The capacitated multi-echelon inventory system with serial structure. 2. An average cost approximation method

Document status and date:
Published: 01/01/1991

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Memorandum COSOR 91-40

The capacitated multi-echelon inventory
system with serial structure:
2. An average cost approximation method

C.J. Speck
J. van der Wal

THE CAPACITATED MULTI-ECHELON INVENTORY SYSTEM WITH SERIAL STRUCTURE :
2. AN AVERAGE COST APPROXIMATION METHOD

C.J. Speck and J. van der Wal

Eindhoven University of Technology

This paper considers a multi-echelon, periodic review inventory model with discrete demand. We assume finite capacities on various production/order sizes and backordering of excess demand. We have already seen that modified base-stock policies work quite well under an average cost criterion. Here a method will be presented which provides an approximation of the average costs corresponding to a modified base-stock policy in a certain class of multi-echelon serial systems. In this the moment-iteration method developed by De Kok plays a central role.

1. Introduction. In this paper we consider a basic production (or inventory) model, in which the stock of a single item must be controlled under periodic review. We assume demands in each period to be independent, nonnegative, and integer-valued. Further, all stockouts are backordered and production, holding, and shortage costs are linear. There are no fixed order costs.

For the long-run-average cost criterion Federgruen and Zipkin[1986] have shown that in the capacitated 1-echelon inventory system with serial structure a modified base-stock policy is the optimal periodic review strategy: If the echelon stock has dropped below a certain level, enough should be produced to raise total stock to that level if attainable; otherwise, the maximum feasible amount should be produced. On the other hand, if the echelon stock is above that level, nothing should be produced.

Analyzing the capacitated N-echelon serial system with $N \geq 2$ we numerically proved in a preceding manuscript (Speck and Van der Wal[1991]) the possible appearance of a so-called 'push ahead'-effect within the optimal policy: Due to certain combinations of already known yet to arrive shipments, magnitude of demand, and finite capacities it might sometimes be

1

profitable to exploit the maximum capacity in order to possibly avoid future ordering limitations.

Neither of the aforementioned papers gives an exact calculation of the average cost for the optimal policy. But already a few approximation methods have been developed, that successfully cope with the difficulties arising from the finite capacities, such as an application of the moment-iteration method of De Kok[1989], and a related method of Zijm (unpublished manuscript). Both methods however are as yet only applicable to the capacitated 1-echelon serial system.

This paper focuses on the development of an average cost approximation method on behalf of the N-echelon serial system with $N \geq 2$. In section 2 we introduce notation, definitions, and assumptions. Section 3 contains a detailed description concerning the average cost calculation when employing a modified base-stock policy. The moment-iteration method of De Kok will be brought forward, which fills an essential part in our approximation method. For clarity reasons we will illustrate our approximation method on the basis of the capacitated 2-echelon serial system.

2. Definitions and assumptions. As already mentioned we will restrict ourselves to the capacitated 2-echelon serial system. Extension of the method given in section 3 to the N-echelon serial system with $N > 2$ is possible, but very laborious.

First of all we recapitulate the model definitions as given in Speck and Van der Wal[1991]:

1. We define the *echelon stock* of a given installation as the stock at that installation plus all the stock in transit to or on hand at any installation downstream minus the backlogs at the most downstream installation.

2. Next, the *echelon inventory position* of an installation denotes the echelon stock plus all items heading for that installation, that already left the preceding installation (or the external supplier).

The capacitated 2-echelon model we analyzed is characterized by the following parameters:

U_i = maximal production/order size, $i = 1, 2$.
l_1 = leadtime of the route from installation 2 to installation 1.
l_2 = leadtime of the route from external supplier to installation 2.
p = shortage cost per unit per period at echelon 1.

2

h_i = additional holding cost per unit per period at echelon i, $i = 1, 2$.

x_i = stock at echelon i at the beginning of a period, $i = 1, 2$.

y_i = inventory position at echelon i at the beginning of a period, $i = 1, 2$.

D_t = demand in period t, $t \in I\!N$.

q_w = $I\!P\{D = w\}$, $w = 0, 1, \ldots$, the demand probabilities.

$F(u)$ = $\sum_{w=0}^{u} q_w$, $u \in I\!N$, the demand distribution.

