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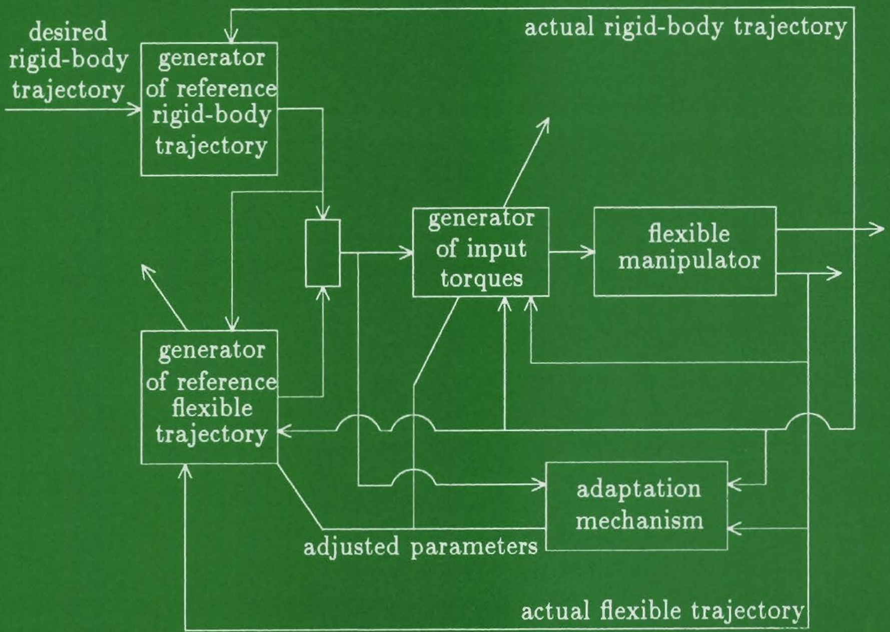

Ivonne M.M. Lammerts

ADAPTIVE COMPUTED REFERENCE COMPUTED TORQUE CONTROL OF FLEXIBLE MANIPULATORS

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Adaptive Computed Reference Computed Torque Control of Flexible Manipulators

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Contents

Chapter 1

Introduction.

1.1 Control of flexible manipulators with unknown parameters.

This thesis is concerned with the motion control of flexible manipulators with unknown parameters. *Manipulators*[1], whether rigid or flexible, are programmable to perform a variety of tasks. These tasks are defined in terms of motions of the end-effector or forces between the end-effector and the environment. In this thesis, we concentrate on tasks that a desired trajectory for the end-effector is known for. Examples of specific tasks are paint spraying, packing, arc welding and parts assembling.

The main feature that distinguishes a manipulator from a numerically *steered* servomechanism is the use of *feedback* control to perform accurate and fast movements and to reduce the sensitivity to disturbances from the environment. This requires the introduction of sensors to collect appropriate information on the actual manipulator performance. However, controlling a manipulator means controlling a multi-input multi-output system with highly nonlinear, strongly coupled dynamics.

An important design objective of current industrial manipulators is to achieve maximal stiffness of the various parts in order to avoid significant deformations and vibrations, and, thus, to improve the obtainable positioning and tracking accuracy. As a consequence, these manipulators are quite heavy and unwieldy. Increasing demands on the accelerations, on the precision and on the energy consumption require lightweight manipulators. However, significant deformations can occur in manipulators of this kind and it turns out that they cannot be accurately controlled by conventional controllers. Therefore, more sophisticated control techniques are required.

A common problem in the control of both rigid and flexible manipulators is that the structure of the describing equations is often known, whereas only imprecise knowledge of the parameters in these equations is available. This so-called *parametric uncertainty* may be caused by an unknown load at the end-effector, poorly known inertias, uncertain and slowly time-varying friction parameters, etc.

The overall objective in this thesis is to design a manipulator control scheme to

[1]The term 'manipulators' denotes both manipulators and robots.

cope with both flexibility and parametric uncertainty. In the next subsection, the control objective of the trajectory tracking of a manipulator will be discussed. Subsection 1.3 will follow with a global sketch of the specific problems in controlling flexible manipulators. Two kinds of flexibility are distinguished: the distributed flexibility in links and, more or less concentrated, flexibility of drives in joints (such as in the transmissions). In general, both link and joint flexibility occur. In literature, joint flexibility is often referred to as joint elasticity. Both types of flexibility will be considered in this thesis. First, the problem of modeling and control of manipulators with known parameters will be discussed and attention will be given to the control of *rigid* manipulators (Subsection 1.4.1), to the control of manipulators with *flexible joints* and rigid links (Subsection 1.4.2), to the control of manipulators with rigid joints and *flexible links* (Subsection 1.4.3) and to the control of manipulators with *flexible joints and flexible links* (Subsection 1.4.4). Next, the problem of *parametric uncertainty* will be considered. This problem will be overcome by an adaptive technique (for a brief introduction to adaptive control, see Appendix B). After the introduction to adaptive manipulator control in Subsection 1.5.1, *convergence*, *stability* and *robustness* will be briefly discussed in Subsection 1.5.2. Subsequently, a review on adaptive control concepts will be given for *rigid* manipulators (Subsection 1.5.3), for manipulators with *flexible joints* and rigid links (Subsection 1.5.4), and for manipulators with rigid joints and *flexible links* (Subsection 1.5.5). No concept for the adaptive control of manipulators with both flexible joints and flexible links and with unknown parameters was found in literature. The conclusion may be that the control of flexible manipulators with unknown parameters is still a challenge.

1.2 The tracking control objective.

In general, a *manipulator* is a multi-degree-of-freedom mechanism with a tree structure of rigid and/or flexible links connected by prismatic and/or revolute joints[2] A joint can be either rigid or flexible. A possible flexibility is elasticity of the drive, e.g. in the gear mechanism. The base link of the tree structure is usually fixed to the ground, whereas the *end-effector* is attached to the last link. This end-effector has to transport an object along a prescribed trajectory. The control task to realize this is referred to as motion control or *tracking control.* For the moment, a rigid manipulator is considered. To realize tracking control, the desired motion of each link has to be determined. For a given non-redundant rigid manipulator, the desired end-effector path can be translated in desired paths for the joint coordinates, which may each represent an absolute or a relative translation or rotation of the joint (due to the absolute motion of the actuated link and the relative motion of the two adjacent links, respectively). Because this motion is caused by a motor that drives the corresponding joint, the tracking control problem is to determine suitable actuator torques and/or forces, such that the manipulator follows the desired trajectory as well as possible with a speed large enough for 'gross motion' to perform the task within an acceptable time and small enough for 'fine motion' to avoid unacceptable overshoot,

[2]A prismatic joint (sliding joint) allows a translation of one link with respect to another link, whereas a revolute joint allows a relative rotation of the connected links.

for instance, to avoid collisions. The controller has to realize this objective in such a way that the controlled manipulator is fairly insensitive to external disturbances and to variations and uncertainties in the dynamic characteristics of the mechanism. This requires the use of feedback and, therefore, the use of suitable sensors to provide the necessary information on the actual manipulator performance. The next subsection will give a global sketch of the special problem of controlling flexible manipulators with unknown parameters.

1.3 A global sketch of the problem.

Precise control of fast manipulators in a wide range of configurations calls for accurate knowledge of the relevant dynamics. This problem has been extensively studied in literature, often under the crucial assumption that the manipulator can be modeled as a rigid-body system and that the dynamics are completely known. In reality, manipulators usually operate with varying and unknown loads. Besides, often only approximate values for the model parameters like masses, moments of inertia, friction constants, etc. are available. This uncertainty is referred to as *parametric uncertainty*. Furthermore, actual manipulators usually show considerable (elastic) deformations and vibrations if they are forced to follow a trajectory rather quickly. Unmodeled flexibility and parametric uncertainty have a negative effect on the tracking performance and can even lead to instability. Hence, for successful control of manipulators both parametric uncertainty and flexibility should be taken into account simultaneously, such that a stable closed-loop performance is guaranteed (for a brief discussion on stability of a control system, see Appendix A).

1.4 Control without parametric uncertainty.

1.4.1 Control of rigid manipulators.

Introduction.

Most of today's manipulators are designed for mechanical stiffness because of the difficulty to control flexibility and not because rigidity itself is inherently attractive. As a consequence, these manipulators are heavy, which results in a low payload to weight ratio. This not only limits the accelerations, but also requires large driving torques and, hence, this increases the size of the actuators needed. The modeling of rigid manipulators will be briefly discussed in Subsection 2.5.2.

Until now, practical manipulator control has been based on the assumption that all links and joints are rigid bodies and that the number of degrees of freedom equals the number of actuator inputs. Also, the majority of the manipulators nowadays use sensors that are located at the actuators. For these manipulators, the equivalent desired trajectories for all joints can be determined via inverse kinematics and *joint-level control* can be applied. If the manipulator is stiff enough and it moves slowly, the interactions between the degrees of freedom are small and each actuator-link combination can be regarded as a single-input single-output system. Then, classical control methods can be applied to each of these systems.

Position control of rigid manipulators.

Joint-level control for rigid manipulators has been widely investigated (e.g. Asada and Slotine, 1986; Spong and Vidyasagar, 1989; Slotine and Li, 1991). Manipulators are actuated by electric or hydraulic motors. Commonly used are *indirect-drives*, where the actuator is coupled to the driven link via a transmission, e.g. a gear mechanism, with a large transmission ratio. This transmission increases the driving torque at the link and decreases the dynamic coupling with the other links. The latter implies that the problem of controlling a slow, indirect-driven manipulator can be overcome with decoupled joint control. Several control schemes currently applied rely on this assumption. Among them, the simple *PD-* and *PID*-methods (e.g. Arimoto and Miyazaki, 1983; Kuo, 1987) are best known. These schemes have originally been designed to stabilize the controlled manipulator around a set-point or operating point. This is of interest for many industrial operations, e.g. pick-and-place tasks, simple spot welding and assembly tasks. This control problem is referred to as *position, regulation* or *set-point control.* Although PD-control assumes a certain type of second-order behavior, it is not based on a manipulator model explicitly. Therefore, PD-controllers are simple to implement, also because the control parameters are relatively easy to tune. Because PD-control is relatively robust for disturbances, PD-control is still most commonly used for industrial applications. PD-control will be briefly discussed in Section 3.3.

Tracking control of rigid manipulators.

Many manipulator tasks require tracking of a trajectory, i.e. a continuous function of time, instead of a discrete set of set-points. *Trajectory tracking* is needed in applications such as arc welding and paint spraying, where both the position and the velocity of the end-effector have to be controlled at all points on the specified trajectory. Because the nonlinear character of manipulators becomes more and more significant for high speed trajectory tracking, linear control approaches such as PD are not satisfactory anymore. Hence, a model-based controller has to be designed that minimizes both the position error and the velocity error in all points of the trajectory. This *motion control* or *tracking control* is the main subject of this thesis.

Many improved controllers proposed in literature are based explicitly on a *mathematical model* of the manipulator dynamics. Such a model consists of a set of second-order differential equations for the generalized coordinates as a function of time in response to the actuator torques[3]. In general, these *equations of motion* are highly nonlinear and strongly coupled, which complicates control design. Originally, this complication was surmounted by the use of conventional, linear control methods, based on a *linearized model.* However, this results in poor performance because of the neglected nonlinear dynamics. Other control methods have been proposed, based on a model that is otherwise *simplified*, e.g. by neglecting nonlinear terms due to Coriolis

[3]The word 'torques' is used in a general sense and denotes both torques and forces.

and centrifugal accelerations. These velocity dependent terms may be insignificant in low speed motions, but they have to be taken into account in fast control tasks.

Model-based control of rigid manipulators.

For higher operational speeds, a recent tendency in industry is to drive the links directly by high-torque motors. For these *direct-driven manipulators*, however, the disturbing effects of highly nonlinear and strongly coupled manipulator dynamics are not reduced by large transmission ratios. Hence, especially for high speed motions, these disturbance effects affect the tracking performance negatively, more than for indirect-driven manipulators. This requires modern, nonlinear control concepts such as optimal control (e.g. Kahn and Roth, 1971), robust control (e.g. Sastry and Slotine, 1983; Slotine, 1985; Spong and Vidyasagar, 1985; Asada and Slotine, 1986), computed torque control (e.g. Luh et al., 1980; Craig, 1986), and adaptive control (e.g. Landau, 1985, 1988; Hsia, 1986; Middleton and Goodwin, 1986; Ortega and Spong, 1988; Slotine and Li, 1986, 1988, 1989).

Although the idea of *model-based control* (see Fig. 1.1) is of major importance in this thesis, it is not our purpose to extensively discuss all proposed control concepts (e.g. Craig, 1986; Asada and Slotine, 1986; Spong and Vidyasagar, 1989; Heeren, 1989; De Jager et al. 1991, Slotine and Li, 1991; De Jager, 1992c, Berghuis, 1993). *Computed torque control* (CTC) is perhaps the best known control concept for rigid

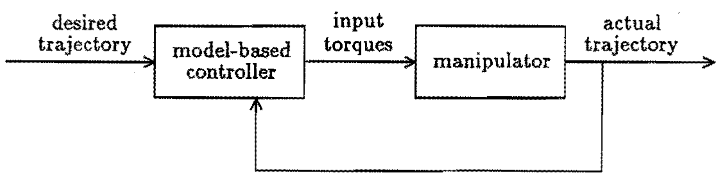


Figure 1.1: Model-based control of a manipulator.

manipulators (e.g. Luh et al. 1980; Craig, 1986; Asada and Slotine, 1986). It will be briefly discussed in Section 3.4. Computed torque control — sometimes called inverse dynamics control or static nonlinear state feedback control — is based explicitly on a model of the rigid manipulator. Via inverse dynamics, it compensates the nonlinear dynamic terms in the model and decouples the interactions between the degrees of freedom. In this way, the nonlinear and coupled equations of motion are transformed into a set of linear, decoupled second-order differential equations that relate the tracking error to a new input. This implies that the tracking error can be controlled by this new input, which is commonly based on classical linear control theory. A CTC scheme usually consists of a compensation part for the nonlinear dynamic terms (i.e. a kind of feedforward control) plus an "internal" or "external" PD- or PID-term (see Subsections 3.4.2 and 3.4.3, respectively). If the model matches the actual manipulator dynamics exactly, the CTC method theoretically guarantees globally, asymptotically stable trajectory tracking. However, in practice the tracking performance substantially decreases if the model structure is imperfect or if the parameters are unknown or time-varying. This sensitivity to model uncertainty can even lead to instability. One of the robust control concepts that account for model uncertainty is the concept of *sliding control*, which will be discussed in Section 3.5. The concept of *adaptive control* is a potential solution to the control of manipulators in the case of parametric uncertainty (for an introduction, see Subsection 1.5 and Appendix B). In Section 3.6, special attention will be given to an extension of the CTC scheme and to an adaptive version in which the unknown model parameters are adjusted on-line (e.g. Slotine and Li, 1986). In this thesis, the (adaptive) version of the CTC scheme will be the starting point for the design of an (adaptive) controller for flexible manipulators.

1.4.2 Control of manipulators with flexible joints and rigid links.

Introduction.

Control of flexible manipulators is presently still an open problem. Mostly, only flexibility at revolute joints is considered, which is acceptable for the majority of the manipulators nowadays, where the effects of joint flexibility dominate the effects of link flexibility. To illustrate this, it is noted that many manipulators are driven by harmonic drives, i.e. by DC or AC motors via geared transmissions with large reduction ratios. Drives of this kind are frequently used for their low weight, their compactness and for their ability to transmit large torques with almost negligible backlash. However, flexibility at the motor side of the drive can have a dramatic influence on the behavior of the manipulator.

For a large class of manipulators, it has been established experimentally (e.g. Sweet and Good, 1984; Rivin, 1985) and theoretically (e.g. Widmann and Ahmad, 1987) that joint flexibility has a significant influence on the tracking performance. Joint (or, more precisely, drive) flexibility arise from several sources, such as deformations in gears, belts, shafts, hydraulic lines, etc. It has been shown that control laws that neglect this kind of flexibility lead to degradation of the tracking accuracy and that they may be unstable when applied to real manipulators. Furthermore, control laws based on rigid manipulator models have to be limited in their bandwidth (and, hence, in the achievable speed) because of the vibrations at the joints. For this reason, joint flexibility cannot be neglected in control design for fast manipulators.

If joint flexibility is taken into account in the model, the structure and the number of the equations of motion alter. Joint flexibility can often be modeled by a linear, torsional spring at each joint (see Fig. 1.2). The flexible model is slightly more complex than the rigid model, e.g. it has more degrees of freedom. The derivation of a general model for manipulators with flexible joints will be given in Subsection 2.5.3. If all joints are flexible and each flexibility is modeled with one extra degree of freedom, the number of degrees of freedom becomes twice that of the equivalent rigid

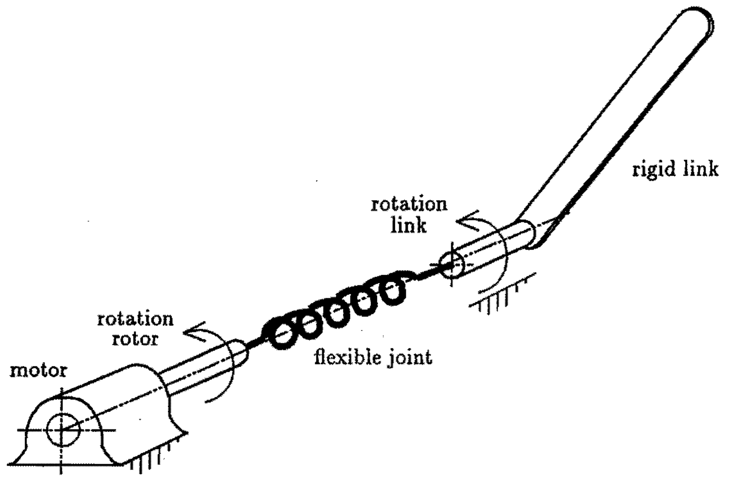


Figure 1.2: A schematic representation of a flexible joint.

manipulator while the number of control inputs is still the same. As a consequence, control of manipulators with flexible joints is considerably more complicated. However, this kind of flexibility is much easier to tackle than distributed link flexibility. Because link flexibility can be modeled approximately by (a chain of) rigid sublinks connected by flexible joints, incorporation of joint flexibility may be seen as a first step to take distributed link flexibility into account.

Control of flexible-joint manipulators.

Although, yet, no general control methods are available for flexible manipulators, some successful control strategies for the special class of flexible-joint manipulators have been developed (e.g. Marino and Spong, 1986; Hung et al., 1989; Spong, 1987, 1990). These strategies are usually based on the assumption that joint flexibility can be modeled by a torsional spring between each actuator and the actuated, rigid link. Then, each flexible joint leads to two second-order differential equations in the model of the manipulator. This results in a complex model for a multi-link flexible-joint manipulator. Known results on the control of flexible-joint manipulators usually rely either on a simplified model (e.g. Spong, 1987) or on special configurations (e.g. Marino and Spong, 1986). Analytical and experimental studies indicate that there are two important concepts to control flexible-joint manipulators (e.g. Hung et al., 1989; Marino and Spong, 1986; Spong, 1987, 1990). For manipulators with relatively flexible joints, the *input-output linearization* approach yields good results because it explicitly incorporates joint flexibility in the control design. For manipulators with sufficiently large joint stiffnesses (i.e. weak joint flexibility), the *singular perturbation* approach gives good performance and is easier to implement.

Input-output linearization control.

Input-output linearization (in literature often referred to as *feedback linearization*) (Spong, 1987; Spong and Vidyasagar, 1989; Nijmeijer and Van der Schaft, 1990; Slotine and Li, 1991) can be seen as a generalization of the classical CTC method for rigid manipulators (for a general block scheme, see Fig. 1.3). In the ideal case,

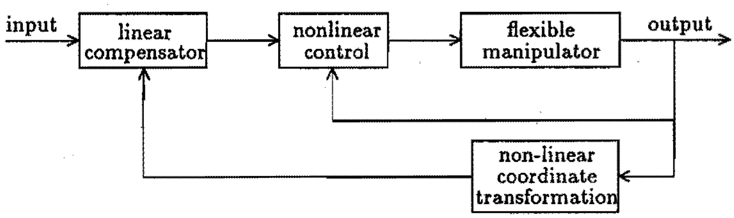


Figure 1.3: Input-output linearization control.

input-output linearization of a nonlinear system results in a decoupled, linear system that can be easily controlled. Although much attention has recently been given to this approach, its application to flexible manipulators is still an open problem.

A number of input-output linearization schemes for manipulators with flexible joints has been proposed. Marino and Spong (1986) showed that a single-link manipulator with one flexible joint is linearizable. Cesareo and Marino (1984) have investigated the more general class of flexible-joint manipulators and showed that linearization is not always possible. Spong (1987) has also studied manipulators with flexible joints and he suggested a *simplified model* that is always linearizable. This model can be viewed as an n-degrees-of-freedom version of the single-link manipulator model of Marino and Spong (1986). However, it is simplified in comparison with other models for flexible-joint manipulators used in literature (Cesareo and Marino, 1984; Khorasani and Spong, 1985; Marino and Nicosia, 1985; Spong et al., 1987). In the model of Spong, there is no inertia coupling between the 'rigid' coordinates (i.e. the link coordinates) and the 'flexible' coordinates (i.e. the coordinates of the motor rotors). This model is quite often used (e.g. Nicosia and Tomei, 1991; Spong and Vidyasagar, 1989; Ghorbel et al., 1989) and it is said to be justified for a large class of manipulators where, roughly speaking, the diagonal terms in the inertia matrix are dominant to the coupling terms. However, it depends on the motion speed and on the trajectory to be followed whether the coupling terms can be neglected (Nicosia and Tomei, 1991). For some special manipulators, these terms are exactly zero, for example, for a one-link manipulator with one revolute joint and for a two-link manipulator with revolute joints and perpendicular axes of rotation.

Spong and Vidyasagar (1989) showed that the input-output linearization approach is identical to the classical CTC method for multi-link rigid manipulators. They have considered multi-link flexible-joint manipulators modeled according to Spong (1987) and they have discussed the design of an appropriate input-output linearization control law. Implementation of an input-output linearization control scheme requires that the state variables are available for feedback. Positions, velocities and/or accelerations can be measured. Some schemes also use the jerks, but measurement of jerks is not possible. Various proposals for observers to estimate the state variables can be found, but problems arise because the theory of nonlinear, robust observers is not sufficiently developed yet. This is also the problem in the design of observers for the estimation of eventually unknown parameters. Furthermore, to our knowledge, there is yet no robust or adaptive control version of the input-output linearization scheme for flexible manipulators, so that robustness for parametric uncertainty and unmodeled dynamics cannot be guaranteed. Another disadvantage of input-output linearization schemes is that they turn out to be computationally quite laborious for multi-link flexible-joint manipulators. This is the case even for a single-link manipulator with one flexible joint (e.g. Spong, 1987). Hence, implementation problems are likely to arise.

Composite control based on singular perturbation.

A well-known alternative for input-output linearization is the composite control method based on the singular perturbation theory (e.g. Kokotovic, 1984; Kokotovic and Khalil, 1986; Slotine and Hong, 1986; Hung et al., 1989; Spong, 1987, 1990). The last few years, the composite control approach has been strongly developed for manipulators with rather stiff joints (i.e. weak joint flexibility). Basically, it reflects the intuitive idea to use a *composite control* scheme that consists of a part based on the rigid manipulator model plus a part with the objective to damp the flexible vibrations (i.e. the high-frequency dynamics). Also in this approach, flexibility at a joint is commonly modeled by a flexible spring between the actuator and the driven link. It can be shown (e.g. Marino and Spong, 1986) that in the case of weak joint flexibility the model can be formulated in a singular perturbation form, with a small perturbation parameter being the inverse of the smallest joint stiffness. Then, according to the *singular perturbation theory* (e.g. Kokotovic, 1984; Kokotovic and Khalil, 1986), the model can be decomposed in a "slow" (i.e. low frequency) submodel that represents the equivalent rigid manipulator dynamics, and a "fast" (i.e. high-frequency) linear submodel that represents the flexible dynamics. The state variables of the slow submodel usually are the link positions and velocities. They are assumed to be constant in the fast submodel. The state variables of the fast submodel usually are the flexible-joint torques and their time-derivatives. They are assumed to change on a much smaller time scale than the 'slow' variables. So, this approach relies on a two-time scale separation of the flexible model. A composite control design on the basis of these two submodels involves two steps: firstly, a 'slow', nonlinear feedback control law is designed to solve the tracking control problem for the slow submodel; secondly, a 'fast', linear feedback control term is added to stabilize the fast submodel along an equilibrium trajectory that depends on the slow control (see Fig. 1.4). This means

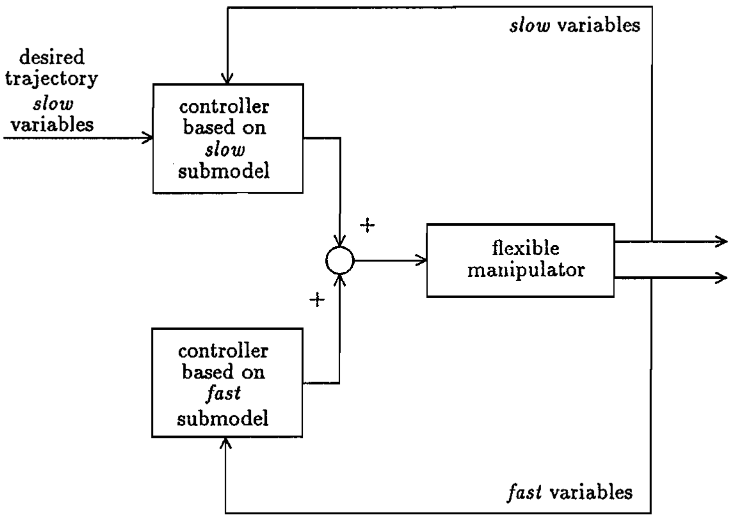


Figure 1.4: Composite control based on singular perturbation.

that the flexible-joint torques are controlled such that they converge asymptotically to the internal torques for the rigid model. Under the assumption of uniform stabilizability of the fast subsystem, Tikhonov's theorem (Kokotovic, 1984) ensures that the closed-loop overall system will track the desired trajectory asymptotically if the joint stiffnesses tend to infinity. If the perturbation parameter is not sufficiently small, the concept of *integral manifolds* (e.g. Sobolev, 1984; Khorasani and Spong, 1985) can be adopted to obtain a more accurate, slow submodel of the flexible-joint manipulator. This submodel represents the rigid manipulator dynamics plus the effects of joint-flexibility up to the first (or possibly higher) order. However, it remains of the same order as the equivalent rigid manipulator model. The main advantage of this strategy is that the slow controller can be based on any well-known control concept for rigid manipulators. Khorasani and Spong (1985) and Slotine and Hong (1986) have designed a sliding control law on the basis of the slow submodel derived by Marino and Nicosia (1985), in order to obtain robustness for parametric uncertainty. It is also possible to use adaptive CTC in the composite control strategy to account for both weak joint flexibility and parametric uncertainty (see Subsection 1.5.4).

Much research on singular perturbation control based on the concept of integral manifolds has been reported on (for instance, Khorasani and Spong, 1985; Spong et al., 1987; Marino and Spong, 1986; Spong, 1990, for a recent survey). However, the stability of the closed-loop system has rarely been addressed. Recently, a complete derivation of composite control with a rigorous stability analysis has been presented by Ghorbel (1991) and by Ghorbel and Spong (1991). They also gave an explicit expression for the range of joint stiffnesses that the closed-loop system is stable for. Their tools for control design are applied to the flexible-joint manipulator model given by Spong (1987). The singular perturbation approach has also led to some promising experimental results (e.g. Hung et al., 1989) and it appears to be more attractive for implementation than the input-output linearization approach. However, both approaches are hampered by the requirement that all positions, velocities, and often also accelerations and jerks must be available. The main drawback of the composite control approach based on singular perturbation is that it relies upon the time-scale separation of the flexible model. This limits its application to weakly flexible manipulators.

1.4.3 Control of manipulators with rigid joints and flexible links.

Introduction.

The drawbacks of a stiff (and therefore heavy) manipulator can, to some extent, be avoided by using lightweight links and drives. The result is a relatively large ratio of payload and manipulator weight. In comparison with a stiff, heavy manipulator, a lightweight manipulator can respond faster with reduced motor power. Other advantages of lightweight manipulators can be lower construction costs, safer operation and improved mobility (Book, 1985). However, the main drawback is that the links of lightweight manipulators usually deform significantly (see Fig. 1.5).

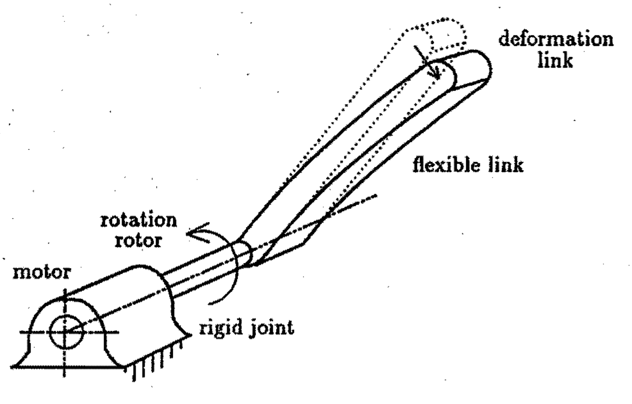


Figure 1.5: A schematic representation of a flexible link.

Modeling and control of flexible-link manipulators has recently got increasing attention (e.g. Book et al., 1975; Cetinkunt et al., 1986; Koppens, 1986; Schmitz, 1986; Cetinkunt and Book, 1987; Koppens et al., 1988; Asada et al., 1990; Chedmail et al., 1991; Denissen, 1992, for a literature study). It turns out that the design of controllers for these systems is very complicated, because the models are highly nonlinear with (far) more degrees of freedom than actuator inputs. In principle, an infinite number of degrees of freedom is needed to describe the motion of a flexible link (e.g. Book, 1985). With finite element methods (e.g. Zienkiewicz, 1979; Usoro et al., 1986) or boundary integral methods (e.g. Hurty and Rubinstein, 1964; Shabana, 1989) flexible links can be modeled approximately by finite dimensional systems. For control purposes, it is often necessary to decrease the number of degrees of freedom via a model reduction method, such as the Guyan reduction (e.g. Guyan, 1965). A procedure for the modeling of flexible-link manipulators is sketched In Subsection 2.5.4.

Control of flexible-link manipulators.

Control design for manipulators with flexible links is complex, because it has to cope with distributed flexibility, which results in a large number of coordinates to be controlled. During the last years, basic research has been devoted to this problem, often specialized for one-link manipulators (e.g. Cannon and Schmitz, 1984; Fukuda, 1985; Sakawa et al., 1985) and focused on suppressing the elastic vibrations via active control. Asada et al. (1990), for instance, applied a feedforward scheme that attempts to minimize the deformations of the links in order to reduce the tracking error of the end-effector. Manipulator control is usually performed without additional measurements of the deformations in the construction. However, for precise trajectory tracking of flexible-link manipulators, it seems obvious to perform measurements of the link deformations and/or the end-effector position. As an example, Fukuda (1985) and Sakawa et al. (1985) proposed controllers to suppress link vibrations that are measured with strain gauges. However, because strain gauges can detect only local deformations, this can lead to global tracking accuracy problems. In order to realize end-effector trajectory tracking as accurate as possible, the best thing to do is to measure the end-effector position, for instance optically (e.g. Harashima and Ueshiba, 1986; Heeren, 1989). Among other proposed control methods that use measurements of both the strains and the end-effector position and methods that rely on observers to estimate the link vibrations, the method proposed by Matsuno and Sakawa (1990) uses accelerometers at the end-effector.

Often, a linear model of the flexible manipulator is derived and a control concept is proposed for stabilizing the overall system (e.g. Book et al., 1975; Chalhoub and Ulsoy, 1987; Kruise, 1990). In the study of Book et al., two linear models of a two-link flexible manipulator have been derived: the first one by using a frequency-domain technique and the second one by linearization of the nonlinear model around a point at the desired trajectory. Several types of feedback control based only on joint angle measurements or based also on link deformation measurements have been investigated in the simulation studies of Book et al. In general, control concepts that rely on linear models of flexible manipulators are attractive by their simplicity, but they cannot cope with fast trajectory tracking because they have actually been designed for set-point control.

Another approach to the control of flexible-link manipulators is to use a primary control law, based on exact linearization of the corresponding rigid-link manipulator, and to add an extra full state feedback term, based on a linearized model and designed to realize both trajectory tracking and active damping of the elastic vibrations (e.g. Cannon and Schmitz, 1984; Hastings and Book, 1985; Chedmail et al., 1991). As an example, Cannon and Schmitz have designed a LQR (linear quadratic regulator) for the end-point control of a one-link flexible manipulator. Using feedback control of the motor velocity and optically measuring the end-effector position, they achieved good positioning results in experiments despite the link flexibility. However, the feasibility of their approach for multi-link flexible manipulators is questionable due to the required computation time. In fact, optimal controllers have been often found to be computationally burden even for controlling rigid manipulators.

