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Capacity reservation and utilization for a manufacturer with uncertain capacity and demand

Y. Boulaksil¹, J.C. Fransoo, T. Tan

School of Industrial Engineering, Eindhoven University of Technology, The Netherlands

Abstract

We consider an OEM (Original Equipment Manufacturer) that has outsourced the production activities to a CM (Contract Manufacturer). The CM produces on a non-dedicated capacitated production line, i.e., the CM produces for multiple OEMs on the same production line. The CM requires that all OEMs reserve capacity slots before ordering and responds to these reservations by acceptance or partial rejection, based on allocation rules that are unknown to the OEM. Therefore, the allocated capacity for the OEM is not known in advance, also because the OEM has no information about the reservations of the other OEMs. We study this problem from the OEM's perspective who faces stochastic demand and stochastic capacity allocation from the contract manufacturer. A single-item periodic review inventory system is considered and we assume linear inventory holding, backorder, and reservation costs. We develop a stochastic dynamic programming model for this problem and characterize the optimal policy. We conduct a numerical study where we also consider the case that the capacity allocation is dependent on the demand distribution. For this case, we show the structure of the optimal policy based on a numerical study. Further, the numerical results reveal several interesting managerial insights, such as the optimal reservation policy is being little sensitive to the uncertainty of capacity allocation. In that case, the optimal reservation quantities hardly increase, but the optimal policy suggests increasing the utilization of the allocated capacity. Moreover, we show that for the contract manufacturer, to achieve the desired behavior, charging small reservation costs is sufficient.

Keywords: Capacity reservation, Stochastic capacity and demand, Outsourcing, Dynamic programming

¹ Corresponding author. Address: Eindhoven University of Technology, School of Industrial Engineering, Pav F18, P.O. Box 513, 5600 MB Eindhoven, The Netherlands. E-mail address: Y.Boulaksil@tue.nl

1. Introduction

Outsourcing has been defined by Chase *et al.* (2004, p.372) as an 'act of moving some of a firm's internal activities and decision responsibilities to outside providers'. In the last few years, many papers appeared on the development of outsourcing in many industries (Kremic *et al.*, 2006). A survey in 1997 of more than 600 large companies by the American Management Association finds that substantial numbers of companies are now outsourcing in many areas: information systems, finance, accounting, manufacturing, maintenance, and personnel. Among manufacturing companies, more than half had outsourced at least one component of their production process (Bryce and Useem, 1998).

Due to contractual agreements and limited information transparency, outsourcing complicates the order placing process for the OEM, especially when the contract manufacturer serves a number of OEMs on the same production line (Boulaksil and Fransoo, 2008). It is common in practice to have a contractual agreement that obliges the OEMs to reserve capacity prior to ordering (Zhao *et al.*, 2007). Capacity reservation offers several benefits to supply chain members such as mitigating the bullwhip effect (Lee *et al.*, 1997), providing flexibility to deal with uncertain demand and helps the contract manufacturer with his capacity planning, i.e., to secure capacity prior to receiving orders from the OEMs (Serel *et al.*, 2001).

In this paper, we consider an OEM that has outsourced the production activities for a long-term to a contract manufacturer, who is the only source of supply for that specific product. The contract manufacturer performs the production activities on a non-dedicated capacitated production line on which multiple OEMs are served. Basically, the contract manufacturer does not have his own product portfolio, but only produces by offering outsourcing services to the OEMs.

According to the contractual agreement, the contract manufacturer requires from the OEM to reserve capacity slots in advance. Once the reservations are collected, the contract manufacturer plans his capacity based on allocation rules and priorities that are unknown to the OEM. Therefore, the available capacity for each OEM is not known in advance. Later, the contract manufacturer responds to the OEM with the accepted reservation quantity, which is the upper bound for the order quantity, which is placed by the OEM to meet the uncertain demand.

From the OEM's perspective, it is not obvious what the optimal strategy is to control such a system. Reservation secures capacity for future orders, but also increases costs. A large body of literature deals with production planning models and inventory systems that considers capacitated supply (Federgruen and Zipkin, 1986; Ciarallo *et al.* 1994), but does not consider capacity reservation in their models. Therefore, our main contribution to this line of research is that we include the capacity reservation problem to the production planning model in case of uncertain capacity and demand.

We study this multi-period inventory system from the OEM's perspective that faces uncertain capacity from the contract manufacturer and uncertain customer demand. The OEM has to decide on the reservation and order quantities (to release to the contract manufacturer) in order to minimize the expected costs. We develop a stochastic dynamic programming model for this problem and characterize the optimal policy. We also conduct a numerical study in which we extend the problem by considering dependency between the demand and capacity distributions.

The numerical results reveal several interesting managerial insights, such as that the utilization of the reservations (the order quantity divided by the accepted reservation quantity) is increased when the capacity uncertainty or the reservation costs increase, while the optimal reservation policy is little sensitive to the level of capacity uncertainty. We also show that the desired reservation and order behavior is achieved when small reservation costs are charged. Moreover, we show the structure of the optimal reservation policy in case of dependency between the distributions, which is characterized by two parameters.

This paper is organized as follows. In section 2, we discuss the literature review and show our contribution to the literature. In section 3, we present the model and some analytical results. In section 4, we present the optimal policies. Then, in section 5, we present and discuss the numerical results and the managerial insights. Finally, in section 6, we draw some conclusions and discuss some managerial insights.

