

Towards spatial reasoning on building information models

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Towards spatial reasoning on building information models

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ABSTRACT: The paper presents a conceptual study on the application of spatial reasoning on building information models. In many cases, building regulations and client demands imply constraints on the building design with inherent spatial semantics. If we are able to represent these spatial constraints in a computer-interpretable way, the building design can be checked for fulfilling them. In this context, spatial reasoning technology can be applied in two different ways. First, we can check the consistency of the spatial constraints in effect, i.e. find out whether there are contradictions between them. Second, we can check whether a concrete building design is compliant with these constraints. The paper gives a detailed overview on the currently available spatial calculi and introduces two possible implementation approaches.

1 INTRODUCTION

The architectural and structural design of buildings is a complex task where numerous rules have to be taken into account. These rules reflect technological constraints, national regulations and client demands. Among them spatial rules play an extraordinarily important role, since the objects to be designed are of intrinsic geometric nature.

To better support engineers and architects in the design process, it is desirable to create software applications which are able to check a concrete building design against the aforementioned rules. Significant scientific results have been achieved for formalizing and checking rules which are based on a comparison of alphanumeric values of individual attributes, such as the thickness of a house's outer walls or a slab's thickness (Ding et al. 2006, Kim & Grobler 2009, Nisbet et al. 2009).

However, the possibility to define and check rules which comprise qualitative spatial relationships between building components (such as *above*, *below*, *touch*, *within* etc.) has been investigated only by few researchers. A first approach for implementing spatial constraint checking technology on the basis of a spatial constraint language has been presented in (Borrmann et al. 2009). However, a main issue remained unsolved: If there are contradictions between different spatial constraints, the solution space for a valid building design may be empty. This has to be detected before the architect or engineer starts trying to fix his design, complying with one rule and violating another in an endless loop.

This paper presents a concept on how spatial reasoning technology can be applied to resolve this issue. Computational reasoning in general is a well-known technology for (1) deriving new knowledge from existing facts by the application and concatenation of rules and (2) checking the consistency of these rules. Spatial reasoning, in particular, provides the possibility to derive new knowledge regarding spatial relationships between objects and to check the consistency of spatial constraints.

2 RELATED WORK

A very important application of formalizing and checking constraints in the context of building information modelling is *Automated Code Checking*. Here, the vision is to encode regulations and building design codes in a computer-interpretable way such that the digital building can be checked against these rules (Han et al. 1997). The International Code Council (ICC) has started to work intensively in this direction and has created the SmartCodes initiative (Nisbet et al. 2009).

Ding et al. have implemented the Australian disabled access code on the basis of IFC models (Ding et al. 2006). In their approach, first a simplified model is created from the IFC model by applying an EXPRESS-X mapping. In a second step, building codes are encoded into object-based rules using the EXPRESS-based rule schema.

A suitable basis for reasoning on high-level concepts, such as the semantic (non-spatial) part of a

building information model, is an ontology. In general, an ontology is defined as a “formal specification of a shared conceptualization” (Gruber 1993). More precisely, it is used to capture the semantics of a domain’s concepts and the relationships among them.

Using the Ontology Web Language (OWL), standardized by W3C, an ontology can be formally specified. OWL distinguishes classes, properties and instances, comparable to the object-oriented paradigm. Additionally, OWL provides property characteristics (*transitive, symmetric, functional, inverse*) and property restrictions (*allValuesFrom, someValuesFrom*) as well as *is-a* relationships with generalisation/specialisation semantics

There are three language flavours of OWL (Lite, DL, and Full). The one relevant for the work presented here is OWL DL (description logic), which is based on the logic SHOIN(D) (Horrocks et al., 2003). It provides a maximum expressiveness while retaining computational completeness, decidability, and the availability of practical reasoning algorithms. Beetz et al. (2009) show how the Industry Foundation Classes (IFC), the most mature and well established data model for building information models can be transformed into an OWL ontology. The resulting IfcOWL ontology forms part of the concept presented in Section 3.

In (Kim & Grobler 2009) an ontology-based approach is presented for representing requirements and constraints of a project. The authors propose to employ an ontology reasoning mechanism to detect conflicts between diverging participants’ requirements in collaborative design scenarios. Unfortunately, the paper discusses only very basic quantitative constraints, such as limits on a slab’s thickness.

The work closest to the approach presented in this paper is (Bhatt et al. 2009) where spatio-terminologic inference has been applied for the design of ambient environments. The authors employ the reasoning engine RacerPro which supports spatial representation and reasoning based on the Region Connection Calculus (RCC, see Section 4). Besides that, the IFC data model is applied for terminological representation and reasoning. However, the presented spatial reasoning approach is restricted to 2D space and directional relationships are not taken into account.