We assume U_i to be finite for $i = 1, 2$, while x_i and y_i are always integer-valued and may be negative since stockouts are backordered.
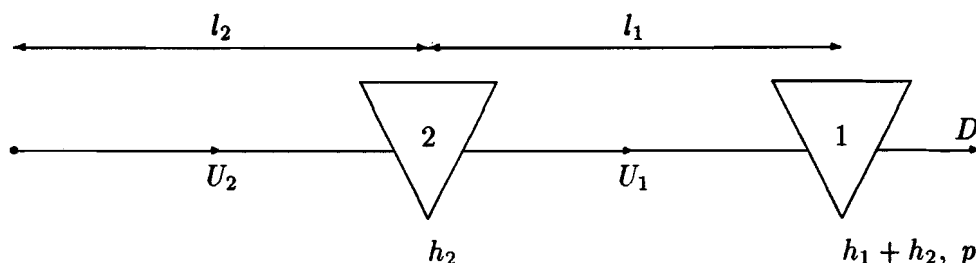
We distinguish the following activities during every period: At the beginning of the period (a) each echelon inventory position is increased, next (b) the external demand is met, and at the end of the period (c) a cost determination takes place.
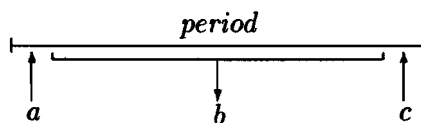
The expected costs at the end of a period consist of linear holding and shortage costs (linear ordering costs are not taken into account). Hence, the expected costs at the end of a period attached to an echelon stock x_i, $i = 1, 2$, at the beginning of that period are described by the well-known *Newsboy*-formulas, here displayed in the discrete form:

For $x_i \in Z\!\!\!Z$, $i = 1, 2$,

$$L_1(x_1) \quad := \quad \sum_{w=0}^{\infty} q_w[(h_1 + h_2)(x_1 - w)^+ + p\,(x_1 - w)^-] - h_2 x_1 \qquad (1)$$

$$L_2(x_2) \quad := \quad h_2 x_2. \qquad (2)$$

3

Finally, referring again to Speck and Van der Wal[1991] we assume without loss of generality

$$0 < U_2 \leq U_1. \tag{3}$$

3. Construction of an approximation method. De Kok[1989] has shown that an approximation of the average costs corresponding to a modified base-stock policy in the capacitated N-echelon serial system with $N = 1$ and $l_1 \in I\!N$ can be obtained by reformulating the problem as a $D|G|1$ queue. This is done by comparing the shortfall on the desired echelon inventory level to the length of a waiting-queue.

Suppose we want to determine the average costs associated with the modified base-stock policy d_1, with $d_1 \in I\!N$. Let $W_t \in I\!N$ be the shortfall on the desired echelon inventory position level d_1 at the beginning of period t. Then, starting from a shortfall of W_t on d_1 at the beginning of period t and meeting demand D_t leads to a shortfall of $W_t + D_t$ at the end of period t. If this amount exceeds the capacity U_1, then the shortfall at the beginning of period $t + 1$ shrinks to $W_t + D_t - U_1$; on the contrary, if $W_t + D_t \leq U_1$, then the shortfall is neutralized and the ideal level, d_1, again is achieved. Hence

$$W_{t+1} = \max(0, W_t + D_t - U_1). \tag{4}$$

This is the well-known relation for the waiting-time of the $(t+1)$st customer in the standard $D|G|1$ queue, for which De Kok has developed a moment-iteration method. The algorithm will be shown later on; it enables us to derive approximations for the waiting-time distribution for instance.

So if we take the collection of echelon inventory positions associated with the modified base-stock policy d_1,

$$\mathcal{Z}[\, d_1\,] = \{\, d_1, d_1 - 1, d_1 - 2, \ldots \}, \tag{5}$$

or, equivalently, the set of shortfall positions,

$$\mathcal{W} = \{\, 0, 1, 2, \ldots \}, \tag{6}$$

as state space we are able to form the enclosed transition matrix, say $\mathcal{Q}_1$ by

means of (4):

$$\mathcal{Q}_1 := \begin{pmatrix} F(U_1) & q_{U_1+1} & q_{U_1+2} & \cdots & \cdots \\ F(U_1-1) & q_{U_1} & q_{U_1+1} & \cdots & \cdots \\ F(U_1-2) & q_{U_1-1} & q_{U_1} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \\ \vdots & \vdots & \vdots & & \ddots \end{pmatrix} \tag{7}$$