2. Literature review

Federgruen and Zipkin (1986) were one of the first to study a periodic review inventory model with a finite but certain capacity level. They proved the optimality of modified basestock policies. An extension of this work is that of Ciarallo *et al.* (1994) who study the stochastic demand and stochastic capacity setting. They show that in a single-period setting, the optimal policy is not affected by the capacity uncertainty, but in the multi-period setting, order-up-to policies that are dependent on the distribution of the capacity are optimal. Several other papers extended this problem (Güllü, 1998; Hwang and Singh, 1998; Wang and Gerchak, 1996; Iida, 2002; Jaksic *et al.*, 2008). Our main contribution to this line of research is that we add the reservation problem to the stochastic demand and stochastic capacity case.

In our model, the OEM makes a reservation by sharing advance demand information (Karaesmen *et al.*, 2002) without knowing exactly what the supply quantity will be, which can be considered as a form of supply uncertainty. A large stream of papers studies the supply uncertainty problem (Bassok and Akella, 1991; Parlar *et al.*, 1995; Güllü *et al.*, 1999; Pac *et al.*, 2009). Most of these papers consider completely uncertain supply quantities, whereas in our case, the supply uncertainty can be partly controlled by the reservation decisions.

Another related part of the literature is about capacity reservation, which has been studied at both the tactical and the operational level. At the tactical level, the main objective is to study contract types and the conditions under which coordination in the supply chain can be achieved. Erkoc and Wu (2005) study the so-called deductible reservation contract, which means that the buyer pays a fee in advance for each reserved unit of capacity. When the buyer places a firm order, the reservation fee is deducted from the order payment, but the fee is not refundable in case the reserved capacity is not fully utilized within the specified time.

At the operational level, many papers have studied capacity reservation (Bonser and Wu, 2001; Hazra and Mahadevan, 2009; Serel *et al.*, 2001; Serel, 2007; Van Norden and Van de Velde, 2005; Mincsovcics *et al.*, 2009). The main objective of these studies is to decide on getting materials supplied either at a lower price by reserving capacity in advance with the long-term supplier or at a higher price from the spot market (Hazra and Mahadevan, 2009) or making reservations to *guarantee* the delivery of (a portion of) the reserved quantity,

given the existence of the more expensive spot market (Serel *et al.*, 2001) or given the uncertain availability of the item in the spot market (Serel, 2007).

Serel *et al.* (2001) show that the existence of the spot market alternative significantly reduces the capacity reservation quantity from the long-term supplier. A similar case is considered by Hazra and Mahadevan (2009), who derive the supplier's optimal capacity reservation price in such a setting. Another paper that has studied capacity reservation is that of Jain and Silver (1995). They consider a single-period setting with stochastic demand and supplier's capacity, but dedicated capacity can be ensured by paying a premium to the supplier. The paper shows that the cost function is not convex in the dedicated capacity, but an algorithm is developed for finding the best level of dedicated capacity.

The literature on capacity reservation considers a single-period (or two-period) dual sourcing setting. We contribute to this line of research by considering a multi-period setting where the reservation problem is integrated with the inventory control problem. The paper that is the closest to our work is that of Costa and Silver (1996). A multi-period inventory problem is considered where the supplier capacity and the customer demand are uncertain. In that paper, the decision maker has the option to reserve some capacity for one or more periods, but the reservations have to be made prior to the start of the planning horizon, whereas in our model, the reservations can be done in each period of the planning horizon, based on more updated information. Furthermore, we characterize the optimal policy for our setting.

3. Model

Table 1. Notation

T	number of periods in the planning horizon
h	inventory holding cost per unit per period
b	backorder cost per unit per period
s	reservation cost per unit per period
x_t	inventory position in period t before ordering
y_t	inventory position in period t after ordering
r_t	reservation quantity in period t for period $t+1$
z_t	reservation position in period t after reserving
a_t	actual accepted reservation quantity in period t
A_t	(random) accepted reservation quantity in period t
q_t	order quantity in period t
D_t	(random) demand in period t
$f_t(d_t)$	probability density function of the demand in period t
d_t	actual demand in period t
C_t	(random) capacity in period t
c_t	actual capacity in period t
α	discount factor ($0 < \alpha \leq 1$)

As discussed earlier, we consider an OEM that has outsourced the production activities for a long-term to a contract manufacturer who serves a number of OEMs on the same production line. The contract manufacturer is the only source of supply for the product. According to the contractual agreement, the OEM reserves capacity before ordering. The reservations are needed by the contract manufacturer for his capacity planning and the contract manufacturer responds to a reservation r_{t-1} one period later by the accepted reservation quantity a_t .

At the moment of reservation, the OEM does not know what the actual allocated capacity c_t will be, because the contract manufacturer decides on the capacity allocation based on rules and priorities that are unknown to the OEM. Further, the OEM also has no information about the reservations of other OEMs. Therefore, from the OEM's perspective, a_t is the minimum of r_{t-1} and the uncertain allocated capacity C_t at the contract manufacturer.

