned} \tag{9}$$

The Matlab program written for solving this system of equations is presented in appendix A.

4 Numerical Examples and Results

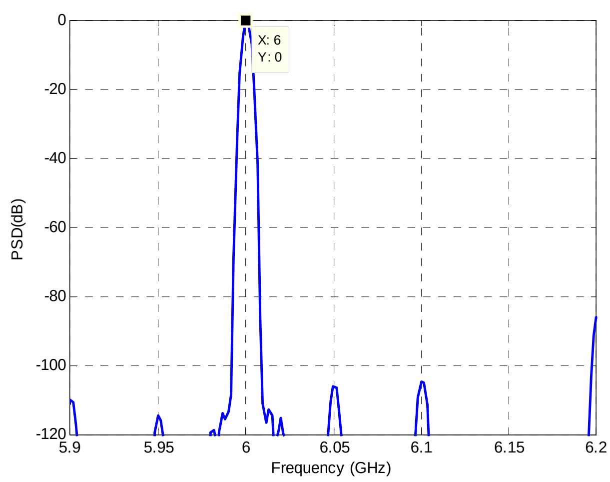
The value of parameters for oscillators, which provided by Dirana3 team, are as follow

$L = 0.53$ nH, $C = 1.33$ pF, $R = 250$ ohm, $S = 1/R$, and $G_n = -1.1/R$.

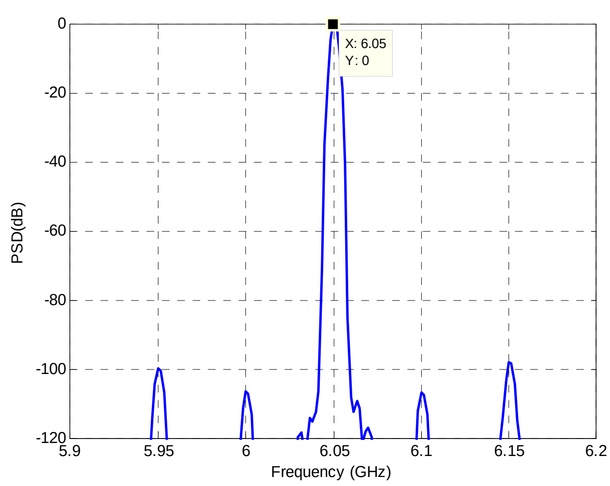
With these parameters oscillator frequency is 6 GHz, $V = 0.6$ and $A = 0.03$.

In all the numerical experiments the simulations are run until $T_{final} = 6\times10^{-7}$ s with the fixed time step $\tau = 10^{-11}$.

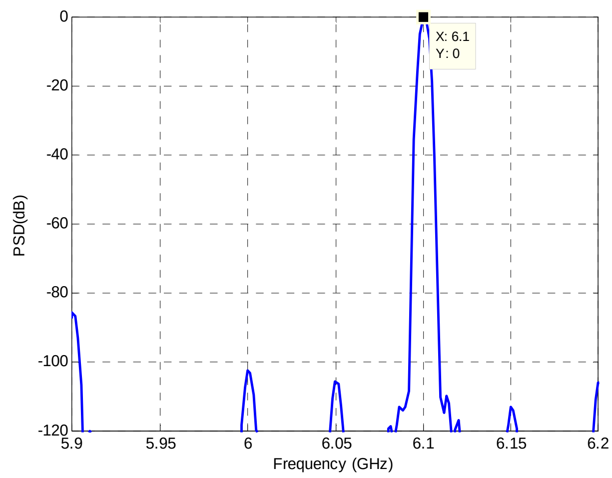
As we have three frequencies, some different situations can occur. First consider the state that the oscillators have equal difference. For example oscillators are at frequency 6 GHz, 6.05 GHz and 6.10 GHz (first two oscillators are in FM and the third DAB). Simulation results of this state for different coupling factors k_{12} and k_{13} are shown in Figs. 4-6, where the power spectral density (PSD) of the output voltage of each oscillator is plotted versus the frequency. We note that for small value of the coupling factor the frequency of the oscillators is not changed. If we increase k_{ij} to 0.01, we observe that the oscillators are pulled, and the frequency of output is slightly changed.