Nonlinear control techniques based on inverse dynamics and stabilization techniques have been proposed e.g. by Singh and Schy (1986), DeLuca and Siciliano (1988) and Das and Singh (1989). Singh and Schy have considered nonlinear control of a two-link manipulator where only the second link (attached to the end-effector) is flexible. They split the set of equations of motion into a set describing the motion of the equivalent rigid-link manipulator, and a set describing the flexibility effects. The latter is linearized under the assumption that only small deviations from the rigid manipulator motion will occur. Finally, a nonlinear decoupling control law is defined for the first set of equations and a linear state feedback control term is added to stabilize the overall system. However, in order to damp the oscillations, extra torques that act at the end-effector had to be introduced.

The idea to consider the total motion of a flexible-link manipulator as the motion of the equivalent rigid-link manipulator plus a small deformation is, to some extent, related to the approach based on singular perturbation. If the deformations and the rigid motions are time separable, this theory can also (Subsection 1.4.2) be used to control manipulators with rigid joints and fairly stiff links (e.g. Kokotovic, 1984; Siciliano and Book, 1988; Siciliano et al., 1989). Usually, the state variables of the slow submodel are the joint positions and velocities, and the state variables of the fast submodel are the bending torques in the links and their time-derivatives. If the link stiffnesses are known, these torques and their derivatives can be determined from strain gauge measurements (e.g. Siciliano and Book, 1988). Two essential assumptions are made in this approach: 1) the perturbation parameter must be "sufficiently" small, i.e. the links have to be "sufficiently" stiff, and 2) the state variables of the slow submodel are assumed to be constant in the fast submodel.

1.4.4 Control of manipulators with flexible joints and flexible links.

Introduction.

The equations of motion for multi-link manipulators with both joint and link flexibility are extremely complex, and the number of actuators is much smaller than the number of degrees of freedom. Furthermore, the desired trajectory for the degrees of freedom is not uniquely determined by the desired trajectory of the end-effector. The control objective for flexible manipulators is to achieve trajectory tracking of the end-effector and to limit the vibrations as much as possible. As mentioned in Subsection 1.3, numerous studies in literature have focused on manipulators with either joint or link flexibility, and have treated single-link examples only. The combined problem of joint flexibility and link flexibility has rarely been discussed in literature. Here, a brief review is given.

Control of flexible-joint flexible-link manipulators.

Joint and link flexibility may be significant sources of tracking errors and undesirable oscillations. Recent simulation studies for one-link and two-link flexible manipulators showed that both distributed and concentrated flexibility have to be considered in control design (e.g. Yang and Donath, 1988a, 1988b). However, because of the extreme complexity, only few reported studies address the problem of modeling and control of manipulators with both joint and link flexibility. Investigations on the control of manipulators with two flexible joints and two flexible links were reported by Fukuda (1985) and Gebler (1987).

Yuan and Lin (1990) and Lin (1991) proposed a two-stage control for multi-link manipulators with flexible joints and flexible links. At first, a feedback control law is designed to perform input-output decoupling. This results in a closed-loop system with a decoupled, linear, time-invariant part that relates inputs to outputs, and a nonlinear part that does not affect the linear part. A linear controller can then be applied to the linear part. However, elastic vibrations in the system will induce undesired disturbances in the nonlinear part, which results in inaccurate end-effector trajectory tracking. Without a stabilizing controller, the overall system can become unstable. In order to suppress the elastic deflections, a perturbed stabilization controller is added, based on a robust, linear feedback control technique, such as LQR (Yuan and Lin) or H_∞ control (Lin). The suggested control strategy offers a compromise between stabilization and tracking accuracy. Although the proposed control law is linear and quite simple, it requires the availability of all states. No experiments have been carried out, but the simulation studies of Yuan and Lin, and Lin on a two-link flexible manipulator indicate that reasonable joint-angle control can be obtained while a significant reduction of the elastic vibrations is being achieved.

Henrichfreise (1988, 1989) has investigated the modeling, analysis and control of a laboratory, three-link manipulator with both flexible joints and flexible links. His model essentially consists of three single-axis modules. In Henrichfreise (1985), some basic research results were reported for a representative, industrial single-axis module with a harmonic drive connected to a flexible link with a payload at the end. Besides joint and link flexibility, other nonlinearities such as friction, gear backlash and actuator torque saturation have been taken into consideration. In Henrichfreise (1988, 1989), the complete dynamic model of the three-link flexible manipulator has been obtained by systematically combining the symbolic equations of motion of the three separate single-axis modules. Henrichfreise stresses that the use of a symbolic manipulation program package like MAPLE (1988) is essential in modeling multi-link flexible manipulators. However, the nonlinear equations of motion that result in symbolic form are extremely bulky and complex and they have to be simplified. Simplification, however, leads to a nonlinear model with parameters that do not have a physical meaning (hence, Henrichfreise has estimated these parameters by recursively comparing simulation results with suitable measurements on the real system). For control purposes, the nonlinear model is linearized for small changes around a chosen operating point. Terms due to Coriolis and centrifugal accelerations as well as the gravity terms are neglected because they are considered to be of minor importance. The unknown model parameters are identified using a linear, frequency-domain technique. The control objective is to achieve a fast, vibration-free and precise positioning of the end-effector in the neighborhood of the linearization point. For each single-axis module, undesirable deflections are eliminated by a two-level controller (Henrichfreise, 1985), while an additional, higher-level controller coordinates the coupled motion of the three modules. This results in a proportional, joint-level feedforward control law for the set-point tracking, together with a feedback term for stabilization. The control is based on measured or reconstructed state variables. The motor positions and velocities as well as the strains in the deforming links are measured, while the unknown state variables are reconstructed by linear, first-order observers. Simulations and experiments showed improved performance in comparison with conventional controllers in the sense that the elastic vibrations were considerably damped. Implementation of the developed control law in laboratory or industrial practice requires quite sophisticated software and hardware. Unfortunately, in the control scheme proposed by Henrichfreise, a large number of model parameters has to be identified and multiple controller parameters have to be tuned. Other drawbacks of this control scheme are that it performs well only in a small region around the chosen operating point in workspace and that it is limited to low speeds because the Coriolis and centrifugal terms have been neglected in the control design.

1.5 Control with parametric uncertainty.

1.5.1 Introduction.

A common characteristic of the control techniques discussed in the previous sections is that they are non-adaptive and result in control schemes with a fixed structure and with fixed parameters. They require the exact knowledge of the manipulator dynamics, whereas, in practice, uncertainties always arise due to unpredictable disturbances and/or parameter variations. *Model uncertainty* can be divided into *parametric uncertainty* (caused by a variable payload and its imprecise location in the end-effector, by poorly known masses and inertias, unknown and changing friction parameters, etc.) and *structure uncertainty* due to unmodeled dynamics (as a result of neglected friction in the drives, backlash in the gears, flexibility in joints and links, etc.). Unmodeled dynamics are usually assumed to be irrelevant or too complex to account for in control design. However, in the presence of model uncertainty, non-adaptive controllers can often not guarantee acceptable performance and can even become unstable. In the field of *robust control* theory, unmodeled flexibility is considered as an unknown disturbance with known bounds (e.g. Sastry and Slotine, 1983; Slotine, 1985; Spong and Vidyasagar, 1985; Asada and Slotine, 1986; De Jager, 1992c). Nathan and Singh (1989) have treated the control problem of a manipulator with two flexible links and have derived a discontinuous robust control law based on VSC (variable structure control). Other authors who have applied the VSC approach to flexible-link manipulators are Yeung and Chen (1989). Among this method and other methods to cope with model uncertainty, *adaptive control* offers an effective approach to improve the control performance in the presence of parametric uncertainty (Narendra and Annaswamy, 1989; Åstrom and Wittenmark, 1989; Landau, 1979). An introduction to adaptive control is given in Appendix B and can also be found in De Jager et al. (1991). If manipulator parameters are poorly known and/or time-varying, the adaptive controller adjusts one or more parameters of the control system (i.e. control gains and/or model parameters) on-line.

In this section, we successively discuss adaptive control schemes that have been proposed in literature for rigid manipulators, for manipulators with flexible joints and rigid links, and for manipulators with rigid joints and flexible links. No literature has been found on practical adaptive control schemes for manipulators with both flexible joints and flexible links. First, we will discuss the issues of convergence, stability and robustness, which play an important role in the design of (adaptive) control schemes.

1.5.2 Convergence, stability and robustness.

An adaptive controller has to guarantee (asymptotic) convergence of the tracking error to zero and has to ensure boundedness of all internal signals, such that the performance of the closed-loop system is robust for parametric uncertainty. Several studies on stability in the context of adaptive manipulator control are known (e.g. Landau, 1979, Narendra and Annaswamy, 1989). A brief review is given in Appendix A and in De Jager et al. (1991). The nonlinearities in manipulator dynamics have caused great problems in proving the *global, asymptotic stability* of closed-loop systems with adaptive controllers. The earliest adaptive algorithms, such as the gradient algorithm, were based on local stability arguments. After that, it has become common practice to use Lyapunov's second method (Lyapunov, 1949; LaSalle and Lefschetz, 1961; Narendra and Annaswamy, 1989; Slotine and Li, 1991) and the hyperstability theory of Popov (Popov, 1973; Landau, 1979; Narendra and Annaswamy, 1989) as powerful tools to develop stable adaptive algorithms.

For rigid manipulators, many stable adaptive control schemes have been proposed (e.g. Landau, 1985; Hsia, 1986; Middleton and Goodwin, 1986; Ortega and Spong, 1988; Slotine and Li, 1986, 1988, 1989). In these schemes, an adaptation mechanism adjusts the unknown parameters on-line. Three categories of adaptive control schemes can be distinguished (see also Appendix B):

1. *Direct adaptive control* (tracking-error based control) schemes, where the parameter adaptation is based on the actual tracking errors (Craig et al., 1986; Slotine and Li, 1986; Sadegh and Horowitz, 1987; Craig, 1988). The main objective of the adaptation is a reduction of the tracking errors. Often, this can be realized by adjusted parameters that do not match the real values at all.

2. *Indirect adaptive control* (prediction-error based control) schemes, where parameter estimates are determined from the prediction errors (Middleton and Goodwin, 1986; Slotine and Li, 1988). The main objective is to find the real parameter values, rather than to adjust the parameters for trajectory tracking purposes.

3. *Composite adaptive control* schemes, where both the tracking errors and the prediction errors are used to drive the parameter adaptation mechanism (Slotine and Li, 1989). These schemes are based on the observation that information on the parameters can be obtained from both sources and that the significance of trajectory tracking can be weighted against the significance of identification by tuning of the adaptation parameters.

Parameter convergence may be important to improve the robustness of an adaptive control scheme for model uncertainty. This can be seen in Vijverstra (1992), where theoretical analysis, simulations and experiments showed that improved robustness and better tracking accuracy can be obtained with the indirect and composite adaptive controller of Slotine and Li (1988 and 1989, respectively) than with the direct adaptive controller of Slotine and Li (1986). With indirect and with composite adaptive control, the parameter estimates often converge to the real values. However, indirect adaptive control requires inversion of the estimated inertia matrix, which, therefore, must be invertible during the whole adaptation process. In recent literature (e.g. Narendra and Annaswamy, 1989; Slotine and Li, 1991), it has been shown that parameter convergence is ensured if the input signals are 'persistently exciting' or 'sufficiently rich'. A detailed exposition of this topic can be found in Butler (1990) for MRAC systems (i.e. model reference adaptive control systems, see Appendix B and Landau, 1979), in Narendra and Annaswamy (1989) for the theory of persistent excitation and in Slotine and Li (1991) for the analysis of parameter convergence in adaptive manipulator control systems.

Roughly spoken, a system is called *robust* if it remains stable in the presence of model uncertainty and bounded external disturbances. Because adaptive control systems are robust for parametric uncertainty, questions may arise about the robustness of the adaptively controlled system for unmodeled dynamics. Most of the adaptive control schemes for rigid manipulators do not provide robustness for unmodeled flexibility in joints and links. The robustness of various proposed adaptive control schemes for unmodeled dynamics has been investigated by means of simulation studies and experiments (e.g. De Jager, 1992a, 1992c; Vijverstra, 1992). Butler (1990) has discussed the robustness of MRAC systems.

1.5.3 Adaptive control of rigid manipulators.

First approaches.

In the last decade, many adaptive schemes have been suggested to solve the control problem for rigid manipulators with unknown parameters (for a survey see e.g. Landau, 1985; Hsia, 1986; Ortega and Spong, 1988). The earliest schemes neglected the fact that the manipulator dynamics are highly nonlinear and strongly coupled, and/or relied on a local stability proof. They are often based on MRAC and often rely on a model that neglects the coupling between the links (Dubowsky and DesForges, 1979; Horowitz and Tomizuka, 1980; Nicosia and Tomei, 1984). In that case, each manipulator-axis module is treated as a single-input single-output, second-order system and the coupling effects to the other modules are considered as disturbances. Dubowsky and DesForges (1979) have used a gradient approach, the so-called *local parametric optimization approach*, which was based on the minimization of a quadratic cost function of the tracking error. Their adaptive control scheme requires the availability of the accelerations as well as the on-line inversion of the adjusted inertia matrix. Furthermore, adaptive controllers based on the local parametric optimization approach do not necessarily result in a globally stable system. To overcome these difficulties, Horowitz and Tomizuka (1980) proposed an improved MRAC scheme that does not require acceleration feedback and is based on the *hyperstability theory of Popov* (1973) in order to guarantee global stability of the closed-loop system. In their scheme, the non-zero, configuration-dependent elements of the model matrices are adapted under the assumption that these quantities vary slowly in time in comparison with the speed of adaptation. For a multi-degree-of-freedom manipulator, this leads to a considerable number of parameters to be updated on-line. However, the main drawbacks of their scheme are that it is based on the rather questionable assumption of slowly time-varying parameters and that no knowledge of the dynamics structure is used. With regard to rigid manipulators, Lammerts (1989) has investigated the MRAC schemes of Landau (1979) and Horowitz and Tomizuka (1980) by means of simulations.

Direct adaptive CTC schemes.

Considerable research has been done to develop stable, adaptive model-based control schemes for rigid manipulators with an adaptation mechanism to adjust the unknown parameters (see Fig. 1.6). To achieve asymptotic trajectory tracking, modern adaptive control schemes fully take into account the *nonlinear* character of the manipulator dynamics. The main category of adaptive schemes for rigid manipulators is, therefore, based on computed torque control (e.g. Craig, 1988; Spong and Vidyasagar, 1989; Slotine and Li, 1991).

Craig et al. (1986) and Craig (1988) presented an adaptive version of classical CTC for rigid manipulators (see Subsection 3.6.2). The key point in their paper is the assumption that the nonlinear equations of motion are linear in the unknown parameters if these are suitably selected. These parameters are assumed to be constant and to have a physical meaning, for instance, the load and link mass, a friction constant, etc. They are adjusted on-line by an adaptation algorithm and are, subsequently, used in the CTC law. The main drawbacks of the scheme proposed by Craig et al. are that it requires acceleration feedback and that it uses the inverse of the estimated inertia matrix. To tackle the problem that the estimated inertia matrix may become singular, Craig et al. require that the parameter estimates lie within regions that the inverse of the inertia matrix exists in. If the adaptation algorithm

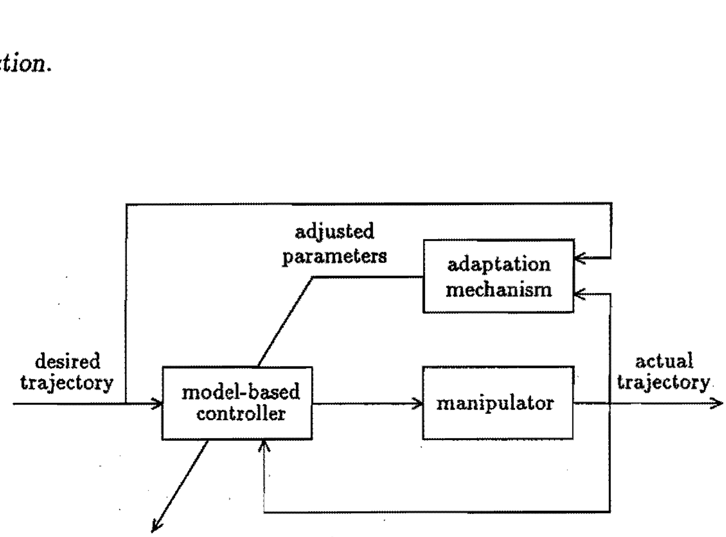


Figure 1.6: Adaptive model-based control of a manipulator.

results in a parameter outside this region, the parameter is set to the boundary value of this region (Craig, 1988). In this way, however, stability becomes questionable and calculation of the regions is not straightforward.

To arrive at globally, asymptotically stable closed-loop systems without the need to measure or reconstruct the accelerations and without the need to compute the inverse of the estimated inertia matrix, many direct adaptive control schemes have been proposed on the basis of CTC. These schemes exploit the *passivity property* of manipulators (e.g. Sadegh and Horowitz, 1987; Slotine and Li, 1986). This property follows on a relation between the inertia matrix and the matrix with Coriolis and centrifugal terms (see, e.g. Spong and Vidyasagar, 1989, and Chapter 2). Based on this property, Sadegh and Horowitz (1987) presented a modified version of the adaptive control scheme of Horowitz and Tomizuka (1980). Like Craig et al. (1986), they assume that the manipulator model is linear in the unknown but constant parameters. This clears the main drawback of the scheme of Horowitz and Tomizuka, i.e. the assumption that the elements of the model matrices vary slowly in time. Furthermore, the number of parameters that must be adjusted on-line is substantially reduced. Finally, the scheme of Sadegh and Horowitz neither requires acceleration feedback nor inversion of the estimated inertia matrix. A similar scheme was proposed by Slotine and Li (1986) and will be shown in Subsection 3.6.3. Lammerts (1990) has given a brief review of the schemes proposed by Landau (1979), Horowitz and Tomizuka (1980), Craig et al. (1986), Slotine and Li (1986) and Sadegh and Horowitz (1987), and has illustrated them by some simulation results.

Despite the great interest in adaptive control, few experimental results (only for manipulators with just one or two degrees of freedom) have been reported in literature (e.g., Tomizuka et al., 1988; Butler, 1990; Vijverstra, 1992; De Jager, 1992a, 1992c, 1993). Tomizuka et al. (1988) are among the few who have carried out extensive simulations and experiments with their adaptive control scheme on both laboratory and industrial manipulators. They confirm that the adaptive controller exhibits a consistently better performance than the non-adaptive counterpart, especially when large parametric uncertainties occur.

1.5.4 Adaptive control of flexible-joint manipulators.

It was mentioned earlier in Subsection 1.4.2 that many composite control concepts based on the singular perturbation theory have been proposed for flexible-joint manipulators with large joint stiffnesses. However, in these concepts the manipulator parameters are assumed to be known exactly. For the model of Spong (1987), Ghorbel et al. (1989) introduced an approach for the control of manipulators with rather stiff joints and unknown parameters. This approach is closely related to the non-adaptive composite control approach of Spong et al. (1987) and Spong (1987). Ghorbel et al. showed that recent adaptive control schemes for rigid manipulators can be used in their scheme (such as the one of Slotine and Li, 1986), provided that a simple, linear correction term is added to stabilize the elastic oscillations. Ghorbel and Spong (1992a) extended the composite control based on the concept of integral manifolds from the known parameter case to the parameter adaptation case. This yields a slow, adaptive control law, which consists of a part based on the rigid model and of a part with corrective terms for the joint flexibility, and a fast, adaptive control law, which is meant to stabilize the oscillations on the integral manifold. With the adaptive control scheme based on the concept of integral manifolds, a larger range of joint stiffnesses is allowed and a better performance is obtained than with the slow, adaptive control law based only on the rigid model. However, the scheme of Ghorbel and Spong requires the computation of the inverse of the estimated inertia matrix and relies on assumptions with respect to the behavior of the parameter estimates. Finally, Ghorbel and Spong (1992b) proposed an adaptation algorithm that accounts for joint flexibility and that does not require the computation of the inverse of the estimated inertia matrix. However, assumptions have still to be made to ensure asymptotic trajectory tracking, and the resulting adaptive composite controller is quite complex, both in its derivation and its implementation.

Tomei et al. (1986) proposed a simpler method. They extend the adaptive model-following control law of Nicosia and Tomei (1984) to an adaptive control law for flexible-joint manipulators. This law has been tested by means of simulations on a nonlinear model of a two-link flexible-joint manipulator.

Chen and Fu (1989) have used the model of Spong (1987) for the two-stage control design of a feedback control scheme with an adaptation mechanism to adjust the unknown parameters. First, a suitable feedback linearization control law is proposed for the flexible-joint torques. Then, an appropriate control law is defined for the motor torques, such that the flexible-joint torques converge asymptotically to the desired, CTC-like computed torques. The resulting, two-stage control law was shown to be locally stable. In contrast to the composite control approach based on the singular perturbation theory, the stiffness of the joints does not have to be large and the resulting control scheme is conceptually simpler and easier to implement.

Recently, a similar idea has been suggested by Brogliato and Lozano-Leal (1991) and Lozano-Leal and Brogliato (1992). They fully exploit the inherent properties of flexible-joint manipulator dynamics, in order to obtain a globally stable adaptive control scheme even if the joint stiffnesses are unknown. However, they assume that the upper bounds of the model parameters are known beforehand. In contrast to Chen and Fu (1989), they define a desired trajectory for the flexible coordinates (i.e. the motor coordinates) instead of for the flexible-joint torques. The computation of the input torques is based on the motor dynamics and on the desired trajectories for the motor coordinates that are computed on-line. However, similarly to the approach of Chen and Fu (1989), the considered dynamic model, defined according to Spong (1987), does not represent a general class of flexible-joint manipulators.

1.5.5 Adaptive control of flexible-link manipulators.

Only recently, research workers started to investigate adaptive control schemes for manipulators with flexible links. Koivo and Lee (1989) have designed a STR (self-tuning regulator, Appendix B) for a two-link manipulator with one rigid and one flexible link. Both simulations and laboratory experiments have been described to demonstrate their approach. Yang and Gibson (1989) have designed and simulated an indirect adaptive control scheme for a manipulator with one rigid and one flexible link. Their adaptation algorithm is based on a linear prediction model, which does not consider all flexible degrees of freedom of the manipulator, and on the control law that minimizes a quadratic criterion. The explicit use of MRAC in controlling the end-effector position of a flexible-link manipulator was reported by, e.g., Meldrum and Balas (1986) and Siciliano et al. (1986). The reference model may have less states than the manipulator model, but one of the conditions is that the output of the reference model must be of the same dimension as the manipulator output. The parameters of the controller (e.g. a simple PD-controller) are changed by the adaptation algorithm, such that the closed-loop dynamics match approximately the behavior of the chosen reference model. Encouraging simulations using MRAC for the control of a single-link flexible manipulator have been reported, but no experiments have been realized. In the field of adaptive control design for rigid and flexible manipulators, Landau (1985, 1988) has given a brief discussion on the state of the art and on future trends. With regard to flexible manipulators, Landau proposed adaptive pole placement and adaptive LQ techniques to be investigated.

1.6 Setup of this thesis.

The final conclusion of this chapter is that, although many different attempts have recently been made to deal with the control problem of flexible manipulators, this problem remains a challenge, especially for the combined problem of both *joint flexibility and link flexibility.* To our knowledge, the control problem of manipulators with both flexible joints and flexible links in the presence of *parametric uncertainty* has not even been addressed yet. In this thesis, we attempt to design a control scheme for manipulators with unknown parameters, on the basis of a dynamic model that takes flexibility in joints and/or links into account. After the presentation of a suitable control scheme for a flexible manipulator, an extension will be given to an adaptive control scheme that adjusts the unknown parameters on-line. For the proof of global, asymptotic stability of the closed-loop system, the second method of Lyapunov (1949) will be extensively used in this thesis.

Numerous direct adaptive CTC schemes that result in globally, asymptotically stable closed-loop systems have been proposed for rigid manipulators. Quite often, the main distinction between these schemes is a different definition of the reference trajectory for the generalized coordinates. Because of its conceptual elegance and simplicity, in this thesis the scheme of Slotine and Li (1986) will be considered as a basic adaptive CTC scheme for further investigation. In Chapter 3, besides this scheme other control concepts will be discussed that are also based on the general model of a rigid manipulator (as derived in Subsection 2.5.2). However, like most other control concepts for manipulators used until now, they rely on the assumption that all manipulator elements are perfectly rigid and that the number of actuator inputs equals the number of degrees of freedom. Therefore, these concepts are generally not applicable to manipulators with significant flexibilities. They usually result in less tracking accuracy and can even lead to instability. The elastic deformations in flexible manipulators give rise to a considerable number of extra degrees of freedom to be controlled. Hence, in order to improve the performance of a flexible manipulator, the control objective has to be reformulated: the controller must achieve trajectory tracking and at the same time stabilization of the elastic deflections. For that purpose, a model of the manipulator has to be considered that contains flexibility in joints and/or links. In Chapter 2, a general model of a flexible manipulator will be derived. Subsequently, flexible-joint manipulators (Subsection 2.5.3), flexible-link manipulators (Subsection 2.5.4) and manipulators with flexible joints and flexible links (Subsection 2.5.5) will be discussed.

In literature, it has been often proposed to actively damp out the undesired deformations as well as possible (in contradiction with the earliest approach to avoid the undesired deformations via a stiff construction). In that case, however, high internal mechanical stresses in the structure will force the flexible manipulator in a quite unnatural way to resemble the dynamic behavior of a stiff manipulator. These stresses may become inadmissibly high because of a limited strength of the lightweight material. Furthermore, the torques that control the resultant, rigid-like manipulator construction are disturbed by the torques that are needed to suppress the deformations. Hence, it seems very difficult to yield accurate end-effector trajectory tracking in this way. Therefore, the purpose of the research in this thesis is to design a controller that realizes asymptotic trajectory tracking of the end-effector, while *bounded* deformations in the construction are allowed to occur. In the approach of composite control based on the singular perturbation theory, this has, more or less, been attempted by using the concept of integral manifolds. Nevertheless, this approach still relies on the limiting assumption of rather stiff joints and links. This has motivated us to design a controller for flexible manipulators that assures global, asymptotic stability of the closed-loop system without restrictions on the magnitudes of the stiffnesses. Quite often, schemes like the input-output linearization scheme and the composite control scheme based on singular perturbation have been proposed only for flexible-joint manipulators. These schemes often rely on a model where the rigid-link dynamics and the motor dynamics are coupled only by the (linear) flexible-joint torques (similarly to the model of Spong, 1987). In this research project, the purpose is to design a control scheme for flexible manipulators with arbitrary configurations, such that no restriction has to be made on any special class of manipulators. More generally, the control scheme to be designed must be applicable not only to flexible-joint manipulators but also to manipulators with flexible links (and/or flexible joints). Moreover, the control scheme must be extended to an adaptive control scheme in order to account for parametric uncertainty. Finally, the new scheme has to offer the possibility for direct end-effector trajectory tracking as well. This aspect has rarely been investigated in literature.

In Chapter 4, a new model-based control scheme will be proposed for flexible manipulators with unknown parameters. The problem of controlling flexible manipulators is tackled such that no distinction has to be made between flexible joints and flexible links. It is assumed that the control model contains all relevant dynamics, but constant parameters may be unknown. The issue of robustness of the proposed control scheme for unmodeled dynamics is not investigated explicitly. The proposed control strategy probably offers a solution to many of the problems encountered in practice. In Chapter 5, we will discuss some of the promising results that are obtained by means of simulations and experiments. Chapter 6 will show the conclusions and put forward some suggestions for future work.

Chapter 2

Modeling of flexible manipulators.

2.1 Introduction.

Today, industrial manipulators are used for various purposes. Because of hardware limitations, until now practical manipulator control has been based on the assumption that the joints are stiff and that the links can be modeled as rigid bodies. Therefore, most of today's manipulators are quite stiff and thus heavy in order to avoid deformations and vibrations. For higher operational accelerations, industrial manipulators should be *lightweight* constructions to reduce the driving forces/torques and to enable the manipulator arm to respond faster. However, flexibility then becomes important and to improve the performance, control design should be based on a model that accounts for flexibilities. Nowadays, the application of more complex control algorithms is possible, because of the availability of advanced multiprocessor equipment.

The flexibility of a manipulator can consist of flexibility in the links and/or flexibility in the joints (i.e. in the drives, such as in the transmissions between the actuators and the links, in the drive shafts, in gear wheels, etc.). Many manipulators are currently powered by harmonic drives with compact gear mechanisms that have high transmission ratios. It has been shown experimentally that flexibilities in these drives result in lightly damped oscillations, which can cause stability problems if the control system is designed under the assumption of rigidity. Besides this so-called joint flexibility, especially for fast, lightweight manipulators also the links may deform. Because of the complexity of the equations of motion for multi-link manipulators with flexibilities in both joints and links, the modeling and control of flexible manipulators has been studied mainly for manipulators with either joint ór link flexibilities, and/or for special configurations, such as single-link manipulators (see Chapter 1). In this thesis, a technique will be introduced for controlling manipulators with both joint and link flexibilities. This technique is based on a model of the flexible manipulator. Although a large variety of schemes for formulating the dynamic equations of rigid manipulators is available (Wittenburg, 1977; Vukobratovic and Kircanski, 1985; Asada and Slotine, 1986; Craig, 1986; Spong and Vidyasagar, 1989; Veldpaus, 1993), an efficient method for modeling flexible manipulators is not commonly known. The modeling of flexible manipulators (with special configurations) has been discussed, for instance, by Sunada and Dubowsky (1983), Book (1984), Sakawa et al. (1985), Cetinkunt et al. (1986), Schmitz (1986), Usoro et al. (1986), Cetinkunt and Book (1987), Spong (1987), Yang and Donath (1988a, 1988b), Shabana (1989), Asada et al. (1990), Kruise (1990), Matsuno and Sakawa (1990), Chedmail et al. (1991). In this chapter, we will use Lagrange's formalism to derive a general dynamic model of a flexible manipulator (see also, e.g., Veldpaus, 1992a, 1992b, 1993; Shabana, 1989). Furthermore, we will discuss some specific models for manipulators with increasing degree of flexibility. This offers more insight into the structure of models of flexible manipulators.

2.2 The kinematics of a multi-link manipulator.

We consider a manipulator that can be modeled as an open chain of m links, connected by cylindrical, revolute or prismatic joints and with one degree of freedom per joint[1] (see Fig 2.1). One end of this *serial-linked manipulator* is fixed to the ground and the

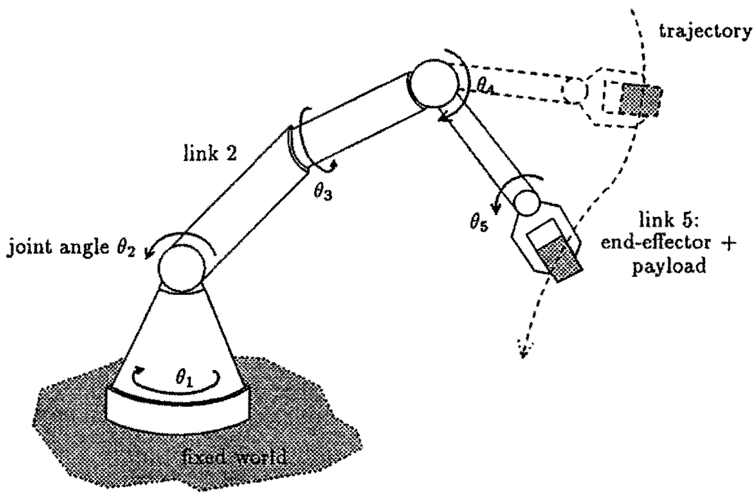


Figure 2.1: A multi-link manipulator.

other end with end-effector and payload has to follow a specified trajectory in space. As will be argued in Section 2.4, it is assumed that the desired motion of each link can be determined from this trajectory if all links are rigid.

In each joint, a drive unit is located and each link is connected to its drive unit via a transmission. Each drive unit typically consists of a servo amplifier and a servo

[1]One degree of freedom per joint is common in practice, but the assumption here does in fact not strictly reject other possible manipulator configurations.

actuator, such as a DC motor. Joint sensors such as encoders, potentiometers and resolvers are attached directly to the servo motor. The dynamics of the motors are not considered in detail and we use the actuator torques[2] as the input variables of the manipulator.

We are interested in manipulators with significant *flexibility* in the joints and in the links. Although different modeling techniques are known, in this thesis, the deformation of the links is chosen to be approximated by a linear combination of chosen displacement functions, whereas flexibilities in the joints will be modeled by means of massless, elastic springs. Drive-side friction and structural damping can be taken into account. The effects of Coulomb friction and of backlash in the gear boxes are not included, but can be inserted, too.

The equations of motion for this kind of multi-link manipulators can be derived using Lagrange's formalism (e.g. Veldpaus, 1992a, 1992b, 1993; Shabana, 1989). This results in a set of second-order differential equations that relate the selected generalized coordinates, i.e. the degrees of freedom, to the torques generated by the actuators. If all joints and links are rigid, the number of degrees of freedom, n, equals the number of actuator inputs, m. If deformations are taken into account, additional degrees of freedom are necessary and, thus, $n > m$.

