COMPUTATION OF FINITE ARRAY EFFECTS IN THE FRAMEWORK OF THE SQUARE KILOMETER ARRAY PROJECT.

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CONTEXT

The Square Kilometer Array (SKA) is a very large radio telescope being planned by an international consortium. It would operate in a very broad frequency band and have a collecting area of one square kilometer. In order to achieve a good resolution, this area will be spread over a few tens of stations, located several tens or hundreds of kilometers apart.

The Netherlands Foundation for Research in Astronomy (ASTRON) is studying the possibility of covering the mid-range frequencies ($\sim$ 0.2 to 2 GHz) with an instrument based on the phased-array technology. This technology presents the major advantages of avoiding mechanically moving structures and of enabling very flexible beamforming. One of the envisaged broadband antenna elements is the tapered slot antenna, also called Vivaldi antenna.

The design of these antennas is based on infinite array models [1], which automatically include the mutual coupling effects. As each station will probably be made of a very large number of small arrays, it is important to know how these arrays will behave when they are truncated. We developed a computation scheme for arrays of antennas made of metallic fins. In the next section, we justify the adopted approach, then details are given for the fast resolution of the resulting equation system. Finally, examples are shown for wide dipoles and comments are made about the extension to Vivaldi antennas.

FINITE ARRAY APPROACH

Several techniques are described in the literature for the rapid computation of finite array effects based on infinite array data. Several of them [2] are based on the computation of coupling coefficients (impedance, admittance or scattering matrix) obtained from infinite array solutions. These coefficients are assumed to remain constant in the finite array environment, considering absent antennas as antennas with either zero input current, zero input voltage, or zero incident power

wave. This kind of assumption proves very good for some particular types of antennas, but they are not well suited to tapered slot arrays, probably because of their three-dimensional structure.

Another technique [3] consists of assuming in a first stage of the analysis that the unknown current distributions on successive antennas just differ by the excitation law. When the Method of Moments is used to solve for the currents, this assumption leads to a different equation system for each antenna of the array, the dimension of these systems being the same as for a single antenna. This technique yields good results when the current distributions on successive antennas do not vary too fast, which is not guaranteed for the antennas under study here.

In further applications, the metallic fins may be electrically connected to each other. This enables currents to flow from one antenna to the next, which would make the couplings stronger in that direction ($\hat{y}$ direction in Figure 1). This means that truncation effects may also be most severe in the plane containing the metallic fins. That is why we chose to model arrays that are infinite in the $\hat{x}$ direction, and finite in the $\hat{y}$ direction.

In the following, the adopted methodology is described, and examples are shown for wide plate dipoles, as those represented in Figure 1. The antennas are assumed to be fed by delta-gap generators, and scanning is performed by considering sources with constant amplitudes and a linear phase progression in both directions. The currents are computed using the Method of Moments (MoM). As the antennas do not contain dielectric material, the free space Green's function may be used. For structures that are periodic in one direction, the Green's function contains an infinite sum of terms. Using the Poisson sum formula, this slowly converging sum can be transformed into a rapidly converging one, which corresponds to a series of propagating and evanescent cylindrical waves [4],[5]. Given that this function is evaluated only in the planes containing the antennas, it only depends on the radial distance to the source (in the YZ plane).
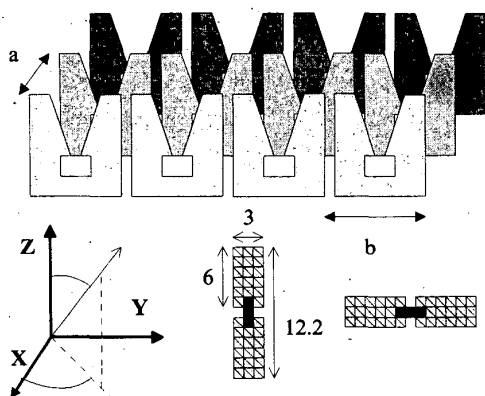
Hence, it can easily be tabulated for later use.

Figure 1 - Array configuration and antennas under test.

The currents are decomposed into triangular basis functions [6], and Galerkin testing is considered. For the computation of the MoM impedance matrix, the convolution integrations are carried out numerically in the space domain, with a separate treatment of the 1/R singularity, for which the integration over the basis functions is performed analytically [6]. The same approach has been adopted for the infinite-by-infinite array computations. In this case, the Green's function (without the singular component) is tabulated as a function of the $\hat{y}$ and $\hat{z}$ coordinates, within a single periodic cell. In general, a table containing 50 points in both directions provides sufficient accuracy.

For structures with a large number of elements in the finite array direction, the computation time rapidly becomes prohibitive. However, a lot of computation time can be saved by exploiting the periodicity of the structure. When filling the MoM impedance matrix, an obvious way to save time is by exploiting the translational symmetry of the array. Hence, the filling time increases only as $n_b^2 n_a$, where n_b is the number of basis functions on each antenna, and n_a is the number of antennas in the $\hat{y}$ direction.

