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Using Imperfect Advance Demand Information in Ordering and Rationing Decisions

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In this paper, we consider an inventory problem with two demand classes having different priorities. The appropriate policy of rationing the available stock, that is reserving some stock for meeting prospective future demand of preferred customers at the expense of deliberately losing some of the currently materialized demand of lower demand class(es), relies on the estimation of the future demand. Utilizing current signals on future demand, which we refer to as imperfect Advance Demand Information (ADI), decreases uncertainty on future demand and hence it may help making better decisions on when to start rejecting lower class demand. We develop a model that incorporates imperfect ADI with inventory ordering (replenishment) decision and rationing available stock. In a two-period setting, we show some structural properties, solve the rationing problem, and propose a solution methodology based on Monte Carlo simulation for the ordering problem. We conduct empirical tests to measure the impact of system parameters on the expected value of imperfect ADI, and consequently we provide managerial insights as to when the utilization of imperfect ADI becomes more valuable.

Keywords: Inventory/production; Advance Demand Information; Customer Reliability; Periodic Review; Demand Classes; Rationing; Monte Carlo Simulation

1 Introduction and Related Literature

Consider a style goods manufacturer delivering shipments for two distinct markets (say, to local retailers and to overseas buyers). Local retail stores have priority over the overseas buyers. The manufacturer's planning horizon is simply divided into two periods with possibly different lengths. At the beginning of the first period the manufacturer has pre-season order information from its customers in the form of soft commitments, that are subject to revisions

within terms of their mutual contracts. Based on the pre-season order information and the on-hand available inventory produced during the pre-season, the manufacturer makes a decision on how much additional goods to produce within the first period that will be available for the second period. After the customer order information are transformed into hard order commitments, the manufacturer decides on how much of the overseas customers' demand to satisfy and how much stock to reserve for the second period, by also taking into account the order information for the second period. This scheme fits into quality flexibility environments with different customer priorities and availability of order information.

The order information in the example above is a form of Advance Demand Information (ADI), which is a term that refers to the information on future demand in general. If the customers place their orders prior to their requirements, this constitutes perfect ADI. In many cases however, the available information on future demand -or the information that can be collected and processed in rather easy and inexpensive ways particularly due to advances in information technologies- includes impurity and uncertainty. We refer to this kind of ADI where there is an early indication of prospective future orders as "imperfect ADI". A simple example is a company that uses sales representatives to market its products, in which case the collection of sales representatives' information as to the number of customers interested in a product can generate an indication about the future sales of that product, hence it constitutes imperfect ADI. Other applications include internet retailing, Vendor Managed Inventory (VMI) applications and Collaborative Planning, Forecasting, and Replenishment (CPFR) environments. In the remainder of the text we use the terms ADI and imperfect ADI interchangeably -unless noted otherwise-, since we consider the imperfect case in this paper.

In today's competitive market conditions, customer differentiation is becoming increasingly important. In our style goods manufacturer example, local retailers may have priority over the overseas customers -or the other way around- for reasons such as contractual agreements that guarantee some high service level, or higher sales volume. Differentiated customer classes not only may have relative priority over each other, but they may also have different ADI structures.

The concept of differentiated customer priorities, or different demand classes, indeed exists in almost all kinds of service or goods production systems. Among many other examples, consider a spare part in a production environment that is of vital importance for a machine, which can also be used for another machine for rather minor purposes. In this case, these

two machines constitute two different demand classes for the spare part. In a restaurant with limited seats, when the only remaining space is a large table, a single customer is probably of not the same value (in economic terms) as a group of customers. Because, if the single customer is seated, there is a risk of losing a group of customers that might arrive while all the tables are occupied.

In our problem environment, there are two demand classes (or customer classes). We refer to the preferred customer class as class-1 and the other one as class-2.

An appropriate policy to handle the problem of facing demand from different classes is to reserve some part of the stock for the use of higher priority customers only, which is known as "(inventory) rationing policy". Demands from both classes are to be met until the inventory level drops down to a critical rationing level, but only class-1 demand is to be met if the inventory level is below that critical rationing level. This results in backordering some class-2 customers with the intention of avoiding (or decreasing the number of) probable class-1 backorders. However, an optimal rationing policy, that is the amount to be rationed and its dynamic relation with some other possible factors (time, lead times, remaining lead times, ADI, number and importance of demand classes, etc.) depends on the problem environment and it is yet an open question in a general context.

While making the critical decision of starting to reject customers with an expectation of future demand from preferred customers, it may be very important to know more about future demand. Therefore, current signals on future demand could be utilized for making better decisions on when to start rejecting lower class demand. In this paper we investigate the impact of using imperfect ADI when two distinguished demand classes exist. The questions that we attempt to answer are: What is the optimal way of allocating the inventory among the demand classes under imperfect ADI? How can the ordering policies be determined in that case? How do the system parameters affect the value of imperfect ADI?

In what follows, we review the related literature on ADI and inventory rationing briefly. Hariharan and Zipkin (1995) show that perfect ADI improves the performance of a continuous-time inventory system in the same way as a reduction in lead-times. Gallego and Özer (2001) model perfect ADI through a vector of future demands and show the optimality of a state-dependent order-up-to policy in a discrete-time setting. Dellaert and Melo (2003) deal with the lot-sizing problem in a similar environment. Karaesmen et al. (2002) consider a capacitated problem under perfect ADI and stochastic lead times. They model the problem via a discrete time make-to-stock queue. We refer the reader to Karaesmen et al. (2003) for a

recent literature survey and treatment of perfect ADI in production/inventory systems.

The literature on different forms of (imperfect) ADI has been rapidly increasing in recent years. Treharne and Sox (2002) consider a problem where the demand in any given period arises from one of a finite collection of probability distributions. They model the demand as a composite-state, partially observed Markov Decision Process and show that a state-dependent base stock policy is optimal for their problem environment. DeCroix and Mookerjee (1997) consider a periodic-review problem in which there is an option of purchasing demand information at the beginning of each period. They consider two levels of demand information: Perfect information allows the decision maker to know the exact demand of the coming period, whereas the imperfect one identifies a particular posterior demand distribution. They characterize the optimal policy for the perfect information case. Van Donselaar et al. (2001) investigate the effect of sharing uncertain ADI between the installers of a project and the manufacturers, in a project-based supply chain. The uncertainty in their setting arises from incompleteness of the selection of installers and manufacturers. Thonemann (2002) elaborates further on a similar problem in which there is a single manufacturer and a number of installers. He considers two types of ADI: Information on whether or not the installers will place an order, and information on which product they will order. Zhu and Thonemann (2004) consider a problem that consists of a number of customers that may provide their demand forecasts. These forecasts are employed to improve the demand forecast of the retailer through an additive Martingale model of forecast evolution. Assuming a linear cost associated with the number of customers that share information, they investigate the relation between the optimal number of customers to contact and the problem parameters.

Although different priorities for some classes of customers is a commonly faced situation, the literature in this field only recently started to expand. Among the existing studies, none considers employing ADI, to the best of our knowledge. In the content of supply chain contracts, Tsay et al. (1999) refer to inventory rationing problems as extremely difficult and consider them as generally intractable. Vericourt et al. (2002) state, as an addition to the reason of the limited amount of research on this topic, that rationing problem is often viewed as an operational decision rather than a strategic one, and hence disregarded in the contracts. According to their empirical results, however, inventory rationing turns out to be important.

One of the pioneering works that models different demand classes is by Veinott (1965). He considers a periodic review model in which each period is divided into small subperiods.

Production or procurement decisions can only take place at the beginning of a (major) period. All demand in a subperiod is met as long as there is enough stock, in decreasing priorities. He introduces the concept of "critical levels" describing a possible critical level rationing policy without analyzing it. Topkis (1968) elaborates on this idea and builds a similar periodic review model, again made up of small subperiods. He proves that the optimal policy can be described by a base stock ordering amount in a period and a set of critical rationing levels in each subperiod. Another study by Evans (1968) confirms the results of Topkis in an environment with two customer classes. They both assume zero lead time for ordering, and the critical rationing levels in each subperiod depend on the remaining time until the end of the period. Nahmias and Demmy (1981) compare the effect of rationing on the fill rate against a traditional system with no rationing, for two demand classes. Cohen et al. (1988) consider a discrete time (s,S) inventory model with two demand classes, but without rationing (that is, upon observing demands, class-1 demand is attempted to be met, followed by class-2 demand, without reserving any amount to avoid possible class-1 backorders in the following periods). They develop a heuristic that generates reasonable s and S values.

Nahmias and Demmy (1981) are the first to build a continuous review model -an (s,Q) model- with rationing. Their purpose in that model is again to compare fill rates, rather than optimization. Moon and Kang (1998) extend the research of Nahmias and Demmy (1981) to several demand classes and compound Poisson demand. Melchiors et al. (2000) also extend the work of Nahmias and Demmy (1981). They evaluate expected cost terms for a similar (s,Q) model. They propose a procedure for computing parameters that minimizes the cost function they derive, based on enumeration and bounding. Deshpande et al. (2003) consider a similar (s,Q) problem with rationing, but unsatisfied demand being backordered instead of lost, and without assuming at most one outstanding order. They propose an efficient algorithm to determine the near-optimal solution. Teunter and Klein Haneveld (1999) consider another continuous review model, a continuous review variant of the model of Topkis (1968) and Evans (1968), for two demand classes with a backorder cost proportional to the length of the backorder period. They propose "remaining time policies" in which the amount of inventory that should be rationed depends on the remaining time until the next procurement opportunity if the lead time is zero, and on the remaining time until the orders to arrive if there is a positive deterministic lead time.

Sobel and Zhang (2001) consider a model with two demand classes with the difference that class-1 demand is deterministic and must be met whereas class-2 demand is stochastic

and can be backordered (without rationing). For a fixed setup cost, they show that the optimal replenishment policy is of a modified-(s,S) type. Frank et al. (2003) extend their analysis to the case where rationing of the stochastic demand is possible. They characterize the optimal replenishment policy in the lost sales case, which turns out to have a complicated structure.

Another branch of rationing literature relies on queuing theory. Ha (1997b) models a rationing problem with several demand classes and lost sales as a single server make-to-stock queue and shows the optimality of a set of monotone rationing levels with Poisson arrivals and exponential production times, i.e. in an M/M/1 setting. Ha (1997a) conducts a similar analysis for the backordering case with two demand classes. He shows that in that case the optimal ordering policy is of order-up-to type and optimal rationing policy is given by a monotone switching curve, such that the critical rationing level is decreasing in the number of the class-2 backorders in the system, again for an M/M/1 setting. Dekker et al. (2002) derive fill rate and average cost terms for Poisson demand and general lead time, for a lot-to-lot model with lost demand and several demand classes. They also propose some efficient solution methods without assuring optimality. Vericourt et al. (2002) extend the study of Ha (1997a) to a several demand classes case. They characterize the optimal rationing policy, which turns out to be a set of critical rationing levels. They also propose an algorithm to compute those critical rationing levels. Vericourt et al. (2001) compare this optimal policy with two other policies. They also compute the optimal parameters under a fill rate constraint.