For control purposes, we assume that all generalized coordinates and velocities are known at each moment of time by filtering and reconstruction of measured signals. Furthermore, we assume that the structure of the mathematical model is correct, i.e. the model will perfectly match the real manipulator dynamics if we know the exact parameter values. However, in practice, parameters are not always known beforehand, for instance, due to an unknown mass of the load and its imprecise position at the end-effector, poorly known masses and inertias, inaccuracies in friction parameters, etc. In the control scheme that has to be designed, *parametric uncertainty* should be taken into account. The unknown parameters are considered to be either perfectly constant, piecewise constant with large intervals between the discontinuities or nearly constant, i.e. they vary on a much larger time-scale than the generalized coordinates.

[2]In this thesis, the notion "torque" is used in a generalized sense, i.e. denoting both torques and forces.

2.3 Lagrange's formalism.

2.3.1 Introduction.

We consider a mechanical system with n degrees of freedom $q_1, q_2, ..., q_n$, being the elements of the column[3] $\underline{q} \in \mathrm{R}^n$ of *generalized coordinates*:

$$\underline{q} = [\ q_1 \ \ \ldots \ \ q_n\ ]^T . \tag{2.1}$$

Let $\underline{p} \in \mathrm{R}^p$ be the column with constant, but unknown manipulator parameters and let $\mathcal{T} = \mathcal{T}(\underline{q}, \underline{\dot{q}}, \underline{p})$ be the kinetic energy. Now, the equations of motion can be found by using *Lagrange's equations*:

$$\frac{d}{dt}(\frac{\partial \mathcal{T}}{\partial \dot{q}_i}) - \frac{\partial \mathcal{T}}{\partial q_i} = F_i , \qquad i = 1, ..., n , \tag{2.2}$$

where the first two terms account for the inertia effects and F_1, F_2, ..., F_n are the generalized torques on the manipulator. These torques, the elements of a column $\underline{F} \in \mathrm{R}^n$, can be determined from the virtual work δW, done by all internal and external torques during a variation δq of $\underline{q}$:

$$\delta W = \delta \underline{q}^T \underline{F} . \tag{2.3}$$

The kinetic energy $\mathcal{T}$ is a homogeneous, quadratic function of the generalized velocities $\underline{\dot{q}}$ and can be written as

$$\mathcal{T} = \frac{1}{2} \underline{\dot{q}}^T M \underline{\dot{q}} , \tag{2.4}$$

where $M \in \mathrm{R}^{n \times n}$ is the symmetric inertia matrix. This matrix is positive definite, because the kinetic energy $\mathcal{T}$ is strictly positive unless the manipulator is at rest. M can also depend explicitly on time t, as will be the case if parameters vary in time. Then, Lagrange's formalism in the form as stated here cannot be used. However, M does often depend only on the coordinates $\underline{q}$ and on the constant parameters $\underline{p}$, and we omit the case in which M depends explicitly on t. Although it is not trivial, the number of selected parameters is often far smaller than the number of matrix elements. If these constant parameters are assumed to be unknown instead of all (time-varying) matrix elements, this is advantageous for adaptive control purposes (Section 3.6, see also the discussion about adaptive control of rigid manipulators in Subsection 1.5.3).

To resume, the *inertia matrix* M satisfies

$$M = M(\underline{q}, \underline{p}) \ ; \quad M = M^T \ ; \quad M > 0 \ ; \quad M \ \text{ linear in } \underline{p} . \tag{2.5}$$

[3]In this thesis the words "vector" and "column" have a different meaning. A vector $\vec{a}$ in the three-dimensional Euclidean space $\mathcal{E}$ is characterized by its direction, its sense and its length $\| \vec{a} \|$. A vector with length 1 is called a unit vector. A column $\underline{a} = [\ a_1 \ \ a_2 \ \ a_3\ ]^T$ is the matrix representation of $\vec{a}$ with respect to a chosen, orthogonal vector basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ in $\mathcal{E}$, if $\vec{a} = a_1\vec{e}_1 + a_2\vec{e}_2 + a_3\vec{e}_3$. This vector basis will not be specified here. The scalar product $\vec{a} \circ \vec{b}$ of $\vec{a} \in \mathcal{E}$ and $\vec{b} \in \mathcal{E}$ is equal to $\vec{a} \circ \vec{b} = \| \vec{a} \| . \| \vec{b} \| .cos(\varphi(\vec{a}, \vec{b}))$, where $\varphi(\vec{a}, \vec{b})$ is the smallest angle between $\vec{a}$ and $\vec{b}$. For more information on the use of vectors, columns, etc. in modeling multi-body systems, we refer to Wittenburg (1977).

2.3.2 The inertia terms.

From Eq. (2.4) for the kinetic energy and from the symmetry of M, it is easily seen that the first term of Eq. (2.2) yields

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_i}) = (M\ddot{\underline{q}})_i + (\dot{M}\dot{\underline{q}})_i \ . \tag{2.6}$$

To elaborate the term $(\dot{M}\dot{q})_i$, it is noted that M is a function of $\underline{q}$ and $\underline{p}$ and that $\underline{p}$ is constant. Hence, this term becomes

$$(\dot{M}\dot{\underline{q}})_i = \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\partial M_{ik}}{\partial q_j}\dot{q}_k\dot{q}_j = (G\dot{\underline{q}})_i \ , \tag{2.7}$$

where the elements of G depend on $\underline{q}$ and are linear in $\dot{\underline{q}}$ and $\underline{p}$:

$$G_{ij} = \sum_{k=1}^{n} \frac{\partial M_{ik}}{\partial q_j}\dot{q}_k \ , \qquad i,j = 1,...,n \ . \tag{2.8}$$

The second term in Eq. (2.2) can be written as

$$\frac{\partial T}{\partial q_i} = \frac{1}{2}\dot{\underline{q}}^T \frac{\partial M}{\partial q_i}\dot{\underline{q}} = \frac{1}{2}(G^T\dot{\underline{q}})_i \ , \tag{2.9}$$

and, hence, the left-hand side of Eq. (2.2) results in

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_i}) - \frac{\partial T}{\partial q_i} = (M\ddot{\underline{q}})_i + (\dot{M}\dot{\underline{q}})_i - \frac{1}{2}(G^T\dot{q})_i \ . \tag{2.10}$$

With $\dot{M}\dot{\underline{q}} = G\dot{\underline{q}} = \frac{1}{2}(\dot{M} + G)\dot{\underline{q}}$, it is seen that the equations of motion (2.2) can be written as

$$M\ddot{\underline{q}} + C\dot{\underline{q}} = \underline{F} \ , \qquad C = \frac{1}{2}[\dot{M} + (G - G^T)] \ , \tag{2.11}$$

in which $(C - \frac{1}{2}\dot{M}) = \frac{1}{2}(G - G^T)$ is a *skew-symmetric* matrix. The term $C\dot{\underline{q}}$ accounts for the Coriolis and centrifugal torques, which arise because the inertia matrix is configuration dependent.

2.3.3 The generalized torques.

The main problem in determining the equations of motion is the specification of the generalized torques $\underline{F}$. If δW_u is the virtual work of the torques $\underline{u} \in \mathrm{R}^m$ generated by the servo motors and δW_v is the virtual work of all other torques in and on the manipulator, the total virtual work δW during a variation $\delta\underline{q}$ of $\underline{q}$ is given by

$$\delta W = \delta W_u + \delta W_v = \delta\underline{q}^T\underline{F} \ . \tag{2.12}$$

The torques $\underline{u}$ are the *input variables*. The work δW_u of these torques is given by

$$\delta W_u = \delta\underline{q}^T H\underline{u} \ , \qquad H \in \mathrm{R}^{n\times m} \ , \quad H = H(\underline{q}) \ , \tag{2.13}$$

where H is the distribution matrix. Without loss of generality, we may assume that the rank of H is maximal, i.e. rank$(H) = m$.

The non-controllable torques $\underline{F} - H\underline{u}$ in $\delta W_v = \delta \underline{q}^T(\underline{F} - H\underline{u})$ are split in conservative torques $-\underline{v}$ and non-conservative torques $-\underline{f}$. The conservative torques follow from a potential $\mathcal{V}(\underline{q}, \underline{p})$ by

$$\underline{v} = \frac{\partial \mathcal{V}}{\partial \underline{q}} ; \quad \underline{v} = \underline{v}(\underline{q}, \underline{p}) . \tag{2.14}$$

Conservative torques are due to, for example, gravity and elasticity.

The column $-\underline{f} = -\underline{f}(\underline{q}, \dot{\underline{q}}, \underline{p}, t)$ accounts for all other torques acting in and on the manipulator, for instance, torques due to (viscous) friction at the motor axes or due to internal damping of flexible links. With $\underline{v}$ and $\underline{f}$, the virtual work ∂W_v can be written as

$$\delta W_v = \delta \underline{q}^T(-\underline{v} - \underline{f}) , \tag{2.15}$$

and the total virtual work δW is given by

$$\delta W = \delta W_u + \delta W_v = \delta \underline{q}^T(H\underline{u} - \underline{v} - \underline{f}) = \delta \underline{q}^T \underline{F} . \tag{2.16}$$

This must hold for every $\delta \underline{q}$ and, therefore, the generalized torques $\underline{f}$ are related to the input $\underline{u}$, to the conservative torques $-\underline{v}$ and to the non-conservative torques $-\underline{f}$ by

$$\underline{F} = H\underline{u} - \underline{f} - \underline{v} . \tag{2.17}$$

Similarly to the inertia matrix M, for a suitably selected set of parameters $\underline{p}$ the columns $\underline{v}$ and $\underline{f}$ are assumed to be linear in $\underline{p}$.

2.3.4 The equations of motion.

Substitution of the generalized torques $\underline{F}$ according to Eq. (2.17) in Eq. (2.11) yields the *equations of motion* in the form

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + \underline{v} + \underline{f} = H\underline{u} , \tag{2.18}$$

or, in a more compact form often used, as

$$M\ddot{\underline{q}} + \underline{n} = H\underline{u} , \qquad \underline{n} = C\dot{\underline{q}} + \underline{v} + \underline{f} . \tag{2.19}$$

Although the equations of motion are nonlinear in $\underline{q}$, the matrices M and C and the columns $\underline{v}$ and $\underline{f}$ are, as mentioned, linear functions of the unknown parameters $\underline{p}$, so that the left-hand side of the equations of motion (2.18) is linear in $\underline{p}$ and can be written as

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + \underline{v} + \underline{f} = \underline{w} + W\underline{p} , \tag{2.20}$$

where $\underline{w} = \underline{w}(\underline{q}, \dot{\underline{q}}, \ddot{\underline{q}}, t)$ and $W = W(\underline{q}, \dot{\underline{q}}, \ddot{\underline{q}}, t)$ do not depend on $\underline{p}$. This property of *linearity in the parameters* (e.g. Spong and Vidyasagar, 1989; Slotine and Li, 1991) will be later used in parameter adaptation algorithms.

Premultiplication of the equations of motion (2.18) by $\underline{\dot{q}}^T$ yields, after some manipulation,

$$\frac{d}{dt}(\frac{1}{2}\underline{\dot{q}}^T M \underline{\dot{q}}) + \underline{\dot{q}}^T (C - \frac{1}{2}\dot{M})\underline{\dot{q}} + \underline{\dot{q}}^T \underline{v} = \underline{\dot{q}}^T (H\underline{u} - \underline{f}) \ . \tag{2.21}$$

The term $\underline{\dot{q}}^T(C - \frac{1}{2}\dot{M})\underline{\dot{q}}$ is zero, because the matrix $(C - \frac{1}{2}\dot{M})$ is skew-symmetric. Furthermore, the term $\underline{\dot{q}}^T \underline{v}$ is equal to the time derivative of the potential $\mathcal{V}$, as can be seen from the definition of the conservative torques in Eq. (2.14). Hence, it is seen that

$$\underline{\dot{q}}^T(H\underline{u} - \underline{f}) = \dot{\mathcal{L}} \ \ ; \quad \mathcal{L} = \mathcal{T} + \mathcal{V} \ , \tag{2.22}$$

where $\mathcal{L}$, which is the sum of the kinetic energy $\mathcal{T}$ and the potential $\mathcal{V}$, is called the mechanical energy. After integration, this yields

$$\int_{t_0}^{t} \underline{\dot{q}}^T (H\underline{u} - \underline{f}) d\tau = \mathcal{L}(t) - \mathcal{L}(t_0). \tag{2.23}$$

A direct consequence of this so-called balance law of mechanical energy is that

$$\int_{t_0}^{t} \underline{\dot{q}}^T (H\underline{u} - \underline{f}) d\tau + \mathcal{L}(t_0) \geq 0 \ , \quad \forall t, \tag{2.24}$$

because $\mathcal{L}$ is positive definite. In literature, this property is referred to as the *passivity property* of mechanical systems (e.g. Spong and Vidyasagar, 1989; Slotine and Li, 1991).

In general, Eq. (2.18) represents a set of n strongly coupled, highly nonlinear equations. Often, it is preferable to simplify the structure of the model (for instance, the structure of H) or to use explicitly another selected set of generalized coordinates (for instance, the output $\underline{y}$). For this purpose, a coordinate transformation is needed to be carried out on the model. Let $\underline{q}^* \in \mathrm{R}^n$ be a new set of generalized coordinates, related to the original coordinates by a static, time-invariant global diffeomorphism[4] $\underline{\psi} : \mathrm{R}^n \rightarrow \mathrm{R}^n$:

$$\underline{q} = \underline{\psi}(\underline{q}^*) \ . \tag{2.25}$$

Hence, the derivative Ψ of $\underline{\psi}$ with respect to $\underline{q}$ is a square, regular matrix with elements Ψ_{ij} defined by

$$\Psi_{ij} = \frac{\partial \psi_i}{\partial q_j^*} \ , \quad i,j = 1,2,...,n. \tag{2.26}$$

[4]Roughly spoken, a *diffeomorphism* $\underline{\psi} : \mathrm{R}^n \rightarrow \mathrm{R}^n$ is differentiable with a square, regular derivative Ψ (and is, thus, also invertible with inverse $\underline{\psi}^{-1}$). For a rather mathematical definition of a diffeomorphism, see Nijmeijer and Van der Schaft (1990).

In terms of the new coordinates $\underline{q}^*$, the equations of motion are given by

$$M^*\ddot{\underline{q}}^* + C^*\dot{\underline{q}}^* + \underline{v}^* + \underline{f}^* = H^*\underline{u} \,, \tag{2.27}$$

where M^*, C^*, H^*, $\underline{v}^*$ and $\underline{f}^*$ are related to the original quantities by

$$M^* = \Psi^T M \Psi \ ; \quad C^* = \Psi^T M \dot{\Psi} + \Psi^T C \Psi \ ; \quad H^* = \Psi^T H \ ; \quad \underline{v}^* = \Psi^T \underline{v} \ ; \quad \underline{f}^* = \Psi^T \underline{f} \tag{2.28}$$

It is easily seen that the relation between C^* and $\dot{M}^*$ can be written as

$$C^* = \frac{1}{2}\dot{M}^* + \frac{1}{2}(\Psi^T M \dot{\Psi} - \dot{\Psi}^T M \Psi) + \frac{1}{2}\Psi^T (G - G^T)\Psi \,,$$

and this implies that $C^* - \frac{1}{2}\dot{M}^*$, like $C - \frac{1}{2}\dot{M}$, is a skew-symmetric matrix.

As mentioned, practical coordinate transformations of this type are often used to simplify, for instance, the distribution matrix. For mechanical systems, it is always possible to choose the coordinates $\underline{q}^*$ such that the corresponding matrix H^* has the following simple and attractive structure:

$$H^* = \begin{bmatrix} 0 \\ I \end{bmatrix} , \tag{2.29}$$

where I is the $(m \times m)$ identity matrix.

In order to differentiate the term 'generalized coordinates' used in this thesis from the term 'state variables' often used elsewhere, here we give the definition of the state $\underline{x} \in \mathrm{R}^{2n}$ that consists of the column of generalized coordinates $\underline{q}$ and of the column of velocities $\dot{\underline{q}}$ as follows:

$$\underline{x} = \begin{bmatrix} \underline{q} \\ \dot{\underline{q}} \end{bmatrix} . \tag{2.30}$$

Now, the manipulator model (2.18) can be easily transformed to the following *state space description* of the model:

$$\dot{\underline{x}} = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}C \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ M^{-1}(H\underline{u} - \underline{v} - \underline{f}) \end{bmatrix} . \tag{2.31}$$

Except for the appendices, in this thesis the state space approach will not be used.

2.4 The desired trajectory.

The elements of $\underline{q} \in \mathrm{R}^n$ represent all generalized coordinates required to describe the kinematic behavior of a flexible or rigid manipulator. For a flexible manipulator, we require that the coordinates are selected in such a way that $\underline{q}$ can be partitioned in the columns $\underline{\theta} \in \mathrm{R}^s$ and $\underline{\varrho} \in \mathrm{R}^{n-s}$, where $\underline{\theta}$ contains the coordinates that are necessary and sufficient to describe the kinematic behavior if no deformations occur in the manipulator. Therefore, the elements of $\underline{\theta}$ are called the *rigid-body coordinates* or *rigid-body degrees of freedom*. The remaining coordinates, the elements of $\underline{\varrho}$, are necessary and sufficient to characterize deformations in the manipulator. They are called the *flexible coordinates* or *flexible degrees of freedom.*

Without practical loss of generality, we may assume that the number of rigid-body degrees of freedom s is equal to the number of input signals m. Now, $\underline{q}$ can be written as

$$\underline{q} = \begin{bmatrix} \underline{\theta} \\ \underline{\varrho} \end{bmatrix} ; \qquad \underline{\theta} \in \mathrm{R}^m \; ; \qquad \underline{\varrho} \in \mathrm{R}^{n-m} \; . \tag{2.32}$$

Let the quantities to be controlled be the elements of the *output* column $\underline{y} \in \mathrm{R}^d$ and let the desired trajectory of these quantities be given by a specified function $\underline{y}_d : \mathrm{R} \rightarrow \mathrm{R}^d$ of time t. In practice, the number of output quantities is smaller than or at the most equal to the number of input signals, so $d \leq m$.

We assume that the output quantities in $\underline{y}$ are related to the generalized coordinates by a static relation of the form $\underline{y} = \underline{\phi}^*(\underline{q})$. With a suitable choice of the generalized coordinates and a suitable partitioning of $\underline{q}$ in $\underline{\theta}$ and $\underline{\varrho}$, for the considered class of manipulators it can be achieved that $\underline{y}$ depends on $\underline{\theta}$ but not on $\underline{\varrho}$. So,

$$\underline{y} = \underline{\phi}(\underline{\theta}) \; . \tag{2.33}$$

It is assumed that $\underline{\phi} : \mathrm{R}^m \rightarrow \mathrm{R}^d$ is continuous and differentiable with derivative Φ. The elements of this $(d \times m)$ matrix are given by

$$\Phi_{ij} = \frac{\partial \phi_i}{\partial r_j} \, , \qquad i = 1, 2, ..., d \, , \qquad j = 1, 2, ..., m \, , \tag{2.34}$$

and the rank of this matrix is at the most equal to d because $d \leq m$. We assume that the rank is maximal for all relevant $\underline{\theta}$, i.e.

$$rank(\Phi) = d \; . \tag{2.35}$$

As a consequence, kinematic singularities are not taken into account.

If $d = m$ and $\mathrm{rank}(\Phi) = d$, then $\underline{\phi} : \mathrm{R}^m \rightarrow \mathrm{R}^m$ is a diffeomorphism and $\underline{\theta}$ can be solved from Eq. (2.33), resulting in $\underline{\theta} = \underline{\phi}^{-1}(\underline{y})$. The desired output trajectory $\underline{y}_d$ now determines uniquely the desired trajectory $\underline{\theta}_d$ of the rigid-body degrees of freedom:

$$\underline{\theta}_d = \underline{\phi}^{-1}(\underline{y}_d) \, , \qquad \text{if} \quad d = m \quad \wedge \quad \mathrm{rank}(\Phi) = d. \tag{2.36}$$

If $d < m$, the manipulator is said to be redundant. In that case, the set of d equations $\underline{y}_d = \underline{\phi}(\underline{\theta}_d)$ does not yield a unique solution for the m elements of $\underline{\theta}_d$. To arrive at a unique solution, it is necessary to formulate $m - d$ extra requirements for the elements of $\underline{\theta}_d$. These extra requirements may, for instance, represent the desire to minimize the energy consumption of the servo motors or to minimize the maximum torques of the motors (Asada and Slotine, 1986). In the sequel, also for $d < m$ it is assumed that the desired trajectory $\underline{\theta}_d$ for the rigid-body degrees of freedom can be determined in one way or another, but such that $\underline{y}_d(t) = \underline{\phi}(\underline{\theta}_d(t))$ for all $t \geq 0$.

The flexible coordinates $\underline{\varrho}$ do not occur in the output equation (2.33) and, thus, the desired output trajectory $\underline{y}_d$ does not impose any requirement on the desired trajectory $\underline{\varrho}_d$ of the flexible coordinates. The definition and determination of $\underline{\varrho}_d$ will be elaborated in detail in Chapter 4. For the moment, it is assumed that $\underline{\varrho}_d$ is known. Hence, the desired trajectory $\underline{q}_d$ of the generalized coordinates can be determined from

$$\underline{q}_d = \begin{bmatrix} \underline{\theta}_d \\ \underline{\varrho}_d \end{bmatrix} . \tag{2.37}$$

In terms of $\underline{q}$ and $\underline{q}_d$, the *control objective* is to generate such an input $\underline{u}$ that the coordinates $\underline{q}$ track the desired trajectory $\underline{q}_d$ asymptotically, i.e.

$$\lim_{t \to \infty} \underline{e} = \underline{0} , \tag{2.38}$$

where $\underline{e} = \underline{q}_d - \underline{q}$ is called the *tracking error.* As argued in Chapter 1, for motion control of manipulators it is desired to let the manipulator track a desired trajectory for both the positions and the velocities asymptotically, i.e. such that

$$\lim_{t \to \infty} \underline{e} = \underline{0} \quad ; \quad \lim_{t \to \infty} \dot{\underline{e}} = \underline{0} . \tag{2.39}$$

For completeness, this can be translated to the next state space description of the manipulator control objective:

$$\lim_{t \to \infty} \underline{e}_x = \underline{0} , \tag{2.40}$$

where $\underline{e}_x = \underline{x}_d - \underline{x}$ is the state error and $\underline{x}_d^T = [\ \underline{q}_d^T \quad \dot{\underline{q}}_d^T\ ]$ is the desired state.

2.5 Some special manipulator models.

2.5.1 Introduction.

In Section 2.3, Lagrange's formalism to derive the equations of motion has been sketched. In the following subsections, some special models for manipulators with increasing degree of flexibility will be derived. First, the dynamic model for a rigid manipulator will be discussed, i.e. for the case in which all flexibilities are negligible small. Next, attention will be given to manipulators with rigid links but with flexible joints, which represent flexibility in the drives (bearings, gear teeth, drive shafts, etc.). Subsequently, manipulators with stiff joints but flexible links will be considered. Finally, a model that incorporates both joint and link flexibilities is discussed. Emphasis will be put on the determination of the number of degrees of freedom n, and on the choice of an appropriate set of generalized coordinates, $\underline{q} \in \mathcal{R}^n$. Then, from the geometry and the mass distribution of the parts of the manipulator, the total kinetic energy can be determined as a function of $\underline{q}$, $\dot{\underline{q}}$ and, possibly, of the unknown parameters $\underline{p}$: $\mathcal{T} = \mathcal{T}(\underline{q}, \dot{\underline{q}}, \underline{p})$. Finally, the total virtual work $\delta W = \delta \underline{q}^T H \underline{u} - \delta \mathcal{V} - \delta \underline{q}^T \underline{f}$ will be specified in terms of the variations $\delta \underline{q}$. The last term in this relation, $\delta \underline{q}^T \underline{f}$, will be given hardly any attention. The resultant torques $\underline{f}$, which cannot be included in $\delta \underline{q}^T H \underline{u}$ or $\delta \mathcal{V}$, often stem from phenomena that are difficult to model with sufficient accuracy, such as friction and damping in contact areas, external disturbances, etc. They strongly depend on the situation at hand and can, in general, be specified only on an ad hoc basis. An accurate and useful specification of the elements of $\underline{f}$ is usually one of the main problems in the derivation of the equations of motion of a mechanical system (see, e.g. De Jager, 1992c, 1993, for a discussion on friction modeling).

As soon as the relations for the kinetic energy and the virtual work are determined, the equations of motion can be derived via Lagrange's formalism by hand or via the use of a symbolic manipulation program like MAPLE (1988) (for the modeling of flexible-link manipulators with the use of a symbolic manipulation program, see, for instance, Cetinkunt et al., 1986; Cetinkunt and Book, 1987).

2.5.2 Rigid manipulator.

Introduction.

As mentioned before, we only consider manipulators that can be modeled as an open chain of m links, connected by joints with only one rotational (or translational) degree of freedom per joint (see Fig 2.2). The links are numbered consecutively, starting with

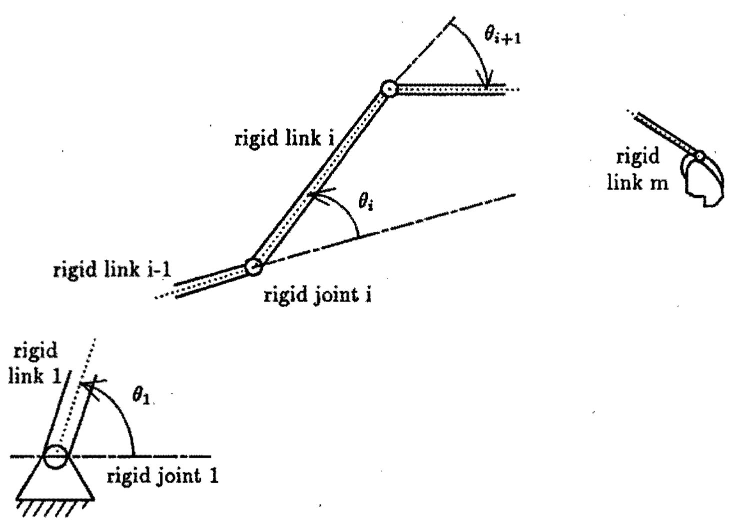


Figure 2.2: The kinematics of a rigid manipulator.

1 for the link connected to the fixed world and ending with number m for the link with end-effector and payload. The joints are numbered accordingly, such that joint i is the joint between link $i-1$ and link i ($i = 1, 2, \ldots, m$). Furthermore, the drives

are numbered such that the number of the joint and the number of the drive in that joint coincide.

Let θ_i be the relative coordinate that characterizes the relative rotation (or translation) in joint i of link i with respect to link $i-1$. Now, among the geometry of the joints and the links, the relative coordinates $\theta_1, \theta_2, \ldots, \theta_m$ determine completely the position and orientation of the material points of all drives and links. Because no extra degrees of freedom are required to describe the kinematics of the manipulator, $\theta_1, \theta_2, \ldots, \theta_m$ can be seen as the elements of the column of rigid-body degrees of freedom $\underline{\theta} \in \mathcal{R}^m$ (as introduced in Section 2.4,) and the column $\underline{q}$ of all generalized coordinates can be chosen equal to $\underline{\theta}$:

$$\underline{q} = \underline{\theta} \, . \tag{2.41}$$

The kinetic energy, virtual work and potential energy.

In general, drive i ($i = 1, 2, \ldots, m$) consists of a frame, a motor and a (gear) transmission (see Fig. 2.3). The stator and the housing of the motor and of the transmission are connected rigidly to this frame, whereas the frame itself is connected rigidly to link $i-1$. Thus, the motion of these parts is determined completely by the motion of link $i-1$, and their contribution $\mathcal{T}_{si}$ to the kinetic energy of the manipulator can be taken into account in the kinetic energy of link $i-1$.

In this subsection, we assume that no deformations occur in the drives and, therefore, the rotation of the rotor, gear wheels, etc. of drive i with respect to the frame of that drive is determined by the motion of link i with respect to link $i-1$, i.e. by the relative coordinate θ_i of joint i. This implies that the kinetic energy $\mathcal{T}_{ri}$ due to the rotation of these parts can be taken into account in the kinetic energy of link i.

Let $\mathcal{T}_{li}$ be the kinetic energy of link i without the drive. Now, the total kinetic energy $\mathcal{T}$ of a rigid manipulator is given by

$$\mathcal{T} = \mathcal{T}_l + \mathcal{T}_d \; ; \qquad \mathcal{T}_l = \sum_{i=1}^{m} \mathcal{T}_{li} = \frac{1}{2}\dot{\underline{\theta}}^T M_l \dot{\underline{\theta}} \, , \quad \mathcal{T}_d = \sum_{i=1}^{m} (\mathcal{T}_{ri} + \mathcal{T}_{si}) = \frac{1}{2}\dot{\underline{\theta}}^T M_d \dot{\underline{\theta}} \, , \tag{2.42}$$

where M_l is the inertia matrix of the links without the drives and M_d is the inertia matrix of the drives. From these results, it is seen that $\mathcal{T}$ is a homogeneous, quadratic form in $\dot{\underline{\theta}}$ and, therefore, in $\dot{\underline{q}} = \dot{\underline{\theta}}$:

$$\mathcal{T} = \frac{1}{2}\dot{\underline{q}}^T M \dot{\underline{q}} \; ; \qquad M = M(\underline{q}, \underline{p}) \, , \quad M = \sum_{i=1}^{m} (M_{li} + M_{di}) \, , \tag{2.43}$$

where M is the symmetric, positive definite inertia matrix of the manipulator. In general, M has no special structure.

The total virtual work δW of all torques in and on the manipulator is the virtual work δW_u of the motor torques minus the variation $\delta \mathcal{V}$ of the potential $\mathcal{V}$ and minus a rest term $\delta \underline{\theta}^T \underline{f}$. As mentioned before, this last term is not considered in any detail.

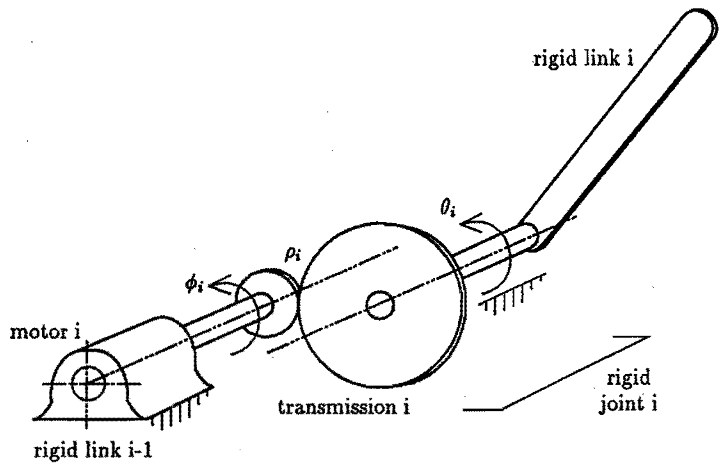


Figure 2.3: Rigid joints and rigid links.

The contribution of the motor in joint i to δW_u equals $\delta\theta_i H_i u_i$, where u_i is the torque generated by the motor and H_i is a constant scalar that accounts for the reduction ratio of the drive. The virtual work δW_u of all motors is given by

$$\delta W_u = \delta\underline{\theta}^T H \underline{u} = \delta\underline{q}^T H \underline{u}\ , \tag{2.44}$$

where H is a diagonal, regular distribution matrix with diagonal elements $H_1, H_2, \ldots, H_m$.

The potential $\mathcal{V}$ can be a function of the generalized coordinates $\underline{q} = \underline{\theta}$, of the unknown parameters $\underline{p}$ and possibly of time t. For rigid manipulators, $\mathcal{V}$ can consist of a contribution of conservative external loads (due to, for instance, gravity) and can, in general, be specified fairly straightforward. The conservative torques $\underline{v}$ can then be derived from Eq. (2.14).

The equations of motion.

As soon as the generalized torques $\underline{f}$ are specified, the equations of motion for a rigid manipulator can be derived via Lagrange's formalism. This results in

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + \underline{v} + \underline{f} = Hu; \qquad \underline{q} = \underline{\theta}\ . \tag{2.45}$$

The most significant characteristics of this set of equations are the diagonal structure of the distribution matrix H and the fact that this matrix is regular.

The use of relative coordinates as the generalized coordinates, i.e. the choice $\underline{q} = \underline{\theta}$, implies that the end-effector position and orientation is determined by the relative coordinates of all preceding links. Because of the accumulation of errors, this can cause numerical problems in solving the equations of motion, especially if the number m of links is large. These problems can be eliminated by performing a suitable coordinate transformation (Subsection 2.3.4), for instance from relative to absolute link coordinates. The coordinate transformation can be carried out either on-line by numerical procedures or off-line by using a symbolic manipulation program. A disadvantage of such transformations, especially for control purposes, is that the earlier-mentioned structure of the distribution matrix H is destroyed.

2.5.3 Manipulator with flexible joints and rigid links.

Introduction.