Notice its fine diagonal structure, apart from the first column. Then, the exact average costs going with the modified base-stock policy d_1 is given by

$$g = \mathcal{Q}_1^\infty r[\ d_1\ ], \tag{8}$$

in which $\mathcal{Q}_1^\infty$ denotes the equilibrium matrix, and $r[\ d_1\ ]$ the cost vector consisting of the expected costs per state from $\mathcal{Z}[\ d_1\ ]$. The equilibrium matrix $\mathcal{Q}_1^\infty$, here consisting of identical rows of equilibrium probabilities $\pi_0^*, \pi_1^*, \pi_2^*, \ldots$, can be constructed by solving the system

$$\begin{cases} \pi \mathcal{Q}_1 & = \pi \\ \sum_{i=0}^\infty \pi_i & = 1, \end{cases} \tag{9}$$

in which π denotes a vector of probabilities. The fact of having a non-finite transition matrix $\mathcal{Q}_1$ however, causes the system (8) to be infinite and therefore does not permit an exact calculation of the equilibrium probabilities π_i^*. But we can derive approximations $\tilde{\pi}_i^*$ by using De Kok's moment-iteration method: Every π_i^*, denoting the equilibrium probability on a gap of i items on level d_1, in fact specifies the probability on a customer's waiting-time of i units in a $D|G|1$ equilibrium system!

Algorithm 1 (Moment-iteration)

Initialisation: *Choose* $I\!E\{W_0\} = 0$ *and* $I\!E\{W_0^2\} = 0$.
Iteration: *Compute*

$$\begin{aligned} I\!E\{W_n + D_n\} &= I\!E\{W_n\} + I\!E\{D_n\}; \\ I\!E\{(W_n + D_n)^2\} &= I\!E\{W_n^2\} + 2I\!E\{W_n\}I\!E\{D_n\} + I\!E\{D_n^2\}. \end{aligned}$$

*Fit a distribution function $\tilde{F}_{W_n+D_n}$ to the first two moments of $W_n + D_n$.
Then calculate*

$$\mathbb{E}\{W_{n+1}\} = \int_{U_1}^{\infty} (y - U_1) \, d\tilde{F}_{W_n+D_n}(y), \tag{10}$$

$$\mathbb{E}\{W_{n+1}^2\} = \int_{U_1}^{\infty} (y - U_1)^2 \, d\tilde{F}_{W_n+D_n}(y). \tag{11}$$

Termination: *Stop whenever*

$$|\mathbb{E}\{W_{n+1}\} - \mathbb{E}\{W_n\}| < \varepsilon_1 \quad and \quad |\mathbb{E}\{W_{n+1}^2\} - \mathbb{E}\{W_n^2\}| < \varepsilon_2,$$

for some prespecified $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$. Then

$$F_W(y) \approx \tilde{F}_{W_n+D}(U_1 + y) \quad , y > 0. \tag{12}$$

□

This method has been grafted upon fitting distributions to the first two moments of an arbitrary distribution. E.g. Tijms[1986] provides us a method in which mixtures of Erlang distributions yield excellent results.

Thus, applying of the moment-iteration method of De Kok provides an accurate approximation for the waiting-time distribution of the equilibrium system, or translated to the inventory system, an approximation for the equilibrium distribution of the shortfall. Then, discretizing the generated waiting-time, or shortfall-amount, distribution we obtain the estimated equilibrium probabilities $\tilde{\pi}_i^*$. Truncating (8) finally results in an adequate approximation of the average cost associated with a modified base-stock policy d_1.
So far the recapitulation of an application of De Kok's moment-iteration method to the capacitated 1-echelon serial system.

We have been able to extend the idea of De Kok to the N-echelon inventory serial system with $N \geq 2$, but in which $l_i = 1$ for all $1 \leq i \leq N$. Suppose we want to determine the average cost corresponding to the modified base-stock policy characterized by echelon inventory position levels (d_2, d_1). We explicitely assume the system to start from the ideal echelon inventory stock levels d_i, $i = 1, 2$. This leads to the definition of the state space $\mathcal{Z}[\,(d_2, d_1)\,]$ belonging to the modified base-stock policy (d_2, d_1) as the collection of possible echelon inventory positions (z_2, z_1) at the beginning of a period before meeting demand:

$$\mathcal{Z}[\,(d_2, d_1)\,] := \{\,(z_2, z_1) \in \mathbb{Z}^2 \mid z_1 \leq d_1 \,, \, z_1 \leq z_2 \leq d_2 \,\}. \tag{13}$$