Once a_t is announced, the reservation costs are charged, which are equal to sa_t with s being the unit reservation cost. These costs are introduced by the contract manufacturer to avoid that the OEMs inflate their reservations. The OEM is not charged for each unit reserved r_t , as this would be 'unfair' if part of the reservation is rejected. In essence, our reservation cost structure is similar to the deductible reservation fee of Erkoc and Wu (2005), as in both cases, a premium is also paid for reservations that are not utilized. After knowing a_t , the OEM decides on the order quantity q_t to meet the uncertain demand D_t . The order quantity q_t cannot exceed a_t and is delivered by the contract manufacturer just before the real demand d_t is observed.

To model this finite-horizon planning problem, we use a stochastic dynamic programming approach with two state variables: the inventory position before ordering x_t and a_t , which forms an upper bound on q_t . These state variables are needed to make decisions on r_t and q_t . We assume a periodic review inventory system with stochastic demand and stochastic capacity. As far as the model is concerned, we do not need any assumptions on the probability distributions of D_t and C_t . However, to show some optimality results in section 4, we assume that the distributions of D_t and C_t are independent of each other. In section 5.2, we relax this assumption and investigate the effects of dependency between the distributions.

We consider the following sequence of events. At the start of period t , the decision maker reviews x_t and a_t , where $a_t = \min\{r_{t-1}, c_t\}$. Then, the reservation costs are incurred: sa_t . Based on the current state of the system (x_t, a_t) , the decision maker decides on $r_t \geq 0$. The decision maker also decides on q_t , which raises the inventory position to $y_t = x_t + q_t$, where $0 \leq q_t \leq a_t$. Then at the end of period t , q_t that was ordered at the beginning of period t arrives and d_t is observed and satisfied as much as possible from inventory; unsatisfied demand is backordered. Then, inventory holding and backorder costs are incurred.

The state variables of the dynamic programming model (x_t, a_t) are updated at the start of period $t+1$ in the following way:

$$x_{t+1} = x_t + q_t - d_t \quad (1)$$

$$a_{t+1} = \min\{r_t, c_{t+1}\} \quad (2)$$

We assume linear inventory holding, backorder and reservation costs. Let $g_t(x_t, a_t)$ denote the minimum expected cost function, optimizing the cost over the finite planning horizon T from t onward and starting in the initial state (x_t, a_t) . Then, we have the following DP recursion:

$$g_t(x_t, a_t) = sa_t + \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t + a_t}} \{\mathcal{L}(y_t) + \alpha E_{D_t, C_{t+1}}[g_{t+1}(y_t - D_t, A_{t+1})]\}, 1 \leq t \leq T \quad (3)$$

where

$$\mathcal{L}(y_t) = h \int_0^{y_t} (y_t - d_t) f_t(d_t) dd_t + b \int_{y_t}^{\infty} (d_t - y_t) f_t(d_t) dd_t \quad (4)$$

and

$$A_{t+1} = \min\{r_t, C_{t+1}\} \quad (2')$$

The last part of (3) is the expected future cost, which is derived by taking the expectation over D_t and C_{t+1} . Further, the stopping condition is $g_{T+1}(\cdot) = 0$.

4. The optimal order and reservation policy

In this section, we characterize the optimal solution of (3) by assuming that the demand (D_t) and capacity (C_t) are identically and independently distributed across periods and are independent of each other. We prove the optimality of a state-dependent reservation policy and a modified basestock policy.

Let $h_t(y_t, a_t) = \mathcal{L}(y_t) + \alpha E_{D_t, C_{t+1}}[g_{t+1}(y_t - D_t, A_{t+1})]$ denote the cost-to-go function in period t . Accordingly, the minimum expected cost function $g_t(\cdot)$ can be rewritten as

$$g_t(x_t, a_t) = sa_t + \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t + a_t}} h_t(y_t, a_t), \quad 1 \leq t \leq T \quad (3')$$

In order to describe the optimal reservation policy, we introduce the "reservation position": $z_t = x_t + r_t$. Let (\hat{y}_t, \hat{z}_t) be the unconstrained minimizers of $h_t(y_t, a_t)$ for given state variables (x_t, a_t) . We first show the convexity results that allow us to find the structure of the optimal policy. Note that the loss function $\mathcal{L}(y_t)$ is convex in y_t (Porteus, 2002). The optimal decisions at any period t (y_t, z_t) are made by minimizing $h_t(\cdot)$ over the feasible region.

Theorem 1:

- a. For any period $1 \leq t \leq T$, $g_t(x_t, a_t)$ and $h_t(y_t, a_t)$ are (jointly) convex functions.
- b. For any period $1 \leq t \leq T$, the optimal order policy is given by:

$$y_t^* = \begin{cases} x_t + a_t & \text{if } x_t < \hat{y}_t - a_t \\ \hat{y}_t & \text{if } \hat{y}_t - a_t \leq x_t \leq \hat{y}_t \\ x_t & \text{if } x_t > \hat{y}_t \end{cases} \quad (5)$$

- c. For any period $1 \leq t \leq T$, the optimal reservation policy is given by:

$$z_t^*(a_t) = \begin{cases} \hat{z}_t(a_t) & \text{if } x_t \leq \hat{z}_t(a_t) \\ x_t & \text{if } x_t > \hat{z}_t(a_t) \end{cases} \quad (6)$$

See appendix A for the proof.