Deformations in the drives often significantly affect the response of a manipulator (see, for instance, Sweet and Good, 1984; Widmann and Ahmad, 1987). For present-day manipulators commonly used, these deformations are dominant compared to the deformations of the links. Therefore, as a first extension to rigid manipulators, manipulators with rigid links but flexible drives are considered. For the moment, it is assumed that all drives are flexible.

Because the links are rigid, their kinematics can be described by the relative coordinates $\theta_1, \theta_2, \ldots, \theta_m$, as in the previous subsection, and, therefore, we use the same definition for $\underline{\theta} \in \mathcal{R}^m$. But, unlike the situation in the previous subsection, the column $\underline{q}$ of generalized coordinates can now not be equal to $\underline{\theta}$ because extra coordinates are required to describe the deformations in the drives.

The number of extra degrees of freedom depends on the design and construction of the drives. A simple example of a procedure to model a flexible drive is sketched in Veldpaus (1992a). At least one extra degree of freedom per drive is required. We assume that it is not necessary to introduce more than one extra degree of freedom per drive[5]. With regard to drive i, this extra degree of freedom denoted by φ_i may, for instance, be the rotation of the rotor with respect to the axis of link $i-1$ (see Fig 2.4). If the extra degrees of freedom $\varphi_1, \varphi_2, \ldots, \varphi_m$ are seen as the elements of a column $\underline{\varphi} \in \mathcal{R}^m$, then the column $\underline{q}$ of all generalized coordinates can be defined by

$$\underline{q} = \begin{bmatrix} \underline{\theta} \\ \underline{\varphi} \end{bmatrix}, \tag{2.46}$$

where $\underline{\theta}$ represent the rigid-body coordinates and $\underline{\varphi}$ represent the flexible coordinates.

[5]This assumption is not as restrictive as it may seem at first sight. In the first stage of modeling a flexible drive, it may be necessary to introduce more than one extra degree of freedom. In the next stage, it is often possible to reduce the number of extra degrees of freedom, for instance by Guyan reduction (see Guyan, 1965). In general, and especially in controller design, a reduction of this type to only one extra degree of freedom or to a very small number of extra degrees of freedom per flexible link is unavoidable.

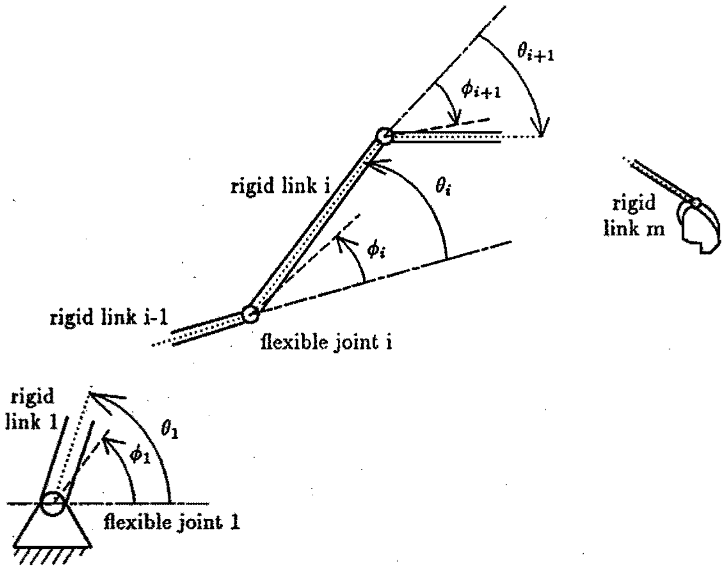


Figure 2.4: The kinematics of a manipulator with flexible joints and rigid links.

The kinetic energy, virtual work and potential energy.

In Veldpaus (1992a), a relation for the kinetic energy $\mathcal{T}_{di}$ of drive i as a function of $\dot{\underline{\theta}}$ and $\underline{\dot{\varphi}}$ is derived. This result, valid for a broad class of flexible drives, is of the form

$$\mathcal{T}_{di} = \frac{1}{2}\dot{\underline{\theta}}^T M_{si}\dot{\underline{\theta}} + \dot{\varphi}_i \underline{m}_{sri}^T \dot{\underline{\theta}} + \frac{1}{2} m_{ri}\dot{\varphi}_i^2 \ , \tag{2.47}$$

where M_{si} is the inertia matrix of drive i such that $\frac{1}{2}\dot{\underline{\theta}}^T M_{si}\dot{\underline{\theta}}$ is the kinetic energy of that drive if the angular velocity $\dot{\varphi}_i$ of the rotor with respect to the stator is equal to zero. In that case, the motions of all parts of the drive are determined completely by the motion of link $i-1$, which, in turn, is determined by $\dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_{i-1}$ and $\theta_1, \theta_2, \ldots, \theta_{i-1}$. This implies that all elements of the matrix M_{si} in row and column $i, i+1, \ldots, m$ are zero. The column $\underline{m}_{sri}$ in Eq. (2.47) accounts for the dynamic coupling between the angular velocity $\dot{\varphi}_i$ and the velocity of the frame of drive i. Because this velocity is determined by $\dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_{i-1}$, all elements of $\underline{m}_{sri}$ in row $i, i+1, \ldots, m$ are zero. Finally, the scalar m_{ri} in Eq. (2.47) is the moment of inertia of the rotating parts of drive i, such that $\frac{1}{2}m_{ri}\dot{\varphi}_i^2$ is the kinetic energy of the drive if the motion of the frame of that drive is suppressed, i.e. if $\dot{\theta}_1 = \dot{\theta}_2 = \cdots = \dot{\theta}_{i-1} = 0$.

The kinetic energy $\mathcal{T}_d$ of all drives is the sum over all drives of the kinetic energy per drive and is, therefore, given by

$$\mathcal{T}_d = \sum_{i=1}^{m} \mathcal{T}_{di} = \frac{1}{2}\dot{\underline{\theta}}^T M_s \dot{\underline{\theta}} + \dot{\underline{\theta}}^T M_{sr} \dot{\underline{\varphi}} + \frac{1}{2}\dot{\underline{\varphi}}^T M_r \dot{\underline{\varphi}} \ , \tag{2.48}$$

with $M_s = \sum_{i=1}^{m} M_{si}$ being an upper triangular matrix, $M_{sr} = [\, \underline{m}_{sr1} \ \underline{m}_{sr2} \ \ldots \ \underline{m}_{srm} \,]$ with zero main diagonal elements, and diagonal matrix $M_r = diag[\, m_{r1} \ m_{r2} \ \ldots \ m_{rm} \,]$.

The total kinetic energy $\mathcal{T} = \mathcal{T}_l + \mathcal{T}_d$ in the manipulator now becomes

$$\mathcal{T} = \frac{1}{2}\dot{\underline{\theta}}^T (M_l + M_s)\dot{\underline{\theta}} + \dot{\underline{\theta}}^T M_{sr} \dot{\underline{\varphi}} + \frac{1}{2}\dot{\underline{\varphi}}^T M_r \dot{\underline{\varphi}} \ , \tag{2.49}$$

or, written in terms of the column $\underline{q}$ of generalized coordinates,

$$\mathcal{T} = \frac{1}{2}\dot{\underline{q}}^T M \dot{\underline{q}}; \qquad \underline{q} = \begin{bmatrix} \underline{\theta} \\ \underline{\varphi} \end{bmatrix}; \quad M = \begin{bmatrix} M_l + M_s & M_{sr} \\ M_{sr}^T & M_r \end{bmatrix} . \tag{2.50}$$

The matrix M_{sr} shows that there is a dynamic coupling between the degrees of freedom $\underline{\varphi}$ and $\underline{\theta}$. For some special manipulator configurations, the coupling terms are exactly zero (Subsection 1.4.2). With regard to other configurations of flexible-joint manipulators, the coupling is often neglected by setting all elements of M_{sr} equal to zero (e.g., the model of Spong, 1987).

To determine the virtual work δW of all forces in and on the manipulator, we have to relate the virtual work δW_u of the motor torques, the variation $\delta \mathcal{V}$ of the potential $\mathcal{V}$ and the rest term $\delta \underline{q}^T \underline{f}$ to variations $\delta \underline{q}$ of the generalized coordinates. Again, we will not consider the rest term in any detail.

The contribution of the motor in drive i to δW_u is given by the product of the torque u_i, generated by this motor, and the variation $\delta\varphi_i$ of the rotation φ_i of the rotor with respect to the frame of this drive. Hence, the virtual work δW_u of all motors is equal to

$$\delta W_u = \delta \underline{\varphi}^T \underline{u} = \delta \underline{q}^T H \underline{u}; \qquad H = \begin{bmatrix} 0 \\ I \end{bmatrix} , \tag{2.51}$$

where 0 is the $(m \times m)$ null matrix and I the $(m \times m)$ unit matrix. It turns out that, with the selected degrees of freedom, the distribution matrix H has a quite specific structure.

The potential $\mathcal{V}$ for a manipulator with flexible joints consists of a contribution $\mathcal{V}_d$ due to the deformation of the joints plus a contribution $\mathcal{V}_r = \delta \underline{q}^T \underline{v}_n$ as for rigid manipulators (due to conservative external loading, such as gravity). Although, in general, torsional flexibility of drive i with a gear transmission is distributed over gear teeth and compliant drive shafts, it is assumed that the flexibility can be modeled by means of a linear elastic, torsional spring with stiffness k_{ri} on the rotor side of an ideal gear transmission plus a linear elastic spring with stiffness k_{li} on the link side of that transmission (see Fig. 2.5). Now, it can be shown that the elastic energy $\mathcal{V}_{di}$

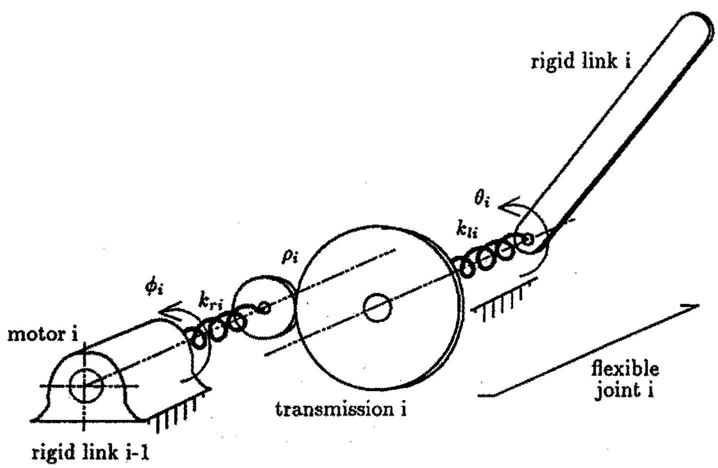


Figure 2.5: Flexible joints and rigid links.

of drive i is given by

$$\mathcal{V}_{di} = \frac{1}{2} k_i \varepsilon_i^2 \; ; \qquad k_i = \frac{k_{ri} k_{li}}{k_{ri} + \rho_i^2 k_{li}} \, , \tag{2.52}$$

where k_i is the equivalent stiffness, $\varepsilon_i = \rho_i \theta_i - \varphi_i$ characterizes the deformation and ρ_i is the reduction ratio such that $\varphi_i = \rho_i \theta_i$ if no deformation occurs in drive i. If the deformations $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m$ are seen as the elements of a column $\underline{\varepsilon} \in \mathcal{R}^m$, the relation for the elastic energy in all drives can be written as

$$\mathcal{V}_d = \sum_{i=1}^{m} \mathcal{V}_{di} = \frac{1}{2} \underline{\varepsilon}^T K_d \underline{\varepsilon} \; ; \qquad K_d = diag[k_1, k_2, \ldots, k_m] \, , \tag{2.53}$$

where K_d is the $\underline{\varepsilon}$-related, diagonal, positive definite stiffness matrix. With $R = diag[\, \rho_1 \;\; \rho_2 \;\; \ldots \;\; \rho_m \,]$ and $\underline{\varepsilon} = R\underline{\theta} - \underline{\varphi} = [\, R \;\; -I \,]\underline{q}$, the relation for this potential becomes

$$\mathcal{V}_d = \frac{1}{2} \underline{q}^T K \underline{q}; \qquad K = \begin{bmatrix} R^T K_d R & -R^T K_d \\ -K_d R & K_d \end{bmatrix} \, , \tag{2.54}$$

where K is the q-related, symmetric, semi-positive definite stiffness matrix.

The equations of motion.

With the given results for the kinetic energy, the virtual work and the potential energy, the use of the formalism of Lagrange results in

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f} = H\underline{u} \, . \tag{2.55}$$

In contrast to the set of equations of motion obtained in the previous subsection for rigid manipulators, the matrices in this set all have a special structure. However, now there are $2m$ equations instead of m.

In the given analysis, the relative coordinates in $\underline{\theta}$ and the rotor rotations $\underline{\varphi}$ are used as degrees of freedom. Because $\underline{\varphi} = R\underline{\theta} - \underline{\varepsilon}$, also the elements of $\underline{\theta}$ and $\underline{\varepsilon}$ can be used as the degrees of freedom. With

$$\underline{q} = E\underline{q}^* , \qquad \underline{q}^* = \begin{bmatrix} \underline{\theta} \\ \underline{\varepsilon} \end{bmatrix} , \quad \underline{q} = \begin{bmatrix} \underline{\theta} \\ \underline{\varphi} \end{bmatrix} , \quad E = \begin{bmatrix} I & 0 \\ R & -I \end{bmatrix} , \tag{2.56}$$

the equations of motion then become (Section 2.3.4):

$$M^*\ddot{\underline{q}}^* + C^*\dot{\underline{q}}^* + K^*\underline{q}^* + \underline{v}_n^* + \underline{f}^* = H^*\underline{u} , \tag{2.57}$$

where M^*, C^*, etc. can be determined using Eq. (2.28). With respect to M^*, this results in

$$M^* = \begin{bmatrix} M_l + M_s + R^TM_{sr} + M_{sr}R + R^TM_rR & -(M_{sr} + R^TM_r) \\ -(M_{sr} + R^TM_r)^T & M_r \end{bmatrix} , \tag{2.58}$$

and it can be shown that $M_l + M_s + R^TM_{sr} + M_{sr}R + R^TM_rR$ is equal to the total inertia matrix of the equivalent rigid manipulator. Furthermore, it is seen that the stiffness matrix K^* and distribution matrix H^* are given by

$$K^* = E^TKE = \begin{bmatrix} 0 & 0 \\ 0 & K_d \end{bmatrix} \quad ; \quad H^* = E^TH = \begin{bmatrix} R^T \\ -I \end{bmatrix} . \tag{2.59}$$

This form of the equations of motion, with $\underline{\theta}$ and $\underline{\varepsilon}$ as the degrees of freedom, is of particular interest if the deformation of some of the drives may be neglected. Suppose that drive i does not deform, i.e. that $\varepsilon_i(t) = 0$ for all t. Then, ε_i is not a degree of freedom in the sense of Lagrange and it should be removed from the column $\underline{q}$. The associated reduced set of equations of motion then follows from the set, given above, by removing equation $m + i$ and column $m + i$ from the matrices M^*, C^* and K^*. For the equivalent rigid manipulator, the distribution matrix H^* equals $R = R^T$.

2.5.4 Manipulator with rigid joints and flexible links.

Introduction.

Modeling of manipulators with flexible links is far more complex than of manipulators with the flexibility concentrated in the joints. Many procedures to model single flexible links have been given in literature (see, for instance, Book, 1984; Cannon and Schmitz, 1984; Fukuda, 1985). However, the modeling of multi-link flexible manipulators is far more difficult (e.g. Cetinkunt et al., 1986; Cetinkunt and Book, 1987; Asada et al. 1990). The number of degrees of freedom substantially increases and, due to the coupling between the links, the manipulator dynamics can no longer be represented by independent, single-beam equations.

Especially in the field of continuum mechanics, several classes of very powerful methods to model and analyze bodies with complex geometry, with an elastic or non-elastic material behavior, and with arbitrary loading have been developed during the last decades. Among them are the class of Finite Element Methods (see, for instance, Zienkiewicz, 1979; Usoro et al., 1986) and the class of Boundary Integral Methods (see, for instance, Hurty and Rubinstein, 1964; Shabana, 1989). Two common characteristics of these methods are their numerical approach and the approximate nature of the obtained solutions: the continuous displacement field in a body is approximated by a combination of a number of *chosen* displacement functions with time-dependent coefficients (e.g. Koppens, 1986; Koppens et al., 1988). The application of these methods in the field of robotics, in order to integrate manipulator models in the control loops, only recently started. As shown by examples investigated in literature, often no more than one (e.g. Cannon and Schmitz, 1984; Fukuda, 1985; Sakawa et al., 1985) or two (e.g. Book et al., 1975; Schmitz, 1986; Nathan and Singh, 1989; Das and Singh, 1989) flexible links in planar motion are considered. Modeling of flexible links with eigenfunctions as the displacement functions is mostly done using only those eigenfunctions that correspond to the lowest two eigenfrequencies. For control purposes, it is necessary to keep the number of degrees of freedom as small as possible. Therefore, the number of degrees of freedom of flexible manipulator models derived via one of the numerical approaches has to be considerably reduced, for instance, by Guyan reduction (Guyan, 1965).

It is not the intention here to review the literature on the modeling of flexible-link manipulators. In this subsection, only one possible procedure will be sketched. First, the kinematics of a multi-link flexible manipulator will be considered and a suitable set of generalized coordinates for the rigid-body motion of the manipulator will be introduced. Then, we will concentrate on one flexible link in two-dimensional space and specify how the equations of motion of this single link can be derived (Veldpaus, 1992b). We do not strive for generality. One restriction is that only *elastic* material behavior will be considered. The choice of the modeling method is determined by the fact that Lagrange's formalism is used in this thesis as a tool to derive the equations of motion of a dynamic system. Therefore, the procedure in this subsection will also be based on the formalism of Lagrange. It involves an approximation of the displacement field in the link by a linear combination of a finite number of known, smooth functions. The coefficients of these chosen functions are unknown functions of time and are interpreted as the extra degrees of freedom of the deforming link. The kinetic energy, the potential energy and the virtual work of the torques on the links are functions of these degrees of freedom. Finally, the formalism of Lagrange will be used to derive a mathematical model for multi-flexible-link manipulators.

The kinematics of a flexible-link manipulator.

We consider the planar manipulator in Fig. 2.6 as an open chain of rigid drives and m flexible links. The links may deform as shown by the curved, solid lines in this figure. The total motion of link i can be decomposed into a part due to the motion of the link as a rigid body, and a part due to the deformation of the link. The main problem with this decomposition is that the notion "motion of the link as a rigid

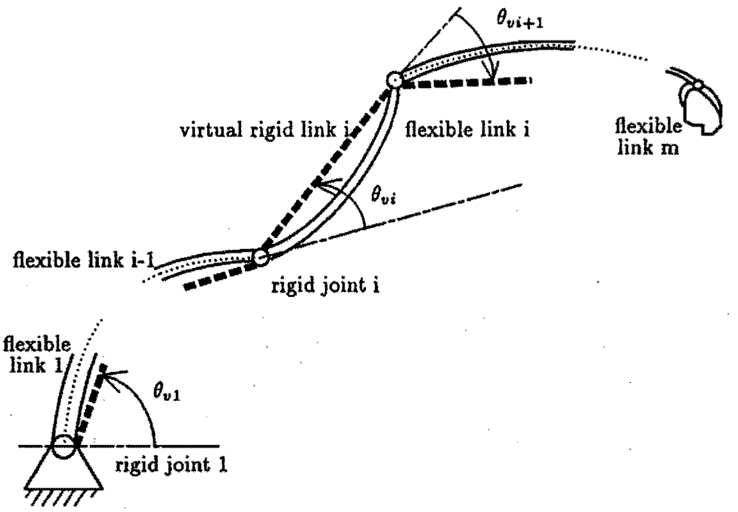


Figure 2.6: The kinematics of a manipulator with rigid joints and flexible links.

body" is not uniquely defined if the link deforms. One possibility to get around this problem, is to introduce a so-called *virtual rigid link* for each deforming link. The virtual rigid link i ($i = 1, 2, \ldots, m$) is an imaginary straight link between the adjacent joints i and $i+1$ of the real link i. In Fig. 2.6, the virtual rigid links are represented by straight, dashed lines. The displacements of all joints can now be represented by the relative or absolute coordinates of the virtual rigid links, just as in the case of a rigid manipulator. In accordance with the selected generalized coordinates for a rigid manipulator, we will use the relative coordinates $\theta_{v1}, \theta_{v2}, \ldots, \theta_{vm}$, where θ_{vi} is the rotation (or translation) of the virtual rigid link i with respect to the virtual rigid link $i-1$. The use of these coordinates reduces the computational complexity in comparison with other coordinates as, for example, the tangents used by Book et al. (1975) and others (see Fig. 2.7). There, the tangent coordinate θ_{ti} of link i is the relative rotation of the tangent of that link in joint i with respect to the tangent of link $i-1$ in the same joint. The use of the above introduced, so-called virtual rigid link coordinates $\underline{\theta}_v = [\ \theta v1 \ \ \theta_{v2} \ \ \ldots \ \ \theta_{vm}\ ]^T$ has advantages, for instance, in solving the inverse dynamics problem because the position and orientation of link i and, more important, the position of joint i depend only on $\theta_{v1}, \theta_{v2}, \ldots, \theta_{v(i-1)}$. In contrast, the kinematics of link i in the tangent coordinate system does not only depend on the tangent coordinates $\theta_{t1}, \theta_{t2}, \ldots, \theta_{ti}$ but also on the deformations of the preceding links, which results in more complex equations of motion.

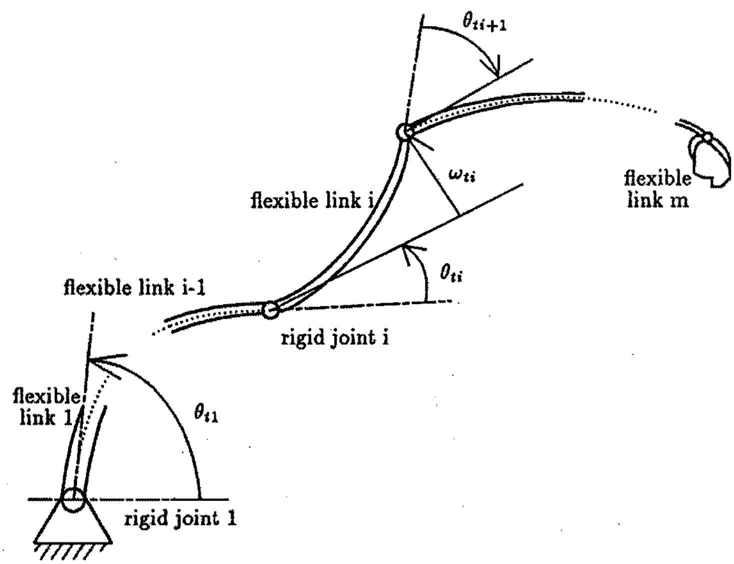


Figure 2.7: A tangent coordinate system.

The kinematics of one flexible link.

We consider a flexible link i, which deforms as shown by the curved, solid line in Fig. 2.8 (Veldpaus, 1992b). In the stress-free configuration, the link is cylindrical and homogeneous with a length l_i, a cross sectional area A_i and a density d_i. We assume that the material of the link is linear elastic with Young's modulus E_i and that the axis through the geometrical center of the cross section is inextensible and is a central principal axis of that cross section. The cross sectional area moment with respect to this axis is denoted by I_i. Classical beam theory is applied, i.e. it is assumed that planes, perpendicular to the beam axis in the stress-free configuration, remain plane and perpendicular to the deformed axis in the current configuration and that the mass of the link may be concentrated on that axis. Therefore, only the mass $\mu_i = d_i A_i$ per unit length is relevant.

The total motion of any material point of link i can be decomposed into a part due to the *motion of the link as a rigid body* and a part due to the *deformation of the link*. Because linear beam theory is adopted, only small deformations are allowed and axial displacements can be neglected. The rigid-body motion of link i is considered to be the motion of the virtual rigid link i with length l_i, which is shown by the dashed, straight line in Fig 2.8. The end points of the link axis are joint i and joint $i+1$. Let ξ_i be the axial coordinate of a material point $\mathcal{P}(\xi_i)$ on the axis of the link, such that $\xi_i = 0$ in joint i and $\xi_i = l_i$ in joint $i+1$. Furthermore, let $\vec{\chi}_i(\xi_i, t)$

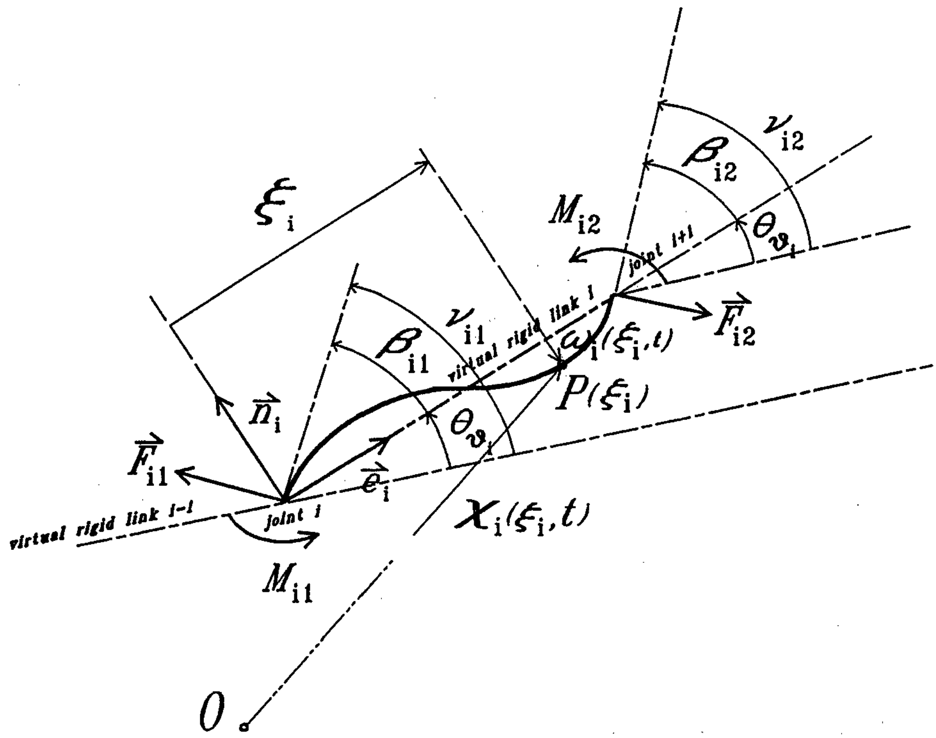


Figure 2.8: The kinematics of one flexible link.

be the absolute position vector of $\mathcal{P}(\xi_i)$ of the deformed link i at time t and let $\vec{e}_i(t) = (\vec{\chi}_i(l_i,t) - \vec{\chi}_i(0,t))/l_i$ be the unit vector along the axis of the virtual rigid link i and let $\vec{n}_i(t)$ be the unit normal on this axis. Now, the deformation of link i can be written as $\vec{w}_i(\xi_i,t) = \vartheta_i(\xi_i,t)\vec{e}_i(t) + w_i(\xi_i,t)\vec{n}_i(t)$. Because linear beam theory is adopted, only small deformations are allowed and the axial displacement $\vartheta_i(\xi_i,t)$ can be neglected. Hence, $\vec{w}_i(\xi_i,t) = w_i(\xi_i,t)\vec{n}_i(t)$ and the position vector $\vec{\chi}_i(\xi_i,t)$ of $\mathcal{P}(\xi_i)$ can be written as

$$\vec{\chi}_i(\xi_i,t) = \vec{\chi}_i(0,t) + \xi_i\vec{e}_i(t) + \vec{w}_i(\xi_i,t) = \vec{\chi}_i(0,t) + \xi_i\vec{e}_i(t) + w_i(\xi_i,t)\vec{n}_i(t) \; ; \qquad (2.60)$$
$$w_i(0,t) = w_i(l_i,t) = 0 \; .$$

The relation for the velocity $\dot{\vec{\chi}}_i(\xi_i,t)$ follows with $\dot{\vec{e}}_i(t) = \dot{\theta}_{vi}(t)\vec{n}_i(t)$ and $\dot{\vec{n}}_i(t) = -\dot{\theta}_{vi}(t)\vec{e}_i(t)$. This gives

$$\dot{\vec{\chi}}_i(\xi_i,t) = \dot{\vec{\chi}}(0,t) + \dot{\theta}_{vi}(t)[\xi_i\vec{n}_i(t) - w_i(\xi_i,t)\vec{e}_i(t)] + \dot{w}_i(\xi_i,t)\vec{n}_i(t) \; . \qquad (2.61)$$

Due to the inextensibility condition of the link axis, the variation $\delta\vec{\chi}_i(l_i,t)$ of the position vector $\vec{\chi}_i(l_i,t)$ of joint $i+1$ has to satisfy $\delta\vec{\chi}_i(l_i,t) = \delta\vec{\chi}_i(0,t) + l_i\delta\theta_{vi}\vec{e}_i$.

Discretization of the deformation field.

Classical beam theory yields a partial differential equation for the field $w_i = w_i(\xi_i, t)$. In order to transform the partial differential equation for a flexible link into a set of ordinary differential equations, the field $w_i(\xi_i, t)$ is approximated by a linear combination of appropriately chosen functions of the axial coordinate ξ_i, with coefficients that can be functions of time t (e.g. Hurty and Rubinstein, 1964):

$$w_i(\xi_i, t) = \sum_{j=1}^{a_i} \psi_{ij}(\xi_i)\alpha_{ij}(t) = \underline{\psi}_i^T(\xi_i)\underline{\alpha}_i(t) \ . \tag{2.62}$$

Here, the elements of $\underline{\psi}_i = \underline{\psi}_i(\xi_i)$ are the chosen, so-called interpolation functions and the elements of $\underline{\alpha} = \underline{\alpha}(t)$ are the new flexible degrees of freedom. The choice of the functions $\underline{\psi}_i$ is of great importance for the quality of the final results and there is a lot of literature on this subject (see, for instance, Hurty and Rubinstein, 1964; Koppens, 1986; Koppens et al., 1988). Often, a set of so-called eigenmodes are used, for instance, a combination of the lowest eigenmodes of a hinged-clamped beam, a clamped-hinged beam and of a clamped-clamped beam. For more information on these eigenmodes, we refer to Flűge (1967). However, using eigenfunctions as the interpolation functions is quite complicated and, in reality, the boundary conditions of a manipulator link are neither those of a clamped beam nor those of a hinged beam. Therefore, it is at least disputable whether the use of eigenfunctions instead of *simple polynomials* is preferable.