(a) First Oscillator,

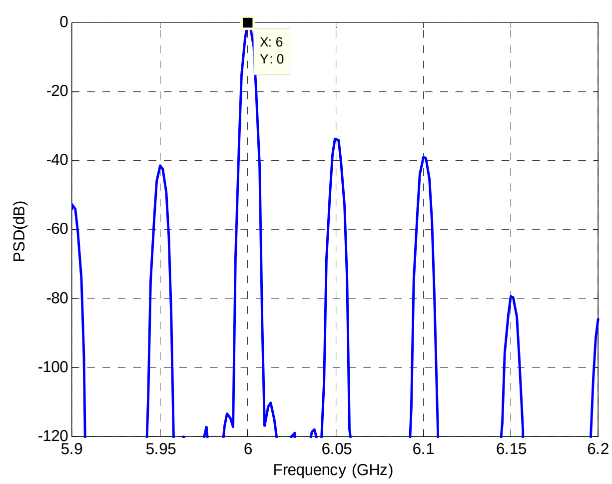


(b) Second Oscillator,

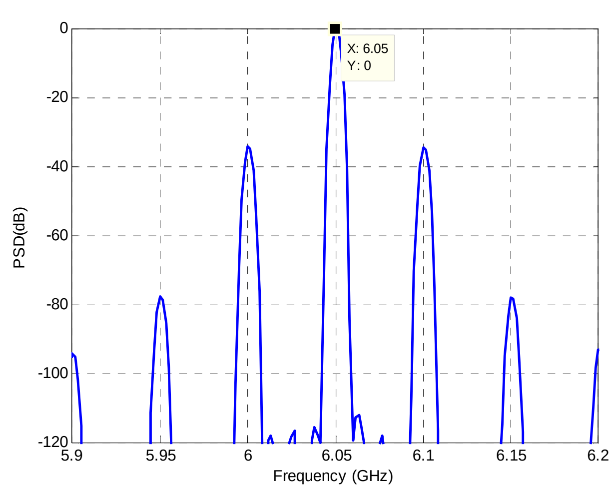


(c) Third Oscillator.

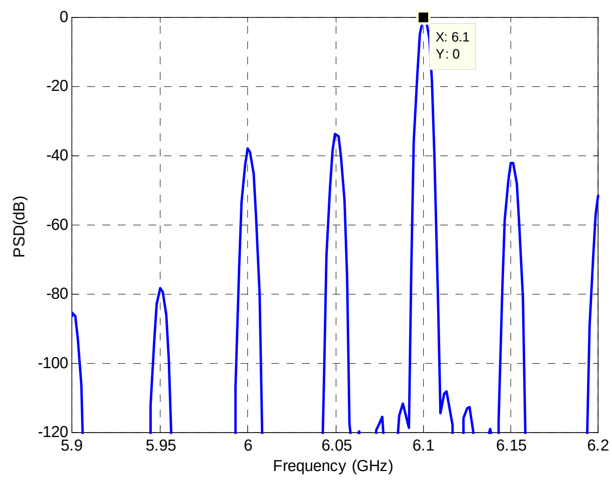
Figure 4: PSD of the output voltage of three mutually coupled oscillators (f_{Osc1} = 6 GHz, f_{Osc2} = 6.05 GHz and f_{Osc3} = 6.1 GHz) versus frequency for $k_{12} = k_{13} = 10^{-7}$.