The optimal order policy (5) is a modified basestock policy, as the order quantity is bounded by a_t if $x_t < \hat{y}_t - a_t$. The optimal reservation policy (6) is a state-dependent reservation-up-to policy. This policy implies that at a given a_t , the reservation quantity should bring the reservation position $z_t = x_t + r_t$ to the optimal reservation position $\hat{z}_t(a_t)$ if $x_t \leq \hat{z}_t(a_t)$. Otherwise, $z_t^*(a_t) = x_t$, which means to reserve nothing.

To summarize, the inventory system can be optimally controlled by two critical parameters: the optimal order-up-to level y_t^* and the optimal reservation-up-to level $z_t^*(a_t)$ according to policies (5) and (6).

5. Numerical study

In this section, we present and discuss a numerical study that we conducted by solving the stochastic dynamic programming formulation given in (3). We construct a number of experiments and we are mainly interested in the effects of:

- different levels of demand and capacity uncertainty and different reservation costs in case of stationary distributions (section 5.1), and
- dependency between the demand and capacity distributions (section 5.2)

on the optimal decisions and the system performance.

The following parameters are set at fixed values: $T=12$, $\alpha=0.99$, $h=1$, and $b=10$. Furthermore, we assume a Gamma distribution for the demand and a Uniform distribution for the capacity (Burgin, 1975).

5.1. Stationary demand and capacity availability

In this section, we consider different levels of demand and capacity uncertainty and we vary the unit reservation cost. In experiments 1-24 (see Table 2), $E[D_t] = E[C_t] = 5$, but we vary:

- the coefficient of variation of the demand $CV(D_t)$ between 0.5, 1, 1.5, 2, and 3;
- the coefficient of variation of the capacity $CV(C_t)$ between 0.28 and 0.52;
- the unit reservation cost between 0, 2, 5, and 10.

Table 2 shows the results of these experiments, where $\hat{z}_1(a_1)$ is shown as a vector in \vec{a}_1 . Moreover, the expected costs are shown for $(x_t, a_t) = (0,6)$, as this is a feasible state for all experiments.

Table 2. Results with varying demand uncertainty, capacity uncertainty, and reservation cost

Exp	$E[D_t]$	$CV(D_t)$	$E[C_t]$	$CV(C_t)$	s	\hat{y}_1	$\hat{z}_1(a_1)$	$E[Cost]$
1	5	0.5	5	0.28	0	19	36	306.32
2	5	0.5	5	0.28	2	22	{20,...,16}	425.00
3	5	0.5	5	0.28	5	25	{19,...,15}	598.52
4	5	0.5	5	0.28	10	28	{18,...,14}	882.65
5	5	1	5	0.28	0	30	59	573.06
6	5	1	5	0.28	2	33	{30,...,26}	690.27
7	5	1	5	0.28	5	35	{27,...,23}	860.26
8	5	1	5	0.28	10	38	{24,...,20}	1136.68
9	5	1.5	5	0.28	0	37	77	704.83
10	5	1.5	5	0.28	2	39	{36,...,32}	820.01
11	5	1.5	5	0.28	5	41	{31,...,27}	985.60
12	5	1.5	5	0.28	10	43	{27,...,23}	1250.49
13	5	0.5	5	0.52	0	22	39	379.69
14	5	0.5	5	0.52	2	28	{25,...,17}	495.67
15	5	0.5	5	0.52	5	36	{25,...,17}	663.92
16	5	0.5	5	0.52	10	45	{24,...,16}	937.70
17	5	1	5	0.52	0	32	60	614.62
18	5	1	5	0.52	2	38	{34,...,26}	729.24
19	5	1	5	0.52	5	43	{33,...,25}	892.21
20	5	1	5	0.52	10	49	{32,...,24}	1154.19
21	5	1.5	5	0.52	0	38	78	735.05
22	5	1.5	5	0.52	2	43	{40,...,32}	847.08
23	5	1.5	5	0.52	5	47	{37,...,29}	1003.41
24	5	1.5	5	0.52	10	51	{34,...,26}	1251.89

The results from Table 2 show that higher demand uncertainty increases \hat{y}_1 , $\hat{z}_1(a_1)$ and leads to higher costs. However, the higher the unit reservation cost s , the lower the effect of an increase of the demand uncertainty, because the incremental increase of the optimal reservation quantities decreases. Figure 1 shows the optimal order-up-to and reservation-up-to levels for different unit reservation cost and different levels of demand uncertainty. From the results, we see that when s increases, the optimal order-up-to level increases much more than the reservation quantities. In other words, when the unit reservation costs increases, it is optimal to increase the order quantity (much more than the reservation quantity) such that a larger part of the accepted reservation is utilized, instead of increasing the reservation quantities. See Figure 2 that confirms this insight by showing the optimal ratio $\frac{z_t}{y_t}$.