The interpolation functions have to be continuous and at least twice differentiable. Furthermore, they have to satisfy $w_i(0,t) = w_i(l_i,t) = 0$ for all $\underline{\alpha}_i$, so

$$\underline{\psi}_i(0) = \underline{\psi}_i(l_i) = \underline{0} \ . \tag{2.63}$$

In practice, it is very unattractive to formulate the approximation for the deformation field in terms of quantities α_{i1}, α_{i2}, ... without physical interpretation. The reason is that the considered beam is a model for just one link i of a manipulator. In determining, for instance, the kinetic energy $\mathcal{T}$ of the manipulator from the sum $\mathcal{T}_1 + \mathcal{T}_2 + \cdots + \mathcal{T}_m$ of the kinetic energy of all links, compatibility between the links must be guaranteed. Because each joint allows exactly one relative motion of the coupled links, each joint introduces five (for three dimensional problems) or two (for two dimensional problems, as in this section) constraint equations for the displacements and rotations of the coupled links. To take these constraints into account, it is advantageous to relate the unknown parameters in the relation for the discretized deformation field to the position vector and to the total rotation of the nodal points, i.e. of those points of the link in which connections with other links can be realized. With respect to $w_i(\xi_i, t)$, this means that we want to use ν_{i1} and ν_{i2} as parameters in the discretized field, where ν_{i1} is the angle at joint i and ν_{i2} is the angle at joint $i+1$ between the tangent of the deformed axis of link i and the axis of the virtual rigid link $i-1$ (see Fig. 2.8). A trivial alternative is to use the angles $\beta_{i1} = \nu_{i1} - \theta_{vi}$ and $\beta_{i2} = \nu_{i2} - \theta_{vi}$, instead of ν_{i1} and ν_{i2}, as parameters. β_{i1} is the angle at joint i and β_{i2} is the angle at joint $i+1$ between the tangent of the deformed axis of link i and the axis of the virtual rigid link i (see also Fig. 2.8). Now, the discretized deformation field can be written as

$$\begin{aligned} w_i(\xi_i,t) &= \psi_{i1}(\xi_i)\beta_{i1}(t) + \psi_{ia_i}(\xi_i)\beta_{i2}(t) + \sum_{j=2}^{a_i-1}\psi_{ij}(\xi_i)\epsilon_{ij}(t) = \\ &= \underline{\psi}_{\beta i}^T(\xi_i)\underline{\beta}_i(t) + \underline{\psi}_{\epsilon i}^T(\xi_i)\underline{\epsilon}_i(t) = \underline{\psi}_i^T(\xi_i)\underline{\alpha}_i(t) \ , \end{aligned} \tag{2.64}$$

where $\underline{\epsilon}_i$ is a column with $a_i - 2$ elements, while $\underline{\psi}_i$, $\underline{\psi}_{\beta i}$, $\underline{\alpha}_i$ and $\underline{\beta}_i$ are defined by

$$\underline{\psi}_i = \begin{bmatrix} \psi_{i1} \\ \underline{\psi}_{\epsilon i} \\ \psi_{ia_i} \end{bmatrix}; \quad \underline{\psi}_{\beta i} = \begin{bmatrix} \psi_{i1} \\ \psi_{ia_i} \end{bmatrix}; \quad \underline{\alpha}_i = \begin{bmatrix} \beta_{i1} \\ \underline{\epsilon}_i \\ \beta_{i2} \end{bmatrix}; \quad \underline{\beta}_i = \begin{bmatrix} \beta_{i1} \\ \beta_{i2} \end{bmatrix} . \tag{2.65}$$

Without loss of generality, we normalize the elements of $\underline{\psi}_i = \underline{\psi}_i(\xi_i)$ such that their maximum absolute value is equal to 1. Apart from the condition (2.63), extra requirements have to be imposed because the elements β_{i1} and β_{i2} of $\underline{\alpha}_i$ have a well defined physical meaning. In the adopted linear theory, the angle $\nu_i(\xi_i,t)$ of link i is equal to $\theta_{vi}(t) + \partial w_i(\xi_i,t)/\partial\xi_i$. Therefore, from the physical meaning of β_{i1} and β_{i2}, it is seen that $\partial w_i(\xi_i,t)/\partial\xi_i$ has to satisfy

$$\left(\frac{\partial w_i(\xi_i,t)}{\partial\xi_i}\right)_{\xi_i=0} = \beta_{i1}; \quad \left(\frac{\partial w_i(\xi_i,t)}{\partial\xi_i}\right)_{\xi_i=l_i} = \beta_{i2} \ . \tag{2.66}$$

This means that the derivatives of the elements of $\underline{\psi}_i = \underline{\psi}_i(\xi_i)$ with respect to ξ_i have to verify the extra conditions

$$\left(\frac{\partial\underline{\psi}_i(\xi_i)}{\partial\xi_i}\right)_{\xi_i=0} = \underline{a}_1; \quad \left(\frac{\partial\underline{\psi}_i(\xi_i)}{\partial\xi_i}\right)_{\xi_i=l_i} = \underline{a}_{a_i} \ , \tag{2.67}$$

where $\underline{a}_1 = [\,1 \;\; 0 \;\; \ldots \;\; 0\,]^T$ and $\underline{a}_{a_i} = [\,0 \;\; 0 \;\; \ldots \;\; 1\,]^T$.
As a consequence, neither the displacements nor the rotations of the end planes of the beam are influenced by $\underline{\epsilon}_i$, and the elements of this column do not play any role in the joint-induced constraint conditions mentioned earlier. For this reason, $\underline{\epsilon}_i$ are called the *internal degrees of freedom* or the *internal coordinates* of the link i. It is clear that the use of virtual rigid links has another advantage among the fact that it results in a less complex model: the boundary conditions each flexible link i has to satisfy can be simply formulated, such that the begin and end position and the orientation of link i have to correspond with the position and orientation of the joints at the begin and end of link i, which in turn depend only on the coordinates of the preceding virtual rigid links and not on the internal coordinates.

The kinetic energy, virtual work and potential energy.

The kinetic energy $\mathcal{T}_i$ in a bending link i is given by the relation

$$\mathcal{T}_i = \frac{1}{2}\int_{\xi_i=0}^{l_i} \mu_i \dot{\vec{\chi}}_i(\xi_i,t)\cdot\dot{\vec{\chi}}_i(\xi_i,t)d\xi_i \ . \tag{2.68}$$

The potential $\mathcal{V}$ for a manipulator with flexible links consists of a contribution $\mathcal{V}_e$ due to the deformation of the links plus a contribution $\mathcal{V}_r = \partial \underline{q}^T \underline{v}_n$ as for rigid manipulators. The elastic energy $\mathcal{V}_{ei}$ for a linear elastic link i with Young's modulus E_i and cross sectional area moment I_i is given by

$$\mathcal{V}_{ei} = \frac{1}{2}\int_{\xi_i=0}^{l_i} E_i I_i (\frac{\partial^2 w_i(\xi_i,t)}{d\xi_i^2})^2 d\xi_i \ . \tag{2.69}$$

Link i is loaded by a force $\vec{F}_{i1}$ and a torque M_{i1} in joint i and by a force $\vec{F}_{i2}$ and a torque M_{i2} in joint $i+1$ (see Fig 2.8). Thus, the expression for the virtual power δW_i of all external loading on link i is given by

$$\delta W_i = \vec{F}_{i1} \circ \delta\vec{\chi}_i(0,t) + M_{i1}\delta\nu_{i1} + \vec{F}_{i2} \circ \delta\vec{\chi}_i(l_i,t) + M_{i2}\delta\nu_{i2} \ . \tag{2.70}$$

With the discretized deformation field (2.64), the relation (2.68) for the kinetic energy $\mathcal{T}_i$ of link i can be elaborated and $\mathcal{T}_i$ can be determined as a function of $\underline{\alpha}_i$ and $\underline{\dot{\alpha}}_i$ (Veldpaus, 1992b):

$$\mathcal{T}_i = \mathcal{T}_{ri} - \dot{\theta}_{vi}(\dot{\vec{\chi}}_i(0,t) \circ \vec{e}_i)\underline{\gamma}_i^T\underline{\alpha}_i + (\dot{\vec{\chi}}_i(0,t) \circ \vec{n}_i)\underline{\gamma}_i^T\underline{\dot{\alpha}}_i + \dot{\theta}_{vi}\underline{h}_i^T\underline{\dot{\alpha}}_i + \frac{1}{2}\underline{\dot{\alpha}}_i^T M_i \underline{\dot{\alpha}}_i \ , \tag{2.71}$$

where $\mathcal{T}_{ri}$ represents the kinetic energy of link i if no deformations occur:

$$\mathcal{T}_{ri} = \frac{1}{2}m_i[\dot{\vec{\chi}}_i(0,t) \circ \dot{\vec{\chi}}_i(0,t) + l_i\dot{\theta}_{vi}\dot{\vec{\chi}}_i(0,t) \circ \vec{n}_i + \frac{1}{3}l_i^2\dot{\theta}_i^2] \ . \tag{2.72}$$

The columns $\underline{\gamma}_i$ and $\underline{h}_i$ and the so-called $\underline{\alpha}_i$-related inertia matrix M_i are given by

$$\begin{aligned} \underline{\gamma}_i &= \int_{\xi_i=0}^{l_i} \mu_i \underline{\psi}_i(\xi_i) d\xi_i; \quad \underline{h}_i = \int_{\xi_i=0}^{l_i} \mu_i \xi_i \underline{\psi}_i(\xi_i) d\xi_i; \\ M_i &= \int_{\xi_i=0}^{l_i} \mu_i \underline{\psi}_i(\xi_i)\underline{\psi}_i^T(\xi_i) d\xi_i \ . \end{aligned} \tag{2.73}$$

They can be partitioned similarly to the partition of $\underline{\alpha}_i$ in β_{i1}, β_{i2} and the column $\underline{\epsilon}_i$, resulting in

$$\underline{\alpha}_i = \begin{bmatrix} \beta_{i1} \\ \underline{\epsilon}_i \\ \beta_{i2} \end{bmatrix}; \quad \underline{\gamma}_i = \begin{bmatrix} \gamma_{\beta i1} \\ \underline{\gamma}_{\epsilon i} \\ \gamma_{\beta i2} \end{bmatrix}; \quad \underline{h}_i = \begin{bmatrix} h_{\beta i1} \\ \underline{h}_{\epsilon i} \\ h_{\beta i2} \end{bmatrix}; \quad M_i = \begin{bmatrix} m_{11} & \underline{m}_{\epsilon 1}^T & m_{12} \\ \underline{m}_{\epsilon 1} & M_{\epsilon\epsilon} & \underline{m}_{\epsilon 2} \\ m_{12} & \underline{m}_{\epsilon 2}^T & m_{22} \end{bmatrix}_i . \tag{2.74}$$

The elastic energy for link i follows from Eq. (2.69). With the discretized deformation field according to Eq. (2.64), it is seen that

$$\mathcal{V}_{ei} = \frac{1}{2}\underline{\alpha}_i^T K_i \underline{\alpha}_i \ , \tag{2.75}$$

where K_i is the so-called $\underline{\alpha}_i$-related stiffness matrix, which is given by

$$K_i = \int_{\xi_i=0}^{l_i} E_i I_i \frac{d^2\underline{\psi}_i(\xi_i)}{d\xi_i^2}(\frac{d^2\underline{\psi}_i^T(\xi_i)}{d\xi_i^2}) d\xi_i \ . \tag{2.76}$$

Partition of this matrix, similarly to the partition of the inertia matrix, results in

$$K_i = \begin{bmatrix} k_{11} & \underline{k}_{\epsilon 1}^T & k_{12} \\ \underline{k}_{\epsilon 1} & K_{\epsilon\epsilon} & \underline{k}_{\epsilon 2} \\ k_{12} & \underline{k}_{\epsilon 2}^T & k_{22} \end{bmatrix}_i . \tag{2.77}$$

None of the columns and matrices $\underline{\gamma}_i$, $\underline{h}_i$, M_i and K_i depend on the degrees of freedom. They are constant and are determined completely by the properties μ_i and E_iI_i of link i together with the chosen interpolation functions.

Elimination of internal degrees of freedom of one link.

Using the formalism of Lagrange and using the given relations for the kinetic energy $\mathcal{T}_i$, the potential energy $\mathcal{V}_i$ and the virtual work δW_i, the equations of motion for link i can be derived. This results in a set of equations in $\dot{\vec{\chi}}_i(0,t)$ and $\ddot{\vec{\chi}}_i(0,t)$, $\dot{\theta}_{vi}$ and $\ddot{\theta}_{vi}$, $\underline{\beta}_i$ and $\ddot{\underline{\beta}}_i$, $\underline{\epsilon}_i$ and $\ddot{\underline{\epsilon}}_i$. Most of these equations contain the forces and torques in the end planes of the beam. However, the equations for the internal degrees of freedom $\underline{\epsilon}_i$ do not depend on these forces and torques. They are given by (Veldpaus, 1992b):

$$\underline{\gamma}_{\epsilon i}(\vec{n}_i \circ \ddot{\vec{\chi}}_i(0,t)) + \underline{h}_{\epsilon i}\ddot{\theta}_{vi} + M_{\beta\epsilon i}\ddot{\underline{\beta}}_i + M_{\epsilon\epsilon i}\ddot{\underline{\epsilon}}_i - 2\underline{\gamma}_{\epsilon i}(\vec{e}_i \circ \dot{\vec{\chi}}_i(0,t))\dot{\theta}_{vi} + K_{\beta\epsilon i}\underline{\beta}_i + K_{\epsilon\epsilon i}\underline{\epsilon}_i = \underline{0} , \tag{2.78}$$

where $M_{\beta\epsilon i} = [\, \underline{m}_{\epsilon 1} \;\; \underline{m}_{\epsilon 2} \,]_i$ and $K_{\beta\epsilon i} = [\, \underline{k}_{\epsilon 1} \;\; \underline{k}_{\epsilon 2} \,]_i$. In most of the applications, the inertia term $\underline{\gamma}_{\epsilon i}(\vec{n}_i \circ \ddot{\vec{\chi}}_i(0,t)) + \cdots - 2\underline{\gamma}_{\epsilon i}(\vec{e}_i \circ \dot{\vec{\chi}}_i(0,t))\dot{\varphi}_i$ will be quite small compared to the stiffness terms $K_{\epsilon\epsilon i}\underline{\epsilon}_i$ and $K_{\beta\epsilon i}\underline{\beta}_i$. If this inertia term is neglected, a linear, static equation for the internal degrees of freedom results. This can be solved for $\underline{\epsilon}_i$, which yields

$$\underline{\epsilon}_i = -K_{\epsilon\epsilon i}^{-1} K_{\beta\epsilon i}\underline{\beta}_i . \tag{2.79}$$

In this way, the column $\underline{\alpha}_i = [\, \beta_{i1} \;\; \underline{\epsilon}_i^T \;\; \beta_{i2} \,]^T$ can be written as a function of $\underline{\beta}_i$:

$$\underline{\alpha}_i = G_i\underline{\beta}_i , \qquad G_i = \begin{bmatrix} h_1 \\ -K_{\epsilon\epsilon i}^{-1} K_{\beta\epsilon i} \\ h_2 \end{bmatrix} , \quad h_1 = [\, 1 \;\; 0 \,] , \quad h_2 = [\, 0 \;\; 1 \,] , \tag{2.80}$$

where G_i is called the Guyan or static reduction matrix of link i (see, for instance, Guyan, 1965).

With this result for $\underline{\alpha}_i$, the final relation for the kinetic energy of a flexible link i, after the elimination of the internal degrees of freedom, is given by

$$\mathcal{T}_i = \mathcal{T}_{ri} - \dot{\theta}_{vi}(\dot{\vec{\chi}}_i(0,t) \circ \vec{e}_i)\underline{\gamma}_{\beta i}^T\underline{\beta}_i + (\dot{\vec{\chi}}_i(0,t) \circ \vec{n}_i)\underline{\gamma}_{\beta i}^T\dot{\underline{\beta}}_i + \dot{\theta}_i\underline{h}_{\beta i}^T\dot{\underline{\beta}}_i + \frac{1}{2}\dot{\underline{\beta}}_i^T M_{\beta i}\dot{\underline{\beta}}_i , \tag{2.81}$$

where

$$\underline{\gamma}_{\beta i} = G_i^T\underline{\gamma}_i; \quad \underline{h}_{\beta i} = G_i^T\underline{h}_i; \quad M_{\beta i} = G_i^T M_i G_i . \tag{2.82}$$

Besides, the elastic energy $\mathcal{V}_{ei}$ can be written as a function of $\underline{\beta}_i$. This yields

$$\mathcal{V}_{ei} = \frac{1}{2}\underline{\beta}_i^T K_{\beta i}\underline{\beta}_i ; \quad K_{\beta i} = G_i^T K_i G_i . \tag{2.83}$$

The equations of motion.

In the previous part of this subsection, a possible procedure to derive the expressions for the kinetic energy $\mathcal{T}_i$, the elastic energy $\mathcal{V}_{ei}$ and the virtual power ∂W_i of one flexible link i is sketched. For more details we refer to Veldpaus (1992b). In this last part, we show the global model structure of a multi-link flexible manipulator, which can be obtained by summarizing the energy quantities per link and using Lagrange's formalism.

The total energy $\mathcal{T}$ in the manipulator with m flexible links can be determined, as mentioned, by summarizing the equivalent quantities per link i:

$$\mathcal{T} = \sum_{i=1}^{m} \mathcal{T}_i = \frac{1}{2}\dot{\underline{\theta}}_v^T M_{vv} \dot{\underline{\theta}}_v + \dot{\underline{\theta}}_v^T M_{\beta v} \dot{\underline{\beta}} + \frac{1}{2}\dot{\underline{\beta}}^T M_{\beta\beta} \dot{\underline{\beta}} = \frac{1}{2}\dot{\underline{q}}^T M \dot{\underline{q}} \,, \qquad (2.84)$$

$$\underline{q} = \begin{bmatrix} \underline{\theta}_v \\ \underline{\beta} \end{bmatrix} , \quad M = \begin{bmatrix} M_{vv} & M_{v\beta} \\ M_{v\beta}^T & M_{\beta\beta} \end{bmatrix} ,$$

where M is the symmetric, positive definite inertia matrix. In general, this matrix has no special structure, because there is a strong dynamic coupling between all degrees of freedom $\underline{q}$.

As already mentioned, the potential $\mathcal{V}$ for a manipulator with flexible links consists of a contribution $\mathcal{V}_e$ due to the deformation of the links plus a contribution $\mathcal{V}_r = \partial \underline{q}^T \underline{v}_n$ as for the equivalent rigid manipulator (for instance, due to conservative external loading). The relation for the elastic energy in all flexible links, $\mathcal{V}_e$, can be written as

$$\mathcal{V}_e = \sum_{i=1}^{m} \mathcal{V}_{ei} = \frac{1}{2}\underline{\beta}^T K_\beta \underline{\beta} = \frac{1}{2}\underline{q}^T K \underline{q} \,; \qquad K = \begin{bmatrix} 0 & 0 \\ 0 & K_\beta \end{bmatrix} , \qquad (2.85)$$

where K is the $\underline{q}$-related stiffness matrix. This matrix has a special structure, because its non-zero elements are related only to the extra degrees of freedom $\underline{\beta}$ and not to the coordinates $\underline{\theta}_v$ of the virtual rigid links. The symmetric, semi-positive definite matrix K_β is composed of the individual stiffness matrices $K_{\beta i}$ per flexible link.

Finally, via Lagrange's formalism a set of strongly nonlinear and coupled equations of motion is obtained in the next form:

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f} = H\underline{u} \,. \qquad (2.86)$$

In this dynamic model, only the stiffness matrix K has a special structure.

2.5.5 Flexible manipulator.

Introduction.

In the previous subsections, we showed the global structures of the models of manipulators with increasing degree of flexibility. If flexibility in both joints and links play an important role, we have to account for them in the manipulator model. In this last subsection, we also make a distinction between rigid and flexible joints, as well between rigid and flexible links.

Assume, that there are d flexible joints and $m-d$ rigid joints. Let $\underline{\theta} \in \mathcal{R}^m$ be the column of relative coordinates of the (virtual) rigid links. Now, for the sequel, it is advantageous to regroup these coordinates in two columns $\underline{\theta}_s \in \mathcal{R}^{m-d}$ and $\underline{\theta}_e \in \mathcal{R}^d$, where $\underline{\theta}_s$ contains the coordinates of the direct-driven (virtual) rigid links (i.e. driven via the rigid joints) and $\underline{\theta}_e$ contains the coordinates of the elastically-driven links (i.e. driven via the flexible joints). The coordinates of the rotors at the flexible joints are the elements of the column $\underline{\varphi} \in \mathcal{R}^d$, so that the column of deformations in the flexible joints is defined by

$$\underline{\varepsilon} = R\underline{\theta}_e - \underline{\varphi} \, . \tag{2.87}$$

Further, we regroup the column $\underline{u}$ of actuator torques in two subcolumns $\underline{u}_s \in \mathcal{R}^{m-d}$ and $\underline{u}_e \in \mathcal{R}^d$, which contain the torques exerted at the rigid and the flexible joints, respectively.

Another assumption is that v links are relatively flexible while $m-v$ links are relatively stiff. Now, we introduce, for instance, one extra degree of freedom per flexible link and define $\underline{\beta}_f \in \mathcal{R}^v$ to be the column of extra degrees of freedom due to the bending of v links[6]. In this way, we get a total column of degrees of freedom which consists of m (virtual) rigid link coordinates $\underline{\theta} = [\ \underline{\theta}_s^T \ \ \underline{\theta}_e^T\ ]^T$and $d+v$ extra degrees of freedom $\underline{\varrho} = [\ \underline{\beta}_f^T \ \ \underline{\phi}^T\ ]^T$ due to the deformation of links and joints. One possible definition of the column of generalized coordinates $\underline{q} \in \mathcal{R}^n$ may now be

$$\underline{q} = \begin{bmatrix} \underline{\theta}_s \\ \underline{\theta}_e \\ \underline{\beta}_f \\ \underline{\varphi} \end{bmatrix} , \tag{2.88}$$

where $n = m + d + v$ is the number of generalized coordinates.

The equations of motion.

Applying Lagrange's formalism yields a dynamic model for the flexible manipulator, which is, in general, of the next form:

$$M\underline{\ddot{q}} + C\underline{\dot{q}} + K\underline{q} + \underline{v}_n + \underline{f} = H\underline{u} \, , \tag{2.89}$$

[6]If β_{i1}, being the angle between the axis of the virtual rigid link i and the tangent of the deformed link i in joint i, is assumed to be equal to $\beta_{(i-1)2}$, being the angle between the axis of the virtual rigid link $i-1$ and the tangent of the deformed link $i-1$ in the same joint i (i.e. we assume that all adjacent virtual rigid links are connected rigidly to each other), this means that β_{i2} of link i is the only extra coordinate for approximating the deformation of link i if we also have eliminated the internal degrees of freedom of this link (for instance, via the Guyan reduction method as indicated in the previous subsection). In this way, for the sake of simplicity, we can assume that $\beta_{12}, \beta_{22}, \ldots, \beta_{v2}$ are the extra degrees of freedom for approximately modeling the manipulator with v flexible links.

where the matrices and columns have no special structures except the stiffness matrix K:

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & R^T K_d R & 0 & -R^T K_d \\ 0 & 0 & K_\beta & 0 \\ 0 & -K_d R & 0 & K_d \end{bmatrix} . \qquad (2.90)$$

Considering the control problem of flexible manipulators, this special structure of the stiffness matrix can be useful for control purposes, as will be explained in Chapter 4.

2.6 Properties of manipulator dynamics.

In this chapter, Lagrange's formalism has been used to derive the dynamic model of a flexible manipulator. To resume, the dependence of the model matrices and columns on the generalized coordinates $\underline{q}$, on the generalized velocities $\dot{\underline{q}}$, on the unknown but constant parameters $\underline{p}$ and, possibly, on time t is shown here explicitly:

$$M(\underline{q},\underline{p})\ddot{\underline{q}} + C(\underline{q},\dot{\underline{q}},\underline{p})\dot{\underline{q}} + K(\underline{p})\underline{q} + \underline{v}_n(\underline{q},\underline{p}) + \underline{f}(\underline{q},\dot{\underline{q}},\underline{p},t) = H(\underline{q})\underline{u} \; , \qquad (2.91)$$

where $K(\underline{p})\underline{q} + \underline{v}_n(\underline{q},\underline{p}) = \underline{v}(\underline{q},\underline{p})$ corresponds to the model (2.18).

In general, the equations of motion for a rigid or flexible manipulator are highly nonlinear. Nevertheless, they have several properties that can be exploited in control design. These properties have already been pointed out, but are summarized here because they will be very important in the rest of this thesis:

1. The distribution matrix H of a manipulator with n degrees of freedom and m ($m \leq n$) input signals is of *full rank* m. This implies that m degrees of freedom can be controlled directly and that $n - m$ degrees of freedom have to be controlled indirectly in case $m < n$.

2. The inertia matrix M is *symmetric* and *positive definite*. Furthermore, both M and M^{-1} are uniformly bounded as a function of $\underline{q}$.

3. The left-hand side of the equations of motion (2.91) (or (2.18)) depends *linearly* on the set of suitably selected, constant parameters $\underline{p}$.

4. The matrix $(\frac{1}{2}\dot{M} - C)$, with C as given by Eq. (2.11) is *skew-symmetric*. This property enables so-called passivity-based control schemes, as will be shown in the next chapters.

The above properties of manipulator dynamics hold for every arbitrary selected set of generalized coordinates (Subsection 2.3.4).

Chapter 3

Control of rigid manipulators.

3.1 Introduction.

Control of manipulators is a current research issue, because manipulators are highly nonlinear, multi-body systems with strong interactions between the degrees of freedom. Traditionally, this problem has been overcome by the PD- (or PID-) algorithm, which does not account for nonlinear dynamics explicitly (e.g. Arimoto and Miyazaki, 1983; Kuo, 1987). However, PD-control is not satisfactory for *trajectory tracking* in the sense that a desired, continuous trajectory has to be approximated by a limited set of desired set-points in order to prove stability. Furthermore, the nonlinear character of manipulators becomes more and more significant for increasing speed, so that linear control approaches such as PD and PID are not powerful enough and more advanced techniques are required (for a review of advanced control concepts, we refer to De Jager et al. 1991; Slotine and Li, 1991). One of the sophisticated schemes proposed so far is the computed torque control (CTC) scheme (Luh et al. 1980; Craig, 1986; Spong and Vidyasagar, 1989; Asada and Slotine, 1986), which aims at exact compensation of the nonlinearities. In the ideal case without model uncertainty, the resulting closed-loop dynamics are linear. In case of model uncertainty, the control problem can be addressed by using concepts of e.g. robust control theory (e.g. Sastry and Slotine, 1983; Slotine, 1985; Spong and Vidyasagar, 1989; Asada and Slotine, 1986) and adaptive control theory (e.g. Hsia, 1986; Middleton and Goodwin, 1986; Ortega and Spong, 1988; Slotine and Li, 1986, 1988, 1989). A robust controller, such as a sliding controller, is typically composed of a nominal trajectory tracking control part, for instance, a computed torque controller, and an additional part dealing with model uncertainty. The structure of an adaptive controller may be similar, but in addition the model parameters (or, possibly, the control parameters) are updated during the control process on the basis of the measured performance.

In Section 3.2, the model for a rigid manipulator and the control objective will be briefly reconsidered and the idea of feedback control will be introduced. In the rest of this chapter, we will focus on five approaches to the control problem of rigid manipulators, i.e. *PD-control* in Section 3.3, *computed torque control (CTC)* in Section 3.4, *sliding CTC* in Section 3.5, *adaptive CTC* in Section 3.6 and *output CTC* in Section 3.7. These approaches will be discussed from a theoretical point of view, using the second (or direct) method of Lyapunov (1949).

3.2 Control objective.

In this chapter, we consider the problem of controlling *rigid manipulators* to make them track a desired output trajectory $\underline{y}_d = \underline{y}_d(t)$. The inputs to the manipulator are the torques $\underline{u} \in \mathrm{R}^m$ generated by the joint actuators. At first, we assume that the manipulator parameters in the column $\underline{p} \in \mathrm{R}^r$ are well-known and we investigate, therefore, non-adaptive manipulator control. Because manipulator parameters are often not known exactly, in the second part of this chapter we will modify these concepts to arrive at a sliding controller in order to robustify the closed-loop system against model uncertainty, and to arrive at an adaptive controller in which the unknown parameters are adjusted on-line.

The dynamic model of a rigid manipulator has been derived in Subsection 2.5.2 and is given by

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + \underline{v} + \underline{f} = H\underline{u} \ , \qquad \underline{y} = \underline{\phi}(\underline{q}) \ , \tag{3.1}$$

where $\underline{q}$ is the column of generalized (joint) coordinates, $\underline{u}$ is the column of input torques, $\underline{y}$ is the output, M is the symmetric, positive definite inertia matrix, C accounts for the Coriolis and centrifugal effects, $\underline{v}$ is the column with conservative torques, $\underline{f}$ is the column of all other torques and H is the square, regular distribution matrix.

As argued in Section 2.4, we assume that it is possible to determine the desired path $\underline{q}_d = \underline{q}_d(t)$ of the generalized coordinates from the given, desired output column $\underline{y}_d = \underline{y}_d(t)$ via the kinematic relationship $\underline{y}_d = \underline{\phi}(\underline{q}_d)$ and, possibly, via optimizing criteria (in case of redundancy). We assume that the desired trajectory $\underline{q}_d(t)$ is smooth with bounded first and second time-derivatives. Now, the tracking control objective is to let $\underline{q}$ track $\underline{q}_d$ asymptotically, i.e. guarantee that the tracking error $\underline{e} = \underline{q}_d - \underline{q}$ satisfies

$$\lim_{t\to\infty} \underline{e} = \underline{0} \ , \qquad \lim_{t\to\infty} \dot{\underline{e}} = \underline{0} \ . \tag{3.2}$$

If this objective is achieved with bounded internal signals (such as the input torques, positions, velocities, accelerations, etc.), the closed-loop system is said to be *globally, asymptotically stable* (see Appendix A). In general, two types of control are known:

- Open-loop control, where the control relies on a *feedforward* scheme based only on the desired path $\underline{q}_d$[1].

[1] Hence, dynamic responses need to be predicted on the basis of an appropriate model of the manipulator. Yet, applications of feedforward control are still limited to rigid manipulators (e.g. Vukobratovic and Kircanski, 1985). Feedforward control of flexible manipulators has rarely been investigated. Asada et al. (1990), for instance, attempted to minimize the tracking error at the end-effector of a flexible-link manipulator by reducing the elastic deformations of the links via feedforward control.

- Closed-loop control, where the current $\underline{q}$ and $\dot{\underline{q}}$ are determined from measurements and/or reconstruction, and compared with $\underline{q}_d$ and $\dot{\underline{q}}_d$. Then, the control is essentially a *feedback control*, based explicitly on the tracking errors $\underline{e}$ and $\dot{\underline{e}}$ to reduce the sensitivity to disturbances and, possibly, to model uncertainty.

In this thesis, feedback control is considered, mostly in combination with a kind of feedforward control. It is assumed that all coordinates $\underline{q}$ and velocities $\dot{\underline{q}}$ are available at each time, by measurement and/or reconstruction. In the last section of this chapter, direct control of the end-effector will be considered, which means that the feedback control is based explicitly on the measured and/or reconstructed quantities $\underline{y}$ and $\dot{\underline{y}}$ only.

3.3 PD-control.

3.3.1 Introduction.

A basic task in the control of manipulators is to move objects from one position to another (for instance, pick-and-place or spot-welding). The classical approach to solve this so-called position control problem is to use a control law that totally ignores all nonlinear dynamic effects and results in an input $\underline{u}$ that is a linear combination of the position error $\underline{e} = \underline{q}_d - \underline{q}$ (the proportional or P-action), the differential of this error (the differential or D-action) and, possibly, the integral of this error (the integral or I-action). The justification of the use of PD- (or PID-) control for nonlinear mechanical systems was originally based on linearization or on a local stability argument (e.g. Arimoto and Miyazaki, 1983). Therefore, its application was limited to small motions around a set point. However, manipulators are typical examples of mechanical systems that have to make large movements, which usually requires the introduction of intermediate points on the desired trajectory. PD-control[2] is used then to drive the manipulator from one intermediate point to another (*set-point tracking*). More recently, global, asymptotic stability of PD-control has been proven for this kind of manipulator control.

In this section, two PD-type controllers will be considered. The main intention of this section is not a thorough discussion of these controllers but an introduction of Lyapunov's second (or direct) method for stability analysis. This method is a rather important tool for the design of the model based controllers that are investigated in the rest of this thesis. For more information on this method, we refer to Appendix A, to Narendra and Annaswamy (1989) and to Slotine and Li (1991).

3.3.2 Set-point tracking control.

In this section, we take into account only the torques due to inertia and the input torques $\underline{u}$. This implies that $\underline{v} = \underline{f} = \underline{0}$, i.e. there are no conservative torques due

[2]Except for Subsection 3.3.4, PID-control instead of PD-control is not mentioned especially nor used explicitly elsewhere in this thesis.

to, for instance, gravity and no torques due to, for instance, friction and damping. Consequently, the equations of motion become

$$M\underline{\ddot{q}} + C\underline{\dot{q}} = H\underline{u} \; . \tag{3.3}$$

The task is to bring the manipulator from a given initial state, characterized by $\underline{q}(0)$ and $\underline{\dot{q}}(0)$, to a desired position $\underline{q}_d$ and, subsequently, to hold the manipulator in that position, i.e. $\underline{\dot{q}}_d = \underline{0}$. Since the position error $\underline{e}$ is given by $\underline{e} = \underline{q}_d - \underline{q}$, it is seen that $\underline{\dot{e}} = -\underline{\dot{q}}$ and $\underline{\ddot{e}} = -\underline{\ddot{q}}$, so the error equation is given by

$$M\underline{\ddot{e}} + C\underline{\dot{e}} = -H\underline{u} \; . \tag{3.4}$$

In literature, several control laws for this task were presented (see, for instance, Spong and Vidyasagar, 1989; Slotine and Li, 1991). Here, only two possibilities are considered.

The first law to be considered is given by

$$H\underline{u} = M(L_D\underline{\dot{e}} + L_P\underline{e}) - C\underline{\dot{e}} \; , \tag{3.5}$$

and results in the quite simple error equation

$$\underline{\ddot{e}} + L_D\underline{\dot{e}} + L_P\underline{e} = \underline{0} \; . \tag{3.6}$$

It is easily seen that $\underline{e} = \underline{0}$ is a unique, globally, asymptotically stable equilibrium of this error system if L_D and L_P are constant matrices with a positive definite symmetric part. This condition is satisfied, for instance, if L_D and L_P are chosen diagonal with constant, positive diagonal elements. In that case, the equations in (3.6) decouple and can be written as

$$\ddot{e}_i + 2\beta_i\omega_i\dot{e}_i + \omega_i^2 e_i = 0 \; , \qquad i = 1, 2, \ldots, m \; , \tag{3.7}$$

where β_i and ω_i represent the relative damping and the natural frequency, respectively, of a constant, linear second-order system. If there are no specific demands with respect to the path of the various errors, it is obvious to assign the same dynamics to each of these errors, i.e. to choose $\omega_1 = \omega_2 = \cdots = \omega_m = \omega_o$ and $\beta_1 = \beta_2 = \cdots = \beta_m = \beta_o$. Now, $L_D = 2\beta_o\omega_o I$ and $L_P = \omega_o^2 I$, and

$$\ddot{e}_i + 2\beta_o\omega_o\dot{e}_i + \omega_o^2 e_i = 0 \; , \qquad i = 1, 2, \ldots, m. \tag{3.8}$$

The choice of the remaining controller parameters ω_o and β_o can be based on well known concepts for linear, constant second-order systems, taking into account aspects like overshoot, desired damping, rise time, etc. (see, for instance, Kuo, 1987).