6

Thus, at the beginning of the first period the system finds itself in the situation as depicted underneath.
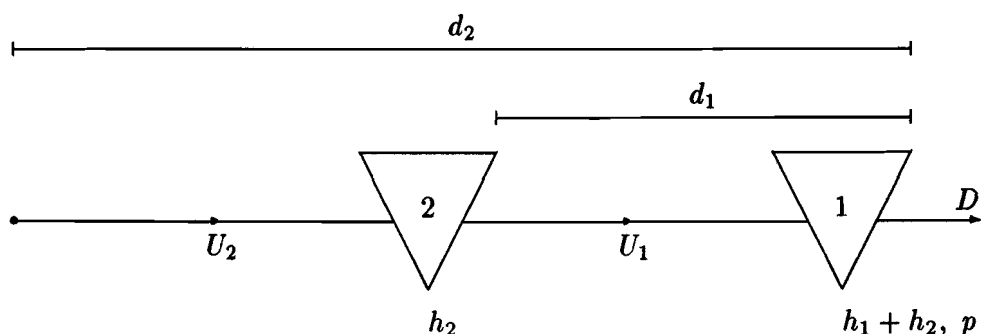
Consequently, when demand equals D in the first period the stock at echelon 1 and 2 amounts to $d_1 - D$ respectively $d_2 - D$ (remember $l_2 = l_1 = 1$), and thus there is a fysical stock of $d_2 - d_1$ on hand at installation 2!
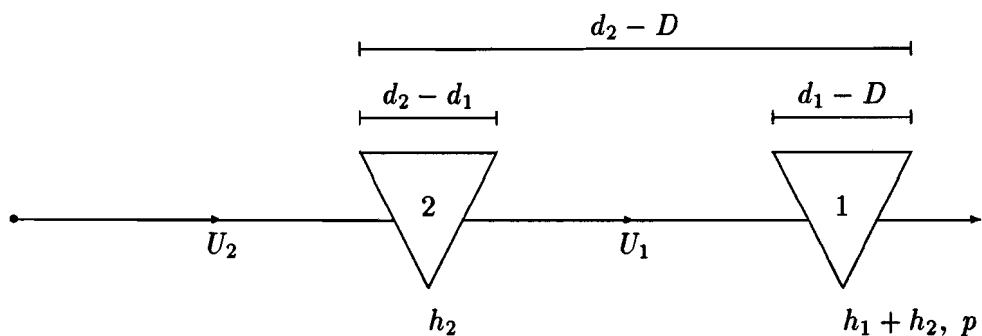
Then, by looking at the amount $d_2 - d_1$ available at installation 2 there are three separate cases distinguishable:

1. $0 \leq d_2 - d_1 \leq U_2$

2. $U_2 < d_2 - d_1 \leq U_1$

3. $U_1 < d_2 - d_1$.

First of all we can skip the case $U_1 < d_2 - d_1$. Namely, every modified base-stock policy (d_2, d_1) aims at a stock of $d_2 - d_1$ on hand at installation 2

7

at the end of an arbitrary period. In the case $U_1 < d_2 - d_1$ this means going for a strictly positive stock at installation 2 during the next period, for at most U_1 can be shipped. For this stock amount holding costs are charged, while no advantage is yielded by this stock, see the analoguous argument that established the preliminary result in Speck and Van der Wal[1991].

In the other two cases the state space $\mathcal{Z}[\ (d_2, d_1)\ ]$ reduces to an irreducible state space in each case. Suppose we practise a modified base-stock policy (d_2, d_1) in which $0 \leq d_2 - d_1 \leq U_2$. Then we are able to construct a partial flow scheme containing the possible transitions from the ideal state (d_2, d_1):
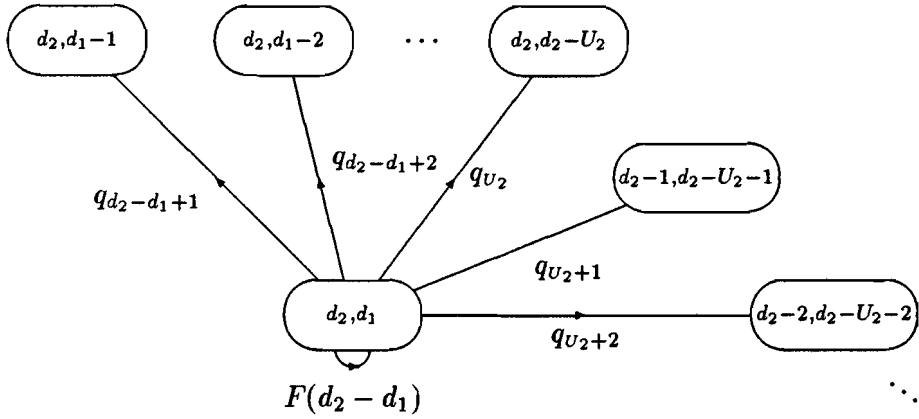
By considering the possible transitions from the other states depicted above we finally attain an irreducible state set $\mathcal{Z}_1[\ (d_2, d_1)\ ]$,

$$\mathcal{Z}_1[\ (d_2, d_1)\ ] \quad := \quad \{\ (d_2, d_1), (d_2, d_1 - 1), \ldots, (d_2, d_2 - U_2),$$
$$(d_2 - 1, d_2 - U_2 - 1), (d_2 - 2, d_2 - U_2 - 2), \ldots\ \}.$$