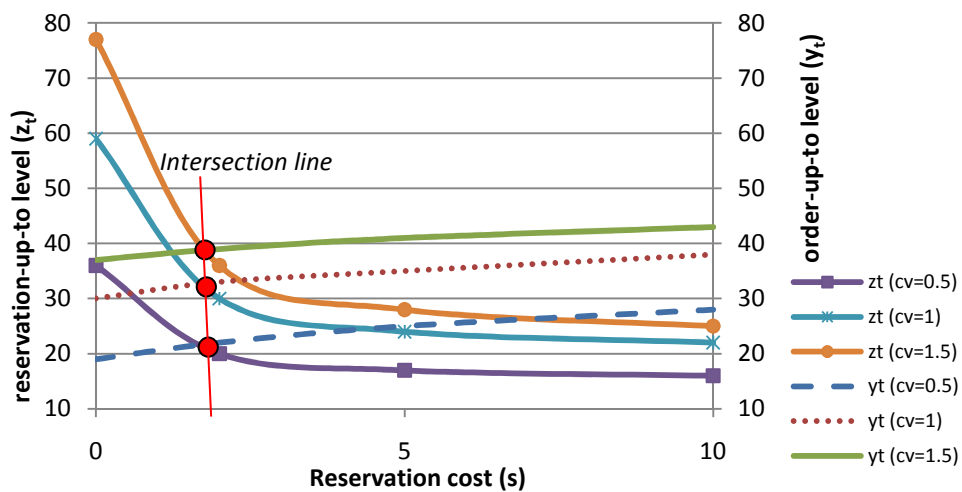


Figure 1. Order-up-to (y_t) and reservation-up-to (z_t) levels for different levels of demand uncertainty and reservation cost and the intersection line.

From the other side, the contract manufacturer would prefer a situation where the difference between the reservation and order quantities is minimal, ideally zero. In Figure 1, we show that these ideal situations are reached at small unit reservation cost. The intersection line that connects the intersection points is almost vertical, which means that the optimal s is independent of the level of demand uncertainty. This means that the contract manufacturer should incorporate small unit reservation cost in the contract to avoid large discrepancies between the reservation and order behavior.

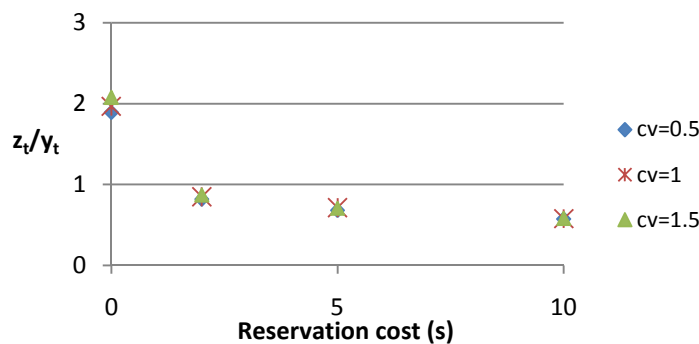


Figure 2. The optimal ratio z_t/y_t at different unit reservation cost and different levels of demand uncertainty.

Another insight from Table 2 is that the higher the demand uncertainty, the lower the effect of an increase of capacity uncertainty on the optimal cost (see Figure 3, where $\Delta Cost$ is given by (7)).

$$\Delta Cost = \frac{E[Cost|CV(C_t) = 1.5] - E[Cost|CV(C_t) = 0.5]}{E[Cost|CV(C_t) = 0.5]} \quad (7)$$

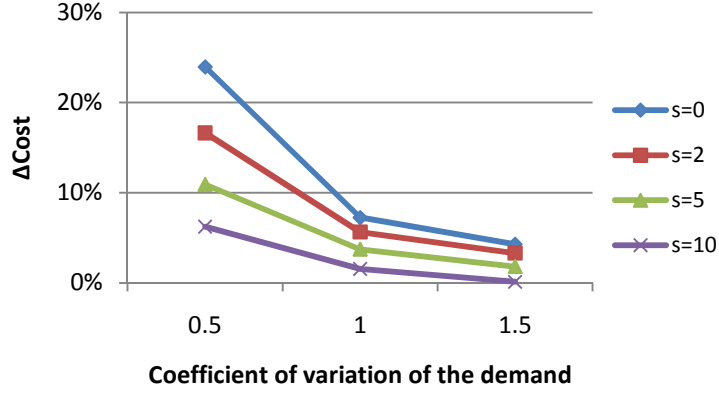


Figure 3. Relative cost increase due to increased capacity uncertainty.

The explanation for this effect is that when the capacity uncertainty increases, $\hat{z}_1(a_1)$ increases little compared with \hat{y}_1 , i.e., the order quantity increases much more than the reservation quantity (see Figure 5.4). Therefore, when the capacity uncertainty increases, it is optimal to increase the order-up-to level much more than the reservation-up-to level. Therefore, when the capacity uncertainty increases, the optimal ratio $\frac{z_t}{y_t}$ decreases.

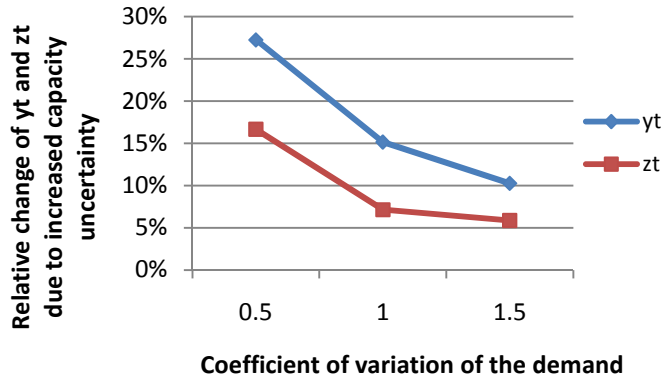


Figure 4. Relative change in y_t and z_t due to increased capacity uncertainty (when $s=2$).