The advantage of the resulting control law $H\underline{u} = M(2\beta_o\omega_o\underline{\dot{e}} + \omega_o^2\underline{e}) - C\underline{\dot{e}}$ is that it is easy to specify the desired error dynamics. The price to be paid is that each of the input signals in $\underline{u}$ depends on each element of $\underline{e}$ and $\underline{\dot{e}}$ and that the position dependent matrix M and the position and velocity dependent matrix C have to be calculated to determine $\underline{u}$ (see Fig. 3.1). In fact, this control scheme represents a PD-controller with time-varying control gains ML_P and $(ML_D - C)$, which requires

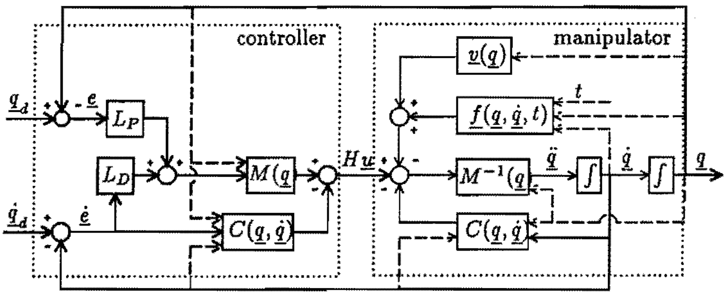


Figure 3.1: PD-control with time-varying control gains.

the configuration-dependent matrices M and C to be known and computed on-line.

The second law to be considered in this section is a PD-controller with constant control gains (see Fig. 3.2):

$$H\underline{u} = K_D\dot{\underline{e}} + K_P\underline{e} \ , \tag{3.9}$$

where K_P is a constant, symmetric, positive definite matrix and K_D is a constant matrix with positive definite, symmetric part. The associated error system becomes

$$M\ddot{\underline{e}} + (C + K_D)\dot{\underline{e}} + K_P\underline{e} = \underline{0} \ . \tag{3.10}$$

In the next subsection, it will be shown that the equilibrium point $\underline{e} = \underline{0}$ of this set of nonlinear differential equations is globally, asymptotically stable. Here, we only note that determining the input $\underline{u}$ with this control law does not require the calculation of the matrices M and C and that the control law can be simplified even more if diagonal matrices are chosen for K_P and K_D. However, the resulting error dynamics are quite difficult to judge without extensive (numerical) experiments.

3.3.3 Lyapunov stability.

In this subsection, the passivity property of manipulator dynamics (Subsection 2.3.4) will be used in an augmented version of Lyapunov's second (or direct) method, in order to prove the stability of the equilibrium point $\underline{e} = \underline{0}$ of the error equation Eq. (3.10), which is associated with the control law $H\underline{u} = K_D\dot{\underline{e}} + K_P\underline{e}$.

First of all, it is noted that $\underline{e} = \underline{0}$ is a unique equilibrium point of Eq. (3.10): in equilibrium this equation reduces to $K_P\underline{e} = \underline{0}$, with $\underline{e} = \underline{0}$ as the unique solution as K_P is positive definite and thus regular. Next, the error equation is premultiplied

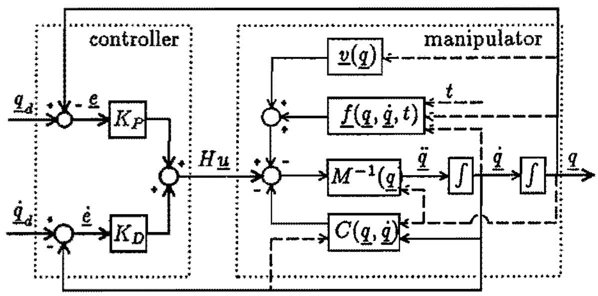


Figure 3.2: PD-control with constant control gains.

with $\dot{\underline{e}}^T$. Elaboration of the resulting scalar equation yields

$$\frac{d}{dt}(\frac{1}{2}\dot{\underline{e}}^T M\dot{\underline{e}} + \frac{1}{2}\underline{e}^T K_P \underline{e}) + \dot{\underline{e}}^T(C - \frac{1}{2}\dot{M})\dot{\underline{e}} = -\dot{\underline{e}}^T K_D \dot{\underline{e}} \ , \tag{3.11}$$

where the symmetry of M and K_P is used. Because $C - \frac{1}{2}\dot{M}$ is skew-symmetric (Subsection 2.3.2), the term $\dot{\underline{e}}^T(C - \frac{1}{2}\dot{M})\dot{\underline{e}}$ is equal to zero for each $\dot{\underline{e}}$ and the above equation can be rewritten as

$$V = \frac{1}{2}\dot{\underline{e}}^T M\dot{\underline{e}} + \frac{1}{2}\underline{e}^T K_P \underline{e} \ ; \qquad \dot{V} = -\dot{\underline{e}}^T K_D \dot{\underline{e}} \ . \tag{3.12}$$

With some reserves, the terms $\frac{1}{2}\dot{\underline{e}}^T M\dot{\underline{e}}$ and $\frac{1}{2}\underline{e}^T K_P\underline{e}$ can be interpreted as the kinetic energy and the elastic energy, respectively, in a mechanical system with degrees of freedom $\underline{e}$ and with inertia matrix M and stiffness matrix K_P. Again with some reserves, the symmetric part $\frac{1}{2}(K_D + K_D^T)$ of the matrix K_D can be seen as the viscous damping matrix of this error system. Then, the term $\dot{\underline{e}}^T K_D\dot{\underline{e}}$ is the dissipated power and $V = \frac{1}{2}\dot{\underline{e}}^T M\dot{\underline{e}} + \frac{1}{2}\underline{e}^T K_P\underline{e}$ is the total mechanical energy of the error system.

The function V of $\underline{e}$ and $\dot{\underline{e}}$ is not a Lyapunov function in the strict sense (Appendix A): although V is a positive definite function of $\underline{e}$ and $\dot{\underline{e}}$, the time derivative $\dot{V}$ is not negative definite but only semi-negative definite as $\dot{V} = 0$ for each $\underline{e}$ if $\dot{\underline{e}} = \underline{0}$. With Barbalat's lemma (e.g. LaSalle and Lefschetz, 1961; Slotine and Li, 1991), it can be shown that both $\dot{\underline{e}}(t)$ and $\ddot{\underline{e}}(t)$ approach $\underline{0}$ for $t \to \infty$ and, hence, it is follows from the error equation (3.10) and from the fact that K_P is regular that $\underline{e}(t)$ approaches $\underline{0}$ for $t \to \infty$. Furthermore, V is a monotone, non-increasing function of time with a bounded value at the initial time $t = 0$, so the error $\underline{e}(t)$ is bounded for all $t \geq 0$. The final conclusion is that $\underline{e} = \underline{0}$ is a globally, asymptotically stable equilibrium point of the error system given by Eq. (3.10).

Stability analyses of the given type, based on the passivity property of manipulator dynamics, are very important and play a major role in the design of controllers in the sequel of this thesis. This kind of stability analyses can be seen as an augmented version of the second (or direct) method of Lyapunov.

3.3.4 Compensation of external loads.

If external loads cannot be neglected, the model of the manipulator can be written as

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + \underline{v} + \underline{f} = H\underline{u}\ . \tag{3.13}$$

In the absence of model uncertainty, the external loads can be taken into account by modifying the given PD-control law in

$$H\underline{u} = K_D\dot{\underline{e}} + K_P\underline{e} + \underline{v} + \underline{f}\ . \tag{3.14}$$

This modified law results in the error system $M\ddot{\underline{e}} + (C + K_D)\dot{\underline{e}} + K_P\underline{e} = \underline{0}$ discussed earlier, with the globally, asymptotically stable equilibrium point $\underline{e} = \underline{0}$.

In the presence of uncertainties, with regard to the external loads only estimates $\hat{\underline{v}}$ and $\hat{\underline{f}}$ for $\underline{v}$ and $\underline{f}$, respectively, are available. The PD-control law, therefore, has to be modified in

$$H\underline{u} = K_D\dot{\underline{e}} + K_P\underline{e} + \hat{\underline{v}} + \hat{\underline{f}}\ , \tag{3.15}$$

resulting in the error system $M\ddot{\underline{e}} + (C + K_D)\dot{\underline{e}} + K_P\underline{e} = \underline{v} + \underline{f} - (\hat{\underline{v}} + \hat{\underline{f}})$. In general, no statements can be made on whether $\underline{e} = \underline{0}$ is an equilibrium point, nor on the stability of a possible equilibrium point. For the moment, it is assumed that there are such uncertainties that $(\underline{v} + \underline{f}) - (\hat{\underline{v}} + \hat{\underline{f}}) = \underline{c}$ with $\underline{c}$ being constant. The given control law now results in a steady-state error $\underline{e}_\infty$, given by

$$\underline{e}_\infty \equiv \lim_{t\to\infty} \underline{e}(t) = K_P^{-1}\underline{c}\ , \tag{3.16}$$

and it is seen that large proportional gains are required to reduce this error. However, this can conflict with the desire to keep the control bandwidth bounded.

A possible solution to this dilemma is to add an integral term to the control law. The modified PID-control law

$$H\underline{u} = K_D\dot{\underline{e}} + K_P\underline{e} + K_I\int_0^t \underline{e}(\tau)d\tau + \hat{\underline{v}} + \hat{\underline{f}} \tag{3.17}$$

leads to the error system

$$M\ddot{\underline{e}} + (C + K_D)\dot{\underline{e}} + K_P\underline{e} + K_I\int_0^t \underline{e}d\tau = \underline{c}\ ; \qquad \underline{c} = (\underline{v} + \underline{f}) - (\hat{\underline{v}} + \hat{\underline{f}})\ . \tag{3.18}$$

In this case, $\underline{e} = \underline{0}$ is a unique equilibrium point for all constant $\underline{c}$ if K_I is regular. The investigation of the stability of this equilibrium point is much more difficult than in the former case without integral action. However, a sufficient condition on the control matrices to guarantee global, asymptotic stability can be derived with the second method of Lyapunov. To arrive at this condition, we define a quantity $\underline{e}_r$ by

$$\underline{e}_r = \underline{e} + \Lambda\int_0^t \underline{e}d\tau\ ; \qquad \Lambda = \Lambda^T\ ;\ \ \Lambda > 0\ . \tag{3.19}$$

With $\underline{c}$ as given before, the error equation can now be written in the standard second-order form

$$M\ddot{\underline{e}}_r + N\dot{\underline{e}}_r + P\underline{e}_r = \underline{c} \ , \tag{3.20}$$

if and only if Λ is such that

$$N = C + K_D - M\Lambda \ , \tag{3.21}$$

$$P = K_P - (C + K_D - M\Lambda)\Lambda \ , \qquad K_I = P\Lambda \ . \tag{3.22}$$

As a first condition, this implies that Λ has to be a constant solution of $M\Lambda^3 - (C + K_D)\Lambda^2 + K_P\Lambda - K_I = 0$, such that $-\Lambda$ is a Hurwitz matrix (i.e. $\Lambda = \Lambda^T$ and $\Lambda > 0$). Hence, a stable, first-order system in $\underline{e}$ is obtained by differentiating Eq. (3.19) once: $\dot{\underline{e}}_r = \dot{\underline{e}} + \Lambda\underline{e}$. Considering the Lyapunov function candidate

$$V = \frac{1}{2}\dot{\underline{e}}_r^T M\dot{\underline{e}}_r + (\underline{e}_r + P^{-1}\underline{c})^T P(\underline{e}_r + P^{-1}\underline{c}) \ , \tag{3.23}$$

the second condition is that $P = K_P - (C + K_D - M\Lambda)\Lambda$ has to be a constant, symmetric and positive definite matrix. Finally, the skew-symmetry of $(C - \frac{1}{2}\dot{M})$ is used to derive the time-derivative of V, which yields

$$\dot{V} = \dot{\underline{e}}_r^T(K_D - M\Lambda)\dot{\underline{e}}_r \ . \tag{3.24}$$

This leads to the third condition, which states that the symmetric part of $K_D - M\Lambda$ has to be a positive definite matrix. The conclusion is that if these three conditions are satisfied, it can be easily shown that $\underline{e}_r - P^{-1}\underline{c} = \underline{0}$ is a globally, asymptotically stable equilibrium point. Differentiated with respect to time, Eq. (3.19) shows that $\underline{e} = \underline{0}$ is a globally, asymptotically stable equilibrium point of the error system (3.20).

3.3.5 Conclusions.

In this section, the second method of Lyapunov has been used to prove the stability of a PD- (and PID-) controller for set-point control of a nonlinear mechanical system. Although it is, in general, hard to find a suitable Lyapunov function, in this case it is rather easy by considering the physical properties of the closed-loop system. The conclusion is that a PD-controller imitates the behavior that would result from inserting mechanical elements, such as springs (P-controller) and dampers (D-controller) in the system. This explains, why the PD-approach performs well in spite of strong nonlinearities and the presence of strongly coupled interactions. Its main advantages are its simplicity, the ease of implementation and tuning, its low costs and, last but not least, its stability and robustness for model uncertainty and disturbances. Therefore, PD-control still is the most common control method in industry.

In the sequel of this thesis, it will be assumed that the manipulator has to follow a desired trajectory rather than to reach a desired position. This situation may arise even when the task is to move a load from an initial position to a desired end position. Asada and Slotine (1986), for instance, point out the fact that overshoot is a common problem in PD-control of manipulators. This is especially the case for direct-drive manipulators, where there is no gearing and, thus, very little damping in the drives to reduce the dynamic effects. Overshoot may be quite undesirable, especially if it causes damaging contact with the environment. The use of PD-control then requires to move slowly through a number of intermediate set-points and introduces a considerable extra time to complete the task. More generally, approximation of a desired trajectory by a series of set-points is not sufficient for demanding tasks such as plasma-welding, laser-cutting or high-speed operations in the presence of obstacles. Because PD-control cannot fully exploit the capabilities of high-performance manipulators, a more sophisticated control concept is needed. Therefore, modern control techniques have been proposed that use explicitly a model of the manipulator in a kind of *feedforward control*, in order to be able to follow a specified, continuous trajectory in space with smaller errors. However, in these new control techniques, the *feedback* part of the control is usually done also by a PD controller.

3.4 Computed torque control (CTC).

3.4.1 Introduction.

Many modern control schemes have been proposed so far to handle the trajectory tracking of rigid manipulators (see Subsection 1.4.1). Among them, the computed torque control (CTC) method or inverse dynamics method (e.g. Luh et al. 1980; Craig, 1986; Asada and Slotine, 1986) accounts explicitly for the nonlinear manipulator dynamics, which are assumed to be known exactly. All nonlinearities in the manipulator dynamics are compensated and, in the ideal case of no model uncertainty, the closed-loop system is linear and decoupled and can be controlled by a linear controller, for instance, a simple PD- (or PID-) controller[3]. The computed torque control method is also known as the static nonlinear state feedback approach, because it achieves the tracking objective via a special kind of feedback linearization (e.g. Spong and Vidyasagar, 1989; Slotine and Li, 1991; Nijmeijer and Van der Schaft, 1990).

In the next subsection, a classical approach to CTC will be discussed, which leads to the familiar second-order, linear error system that is easily shown to be stable under certain conditions for the control gains. Stability can be proven via the second method of Lyapunov, but with another Lyapunov function than in the previous section because the passivity property of manipulator dynamics is not used. In Subsection 3.4.3, instead, the same Lyapunov function will be considered as in the previous section, which yields a CTC scheme that is based explicitly on the passivity property of manipulator dynamics. Other versions of CTC laws are possible, based on a different choice of the Lyapunov function candidate. The main drawback of the CTC approach is that it assumes a perfect model of the manipulator. In Subsection 3.4.4, this problem will be sketched for the passivity-based CTC scheme.

[3]Another criterion in the choice of a linear controller could be the robustness of the computed torque scheme for model uncertainty (e.g. Slotine and Li, 1991; Spong and Vidyasagar, 1989). This issue will be discussed in Section 3.5 on sliding control.

3.4.2 Classical computed torque control.

The original idea behind computed torque control method is to define the control torque $\underline{u} = \underline{u}(t)$ on the basis of the manipulator model (3.1), as follows:

$$H\underline{u} = M\underline{a} + C\underline{\dot{q}} + \underline{v} + \underline{f} \ . \tag{3.25}$$

This expression for $\underline{u}$, with $\underline{a}$ as the new control input, is known as the "computed torque". It compensates exactly for the nonlinear manipulator dynamics if the model is perfect (see Fig. 3.3). Substitution of this expression for $\underline{u}$ into (3.1) results in

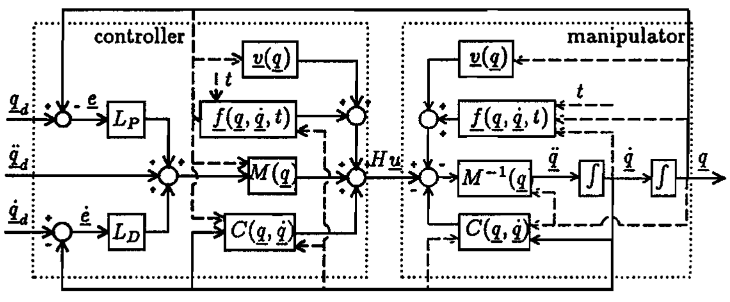


Figure 3.3: Classical computed torque control.

$M(\underline{\ddot{q}} - \underline{a}) = \underline{0}$ and, because M is regular, also in

$$\underline{\ddot{q}} = \underline{a} \ . \tag{3.26}$$

This represents a set of n decoupled double integrators. Asymptotic tracking of the desired trajectory $\underline{q}_d = \underline{q}_d(t)$ can be achieved, for instance, by the following simple law for the new input $\underline{a}$ (e.g. Craig, 1986; Asada and Slotine, 1986):

$$\underline{a} = \underline{\ddot{q}}_d + L_D\underline{\dot{e}} + L_P\underline{e} \ . \tag{3.27}$$

This control law leads to a similar second-order error system as in the preceding section for the PD-controller with time-varying control gains (Eq. (3.6)), i.e.

$$\underline{\ddot{e}} + L_D\underline{\dot{e}} + L_P\underline{e} = \underline{0} \ . \tag{3.28}$$

Consequently, all corresponding remarks in the preceding section concerning stability, the choice of the control gains, bandwidth limitation and the addition of integral action are also relevant to this classical version of the computed torque control law.

If, instead, Lyapunov's second method is used to prove the stability of the equilibrium $\underline{e} = \underline{0}$, the following Lyapunov function can be chosen:

$$V = \frac{1}{2}\underline{\dot{e}}^T\underline{\dot{e}} + \frac{1}{2}\underline{e}^T L_P \underline{e}\ ; \qquad \dot{V} = -\underline{\dot{e}}^T L_D \underline{\dot{e}}\ . \tag{3.29}$$

This Lyapunov function differs from Eq (3.12), where the inertia matrix M is needed in the energy-like Lyapunov function. Because the nonlinear term $C\underline{\dot{q}}$ is compensated exactly here, the passivity property of manipulator dynamics is not used explicitly in the stability proof. The conditions for the control gains are $L_P = L_P^T$, $L_P > 0$, L_P constant and $L_D = L_D^T > 0$. This *classical CTC* scheme, i.e.

$$H\underline{u} = M(\underline{\ddot{q}}_d + L_D\underline{\dot{e}} + L_P\underline{e}) + C\underline{\dot{q}} + \underline{v} + \underline{f}\ , \tag{3.30}$$

has a so-called "internal" PD-controller. In the next subsection, a CTC scheme with a so-called "external" PD-controller will be discussed. There, the passivity property of manipulator will be exploited in the stability proof.

3.4.3 Passivity-based computed torque control.

This subsection deals with a computed torque control method that exploits the passivity property of mechanical systems to guarantee asymptotic stability of the closed-loop system. For set-point control, it has been shown in Subsection 3.3.2 that the PD-control law $H\underline{u} = K_D\underline{\dot{e}} + K_P\underline{e}$ results in the error system $M\underline{\ddot{e}} + (C + K_D)\underline{\dot{e}} + K_P\underline{e} = \underline{0}$ if external loads are not taken into consideration. In order to extend the set-point controller (where $\underline{\dot{q}}_d = \underline{\ddot{q}}_d = \underline{0}$) to a trajectory tracking controller (where $\underline{\dot{q}}_d \neq \underline{0}$ and $\underline{\ddot{q}}_d \neq \underline{0}$), and in order to take external loads into account, the following CTC-like control law is proposed (see Fig. 3.4):

$$H\underline{u} = M\underline{\ddot{q}}_d + C\underline{\dot{q}}_d + \underline{v} + \underline{f} + K_D\underline{\dot{e}} + K_P\underline{e}\ . \tag{3.31}$$

This control law results in the error system

$$M\underline{\ddot{e}} + (C + K_D)\underline{\dot{e}} + K_P\underline{e} = \underline{0}\ , \tag{3.32}$$

and global, asymptotic stability can be proven in the same way as in Subsection 3.3.3. Again, the Lyapunov function $V = \frac{1}{2}\underline{\dot{e}}^T M\underline{\dot{e}} + \frac{1}{2}\underline{e}^T K_P\underline{e}$ is used in the stability proof that exploits the passivity property, i.e. the property that $C - \frac{1}{2}\dot{M}$ is skew-symmetric. Note that the same conditions for the control gains are given, i.e. $K_P = K_P^T$; $K_P > 0$; K_P constant and $K_D + K_D^T > 0$ and that both K_P and K_D are usually chosen to be diagonal and constant. In literature, these control schemes are referred to as *passivity-based control* schemes (e.g. Ortega and Spong, 1988, Paden and Panja, 1988). In contradiction to the classical CTC scheme (3.30), the passivity-based CTC scheme contains an "external" PD-controller. Other differences are that the term $C\underline{\dot{q}}_d$ does not compensate exactly the actual Coriolis and centrifugal torques $C\underline{\dot{q}}$ and that the passivity-based scheme does not linearize nor decouple the equations of motion.

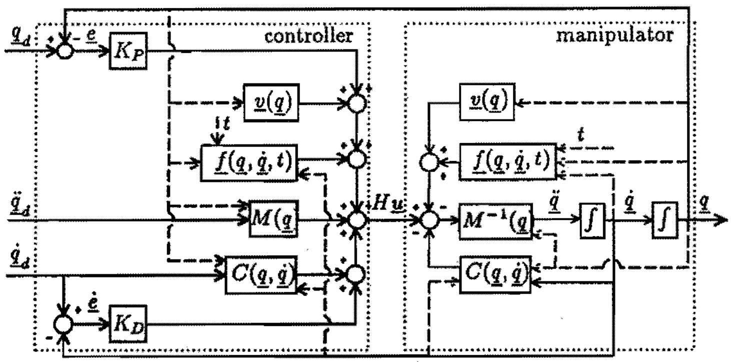


Figure 3.4: Passivity-based computed torque control.

3.4.4 Model uncertainty.

Computed torque control is a good approach for the motion control of rigid manipulators. However, a major limitation is that only estimates $\hat{M}$, $\hat{C}$, $\hat{\underline{v}}$ and $\hat{\underline{f}}$ of M, C, $\underline{v}$ and $\underline{f}$, respectively, are available in practice. This means that the control law (3.31) takes the form

$$H\underline{u} = \hat{M}\ddot{\underline{q}}_d + \hat{C}\dot{\underline{q}}_d + \hat{\underline{v}} + \hat{\underline{f}} + K_D\dot{\underline{e}} + K_P\underline{e}\ , \tag{3.33}$$

which leads to the closed-loop dynamics

$$M\ddot{\underline{e}} + (C + K_D)\dot{\underline{e}} + K_P\underline{e} = (M - \hat{M})\ddot{\underline{q}}_d + (C - \hat{C})\dot{\underline{q}}_d + (\underline{v} - \hat{\underline{v}}) + (\underline{f} - \hat{\underline{f}})\ . \tag{3.34}$$

If model errors are nonzero, performance is difficult to predict and stability can, in general, not be guaranteed. Therefore, questions about *robustness* arise, with respect to both parametric uncertainty and structure uncertainty due to unmodeled dynamics. Craig (1988) has discussed the robustness of CTC for parametric uncertainty, assuming that uncertainty bounds for the parameter values are known. If these bounds are small, the closed-loop system performs well. If the bounds are large, the system can still be shown to be stable, but the quality of control will degrade. Other robustness analyses have been carried out by, for instance, Spong and Vidyasagar (1985) and Slotine (1985). Roughly speaking, the control gains must be large enough to reduce the effects of model uncertainty (as for PD-control), but this is in conflict with the objective to keep the control bandwidth low and to avoid actuator saturation.

3.4.5 Conclusions.

Classical CTC compensates exactly for nonlinear manipulator dynamics and decouples the interactions, such that the closed-loop system is a decoupled, linear system that can be controlled by simple, classical linear feedback controllers. The passivity-based CTC, instead, does not linearize and decouple the equations of motion. Nevertheless, it leads to a globally, asymptotically stable closed-loop system if there is no model uncertainty. For a more detailed exposition of these two approaches, we refer to Berghuis (1993). CTC is a good approach to track desired trajectories in all possible configurations of rigid manipulators. However, the desired performance is achieved only if a perfect manipulator model without parametric uncertainty is available. In practice, this is never the case and, as a consequence, the tracking accuracy decreases and the controlled system can even become unstable. Therefore, more robust methods, such as sliding CTC and adaptive CTC, have been proposed. They combine the fine qualities of CTC with robustness for model uncertainty. In the next section, a sliding controller will be proposed, which consists of a CTC law plus an extra term that switches the control input as long as the closed-loop system behavior does not match the desired behavior. In Section 3.6, the concept of adaptive control for rigid manipulators will be discussed in order to deal with parametric uncertainty.

3.5 Sliding computed torque control.

3.5.1 Introduction.

A simple approach to robust control is the sliding surface or variable structure system (VSS) control method (Utkin, 1978). The simplest example of a VSS is a relay or on-off system, in which the control input can take only two values: on or off. The concept of VSS has been developed in the Soviet Union more than 30 years ago (e.g. Filippov, 1960). Since then, VSS and the allied variable structure control (VSC) have been extensively studied (e.g. Utkin, 1977, for a survey until 1972). VSC is based on the operation of logical switches in a VSS, but differs from simple relay control in that it provides extremely high speed switching around the control values. The enormous improvement in power electronics and control computers has made high speed switching control realizable for a broad class of systems. One of the areas in which VSC receives increasing attention is robotics (e.g. Sastry and Slotine, 1983; Slotine, 1985; Asada and Slotine, 1986) especially if the model structure is inaccurate and the model parameters are imprecise with known uncertainty bounds. The concept of sliding control for nonlinear systems has also been discussed in De Jager et al. (1991).

In this section, a sliding controller will be discussed, which is based on a rigid manipulator model with parametric uncertainty and unknown but bounded disturbances. The basic notion in sliding control is the sliding surface S. It is a subspace of the error state space, i.e. of the set of columns $\underline{e}_x$ with the tracking error $\underline{e}$ and the velocity error $\dot{\underline{e}}$ as its partial columns:

$$\underline{e}_x^T = [\underline{e}^T \;\; \dot{\underline{e}}^T\,] \, . \tag{3.35}$$

Here, only sliding surfaces that do not depend explicitly on time t are considered. Now, the *sliding surface* can be seen as the set of all error states $\underline{e}_x$ that satisfy constraints of the type $\underline{s}^*(\underline{e}_x) = \underline{0}$, or, formulated in terms of $\underline{e}$ and $\dot{\underline{e}}$, constraints of the type

$$\underline{s}(\underline{e}, \dot{\underline{e}}) = \underline{0} \ . \tag{3.36}$$

The closed-loop system is said to be in *sliding motion* at time t if both $\underline{s}$ and $\dot{\underline{s}}$ are equal to $\underline{0}$ at time t, i.e. if the error state is on S at time t and, at least for the moment, remains on S.

The crucial requirements in the definition of the sliding surface, i.e. in the choice of the sliding functions $\underline{s} = \underline{s}(\underline{e}, \dot{\underline{e}})$, are:

1. *the existence condition*: there must exist at least one input $\underline{u}$ that keeps the closed-loop system in sliding motion once it is in sliding motion;
2. *the reachability condition*: there must exist at least one input $\underline{u}$ that puts the system from any error state into sliding motion;
3. *the convergence condition*: if the system remains in sliding motion, the error $\underline{e}$ has to converge to $\underline{0}$.

The convergence condition is satisfied if $\underline{s}(\underline{e}, \dot{\underline{e}}) = \underline{0}$ for $t \geq t_s$ implies that $\underline{e}(t)$ converges to $\underline{0}$. This is the case, if $\underline{e} = \underline{0}$ is a globally, asymptotically stable equilibrium point of $\underline{s}(\underline{e}, \dot{\underline{e}}) = \underline{0}$. A suitable choice for the sliding functions $\underline{s} = \underline{s}(\underline{e}, \dot{\underline{e}})$ is, for example,

$$\underline{s} = \dot{\underline{e}} + \Lambda \underline{e} \ , \tag{3.37}$$

where Λ is a constant matrix with a positive definite symmetric part, i.e. $\Lambda + \Lambda^T > 0$. This choice for the sliding functions is by far not the only possibility to satisfy the convergence condition (e.g. Asada and Slotine, 1986; Slotine and Li, 1991). However, for reasons of simplicity, this choice will be used in this section. Generalization to other suitable choices, especially choices of the type $\underline{s} = \dot{\underline{e}} + \underline{\alpha}(\underline{e})$, is fairly straightforward.

In the following subsections, first, attention will be given to the inputs that are required to put the closed-loop system in sliding motion and to keep it in sliding motion. After that, some special aspects, such as stability of the closed-loop system and model uncertainty, will be discussed in some detail.

3.5.2 Sliding CTC for perfect models.

The mathematical model of a rigid manipulator as determined in Subsection 2.5.2 is given by Eq. (3.1). With $\dot{\underline{q}} = \dot{\underline{q}}_d - \dot{\underline{e}} = \dot{\underline{q}}_d + \Lambda \underline{e} - \underline{s}$, this model can be written as

$$M\dot{\underline{s}} + C\underline{s} = M(\ddot{\underline{q}}_d + \Lambda \dot{\underline{e}}) + C(\dot{\underline{q}}_d + \Lambda \underline{e}) + \underline{v} + \underline{f} - H\underline{u} \ , \tag{3.38}$$

or, in a more compact form, as

$$M\dot{\underline{s}} + C\underline{s} = M\ddot{\underline{q}}_r + C\dot{\underline{q}}_r + \underline{v} + \underline{f} - H\underline{u} \ . \tag{3.39}$$

Here, $\underline{q}_r = \underline{q}_r(t)$ is a so-called *reference trajectory* defined by the differential equation

$$\dot{\underline{q}}_r = \dot{\underline{q}}_d + \Lambda \underline{e} \ , \tag{3.40}$$

with a suitable condition for $t = 0$, for instance, $\underline{q}_r(0) = \underline{q}_d(0)$.

If the closed-loop system is in sliding motion, then $\underline{s} = \underline{0}$ and $\dot{\underline{s}} = \underline{0}$, and it is easily seen from Eq. (3.39) that the corresponding input, the *equivalent input* $\underline{u}_{eq}$, has to satisfy

$$H\underline{u}_{eq} = M\ddot{\underline{q}}_r + C\dot{\underline{q}}_r + \underline{v} + \underline{f} \ . \tag{3.41}$$

This yields the equivalent error system $M\dot{\underline{s}} + C\underline{s} = \underline{0}$. If, at a certain moment, $\underline{s} = \underline{0}$ and $\underline{u} = \underline{u}_{eq}$, then $\dot{\underline{s}} = \underline{0}$, which means that $\underline{s}$ remains equal to $\underline{0}$. Hence, the existence condition is satisfied for the chosen sliding functions $\underline{s} = \dot{\underline{e}} + \Lambda\underline{e}$. The remaining problem is to determine an input that forces the error state towards the sliding surface if at any moment $\underline{s} \neq \underline{0}$.