The corresponding transition matrix has the following structure:

$$
\begin{pmatrix}
F(d_2 - d_1) & q_{d_2-d_1+1} & \cdots & q_{U_2} & q_{U_2+1} & q_{U_2+2} & \cdots \\
F(d_2 - d_1) & q_{d_2-d_1+1} & \cdots & q_{U_2} & q_{U_2+1} & q_{U_2+2} & \cdots \\
F(d_2 - d_1) & q_{d_2-d_1+1} & \cdots & q_{U_2} & q_{U_2+1} & q_{U_2+2} & \cdots \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \\
F(d_2 - d_1) & q_{d_2-d_1+1} & \cdots & q_{U_2} & q_{U_2+1} & q_{U_2+2} & \cdots \\
F(d_2 - d_1 - 1) & q_{d_2-d_1} & \cdots & q_{U_2-1} & q_{U_2} & q_{U_2+1} & \cdots \\
F(d_2 - d_1 - 2) & q_{d_2-d_1-1} & \cdots & q_{U_2-2} & q_{U_2-1} & q_{U_2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{pmatrix}
$$

For the remaining case ($U_2 < d_2 - d_1 \le U_1$) a similar flow scheme can be drawn, which shows the reduction of $\mathcal{Z}[\,(d_2, d_1)\,]$ to an irreducible state space $\mathcal{Z}_2[\,(d_2, d_1)\,]$,

$$
\begin{aligned}
\mathcal{Z}_2[\,(d_2, d_1)\,] \;:=\; &\{\,(d_2, d_1), (d_2 - 1, d_1), \ldots, (d_1 + U_2, d_1), \\
&(d_1 + U_2 - 1, d_1 - 1), (d_1 + U_2 - 2, d_1 - 2), \ldots\,\}
\end{aligned}
$$

In this case we obtain a nicely structured transition matrix, reminiscent of the one in the 1-echelon system.

$$
\begin{pmatrix}
F(U_2) & q_{U_2+1} & \cdots & q_{d_2-d_1} & q_{d_2-d_1+1} & q_{d_2-d_1+2} & \cdots \\
F(U_2 - 1) & q_{U_2} & \cdots & q_{d_2-d_1-1} & q_{d_2-d_1} & q_{d_2-d_1+1} & \cdots \\
F(U_2 - 2) & q_{U_2-1} & \cdots & q_{d_2-d_1-2} & q_{d_2-d_1-1} & q_{d_2-d_1} & \cdots \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \\
F(a) & q_{a+1} & q_{a+2} & \cdots & q_{U_2+1} & q_{U_2+2} & \cdots \\
F(a - 1) & q_a & q_{a+1} & \cdots & q_{U_2} & q_{U_2+1} & \cdots \\
F(a - 2) & q_{a-1} & q_a & \cdots & q_{U_2-1} & q_{U_2} & \cdots \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots
\end{pmatrix}
$$

in which $a := 2U_2 - d_2 + d_1$.

By joining the states $(z_2, z_1) \in \mathcal{Z}_1[\,(d_2, d_1)\,]$ in which $z_2 = d_2$ to one state

$(d_2, *)$, and carrying out a corresponding clustering within the transition matrix associated with $\mathcal{Z}_1[\ (d_2, d_1)\ ]$, we achieve both transition matrices being the same:

$$
\mathcal{Q}_2 := \begin{pmatrix}
F(U_2) & q_{U_2+1} & q_{U_2+2} & \cdots & \cdots \\
F(U_2-1) & q_{U_2} & q_{U_2+1} & \cdots & \cdots \\
F(U_2-2) & q_{U_2-1} & q_{U_2} & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \\
\vdots & \vdots & \vdots & & \ddots
\end{pmatrix}
\tag{14}
$$

Inherently, $\mathcal{Z}_1[\ (d_2, d_1)\ ]$ changes to

$$
\mathcal{Z}_1[\ (d_2, d_1)\ ] =
$$
$$
\{\ (d_2, *), (d_2 - 1, d_2 - U_2 - 1), (d_2 - 2, d_2 - U_2 - 2), \ldots\ \}
$$