5.2. Dependency between the distributions

In this section, we show the results of a numerical study in which we consider dependency between the demand and capacity distributions. In particular, we assume that the capacity allocation C_t of the contract manufacturer is dependent on the OEM's demand D_t and therefore, the results of section 4 do not hold anymore. We consider both the situations where the dependency is positive (section 5.2.1.) and negative (section 5.2.2.). We also show the structure of the optimal policy for these two situations. In appendix B, we show how the conditional probability distributions are determined.

5.2.1. Positive dependency

In this section, we consider the case where the contract manufacturer is allocating more capacity when the OEM's demand is higher. The idea is that when the OEM's demand is higher, the OEM will request more (in terms of reservations and orders) and the contract manufacturer is then willing to allocate more capacity to the OEM to avoid the OEM searching for another source of supply. This situation is also possible when the OEMs' demand quantities are negatively correlated, which means that the more the OEM reserves and orders the less the other OEMs reserve and order, the more capacity is available for the OEM. This could happen when the total demand of all OEMs is relatively stable due to a fixed market size and an increase in the OEM's demand represents an increased market share.

First, we show the structure of the optimal policy as we observed during the numerical studies. Then, we discuss the numerical results and compare them with non-correlated case. Based on the numerical studies, we observed that the optimal order policy remains a modified basestock policy with the same structure as (5). However, the reservation policy does not remain the same. Figure 5 shows the structure of the optimal reservation policy in case of positive dependency, as we observed in our numerical study. The policy can be characterized by two optimal reservation-up-to levels, where the second level is lower than the first one. When the (starting) inventory position exceeds some point, it is optimal to target for a lower reservation-up-to level, i.e., to reserve much less. Due to the positive dependency, less has to be reserved (which also limits the reservation costs) to get the same amount of capacity allocated.

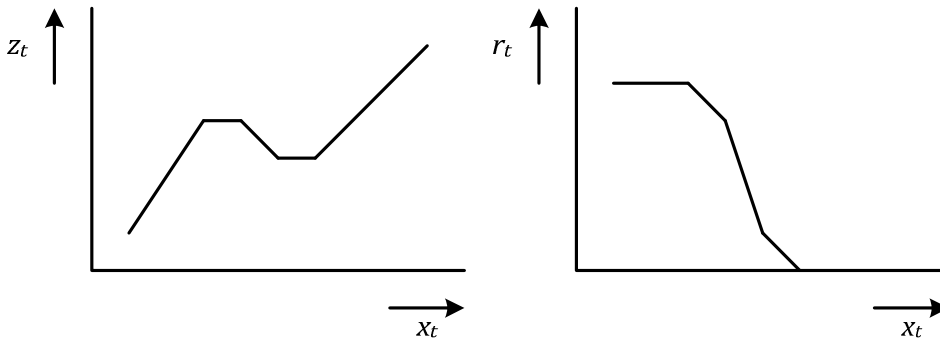


Figure 5. The structure of the optimal reservation policy in case of positive dependency

Table 3 shows the numerical results for 9 experiments that we conducted with positive dependency between the demand and capacity distributions. The results show that for all experiments, the expected cost is lower than in case with no dependency (on average 20.6 %). Due to the positive dependency, less has to be reserved with lower risk of getting too high accepted reservations, which results in lower (reservation) costs.

This result suggests that it is worthwhile to collect market information of the competitors (that produce at the same contract manufacturer) and to assess the dependency between the own demand and that of the competitors. In case of a negative dependency between OEMs' demand (which means there is a positive dependency between the own demand and the available capacity at the contract manufacturer), which is for example the case when the competitors operate in different market sectors, it is wise to adapt the reservation policy towards the contract manufacturer.

Table 3. Results with positive dependency

Exp	$E[D_t]$	$CV(D_t)$	$E[C_t]$	$CV(C_t)$	s	\hat{y}_1	$E[Cost]$
25	5	0.5	5	0.28	0	17	234.74
26	5	0.5	5	0.28	2	20	358.88
27	5	0.5	5	0.28	5	24	538.13
28	5	0.5	5	0.28	10	28	830.40
29	5	1	5	0.28	2	31	598.66
30	5	1.5	5	0.28	2	38	717.91
31	5	0.5	5	0.52	2	25	370.53
32	5	1	5	0.52	2	35	554.45
33	5	1.5	5	0.52	2	40	684.23
34	5	2	5	0.28	2	41	750.22
35	5	2	5	0.52	2	45	769.50
36	5	3	5	0.28	2	42	787.79
37	5	3	5	0.52	2	54	864.16

5.2.2. Negative dependency

In this numerical study, we consider the negative dependency case. Such a situation is likely when the different OEMs who all reserve and order at the same contract manufacturer operate in the same market, which results in a positive correlation between the demand D_t of the different OEMs. That means that all OEMs will increase their reservations and orders in case of a demand increase and vice versa. In such a situation, the contract manufacturer faces increased demand from all OEMs simultaneously, which results in a smaller capacity allocation for each OEM. Based on the results of the numerical studies, we observe that the optimal order policy remains the same as (5), but the structure of the reservation policy changes and is shown in Figure 6.