3 CONCEPT

The application of computational reasoning technology can help to facilitate the design task and support the designing architects and engineers. There are two important applications of inference techniques in the context of building design and engineering. The first application is the detection of contradic-

tions between individual requirements and/or regulations, i.e. checking the consistency of all effective constraints. The second one is to check a concrete building information model for compliance with the client’s requirements or with certain regulations.

3.1 Application 1: Constraints consistency checking

A simple example for inconsistent spatial constraints would be the following:

- C1: “The heating equipment must be within Room1.”
- C2: “Room2 must be directly above Room1.”
- C3: “The heating equipment must not be directly below Room2.”

For real world projects, the network of spatial constraints is much more complex and detecting inconsistencies between them is very difficult. Here, the application of spatial reasoning technology can significantly support the architects and engineers. To verify the consistency of the spatial constraints, a reasoning engine supporting spatial calculi is applied.

3.2 Application 2: Compliance checking

For checking a concrete building model for compliance with the effective spatial constraints, qualitative spatial relations between individual building components are required as facts. They can be retrieved using the Spatial Query Language presented in (Borrmann & Rank 2010). Using the Spatial Query Language, we can automatically identify the spatial relationship holding between any two building components. This includes topological and directional relationships.

The resulting set of spatial facts can then be checked for compliance with the effective spatial constraints representing regulations, client demands, or construction rules. Here again, a reasoning engine can be applied, in this case to prove the consistency of the instance population with the spatial constraints in effect.

4 SPATIAL CALCULI

The basis for any formal reasoning is a *calculus*. A calculus is a system of rules which allows to derive new knowledge from given facts (axioms) in a logically consistent way. This process is called inference.

In the domain of spatial reasoning different qualitative properties of and relations between spatial objects are of interest, including topology, orientation, shape, size and distance.

To allow reasoning, a suitable qualitative representation of facts is necessary, i.e. continuous properties have to be mapped to a discrete set of symbols. Moreover, these symbols have to meet the

requirement that represent *jointly exhaustive and pairwise disjoint* (JEPD) facts. In the next subsections we will have a closer look on available calculi for topological and directional reasoning.

4.1 Calculi for topological reasoning

The Region Connection Calculus (RCC) is the most established calculus for topological reasoning. Since RCC also covers part-whole relationships between objects, it is also referred to as mereo-topological calculus (Randell et al. 1992). It has been designed for reasoning on regions in n -dimensional space, a region being defined as a set of points delimited by a continuous boundary curve.

The RCC-8 version defines eight different topological relations between two regions:

- *disconnected* (DC),
- *externally connected* (EC),
- *partial overlap* (PO),
- *equal* (EQ),
- *tangential proper part* (TPP) and its inverse (TPPi),
- *non-tangential proper part* (NTPP) and its inverse (NTPPi).

These different relations are illustrated in Figure 1.

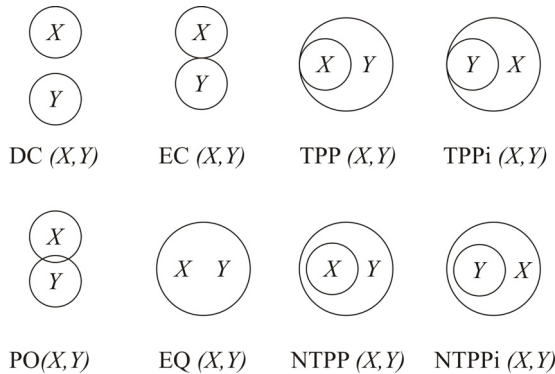


Figure 1: The eight topological relationships defined by RCC-8.

o	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	*	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC	DC	DC
EC	DC,EC,PO,TPPi,NTPPi	*	DC,EC,PO,TPP,TPPi,EQ	EC,PO,TPP,NTPP	PO,TPP,NTPP	DC,EC	DC	EC
PO	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPPi,NTPPi	*	PO,TPP,NTPP	PO,TPP,NTPP	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPPi,NTPPi	PO
TPP	DC	DC,EC	DC,EC,PO,TPP,NTPP	*	NTPP	DC,EC,PO,TPP,TPPi,EQ	DC,EC,PO,TPPi,NTPPi	TPP
NTPP	DC	DC	DC,EC,PO,TPP,NTPP	NTPP	*	DC,EC,PO,TPP,NTPP	*	NTPP
TPPi	DC,EC,PO,TPPi,NTPPi	EC,PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPP,TPPi,EQ	PO,TPP,NTPP	TPPi,NTPPi	NTPPi	TPPi
NTPPi	DC,EC,PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPP,NTPP,TPPi,NTPPi,EQ	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

Figure 2: Composition table of the Region-Connection Calculus (Randell et al. 1992). * denotes the universal relation.