Our work weakly relates with the literature on dynamic pricing with inventory considerations (or similarly to revenue management), as a part of that literature focus on market environments where there is no opportunity for inventory replenishment over the remaining part of the period or selling season (see Elmaghraby and Keskinocak (2003) for a recent review). Under such an environment, the rationing policy reserves the available stock for the preferred customers, without considering a price change.

In this study we consider a two-stage problem where an ordering quantity decision is made in the initial stage, and how much of the low-priority demand to ration is decided in the second stage. We consider two streams of customers (high priority and low priority), each facing independent stochastic demand. Imperfect information on the demand for each stage is available. The objective is to minimize the expected total inventory-related costs.

The contribution of this study can be summarized as follows:

a. We present and analyze a model that incorporates use of imperfect ADI in the presence of customer classes. Specifically, we model a decision problem in which rationing decision follows the ordering decision (with a time lag).

b. We show the effect of using imperfect ADI on the rationing decision.

c. We characterize the behavior of the optimal rationing policy under imperfect ADI. Additionally, we obtain useful structural properties of the problem posed.

d. We present computational analysis that provides valuable managerial insight for the design and operation of such systems.

The rest of this paper is organized as follows. In Section 2 we discuss our problem environment and present the solution to stock rationing problem under imperfect ADI when two demand classes exist. We develop a Monte Carlo simulation-based solution to inventory ordering problem in Section 3. We examine the value of information aspect of ADI on rationing decision in Section 4. Finally, we present our conclusions and discuss possible extensions in Section 5.

2 Modeling Framework and Rationing Problem

In this study we explore the characteristics of the solution to the complicated question of integrating imperfect ADI in inventory rationing and replenishment. In specific, we focus on a problem with one ordering (replenishment) decision and one rationing decision. The objective is to minimize the expected total inventory-related costs. We assume linear holding and lost sale costs. The notation is introduced as need arises, but we summarize our major notation in Table 1 for the ease of reference.

2.1 Description of the Model

Class-1 demand (the demand class with higher priority) is either immediately satisfied or lost. Class-2 demand is accumulated until the end of the period and unmet demand at the end of the period is lost. Each unit of lost demand from a class-1 customer incurs a cost of b_1 to the system, and each unit of lost demand from a class-2 customer incurs a cost of b_2, such that $b_1 > b_2$.

Table 1: Relevant Notation

K_n	:	Generic random variable denoting the size of the ADI available at the beginning of period n $(n = 1, 2)$
k_n	:	Size of the ADI available (observed) at the beginning of period n $(n = 1, 2)$
$D_n^i(k_n)$	:	Random variable that denotes the demand from class-i customers that occurs in period n, if the size of ADI that is available at the beginning of period n is k_n, for $i = 1, 2$, $n = 1, 2$
$D_n(k_n)$	:	Random variable denoting the total demand in period n, for $n = 1, 2$.
d_n^i	:	Realization of $D_n^i(k_n)$, that is the actual demand from class-i customers that occurs in period n, for $i = 1, 2$, $n = 1, 2$
d_n	:	Realization of $D_n(k_n)$
$G_n^i(w|k_n)$	:	Distribution function of $D_n^i(k_n)$, for $i = 1, 2$, $n = 1, 2$
$G_n(w|k_n)$	:	Distribution function of $D_n(k_n)$, for $n = 1, 2$
x	:	Inventory on-hand at the beginning of period 1
Q	:	Amount ordered at the beginning of the first period
y	:	Inventory position at the beginning of the second period
b_i	:	stockout cost per unit lost sale for demand class i
h	:	inventory holding cost per unit per period

There are two decision epochs in the planning horizon. After collecting the initial ADI, k_1, an ordering decision is made at the beginning of the first period. Production (or procurement) lead-time is one period and there is a starting inventory of x on hand. Therefore, the other decision to make is how much of the class-2 demand to ration at the end of the first period. The amount ordered at the beginning of the first period is not available at the instance of this rationing decision. During the first period, imperfect ADI on the demand of the second period, k_2, is collected and this information is available while making the rationing decision at the end of the first period. The end of the second period is the end of the planning horizon, therefore all of the demand from class-2 customers are attempted to be met at the end of the second period. We note that the periods are not necessarily of the same duration.

Let $D_n^i(k_n)$ be the random variable that denotes the demand from class-i customers that occurs in period n, if the size of ADI that is available at the beginning of period n is k_n, for $i = 1, 2$, and $n = 1, 2$, and let d_n^i denote the realization of $D_n^i(k_n)$, that is the actual demand. Note that we consider k_n as all of the available ADI on both class-1 and class-2 demands. In other words, there exists information that affects demand from both classes. This generalization makes it possible to cover cases such as ADI for the demand classes being

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separately available, or ADI being available on only one demand class. In order to simplify the notation, we suppress k_n and denote the random variable for demand as D_n^i. The model can be adjusted depending on the most likely relation between imperfect ADI and demand to be realized. For example, if ADI is separable for the demand classes, then D_n^i would stand for $D_n^i(k_n^i)$.

We define $D_n = D_n^1 + D_n^2$ to denote the total demand in period n, for $n = 1, 2$. Let Q be the quantity ordered at the beginning of the first period, and let x be the initial inventory level in the system. The following order of events take place: At the beginning of the first period, k_1 is observed, and Q is ordered. At the end of the first period, d_1^1, d_1^2, and k_2 are observed, x is made available to meet d_1^1 as much as possible, the rationing decision is made, and accordingly d_1^2 is met to the extent allowed by the rationing decision. At the beginning of the second period, Q is received. Finally, at the end of the second period, d_2^1 and d_2^2 are observed, d_2^1 is met as much as possible, and the remaining items on hand, if any, are made available to meet d_2^2.

Note that information as to the future demand is available in the system at both decision epochs, namely k_1 at the point of ordering decision, and k_2 at the point of rationing decision. This information should be utilized to make better ordering and rationing decisions.

This model fits better to products with a short life cycle, such as the style goods example described in Section 1. Demand signals for such products may be of crucial importance, especially if the possible number of replenishments is few. Our model considers such a product with two epochs of ordering. The first ordering decision that is made with little or no information on future demand (or future fashion), determines the initial inventory. As more information on demand is collected, the decisions of both the second replenishment and rationing available inventory during the lead time of the second replenishment are made by utilizing this information. In such environments it is more likely that unmet demand is lost due to competitive market conditions.

We also note that any kind of relation between ADI and demand can be considered in our modeling framework. For instance, there can be separate information on each demand class, which might be correlated as well. While explicit solutions may become difficult to compute for some distributions and correlation structures, it is possible to apply numeric methods and/or use approximations in those cases.

While we discuss our model in detail and show some structural properties in the rest of the paper, the objective function can be summarized as:

$$\min_{Q} E_{K_2, D_1^1, D_1^2} \left[\min_{y} \left\{ TRC_1 + E_{D_2^1, D_2^2}[TRC_2] \right\} \right]^{\dagger} \tag{1}$$

where y, as will be shown later, is an inventory level that defines the rationing level when combined with Q and demand realizations of the first period, and TRC_1 and TRC_2 are the total relevant costs for the system at the end of the first and the second period, respectively. In Sections 2.2 and 2.3 we analyze the rationing problem which is depicted as the inner minimization over y in (1), and subsequently we analyze the rest of the problem in Section 3.

2.2 Rationing Problem and Derivation of the Expected Cost Function

We elaborate on the problem using a backward recursion and first handle the rationing problem at the end of the first period for a given set of parameters: initial inventory level (x), amount ordered at the beginning of the first period (Q), realized demand from class-1 and class-2 customers in the first period $(d_1^1$ and $d_1^2)$, and the size of ADI collected during the first period (k_2). The value of k_1 is irrelevant at this point, since the first period demands that depend on it have already been materialized. Similarly, the random variables D_1^1, D_1^2, and K_2 are also irrelevant, since their realizations have already been observed. Demand is either met fully, or partially, or not met at all, depending on the availability of on hand stock and rationing decision.

Let $\tilde{R}$ be the non-negative critical rationing level such that if the inventory on hand after meeting class-1 demand at the end of the first period is less than $\tilde{R}$, then all of d_1^2 is decided not to be fulfilled, and hence lost; otherwise, class-2 demand is met as long as on hand inventory does not drop to a level less than $\tilde{R}$.

We note that the problem has a trivial solution if $d_1^1 \geq x$, as in this case all of the initial stock will be used to satisfy class-1 customers.

Although it is possible to formulate the cost terms and hence the expected cost by making use of $\tilde{R}$, the formulation gets rather complicated then. Therefore we consider another variable, R, which is the inventory level right after meeting first period demand. Note that R is related to $\tilde{R}$ in the following way:

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†The subscripts of E are the random variables over which the expectation is taken.

$$R = \begin{cases} 0 & \text{if} \quad x < d_1^1 \\ x - d_1^1 & \text{if} \quad x - \tilde{R} \le d_1^1 \le x \\ \tilde{R} & \text{if} \quad x - \tilde{R} - d_1^2 \le d_1^1 < x - \tilde{R} \\ x - d_1 & \text{if} \quad d_1^1 \le x - \tilde{R} - d_1^2. \end{cases} \tag{2}$$

R is the net reserved amount at the end of the first period, either due to deliberate rationing or due to excessive or insufficient demand. Therefore, we refer to R as the 'reserve level'. The relation defined in (2) translates into the following.

$$R = \left(Min\{(x - d_1^1)^+, \tilde{R}\} \right) + \left(x - d_1 - \tilde{R} \right)^+. \tag{3}$$

A closer examination of (2) reveals the following points: First of all, if there is an excessive amount of class-1 demand, then R equals zero, i.e. nothing is left to reserve after satisfying class-1 demand partially. In this case, all of class-2 demand is lost as well as a part of class-1 demand. Secondly, if the inventory level drops under the critical rationing level after satisfying class-1 demand fully, then whatever left on hand will be reserved, losing all of the demand from class-2 customers. Thirdly, if the inventory level is still more than the critical rationing level after fully satisfying class-1 demand, then an amount that equals the critical rationing level will be reserved if there is sufficient class-2 demand, losing class-2 demand partially. And finally, if the total demand is not sufficient to drop the inventory level to a point under the critical rationing level even after fully satisfying both classes of demand, then whatever left after meeting all the demand will have to be reserved to the next period. In this case, all of the demand is met at the end of period 1.

We refer to the optimal value of R as R^*, which is a function of x, Q, d_1^1, d_1^2, k_2, distribution functions of period-2 demands, and cost parameters b_1, b_2, and h, as we derive in what follows.

Upon arrival of Q at the beginning of the second period, the system adjusts its inventory level, which equals inventory position at that point, to

$$y = Q + R. \tag{4}$$

But since Q is known at this point, finding R^* is equivalent to finding y^*, i.e. the optimal value of y. Therefore, rationing problem reduces to a modified newsboy problem with the cost function as derived below.