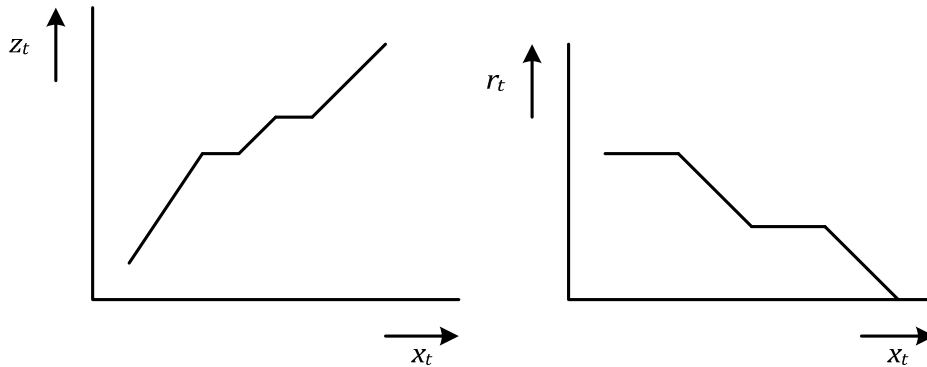


Figure 6. The structure of the optimal reservation policy in case of negative dependency

Like in the positive dependency case, the optimal policy can be characterized by two optimal reservation-up-to levels, but the second one is now higher than the first one. When the starting inventory position exceeds some point, it is optimal to target for a higher reservation-up-to level to get less capacity allocated and consequently not facing too high reservation and inventory holding costs.

Table 4 shows the numerical results of the experiments that we conducted with negative dependency between the demand and capacity distributions. The results show that for all experiments, the costs are higher than in case with no dependency. Due to the negative dependency, the probability of not getting enough supplied to meet the demand increases, which increases the backorder costs.

Like in the positive dependency case, it is worthwhile to assess whether there is dependency between the own demand and that of the competitors, by which the dependency between the own demand and the contract manufacturer's available capacity level can be estimated. If the latter dependency appears to be negative, it is recommended to take measures to eliminate the negative dependency, as this leads to higher costs. The elimination can be done by keeping some safety stock to avoid backorders or by agreeing (contractually) on paying slightly more to make an appeal to (in case needed) a fixed amount of the contract manufacturer's production capacity.

Table 4. Results with negative dependency

<i>Exp</i>	$E[D_t]$	$CV(D_t)$	$E[C_t]$	$CV(C_t)$	s	\hat{y}_1	$E[Cost]$
38	5	0.5	5	0.28	0	20	395.51
39	5	0.5	5	0.28	2	22	507.63
40	5	0.5	5	0.28	5	25	673.77
41	5	0.5	5	0.28	10	28	947.42
42	5	1	5	0.28	0	32	719.15
43	5	1	5	0.28	2	35	825.08
44	5	1.5	5	0.28	2	41	953.94
45	5	0.5	5	0.52	0	24	575.52
46	5	0.5	5	0.52	2	29	681.81
47	5	1	5	0.52	2	40	983.23
48	5	1.5	5	0.52	2	46	1116.38
49	5	2	5	0.28	2	44	999.53
50	5	2	5	0.52	2	50	1163.27
51	5	3	5	0.28	2	48	1040.79
52	5	3	5	0.52	2	53	1207.47

6. Conclusions

In this paper, we consider the case where a manufacturing company has outsourced the production activities to a contract manufacturer. The contract manufacturer produces on a non-dedicated production line on which multiple OEMs are served. For capacity planning purposes, the contract manufacturer requires that the OEM reserves capacity before ordering and responds to the reservations by acceptance or partial rejection based on rules that are unknown to the OEM. Therefore, the allocated capacity to the OEM is not known in advance.

We study this problem from the OEM's perspective who faces stochastic customer demand and stochastic capacity allocation from the contract manufacturer and who has to decide on the reservation and order quantities under uncertainty. We develop a stochastic dynamic programming model for this problem and we characterize the optimal reservation and order policies. The optimal reservation policy is a state-dependent policy, as the optimal target reservation-up-to level is dependent on the accepted reservation quantity. The optimal order policy is a modified basestock policy; the order quantity is bounded by the accepted reservation quantity.

We conduct a numerical study which reveals several interesting (managerial) insights. First, in case the unit reservation cost or the capacity uncertainty increases, it is optimal to increase the order quantity (much more than the reservation quantity) and so, the utilization of the accepted reservation quantity. This might be counterintuitive, as one would expect to mainly increase the reservation quantities in case the capacity uncertainty increases to hedge against the uncertainty faced from the contract manufacturer. Another insight is that

the effect of an increase of the capacity uncertainty decreases substantially when the demand uncertainty increases. When the demand uncertainty increases, the optimal order quantities increase, by which the order will be (closely) equal to the accepted reservation quantity. The action of increasing the order quantity is also required when the capacity uncertainty increases, and therefore, we see that the effect of an increase of the capacity uncertainty is very little when the demand uncertainty increases.

Another managerial insight follows from the fact that from the contract manufacturer's perspective, it is desired to have the reservation equal to the order quantity. We have seen that this can be achieved when little reservation costs are charged. This optimal unit reservation cost is independent of the level of demand uncertainty. Charging no reservation costs leads to over reservation and charging higher reservation costs leads to under reservation. This managerial insight is helpful when having contract negotiations with the OEMs on setting the reservation cost, which is a contract parameter.

