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Stochastic Facility Location with General Long-Run Costs and Convex Short-Run Costs

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Abstract

This paper addresses the problem of minimizing the expected cost of locating a number of single product facilities and allocating uncertain customer demand to these facilities. The total costs consist of two components: firstly linear transportation cost of satisfying customer demand and secondly the costs of investing in a facility as well as maintaining and operating it. These facility costs are general and non-linear in shape and could express both changing economies of scale and diseconomies of scale. We formulate the problem as a two-stage stochastic programming model where both demand and short-run costs may be uncertain at the investment time. We use a solution method based on Lagrangean relaxation, and show computational results for a slaughterhouse location case from the Norwegian meat industry.

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1 Introduction

Mathematical programming approaches to model and solve facility location models have been extensively studied since the 1950's [4, 7]. In this paper we deal with two issues that have been analyzed separately to some degree, but rarely in combination: non-linear facility costs and stochasticity in costs and demand. From a model perspective our work is a generalization of an early paper by Balachandran and Jain [2]. Our approach to solve the problem is completely different from the former. We demonstrate both the model and the solution method in a real life application. We also emphasize the importance of understanding both the long-run cost function for facilities which often are subject to economies of scale that change over the production interval and the short-run cost functions which often are convex as a consequence of diminishing marginal return on production input factors.

Before we move on to a description of the structure of the paper, we will give a short overview of relevant literature. Traditionally, the facility costs are treated as fixed set-up costs and linear variable costs [see e. g. 21, 15, 23]. This is a situation where marginal costs are constant and the economies of scale come from sharing the fixed part on more units. In real-world applications however, both the fixed part of facility costs and the marginal costs often depend on the size of the facility [13, 28, 20]. Usually the degree to which economies of

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scale are experienced changes with volume and even diseconomies of scale are common [see e. g. 3, 26, for a definition].

Several models and algorithms have been developed that can be used to formulate and solve facility location problems subject to changing economies of scale in a deterministic setting: Soland [33] develops an algorithm for a facility location problem with facility costs that are concave in the amount produced and transportation costs that are concave in the amount shipped. Domschke and Voß [11] formulate a multi-product facility location model with concave production costs.

Changing economies of scale and diseconomies of scale have also been represented by means of a deterministic facility location problem with staircase costs (FLSC), see for example Holmberg [16], Holmberg [17], Holmberg and Ling [18] or Harkness and ReVelle [14]. The modular capacitated plant location model (MCPL) can be interpreted as a generalization of the FLSC, see e. g. Correia and Captivo [9] or Correia and Captivo [10]. Van den Broek, Schütz, Stougie, and Tomasgard [34] present an application from the Norwegian meat industry. They use a piecewise linearization of a general non-linear facility cost curve, but employ a solution strategy similar to the framework given by the aforementioned papers.

The aforementioned paper by Balachandran and Jain [2] was an early approach with a general piecewise linear objective preceding the ones in the previous paragraph. This paper also is one of the first that allows for demand uncertainty in the model. Uncertainty has later been incorporated in a number of facility location models. Good overviews over the literature on facility location under uncertainty can be found in the reviews by Louveaux [23] and

Snyder [32]. Louveaux [23] provides a modelling oriented review over location models belonging to the field of stochastic programming, whereas Snyder [32] is more interested in the application of various models for optimization under uncertainty to facility location. Some examples are listed here: Louveaux and Peeters [24] present a two-stage stochastic programming problem with uncertainty in demand, selling prices, as well as in production and transportation cost. Location decisions belong to the first stage while demand is allocated in the second stage. Laporte, Louveaux, and van Hamme [22] include also establishment of transportation channels between a facility and customers in the first-stage decisions. Another class of interesting models focusses on capacity expansion. Eppen, Martin, and Schrage [12] present a two-stage formulation. A formulation for a multi-stage capacity expansion problem under uncertain demand is presented by Ahmed, King, and Parija [1]. They propose a formulation that allows for exploiting the lot-sizing substructure of their problem.

Balachandran and Jain [2] is the only paper that combines modelling of uncertainty with a general objective allowing for changing economies of scale and diseconomies of scale. The model is for single product facilities. They use a piecewise linear, potentially discontinuous, objective for the facility costs. Transportation costs are linear. In the first stage a capacity is chosen for each facility. In the second stage demand is allocated to facilities and deviations from the capacities of facilities are penalized with costs for over-capacity and under-capacity. The solution methodology based on branch-and-bound is general as the objective function is bounded from below by the best linear function and the resulting node problem is a stochastic transportation problem. The branching is done on capacity intervals where the piecewise linear objective has breakpoints. This is similar to how an approach using special ordered sets

[35] in nowadays commercial solvers would work.

We extend this approach in several ways. Firstly, we model the first stage decision as a design capacity interval rather than a fixed point. Inside this interval the variable short-term cost is linear. Secondly, we give a more thorough description and motivation of the short-run cost functions of facilities leading to a general piecewise linear convex second-stage cost function rather than two penalty costs around the design capacity. Hence our approach can be seen as a hybrid between the capacitated and uncapacitated problem, as a capacity interval is decided in the first stage, but volumes outside this interval may be produced subject to a short-term cost function in the second stage. Thirdly, we allow for stochastic costs as well as stochastic demand. Finally, the solution method we choose allows for solving problems of sizes met in real life cases. In our experience, a formulation based on special ordered sets and branch-and-bound is not computationally tractable for large problems with many linepieces in the approximations. For example, the deterministic version of our real life problem still has an optimality gap of 66% after 7 hours running time using a commercial solver and special ordered sets.

For solving this problem, we approximate the facility cost functions by piecewise linear functions, upon which we decompose the problem using Lagrangean relaxation. Relaxing the demand constraints makes the problem separable in the facilities and we apply an efficient algorithm based on a solution method for the continuous knapsack problem to solve the subproblems. The technique of Lagrangean relaxation in combination with a heuristic to generate feasible solutions for the original problem from the Lagrangean relaxation solutions has been successfully applied already in the past for deterministic facility location problems. Examples for this can be found in Cornuejols, Fisher, and

Nemhauser [8], Nemhauser and Wolsey [27], Shetty [31], Beasley [5], or Holmberg, Rönnqvist, and Yuan [19]. Lagrangean relaxation has also been used for solving the FLSP and the MCPL, see e. g. Holmberg and Ling [18], Harkness and ReVelle [14], Correia and Captivo [9], Correia and Captivo [10], and Van den Broek et al. [34]. The approach we have chosen to solve the problem brings us within the framework of the FLSC and MCPL, even if all these papers deal with the deterministic case. Our algorithm is a modification of the one used by Van den Broek et al. [34] on the deterministic version of the problem and makes the problem separable in scenarios by imposing non-anticipativity constraints on the first-stage decision variables.

In Section 2 we provide the stochastic programming formulation for a facility location problem with a non-linear, non convex, non-concave objective function, uncertain short-run costs and uncertain demand. Our solution method is presented in Section 3. A full-size real life case from the Norwegian meat industry is described in Section 4 [see also 34]. Computational results for this problem from practice are shown in Section 5. Conclusions in Section 6 finish the paper.

2 The Mathematical Programming Model

In this section we provide a two-stage stochastic programming formulation for a facility location problem with non-convex non-concave facility costs, linear transportation costs and uncertain demand. The motivation for the shape of the objective is the need to model more realistic situations for economies of scale than what is found in the literature, with very few exceptions that we are aware of. The first-stage decision is to determine the location of facilities and

the capacity to be installed at this location. After observing the demand, we decide in the second stage the allocation of customer demand to the facilities opened in the first stage.