Inventory related costs at the end of the first period, TRC_1, can be stated as

$$TRC_1 = b_1(d_1^1 - x)^+ + b_2 \left[R - \left((x - d_1^1)^+ - d_1^2 \right) \right] + hR. \tag{5}$$

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Note that there is no stochastic term in TRC_1. We can substitute (4) in (5) to obtain

$$TRC_1 = b_1(d_1^1 - x)^+ + b_2\left[(y - Q) - \left((x - d_1^1)^+ - d_1^2\right)\right] + h(y - Q). \tag{6}$$

Inventory related costs at the end of the second period, TRC_2, can be stated as

$$TRC_2 = b_1(D_2^1 - y)^+ + b_2\left[D_2^2 - (y - D_2^1)^+\right]^+ + h(y - D_2)^+. \tag{7}$$

One can use s (salvage cost) instead of h in (7) to take the end-of-horizon effect into consideration. Combining (6) and (7), inventory rationing problem at the end of the first period can be formulated as

$$Minimize_y \quad E[TRC|k_2, d_1^1, d_1^2] = TRC_1 + E[TRC_2|k_2, d_1^1, d_1^2]^\dagger \tag{8}$$

$$subject\ to \quad Q + (x - d_1)^+ \le y \le Q + (x - d_1^1)^+$$

The constraints in (8) define the lower and upper bounds on the inventory level upon arrival of Q at the beginning of the second period, respectively. Equation (3) reveals that $R \ge (x - d_1)^+$ and $R \le (x - d_1^1)^+$. In other words, the minimum amount that must be reserved is what remains after meeting all of the demand in the first period, and the maximum amount that can be reserved is what remains after meeting class-1 demand in the first period, as discussed after defining R. Since $y = Q + R$, the constraints in (8) follow.

We refer to $E[TRC|k_2, d_1^1, d_1^2]$ as the "Expected Total Conditional Cost" ($ETCC$) and for the ease of notation we drop d_1^1, d_1^2 terms in $E[TRC_2|k_2, d_1^1, d_1^2]$. Consequently,

$$ETCC = TRC_1 + E[TRC_2|k_2]. \tag{9}$$

In specific,

$$
\begin{aligned}
E[TRC_2|k_2] &= b_1 \int_y^\infty (w - y)dG_2^1(w|k_2) \\
&\quad + b_2 \int_{-\infty}^y \int_{y-w^1}^\infty (w^1 + w^2 - y)dG_2^2(w^2|k_2)dG_2^1(w^1|k_2) \\
&\quad + b_2 E[D_2^2|k_2]\int_y^\infty dG_2^1(w|k_2) + h\int_{-\infty}^y (y - w)dG_2(w|k_2),
\end{aligned}
\tag{10}
$$

where $G_2^1(w|k_2)$, $G_2^2(w|k_2)$, and $G_2(w|k_2)$ are the distribution functions of D_2^1, D_2^2, and D_2, respectively. We note that while the demand may be discrete, for the ease of exposition we assume that demand distributions are continuous and $E[TRC_2|k_2]$ is twice differentiable.

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†We use the notation "$E[TRC_2|k_2, d_1^1, d_1^2]$" to refer to $E[TRC_2|K_2 = k_2, D_1^1 = d_1^1, D_1^2 = d_1^2]$, and similar notation for the conditional part whenever there is no ambiguity.

2.3 Properties of the Expected Total Conditional Cost Function and the Optimal Reserve Level

Lemma 1 $E[TRC_2|k_2]$ *is convex in y, and hence in R, for a given $Q \geq 0$ and for all k_2.*

Proof : Proof is provided in Appendix A.

Now we state the following theorem, which is used for solving the optimization problem presented in (8).

Theorem 1 *ETCC is convex in y, and hence in R, for a given $Q \geq 0$ and for all k_2.*

Proof : TRC_1 is linear in y, and $E[TRC_2|k_2]$ is convex in y for all k_2 from Lemma 1. Therefore, their sum is convex in y for all k_2. The same argument holds for R for any given $Q \geq 0$, since $y = Q + R$. This completes the proof. □

One can find the optimal reserve level at the end of the first period as stated in the following theorem.

Theorem 2 *The optimal reserve level at the end of the first period is*

$$R^* = Max\left\{Min\left\{(y^* - Q), (x - d_1^1)^+\right\}, (x - d_1)^+\right\}, \tag{11}$$

where

$$y^* = \left\{\min y | (b_2 - b_1)\left(1 - G_2^1(y|k_2)\right) + (b_2 + h)G_2(y|k_2) + h = 0\right\}. \tag{12}$$

Proof : We first note that $ETCC = TRC_1 + E[TRC_2|k_2]$. From (6) we obtain

$$\frac{d\,TRC_1}{d\,y} = b_2 + h.$$

We derive in Equation (26) of Appendix A that

$$\frac{d\,E[TRC_2|k_2]}{d\,y} = (b_1 - b_2)G_2^1(y|k_2) + (b_2 + h)G_2(y|k_2) - b_1.$$

Consequently, the first order condition is sufficient to find y^*, the minimizer of $ETCC$, as stated in (12), due to Theorem 1. This result, combined with the boundary conditions that are defined in (8), result in (11). We also note that there exists a solution to (12), because $\lim_{y \to -\infty} \frac{d\,ETCC}{d\,y} = -b_1 + b_2 + h < 0$, $\lim_{y \to +\infty} \frac{d\,ETCC}{d\,y} = b_2 + 2h > 0$, and due to the convexity of $ETCC$ by Theorem 1. □

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For the special case of perfect ADI on class-1 demand, that is when d_2^1 is known with certainty by the end of the first period, we show in Appendix B that $y^* = d_2^1$, which replaces (12). This means that the ideal inventory level that the system would like to dedicate to the second period is the class-1 demand that will be materialized in the second period, which in this case is known with certainty right before the rationing decision. Reserving below it would result in unmet class-1 demand in the second period, and reserving above it would result in reserving for class-2 demand in the second period (that is, possibly losing actual class-2 customers in the first period with an anticipation of class-2 demand in the second period). Similarly, for the special case of $k_2 = 0$, that is when the ADI realization turns out to be zero, we have $y^* = 0$ and $R^* = (x - d_1)^+$ which means that nothing should be intentionally reserved for the second period.

Theorem 2 enables us to characterize the relation between optimal reserve level R^* versus Q and x, given that the rest of the system remains the same. We enumerate some of those characterizing properties that relate R^* with Q in Corollary 1, and those with x in Corollary 2.

Corollary 1 *The following properties hold.*

1. $\lim_{Q \to \infty} R^* = (x - d_1)^+$

2. If $0 \leq Q < (y^* - (x - d_1^1)^+)^+$, then $R^* = (x - d_1^1)^+$

3. R^* is a non-increasing function of Q

4.
$$\frac{dR^*}{dQ} = \begin{cases} 0 & \text{if } Q < Q_l \text{ or } Q > Q_u \\ -1 & \text{if } Q_l < Q < Q_u, \end{cases} \tag{13}$$

where

$$Q_l = \left(y^* - (x - d_1^1)^+\right)^+, \quad Q_u = \left(y^* - (x - d_1)^+\right)^+. \tag{14}$$

Property 1 states that no intentional rationing should be done if Q is very large. Property 2 states that all what can be rationed should be rationed if Q is not sufficient to reach the desired inventory level at the beginning of the second period. Property 3 states that increased order quantity decreases the necessity to ration. Finally, property 4 states that for insufficient or abundant Q, marginal change in Q does not affect R^*, since R^* is solely determined by

the initial inventory and first period demand in that case. For any Q in between, the system reacts to a unit increase in Q by a unit decrease in R^* to maintain the optimal inventory level at the beginning of the second period, $y^* = Q + R^*$.

Corollary 2 *The following properties hold.*

1. $\lim_{x \to \infty} R^* = \infty$

2. *If $x \leq d_1^1$, then $R^* = 0$*

3. *R^* is a non-decreasing function of x*

4.
$$\frac{dR^*}{dx} = \begin{cases} 0 & if \quad x < x_l \quad or \quad x_m < x < x_u \\ 1 & if \quad x_l < x < x_m \quad or \quad x > x_u, \end{cases} \tag{15}$$

where

$$x_l = d_1^1, \ x_m = d_1^1 + (y^* - Q)^+, \ and \ x_u = d_1 + (y^* - Q)^+.$$

Properties 1 and 3 state that for higher initial inventory levels there will be higher reserve levels, without an upper bound. Property 2 assures that no class-1 demand is rationed. Finally, property 4 translates into the following: If x is less than class-1 demand, then a marginal change in x does not affect R^*, since nothing is reserved anyway. If x is more than class-1 demand but not sufficient to reach the optimal inventory level at the beginning of the second period (y^*), then the system will reserve all that it can. If x is sufficient to reach y^*, then any x in excess will be used to meet class-2 demand and hence will not be reserved, until all the demand is met. If x is any larger than that, then it will have to be left over to the second period.

With the help of Corollary 1 and Corollary 2, the relation between R^* and Q for a given x, and the relation between R^* and x for a given Q can be illustrated as in Figure 1, where $R_u = Max\{(Min\{y^*, (x - d_1^1)^+\}), (x - d_1)^+\}$, $R_l = (x - d_1)^+$, and $R_m = (y^* - Q)^+$. If $Q_l > 0$, as in the illustration, then it turns out that $R_u = (x - d_1^1)^+$.

3   Determination of the Initial Order Quantity

Now we proceed to the problem of deciding how much to order at the beginning of the first period for a given k_1 and x. In other words, we need to find the value of Q that
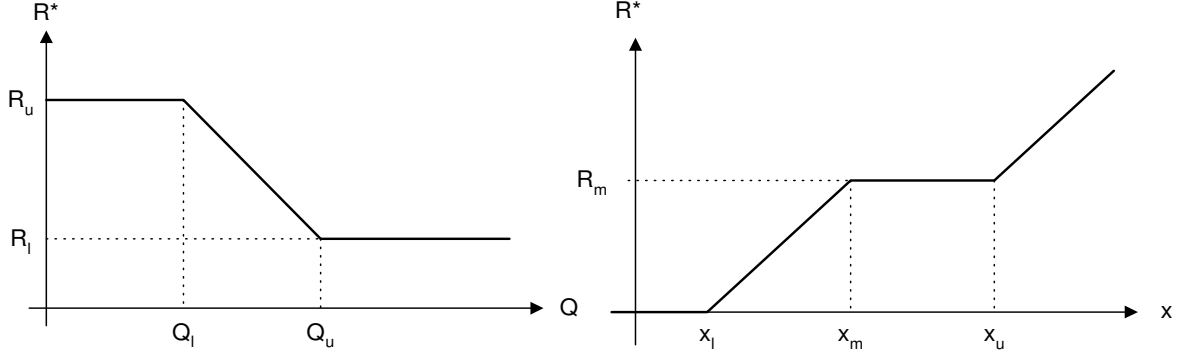
Figure 1: Change of Optimal Reserve Level versus Order Quantity and Initial Inventory

minimizes the expected total inventory related costs for both periods, $E[TRC(Q)]$. Note that $E[TRC(Q)] = E_{K_2,D_1^1,D_1^2}[ETCC] = E_{K_2,D_1^1,D_1^2}[TRC_1 + E[TRC_2|k_2]]$. We first state the following theorem.