The exact average costs associated with the modified base-stock policy (d_2, d_1) is given by

$$
g = \mathcal{Q}_2^\infty r_1[\ (d_2, d_1)\ ]
\tag{15}
$$

or

$$
g = \mathcal{Q}_2^\infty r_2[\ (d_2, d_1)\ ],
\tag{16}
$$

depending on the value of $d_2 - d_1$. In both cases $\mathcal{Q}_2^\infty$ is the equilibrium matrix of the system, and $r_i[\ (d_2, d_1)\ ]$ for $i = 1, 2$ the cost vector consisting of the expected 1-period costs per state out of the state space $\mathcal{Z}_i[\ (d_2, d_1)\ ]$ associated with a modified base-stock policy (d_2, d_1). The costs are defined as

$$
r(z_2, z_1) := \int_0^\infty \{\ L_2(z_2 - w) + L_1(z_1 - w)\ \}\ dF(w).
\tag{17}
$$

Evidently, when at the beginning of a period t the inventory position of echelon i, $i = 1, 2$, is raised to level z_i, then the stock of echelon i equals $z_1 - D_t$ at the beginning of period $t+1$. Then the costs at the end of period $t+1$ associated with echelon i amount to $L_i(z_1 - D_t)$.

The equilibrium probabilities $\pi_0^*, \pi_1^*, \pi_2^*, \ldots$ again can be derived by solving the system

$$
\begin{cases}
\pi \mathcal{Q}_2 & = \pi \\
\sum_{i=0}^\infty \pi_i & = 1.
\end{cases}
\tag{18}
$$

10

Notice that in case of $0 \leq d_2 - d_1 \leq U_2$ the equilibrium probabilities of the states (d_2, d_1), $(d_2, d_1 - 1)$, ... , $(d_2, d_2 - U_2)$ are obtained by multiplying the equilibrium probability π_0^* by the relative probabilities within the junction state $(d_2, *)$.

Though in general it is not possible to determine the equilibrium probabilities π_i^* exactly we can provide estimations $\tilde{\pi}_i^*$. Namely, as $\mathcal{Q}_2$ has the very same structure as the matrix $\mathcal{Q}_1$ in the capacitated 1-echelon serial system, we again invoke the moment-iteration method of De Kok in order to achieve an approximation for the waiting-time (or 'shortfall') distribution F_W of the equilibrium system. Discretisation of this generated distribution F_W then yields us an approximation for the probabilities π_i^*:

$$\tilde{\pi}_0^* := F_W(\tfrac{1}{2}),$$
$$\tilde{\pi}_i^* := F_W(i + \tfrac{1}{2}) - F_W(i - \tfrac{1}{2}), \quad i \geq 1.$$

Truncating (15) or (16) yields an estimation of the average costs associated with the modified base-stock policy (d_2, d_1), provided that the demand distribution F is explicitly given in order to calculate exactly the expected 1-period costs $r(z_2, z_1)$ per state (z_2, z_1) in (17). If only the first two moments of the demand D are given, we can only approximate the expected 1-period costs: As in the first iteration of algorithm 1 a distribution is fitted to the moments of D, we have $\tilde{F}_{W_0 + D_0}$, or $\tilde{F}_D$, at our disposal. By means of this fitted distribution $\tilde{F}_D$ we can easily determine the (approximated) expected 1-period costs.

We have executed the average cost approximation on a class of 2-echelon serial systems, in which, besides $l_2 = l_1 = 1$,

$$\mathbb{E}\{D\} = 100$$
$$\sigma\{D\} = 70$$
$$h_1 = 2$$
$$h_2 = 2$$
$$p = 200$$
$$\varepsilon_1 = 10^{-3}$$
$$\varepsilon_2 = 10^{-3}.$$

Notice that due to the values of $\mathbb{E}\{D\}$ and $\sigma\{D\}$ the demand distribution is approximated by an Erlang mixture with the same scale parameter,

according to Tijms[1986].

First we generated the average cost of an optimal (i.e. base-stock) policy for the *uncapacitated* system by means of the program *LIJNSYSTEEM3* given in Van Houtum and Zijm[1990]. *LIJNSYSTEEM3* provides an *estimation* of the optimal (base-stock) policy by a recursive calculation of incomplete convolutions in which the fit-method described in Tijms[1986] is used, plus an *exact* calculation of the average costs associated with the generated base-stock policy. At the chosen parameter values this happens to be the base-stock policy $(d_2^\infty, d_1^\infty) = (614.1, 498.9)$. Assuming the demand distribution to be exactly an Erlang mixture with the same scale parameter we see that the average cost g^∞ associated with (614.1,498.9) amounts to 1669.03.