2.1 Modelling short-run costs and long-run costs

The cost functions that are underlying the first-stage and second-stage decisions are the long-run and the short-run cost function, respectively (see e. g. Mathis and Koscianski [26] or Perloff [29]). In the long-run, all input factors are variable, i. e. one will always choose the combination of input factors that produces the desired output at minimal cost. To illustrate this, consider a product with two input factors, for example capital and labour. The three isoquants in Figure 1 are the technologically efficient combinations of the two input factors to produce the quantities Q_1, Q_2, and Q_3 respectively. The economically efficient combinations of capital and labour to produce these quantities are given by the points P_1, P_2, and P_3, where the isocost curves C_1, C_2, and C_3 are tangents to the corresponding isoquants (assuming linear costs for the components). These minimal cost combinations constitute the long-run expansion path. Thus, in order to produce Q_2 in the long-run, one would choose the combination of capital and labour as given by P_2.

In the short-run, it is no longer possible to vary all input factors. Consider capital (e. g. the number of machines) as the fixed input in the example above. The decision-maker has implemented the combination of capital and labour as given by P_2. Demand however, is varying and the quantity produced deviates from Q_2 in order to meet demand. The output can only be increased or decreased by adjusting the factor labour, creating the short-run expansion

path in Figure 1. One can see from this figure that the costs for producing quantities Q_1 and Q_3 on the short-run expansion path are higher than the costs for the same quantities on the long-run expansion path.

Both the long-run expansion path and the short-run expansion path translate into total cost functions. The typical S-shape of long-run total cost curves results from a long-run marginal cost function that is decreasing and then increasing in the production interval. The resulting long-run total cost curve then exhibits economies of scale as the average cost function is decreasing. The long-run marginal costs and the long-run average costs are shown in Figure 2. This type of marginal cost function can for example be found in the meat producing industry [20].

In the natural monopoly case the marginal cost approaches the average cost from below without crossing it, as in Figure 2. Usually diseconomies of scale will eventually lead to a situation where average costs start rising. Both the solution method and the model presented here allow for a general, non-convex, non-concave, shape of the objective.

To each installed capacity a short-run cost function is assigned, which is tangent to the long-run cost function at that capacity. The short-run total costs represent the costs of operating a facility given the installed capacity. These cost functions are convex under the assumptions that the marginal returns of the variable input factors are diminishing. The relationship between long-run total costs and short-run costs is depicted in Figure 3.

The first-stage decision is to decide upon the capacities of the facilities. This decision is based on the long-run total cost function and thereby implicitly decides the second-stage short-run cost function.

After the facilities are opened, production is assigned to the open facilities in order to satisfy demand in the second-stage. The second-stage cost function is the short-run total cost function, i. e. a deviation of the production level from the installed capacity is more costly than the long-run total costs.

2.2 Linearized Model Formulation

The objective function consists of linear transportation costs and non-convex, non-concave facility costs. We approximate both the first-stage facility cost function and the second-stage facility cost function by a piecewise linear function, creating a piecewise linear, non-convex, non-concave objective function. The first-stage decision is to determine the designed capacity interval for the facilities. The designed capacity is described by the lower and upper capacity limit of the chosen linepiece k on the first-stage facility cost function.

We model the non-convex, non-concave first-stage cost function using the approach of a special ordered set of type 1 [see e. g. 35], i. e. using an ordered set of binary variables, one for each linepiece of the cost function, that have to add up to one. In a feasible solution, exactly one of the variables will be equal to one, corresponding to the chosen linepiece and defining the design capacity of the facility.

Once customer demand is known, the second-stage decision is to allocate demand to the opened facilities. Depending on the realization of demand, the production level is adjusted according to the short-run expansion path, varying the variable input factors. It is possible to either exceed the upper capacity limit installed in the first stage up to a certain limit, e. g. by using overtime

hours, or to allocate less demand to a facility than its installed capacity. The total facility costs however, always exceed the costs that would have occurred if the right linepiece for the production level had been chosen in the fist stage. We approximate the convex second-stage short-run cost function by a piecewise linear function. Due to the convexity of the short-run cost function we do not need to use special ordered sets of type 2 to do this [35].

Let n be the number of demand points in the problem. We assume that each of these points is also a potential facility location.

By K we denote the number of linepieces used to approximate the non-linear first-stage facility cost function, resulting in $K+1$ breakpoints. $P_1, \ldots, P_{K+1}$ are the per unit cost, and $F_1, \ldots, F_{K+1}$ the volumes at these breakpoints. We define $P_0 = 0$ and $F_0 = 0$, such that the choice of this linepiece for a location means that no facility is opened in that location. In order to properly represent the fixed costs of opening a facility, F_2 is chosen small. Thus, P_2 becomes high as the fixed costs of the facility are distributed only over these few units.

The first-stage decision variables are represented by the $n(K+1)$-dimensional vector y that is made up by all $y_{ik}, i = 1, \ldots, n, k = 0, \ldots, K$. If $y_{ik} = 1, k \neq 0$, the linepiece between F_k and F_{k+1} is chosen for location i.

The second-stage cost function is approximated by a piecewise linear function, depending on the choice of linepiece k in the first stage. The linearized first-stage and second-stage facility cost functions are illustrated in Figure 4. In the following we assume that all facilities have the same cost function. This is not a necessary assumption for the modelling and decomposition approach we have chosen but we have not tested the quality of the heuristic used to find upper bounds in cases where they are different.

The second-stage cost function consists of B linesegments, thus having $B+1$ breakpoints. We denote the breakpoints of this function by Q_{kb}, $\forall k, b = 1, \ldots, B+1$. We define $Q_{k\beta} = F_k$ and $Q_{k(\beta+1)} = F_{k+1}$ such that the linepiece between breakpoints β and $\beta+1$ of the second-stage cost function is equal to the linepiece chosen in the first-stage. The slope of every linepiece is given by u_{kb}, representing the per unit production costs. The total costs at each breakpoint are given by C_{kb}, $\forall k, b = 1, \ldots, B+1$. With $Q_{k\beta}$ and $Q_{k(\beta+1)}$, we get $C_{k\beta} = P_k F_k$, and $C_{k(\beta+1)} = P_{k+1} F_{k+1}$. We introduce the second stage decision variables μ_{jkb}, $\forall k, b = 1, \ldots, B+1$, denoting the weight of breakpoint b given linepiece k at location j. For each j and each k, they should add up to 1, only two of them can be non-zero and the non-zero weights must be adjacent. This will be automatically satisified as the short-run cost curve is convex.

2.3 A Two-stage Recourse Formulation

The uncertainty in short-run facility costs and customer demand is modeled by probability distributions discretized in a set of scenarios S (we abusively use S also for the number of scenarios). Superscript s indicates the scenario and p^s denotes the probability of scenario s. D_i^s is the demand realization at a given location i and C^s the realization of short-run costs for the given scenario s. T_{ij} is the per unit transportation cost from location i to location j. We define the parameters $L_{ij} = 1$ if demand at location i can be served at location j and $L_{ij} = 0$ otherwise.