Theorem 3 $E[TRC(Q)]$ *is convex in Q under the optimal reserve level policy that is defined in Theorem 2.*

Proof : Proof is provided in Appendix C.

Theorem 3 holds for any reserve level policy, provided that the reserve level, R, is twice differentiable (piecewise) with respect to Q and the second derivative is zero, as shown in Appendix C.

It is also interesting to observe that substituting (13) into (35) of Appendix C results in

$$\frac{dE[TRC_2|k_2]}{dQ} = \begin{cases} (b_1 - b_2)G^1(Q + R|k_2) \\ +(b_2 + h)G(Q + R|k_2) - b_1 & \text{if} \quad Q < Q_l \text{ or } Q > Q_u \\ 0 & \text{if} \quad Q_l < Q < Q_u. \end{cases} \tag{16}$$

This result reveals that the marginal contribution of Q to $E[TRC_2|k_2]$ within the limits $Q_l < Q < Q_u$ is zero. In other words, a decrease (or increase) of Q^* by a marginal unit (as long as it is still in the limits mentioned) will result in the same $E[TRC_2|k_2]$, because the system will adjust itself to exactly the same inventory position by reserving a unit more (or less) for the second period.

We note that explicit computation of $E[TRC(Q)]$ is difficult. Therefore, we propose the following method based on Monte Carlo simulation in order to calculate approximate expected total inventory related costs.

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Approximate Cost Evaluation Procedure (ACEP):

- Initialization: Set TotalCost=0

- Main Step: For Counter = 1 to NUM (a large enough number) do

    - Generate a set of realizations k_2, d_1^1, and d_1^2 from relevant distributions

    - Determine the optimal reserve levels for these generated values using Theorem 2 and calculate the corresponding $ETCC$

    - TotalCost = TotalCost + $ETCC$

- Output: Approximate $E[TRC(Q)]$ as TotalCost / NUM.

ACEP approximates the value of $E[TRC(Q)]$ for a given $Q \geq 0$, by generating realizations of K_2, D_1^1, and D_1^2, and taking the average of the optimal $ETCC$ values that are calculated for each realization. The remaining issue is to search for the optimal order quantity, Q^*, that minimizes $E[TRC(Q)]$. Theorem 3 enables the use of any search algorithm that is designed for (quasi)convex functions. In our empirical studies that we present in Section 4, we apply golden section method.

4 Value of Information

In this section we consider the value of information aspect of ADI on rationing decisions. The rationing decision is to determine the amount to reserve for the second period, possibly by not meeting some portion or all of class-2 demand in the first period. At this instance, ADI might help making a better rationing decision by decreasing the uncertainty on the demand of the second period. We conduct empirical tests to find out under which circumstances there exists higher value of this ADI.

4.1 Problem Setting

In the tests that we conduct, we consider a structure in which ADI is available on class-1 customers only. Each individual information on demand is a prospective demand and it has a probability, p, of being materialized as demand in the next period. The probability of demand realization, p, may be referred to as "customer reliability level", as well. While we assume homogeneity of customer reliability levels for simplification, this is not a restrictive

assumption. Consequently, the conditional demand distribution, D_i^1, is Binomial with parameters k_i and p, for $i = 1, 2$, which we approximate by normal distribution with mean $k_i p$ and variance $k_i p(1 - p)$. The distribution of class-2 demand is taken to be normal with mean μ_2 and variance σ_2^2 in both periods, independent of D_i^1 and k_i. We compare two policies for operating the system of concern, in order to reveal the value of information: ADI-case and NoADI-case. In both cases the decisions are made in an optimal manner, but only in the ADI-case the system is operated under advance demand information (on class-1 demand).

For the sake of comparisons, the distribution of ADI is assumed to be known. We assume a normal distribution with mean μ_K and variance σ_K^2 for ADI. Our general approach is to obtain expected inventory-related costs for both of the cases and compare them. Before we continue with discussing the details of our analysis, we first summarize our findings on the value of ADI on rationing decision. The analysis of the test results reveals that ADI turns out to be more valuable when

- demand variance is high

- relative importance of class-1 demand is high

- there is sufficient class-2 demand in the first period and there is sufficient initial inventory

- order quantity plus the initial inventory in excess of the first period demand is close to the expected class-1 demand of the next period plus some safety stock.

While the first three of these findings are rather intuitive, the last result was unexpected to us prior to the experimentation phase. We discuss the intuition behind this result in Section 4.2.

In the remainder of this section, we explain our analysis and present our findings. We start with the issue of optimal policies for the NoADI-case.

Optimal Reserve Level Under Lack of ADI

In order to find the optimal reserve level at the end of period-1, we use Theorem 2 as in the ADI-case. However, since ADI is not collected, the demand distributions in the NoADI-case are observed as $G_2^1(y)$ and $G_2(y)$, instead of $G_2^1(y|k_2)$ and $G_2(y|k_2)$, respectively. Consequently, equation (12) is replaced with

$$y^*_{NoADI} = \left\{ \min y | (b_2 - b_1)\left(1 - G_2^1(y)\right) + (b_2 + h)G_2(y) + h = 0 \right\}. \qquad (17)$$

The mean and the variance of D_2^1 can be evaluated by conditioning on K_2.

$$
\begin{aligned}
E[D_2^1] &= E\left[E[D_2^1|K_2]\right] = E[K_2 p] = p\mu_K \\
V[D_2^1] &= E\left[V[D_2^1|K_2]\right] + V\left[E[D_2^1|K_2]\right] \\
&= E[K_2 p(1-p)] + V[K_2 p] \\
&= p(1-p)\mu_K + p^2 \sigma_K^2. \qquad (18)
\end{aligned}
$$

Thus, class-1 demand distribution under NoADI-case can be reflected by

$$D_2^1 \sim N(p\mu_K \ , \ p(1-p)\mu_K + p^2\sigma_K^2), \qquad (19)$$

and since D_2^2 is assumed to be normally distributed with mean μ_2, and standard deviation σ_2, the total demand in the second period under NoADI-case is normally distributed as well, since it is the sum of two independent normal distributions. Hence,

$$D_2 \sim N(p\mu_K + \mu_2 \ , \ p(1-p)\mu_K + p^2\sigma_K^2 + \sigma_2^2). \qquad (20)$$

Consequently, Theorem 2 is used in NoADI-case with the demand distributions being the ones presented above. As a result, there exists a unique reserve level for any given set of parameters at the end of the first period, as opposed to a variable reserve level that depends on k_2 in the ADI-case. Figure 2 illustrates this point. In both of the graphs in this figure, the change of optimal reserve level as a function of ADI size is plotted for ADI- and NoADI-cases under a certain set of parameters and a given Q. The value of Q is relatively smaller in the left hand side graph compared to the one in the right hand side.[†] Note that information plays a crucial role in making the optimal rationing decision, since the system reacts to available information in the ADI-case by reserving a varying portion of class-2 demand that ranges from none to all, unlike the NoADI-case with a fixed reserve level.

4.2   Experimentation

Computation of the Value of Information

The system state at the instance of the rationing decision is defined by x, d_1^1, d_1^2, Q, and k_2, as well as cost parameters b_1, b_2, h, and distribution parameters $p, \mu_K, \mu_2, \sigma_K$, and σ_2. The

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[†]While Figure 2 gives the impression that the optimal reserve level is piecewise linear in k_2 in the ADI-case, it is indeed (very slightly) nonlinear.
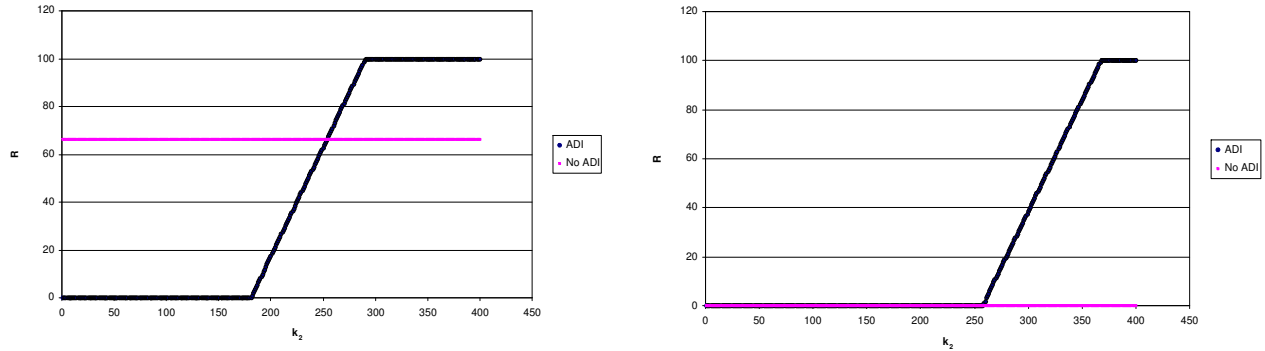
Figure 2: R versus ADI size for two different Q values

approach we follow in order to reveal the value of information aspect of ADI in rationing decisions is to generate k_2 realizations, calculate corresponding optimal reserve levels and relevant costs, average the costs out, and compare. Any realization that does not satisfy $k_2 \geq 0$ should be disregarded or truncated to zero, because it would otherwise result in $Var(D_2^1) < 0$ in the ADI-case.

The algorithm that is used for calculating the value of information is outlined below. We present the algorithm in detail in Appendix D.

Algorithm for VoI on Rationing Decision (AVORD):

- Input: $x, d_1^1, d_1^2, Q, b_1, b_2, h, p, \mu_K, \mu_2, \sigma_K, \sigma_2$

- Calculate TRC_1 for NoADI-case

- Approximate $E[TRC_1|d_1^1, d_1^2]$ for ADI-case and $E[TRC_2|d_1^1, d_1^2]$ for both cases by conditioning on a large number ($NUM1$) of k_2 realizations from $N(\mu_K, \sigma_K^2)$ distribution and then averaging out

- Output:
$$\%VoI = 100 * \frac{E[TRC|d_1^1, d_1^2]_{NoADI} - E[TRC|d_1^1, d_1^2]_{ADI}}{E[TRC|d_1^1, d_1^2]_{ADI}}$$

We note that the performance measure considered above is the percent penalty of not utilizing ADI (which we refer to as $\%VoI$). Since the value of information (VoI) lacks relativity, we report $\%VoI$ in the results.

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Since there is a fixed reserve level for the given input parameters in NoADI-case independent of the value of k_2, TRC_1 in the NoADI-case does not change with k_2, therefore it suffices to compute it once. However, optimal reserve level depends on k_2 in ADI-case, so TRC_1 is a function of k_2. Therefore we take the average of TRC_1 values conditioned on k_2 realizations in this case, which we refer to as $E[TRC_1|d_1^1, d_1^2]_{ADI}$.

Description of the Tests Conducted

We conduct tests to examine the value of ADI on rationing decision as a function of problem parameters. We first investigate which factors influence the value of ADI. Then we conduct tests for a range of values of some parameters in order not only to comprehend the effect of different values of those parameters on the value of information, but also to observe the sensitivity of the $\%VoI$.