Next, for increasing U_2 we have determined the average costs associated with the modified base-stock policy $(\lfloor d_2^\infty \rfloor, \lfloor d_1^\infty \rfloor) = (614, 498)$, see table 1. Thereoff successively can be read the capacity U_2, the average shortfall

$(d_2^\infty, d_1^\infty) = (614.1, 498.9)$				
$g^\infty = 1669.03$				
U_2	$I\!\!E\{W\}$	n	$\tilde{g}$	*ctime*
110	215.42	656	14302.1	6.6 s
125	72.09	121	3110.9	3.7 s
150	26.99	38	1922.1	3.0 s
200	7.45	14	1713.5	2.7 s
250	2.60	9	1681.0	3.0 s
300	0.98	6	1672.6	3.0 s
400	0.15	4	1669.5	3.2 s
500	0.02	3	1669.3	3.4 s

Table 1: Increasing U_2

$I\!\!E\{W\}$ on the desired echelon inventory levels in the equilibrium system, the

12

number of moment-iterations n carried out, the generated average cost approximation $\tilde{g}$ attached to $(\lfloor d_2^\infty \rfloor, \lfloor d_1^\infty \rfloor)$, and an indication of the consumed calculation time in seconds *ctime* on an Olivetti M240 PC with numerical coprocessor.

Probabilities smaller than 10^{-8} were set equal to 0.

We find that both average shortfall and average costs decrease if U_2 increases, which is according to our expectations. Further we see that the average cost $\tilde{g}$ converges towards 1669 ($U_2 = 500$ almost corresponds with the infinite capacity case). But we have to keep in mind, that 1669.03 is the exact average cost belonging to the modified base-stock policy (614.1,498.9), not (614,498)! Further research however pointed out that the exact average cost associated with the base-stock policy $(614,498)$ in the uncapacitated model amounts to 1669.04, thus indicating the average cost function to be very flat near the minimum.

4. Finding a nearly optimal modified base-stock policy. The cost calculation method presented above enables us, referring to our conclusions in Speck and Van der Wal[1991], to derive a nearly optimal modified base-stock policy serving in its turn as an approximation for the optimal periodic review policy.

If for the capacitated 2-echelon serial system an initial average cost value is given associated with an optimal base-stock policy (d_2^∞, d_1^∞) from the uncapacitated model, then the search for a nearly optimal modified base-stock policy can be restricted to the grid points of the upper right quadrant in the 2-dimensional space with $(\lfloor d_1^\infty \rfloor, \lfloor d_1^\infty \rfloor)$ as origin. The extensive scala of numerical procedures then enables us to achieve a further efficient reduction of the searching process.

References

[1] De Kok, A.G. (1989), *A moment-iteration method for approximating the waiting-time characteristics of the $GI|G|1$ queue*, Probability in the Engineering and Informational Sciences **3** : 273-287.

[2] Federgruen, A., and Zipkin, P. (1986), *An inventory model with limited production capacity and uncertain demands : the average-cost criterion*, Mathematics of Operations Research **11** : 193-207.

[3] Speck, C.J., and Van der Wal, J. (1991), *The capacitated multi-echelon inventory system with serial structure: 1. The 'push ahead'-effect*, Dept. of Maths. and Comp. Science, Memorandum COSOR 91-39, Eindhoven University of Technology, submitted for publication.

13

[4] Tijms, H.C. (1986), *Stochastic modelling and analysis : a computational approach*, Wiley, New York : 397-400.

[5] Van Houtum, G.J., and Zijm, W.H.M. (1990), *Computational procedures for stochastic multi-echelon production systems*, International Journal of Production Economics **23** : 223-237.

EINDHOVEN UNIVERSITY OF TECHNOLOGY
Department of Mathematics and Computing Science
PROBABILITY THEORY, STATISTICS, OPERATIONS RESEARCH
AND SYSTEMS THEORY
P.O. Box 513
5600 MB Eindhoven, The Netherlands