The continuous decision variables x_{ij}^s denote the amount of demand at i served

at j in scenario s. This leads to the stochastic programming formulation

$$\min \sum_{s=1}^{S} p^s \mathcal{Q}^s(y) \tag{1}$$

subject to

$$\sum_{k=0}^{K} y_{jk} = 1, \qquad \forall j, \tag{2}$$

$$y_{jk} \in \{0,1\}, \qquad \forall j,k. \tag{3}$$

where the second stage problem is given as

$$\mathcal{Q}^s(y) = \sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij} x_{ij}^s + \sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{b=1}^{B+1} C_{kb}^s \mu_{jkb}^s \tag{4}$$

subject to

$$\sum_{j=1}^{n} x_{ij}^s = D_i^s, \qquad \forall i,s, \tag{5}$$

$$\sum_{i=1}^{n} x_{ij}^s = \sum_{k=1}^{K}\sum_{b=1}^{B+1} Q_{kb}\mu_{jkb}^s, \qquad \forall j,s, \tag{6}$$

$$x_{ij}^s \leq L_{ij} D_i^s, \qquad \forall i,j,s, \tag{7}$$

$$\sum_{b=1}^{B+1} \mu_{jkb}^s = y_{jk}, \qquad \forall j,k,s, \tag{8}$$

$$x_{ij}^s \geq 0, \qquad \forall i,j,s, \tag{9}$$

$$\mu_{jkb}^s \geq 0, \qquad \forall j,k,b,s. \tag{10}$$

Restrictions (2) ensure that only one linepiece is chosen for each location. The objective function of the second stage problem (4) is given as the sum of transportation and production costs. Constraints (5) force all demand at location i to be assigned. Constraints (7) only allow assignment of demand to locations where the demand can be satisfied. Constraints (6) ensure that demand is allocated to open facilities only. Restrictions (8) link the correct

second-stage cost function to the first-stage decision. (9)-(10) are the non-negativity constraints.

For each scenario we create a copy of the first-stage decision variables and add non-anticipativity constraints (13) [30]. This way, the first-stage problem (1)-(3) translates into

$$\min \sum_{s=1}^{S} p^s \mathcal{Q}^s(y) \tag{11}$$

subject to

$$\sum_{k=0}^{K} y_{jk}^s = 1, \qquad \forall j, s, \tag{12}$$

$$y_{jk}^1 = y_{jk}^2 = \cdots = y_{jk}^{S-1} = y_{jk}^S, \qquad \forall j, k, \tag{13}$$

$$y_{jk}^s \in \{0, 1\}, \qquad \forall j, k, s. \tag{14}$$

In addition we change restriction (8) to

$$\sum_{b=1}^{B+1} \mu_{jkb}^s = y_{jk}^s, \qquad \forall j, k, s. \tag{15}$$

3 Lagrangean Relaxation

We define a Lagrangean relaxation by relaxing the demand constraints (5) in the problem formulaton above using $\lambda = \left(\lambda_1^1, \ldots, \lambda_1^S, \ldots, \lambda_n^1, \ldots, \lambda_n^S\right)$ as the vector of Langragean multipliers:

$$LR(\lambda) = \min \sum_{s=1}^{S} p^s \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \left(T_{ij} - \lambda_i^s\right) x_{ij}^s + \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{b=1}^{B+1} C_{kb}^s \mu_{jkb}^s + \sum_{i=1}^{n} \lambda_i^s D_i^s \right]$$

subject to (6)-(7), (9)-(10), and (12)-(15).

For a given λ, $\sum_{s=1}^{S} \sum_{i=1}^{n} p^s \lambda_i^s D_i^s$ is constant. The problem is therefore sepa-

rable in j. We write $LR(\lambda) = \sum_{j=1}^{n} g_j(\lambda) + \sum_{s=1}^{S} \sum_{i=1}^{n} p^s \lambda_i^s D_i^s$ with $g_j(\lambda)$ being the optimal value of the Lagrangean subproblem for each location j:

$$g_j(\lambda) = \min \sum_{s=1}^{S} p^s \left[\sum_{i=1}^{n} (T_{ij} - \lambda_i^s)\, x_{ij}^s + \sum_{k=1}^{K} \sum_{b=1}^{B+1} C_{kb}^s \mu_{jkb}^s \right] \tag{16}$$

subject to

$$\sum_{k=0}^{K} y_{jk}^s = 1, \qquad \forall s, \tag{17}$$

$$y_{jk}^1 = y_{jk}^2 = \cdots = y_{jk}^{S-1} = y_{jk}^S, \qquad \forall k, \tag{18}$$

$$\sum_{i=1}^{n} x_{ij}^s = \sum_{k=1}^{K} \sum_{b=1}^{B+1} Q_{kb} \mu_{jkb}^s, \qquad \forall s, \tag{19}$$

$$x_{ij}^s \leq L_{ij} D_i^s, \qquad \forall i, s, \tag{20}$$

$$\sum_{b=1}^{B+1} \mu_{jkb}^s = y_{jk}^s, \qquad \forall k, s, \tag{21}$$

$$y_{jk}^s \in \{0, 1\}, \qquad \forall k, s, \tag{22}$$

$$x_{ij}^s \geq 0, \qquad \forall i, s, \tag{23}$$

$$\mu_{jkb}^s \geq 0, \qquad \forall k, b, s. \tag{24}$$

3.1 Solving the Subproblem

The first-stage decision is to choose the designed capacity of the facility to open at location j, which corresponds to choosing a linepiece k of the piecewise linear long-run facility cost function. Once the linepiece k is chosen, the second-stage facility cost function is convex piecewise linear with B linepieces, having strictly increasing slopes u_{kb}, where $u_{k\beta} = \frac{P_{k+1}F_{k+1} - P_k F_k}{F_{k+1} - F_k}$ is the slope of the linepiece chosen in the first stage. Choosing a linepiece k for a given location j also takes care of the non-anticipativity constraints (18) as the choice of the linepiece is valid for all scenarios. If we thus consider the problem (16)-(24) for each linepiece $k = 1, \ldots, K$ separately, $g_j(\lambda)$ becomes separable in scenarios.

The case $k = 0$ does not have to be calculated, as no facility will be opened and no costs occur. The subproblem $g_{jks}(\lambda)$ for a given location j, linepiece k and scenario s is:

$$g_{jks}(\lambda) = \min \sum_{i=1}^{n} (T_{ij} - \lambda_i^s)\, x_{ij}^s + \sum_{b=1}^{B+1} C_{kb}^s \mu_{jkb}^s \tag{25}$$

subject to

$$\sum_{i=1}^{n} x_{ij}^s = \sum_{b=1}^{B+1} Q_{kb} \mu_{jkb}^s, \tag{26}$$

$$x_{ij}^s \leq L_{ij} D_i^s, \qquad \forall i, \tag{27}$$

$$\sum_{b=1}^{B+1} \mu_{jkb}^s = 1, \tag{28}$$

$$x_{ij}^s \geq 0, \qquad \forall i, \tag{29}$$

$$\mu_{jkb}^s \geq 0, \qquad \forall b. \tag{30}$$

Problem (25)-(30) is of the same type as the Lagrangean subproblem for a given facility solved in Van den Broek et al. [34] for deterministic facility location problems with a general objective. Their subproblem is a continuous knapsack problem with lower and upper capacity bounds and a linear objective, while our continuous knapsack problem has a piecewise linear convex objective in (25). Still we can adapt the method described by Martello and Toth [25] for solving continuous knapsack problems with linear objective function to the algorithm to find $g_{jks}(\lambda)$ described in Figure 5.