The RCC-8 reasoning allows to derive from the given information on the topological relation $R1$ between two objects A and B and the relation $R2$ between two objects B and C , the topological relationships $R3$ between the objects A and C . The inference

process is realized by applying the composition table shown in Figure 2. For explaining the usage of the table, we suppose that the relation EC holds for objects A and B and $NTPP$ for objects B and C . Using the table we can derive that for A and C the relation DC must hold.

When restricted to simple plane regions, RCC-8 is equivalent to the 9-Intersection Model (Egenhofer 1991), a very influential model in the GIS domain.

4.2 Calculi for directional reasoning

Direction is a binary relation of an ordered pair of objects A and B , where A is the reference object and B is the target object. The third part of a directional relation is formed by the reference frame, which assigns names or symbols to space partitions.

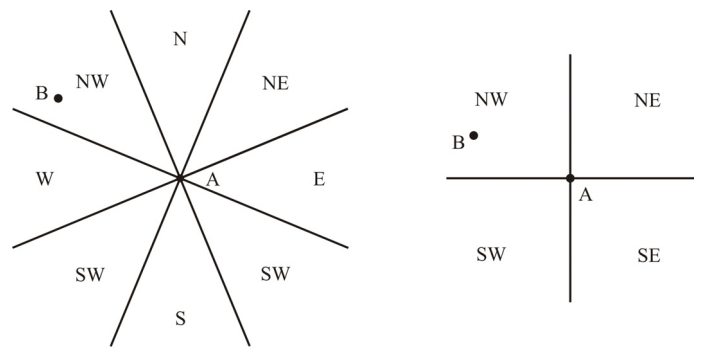


Figure 3: Frank's cone-shaped (left) and projection-based (right) models of directional relationships between points.

In a geographical context, we usually distinguish four (*north, east, south, west*) or eight space partitions (*north, north-east, east, south-east, south, south-west, west, north-west*). In 3D context, normally the additional directional predicates *above* and *below* are used, which may also be employed in conjunction with the aforementioned 2D sub-direction, resulting in *north-east-above, east-above, etc.*

For directional reasoning in two-dimensional space, Frank (1992) has defined two models for defining directional relations between points: the cone-based and the projection-based model. The cone-based model dissects the space around the reference point in either four partitions of 90° or eight partitions of 45° (Figure 3, left-hand side). The direction of the target point with respect to the reference point is defined by the partition in which the target point is located. Figure 4 shows the composition table for the four-partitions case. From the total of 25 different combinations, one can only infer 13 cases exactly and four approximately (lower case letters indicate approximate reasoning).

The projection-based model (Frank 1992) dissects the space by means of horizontal and vertical lines that cross at the reference point (Figure 3, right-hand side). While the horizontal line creates a northern and southern halfspace, the vertical line creates the western and eastern halfspace. Superimposing the

halfspaces produces four directional partitions, namely *north-west*, *north-east*, *south-east* and *south-west*. The composition table for the projection-based model is depicted in Figure 5.

	N	E	S	W	O
N	N		o		N
E		E		o	E
S	o		S		S
W		o		W	W
O	N	E	S	W	O

Figure 4: Composition table for four cone-shaped directions (Frank 1992). *O* denotes the identity relation, i.e. both objects having the same position. Lower case letters indicate approximate reasoning.

	N	NE	E	SE	S	SW	W	NW	O
N	N	N	NE	e	o	w	NW	NW	N
NE	NE	NE	NE	e	e	o	NW	n	NE
E	NE	NE	E	SE	SE	s	o	n	E
SE	e	e	SE	SE	SE	s	s	o	SE
S	o	e	SE	SE	S	SW	SW	w	S
SW	w	o	SE	SE	SW	SW	SW	w	SW
W	NW	n	o	SE	SW	SW	W	NW	W
NW	NW	n	n	o	w	w	NW	NW	NW
O	N	NE	E	SE	S	SW	W	NW	O

Figure 5: Composition table for the projection-based model (Frank 1992).