To solve this problem, the input $\underline{u}$ is written as the sum of $\underline{u}_{eq}$ according to Eq. (3.41) and a new input $\underline{u}_s$:

$$\underline{u} = \underline{u}_{eq} + \underline{u}_s \ . \tag{3.42}$$

Substitution in Eq. (3.39) yields

$$M\dot{\underline{s}} + C\underline{s} = -H\underline{u}_s \ . \tag{3.43}$$

Numerous concepts are known to arrive at such a control law for $\underline{u}_s$ that $\underline{s} = \underline{0}$ is a globally, asymptotically stable equilibrium point of this error system. For instance, an obvious choice is a PD-like control term

$$H\underline{u}_s = K_s\underline{s} \ , \qquad K_s + K_s^T > 0 \ , \tag{3.44}$$

which yields the error system $M\dot{\underline{s}} + (C + K_s)\underline{s} = \underline{0}$. This error system can be proven to be stable in the equilibrium point $\underline{s} = \underline{0}$, by choosing $V = \frac{1}{2}\underline{s}^T M\underline{s}$, which results in $\dot{V} = -\underline{s}^T K_s\underline{s}$. Substitution of law (3.41) for $\underline{u}_{eq}$ and law (3.44) for $\underline{u}_s$ in $\underline{u} = \underline{u}_{eq} + \underline{u}_s$ yields the control law for the complete input $\underline{u}$. In comparison with the control law proposed in Subsection 3.4.3, this computed torque-like control law can also be considered as passivity-based CTC. But the difference is that it uses the reference trajectory $\underline{q}_r$ instead of the desired trajectory $\underline{q}_d$. Hence, in the sequel of this thesis, the method based on this control law is referred to as *passivity-based CTC with reference trajectory.*

In sliding control, it is common practice to take a discontinuous law of the form

$$H\underline{u}_s = G sign(\underline{s}) \ , \tag{3.45}$$

where G is a diagonal matrix with positive, constant main diagonal elements $g_1, g_2, \ldots, g_n$ and $sign(\underline{s})$ is a column with element i equal to $sign(s_i)$. This sliding control law results in the closed-loop system

$$M\dot{\underline{s}} + C\underline{s} = -G sign(\underline{s}) \ . \tag{3.46}$$

The equilibrium point $\underline{s} = \underline{0}$ of this system is indeed globally, asymptotically stable, because the function V, defined by

$$V = \frac{1}{2}\underline{s}^T M\underline{s} \ , \tag{3.47}$$

is a Lyapunov function as

$$\dot{V} = -\underline{s}^T G sign(\underline{s}) + \underline{s}^T(\frac{1}{2}M - C)\underline{s} = -\sum_{i=1}^{n} g_i \mid s_i \mid , \tag{3.48}$$

and, therefore, $\dot{V} < 0$ if $\underline{s} \neq \underline{0}$.

Substitution of law (3.41) for $\underline{u}_{eq}$ and law (3.45) for $\underline{u}_s$ in $\underline{u} = \underline{u}_{eq} + \underline{u}_s$ yields the control law for the complete input $\underline{u}$. This sliding CTC scheme is shown in Fig. 3.5.

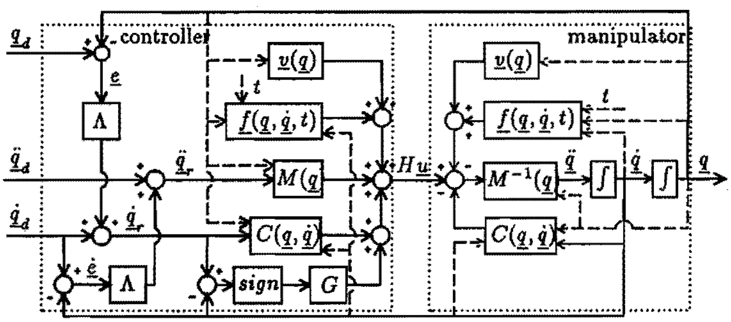


Figure 3.5: Sliding computed torque control.

3.5.3 Sliding CTC for imperfect models.

Because of model uncertainties, only estimates $\hat{M}$, $\hat{C}$, $\hat{\underline{v}}$ and $\hat{\underline{f}}$ for M, C, $\underline{v}$ and $\underline{f}$, respectively, are available in practice. As a consequence, Eq (3.41) is not a suitable relation to determine the equivalent control and, instead, we can only use

$$H\underline{u}_{eq} = \hat{M}\ddot{\underline{q}}_r + \hat{C}\dot{\underline{q}}_r + \hat{\underline{v}} + \hat{\underline{f}} . \tag{3.49}$$

Substitution of this law in $H\underline{u} = H\underline{u}_{eq} + H\underline{u}_s$, followed by substitution in the model (3.39), yields

$$M\dot{\underline{s}} + C\underline{s} = \underline{w} - H\underline{u}_s , \tag{3.50}$$

where the 'disturbance' $\underline{w}$ is defined by

$$\underline{w} = (M - \hat{M})\ddot{\underline{q}}_r + (C - \hat{C})\dot{\underline{q}}_r + (\underline{v} - \hat{\underline{v}}) + (\underline{f} - \hat{\underline{f}}) . \tag{3.51}$$

We assume that at time t for each element w_i ($i = 1, 2, \ldots, n$) of the model imperfection $\underline{w}$ a, possibly time dependent, bound b_i can be determined:

$$\mid w_i \mid \leq b_i , \quad i = 1, 2, \ldots, n. \tag{3.52}$$

The choice of the sliding term $H\underline{u}_s$ is based on the requirement that $\underline{s} = \underline{0}$ is a globally, asymptotically stable equilibrium point of the closed-loop system (for $H\underline{u}_s$ applied to the system (3.50)). Now, the starting-point is the choice of a suitable candidate-Lyapunov function V. Here, again, we choose

$$V = \frac{1}{2}\underline{s}^T M \underline{s} \,. \tag{3.53}$$

With Eq. (3.50), the time-derivative $\dot{V} = \underline{s}^T(M\underline{\dot{s}} + \frac{1}{2}\dot{M})$ can be written as

$$\dot{V} = \underline{s}^T(\underline{w} - H\underline{u}_s) + \underline{s}^T(\frac{1}{2}\dot{M} - C)\underline{s} = \underline{s}^T(\underline{w} - H\underline{u}_s) \,, \tag{3.54}$$

because $\frac{1}{2}\dot{M} - C$ is skew-symmetric. With the sliding term $H\underline{u}_s = Gsign(\underline{s})$, it cannot be guaranteed that $\dot{V} < 0$ for each $\underline{s} \neq \underline{0}$ unless each of the non-trivial elements $g_1, g_2, \ldots, g_n$ of the diagonal matrix G is larger than $b_1, b_2, \ldots, b_n$, respectively. This must hold for all $t \geq 0$, so the condition for the diagonal elements of G can be written as

$$g_i > b_i \,, \qquad i = 1, 2, \ldots, n. \tag{3.55}$$

Since b_i may possibly be time dependent, g_i can be a function of time that has to be computed on-line. However, this can be avoided if the maximum of b_i over the whole time interval is known beforehand and is not unacceptable large.

It is easily seen that the sliding term $H\underline{u}_s = Gsign(\underline{s})$ substituted in Eq. (3.54) results in

$$\dot{V} = -\sum_{i=1}^{n} [g_i \, sign(s_i) - w_i] s_i \,. \tag{3.56}$$

Since g_i is chosen to be larger than b_i and $| \, w_i \, | \leq b_i$ for $i = 1, 2, \ldots, n$, $\dot{V} < 0$ for each $\underline{s} \neq \underline{0}$. Hence, the closed-loop system that is obtained if the complete sliding CTC law

$$H\underline{u} = \hat{M}\underline{\ddot{q}}_r + \hat{C}\underline{\dot{q}}_r + \underline{\hat{v}} + \underline{\hat{f}} + Gsign(\underline{s}) \tag{3.57}$$

is applied to system (3.39) is globally, asymptotically stable.

3.5.4 Conclusions.

The idea behind sliding control is to define a well-behaving function of the sliding surface and to select a control law that makes a Lyapunov-like function satisfy the sliding condition. This assures that the closed-loop system reaches the sliding surface in finite time, despite the presence of bounded disturbances and model uncertainty (for a specification of the reaching time, see, for instance, Asada and Slotine, 1986; Slotine and Li, 1991). Then, if the control switches infinitely fast, the performance of the closed-loop system can be kept in sliding motion despite bounded disturbances and model uncertainty. This means that in the ideal case the sliding control law leads to robust trajectory tracking (under the assumption that the uncertainty bounds are known). However, in practice, the small but nonzero time-delay in control switching causes the system configuration to slightly overshoot the sliding surface each time the control input switches. Then, the closed-loop system does not move exactly along the sliding surface, but switches with high-frequency oscillations across the surface. This *chattering* of the control inputs, which is typically for sliding control, may be acceptable in some special applications (such as power electronics). But especially for manipulator control, chattering is often highly undesirable, because it implies extremely high control activity and may excite unmodeled dynamics. Furthermore, control chattering reduces the insensitivity to model uncertainty and to disturbances, as the system is not in pure sliding motion. One of the possibilities to avoid chattering is to modify the switching control law in the sense that it becomes continuous in a chosen boundary layer around the sliding surface (for a comparison of methods to eliminate chattering, see De Jager, 1992b). For instance, a saturation function may be an appropriate smooth approximation of the discontinuous sign function in the original sliding term (e.g. Slotine, 1985; Asada and Slotine, 1986). However, given the width of the chosen boundary layer, this approach implies a trade-off between the control performance and the robustness property.

Although the sliding control approach provides a systematic procedure to design a stable control system in the presence of modeling imprecision and small disturbances, questions remain whether it is suitable for manipulator control implementation. This is due to the control chattering that can occur.

3.6 Adaptive computed torque control.

3.6.1 Introduction.

The sliding controller of the previous section is composed of a computed torque-like law plus a sliding term that accounts for model uncertainty with known bounds. If the model structure is assumed to be correct but only imprecise knowledge of parameters is available, other advanced control techniques can be applied, such as adaptive control. Many papers have been devoted to the issue of adaptive control for rigid manipulators (see Subsection 1.5.3). For a global introduction to adaptive control we refer to Appendix B.

In adaptive control, one or more control parameters and/or model parameters are modified on-line (Fig. 3.6). Adaptation of control parameters is often used in MRAC (model reference adaptive control; see Appendix B), where the actual performance of the closed-loop system is compared to the behavior of a chosen, stable reference model (e.g. Landau, 1979; Horowitz and Tomizuka, 1980; Balestrino et al., 1983; Nicosia and Tomei, 1984; Butler, 1990, for an overview). In this section, instead, we will focus on manipulator control based on a model of the manipulator, which is assumed to have a perfect structure but unknown parameters $\underline{p}$. Therefore, it seems obvious to adapt model parameters instead of control parameters (e.g. Craig et al. 1986, Sadegh and Horowitz, 1987, Slotine and Li, 1986). The adaptive control law may then be the same as for the non-adaptive case with the difference that the unknown model parameters are replaced by their on-line adjustments. Therefore, the adaptive controller can consist of a computed torque-like version in adaptive form, plus an on-line adaptation algorithm that adjusts the unknown model parameters.

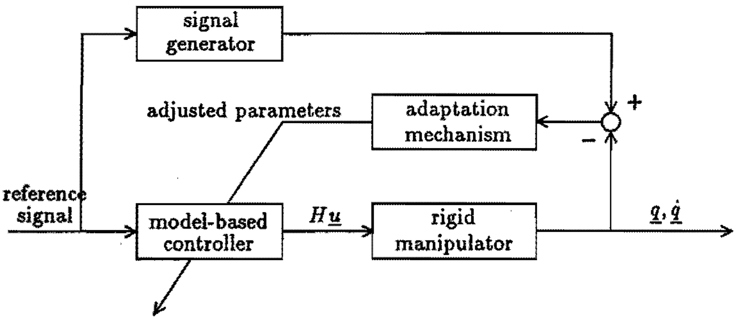


Figure 3.6: A parameter-adaptive control scheme.

As discussed in Subsection 1.5.3, the oldest adaptive control schemes were based on adjustments of configuration-dependent parameters (such as the elements of M and C that were assumed to vary slowly in time; see, for instance, Horowitz and Tomizuka, 1980), on a linearized model of the manipulator or on other assumptions that can endanger stability. Therefore, we concentrate on the more recent proposals for adaptive controllers, which are based on the property that the model is linear in $\underline{p}$ and which result in globally, asymptotically stable closed-loop systems (e.g. Craig et al., 1986; Slotine and Li, 1986).

The adaptive controller design problem encountered in this section can be stated as follows:

> Given a manipulator model with *unknown* but *constant* parameters $\underline{p}$ and given a twice differentiable desired trajectory $\underline{q}_d = \underline{q}_d(t)$, derive such a control law for the input $\underline{u}$ and such an on-line adaptation algorithm for the parameters $\underline{p}$ that global, asymptotic stability of the trajectory tracking is guaranteed after an initial adaptation process.

Many of the properties of manipulator dynamics as discussed in Section 2.6 will be used to solve this problem. Especially the 'passivity property' of mechanical systems will be exploited here and, as already mentioned, the key assumption in this section is that the model equations are linear in the unknown parameters $\underline{p}$. An adaptive controller then adjusts the unknown parameters $\underline{p}$ on-line and uses the most recent estimates $\hat{\underline{p}}$ in the adaptive control law. In the next subsection, we will start from an adaptive version of classical CTC. This version will appear unsuited to implementation. Therefore, a modified CTC law will be used for adaptive control and a parameter adaptation algorithm will be derived that makes the actual trajectory track the desired trajectory asymptotically and that makes the closed-loop system globally, asymptotically stable in the sense of Lyapunov. Finally, tuning of the control and the adaptation gains will be discussed.

3.6.2 An adaptive version of classical CTC.

The classical CTC law (3.30) linearizes and decouples completely the nonlinear manipulator model in the absence of model uncertainty. If the model structure is perfect, but only estimates $\underline{\hat{p}}$ for the unknown parameters $\underline{p}$ are available, the adaptive version of that CTC law is obtained by replacing M, C, $\underline{v}$ and $\underline{f}$ by their estimates $\hat{M} = M(\underline{q}, \underline{\hat{p}})$, $\hat{C} = C(\underline{q}, \underline{\dot{q}}, \underline{\hat{p}})$, $\underline{\hat{v}} = \underline{v}(\underline{q}, \underline{\hat{p}})$ and $\underline{\hat{f}} = \underline{f}(\underline{q}, \underline{\dot{q}}, \underline{\hat{p}}, t)$, respectively:

$$H\underline{u} = \hat{M}(\underline{\ddot{q}}_d + K_D\underline{\dot{e}} + K_P\underline{e}) + \hat{C}\underline{\dot{q}} + \underline{\hat{v}} + \underline{\hat{f}} \; . \tag{3.58}$$

The error equation, corresponding to this law, is given by

$$\hat{M}(\underline{\ddot{e}} + K_D\underline{\dot{e}} + K_P\underline{e}) = -[(\hat{M} - M)\underline{\ddot{q}} + (\hat{C} - C)\underline{\dot{q}} + (\underline{\hat{v}} - \underline{v}) + (\underline{\hat{f}} - \underline{f})] \; . \tag{3.59}$$

Because the model is linear in the unknown parameters, the right-hand side must be proportional to the parameter error $\underline{e}_p = \underline{\hat{p}} - \underline{p}$. Hence, such a matrix $W = W(\underline{q}, \underline{\dot{q}}, \underline{\ddot{q}}, t)$ exists that

$$W\underline{e}_p = (\hat{M} - M)\underline{\ddot{q}} + (\hat{C} - C)\underline{\dot{q}} + (\underline{\hat{v}} - \underline{v}) + (\underline{\hat{f}} - \underline{f}) \;\; ; \qquad \underline{e}_p = \underline{\hat{p}} - \underline{p} \; , \tag{3.60}$$

and the error equation can be rewritten as

$$\hat{M}(\underline{\ddot{e}} + K_D\underline{\dot{e}} + K_P\underline{e}) = -W\underline{e}_p \; . \tag{3.61}$$

If $\underline{e}_p \neq \underline{0}$, this parameter mismatch can result in poor tracking performance and can even lead to instability. In the previous section, a sliding control term was added to the CTC law in order to overcome the effects of this parameter mismatch without adaptation of $\underline{\hat{p}}$. The adaptive control approach, on the other hand, adjusts $\underline{\hat{p}} = \underline{\hat{p}}(t)$ on-line.

To assure global, asymptotic stability of the adaptive control system, the adaptation algorithm for the parameters can be designed, e.g., via Lyapunov's second method. However, the function $V = \frac{1}{2}\underline{\dot{e}}^T\underline{\dot{e}} + \frac{1}{2}\underline{e}^T K_P \underline{e}$ from Section 3.4.2 is not a Lyapunov function for this case, as now $\dot{V} = -\underline{\dot{e}}^T K_D \underline{\dot{e}} - \underline{\dot{e}}^T \hat{M}^{-1} W \underline{e}_p$ and it is not guaranteed that $\dot{V} \leq 0$ for all $\underline{e} \neq \underline{0}$ and $\underline{\dot{e}} \neq \underline{0}$. To overcome this problem, a positive definite quadratic term in $\underline{e}_p$ is added to the function V from Section 3.4.2, resulting in

$$V = \frac{1}{2}\underline{\dot{e}}^T\underline{\dot{e}} + \frac{1}{2}\underline{e}^T K_P \underline{e} + \frac{1}{2}\underline{e}_p^T \Gamma^{-1} \underline{e}_p \; ; \qquad \Gamma > 0 \; . \tag{3.62}$$

Differentiation of this candidate Lyapunov function yields

$$\dot{V} = -\underline{\dot{e}}^T K_D \underline{\dot{e}} + \underline{e}_p^T[-W^T\hat{M}^{-1}\underline{\dot{e}} + \Gamma^{-1}\underline{\dot{e}}_p] \; . \tag{3.63}$$

The most obvious choice to guarantee that $\dot{V} \leq 0$, is to make the last term equal to zero by adaptation of the parameters such that $\underline{\dot{e}}_p = \Gamma W^T \hat{M}^{-1}\underline{\dot{e}}$. Because $\underline{e}_p = \underline{\hat{p}} - \underline{p}$ with constant $\underline{p}$, this results in the following adaptation algorithm for $\underline{\hat{p}}$:

$$\dot{\underline{\hat{p}}} = \Gamma W^T \hat{M}^{-1} \underline{\dot{e}} \; . \tag{3.64}$$

Implementation of this adaptation algorithm is hampered by several serious drawbacks:

- The elements of W depend on $\underline{q}$ and $\dot{\underline{q}}$ but also on $\ddot{\underline{q}}$. Hence, extra measurements or reconstruction are required to determine $\ddot{\underline{q}}$.
- Inversion of the matrix $\hat{M}$ is time consuming. This can limit the sampling rate.
- The approximation $\hat{M}$ may be singular.

Furthermore, the control law (3.58) combined with this adaptation algorithm guarantees only that $\dot{\underline{e}}(t) \to \underline{0}$ for $t \to \infty$ and that $\underline{e}_p(t)$ becomes constant for $t \to \infty$. Hence, application of this adaptive control scheme does not imply $\underline{e}(t) \to \underline{0}$ for $t \to \infty$.

3.6.3 An adaptive version of passivity-based CTC.

Several solutions for the problems mentioned in the previous subsection were proposed (e.g. Middleton and Goodwin, 1988; Ortega and Spong, 1988; Slotine and Li, 1986). One of the most favorable solutions to get an adaptive control scheme without the need of acceleration feedback uses a filtered version of the desired trajectory, the so-called reference trajectory $\underline{q}_r = \underline{q}_r(t)$ that was already introduced in Subsection 3.5.2 as:

$$\dot{\underline{q}}_r = \dot{\underline{q}}_d + \Lambda(\underline{q}_d - \underline{q}) \quad ; \qquad \Lambda + \Lambda^T > 0 \ , \tag{3.65}$$

with an appropriate initial condition. The difference $\underline{q}_r - \underline{q}$, the so-called reference error $\underline{e}_r$, satisfies

$$\dot{\underline{e}}_r = \dot{\underline{q}}_r - \dot{\underline{q}} = \dot{\underline{e}} + \Lambda \underline{e} \, . \tag{3.66}$$

This is a stable, first-order differential equation in the tracking error $\underline{e}$, with $\dot{\underline{e}}_r$ as the input[4]. It is noted that the reference velocity error $\dot{\underline{e}}_r$ resembles the sliding parameter $\underline{s}$ used in the previous section. Hence, if the (adaptive) controller is able to guarantee that $\dot{\underline{e}}_r(t)$ satisfies some weak continuity conditions to realize that $\dot{\underline{e}}(t) \to \underline{0}$ for $t \to \infty$ and to keep $\dot{\underline{e}}_r(t)$ bounded (e.g. Slotine and Li, 1991), then $\underline{e}(t)$ is bounded and $\underline{e}(t) \to \underline{0}$ for $t \to \infty$. This means that the controller objective can be reformulated in terms of the reference velocity error $\dot{\underline{e}}_r$ instead of the usual tracking error $\underline{e}$. Besides, it offers the possibility to use Lyapunov functions in terms of $\dot{\underline{e}}_r$ instead of $\underline{e}$, as will be done here.

The problems, associated with the adaptive version of classical CTC, can be easily handled by a similar modification of that law as in Section 3.5 (e.g. Slotine and Li, 1986). In Subsection 3.5.2, this modified law was shown to be a passivity-based CTC law that uses the reference trajectory $\underline{q}_r$ instead of the desired trajectory $\underline{q}_d$. For imperfect models, as discussed in Subsection 3.5.3, this passivity-based CTC law with reference trajectory can be given by

$$H\underline{u} = \hat{M}\ddot{\underline{q}}_r + \hat{C}\dot{\underline{q}}_r + \hat{\underline{v}} + \hat{\underline{f}} + K_r\dot{\underline{e}}_r \ , \tag{3.67}$$

[4] Other definitions than $\dot{\underline{e}}_r = \dot{\underline{e}} + \Lambda\underline{e}$ for $\underline{e}_r$ are possible, for instance, $\ddot{\underline{e}}_r = \ddot{\underline{e}} + \Lambda_d\dot{\underline{e}} + \Lambda_p\underline{e}$ (e.g. Sadegh and Horowitz, 1987). This results in different adaptation algorithms. However, this issue shall not be considered here.

where $K_r\underline{\dot{e}}_r = K_r\underline{\dot{e}} + K_r\Lambda\underline{e}$ can be considered as an "external" PD-control term. In Section 3.5, the parameter estimates $\underline{\hat{p}}$ have been determined off-line. In this section, they are adjusted on-line by an adaptation algorithm, which is designed in such a way that global, asymptotical stability of the closed-loop system is assured. Therefore, we now consider the associated error equation

$$M\underline{\ddot{e}}_r + C\underline{\dot{e}}_r + K_r\underline{\dot{e}}_r = -W_r\underline{e}_p \,, \tag{3.68}$$

where $W_r = W_r(\underline{q}, \underline{\dot{q}}, \underline{\dot{q}}_r, \underline{\ddot{q}}_r, t)$ is defined by

$$W_r\underline{e}_p = (\hat{M} - M)\underline{\ddot{q}}_r + (\hat{C} - C)\underline{\dot{q}}_r + (\underline{\hat{v}} - \underline{v}) + (\underline{\hat{f}} - \underline{f}) \,. \tag{3.69}$$

An essential difference between this matrix W_r and the matrix W as defined in Eq. (3.60), is that the elements of W_r do not depend on the acceleration $\underline{\ddot{q}}$ while the elements of W are linear functions of $\underline{\ddot{q}}$.

To investigate stability, the same Lyapunov function candidate as in Subsection 3.5.2 is chosen (with $\underline{\dot{e}}_r = \underline{s}$), supplemented with a positive definite, quadratic form in the parameter error $\underline{e}_p$:

$$V = \frac{1}{2}\underline{\dot{e}}_r^T M \underline{\dot{e}}_r + \frac{1}{2}\underline{e}_p^T \Gamma^{-1} \underline{e}_p \,. \tag{3.70}$$

With the skew-symmetry of the matrix $(\frac{1}{2}\dot{M} - C)$, the time-derivative of this function can be written as

$$\dot{V} = -\underline{\dot{e}}_r^T K_r \underline{\dot{e}}_r + \underline{e}_p^T[-W_r^T\underline{\dot{e}}_r + \Gamma^{-1}\underline{\dot{e}}_p] \,. \tag{3.71}$$

Therefore, the *adaptation algorithm*

$$\underline{\dot{\hat{p}}} = \Gamma W_r^T \underline{\dot{e}}_r \tag{3.72}$$

yields $\dot{V} = -\underline{\dot{e}}_r^T K_r \underline{\dot{e}}_r < 0$ if $\underline{\dot{e}}_r \neq \underline{0}$. This guarantees, that $\underline{\dot{e}}_r(t) \to \underline{0}$ for $t \to \infty$ and that both $\underline{\dot{e}}_r$ and $\underline{\hat{p}}$ remain bounded. Hence, $\underline{q}$, $\underline{\dot{q}}$ and $\underline{\hat{p}}$ are bounded and global, asymptotic stability of the closed-loop system is assured.

This approach overcomes the drawbacks of the adaptive control scheme that is proposed in the previous subsection: it does not require the inversion of $\hat{M}$ neither the knowledge of the actual acceleration $\underline{\ddot{q}}$ and it guarantees that both $\underline{e}$ and $\underline{\dot{e}}$ converge asymptotically to $\underline{0}$. For a block diagram of the adaptive CTC scheme proposed here, see Fig. 3.7.

3.6.4 Discussion of the adaptive controller (gains).

The adaptive control law (3.67) computes the input $\underline{u}$ on the basis of the manipulator model, with parameters $\underline{\hat{p}}$ that are adjusted on-line by the adaptation algorithm (3.72). This algorithm adjusts $\underline{\hat{p}}$ as long as $\underline{\dot{e}}_r \neq \underline{0}$. Although trajectory tracking is guaranteed, this does not imply that the adjusted parameters $\underline{\hat{p}}$ will finally match the real parameters $\underline{p}$. Parameter convergence is assured only under certain persistent excitation conditions, i.e. if the reference trajectory is "sufficiently

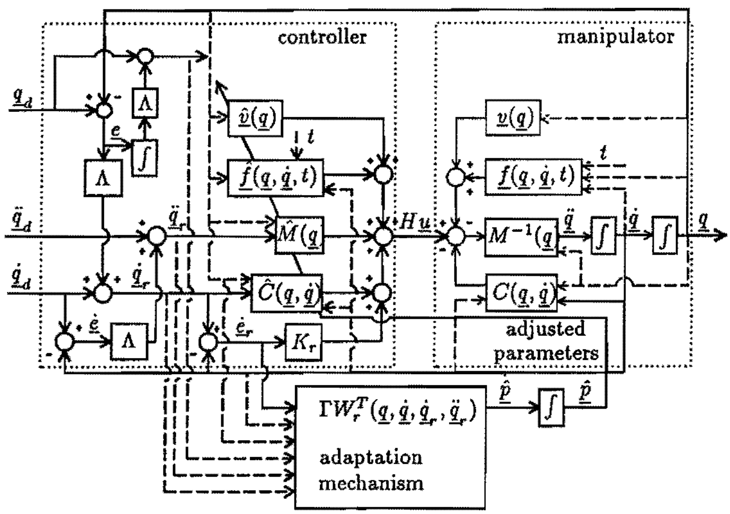


Figure 3.7: Adaptive computed torque control.

rich" (e.g. Narendra and Annaswamy, 1989; Slotine and Li, 1991). Parameter convergence is of minor importance here, as the primary objective is to track a desired trajectory (which can be achieved without parameter convergence).

The adaptation gain Γ must be positive definite in order to assure global, asymptotic stability of the closed-loop system. Besides, Γ has to be constant and symmetric. The larger the gain, the larger the speed of adaptation, and, thus, the better the tracking performance. However, a too large gain Γ can result in fluctuations of the adjusted parameters. This can give rise to oscillations in the input, which may excite unmodeled dynamics. Until now, there is no systematic approach to derive 'optimal' values for the control gains and adaptation gain: tuning is still a matter of trial-and-error.

The control gains K_r and Λ have to be positive definite. Larger values of K_r result in a faster convergence of $\underline{\dot{e}}_r$ to $\underline{0}$, whereas larger values of Λ lead to a faster tracking convergence. However, a compromise has to be found between the tracking performance and the control bandwidth. The latter is limited by unmodeled dynamics and the actuator bandwidth. Often, K_r and Λ are chosen to be constant and diagonal. Another choice may, for instance, be $K_r = \lambda\hat{M}$ and $\Lambda = \lambda I$ with a positive constant λ. This results in

$$H\underline{u} = \hat{M}(\underline{\ddot{q}}_d + 2\lambda\underline{\dot{e}} + \lambda^2\underline{e}) + \hat{C}\underline{\dot{q}}_r + \underline{\hat{v}} + \underline{\hat{f}} \,. \tag{3.73}$$

The time-derivative $\dot{V}$ of the candidate Lyapunov function (3.70) then becomes

$$\dot{V} = -\lambda \dot{\underline{e}}_r^T \hat{M} \dot{\underline{e}}_r + \underline{e}_p^T[-W_r^T \dot{\underline{e}}_r + \Lambda^{-1}\underline{e}_p] \; . \tag{3.74}$$

If the approximation of $\hat{M}$ is always positive definite, no stability problems will occur. However, this cannot generally be guaranteed beforehand. Nevertheless, it is possible to transform this expression for $\dot{V}$ into

$$\dot{V} = -\lambda \dot{\underline{e}}_r^T M \dot{\underline{e}}_r - \lambda \dot{\underline{e}}_r^T (\hat{M} - M)\dot{\underline{e}}_r + \underline{e}_p^T[-W_r^T \dot{\underline{e}}_r + \Lambda^{-1}\underline{e}_p] \; . \tag{3.75}$$

The matrix $(\hat{M} - M)$ is proportional to the parameter error $\underline{e}_p$, which means that $\dot{V}$ can also be written as

$$\dot{V} = -\lambda \dot{\underline{e}}_r^T M \dot{\underline{e}}_r + \underline{e}_p^T[-W_s^T \dot{\underline{e}}_r + \Lambda^{-1}\underline{e}_p] \; , \tag{3.76}$$

where $W_s \underline{e}_p = W_r \underline{e}_p + \lambda(\hat{M} - M)\dot{\underline{e}}_r$. This implies, that asymptotic stability of the closed-loop system with the control law (3.73) is achieved if the matrix W_r in the adaptation algorithm is replaced by the matrix W_s:

$$\dot{\hat{\underline{p}}} = \Gamma W_s^T \dot{\underline{e}}_r \; . \tag{3.77}$$

3.6.5 Conclusions.

In this section, adaptive computed torque control for rigid manipulators has been briefly discussed. For a more detailed exposition of this approach and other versions of the adaptive CTC scheme for rigid manipulators, we refer to Berghuis (1993). It can be concluded that an adaptive version of the classical computed torque control scheme has some important drawbacks, whereas a suitable extension can be derived of the passivity-based CTC scheme to an adaptive version. Furthermore, it can be noted that the second method of Lyapunov has appeared to be quite suitable in designing stable, adaptive and non-adaptive control schemes. The only important problem there may be in designing a control law via the second method of Lyapunov, is the choice of an appropriate candidate Lyapunov function. The hyperstability theory of Popov (1973) can be used as an alternative (see Appendix A). It recasts the nonlinear (adaptive) control system into a "standard" feedback system with a feedforward block and a feedback block. Both blocks have to be "hyperstable" in order to guarantee global, asymptotic hyperstability of the complete system.

The hyperstability theory of Popov offers a systematic solution to the design problem of stable, nonlinear systems. It results in more freedom in the choice of adaptive control schemes. Authors who have used this theory in the design of adaptive controllers for rigid manipulators are, for instance, Landau (1979), Horowitz and Tomizuka (1980) and Balestrino et al. (1983). For a certain class of systems, there seems to be a close relation between the hyperstability theory of Popov and the second method of Lyapunov. For a comparison of these two approaches for the design of stable, adaptive controllers, we refer to Landau (1979). In Appendix A, we attempt to briefly associate Popov's hyperstability theory with the Lyapunov approach and in Appendix B, we give a slight link with adaptive manipulator control as discussed in this section.

3.7 CTC of the end-effector.

The objective in controlling rigid manipulators is to make them track a desired end-effector trajectory $\underline{y}_d = \underline{y}_d(t)$. In the previous sections, control schemes have been discussed that realize this objective indirectly by controlling the generalized coordinates $\underline{q}$. This is possible only if the desired trajectory $\underline{q}_d = \underline{q}_d(t)$ is determined from $\underline{y}_d = \underline{\phi}(\underline{q}_d)$, possibly extended with optimizing criteria if the number of output quantities is smaller than the number of generalized coordinates. In this section, the idea of CTC of the end-effector will be sketched, i.e. the problem of feedback control based on the measured and/or reconstructed output $\underline{y}$.

The quantities to be controlled, such as the position and velocity of the end-effector, are considered as the elements of the output column $\underline{y}$. It is assumed that the desired end-effector trajectory $\underline{y}_d(t)$ is smooth with bounded first and second time-derivatives. Furthermore, it is assumed that the number of output quantities equals the number of control inputs and, hence, also the number of degrees of freedom of the rigid manipulator. Now, the end-effector tracking control problem is to determine such an input $\underline{u}$ that the output tracking error $\underline{e}_y = \underline{y}_d - \underline{y}$ satisfies

$$\lim_{t\to\infty} \underline{e}_y = \underline{0} , \qquad \lim_{t\to\infty} \underline{\dot{e}}_y = \underline{0} , \tag{3.78}$$

and that the generalized coordinates and their derivatives remain bounded.