We try different values of input parameters to investigate which values yield to increased $\%VoI$. We consider every combination of some fixed values of the input parameters, hence our experimental design follows a full-factorial fixed-effects model. The values of the input parameters (i.e. the levels of the factors) that we utilize in the experiment are provided in Table 2.

Table 2: Values of Input Parameters

p	0.1, 0.5, 0.9
Q	0, 100, 200
x	10, 100
b_1/b_2	1.3, 10
$b_2 FR$	0.8, 0.95
μ_K	20, 200
μ_2	10, 100
CV	0.05, 0.25
d_1^2	10, 100

We fix $h = 1$ and set the cost parameters according to two considerations: b_1/b_2 being close to 1 (since $b_1 > b_2 + h$ must hold, we set $b_1/b_2 = 1.3$) and being 10; and the trade-off between class-2 backorders and inventory holding (that is, $b_2/(b_2 + h)$, which we refer to as "$b_2 FR$", resembling approximate "fill rate" as if there are no demand classes) being 0.8 or 0.95. The resulting sets of cost parameters are $(b_1, b_2) = (5.2, 4), (40, 4), (24.7, 19)$, and $(190, 19)$. The coefficient of variation, CV, is taken as common to k_2 distribution and D_2^2 distribution at the same time. That is, $\sigma_K/\mu_K = \sigma_2/\mu_2 = CV$. We set $d_1^1 = 0$ in all sets,

because d_1^1 is to be met from x in all cases and if $d_1^1 > x$ then there will be no inventory left to ration at the end of the first period, and the value of information will trivially be zero.

These values of parameters result in $2^7 * 3^2 = 1152$ sets. Since we are interested in revealing the characteristics of parameter sets that yield high $\%VoI$ rather than obtaining confidence intervals on $\%VoI$ for each set, we only evaluate point estimates. Nevertheless, we note that one might construct confidence intervals as well.

A sufficiently large run length (NUM1), that is the number of k_2 realizations that will suffice to estimate the values determined in AVORD with a specified precision, can be determined by an appropriate statistical method. We followed a method provided by Law and Kelton (1991), pp 538, based on a fixed number of replications. Nevertheless, we conducted the experiments with a significantly larger run length to safely cover for some parameter sets that might yield high variance.

Discussion of the Results

We conduct an analysis of variance (ANOVA) for this experiment on a model that includes all of the main and two-way interaction effects. The main effects that are significant on $\%VoI$ at 95% confidence level (and their respective p-values[†]) are μ_K (0.001), b_1/b_2 (0.003), μ_2 (0.004), x (0.005), d_1^2 (0.017), p (0.023), and CV (0.028).

For this set of parameters, we deduce from main effects plots that there is a higher expected value of information on rationing decision when expected ADI size is high, relative importance of class-1 customers (i.e. b_1/b_2) is high, expected class-2 demand of the second period is low, initial inventory level is high, class-2 demand of the first period is high, customer reliability level is high, and system variability (induced by k_2 and D_2^2) is high. These results are mostly in line with intuition, as we briefly discuss in what follows.

Observation 1. When class-1 demand is not significantly more important than class-2 demand, and especially when both backorder costs are relatively low, then it does not pay off to collect advance demand information, because the rationale for rationing (losing a class-2 demand deliberately and also facing a holding cost to avoid a possible loss of class-1 demand) is low.

Observation 2. Higher values of expected ADI size and customer reliability level stand for a higher expectation for class-1 demand in the second period. In that case, reservation

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[†] "p-value" is the smallest level of significance that would lead to rejection of ANOVA null hypothesis, and it should not be confused with customer reliability level, p.

for the second period gets more critical, hence the value of information. Nevertheless, this statement holds as long as the quantity ordered (Q) plus the initial inventory in excess of the first period demand ($(x - d_1)^+$) is not much less or much more than expected class-1 demand of the second period plus some safety stock. We discuss this issue later in this section. We note that the importance of ADI is not necessarily only high when it signals a higher prospective demand. Indeed, the average of the reserve levels for the ADI-case are mostly lower than the fixed reserve level of NoADI-case in those sets that yield higher expected value of information. In other words, "blindly" reserving in bulk with an expectation of higher class-1 demand without having ADI is not necessarily better, because when the size of ADI is low, it alerts the system for a prospective low class-1 demand, despite high expectation prior to ADI realization (i.e. when $E[K_2] = \mu_K$ is high but realized k_2 is relatively low). These arguments hold in a stronger sense when the variance of the ADI size and the variance of class-2 demand are higher, hence the variance of the second period demand is high in total.

Observation 3. A low level of initial inventory, x, leaves less room for rationing, and therefore the expected value of information is less in that case. A similar reasoning holds for the amount of class-2 demand in the first period, d_1^2. Nevertheless, there are some results contradicting the former observation when d_1^2 is also low. This is because when the initial inventory is low and d_1^2 is high, then the rationing decision changes only a small portion of the first period costs (since d_1^2 is relatively much higher), and it does not affect the second period costs significantly either (since x is low); however, when x and d_1^2 are both low, then the rationing decision changes a significant portion of the first period costs, although the argument for the second period costs still hold.

Observation 4. When the expected class-2 demand of the second period is low, then the second period demand is mostly defined by class-1 demand. Therefore, ADI (which is on class-1 demand in this experiment) becomes more important.

It should be noted that none of the factors that lead to higher $\%VoI$ can be effective only by themselves. For example, if there is no initial inventory, then the value of information will be zero, independent of all other factors, since there will be nothing to ration. Other factors that trivially yield zero value of information themselves are the customer reliability level being zero, coefficient of variation being zero, Q or x assuming a very high value (theoretically when $x + Q - d_1$ approaches infinity), μ_K and hence expected class-1 demand of the second period being zero (if non-negative demand is assumed), class-2 demand of the first period being zero, and the cost parameters being such that $b_1 \leq b_2 + h$. $Q = 0$ is not

in this list, because optimal reserve levels could very much depend on ADI when $Q = 0$ too, depending on other parameters.

We also note that the interaction effects are mostly due to a similar reasoning as pointed in the discussion above. For example, there appears to be an interaction effect for x and CV, with a p-value of 0.028. But this is mainly because $\%VoI$ is small for both values of the CV when the initial inventory is small, i.e. $x = 10$. Larger CV yields larger $\%VoI$ when $x = 100$.

The Impact of the Selection of Q on the $\%VoI$

The previous experiment does not make it very clear how the value of ADI changes as a function of Q. Nevertheless, it appears that Q is close to the expected class-1 demand of the second period ($E[D_2^1]$) in most of the results with higher $\%VoI$. Therefore, we conduct another ANOVA, this time with the factor $E[D_2^1]$ ($= p\mu_K$) instead of the factors p and μ_K. The model includes all of the main and two-way interaction effects again. The main effects that are significant on $\%VoI$ at 95% confidence level (and their respective p-values) are $E[D_2^1]$ (0.000), b_1/b_2 (0.003), μ_2 (0.004), x (0.005), d_1^2 (0.016), and CV (0.026). The most significant interaction effect is between Q and $E[D_2^1]$ with a reported p-value of 0.000.

With the purpose of gaining better insight on the sensitivity of $\%VoI$ with respect to Q we fix a set of input parameters and apply AVORD with varying values of Q. The values of the parameters in this set are $p = 0.9$, $CV = 0.25$, $\mu_K = 200, \mu_2 = 10, x = 100, d_1^1 = 0, d_1^2 = 100, b_1 = 190, b_2 = 19, h = 1$. We note that the initial inventory is equal to the first period demand in this set. The graph depicting the resulting $\%VoI$ versus Q values is presented in Figure 3. A second set of parameters with only cost parameters changed as $b_1 = 12, b_2 = 5, h = 1$ is also depicted on the same graph.

We also present the expected total cost figures, $E[TRC|d_1^1, d_1^2]$, for ADI- and NoADI-cases with $b_1 = 12, b_2 = 5, h = 1$ in Figure 4. Expected total costs when rationing is not allowed (ETRCNo-Rat) is also presented on the same graph for comparison purposes.

Observation 5. As a result of this test, we conclude that $\%VoI$ is especially high within a range of Q, under the problem settings considered here. The reason why $\%VoI$ is not significant for very low or very high values of Q can be explained as follows. For Q values that are much smaller than the expected class-1 demand of the second period (which is 180 in this test), the optimal rationing policy is to reserve a large amount, if possible. But since $x = 100$ in this test, the optimal policy is to ration most of it, if not all, even if k_2 realization is relatively low. Similarly, for Q values that are much higher than the expected
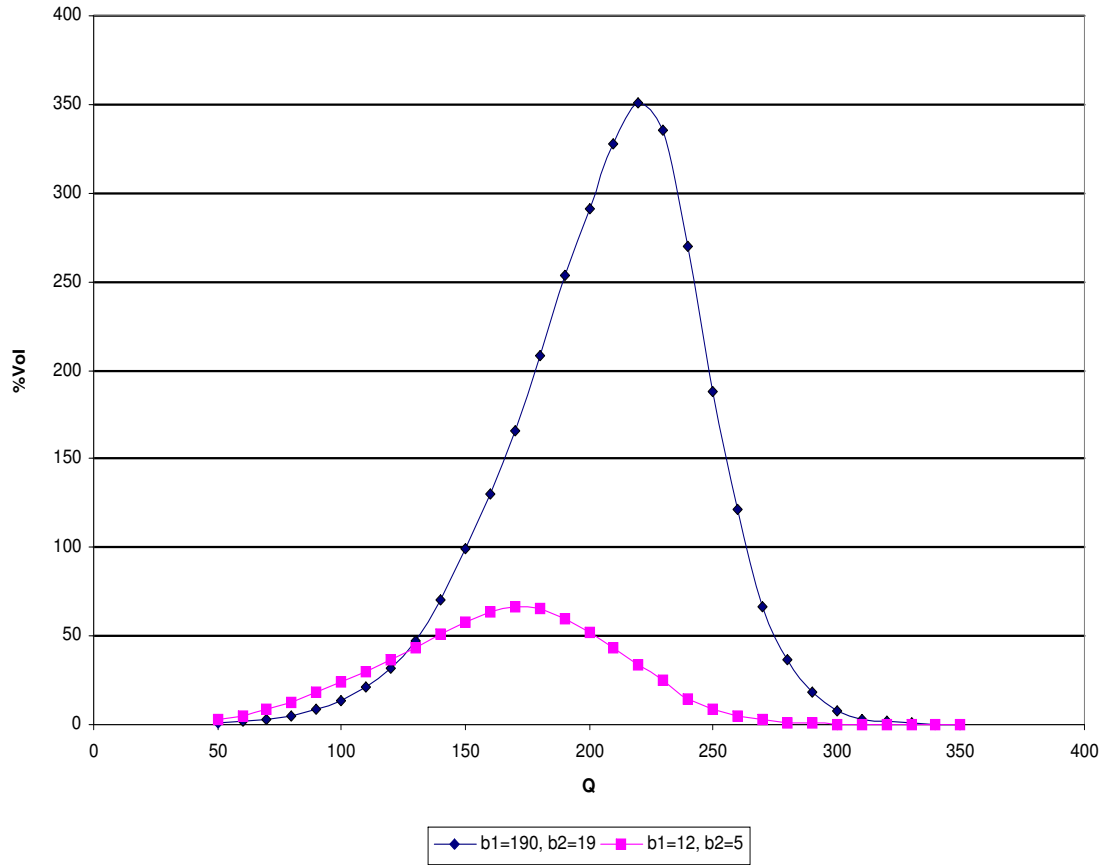
24

Figure 3: $\%VoI$ versus Order Quantity

class-1 demand of period 2, the optimal policy is to reserve very little, if any at all, even if k_2 realization is relatively high. Observe that Figure 4 confirms this point, since the expected costs of both policies converge to that with no rationing as Q increases. Therefore ADI is no longer very critical in such cases. For Q values that are in between, ADI has a high value. For the test with $b_1 = 190, b_2 = 19, h = 1$, the penalty of not utilizing ADI reaches values up to 350.64% (when $Q = 220$), which equivalently corresponds to a cost saving of 77.81% when ADI is utilized. For the test with $b_1 = 12, b_2 = 5, h = 1$, the penalty is as high as 66.93% (when $Q = 170$) -or the cost saving is 40.09%-. We also note that the values of Q for which ADI is more valuable change according to cost parameters.