(a) First Oscillator,

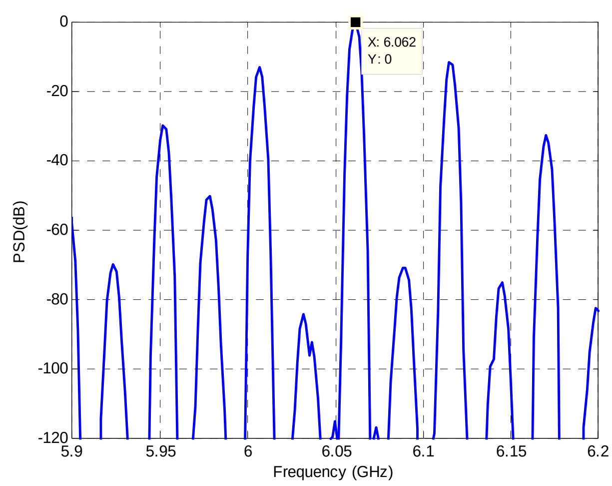


(b) Second Oscillator,

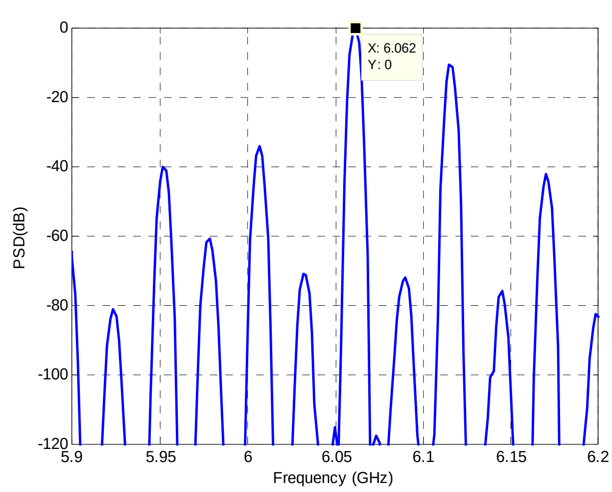


(c) Third Oscillator.

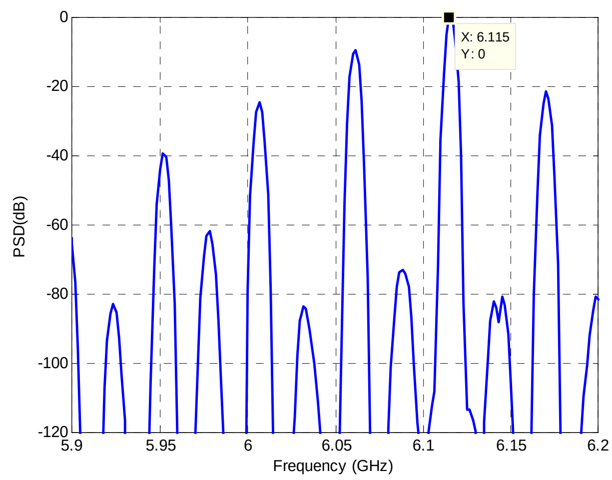
Figure 5: PSD of the output voltage of three mutually coupled oscillators (f_{Osc1} = 6 GHz, f_{Osc2} = 6.05 GHz and f_{Osc3} = 6.1 GHz) versus frequency for $k_{12} = k_{13} = 0.00041$.



(a) First Oscillator,



(b) Second Oscillator,

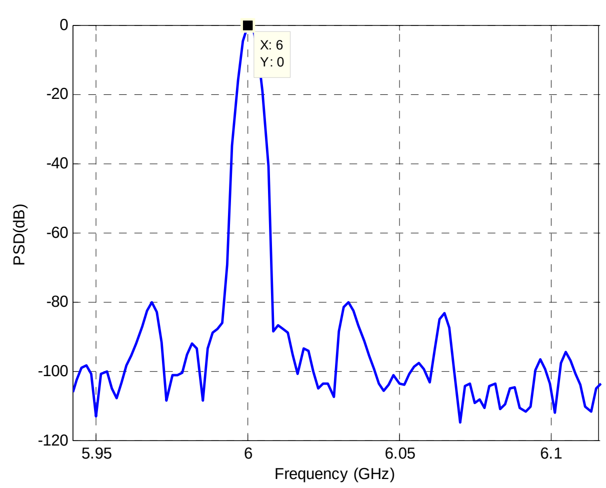


(c) Third Oscillator.

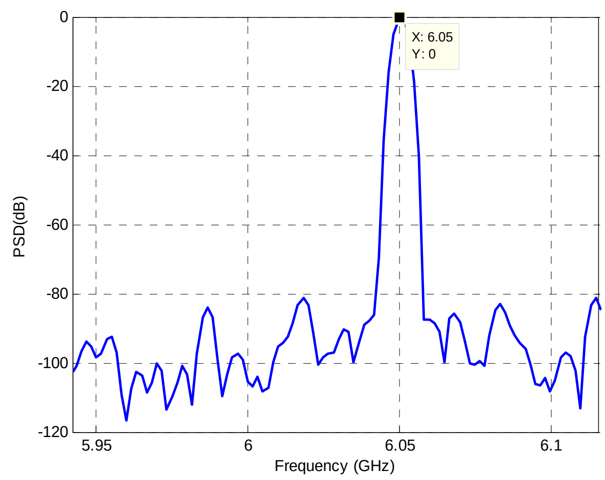
Figure 6: PSD of the output voltage of three mutually coupled oscillators (f_{Osc1} = 6 GHz, f_{Osc2} = 6.05 GHz and f_{Osc3} = 6.1 GHz) versus frequency for $k_{12} = k_{13} = 0.01$.