The customer locations i are first sorted according to increasing $T_{ij} - \lambda_i^s + u_{k1}^s$. In this order the customers are allocated to the facility at location j in Step 1, until either all customers are allocated (a), or if the unit costs of the subproblem $T_{ij} - \lambda_i^s + u_{kb}^s$ become positive, i.e. it is no longer profitable to serve customers from j (b), or if the upper breakpoint of the linepiece is

reached (c). In Step 2 we calculate the objective function of the knapsack problem. The first element of the sum is the value of the previous linepiece, the next element is the value of all customers locations we decided to serve on the current linepiece. The third element corrects the objective function value in case the demand of the last customer on the previous linepiece spans across the two linepieces. In Step 3, we update the costs of serving additional customers for the new linepiece and move to next linepiece (Step 4). The algorithm continues until the solution is optimal or all linepieces have been considered, i.e. the overall capacity limit of the facility has been reached.

After calculating $g_{jks}(\lambda)$, addition gives $g_{jk}(\lambda) = \sum_{s=1}^{S} p^s g_{jks}(\lambda)$, and subproblem (16)-(24) is solved by $g_j(\lambda) = \min_k g_{jk}(\lambda)$. The computational complexity of this procedure is $O(n \cdot K \cdot S)$.

3.2 The Lagrangean Dual

In order to find the best lower bound on the optimal solution value of the original problem, one has to solve the Lagrangean dual problem (*LD*):

$$LD = \max_{\lambda} LR(\lambda).$$

We solve *LD* by a sub-gradient optimization method, which is commonly used for facility location problems. The procedure is for example described in Holmberg et al. [19], but we repeat it here for the sake of completeness. The partial derivative of *LR* is given by

$$\delta_i^s = \frac{\partial LR(\lambda)}{\partial \lambda_i^s} = D_i^s - \sum_{j=1}^{n} x_{ij}^s(\lambda),$$

with $x_{ij}^s(\lambda)$ being the optimal solution of the Lagrangean relaxation given λ. Hence, the gradient of LR is given by $\nabla LR(\lambda) = (\delta_1^1, \ldots, \delta_n^S)$.

Initialize: Choose values for $\epsilon_1 > 0$, $\epsilon_2 > 0$, V, V_1 and η_0.
Set UB equal to the value of some approximate solution. Set $LB = -\infty$.
Set $v = 1$, $v_1 = 1$, choose starting point $\lambda^{(1)}$, and set $\eta = \eta_0$.
Repeat: Until $v = V$,

(1) Determine $LR(\lambda^{(v)})$.
If $LR(\lambda^{(v)}) > LB$, set $LB = LR(\lambda^{(v)})$ and $v_1 = 0$;
else, set $v_1 = v_1 + 1$. If $v_1 = V_1$ set $\eta = \frac{\eta}{2}$ and set $v_1 = 0$.

(2) Derive a feasible solution from $y_{jk}(\lambda^{(v)})$ and $x_{ij}^s(\lambda^{(v)})$, yielding value $G^{(v)}$.
If $G^{(v)} \leq UB$, set $UB = G^{(v)}$ and set $\eta = \eta_0$. If $UB - LB < 1$, stop: UB is the optimal solution value.

(3) Calculate the gradient $s^{(v)} = \nabla LR(\lambda^{(v)})$, determine the step length $t^{(v)} = \eta \frac{UB - LR(\lambda^{(v)})}{||s^{(v)}||^2}$, and set $\lambda^{(v+1)} = \lambda^{(v)} + t^{(v)} s^{(v)}$.

(4) If $||s^{(v)}|| \leq \epsilon_1$ or $||\lambda^{(v+1)} - \lambda^{(v)}|| \leq \epsilon_2$, stop;
else, set $v = v + 1$.

Output: LB is the approximate solution of the Lagrangean dual.

Given λ, $LR(\lambda)$ yields a lower bound on the optimal solution value, but the optimal solution of the Lagrangean relaxation is in general not a feasible solution to the original problem. Step 2 of the algorithm described above uses a heuristic to find an upper bound $G^{(v)}$ by constructing a feasible solution for the original problem based on the solution of the Lagrangean relaxation in iteration v. The heuristic is presented in the next section.

The heuristic for finding a feasible solution, starts from the optimal solution of the Lagrangean relaxation, by installation of facilities at locations j, which have $y_{jk}(\lambda) = 1$ for $k \geq 1$, and allocation of demand for each scenario, given by $x^s_{ij}(\lambda)$. If not all demand is satisfied and there is no more capacity available subject to the short-run capacity limit Q_{kB+1} for any facility with first-stage design capacity between F_k and F_{k+1} (see Figure 4), the heuristic first tries to expand capacity of open facilities. If also this does not create enough total capacity then eventually the heuristic resorts to opening new facilities. The rules used for expanding and opening facilities are described in detail below.

We introduce additional notation: MC_j for the maximum capacity available at location j and UC^s_j for the capacity used at location j in scenario s. In the following we use y, MC, x^s and UC^s to denote the vectors of all y_j, MC_j, x^s_{ij} and UC^s_j, respectively, and x and UC to denote the vectors of all x^s and UC^s.

Initialize:

Set $x^s_{ij} = 0$, $y_{jk} = y_{jk}(\lambda)$, $MC_j = \sum_k y_{jk} Q_{k(B+1)}$, and $UC^s_j = 0$, $\forall i, j, k, s$.

(1) Define $\mathcal{I}^s_1 = \left\{ i \mid x^s_{ij}(\lambda) > 0,\ \forall j \right\},\ \forall s$.

For each scenario s: if $\mathcal{I}^s_1 \neq \emptyset$ do `AssignToExisting`$(\mathcal{I}^s_1, x^s, MC, UC^s)$.

(2) Define $\mathcal{I}^s_2 = \left\{ i \mid \sum_j x^s_{ij} < D^s_i \right\},\ \forall s$ and $\mathcal{I}_2 = \bigcup_s \mathcal{I}^s_2$.

(3) While $\mathcal{I}_2 \neq \emptyset$ do:

(a) For each scenario s, if $\mathcal{I}^s_2 \neq \emptyset$, do `AssignToExisting`$(\mathcal{I}^s_2, x^s, MC, UC^s)$.

(b) Define $\mathcal{I}^s_2 = \left\{ i \mid \sum_j x^s_{ij} < D^s_i \right\},\ \forall s$ and $\mathcal{I}_2 = \bigcup_s \mathcal{I}^s_2$.

If $\mathcal{I}_2 \neq \emptyset$:

(i) Define $\mathcal{J}_1 = \left\{ j \mid \sum_{k=1}^{K-1} y_{jk} \; = \; 1,\, UC_j^s = MC_j \right\}$.

(ii) Do `ExpandExisting`$(\mathcal{I}_2, \mathcal{J}_1, x, MC, UC)$.

end If.

(c) Define $\mathcal{I}_2^s = \left\{ i \mid \sum_j x_{ij}^s < D_i^s \right\}$, $\forall s$ and $\mathcal{I}_2 = \bigcup_s \mathcal{I}_2^s$.

(d) If $\mathcal{I}_2 \neq \emptyset$: set $\mathcal{J}_2 = \{ j \mid y_{jk} = 1, k = 0 \}$

(i) Do `OpenNew`$(\mathcal{I}_2, \mathcal{J}_2, y, x^s, MC, UC^s)$.

(ii) Define $\mathcal{I}_2^s = \left\{ i \mid \sum_j x_{ij}^s < D_i^s \right\}$, $\forall s$ and $\mathcal{I}_2 = \bigcup_s \mathcal{I}_2^s$.

end If.

end While.

Output: UB is the cost of a feasible solution to problem (11)-(10).

The subroutine `AssignToExisting` takes a set of customer locations $\mathcal{I}$ and tries to assign the demand of these customers to existing facilities. For each customer $i \in \mathcal{I}$, the subroutine first determines the facility j that can satisfy demand at location i at lowest cost T_{ij}. It then assigns as much customer demand as possible to facility j. A detailed description of `AssignToExisting` is given in Figure 6.