As for the resolution of the MoM equations system, a global inversion of the impedance matrix requires a computation time proportional to $n_b^3 n_a^3$. In the next section, we show that this time can be dramatically reduced, using techniques that also provide some insight in the finite array effects.

FAST RESOLUTION

Standard current distributions

The number of unknowns can be significantly reduced

if the currents on a given antenna may be considered as the linear combination of a small number of standard distributions. Two such distributions are obviously the infinite-by-infinite array and the single element-by-infinite array distributions. This is generally not enough to model accurately the currents on all the antennas located in a finite array. Typical edge distributions can be obtained by solving the problem explicitly for small arrays. However, if the array is too small, the edge distributions may differ significantly from those of large arrays.

Another solution consists of computing edge distributions of left and right semi-infinite arrays. The semi-infinite array solution is obtained rapidly by replacing the infinite-by-infinite array periodic Green's function by the Green's function related to a semi-infinity of identical electric dipoles having the proper linear phase progression. Techniques for the fast computation of this Green's function can be found in [7],[5], where the technique described in the latter reference has been used here.

It is important to realize that this so-called semi-infinite array solution is not entirely physical. Indeed, given the particular definition of the Green's function used in that case, the currents on successive antennas are forced to be periodic (with the proper phase shift). When the antennas are very large in the $\hat{y}$ direction, this periodicity assumption is not good, even in the purpose of obtaining typical edge distributions. This becomes obvious when the antennas are connected to each other, as it is typically the case for tapered slot antenna arrays [1]. In that case, the edge antennas clearly present different current patterns, compared to the next antennas.

This problem can be overcome by solving explicitly the MoM equation system for the two first antennas, and by assuming that the currents are the same on antennas 2, 3,... to infinity. In this case, the MoM equation system can be written as follows:

$$Z_{11} I_1 + Z_{12}^S I_2 = V_1 \qquad (1)$$
$$Z_{21} I_1 + Z_{22}^S I_2 = V_2 \qquad (2)$$

where I_1 and I_2 are vectors containing the current coefficients on antennas 1 and 2, and V_1 and V_2 are the corresponding excitation vectors. Z_{11} and Z_{21} are portions of the finite-by-infinite array MoM impedance matrix, related to basis functions located on antenna 1, and testing functions located on antennas 1 and 2, respectively. Z_{12}^S and Z_{22}^S are similar matrices for basis functions located on antenna 2, with

however a major difference in the Green's function, which now corresponds to the semi-infinite array case.

This reduces the number of unknowns per antenna to six, corresponding to the coefficients related to six pre-computed current distributions. Hence, the total number of unknowns is $6\,n_a$, while the number of equations is $n_a\,n_b$. Therefore, the system of equations is solved in the least squares sense. If the equation system can be written as $A\,x = b$, the new system to be solved reads $A^H\,A\,x = A^H\,b$, where the (H) superscript stands for conjugate transposed. In this case, the most time consuming step consists of the computation of the $A^H\,A$ matrix, where the number of elementary operations increases as $n_a^3\,n_b$. Compared to the explicit inversion of the whole MoM impedance matrix, this corresponds to a time saving proportional to n_b^2.

Iterative refinement

The technique described above provides accuracies of the order of one percent. This is sufficient for most applications. However, in some cases, the error may be larger, and the solution can be refined iteratively. The simplest technique probably consists of solving explicitly for the currents on a given antenna by considering the approximate currents on the other antennas as an external excitation. The dimension of the matrix to be inverted is then n_b, so that the related computation time is limited. This can be done successively for all antennas, till a satisfactory accuracy is achieved.

This is equivalent to applying a stationary group-iterative technique to solve the MoM equations system [8]. It is interesting to notice that the matrix to be inverted at each step is always the same. Hence, this operation can be performed once and for all before the iterations are started. The most time consuming operation consists of the estimation of the equivalent excitation due to the currents on the other antennas. If the procedure is swept once across the array, the number of operations is proportional to $n_a^2\,n_b^2$. Compared to an explicit inversion, this corresponds to a time saving of the order $n_a\,n_b$.

The convergence of this stationary procedure cannot be guaranteed, and many sweeps may be necessary before the desired accuracy is achieved. The convergence is usually greatly enhanced when, at each step, the currents are solved explicitly for two successive antennas, while the new solution is kept for the first of those antennas only. This ensures a certain minimum distance between the antennas for which the currents are to be updated, and the external sources, due to the (approximately known) currents on the other antennas. In this case, the number of operations per iteration is multiplied by four, but cases of divergence become extremely rare, and the convergence rate is generally much better. Consequently, a few sweeps across the array are sufficient to achieve a 10^{-3} accuracy, which is certainly sufficient, in view of the accuracy to be expected from the MoM approach itself.

RESULTS

In the following, results are shown for wide plate dipoles, with dimensions of 3×12 units, contained in the Y-Z plane (Figure 1). The antennas are fed by a delta gap source of width 1 and the currents are modeled by 89 basis functions with triangular basis.