Another possible situation is one FM and two DAB signals, for example FM is at 50 MHz of oscillator DAB1 and about 8 MHz of DAB2, so: f_{Osc1} = 6 GHz, f_{Osc2} = 6.05 GHz and f_{Osc3} = 6.058 GHz.

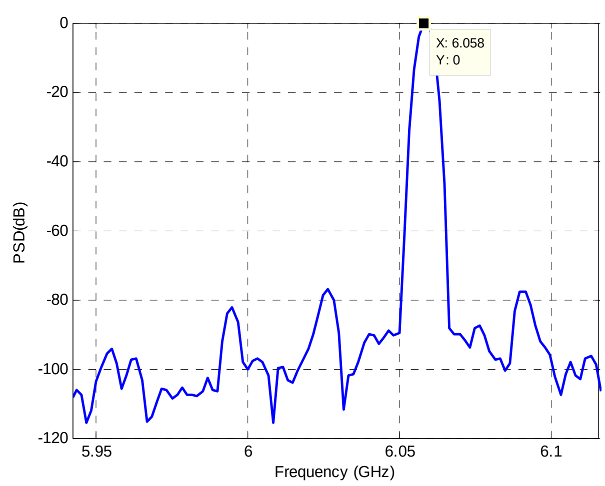
The PSD of the output voltage of each oscillator versus frequency in this state are shown in Figs. 7-9 when coupling factors $k_{12} = k_{13}$ varies from 10^{-7} to 10^{-2}. For very small value of the coupling factors (10^{-7}) the frequency of the oscillators is not changed and there is no side band. But in the case of $k_{12} = k_{13} = 0.00041$ some side bands are appeared and the main frequency lobe is not clear. When coupling factors increase to 0.01, oscillators lock in a frequency which is not equal to the first frequency of each of them. As only one frequency exists on board, the output signals are very clear and there is not any side band.



(a) First Oscillator,

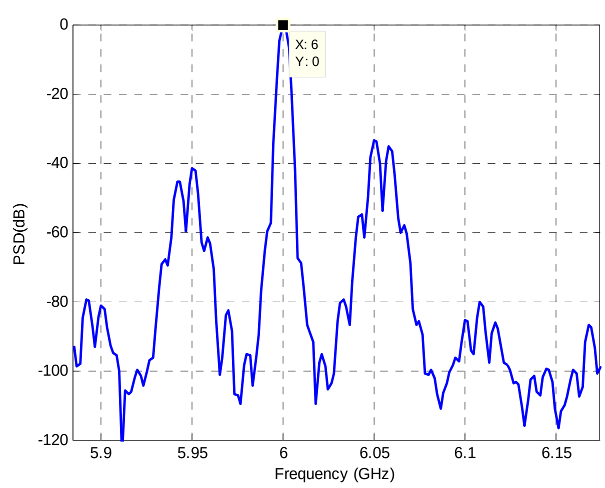


(b) Second Oscillator,

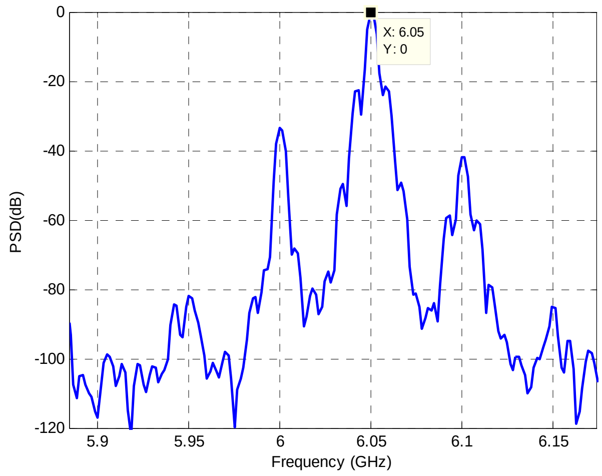


(c) Third Oscillator.