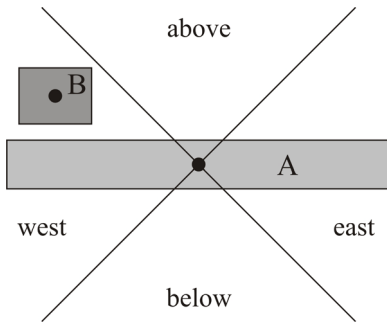


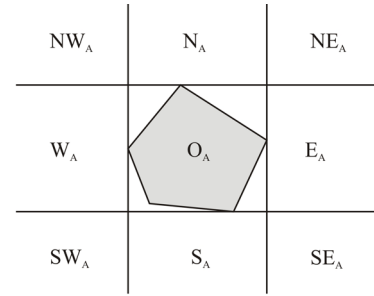
Figure 6: Approximating target and reference object by their centroids may cause results that do not comply with the intuitive expectations of the users. In this example, *B* is classified as being west of *A* and not as being above it.

These calculi are defined only for point-point relationships in 2D space. In the context of building information modeling, however, we mainly deal with extended objects in 3D space. In order to apply the available models on extended 3D objects, a point-based approximation, such as the center of gravity, is normally used. However, this rough approximation often causes results that do not comply with the intuitive expectations of the user (Figure 6).

Only few calculi are available for directional relationships of extended objects. One of them is the Rectangle Algebra (Balbiani et al. 1999) which approximates both the reference and the primary object by their bounding boxes. Another one is the Cardinal Direction Calculus (CDC) introduced in (Goyal &

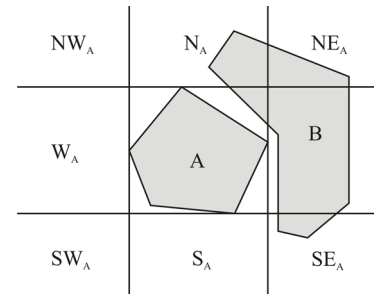
Egenhofer 1997) for representing directional relationships between connected regions. In CDC, the reference object is approximated by a box, while the primary object remains un-approximated. The bounding box of the reference object forms nine direction partitions. For representing a directional relationship, a matrix is employed that captures which of these nine partitions is covered by the primary object (Figure 7). Out of the 512 possible matrix assignments, only 218 exist in reality – they form the basic relations of the calculus. Figure 8 depicts one of these relations as an example.

In (Zhang et al. 2009) an efficient algorithm for checking consistency of basic CDC networks has been introduced.



$$\text{dir}_{RR}(A, B) = \begin{bmatrix} NW_A \cap B & N_A \cap B & NE_A \cap B \\ W_A \cap B & O_A \cap B & E_A \cap B \\ SW_A \cap B & S_A \cap B & SE_A \cap B \end{bmatrix}$$

Figure 7: In the CDC, a 3x3 matrix is used to capture the relationship between the reference and the primary object.



$$\text{dir}_{RR}(A, B) = \begin{bmatrix} \emptyset & \neg \emptyset & \neg \emptyset \\ \emptyset & \neg \emptyset & \neg \emptyset \\ \emptyset & \emptyset & \neg \emptyset \end{bmatrix}$$

Figure 8: An example for a directional relation between the objects *A* and *B*.

4.3 Calculi combining topological and directional relations

Only few works are known which combine topological and directional relationships in one calculus. Among them is (Sun & Li, 2005) which combines RCC-8 with the Cardinal Direction Calculus, and (Li 2007) which combines RCC-8 with the Rectangle Algebra.

5 TECHNICAL IMPLEMENTATION

5.1 Option 1: Using a hard-wired spatial reasoner

One possible option for the technical realization of the presented concept is the application of a reasoner with hard-wired capabilities for spatial reasoning. One example is the reasoning engine RacerPro which provides reasoning over ontologies (TBox) and their instances (ABox) as well region-based spatial reasoning by the so-called S-Box. Using Racer Pro, we can represent the building information model on the one hand as semantic model according to the IfcOWL ontology. This part is stored in the TBox and ABox, respectively. On the other hand, spatial knowledge on the building model can be derived from the 3D geometry representation using the spatial query language developed by the authors. Figure 9 depicts this concept. Links can be established between the ontological objects of the ABox and their spatial representation in the SBox, allowing for combined spatio-ontological reasoning.