If the existing facilities cannot satisfy all customer demand, we try to resolve these infeasibilities by expanding the capacity of the existing facilities. The subroutine `ExpandExisting` takes as input parameters the set $\mathcal{I}$ of customers with unsatisfied demand and the set $\mathcal{J}$ of facilities that have no more capacity available.

It then determines the facility $j \in \mathcal{J}$ that can serve most of the customers in $\mathcal{I}$, expands this facility, and assigns as much customer demand as possible to it. This subroutine is shown in Figure 7.

If the solution is still infeasible, we open new facilities. The subroutine `OpenNew`, described in detail in Figure 8, has as input the set $\mathcal{I}$ of customer locations with unsatisfied demand and the set $\mathcal{J}$ of locations without facility. It determines the location $j \in \mathcal{J}$ that can satisfy most customer demand and assigns this demand to location j. The subroutine then installs a maximum capacity at this facility such that the expected capacity usage is smaller than the installed capacity. If the newly opened facility is not enough, step 3 of the heuristic is redone with the new facility included.

The heuristic returns a feasible solution, y_{jk} and x^s_{jk}, with a solution value denoted by $G^{(v)}$ at iteration v of the sub-gradient optimization routine described in Section 3.2. When the locations are fixed the resulting problem is a linear stochastic transportation problem. To improve the solution we use XpressMP to solve the stochastic transportation problem every 100 iterations or in iterations when a new best solution is found by the heuristic. It can be mentioned that the version of the algorithm where we do not try to improve the heuristic solution using XpressMP, only gives marginally worse results in empirical computational studies.

4 Case Description

In this section we present a case from the Norwegian meat industry regarding the location of slaughterhouses for cattle. Computational results for this case are presented in the next section. The facility costs used here are based on a German study [20]. These costs include both fixed costs (capital cost, personal, insurance) and variable costs (energy, workforce, water, cleaning, repairs, classification, material, waste management). The total cost function is

depicted as the solid line in Figure 9. As the average costs are monotonically decreasing, the total cost curve we use is concave for low volumes and convex for higher volumes. Marginal costs are always below average costs.

There are 435 possible locations for facilities, corresponding to municipalities in Norway. The facility cost functions are equal for all facilities and represents the first-stage cost. The piecewise linear function used to approximate the total cost function is the dashed line in Figure 9.

The transportation data is taken from Van den Broek et al. [34]. The two important elements here are transportation time and transportation cost. Concerning the first, due to legal restrictions, animals cannot be transported for more than 8 hours. This clearly limits the possible transportation distance. The transportation time is defined as the total duration from loading the first animal on the truck, until the last animal has left the truck at the slaughterhouse. This time is approximated by the time to drive from the collection region to the slaughterhouse (travel time) plus the average time of filling up the truck on a collection round-trip (collecting time). Allocation of animal municipalities to slaughterhouse municipalities which do not satisfy the 8 hour rule is eliminated using the binary parameter L_{ij} which is set to 0 for infeasible combinations.

Transportation costs consist of two components. Firstly, there is the driving cost from the slaughterhouse to the region (municipalities) where animals are to be picked up and back to the slaughterhouse (travelling cost). Secondly, there is the cost of collecting animals in the region (collecting cost). For approximating these costs Borgen, Schea, Rømo, and Tomasgard [6] estimated, based on empirical data from the Norwegian meat cooperative, the average

distance driven, the average number of stops at farms, and the average time per stop.

The data from the Norwegian meat cooperative contains no indication of differences between the different regions regarding the velocity of the cars or the costs. Thus, Van den Broek et al. [34] assume equal velocity and equal costs in all regions. As the transportation operator is paid by the travelling distance, and has an additional payment linear in the number of animals on the truck, the transportation costs are linear in the distance to the slaughterhouse and linear in the number of animals transported in the truck.

We approximate the first-stage facility cost function by 6 linepieces. The breakpoints are given in $(tons/year, NOK/kilo)$ and chosen as: $(0, 0)$, $(1.3, 6153.85)$, $(1000, 8.03)$, $(5000, 3.43)$, $(9000, 2.18)$, $(17500, 1.34)$, and $(40000, 1.1)$. In this case the short-run cost functions are assumed to be deterministic. The second-stage facility cost function is represented by a convex piecewise linear function with 3 linepieces. The second linepiece corresponds to the linepiece chosen in the first stage ($y_{jk} = 1$) and has a per unit cost of $u_{k2} = \frac{P_{k+1}F_{k+1} - P_kF_k}{F_{k+1} - F_k}$, see Figure 4. The per unit cost of the first linepiece is given as $u_{k1} = 0.75 \cdot \min\left\{u_{k1}, u_{(k-1)1}\right\}$ and on the last linepiece it is defined as $u_{k3} = u_{k2} + u_{(k+1)2}$ for $k < 6$ and $u_{k3} = 5 \cdot u_{k2}$ for $k = 6$. In addition the upper limit on the capacity usage is set to $Q_{k3} = 1.2 \cdot F_{k+1}$.

We aggregate demand in the same 435 municipalities that are candidates for locations. Demand is here described as a farmer's demand to deliver animals to a slaughterhouse. The demand is aggregated into demand per municipality and year. We generated 3 groups of demand data sets drawing from a multivariate normal distribution with expectation equal to the original animal population

of year 1999. The first group of data sets assumes that demand is varying on a national level, i. e. the demand in all municipalities is perfectly correlated. The second group considers regional demand variations. The municipalities are grouped into 4 regions (Northern Norway, Mid-Norway, Western Norway, and Southern Norway). Demand is perfectly correlated within a region, but uncorrelated between the different regions. The last data sets assume no correlation in demand between the different municipalities. For each of the three groups we generated 2 sets of scenarios, with 100 scenarios each. The first scenario set has a standard deviation equal to 50% of the expected demand, whereas the other set has a standard deviation equal to 20% of the expected value. We also made data sets with 10 scenarios generated from the same distributions.

5 Computational Results

We present the most important results for problem instances from the Norwegian meat cooperative. In all instances the 435 demand regions and possible locations for facilities are equal. All calculations were carried out on a PC running a Linux kernel 2.6.11 with a 3GHz Intel Xeon processor and 6GB RAM. XpressMP 2004D was used as commercial solver whenever stochastic LP's were solved. The parameters ε_1 and ε_2 are set to $1.0 \cdot 10^{-20}$. Test runs indicated that the initial value for the Lagrangean multipliers has almost no influence on the results. The maximum number of iterations is set to $V = 3000$. The parameters η_0 and V_1, the number of iterations without improvement before reducing the step size parameter η, were adjusted for each data set in order to produce reasonable results. Results are given in Table 1. The problem instances are

described as a combination of the correlation level ((N)ational, (R)egional, or (U)ncorrelated), standard deviation of the data set σ (50% or 20% of the expected demand), and the number of scenarios S. The total costs are given in NOK 1000. The gap between lower bound given by the approximation of the Lagrangean dual and the upper bound given by the best solution found is defined as $\frac{UB-LB}{UB}$. We also show for each problem instance the expected cost of implementing the expected value solution: EEV= $\min \sum_{s=1}^{S} p^s \mathcal{Q}^s(\bar{y})$. Here $\bar{y}$ is the optimal solution of the deterministic problem resulting from replacing all stochastic variables with their expectation.