Observation 6. Figure 4 also reveals that the penalty paid for not rationing compared to the optimal rationing policy is significant especially when the order quantity is smaller than optimal. Moreover, for order quantities that are much smaller than the optimal, the majority
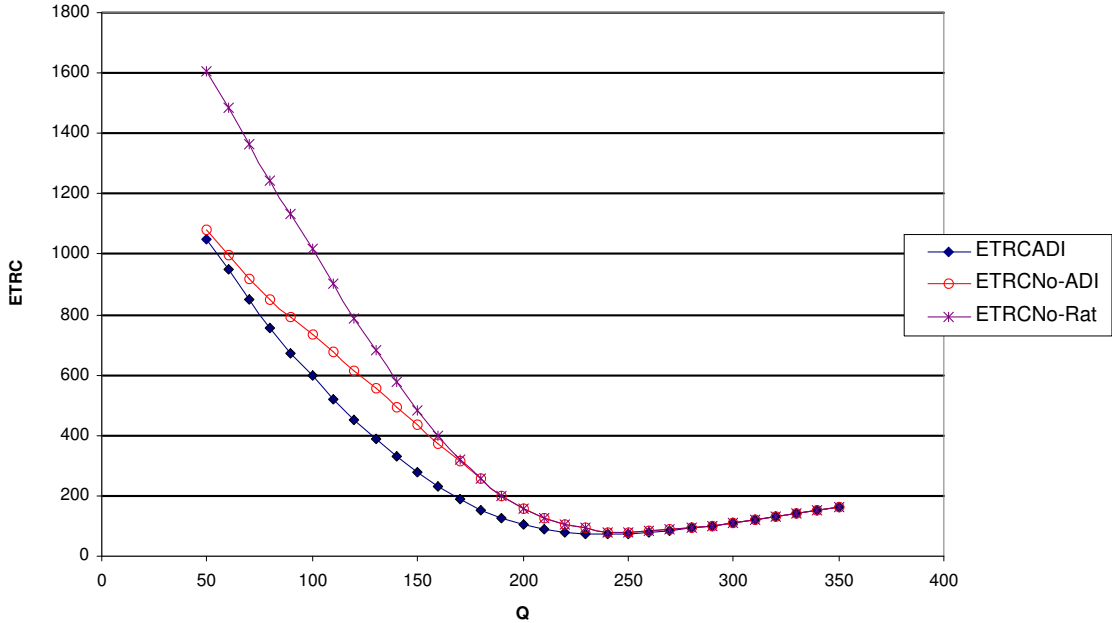
Figure 4: $E[TRC|d_1^1, d_1^2]$ versus Order Quantity

of the penalty is due to not rationing, whereas for the rest the majority of the penalty is due to not employing imperfect ADI in the rationing decision. There are still considerable savings if rationing and imperfect ADI are used with the optimal ordering quantity. For the set of parameters that generated Figure 4, $\%VoI$ is 16.25% when the system operates with the optimal order quantity of 238. (The optimal policy under lack of ADI is not to ration for that Q.)

We extend our analysis by examining the relation between Q and $\%VoI$ for different values of initial inventory level x, because x and Q interact in determining the reserve level. We note that the previous observations in this subsection (namely, Observations 5 and 6) were made for an x value that did not exceed d_1, which means that the only source to satisfy the demand of the second period on top of the order quantity was rationing. In what follows we let the values of the initial inventory vary.

The values of the parameters in this set are the same as the previous one, except for x varying now and $\mu_2 = 100$ instead of 10. That is, $p = 0.9$, $CV = 0.25$, $\mu_K = 200, \mu_2 = 100, d_1^1 = 0, d_1^2 = 100, b_1 = 190, b_2 = 19, h = 1$. The $\%VoI$ versus Q for different values of $x \geq d_1$ are presented in Figure 5, and those for $x \leq d_1$ are presented in Figure 6. We note
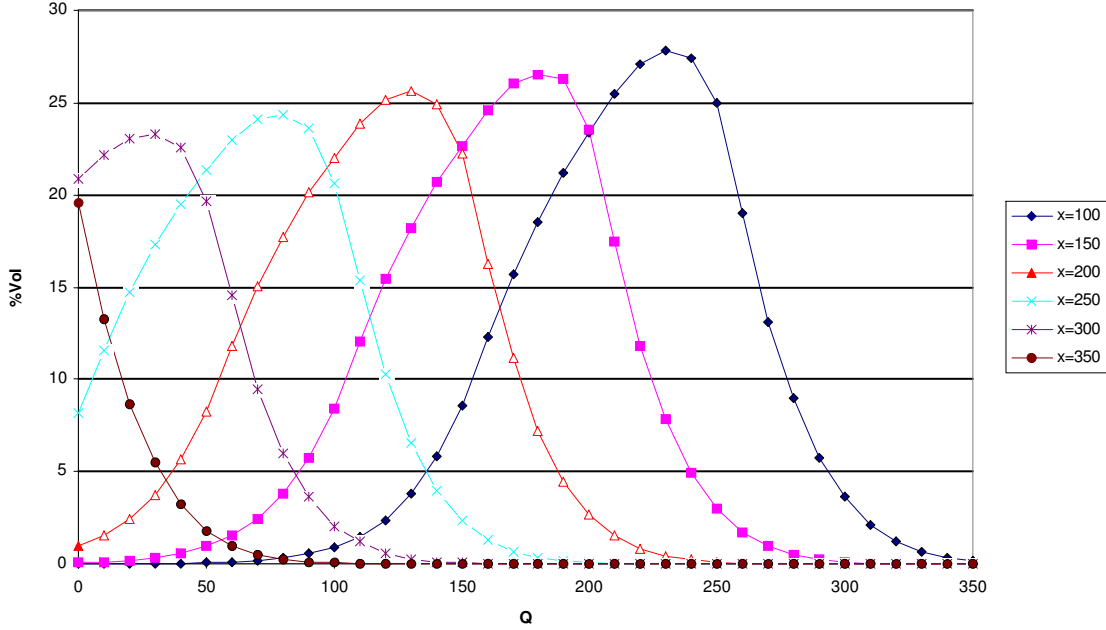
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Figure 5: $\%VoI$ versus Order Quantity for $x \geq d_1$

that when $x \geq d_1$ (where $d_1 = d_1^2$ in our current setting), then the individual values of $x - d_1$ and Q are irrelevant as long as their sum remains unchanged. Because, $x + Q - d_1$ is the lower bound on y, that is $x + Q - d_1$ is the minimum amount that will be available at the beginning of the second period even if all d_1 is met. Therefore, the absolute value of information is the same[†] for the same $x + Q - d_1$ values, and $\%VoI$ changes only due to relativity (since the system cost increases for higher x). Consequently, when $x \geq d_1$, relatively higher $\%VoI$ values seem to be attained within a range of $x + Q - d_1$ that is not too high or too low compared to the expected class-1 demand of the second period plus some safety stock.

For the case where $x \leq d_1$, $\%VoI$ increases as x increases for all values of Q (which is in line with our initial results that has already been discussed). Relatively higher $\%VoI$ values in this case are attained within a range of Q that is not too high or too low compared to the expected class-1 demand of the second period plus some safety stock.

Observation 7. Combining these two cases, we conclude that relatively higher $\%VoI$ values are attained within a range of $Q + (x - d_1)^+$ that is not too high or too low compared to the expected class-1 demand of the second period plus some safety stock, under the ex-

---

[†]there are insignificant differences due to the simulation-based calculation of the expected costs
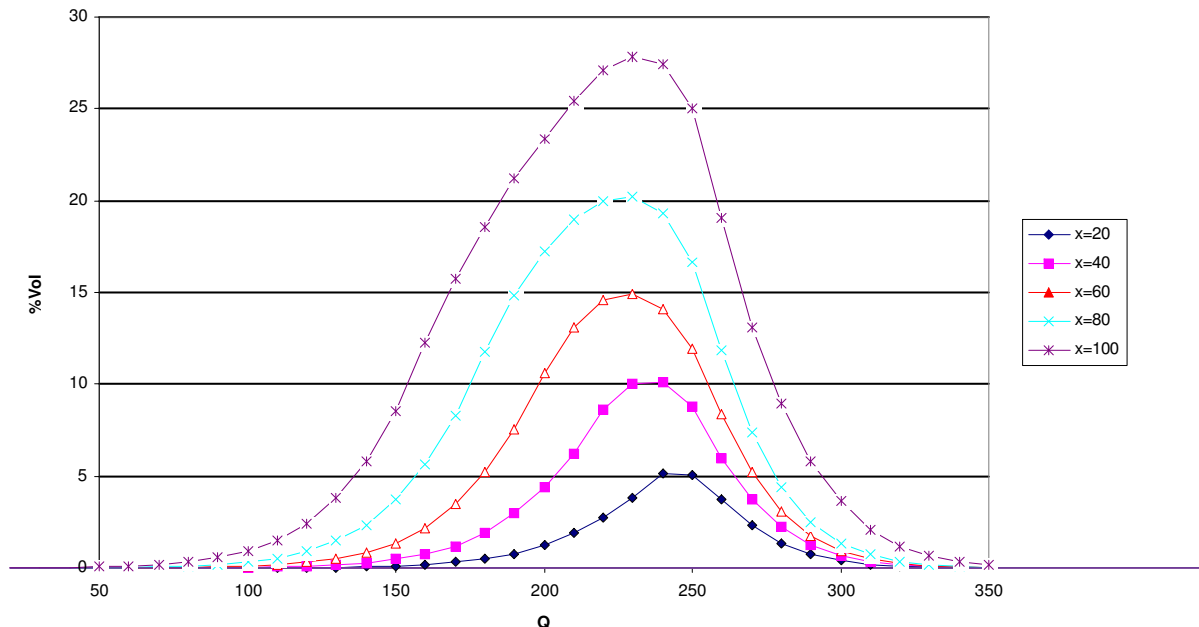
Figure 6: $\%VoI$ versus Order Quantity for $x \leq d_1$

perimental settings that we consider.