In the first case, the dipoles are oriented along $\hat{y}$, and the wavelength is 24 units. The element spacings are 12 units in the $\hat{x}$ direction and 16 units in the $\hat{y}$ direction. The array is scanned at an elevation angle of 10 degrees with respect to the $\hat{z}$ axis and an azimuth angle of 30 degrees with respect to the $\hat{x}$ axis. Figure 2 shows results obtained for an array containing 15 elements in the finite array direction. The antenna index increases in the $+\hat{y}$ direction. The horizontal lines give the active input impedance for the infinite-by-finite array solution. The solution based on six unknowns per antenna (crosses) already provides a sufficient accuracy, which is of the order of 0.1 percent. The impedances obtained based on the periodicity assumption (dotted line) are not as good, but they follow quite well the trend of the finite array effects.

In the second example, the dipoles are oriented along $\hat{z}$, and the wavelength is 20 units. The element spacings are 10 units in the $\hat{x}$ direction and 5 units in the $\hat{y}$ direction. Owing to the orientation of the dipoles, this array does not radiate in the $\hat{z}$ direction, hence results will be shown for scan angles far enough from normal, i.e. an elevation angle of 30 degrees and an azimuth angle of 60 degrees. Results are shown in Figure 3. In this case, the solution based on 6 unknowns per antenna still provides a very good accuracy, while the periodicity assumption yields results that do not exhibit the same trend at all.

The two stars at each end of the arrays stand for the solution obtained for semi-infinite arrays. They already give a good idea of the deviations from the infinite array solutions for the edge antennas.
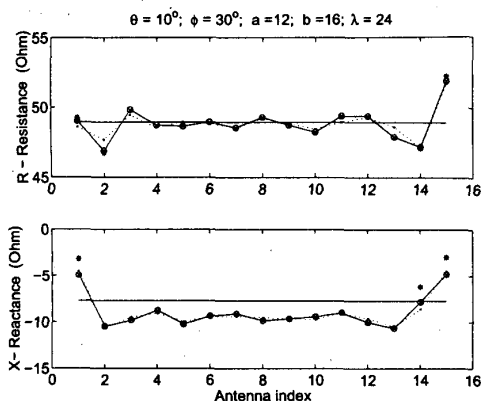
θ = 10°; φ = 30°; a =12; b =16; λ = 24

Figure 2 - Active input impedances in a finite-by-infinite array of wide plate dipoles oriented along the finite array direction. Circles: explicit solution; crosses: with six unknowns per antenna; dots: based on periodicity assumption; solid line: infinite-by-infinite array solution.
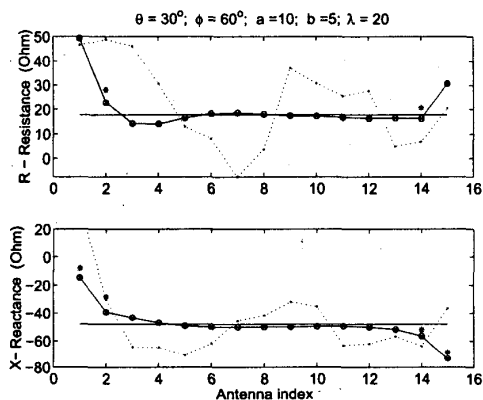
θ = 30°; φ = 60°; a =10; b =5; λ = 20

Figure 3 - Active input impedances in a finite-by-infinite array of wide plate dipoles perpendicular to the array plane.

First attempts have been made with Vivaldi antennas. This case presents the additional difficulty that the antennas are connected to each other, so that currents can flow across the array. Tests were performed with antennas loaded with a resistance equal to the real part of the infinite array solution for broadside scan. In this case, the solution with six unknowns per antenna yields an accuracy of the order of a few percent, and an iterative refinement may be necessary. Further research is being carried out to define a set of standard current distributions that better matches the particular geometry of the Vivaldi antennas.

CONCLUSION

Truncation effects are studied in arrays made of metallic plates perpendicular to the array plane. Given that classical approximations cannot be used, finite-by-infinite arrays are studied, the finite array direction being in the plane of the plates, where the couplings are expected to be the strongest.

With the help of a specific Green's function, the Method of Moments impedance matrix can be computed rapidly. Its resolution is strongly accelerated by exploiting the periodicity of the array in the finite array direction. First, the number of unknowns is reduced by considering the currents on a given antenna as a linear combination of six typical current distributions, resulting from infinite and semi-infinite array solutions. Next, the solution is refined by a stationary iteration technique, where the currents are updated for each antenna successively, by solving the problem explicitly for some neighborhood around that antenna.

Compared to a blind resolution of the MoM equations system, large time savings are achieved. For example when the number of basis functions per antenna is of the same order as the number of antennas ($n_b \simeq n_a \simeq n$), the times saving is proportional to n^2.

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