The deterministic expected value problem is the problem in which all stochastic parameters are set at their expected value. Solving this problem using the algorithm described in section 3 results in a lower bound of NOK 233.2 million and a best feasible solution with cost NOK 241.8 million after 3000 iterations with a computation time of 38 minutes and 5 seconds. The optimality gap is 3.56%. This gap was reduced to 3% after 25000 iterations and around 330 minutes of computation time. Our method solves the stochastic problem instances within 10% of optimality in 3000 iterations for all but one instance and often the gap is around 5%. This is acceptable for practical purposes when solving large real life problem instances which was the target of our investigation. If better accuracy is required our approach may be integrated in a branch-and-bound scheme. If a time speed up is needed the algorithm is suitable for parallelization. The time used to find the solution in the current implementation increases linearly in the number of scenarios and in the number of iterations.

[0] The algorithm stops after 2593 iterations.

In the deterministic model the facility costs account for approximately two thirds of the expected total costs. This ratio between facility costs and transportation costs appear to be the same for the stochastic problem instances. The first-stage decisions for the expected value problem and the stochastic problem instances are shown in Table 2. The numbers correspond to the interval chosen for installed capacity.

Finally let us look more in detail into the characteristics of the solutions and the effect of the scenario representation chosen. When we compare the solution of the stochastic problem instances with the deterministic instance we see that the stochastic version put more weight on flexibility, except for the uncorrelated instances. This is reasonable as the convex second-stage short-run cost function motivates investments in more operational flexibility. In fact the expected value solution is infeasible both for instances with completely correlated demand and regionally correlated demand, as to little flexibility is built into the first-stage solution.

If we look at the uncorrelated instances in Table 1 the expected value solution on the other hand seems to be a good solution, in fact better than the solution provided by the heuristic. This is easy to explain. When demand is uncorrelated high demand in one municipality is more likely to be canceled out by low demand in another municipality within the same slaughterhouse operating region; the demand variance in a slaughterhouse region will be smaller than in the correlated cases. Then less weight is put on flexibility. Although the location patterns of the correlated problem instances are quite similar to those of the expected value problem, the choice of linepieces, i. e. the design capacity, differs. For the correlated and partly correlated cases it seems like the stochastic models ensure that flexible solutions are chosen.

This leads us into a discussion of how good the stochastic models represent the real life decision problem. Clearly the number of scenarios used to represent uncertainty is important here. The cases with completely correlated and partly correlated demand have 1 and 4 stochastic variables respectively. Then the 100 scenarios may give a good enough description of the underlying uncertainty. In general the number of scenarios required to get a good representation of the uncertainty increases both with higher standard deviations for variables and when the variables are uncorrelated. The uncorrelated problem instances have 100 scenarios to represent 435 stochastic variables which is too little to give a good enough description of the underlying uncertainty. When we use too few scenarios to represent uncertainty in our problem relative to the number of variables, we may end up with a solution that is infeasible in reality even if it is feasible for the scenario representation used: For the uncorrelated case we should increase the number of independent scenarios to represent uncertainty properly. This would increase the likelihood of high demand scenarios and thereby increase the probability that the expected value solution is infeasible.

Even in the correlated cases we cannot be sure that the scenario representation we have chosen is such that infeasibilities will never occur in the real life, but at least we see that the chosen solutions put more weight on flexibility. This is also needed for the uncorrelated problem, but we need a higher number of scenarios to capture the uncertainty. The calculation time set a limit to the number of scenarios and thereby the number of stochastic variables we are able to handle in a meaningful way. Parallelization of the algorithm will reduce solution times and increase this number.

6 Conclusions

We have shown how to model and solve a facility location problem with a general objective particularly suited to situations with changing economies of scale or diseconomies of scale, uncertain costs and uncertain demand. The computational results are based on a case from the Norwegian meat industry, but the problem formulation is general enough to apply to other facility location problems as well. We motivate our choices of long-run and short-run cost functions.

Approximating the non-convex non-concave objective function with a piecewise linear function allows us to separate the problem both in facilities and scenarios by applying Lagrangean relaxation. By means of a simple greedy heuristic, we generate feasible solutions from the solution of the Lagrangean subproblem. Based on sub-gradient optimization we solve the Lagrangean dual and achieve acceptable optimality gaps for real-life problems.

Computational results for the real-life case show that modelling uncertainty in demand and correlations between demand in different regions and municipalities are important in practice. In our particular case the situation with uncorrelated demand at first seems to be easier to solve as it can be argued that the expected value solution in the first stage is a good heuristic choice. This is not true. In fact if one increases the number of scenarios used to represent uncertainty, it is likely that the expected value solution will be infeasible. This we also see when demand is positively correlated. Then the expected value solution will lead to infeasibility in the second stage because of capacity problems or induce expensive allocations. The heuristic we have developed seems

to behave well for the stochastic problems, both in terms of solutions times that are linear in the number of scenarios and the quality of the solutions. The solutions provided are better than the ones we get from the expected value problems. It shows the importance of solving the stochastic models as more weight is put on flexibility and on avoiding shortfall situations. It provides solutions with more flexibility, recognizing the variability in demand.

The use of more sophisticated heuristics to generate feasible solutions will most likely improve the quality of the solutions. A topic for future research is the use of the bounds obtained by the method described in this paper in a branch-and-bound scheme. Still in many cases the inaccuracies in both real data and the descriptions of uncertainty are probably larger than the optimality gaps achieved in our approach at the moment. For many practical situations, like the case we investigated for the Norwegian meat cooperative, the suggested Lagrangean relaxation and greedy approach presented here provide good enough solutions to be valuable as decision support in strategic processes.

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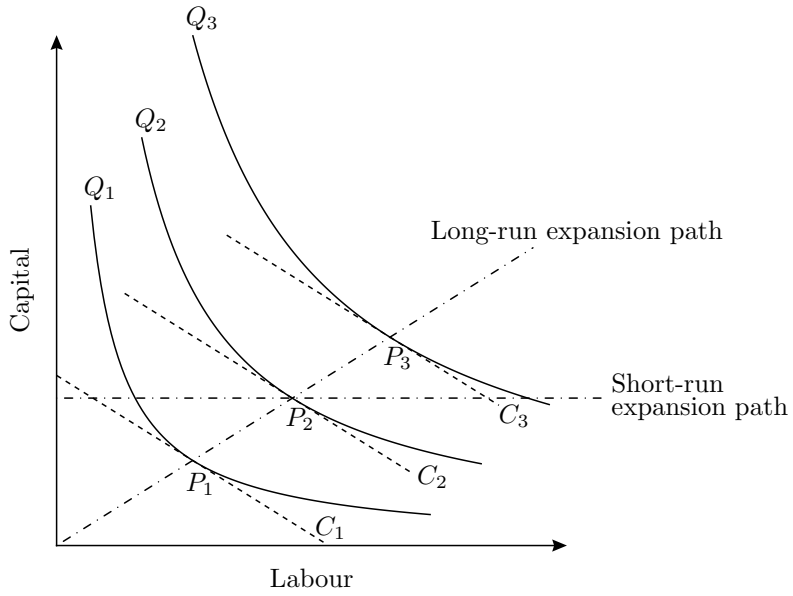