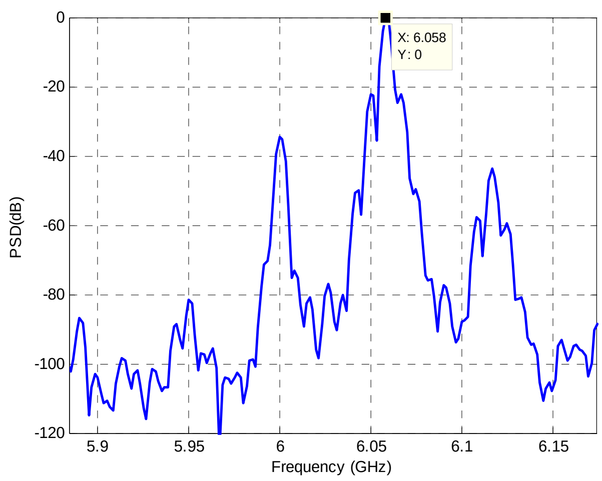
Figure 7: PSD of the output voltage of three mutually coupled oscillators (f_{Osc1} = 6 GHz, f_{Osc2} = 6.05 GHz and f_{Osc3} = 6.058 GHz) versus frequency for $k_{12} = k_{13} = 10^{-7}$.



(a) First Oscillator,

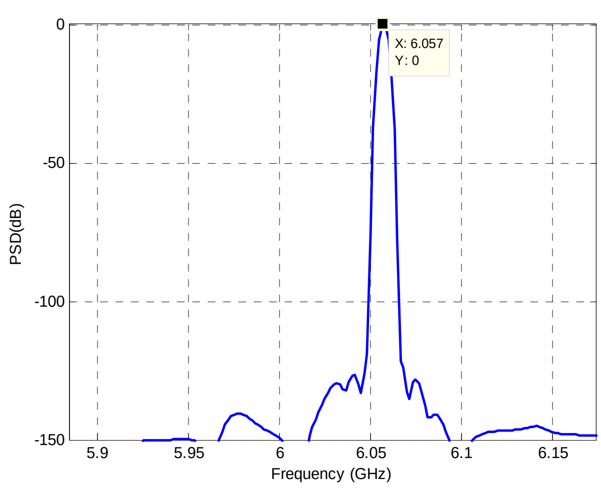


(b) Second Oscillator,

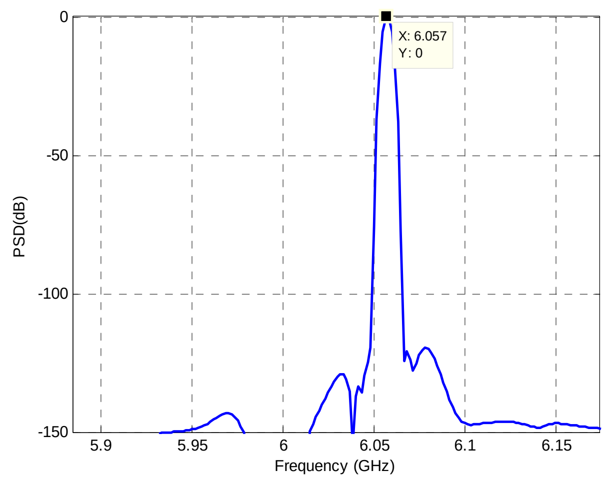


(c) Third Oscillator.

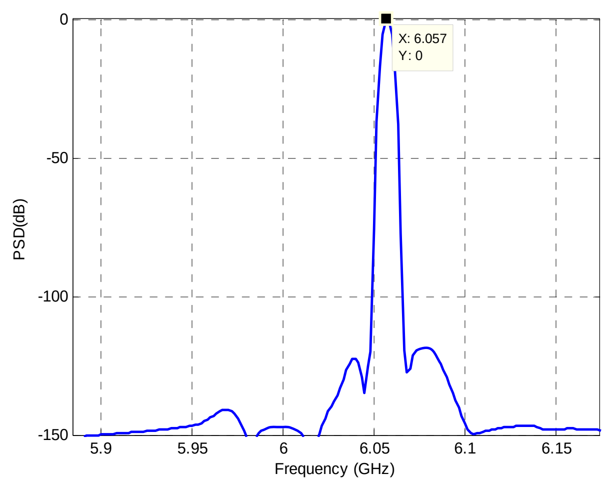
Figure 8: PSD of the output voltage of three mutually coupled oscillators (f_{Osc1} = 6 GHz, f_{Osc2} = 6.05 GHz and f_{Osc3} = 6.058 GHz) versus frequency for $k_{12} = k_{13}$ =0.00041.