The Impact of CV and p on the $\%VoI$

We conduct further tests to investigate the behavior of $\%VoI$ as customer reliability level (p) and the coefficient of variation (CV) changes. We assume the same input parameters as in the first test for $\%VoI$ versus Q, with $b_1 = 12, b_2 = 5, h = 1$, except for p and CV. We let p vary between 0.1 and 1, and CV between 0.05 and 0.25. We also fix Q at 100. The results of these tests are summarized in Figure 7.

Figure 7 confirms our previous observations about p and CV in general:

Observation 8. ADI increases as the coefficient of variation, hence the demand variance, increases.

Observation 9. Other parameter levels are critical in any conclusion to draw about p, as discussed before. $\%VoI$ is zero for small values of p in Figure 7, because the current value of $Q = 100$ covers for expected class-1 demand in the second period (which is less than 40 for $p < 0.2$) and the optimal rationing policy is not to ration. On the other hand, the percent penalty of not utilizing ADI decreases for high values of p in that case, since the gap between
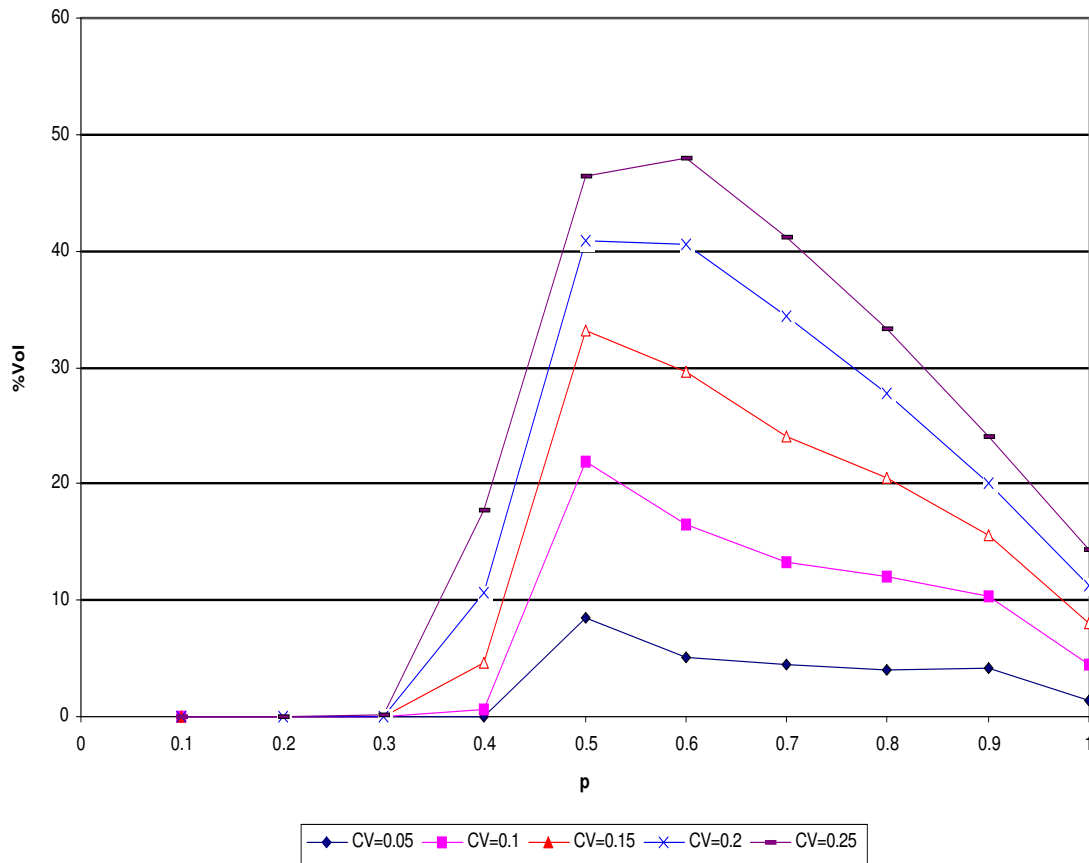
28

Figure 7: $\%VoI$ versus Customer Reliability Level and Coefficient of Variation

the expected class-1 demand of the second period and Q decreases the relative importance of $\%VoI$.

5. CONCLUSIONS AND FUTURE RESEARCH

The main motivation of employing imperfect advance demand information (ADI) in an inventory/production system in general is that it can improve the performance of the system through decreasing uncertainty on future demand. When there are multiple demand classes of different priorities, then the appropriate policy of rationing the available inventory, that is reserving some stock for meeting prospective future demand of preferred customers, comes at the expense of possibly losing some of the currently materialized demand of lower demand classes. This delicate issue, after all, relies heavily on the estimation of the future

demand, therefore utilizing current signals on future demand may be extremely important as to making better decisions on when to start rejecting current demand.

In this paper we have developed a model that helps us investigate this problem in a simplified environment. We analyzed a system that is made up of one ordering and one rationing decision under two demand classes. Consequently, the rationing problem is solved analytically. A Monte Carlo simulation-based procedure is developed to evaluate the total expected inventory-related costs. Showing that expected cost function is convex in terms of order quantity, an approach is suggested for determining the optimal order level. A procedure is built to evaluate the expected value of imperfect ADI on the rationing decision. Empirical tests are conducted for the normally distributed demand case to measure the impact of system parameters on the expected value of imperfect ADI. Under the parameters we considered, the results of these tests revealed that imperfect ADI is more valuable when the demand variance is high, relative importance of class-1 demand is high, there is sufficient class-2 demand at the first period and sufficient initial inventory to increase the flexibility to ration, and the order quantity plus the initial inventory in excess of the first period demand is close to the expected class-1 demand of the next period plus some safety stock. We show that rationing becomes very effective if ADI can be utilized.

This study presents some issues of relevance with respect to the design and operation of such systems. The following is a relevant list of issues for managerial insight:

a. Rationing is a difficult decision to apply in practice, as it may have an undesired influence on low-priority customer demand. On the other hand, if the return of such an action is significant then some incentives can be designed to prevent those undesirable influences. We show that, in the environment we have described -the rationing decision complemented by ADI-, the benefits can be sufficiently large (even though ADI is imperfect).

b. The rationing decision is generally considered to be an operational decision, however as shown in the computational analysis, especially if the system is operating with order quantities that are smaller than optimal, the penalty paid for not applying the optimal rationing policy can be significant. For order quantities that are much smaller than the optimal, the majority of the penalty is due to not rationing, whereas for the rest the majority of the penalty is due to not employing imperfect ADI in the rationing

decision. There are still considerable savings if rationing and imperfect ADI are used with the optimal ordering quantity.

c. Imperfect ADI and rationing are two important characteristics that will improve system performance where uncertainty, non-stationarity and long lead times are important features of the inventory system considered.

We note that although we have assumed no set-up cost in the analysis, our model can easily be extended to cover a positive set-up cost, as well. Nevertheless, extending the model into a longer or infinite horizon multi-period structure is not as straightforward. Under a general lead time assumption, increased dimensionality becomes an important issue in that case. A multi-period structure disallows considering the ordering and rationing decisions distinctly as we do in this paper, because optimality should be on both of those decisions in every period. A possible approach could be to pre-set a rationing policy as a function of ADI and then solve for Q, or vice versa. Another issue in the multi-period setting is the modeling of the ADI structure. While increasing the complexity of the problem, a possible solution or characterization could provide further insight. A natural extension is to handle the case of several customer classes, which may be necessary both for generalization purposes and for some possible applications. Instead of a single reserve level, there would be a reserve level for each class except for class-1 in this case, which is more difficult to handle analytically. A continuous review structure is another possible research direction. Rationing policies that are not only functions of imperfect ADI, but also of the remaining lead times could be foreseen as possible solutions in that case.

APPENDIX A.   PROOF OF LEMMA 1

We show the convexity of $E[TRC_2|k_2]$ in y by showing nonnegativity of its second derivative with respect to y for all k_2.

$$
\begin{aligned}
\frac{d\,E[TRC_2|k_2]}{d\,y} &= -b_1 \int_y^\infty dG_2^1(w|k_2) \\
&\quad + b_2 \left[\left(\int_{-\infty}^\infty D_2^2 dG_2^2(w|k_2) \right) g_2^1(y|k_2) - \int_{-\infty}^y \int_{y-w^1}^\infty dG_2^2(w^2|k_2) dG_2^1(w^1|k_2) \right] \\
&\quad - b_2 E[D_2^2|k_2] g_2^1(y|k_2) + h \int_{-\infty}^y dG_2(w|k_2) \\
&= -b_1(1 - G_2^1(y|k_2)) - b_2 Prob\{D_2^1 \leq y, D_2 \geq y|k_2\} + hG_2(y|k_2), \qquad (21)
\end{aligned}
$$

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where $g_2^1(y|k_2)$ is the density function of D_2^1 conditioned on k_2, evaluated at $D_2^1 = y$. Before deriving the second derivative, we note that-

$$
\begin{aligned}
G_2^1(y|k_2) &= Prob\{D_2^1 \leq y|k_2\} \\
&= Prob\{D_2^1 \leq y, D_2 \geq y|k_2\} + Prob\{D_2^1 \leq y, D_2 \leq y|k_2\} \quad (22) \\
&= Prob\{D_2^1 \leq y, D_2 \geq y|k_2\} + Prob\{D_2 \leq y|k_2\} \quad (23) \\
&= Prob\{D_2^1 \leq y, D_2 \geq y|k_2\} + G_2(y|k_2). \quad (24)
\end{aligned}
$$

Equation (23) follows (22) since all demands are nonnegative. The terms in (24) can be rearranged to yield

$$
Prob\{D_2^1 \leq y, D_2 \geq y|k_2\} = G_2^1(y|k_2) - G_2(y|k_2). \quad (25)
$$

Therefore, (21) can be further simplified as

$$
\begin{aligned}
\frac{d\, E[TRC_2|k_2]}{d\,y} &= -b_1(1 - G_2^1(y|k_2)) - b_2\left(G_2^1(y|k_2) - G_2(y|k_2)\right) + hG_2(y|k_2) \\
&= (b_1 - b_2)G_2^1(y|k_2) + (b_2 + h)G_2(y|k_2) - b_1. \quad (26)
\end{aligned}
$$

Then,

$$
\frac{d^2\, E[TRC_2|k_2]}{d\,y^2} = (b_1 - b_2)g_2^1(y|k_2) + (b_2 + h)g_2(y|k_2). \quad (27)
$$

(27) is nonnegative for all k_2, because $b_1 > b_2$, and $g_2^1(y|k_2)$ and $g_2(y|k_2)$ are density functions for all k_2.

The above result holds for R for any fixed $Q \geq 0$, since $y = Q + R$. This completes the proof.