Fig. 1. Long-run and short-run expansion paths

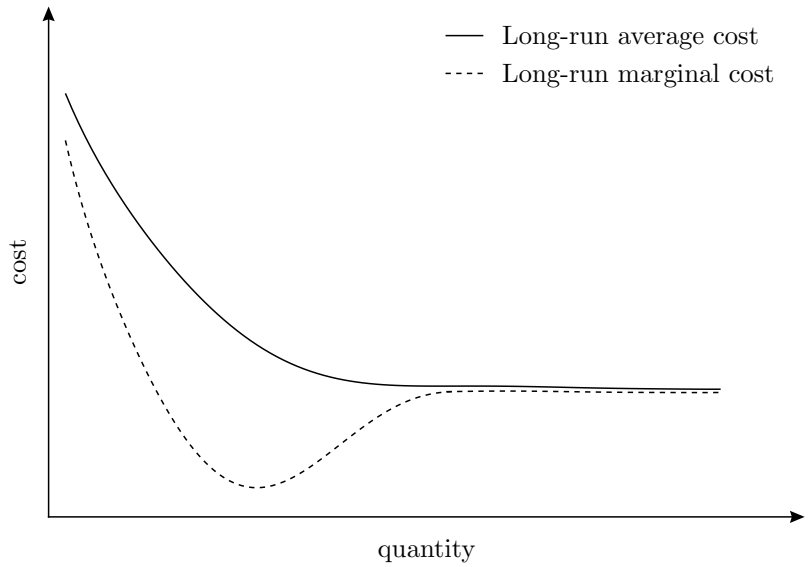

Fig. 2. Long-run average cost function and long-run marginal cost function

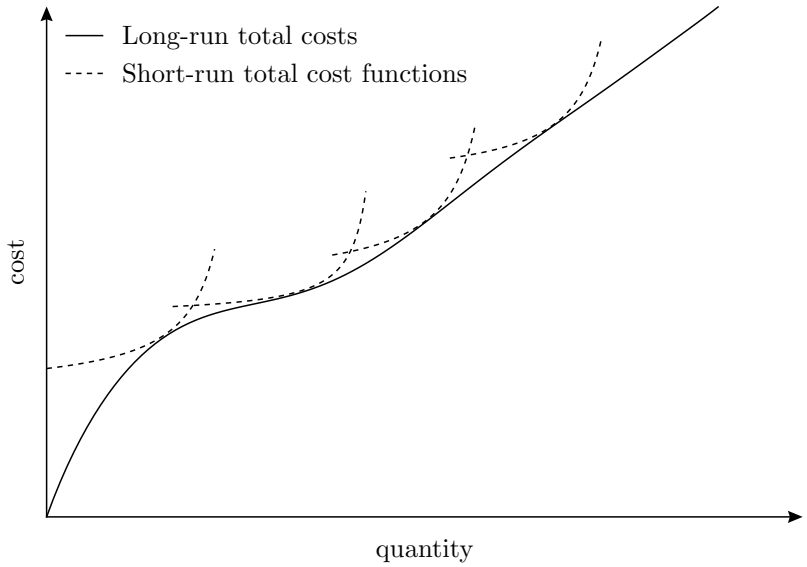

Fig. 3. Long-run and short-run total facility cost function

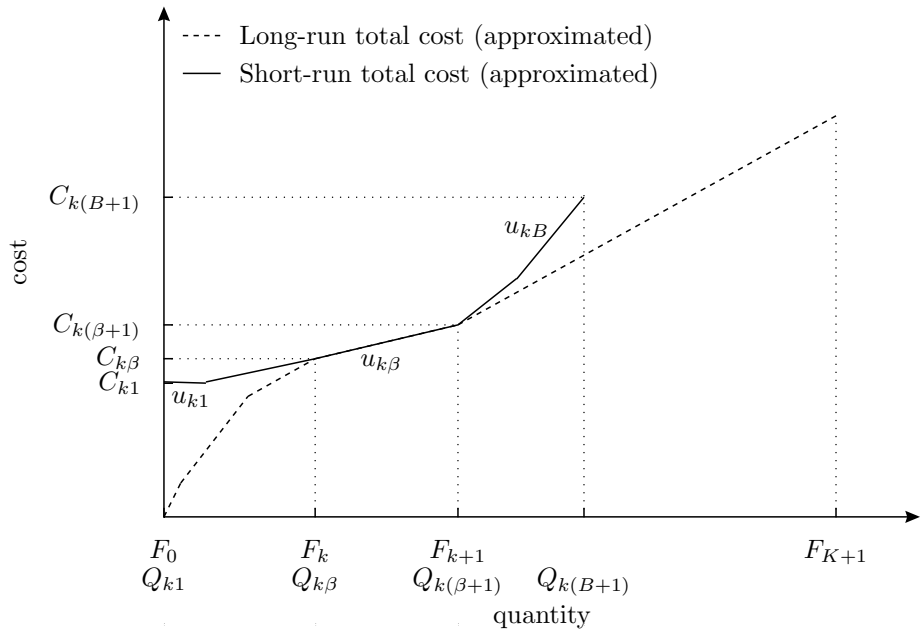

Fig. 4. Approximated first-stage and second-stage facility cost function

Initialize: Set $g^0_{jks}(\lambda) = 0$, $b = 1$, and $i_0 = 1$.
Define $q^s_i = T_{ij} - \lambda^s_i + u^s_{k1}$, $\forall i$.
Sort the locations i in order of increasing q^s_i: $q^s_1 \leq \cdots \leq q^s_n$.
Repeat: Until $b > B$,

(1) Set $x^s_{ij} = L_{ij} D^s_i$, $i = 1, \ldots, n$, until either

(a) $x^s_{ij} = L_{ij} D^s_i$, $\forall i$,
or for the first time for some index (i_b),

(b) $q_{i_b} > 0$, or

(c) $\sum_{m=1}^{i_b} x^s_{mj} > Q_{kb+1}$.

If (a): Set $b = B$ and $i_b = n$. The solution is optimal.
If (b): Set $x^s_{mj} = 0$, $m = i_b, \ldots, n$ and $b = B$. The solution is optimal.
If (c): Set $x^s_{i_b j} = Q_{kb+1} - \sum_{m=1}^{i_b - 1} x^s_{mj}$.

(2) Calculate $g^b_{jks}(\lambda) = g^{b-1}_{jks}(\lambda) + \sum_{m=i_{b-1}}^{i_b} q^s_m x^s_{mj} - q^s_{i_{b-1}} \left(Q_{kb} - \sum_{m=1}^{i_{b-1}-1} x^s_{mj} \right)$

(3) If $b < B$: update $q^s_m = T_{mj} - \lambda^s_m + u_{kb+1}$, $m = i_1, \ldots, n$. The sequence of locations i is not changed.

(4) Set $b = b + 1$.

Output: $g^B_{jks}(\lambda)$ is the solution to (25)-(30).

Fig. 5. Solution algorithm for knapsack problem

`AssignToExisting(Input:` $\mathcal{I}, x^s, MC, UC^s$`; Output:` x^s, UC^s`)`
While $\mathcal{I} \neq \emptyset$:
(1) Choose $i \in \mathcal{I}$.
(2) Define $\mathcal{J} = \left\{ j \mid \sum_{k>0} y_{jk} = 1,\ UC_j^s < MC_j,\ L_{ij} = 1 \right\}$.
(3) While $\mathcal{J} \neq \emptyset$ do:
    (a) Choose the location j^* with lowest transportation cost T_{ij}.
    (b) Set $x_{ij^*}^s = x_{ij^*}^s + \min\left\{ MC_{j^*} - UC_{j^*}^s; D_i^s - \sum_j x_{ij}^s \right\}$.
    (c) Update $UC_{j^*}^s = UC_{j^*}^s + \min\left\{ MC_{j^*} - UC_{j^*}^s; D_i^s - \sum_j x_{ij}^s \right\}$.
    (d) If $\sum_j x_{ij}^s = D_i^s$: do $\mathcal{I} = \mathcal{I} \setminus \{i\}$ else
        $\mathcal{J} = \mathcal{J} \setminus \{j\}$.
    end While.
(4) $\mathcal{I} = \mathcal{I} \setminus \{i\}$.
end While.