Finally, we studied the case where the capacity allocation of the contract manufacturer depends on the OEM's demand distribution. When the distributions are dependent, the structure of the optimal order policy is the same as in the independent case, but the optimal reservation policy changes to a policy with two optimal target reservation-up-to levels. Dependent on whether the dependency is positive (or negative), the second optimal reservation-up-to level is lower (or higher) than the first one by which the model adapts its reservation quantities to the higher (or lower) capacity allocation. We have seen that the expected cost decreases when the dependency is positive and increases when the dependency is negative. These results suggest that it is worthwhile to collect market information of the competitors (that produce at the same contract manufacturer) to assess the dependency between the own demand and the available capacity at the contract manufacturer. In case the dependency is positive, it is wise to adapt the reservation policy towards the contract manufacturer to save costs. In case of negative dependency, one can think of measures like keeping safety stocks to hedge against the little capacity allocation of the contract manufacturer or agreeing on paying an additional premium to ensure (in case needed) a fixed amount of capacity from the contract manufacturer. Of course, these measures should be cheaper than the extra cost due to the negative dependency.

Appendix A. Proof of theorem 1

Let

$$\begin{aligned} g_t(x_t, a_t) &= \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t a_t}} \{s a_t + \mathcal{L}(y_t) + \alpha E_{D_t, C_t} [g_{t+1}(y_t - D_t, A_t)]\} \\ &= \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t a_t}} \{s a_t + h_t(y_t, a_t)\} = s a_t + i_t(x_t, a_t) \end{aligned}$$

- The functions $g_t(x_t, a_t)$ and $h_t(y_t, a_t)$ are jointly convex functions for any $t \in [1, T]$. We prove this by induction. $g_{T+1}(\cdot) = 0$ and is convex. Assume that $h_{t+1}(\cdot)$ is also convex. Then, the function $h_t(y_t, a_t) = \mathcal{L}(y_t) + \alpha E_{D_t, C_t} [g_{t+1}(y_t - D_t, A_t)]$ is also convex, because:
 - $\mathcal{L}(y_t)$ is a convex function;
 - $E[g_{t+1}(y_t - D_t, A_t)]$ is convex due to the convexity of the expected value operator. Rule: If $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is convex, then the function $g(x) = E_w\{f(x + w)\}$ is also a convex function, where w is a random vector in \mathbb{R}^n , provided that the expected value is finite for every $x \in \mathbb{R}^n$ (Bertsekas, 2005);
 - the linear combination of two convex functions remains convex. Rule: Let K be a non-empty index set, X a convex set, and for each $k \in K$ let $f_k(\cdot)$ be a convex function on X and let $p_k \geq 0$. Then $\sum_{k \in K} p_k f_k(x)$, $x \in X$ is a convex function on any convex subset of X , where the sum takes finite values (Heyman and Sobel, 2004).
- $i_t(x_t, a_t) = \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t a_t}} \{h_t(y_t, a_t)\}$ is also convex when $h_t(y_t, a_t)$ is convex. Rule: Let X be a non-empty set with A_x a non-empty set for each $x \in X$. Let $C = \{(x, y): y \in A_x, x \in X\}$, let J be a real-valued function on C , and define $f(x) = \inf\{J(x, y): y \in A_x, x \in X\}$. If C is a convex set and J is a convex function on C , then f is a convex function on any convex subset of $X^* = \{x: x \in X, f(x) > -\infty\}$ (Heyman and Sobel, 2004).
- The function $g_t(x_t, a_t) = s a_t + i_t(x_t, a_t)$ is then also convex.

Appendix B. Conditional probability distribution

In part of the numerical studies, we consider the case where the capacity allocation C_t by the contract manufacturer is positively or negatively dependent on OEM's demand D_t . Therefore, we adapt the probability mass function of C_t to a conditional probability mass function in which C_t is conditioned on D_t : $P\{C_t = c | D_t = d\}$. This function is basically the matrix G of size $m \times n$, where $m = d_{max}$ (the maximum demand) and $n = c_{max} - c_{min} + 1$ (where c_{min} and c_{max} are the bounds of the capacity distribution).

In case of positive dependency, elements $G_{m,1} = G_{1,n} = 0$ and $G_{1,1} = G_{m,n} = \begin{cases} \frac{2}{n} & \text{if } n \geq 2 \\ 1 & \text{if } n = 1 \end{cases}$.

Then, the rows and columns are filled by a *linear* decrease/increase from 0 to $G_{1,1}$ or $G_{m,n}$. Finally, the probabilities are rescaled, such that distribution sums up to 1.

$$G = \begin{bmatrix} G_{1,1} & \frac{2(n-2)}{n(n-1)} & \frac{2(n-3)}{n(n-1)} & \cdots & \frac{2}{n(n-1)} & 0 \\ & \vdots & & \ddots & \vdots & \\ & 0 & \cdots & \cdots & \cdots & G_{m,n} \end{bmatrix}$$

In case of negative dependency, the same procedure is applied, but now $G_{1,1} = G_{m,n} = 0$ and $G_{m,1} = G_{1,n} = \begin{cases} \frac{2}{n} & \text{if } n \geq 2 \\ 1 & \text{if } n = 1 \end{cases}$.

$$G = \begin{bmatrix} 0 & \cdots & G_{1,n} \\ \vdots & \ddots & \vdots \\ G_{m,1} & \cdots & 0 \end{bmatrix}$$

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