(a) First Oscillator,



(b) Second Oscillator,



(c) Third Oscillator.

Figure 9: PSD of the output voltage of three mutually coupled oscillators (f_{Osc1} = 6 GHz, f_{Osc2} = 6.05 GHz and f_{Osc3} = 6.058 GHz) versus frequency for $k_{12} = k_{13}$ =0.01.

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Appendix A
Main program in Matlab

```
% Initiate time step and total time interval
dt = 1e-11;
interval = 6e-7;
% Parameter initiation
kk12 = 0.01;
kk13 = 0.01;
kk23 = 0.01;
frequency_osc1=6e+9;
% offset frequency for the second and third oscillators
f_offset_12 = 50e+6;
f_offset_13 = 100e+6;
% Call function 'MMatrix' to calculate coefficient
% matrix  for system of ODEs
[Mass] = MMatrix(kk12,kk13,kk23);
odeset.Mass = Mass;
TSPAN = 0:dt:interval;
odeset.InitialStep = dt;
% Initiate initial conditions
IC = zeros(6,1);
IC(1,1) =  0.5;
IC(3,1) = -0.5;
IC(5,1) = -0.5;
% Call function 'full_ode' in 'ode45'
% to define differential equations
% 'ode45': Solve ODE
% based on an explicit Runge-Kutta (4,5) formula
[t Y] = ode45(@(t,y) full_ode(t,y),TSPAN,IC,odeset);
% Initiate length of Kaiser window for plotting PSD
kaiser_wind_length=15;
% Plot PSD of desired signal using 'data_plot' function
```

```
[h_osc1,f1_new]=data_plot(frequency_osc1,frequency_osc1,…
          (Y(:,1))',dt,kaiser_wind_length);
```

Appendix B

MMatrix Function in Matlab

```
% Calculate coefficient matrix for system of ODEs
function [M]=MMatrix(k12,k13,k23)

% oscillator 1
L1 = 0.53*1e-9;
f1 = 6e9;
C1 = 1 / (L1*(2*pi*f1)^2);
R1 = 250;

% oscillator 2
L2 = 0.53*1e-9;
f2 = f1 + f_offset12;
C2 = 1 / (L2*(2*pi*f2)^2);
R2 = 250;

% oscillator 3
L3 = 0.53*1e-9;
f3 = f1 + f_offset13;
C3 = 1 / (L3*(2*pi*f3)^2);
R3 = 250;

M12=k12*sqrt(L1*L2);
M13=k13*sqrt(L1*L3);
M23=k23*sqrt(L2*L3);

M=zeros(6,6);
M(1,1)=C1;
M(2,2)=L1;
M(2,4)=M12;
```

```
M(2,6)=M13;

M(3,3)=C2;
M(4,2)=M12;
M(4,4)=L2;
M(4,6)=M23;

M(5,5)=C3;
M(6,2)=M13;
M(6,4)=M23;
M(6,6)=L3;
```

Appendix C

full_ode Function in Matlab

```
% Define differential equations
function dy=full_ode(t,y)


R1=250;

R2=250;

R3=250;

S1=1/R1;

S2=1/R2;

S3=1/R3;


dy=zeros(6,1);

dy(1)=-y(1)/R1-S1*tanh(-1.1*y(1))-y(2);

dy(2)=y(1);

dy(3)=-y(3)/R2-S2*tanh(-1.1*y(3))-y(4);

dy(4)=y(3);

dy(5)=-y(5)/R3-S3*tanh(-1.1*y(5))-y(6);

dy(6)=y(5);
```

Appendix D

Data_plot Function in Matlab

```
% Plot PSD of desired signal
function [h,new_frequency]=data_plot(frequency,finj,V,dt,…
                        kaiser_wind_length)


Tstep = dt;

N  = length(V);

Fs = 1/Tstep;

df = Fs/l_sim;

f   = [df:df:Fs];

f1 = df;

f2 = 10*frequency;


n1 = floor(f1/df);

n2 = floor(f2/df);


win     =  kaiser(N,kaiser_wind_length);
Vw      =  V'.*win;
VwF    =  abs(fft(Vw')).^2;
VwFn =  VwF/max(VwF);


plot(((f(n1:n2)-df),10*log10(VwFn(n1:n2)))
```

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