Fig. 6. Subroutine `AssignToExisting`

`ExpandExisting(Input:` $\mathcal{I}, \mathcal{J}, x^s, MC, UC^s$; `Output:` x^s, UC^s)
While $\mathcal{J} \neq \emptyset$ do
(1) For each $j \in \{\mathcal{J}\}$ define $\mathcal{I}_j = \{i \in \mathcal{I} \mid L_{ij} = 1\}$.
(2) Choose the set $\mathcal{I}_{j'}$ with highest cardinality.
(3) If $\mathcal{I}_{j'} \neq \emptyset$ do:
(a) Expand the facility at location $j^{'}$, i. e. change $y_{j'k} = 1$ into $y_{j'k} = 0$ and $y_{j'k+1} = 1$.
(b) Update $MC_{j'} = \sum_k Q_{k(B+1)} y_{j'k}$.
(c) For each $i \in \mathcal{I}_{j'}$ and for each scenario s do:
(i) Set $x^s_{ij'} = x^s_{ij'} + \min\left\{MC_{j'} - UC^s_{j'}; D^s_i - \sum_j x^s_{ij}\right\}$.
(ii) Update $UC^s_{j'} = UC^s_{j'} + \min\left\{MC_{j'} - UC^s_{j'}; D^s_i - \sum_j x^s_{ij}\right\}$.
(iii) If $\sum_{j'} x^s_{ij'} = D^s_i$, $\forall s$ do: $\mathcal{I} = \mathcal{I} \setminus \{i\}$ else $\mathcal{J} = \mathcal{J} \setminus \left\{j^{'}\right\}$.
end For.
Else set $\mathcal{J} = \emptyset$.
end If.
end While.

Fig. 7. Subroutine `ExpandExisting`

`OpenNew(Input:` $\mathcal{I}, \mathcal{J}, y, x^s, MC, UC$ `; Output:` y, x, UC^s`)`
While $\mathcal{J} \neq \emptyset$ do
(1) For each $j \in \mathcal{J}$ determine $\mathcal{I}_j = \{i \in \mathcal{I} \mid L_{ij} = 1\}$.
(2) Choose the set $\mathcal{I}_{j'}$ with highest cardinality.
(3) If $\mathcal{I}_{j'} \neq \emptyset$, do For each $i \in \mathcal{I}_{j'}$ and for each scenario s:
(a) Set $x^s_{ij'} = D^s_i - \sum_j x^s_{ij}$.
(b) Calculate $UC^s_{j'} = UC^s_{j'} + x^s_{ij'}$, $\forall s$.
end For.
(4) Choose k such that $y_{j'k} = 1$ and k is the smallest number for which $E(UC^s_{j'}) < MC_{j'} = Q_{k(B+1)}$. Do $\mathcal{J} = \mathcal{J} \setminus \{j'\}$.
(5) If $\exists s'$ with $UC^{s'}_{j'} > MC_{j'}$: set $x^s_{ij} = 0$, $UC^s_j = 0$, $\forall i, j, s$, $\mathcal{J} = \emptyset$.
end While.

Fig. 8. Subroutine `OpenNew`

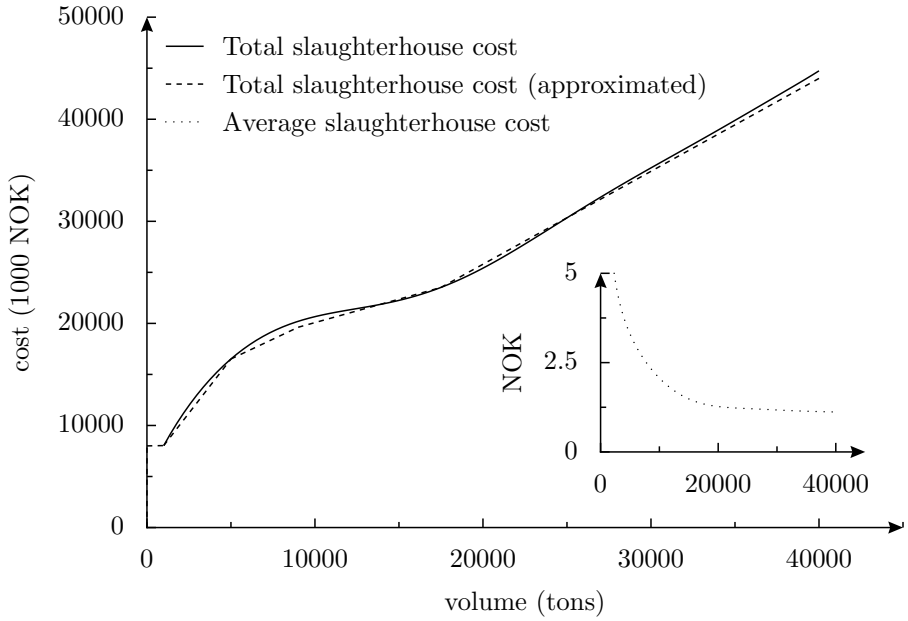

Fig. 9. Total facility costs for slaughterhouses as function of the volume.

Table 1
Computational results

Problem Instance			η_0	V_1	LB	UB	CPU-time	gap	EEV
Corr.	σ	S							
N	50%	100	1.75	325	236372	257412	38:15:06	8.17%	$+\infty$
N	50%	10	2	75	223917	238922	3:30:40	6.28%	$+\infty$
N	20%	100	2	50	234583	248850	35:56:12	5.73%	$+\infty$
N	20%	10	2	125	232688	244620	3:39:37	4.88%	$+\infty$
R	50%	100	1	15	224359	257711	34:38:57	12.94%	$+\infty$
R	50%	10	1	18	226249	248720	3:31:30	9.03% [2]	$+\infty$
R	20%	100	1.75	125	224558	242057	35:19:33	7.23%	$+\infty$
R	20%	10	1.75	100	228726	244972	3:28:44	6.63%	$+\infty$
U	50%	100	1.75	150	228464	245637	35:49:27	6.99%	243906
U	50%	10	1.75	125	228447	243934	3:31:09	6.34%	243960
U	20%	100	1.75	125	229789	242543	34:47:57	5.26%	242166
U	20%	10	2	125	231505	243063	3:33:16	4.76%	242224

Table 2
Slaughterhouse locations and chosen linepieces in the first-stage

Loc.	EV	*N*				*R*				*U*			
		50%		20%		50%		20%		50%		20%	
		100	10	100	10	100	10	100	10	100	10	100	10
29	5		6	6			6	6	6			5	
40		6			6	6							
60												2	
64										6	6		
78													6
137						2							
156													2
172										5	5		
174													5
175		6											
183	5		5	5	5	6	5	5	5			5	
208	2	2	2	2	2	2	2	2	2				
209										2	2		
214												3	
248						6							2
249	5	6	6	6	6		6	5	5	5	5		
272												5	
280		6		3									
292													6
297	5		6		6		6	6	6	6	5		
309						6							
322				6									
329												5	
337						5							4
338		6											
343	3				3				3				
350			6				4	6		6	4		
362	3			3	3	3			3			3	
365		2											
366											2		
367			2					2					
374	3	6		3	3	3	3		3			3	3
376			3					3		3	3		
380							2			2			2
397						3							
412	3	3	2	3	3		3	2	2	2	2	3	3
416						2							
421						1							
426						3							
432	2	2	2	2	2		2	2	2	2	2	2	2
EEV	—	infeasible								feasible			