APPENDIX B.   OPTIMAL RATIONING UNDER PERFECT ADI

Since we consider the perfect ADI on class-1 demand, d_2^1 is known with certainty prior to rationing decision. We can rewrite the total relevant cost function for the second period as

$$
TRC_2|k_2 = \begin{cases} b_1(d_2^1 - y) + b_2 D_2^2 & \text{if } y \leq d_2^1 \\ b_2[D_2^2 + d_2^1 - y]^+ + h[y - d_2^1 - D_2^2]^+ & \text{if } y > d_2^1. \end{cases} \quad (28)
$$

The expectation of (28) is

$$
E[TRC_2|k_2] = \begin{cases} b_1(d_2^1 - y) + b_2 E[D_2^2] & \text{if } y \leq d_2^1 \\ b_2 E[D_2^2 + d_2^1 - y]^+ + hE[y - d_2^1 - D_2^2]^+ & \text{if } y > d_2^1, \end{cases} \quad (29)
$$

whose derivative with respect to y is

$$\frac{dE[TRC_2|k_2]}{dy} = \begin{cases} -b_1 & \text{if } y \le d_2^1 \\ -b_2(1 - G_2^2(y - d_2^1|k_2)) + h(G_2^2(y - d_2^1|k_2)) & \text{if } y > d_2^1. \end{cases} \quad (30)$$

Hence, we have

$$\frac{dE[ETCC]}{dy} = \begin{cases} b_2 + h - b_1 & \text{if } y \le d_2^1 \\ (b_2 + h)(G_2^2(y - d_2^1|k_2)) + h & \text{if } y > d_2^1. \end{cases} \quad (31)$$

This function is negative for the region $y \le d_2^1$ since $b_1 > b_2 + h$ and positive for the region $y > d_2^1$ since G_2^2 is a distribution function. Consequently, $E[ETCC]$ is decreasing in y for $y \le d_2^1$, and increasing in y for $y > d_2^1$, which results in $y^* = d_2^1$.

<h3 style="text-align:center">APPENDIX C.   PROOF OF THEOREM 3</h3>

In order to prove Theorem 3, we first show that $ETCC = TRC_1 + E[TRC_2|k_2]$ is convex in Q for a given set of d_1^1, d_1^2, and k_2 due to the nonnegativity of its second derivative with respect to Q. We have already stated in (10) that

$$\begin{aligned} E[TRC_2|k_2] = \; & b_1 \int_y^\infty (w - y)dG_2^1(w|k_2) \\ & + b_2 \int_{-\infty}^y \int_{y-w^1}^\infty (w^1 + w^2 - y)dG_2^2(w^2|k_2)dG_2^1(w^1|k_2) \\ & + b_2 E[D_2^2|k_2] \int_y^\infty dG_2^1(w|k_2) + h \int_{-\infty}^y (y - w)dG_2(w|k_2). \end{aligned}$$

We replace y by $Q + R$ (for $R = R^*$ in case of optimal reserve policy) in (10) to yield

$$\begin{aligned} E[TRC_2|k_2] = \; & b_1 \int_{Q+R}^\infty (w - (Q + R))\, dG_2^1(w|k_2) \\ & + b_2 \int_{-\infty}^{Q+R} \int_{Q+R-w^1}^\infty \left(w^2 + w^1 - (Q + R)\right) dG_2^2(w^2|k_2)dG_2^1(w^1|k_2) \\ & + b_2 E[D_2^2|k_2] \int_{Q+R}^\infty dG_2^1(w|k_2) \\ & + h \int_{-\infty}^{Q+R} (Q + R - w)\, dG_2(w|k_2). \end{aligned} \quad (32)$$

Note that R is a function of Q, so the derivation of (32) requires derivation of R with respect to Q as well. (The derivative is given in (13) for $R = R^*$.) Let $R' = dR/dQ$. Then,

$$\begin{aligned} \frac{d\, E[TRC_2|k_2]}{d\, Q} = \; & -b_1 \int_{Q+R}^\infty (1 + R')dG_2^1(w|k_2) \\ & + b_2 \left[\left(\int_{-\infty}^\infty (1 + R')w dG_2^2(w|k_2) \right) g_2^1(Q + R|k_2) - \right. \\ & \left. \int_{-\infty}^{Q+R} \int_{Q+R-w^1}^\infty (1 + R')dG_2^2(w^2|k_2)dG_2^1(w^1|k_2) \right] \\ & - b_2 E[D_2^2|k_2](1 + R')g_2^1(Q + R|k_2) + h \int_{-\infty}^{Q+R} (1 + R')dG_2(w|k_2) \end{aligned}$$

<div style="text-align:center">33</div>

$$
\begin{aligned}
&= -b_1(1+R')(1-G_2^1(Q+R|k_2)) - b_2(1+R')Prob\{D_2^1 \le Q+R, D_2 \ge Q+R|k_2\} \\
&\quad + h(1+R')G_2(Q+R|k_2) \tag{33}
\end{aligned}
$$

$$
\begin{aligned}
&= -b_1(1+R')(1-G_2^1(Q+R|k_2)) - b_2(1+R')(G_2^1(Q+R|k_2) - G_2(Q+R|k_2)) \\
&\quad + h(1+R')G_2(Q+R|k_2) \tag{34}
\end{aligned}
$$

$$
= (1+R')\left[(b_1-b_2)G_2^1(Q+R|k_2) + (b_2+h)G_2(Q+R|k_2) - b_1\right]. \tag{35}
$$

Equation (33) translates into (34) due to (25). We also have

$$
d\,TRC_1/dQ = (b_2+h)R'.
$$

Therefore,

$$
\frac{d\,ETCC}{d\,Q} = (1+R')\left[(b_1-b_2)G_2^1(Q+R|k_2) + (b_2+h)G_2(Q+R|k_2) - b_1\right] + (b_2+h)R'.
$$

If $d^2R/dQ^2 = 0$, which is the case for $R = R^*$, the second derivative of $ETCC$ with respect to Q turns out to be

$$
\frac{d^2\,ETCC}{d\,Q^2} = (1+R')^2\left[(b_1-b_2)g_2^1(Q+R|k_2) + (b_2+h)g_2(Q+R|k_2)\right]. \tag{36}
$$

We note that (36) is nonnegative for all k_2, because $(1+R')^2 \ge 0$, $b_1 > b_2$, and $g_2^1(Q+R|k_2)$ and $g_2(Q+R|k_2)$ are density functions for all k_2. Consequently, $ETCC$ is convex in Q for a given set of d_1^1, d_1^2, and k_2.

Finally, we note that $E[TRC(Q)] = E_{D_1^1,D_1^2,K}[ETCC]$. Consequently, since expectations can be written as the limits of Riemann-Stieltjes sums, and the positive-weighted sum of convex functions are convex -see, e.g. Heyman and Sobel (1984)-, we conclude that $E[TRC(Q)]$ is convex over Q as well.

APPENDIX D. ALGORITHM FOR VOI ON RATIONING DECISION

We first note that, for normal distribution

$$
1 - G_2^1(y|k_2) = 1 - \Phi\left(\frac{y - k_2 p}{\sqrt{k_2 p(1-p)}}\right) \tag{37}
$$

and

$$
G_2(y|k_2) = \Phi\left(\frac{y - (k_2 p + \mu_2)}{\sqrt{k_2 p(1-p) + \sigma_2^2}}\right), \tag{38}
$$

where Φ is the standard normal distribution function, which cannot be evaluated explicitly. However, there exists some accurate approximations to evaluate Φ, which can be applied to (37) and (38). We use the approximation that is developed by Waissi and Rossin (1996) in our empirical tests. Now we present our algorithm.

Step 0   Input: $x, d_1^1, d_1^2, Q, b_1, b_2, h, p, \mu_K, \mu_2, \sigma_K, \sigma_2$

              Set $i = 0$, $TC1_{ADI} = 0$, $TC2_{ADI} = 0$, $TC2_{NoADI} = 0$

Step 1   − Calculate y^*_{NoADI} and R^*_{NoADI} by using Theorem 2 with demand distributions as stated in (19) and (20)

             − Calculate $(TRC_1)_{NoADI}$ by using (5) with $R = R^*_{NoADI}$

Step 2   Increment i by 1

Step 3   Generate a k_2 realization from $N(\mu_K, \sigma_K^2)$ distribution

              If $k_2 \leq 0$ then set $k_2 = 0$

Step 4   − If $k_2 = 0$ then $y^*_{ADI} = 0$ and $R^*_{ADI} = (x - d_1)^+$. Else, calculate y^*_{ADI} and R^*_{ADI} by using Theorem 2 with demand distributions as stated in (37) and (38)

             − Calculate $(TRC_1)_{ADI}$ by using (5) with $R = R^*_{ADI}$

             − Calculate $E[TRC_2|k_2]_{ADI}$ from (10) with $y = y^*_{ADI}$

             − Calculate $E[TRC_2|k_2]_{NoADI}$ from (10) with $y = y^*_{NoADI}$

Step 5   Set $TC1_{ADI} = TC1_{ADI} + (TRC_1)_{ADI}$,

              $TC2_{ADI} = TC2_{ADI} + E[TRC_2|k_2]_{ADI}$,

              $TC2_{NoADI} = TC2_{NoADI} + E[TRC_2|k_2]_{NoADI}$,

Step 6   If $i \leq NUM1$ (a large enough number) go to Step 2

Step 7   Set $E[TRC_1|d_1^1, d_1^2]_{ADI} = TC1_{ADI}/NUM1$,

              $E[TRC_2|d_1^1, d_1^2]_{ADI} = TC2_{ADI}/NUM1$,

              $E[TRC_2|d_1^1, d_1^2]_{NoADI} = TC2_{NoADI}/NUM1$,

Step 8   Set $E[TRC|d_1^1, d_1^2]_{ADI} = E[TRC_1|d_1^1, d_1^2]_{ADI} + E[TRC_2|d_1^1, d_1^2]_{ADI}$,

              $E[TRC|d_1^1, d_1^2]_{NoADI} = (TRC_1)_{NoADI} + E[TRC_2|d_1^1, d_1^2]_{NoADI}$

35

Step 9   Output: $VoI = E[TRC|d_1^1, d_1^2]_{NoADI} - E[TRC|d_1^1, d_1^2]_{ADI}$,

$\quad\quad \%VoI = 100 * VoI/E[TRC|d_1^1, d_1^2]_{ADI}$,

Some of the calculations in the above algorithm are conducted by making use of simplifications and the above-mentioned approximation for Normal distribution. Although $dETCC/dy$ can be evaluated accordingly for a given y, it remains to find the value of y^* that solves (12). But due to Theorem 1, $dETCC/dy$ is a monotone non-decreasing function in y. It can be shown that the absolute value of a monotone function is quasiconvex on the region that it is defined. Therefore, $|dETCC/dy|$ is quasiconvex in y and it is possible to apply golden section method (or another appropriate search method) to find the value of y that minimizes $|dETCC/dy|$. But $|dETCC/dy|$ is minimized when $dETCC/dy = 0$ (which is shown to exist in Section 2), hence the minimizer of $|dETCC/dy|$ solves (12).

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