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A Comment on the Radiation Resistance

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First, I would like to thank Dr. John D. Mahony for his valuable comments regarding Q-integrals representation, introduced by Equation (1) in my paper [1]. I also would like to thank him for the observed misprints in the paper: a multiplier $(1/2)$ was missing in Equations (7) and (8), and a superscript must be changed from (1) to (-1) in the last term of Equation (13).

Second, by using the following simple recurrence for the Bessel functions,

$$\frac{2J_1(x)}{x} = J_0(x) + J_2(x),$$

one can obtain an alternative expression for the auxiliary function $T(\xi)$ involved in Equation (16) for the radiation resistance, $R_{r1}(\xi)$, in the case of a sinusoidal excitation of the circular-loop antenna:

$$T(\xi) \approx \frac{1}{2}\left[Q_{00}^{(1)}(\xi) + Q_{22}^{(1)}(\xi) - \frac{2}{\xi^2}Q_{11}^{(1)}(\xi) \right]$$

instead of Equation (15). This expression has the advantage of being in terms of only three $Q_{nn}^{(1)}(\xi)$ functions (with $m = n$). For the particular case of $n = 1$, an explicit expression in terms of Bessel functions was given by Equation (8) in the paper (for the case of a constant excitation). In the very recent paper of Mahony in the same column [2], he offered a generalization of this representation for the case of an arbitrary n in his Equation (4):

$$Q_{nn}^{(1)}(\xi) = \frac{1}{\xi}\sum_{k=n}^{\infty} J_{2k+1}(2\xi),$$

which transforms the equation given above for the $T(\xi)$ function into the following final equation:

$$T(\xi) \approx \frac{1}{2\xi}\left[J_1(2\xi) + \left(1 - \frac{2}{\xi^2}\right)J_3(2\xi) \right.$$

$$\left. +2\left(1 - \frac{1}{\xi^2}\right)\sum_{k=2}^{\infty} J_{2k+1}(2\xi) \right]. \tag{15a}$$

I have performed calculations using both expressions: the old one, Equation (15), and the new one, Equation (15a). They gave very similar results in the interval $(0 < \xi < 15)$, with a relative difference less than 0.2%, so theoretically they are both equivalent. They are both represented by series expressions: Equation (15) is a power series, while Equation (15a) is a Bessel series. However, the number of the terms, N, involved in Equation (15) increases rapidly with an increase of the argument, ξ – particularly because of the gamma functions involved in the calculation of its coefficients, Equations (3) and (5). Typically, for the worst case here ($\xi = 15$), $N = 40$ terms were sufficient. The number of the terms employed in Equation (15a) also increases with an increase of ξ, but not so rapidly. For the same case, here $N = 20$ terms were sufficient. Because of the standard *MATLAB* functions used for the calculation of gamma functions and Bessel functions, the computational time was relatively small for both cases: 0.92 sec for the first case, and 0.72 sec for the second case (150 values of the radiation resistance were computed and plotted).

Conclusion

I have to admit that the second expression for the radiation resistance, Equation (15a), developed with the help of Mahony's comments, has some advantages, being more concise analytically and having more stable and rapidly convergent behavior numerically.

References

1. Sava V. Savov, "An Efficient Solution of a Class of Integrals Arising in Antenna Theory," *IEEE Antennas and Propagation Magazine*, **44**, 5, October 2002, pp. 98-101.

2. John D. Mahony, "Circular Microstrip-Patch Directivity Revisited: An Easily Computable Exact Expression," *IEEE Antennas and Propagation Magazine*, **45**, 1, February 2003, pp. 120-122.