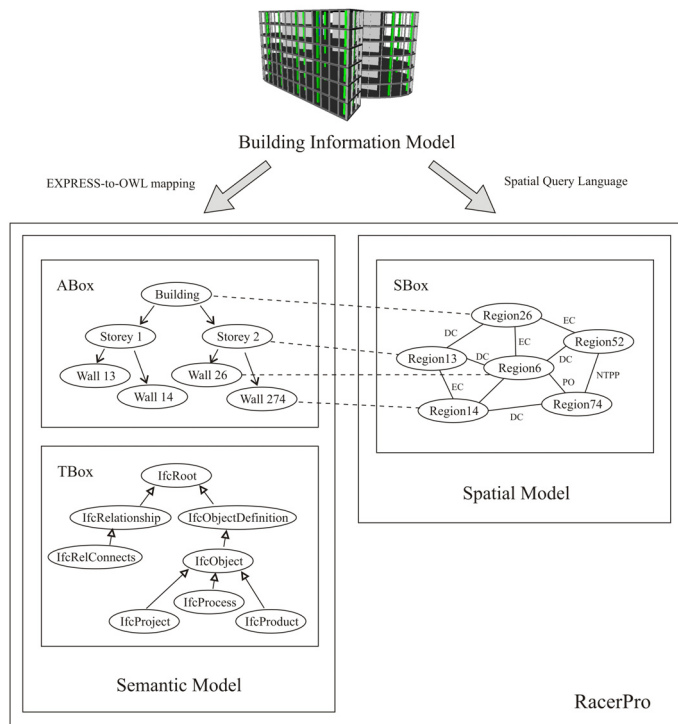


Figure 9: From the building information model, information for the semantic reasoner (ABox) and the spatial reasoner (SBox) is derived.

The spatial constraints to be applied can be expressed as facts of the SBox. Its reasoning capabilities can then be used (1) to check the consistency of these constraints, and (2) to check compliance of the spatial objects with these constraints.

The advantage of this implementation approach is its comparatively easy realization, since the desired spatial reasoning technology can be used out-of-the-box. On the other hand, the RacerPro's S-Box provides only reasoning on topological relationships, reasoning on directional relationships is not possible.

5.2 Option 2: Using an extensible reasoner

Another implementation approach is the application of a reasoner which provides the possibility for a flexible integration of arbitrary spatial calculi. One example for such an extensible reasoner is the SparQ toolbox, which supports binary and ternary spatial calculi (Wallgrün et al. 2007). A new calculus can be specified using a LISP-like syntax. For any defined calculus SparQ provides the following reasoning functionalities:

- qualification (turning a quantitative geometric scene description into a qualitative one)
- computing with relations
- constraint reasoning

The latter refers to solving constraint satisfaction problems (CSP) modeled as constraint graphs; these are complete labeled graphs with a node for each spatial object (also denoted as variable) and each edge labeled with a relation from the calculus. A CSP is consistent, if an assignment for all variables can be found that satisfies all the constraints.

The SparQ toolbox is able to detect inconsistencies of a constraint graph, which in our case can be applied for realizing application scenario 1. The toolbox is also able to 'heal' the constraint graph by removing one or more constraints. For concrete spatial objects, the 'scenario consistency' can be checked (Application 2).

Unfortunately, the provided reference implementations for spatial calculi are exclusively for reasoning about the orientation of point objects or line segments. The available calculi include Allen's Interval Algebra (Allen 1983) and Freksa's Double Cross Calculus (Freksa 1992) among others. An implementation of RCC-8 is unfortunately not yet provided.

The advantage of using an extensible spatial reasoner is the great flexibility of this approach. In principle, any of the available spatial calculi can be integrated. However, the implementation effort should not be under-estimated.

6 DISCUSSION & FUTURE WORK

The paper has introduced a concept for enabling spatial reasoning on building information models. There are two important applications of spatial reasoning in the context of building design and engineering: (1) detecting contradictions between the effective spatial constraints and (2) checking a concrete building information model for compliance with the effective spatial constraints.

The paper has discussed in detail available spatial calculi which form the basis for spatial reasoning. The region-connection calculus (RCC) enables reasoning on topological relations between n -dimensional regions. Frank's directional calculi enable

reasoning on directional relationships, but only between points. An alternative model for expressing directional relationships between extended objects is the Cardinal Direction Calculus by Goyal and Egenhofer.

The paper has further discussed two different implementation concepts. The first one is based on the application of a reasoning engine with hard-wired support for reasoning using a specific spatial calculus. The second one is based on employing reasoner which can be extended by any form of spatial calculus.

The authors see a high potential in using spatial reasoning technology for building information models and will continue their work on this topic. The next steps will be:

- Find suitable calculi for spatial reasoning on building information models. It will be of special importance to investigate which of the available calculi can be applied on extended 3D objects. If suitable calculi are not available, emphasis has to be placed on developing them.
- Experiment with both implementation approaches and decide for the more appropriate one.
- Ensure compatibility between the “spatial facts” about a specific building model generated by applying the spatial query language and the chosen calculus.
- Create a prototypical implementation. Derive a set of spatial constraints from client demands and regulations. Check concrete building models for compliance with these constraints.

It will be a long road of research and development towards realizing spatial reasoning on building information models, but the authors are happily facing this challenge.

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