Secretariate: Dommelbuilding 0.03
Telephone   : 040-473130

-List of COSOR-memoranda - 1991

Number	Month	Author	Title
91-01	January	M.W.I. van Kraaij W.Z. Venema J. Wessels	The construction of a strategy for manpower planning problems.
91-02	January	M.W.I. van Kraaij W.Z. Venema J. Wessels	Support for problem formulation and evaluation in manpower planning problems.
91-03	January	M.W.P. Savelsbergh	The vehicle routing problem with time windows: minimizing route duration.
91-04	January	M.W.I. van Kraaij	Some considerations concerning the problem interpreter of the new manpower planning system formasy.
91-05	February	G.L. Nemhauser M.W.P. Savelsbergh	A cutting plane algorithm for the single machine scheduling problem with release times.
91-06	March	R.J.G. Wilms	Properties of Fourier-Stieltjes sequences of distribution with support in [0,1).
91-07	March	F. Coolen R. Dekker A. Smit	Analysis of a two-phase inspection model with competing risks.
91-08	April	P.J. Zwietering E.H.L. Aarts J. Wessels	The Design and Complexity of Exact Multi-Layered Perceptrons.
91-09	May	P.J. Zwietering E.H.L. Aarts J. Wessels	The Classification Capabilities of Exact Two-Layered Peceptrons.
91-10	May	P.J. Zwietering E.H.L. Aarts J. Wessels	Sorting With A Neural Net.
91-11	May	F. Coolen	On some misconceptions about subjective probability and Bayesian inference.

91-12	May	P. van der Laan	Two-stage selection procedures with attention to screening.
91-13	May	I.J.B.F. Adan G.J. van Houtum J. Wessels W.H.M. Zijm	A compensation procedure for multiprogramming queues.
91-14	June	J. Korst E. Aarts J.K. Lenstra J. Wessels	Periodic assignment and graph colouring.
91-15	July	P.J. Zwietering M.J.A.L. van Kraaij E.H.L. Aarts J. Wessels	Neural Networks and Production Planning.
91-16	July	P. Deheuvels J.H.J. Einmahl	Approximations and Two-Sample Tests Based on $P - P$ and $Q - Q$ Plots of the Kaplan-Meier Estimators of Lifetime Distributions.
91-17	August	M.W.P. Savelsbergh G.C. Sigismondi G.L. Nemhauser	Functional description of MINTO, a Mixed INTeger Optimizer.
91-18	August	M.W.P. Savelsbergh G.C. Sigismondi G.L. Nemhauser	MINTO, a Mixed INTeger Optimizer.
91-19	August	P. van der Laan	The efficiency of subset selection of an almost best treatment.
91-20	September	P. van der Laan	Subset selection for an -best population: efficiency results.
91-21	September	E. Levner A.S. Nemirovsky	A network flow algorithm for just-in-time project scheduling.
91-22	September	R.J.M. Vaessens E.H.L. Aarts J.H. van Lint	Genetic Algorithms in Coding Theory – A Table for $A_3(n,d)$.
91-23	September	P. van der Laan	Distribution theory for selection from logistic populations.

91-24	October	I.J.B.F. Adan J. Wessels W.H.M. Zijm	Matrix-geometric analysis of the shortest queue problem with threshold jockeying.
91-25	October	I.J.B.F. Adan J. Wessels W.H.M. Zijm	Analysing Multiprogramming Queues by Generating Functions.
91-26	October	E.E.M. van Berkum P.M. Upperman	D-optimal designs for an incomplete quadratic model.
91-27	October	R.P. Gilles P.H.M. Ruys S. Jilin	Quasi-Networks in Social Relational Systems.
91-28	October	I.J.B.F. Adan J. Wessels W.H.M. Zijm	A Compensation Approach for Two-dimensional Markov Processes.
91-29	November	J. Wessels	Tools for the Interfacing Between Dynamical Problems and Models withing Decision Support Systems.
91-30	November	G.L. Nemhauser M.W.P. Savelsbergh G.C. Sigismondi	Constraint Classification for Mixed Integer Program- ming Formulations.
91-31	November	J.Th.M. Wijnen	Taguchi Methods.
91-32	November	F.P.A. Coolen	The Theory of Imprecise Probabilities: Some Results for Distribution Functions, Densities, Hazard Rates and Hazard Functions.
91-33	December	S. van Hoesel A. Wagelmans	On the P-coverage problem on the real line.
91-34	December	S. van Hoesel A. Wagelmans	On setup cost reduction in the economic lot-sizing model without speculative motives.
91-35	December	S. van Hoesel A. Wagelmans	On the complexity of post- optimality analysis of 0/1 programs.
91-36	December	F.P.A. Coolen	Imprecise Conjugate Prior Densities for the One- Parameter Exponential Family of Distributions.

91-37	December	J. Wessels	Decision systems; the relation between problem specification and mathematical analysis.
91-38	December	J.J.A.M. Brands R.J.G. Wilms	On the asymptotically uniform distribution modulo 1 of extreme order statistics.
91-39	December	C.J. Speck J. van der Wal	The capacitated multi-echelon inventory system with serial structure: 1. The 'push ahead'-effect
91-40	December	C.J. Speck J. van der Wal	The capacitated multi-echelon inventory system with serial structure: 2. An average cost approximation method