The dynamic model of a rigid manipulator as specified in Subsection 2.5.2 is given by

$$M\underline{\ddot{q}} + C\underline{\dot{q}} + \underline{v} + \underline{f} = H\underline{u} , \qquad \underline{y} = \underline{\phi}(\underline{q}) . \tag{3.79}$$

It is assumed that the output equation is a global diffeomorphism (Section 2.4). In order to design a control scheme for the end-effector that is based explicitly on $\underline{y}$ instead of on $\underline{q}$ as the variables to be controlled, a coordinate transformation is performed on the equations of motion via the kinematic relationship (see also Section 2.4):

$$\underline{q} = \underline{\phi}^{-1}(\underline{y}) = \underline{\psi}(\underline{y}) . \tag{3.80}$$

Here, $\underline{\psi}$ is continuous and differentiable with derivative Ψ:

$$\Psi_{ij} = \frac{\partial \psi_i}{\partial y_j} , \qquad i,j = 1,2,\ldots,n . \tag{3.81}$$

After the coordinate transformation, the new model has the same structure as the transformed model in Subsection 2.3.4 and is given by

$$M^*\underline{\ddot{y}} + C^*\underline{\dot{y}} + \underline{v}^* + \underline{f}^* = H^*\underline{u} . \tag{3.82}$$

Under the conditions stated in Section 2.4, CTC of the end-effector can be realized in the same way as in the previous part of this chapter. Hence, it is quite easy to derive output equivalents of, for instance, both the classical CTC and the passivity-based CTC schemes. This results in

$$H^*\underline{u} = M^*(\underline{\ddot{y}}_d + L_D^*\underline{\dot{e}}_y + L_P^*\underline{e}_y) + C^*\underline{\dot{y}} + \underline{v}^* + \underline{f}^* , \tag{3.83}$$

and

$$H^*\underline{u} = M^*\ddot{\underline{y}}_d + C^*\dot{\underline{y}} + \underline{v}^* + \underline{f}^* + K_D^*\dot{\underline{e}}_y + K_P^*\underline{e}_y \ , \tag{3.84}$$

respectively. Similarly, sliding or adaptive CTC of the end-effector can be applied for a rigid manipulator with model uncertainty.

3.8 Conclusions.

In this chapter, some control laws have been discussed for rigid manipulators. Here, they will be summarized, with the explicit notation of $\underline{q}$, $\dot{\underline{q}}$, $\underline{p}$ and t.

For set-point tracking, in Section 3.3 two PD-type controllers have been considered: a PD-controller with time-varying control gains, i.e.

$$H(\underline{q})\underline{u} = [M(\underline{q},\underline{p})L_D - C(\underline{q},\dot{\underline{q}},\underline{p})]\dot{\underline{e}} + L_P\underline{e} \ , \tag{3.85}$$

and a PD-controller with constant control gains, i.e.

$$H(\underline{q})\underline{u} = K_D\dot{\underline{e}} + K_P\underline{e} \ . \tag{3.86}$$

The latter is most commonly used in industry.

In Section 3.4, trajectory tracking has been considered in the ideal case of no model uncertainty. The most well-known, model-based control method for this task is *classical CTC*:

$$H(\underline{q})\underline{u} = M(\underline{q},\underline{p})[\ddot{\underline{q}}_d + L_D\dot{\underline{e}} + L_P\underline{e}] + C(\underline{q},\dot{\underline{q}},\underline{p})\dot{\underline{q}} + \underline{v}(\underline{q},\underline{p}) + \underline{f}(\underline{q},\dot{\underline{q}},\underline{p},t) \ , \tag{3.87}$$

which yields a linearized and decoupled closed-loop system. The other model-based control method, discussed here, is the *passivity-based CTC*:

$$H(\underline{q})\underline{u} = M(\underline{q},\underline{p})\ddot{\underline{q}}_d + C(\underline{q},\dot{\underline{q}},\underline{p})\dot{\underline{q}}_d + \underline{v}(\underline{q},\underline{p}) + \underline{f}(\underline{q},\dot{\underline{q}},\underline{p},t) + K_D\dot{\underline{e}} + K_P\underline{e} \ , \tag{3.88}$$

which does not compensate exactly for nonlinear manipulator dynamics and, hence, does not yield a linearized and decoupled closed-loop system. Similarly to PD-control with time-varying control gains, this approach needs the explicit use of the second method of Lyapunov in order to prove global, asymptotic stability of the closed-loop system.

In Section 3.5, trajectory tracking has been studied in the practical case of model uncertainty. In the field of robust control, *sliding CTC* is proposed:

$$H(\underline{q})\underline{u} = M(\underline{q},\hat{\underline{p}})\ddot{\underline{q}}_r + C(\underline{q},\dot{\underline{q}},\hat{\underline{p}})\dot{\underline{q}}_r + \underline{v}(\underline{q},\hat{\underline{p}}) + \underline{f}(\underline{q},\dot{\underline{q}},\hat{\underline{p}},t) + Gsign(\dot{\underline{e}}_r) \ , \tag{3.89}$$

where the model parameters $\hat{\underline{p}}$ are estimated <u>off-line</u>. The sign function in this control law results in a switching controller, such that

$$\dot{\underline{e}}_r = \dot{\underline{q}}_r - \dot{\underline{q}} = \dot{\underline{e}} + \Lambda\underline{e} \tag{3.90}$$

converges asymptotically to zero. The reference trajectory $\underline{q}_r$ defined by the above first-order differential equation in $\underline{e}$ will be quite important in the sequel of this thesis, because it can be suitably used in adaptive control schemes.

In Section 3.6, we have especially considered the case of parametric uncertainty. There, an *adaptive CTC* law for rigid manipulators was proposed as follows:

$$H(\underline{q})\underline{u} = M(\underline{q},\hat{\underline{p}})\ddot{\underline{q}}_r + C(\underline{q},\dot{\underline{q}},\hat{\underline{p}})\dot{\underline{q}}_r + \underline{v}(\underline{q},\hat{\underline{p}}) + \underline{f}(\underline{q},\dot{\underline{q}},\hat{\underline{p}},t) + K_r\dot{\underline{e}}_r \ , \qquad (3.91)$$

where the model parameters $\hat{\underline{p}}$ are estimated on-line, via the adaptation algorithm

$$\dot{\hat{\underline{p}}} = \Gamma W_r^T(\underline{q},\dot{\underline{q}},\dot{\underline{q}}_r,\ddot{\underline{q}}_r,t)\dot{\underline{e}}_r \ . \qquad (3.92)$$

Global, asymptotic stability of the closed-loop system is, again, proven via the second method of Lyapunov. A global scheme of adaptive CTC (as shown in Fig. 3.8) shows that the adaptive controller for a rigid manipulator consists of a part that computes the input torques $\underline{u}$, a signal generator that computes the reference trajectory $\underline{q}_r$, and an adaptation mechanism that computes the parameter estimates $\hat{\underline{p}}$. In the next

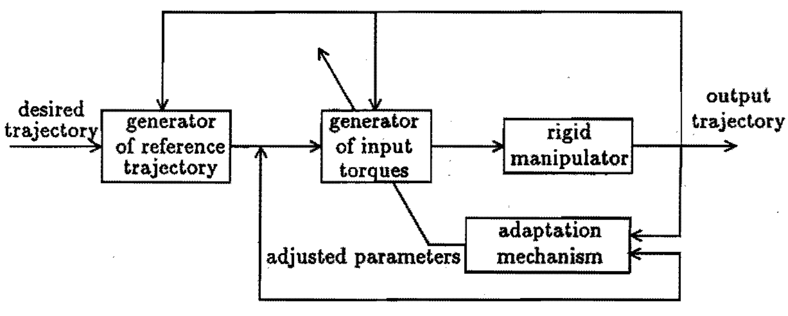


Figure 3.8: Global block scheme of adaptive CTC.

chapter, this controller structure will be extended for the adaptive control of a flexible manipulator.

In Section 3.7, it was shown that the control approaches summarized above can be easily applied to realize asymptotic tracking of the end-effector trajectory directly. However, this cannot be realized if the number of output quantities is smaller than the number of degrees of freedom, which is the case for flexible manipulators. In that case, applying (adaptive) CTC of the end-effector can result in an end-effector trajectory $\underline{y}$ that tracks asymptotically the desired end-effector trajectory $\underline{y}_d$, while some generalized coordinates may become unstable. This is a serious drawback of the 'classical' versions of (adaptive) CTC, because they all rely on the assumption of a perfectly rigid manipulator. This problem shall be dealt with in the following chapter.

Chapter 4

Control of flexible manipulators.

4.1 Introduction.

The (adaptive) control schemes described in the previous chapter are applicable to rigid manipulators. Their usefulness is limited because manipulators are never perfectly rigid. If a high performance is required, flexibility must be taken into account.

The main problems in controlling flexible manipulators are that the number of control inputs is smaller than the number of degrees of freedom, and that the number of degrees of freedom is larger than the number of output quantities for which a desired trajectory is specified. In literature, attempts are often made to derive a control law that results in asymptotic tracking of the desired trajectory and that suppresses the undesired deformations. However, it seems more feasible to search for a control law that results in asymptotic tracking of the desired end-effector trajectory and in a limitation of the undesired deformations to an acceptable level. This is the main subject of this chapter.

4.2 Summary of flexible manipulator models.

In Chapter 2, Lagrange's formalism is used to derive the equations of motion for flexible manipulators. This resulted in the fairly general model (2.91):

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f} = H\underline{u} \; , \tag{4.1}$$

where the number of generalized coordinates n exceeds the number of inputs m. The inertia matrix M is symmetric and positive definite, and can depend on the generalized coordinates $\underline{q}$ and on the constant parameters $\underline{p}$. The term $C\dot{\underline{q}}$ accounts for the Coriolis and centrifugal forces. The elements of C can depend on $\underline{q}$ and $\underline{p}$ and are proportional to $\dot{\underline{q}}$. Without loss of generality we require that $\dot{M} - 2C$ is skew-symmetric, i.e. that the passivity condition is satisfied. The term $K\underline{q}$ represents all internal torques due to linear, elastic deformations. The symmetric stiffness matrix K can depend on $\underline{p}$ but not on $\underline{q}$ or $\dot{\underline{q}}$. This matrix is semi-positive definite and certainly not positive definite. The elements of $\underline{v}_n + \underline{f}$ can be functions of $\underline{q}$, $\dot{\underline{q}}$, $\underline{p}$ and time t. They represent all internal and external torques that are not modeled by the linear stiffness term $K\underline{q}$ or in the driving term $H\underline{u}$. The distribution matrix H has maximum rank, i.e. $\text{rank}(H) = m$. The elements of H may be functions of $\underline{q}$, but it is assumed that they do not depend on $\underline{p}$.

Because H has maximum rank, such regular transformation matrices T exist that

$$TH = \begin{bmatrix} 0 \\ I \end{bmatrix} \; ; \qquad det(T) \neq 0 \; . \tag{4.2}$$

These transformation matrices are of the form

$$T^T = \begin{bmatrix} N & H(H^T H)^{-1} \end{bmatrix} \tag{4.3}$$

where the rank of the $(n \times (n-m))$-matrix N has to be maximum, i.e. $\text{rank}(N) = n - m$, and N must satisfy

$$N^T H = 0 \; . \tag{4.4}$$

Transformation matrices of this kind can be used to split the set (4.1) of n equations in a set of $n - m$ equations that contain the input and a set of m equations that do not contain the input. Premultiplication of Eq. (4.1) by T yields the desired decomposition:

$$N^T[M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f}] \;=\; \underline{0} \; ; \tag{4.5}$$

$$\underline{u} = (H^T H)^{-1} H^T [M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f}] \qquad . \tag{4.6}$$

It is assumed that the structure of the model (4.1) is correct, i.e. that such parameters $\underline{p}$ exist that the model perfectly matches the real manipulator dynamics. Furthermore, it is assumed that M, C, K and $\underline{v}_n + \underline{f}$ depend linearly on $\underline{p}$. Then, if the left-hand side of Eq. (4.1) is denoted by $\underline{h}$,

$$\underline{h}(\underline{q},\dot{\underline{q}},\ddot{\underline{q}},\underline{p},t) = M(\underline{q},\underline{p})\ddot{\underline{q}} + C(\underline{q},\dot{\underline{q}},\underline{p})\dot{\underline{q}} + K(\underline{p})\underline{q} + \underline{v}_n(\underline{q},\underline{p}) + \underline{f}(\underline{q},\dot{\underline{q}},\underline{p},t) \; , \tag{4.7}$$

such a matrix W exists that (see also Subsection 2.3.4):

$$\underline{h}(\underline{q},\dot{\underline{q}},\ddot{\underline{q}},\underline{p}_2,t) - \underline{h}(\underline{q},\dot{\underline{q}},\ddot{\underline{q}},\underline{p}_1,t) = W(\underline{q},\dot{\underline{q}},\ddot{\underline{q}},t)[\underline{p}_2 - \underline{p}_1] \;\; \forall \underline{p}_1, \underline{p}_2 . \tag{4.8}$$

As stated in Section 2.4, the generalized coordinates are selected such that $\underline{q}$ can be partitioned in a column $\underline{\theta} \in \mathrm{R}^m$ of rigid-body coordinates and a column $\underline{\varrho} \in \mathrm{R}^{n-m}$ of flexible coordinates, i.e.

$$\underline{q} = \begin{bmatrix} \underline{\theta} \\ \underline{\varrho} \end{bmatrix} \; . \tag{4.9}$$

The choice of the rigid-body coordinates is free provided that they determine completely the output $\underline{y} \in \mathrm{R}^m$ (see Section 2.5). So

$$\underline{y} = \underline{\phi}(\underline{\theta}) \; . \tag{4.10}$$

It is assumed that this relation is invertible for all relevant values of $\underline{\theta}$, i.e. there is such a function $\underline{\phi}^{-1}$ that $\underline{\theta} = \underline{\phi}^{-1}(\underline{y})$. This implies, for instance, that kinematic singularities are left aside.

For some analyses in this chapter, it is advantageous to rewrite Eq. (4.5) in terms of the partial columns $\underline{\theta}$ and $\underline{\varrho}$ of $\underline{q}$. With

$$
\begin{aligned}
N^T M &= \left[\, M_{N\theta} \;\; M_{N\varrho} \,\right] \; ; \;\; N^T C = \left[\, C_{N\theta} \;\; C_{N\varrho} \,\right] \; ; \\
N^T K &= \left[\, K_{N\theta} \;\; K_{N\varrho} \,\right] \; ; \;\; N^T \underline{v}_n = \underline{v}_{Nn} \; ; \;\; N^T \underline{f} = \underline{f}_N \; ,
\end{aligned} \tag{4.11}
$$

this results in

$$
M_{N\theta}\ddot{\underline{\theta}} + M_{N\varrho}\ddot{\underline{\varrho}} + C_{N\theta}\dot{\underline{\theta}} + C_{N\varrho}\dot{\underline{\varrho}} + K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho} + \underline{v}_{Nn} + \underline{f}_N = \underline{0} \; . \tag{4.12}
$$

In Section 2.5, some special models for flexible manipulators were given. As an example, the model of Subsection 2.5.3 for manipulators with flexible joints and rigid links is considered. With the same rigid-body coordinates $\underline{\theta}$ as in Subsection 2.5.3 and with the actuator rotor rotations $\underline{\varphi}$ as the flexible coordinates $\underline{\varrho}$, the distribution matrix H is given by Eq. (2.51) and it is seen that the transformation matrix T can equal the unit matrix. Then, the partial inertia and stiffness matrices of Eq. (4.11) are given by

$$
\begin{aligned}
\left[\, M_{N\theta} \;\; M_{N\varrho} \,\right] &= \left[\, M_l + M_s \;\; M_{sr} \,\right] \; ; \\
\left[\, K_{N\theta} \;\; K_{N\varrho} \,\right] &= \left[\, R^T K_d R \;\; -R^T K_d \,\right] \; .
\end{aligned} \tag{4.13}
$$

Relations for the partial matrices $C_{N\theta}$ and $C_{N\varrho}$ are not given here but can be determined following the method in Subsection 2.3.

In the literature on control of flexible manipulators, the model of Spong (1987) is often used. In this model, there is no dynamic coupling between the rigid-body coordinates and the flexible coordinates. Then, the partial inertia matrix $M_{N\varrho}$ and the associated matrix $C_{N\varrho}$ in (4.13) are zero, and Eq. (4.12) reduces to

$$
M_{N\theta}\ddot{\underline{\theta}} + C_{N\theta}\dot{\underline{\theta}} + K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho} + \underline{v}_{Nn} + \underline{f}_N = \underline{0} \; . \tag{4.14}
$$

4.3 Control objective and design strategy.

The inverse output equation $\underline{\theta} = \underline{\phi}^{-1}(\underline{y})$, combined with the desired output trajectory $\underline{y}_d = \underline{y}_d(t)$, determines the desired trajectory $\underline{\theta}_d = \underline{\theta}_d(t)$ for the rigid-body coordinates via

$$
\underline{\theta}_d(t) = \underline{\phi}^{-1}(\underline{y}_d(t)) \; . \tag{4.15}
$$

It is assumed that the function $\underline{\theta}_d = \underline{\theta}_d(t)$ is smooth with bounded derivatives up to the required order.

The control objective is to make the rigid-body coordinates track the desired trajectory asymptotically, under the constraint that all flexible coordinates $\underline{\varrho}$ and their derivatives $\dot{\underline{\varrho}}$ remain bounded. This constraint turns out to be the main obstacle in the design of a suitable controller. In this chapter, the strategy to achieve the control objective is to determine a bounded, sufficiently smooth desired trajectory $\underline{\varrho}_d = \underline{\varrho}_d(t)$ for the flexible coordinates, and to formulate a control law that guarantees asymptotic tracking of both the rigid and the flexible coordinates. Let $\underline{q}_d = \underline{q}_d(t)$, with $\underline{q}_d^T = [\ \underline{\theta}_d^T \ \ \underline{\varrho}_d^T\ ]$, represent the desired trajectory of the generalized coordinates, and let $\underline{e} = \underline{q}_d - \underline{q}$ again be the tracking error. To arrive at a set of equations for the unknown desired trajectory $\underline{\varrho}_d$ and for the input $\underline{u}$, the desired behavior of the controlled manipulator is formulated in terms of the tracking error. The error equation is of the form

$$M\ddot{\underline{e}} + \underline{g}(\underline{e}, \dot{\underline{e}}) = \underline{0}\ ; \qquad \underline{g}(\underline{0}, \underline{0}) = \underline{0}\ , \tag{4.16}$$

where the yet unspecified column function $\underline{g} = \underline{g}(\underline{e}, \dot{\underline{e}})$ has to guarantee that the equilibrium point $\underline{e} = \underline{0}$ of Eq. (4.16) is globally, asymptotically stable. In the next section, some possible choices for $\underline{g} = \underline{g}(\underline{e}, \dot{\underline{e}})$ will be considered.

Substitution of $\ddot{\underline{e}} = \ddot{\underline{q}}_d - \ddot{\underline{q}}$ and elimination of $\ddot{\underline{q}}$ by using the equations of motion (4.1) results in

$$M\ddot{\underline{q}}_d + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f} + \underline{g}(\underline{e}, \dot{\underline{e}}) = H\underline{u}\ . \tag{4.17}$$

After premultiplication with the transformation matrix T of the previous section, this yields

$$N^T[M\ddot{\underline{q}}_d + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f} + \underline{g}] \ = \ \underline{0}\ ; \tag{4.18}$$

$$\underline{u} = (H^T H)^{-1} H^T [M\ddot{\underline{q}}_d + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f} + \underline{g}] \quad . \tag{4.19}$$

The set (4.18) of $n - m$ equations is used to determine the desired trajectory $\underline{\varrho}_d = \underline{\varrho}_d(t)$ of the flexible coordinates, which is unknown beforehand. Subsequently, the input $\underline{u}$ follows from Eq. (4.19). This idea is projected in Fig. 4.1, where the model-based controller is split into a signal generator that computes the unknown desired trajectories and a torque generator that computes the input signals for the flexible manipulator. In this approach, the elastic deformations and vibrations are not sup-

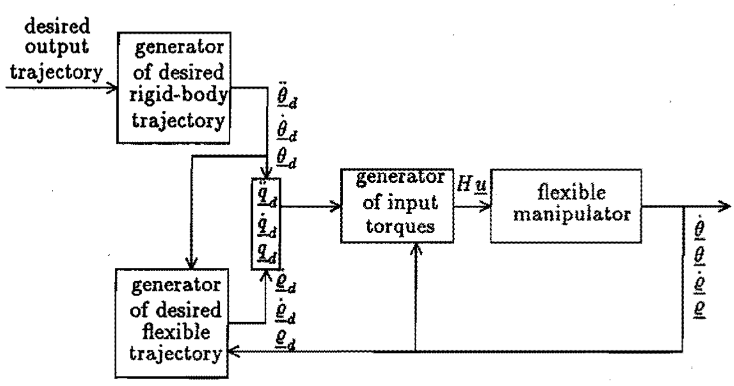


Figure 4.1: A control scheme for flexible manipulators.

pressed during trajectory tracking but they are stabilized on the basis of the equations of motion.

With the abbreviation

$$\underline{g}_N = N^T \underline{g} \ , \tag{4.20}$$

and the component matrices and columns introduced in Eq. (4.11), the equations for $\underline{\varrho}_d$ can be written as

$$M_{N\theta}\ddot{\underline{\theta}}_d + M_{N\varrho}\ddot{\underline{\varrho}}_d + C_{N\theta}\dot{\underline{\theta}} + C_{N\varrho}\dot{\underline{\varrho}} + K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho} + \underline{v}_{Nn} + \underline{f}_N + \underline{g}_N = \underline{0} \ . \tag{4.21}$$

It is not always obvious how a bounded solution for $\underline{\varrho}_d(t)$ can be calculated on-line from this equation. This has little to do with the choice of the column function $\underline{g} = \underline{g}(\underline{e}, \dot{\underline{e}})$ in the error equation (4.16). To illustrate this, it is assumed that the control objective, i.e. asymptotic tracking, is achieved. Then, $\underline{e}(t) \simeq \underline{0}$ and $\dot{\underline{e}}(t) \simeq \underline{0}$ and, hence, $\underline{g}(t) \simeq \underline{0}$ for large values of t. In that situation, $\underline{\theta}(t) \simeq \underline{\theta}_d(t)$ and $\underline{\varrho}(t) \simeq \underline{\varrho}_d(t)$, and Eq. (4.21) reduces to Eq. (4.12), where $\underline{\theta}$, $\underline{\varrho}$ and their derivatives are all replaced by the desired values. In fact, the problem of finding a control law for flexible manipulators boils down to the problem of finding an algorithm to determine a bounded solution for $\underline{\varrho}$ of Eq. (4.12). For some concrete cases, this will be considered in more detail in the next chapter. Here, only some general remarks are given. For the sake of simplicity, for the moment the columns $\underline{v}_{Nn}$ and $\underline{f}_N$ that are unspecified yet are neglected. Then, Eq. (4.12) reduces to

$$M_{N\theta}\ddot{\underline{\theta}} + M_{N\varrho}\ddot{\underline{\varrho}} + C_{N\theta}\dot{\underline{\theta}} + C_{N\varrho}\dot{\underline{\varrho}} + K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho} = \underline{0} \ . \tag{4.22}$$

The character of these equations for $\underline{\varrho}$ strongly depends on the properties of the matrices $M_{N\varrho}$, $C_{N\varrho}$ and $K_{N\varrho}$. If $M_{N\varrho} = 0$ and $C_{N\varrho} = 0$, as is the case for the model of Spong (1987), Eq. (4.22) represents a set of algebraic equations in $\underline{\varrho}$ and it is easy to determine a bounded solution for $\underline{\varrho}$ if the partial stiffness matrix $K_{N\varrho}$ is regular. This is discussed in more detail by Lammerts et. al. (1991) in their paper on the control of manipulators with a mixture of rigid and flexible joints.

If $M_{N\varrho} \neq 0$ and $C_{N\varrho} \neq 0$, then Eq. (4.22) can represent a mixed set of algebraic and second-order differential equations in the flexible coordinates. No numerical problems are to be expected provided that this set has a bounded solution, given a desired trajectory for the rigid-body coordinates and suitable initial conditions. However, this is not always the case, as will be seen in the next chapter. In that case, special algorithms have to be used.

4.4 Some control laws for the non-adaptive case.

4.4.1 Introduction.

In this section, some control laws for the non-adaptive case are derived. Attention is focused on the choice of the column function $\underline{g} = \underline{g}(\underline{e}, \dot{\underline{e}})$ in the error equation (4.16) and on the consequences of this choice for Eq. (4.18), from which the desired trajectory for the flexible coordinates is determined. A detailed discussion on the numerical elaboration of Eq. (4.18) is given in the next chapter.

4.4.2 Computed Desired trajectory Computed Torque Control (CDCTC).

A version of the classical CTC method as discussed in Section 3.4.2, but now for flexible manipulators, is obtained with the following choice of $\underline{g} = \underline{g}(\underline{e}, \underline{\dot{e}})$:

$$\underline{g}(\underline{e}, \underline{\dot{e}}) = M(L_D \underline{\dot{e}} + L_P \underline{e}) \; , \tag{4.23}$$

where L_P is symmetric and where both L_P and $L_D + L_D^T$ are positive definite. Eq. (4.18) then becomes

$$N^T[M(\underline{\ddot{q}}_d + L_D \underline{\dot{e}} + L_P \underline{e}) + C\underline{\dot{q}} + K\underline{q} + \underline{v}_n + \underline{f}] = \underline{0} \; . \tag{4.24}$$

It is assumed that L_D and L_P are block diagonal matrices, given by

$$\begin{aligned} L_D &= \begin{bmatrix} D_{\theta\theta} & 0 \\ 0 & D_{\varrho\varrho} \end{bmatrix} ; \quad D_{\theta\theta} + D_{\theta\theta}^T > 0 \; ; \quad D_{\varrho\varrho} + D_{\varrho\varrho}^T > 0 \; , \\ L_P &= \begin{bmatrix} P_{\theta\theta} & 0 \\ 0 & P_{\varrho\varrho} \end{bmatrix} ; \quad P_{\theta\theta} = P_{\theta\theta}^T \; ; \quad P_{\varrho\varrho} = P_{\varrho\varrho}^T \; ; \quad P_{\theta\theta} > 0 \; ; \quad P_{\varrho\varrho} > 0 \; . \end{aligned} \tag{4.25}$$

Now, Eq. (4.24) can be written as

$$\begin{aligned} M_{N\theta}(\underline{\ddot{\theta}}_d + D_{\theta\theta}\underline{\dot{e}}_\theta + P_{\theta\theta}\underline{e}_\theta) + M_{N\varrho}(\underline{\ddot{\varrho}}_d + D_{\varrho\varrho}\underline{\dot{e}}_\varrho + P_{\varrho\varrho}\underline{e}_\varrho) &\; + \\ + C_{N\theta}\underline{\dot{\theta}} + C_{N\varrho}\underline{\dot{\varrho}} + K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho} + \underline{v}_{Nn} + \underline{f}_N &= \underline{0} \; , \end{aligned} \tag{4.26}$$

where $\underline{e}_\theta = \underline{\theta}_d - \underline{\theta}$ and $\underline{e}_\varrho = \underline{\varrho}_d - \underline{\varrho}$.

It depends on the properties of the matrices in this equation and on the properties of the column functions $\underline{v}_{Nn}$ and $\underline{f}_N$, whether it is possible to determine a bounded solution for $\underline{\varrho}_d$. For instance, in the model of Spong $M_{N\varrho} = 0$ and $C_{N\varrho} = 0$, and it is easily seen that Eq. (4.26) does not yield any relation at all for $\underline{\varrho}_d$. Hence, the choice (4.25) for diagonal control gains L_P and L_D is not usable for this model and for the given error equation (4.23). One of the possible modifications, at least for this model, is to choose:

$$L_P = \begin{bmatrix} P_{\theta\theta} & S \\ 0 & P_{\varrho\varrho} \end{bmatrix} \tag{4.27}$$

where S is a yet unspecified $(n \times (n-m))$ matrix. The error equation is then given by

$$\underline{\ddot{e}}_\theta + D_{\theta\theta}\underline{\dot{e}}_\theta + P_{\theta\theta}\underline{e}_\theta = -S\underline{e}_\varrho \; ; \tag{4.28}$$

$$\underline{\ddot{e}}_\varrho + D_{\varrho\varrho}\underline{\dot{e}}_\varrho + P_{\varrho\varrho}\underline{e}_\varrho = \underline{0} \; , \tag{4.29}$$

and the equilibrium point $\underline{e} = \underline{0}$, i.e. $\underline{e}_\theta = \underline{0}$ and $\underline{e}_\varrho = \underline{0}$, is globally, asymptotically stable as follows from the stability theorem for hierarchical systems (Vidyasagar, 1993). Furthermore, Eq. (4.26) becomes

$$\begin{aligned} M_{N\theta}(\underline{\ddot{\theta}}_d + D_{\theta\theta}\underline{\dot{e}}_\theta + P_{\theta\theta}\underline{e}_\theta) + M_{N\varrho}(\underline{\ddot{\varrho}}_d + D_{\varrho\varrho}\underline{\dot{e}}_\varrho + P_{\varrho\varrho}\underline{e}_\varrho) &\; + \\ C_{N\theta}\underline{\dot{\theta}} + C_{N\varrho}\underline{\dot{\varrho}} + K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho} + M_{N\theta}S\underline{e}_\varrho + \underline{v}_{Nn} + \underline{f}_N &= \underline{0} \; . \end{aligned} \tag{4.30}$$

It is assumed that the rank of the $((n-m)\times m)$-matrix $M_{N\theta}$ is equal to $n-m$. Now, S can be chosen, for example, such that

$$M_{N\theta}S = K_{N\varrho} \ , \tag{4.31}$$

and, by using $\underline{e}_{\varrho} = \underline{\varrho}_d - \underline{\varrho}$, it is easily seen that the term $K_{N\varrho}\underline{\varrho} + M_{N\theta}S\underline{e}_{\varrho}$ reduces to $K_{N\varrho}\underline{\varrho}_d$. For the model of Spong, Eq. (4.30) then yields

$$M_{N\theta}(\ddot{\underline{\theta}}_d + D_{\theta\theta}\dot{\underline{e}}_\theta + P_{\theta\theta}\underline{e}_\theta) + C_{N\theta}\dot{\underline{\theta}} + K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho}_d + \underline{v}_{Nn} + \underline{f}_N = \underline{0} \ , \tag{4.32}$$

from which the unknown desired trajectory $\underline{\varrho}_d$ can be quite easily computed. With Eq. (4.13), it can be seen that a 'computed torque-like' definition has been given for the flexible-joint torques $R^T K_d(R\underline{\theta} - \underline{\varrho})$, namely $R^T K_d(R\underline{\theta} - \underline{\varrho}_d) = K_{N\theta}\underline{\theta} + K_{N\varrho}\underline{\varrho}_d$, according to Eq. (4.32). They are defined such that the tracking objective for the rigid-body coordinates is achieved if the actual flexible-joint torques track these 'desired' flexible-joint torques asymptotically. This is assured for a control input $\underline{u}$ according to Eq. (4.19), which yields the closed-loop performance from Eqs. (4.28) and (4.29). This CTC-like control law will be called Computed Desired trajectory Computed Torque Control (CDCTC). A special version of this control law for flexible-joint manipulators has been discussed by Lammerts (1990) and Lammerts et. al. (1991). It results in quite acceptable behavior of the closed-loop system. Lammerts (1990) and Lammerts et. al. (1991) showed that a procedure as outlined above is also usable for manipulators with a mixture of rigid and flexible joints.

4.4.3 Passivity-based CDCTC.

The discussion on the passivity-based CTC method for rigid manipulators in Subsection 3.4.3 suggests the following choice of $\underline{g} = \underline{g}(\underline{e}, \dot{\underline{e}})$:

$$\underline{g}(\underline{e}, \dot{\underline{e}}) = C\dot{\underline{e}} + K_D\dot{\underline{e}} + K_P\underline{e} \ , \tag{4.33}$$

where K_P and K_D are chosen similarly to the choice of L_D and L_P, respectively, in Eq. (4.25). A similar derivation as in the previous subsection shows that with this choice no relation at all is obtained for the desired trajectory $\underline{\varrho}_d = \underline{\varrho}_d(t)$ if the model of Spong is used. For that reason, Eq. (4.33) is directly modified to

$$\underline{g}(\underline{e}, \dot{\underline{e}}) = C\dot{\underline{e}} + K_D\dot{\underline{e}} + K_P\underline{e} + K\underline{e} \ . \tag{4.34}$$

With this choice, the error equation becomes $M\ddot{\underline{e}} + C\dot{\underline{e}} + K_D\dot{\underline{e}} + K_P\underline{e} = \underline{0}$, which can be shown to be globally, asymptotically stable, with the Lyapunov function $V = \frac{1}{2}\dot{\underline{e}}^T M\dot{\underline{e}} + \frac{1}{2}\underline{e}^T(K_P + K)\underline{e}$ (see further Subsection 3.4.3). Now, Eq. (4.18) becomes

$$N^T[M\ddot{\underline{q}}_d + C\dot{\underline{q}}_d + K\underline{q}_d + K_D\dot{\underline{e}} + K_P\underline{e} + \underline{v}_n + \underline{f}] = \underline{0} \ , \tag{4.35}$$

which can also be written as

$$\begin{aligned} M_{N\theta}\ddot{\underline{\theta}}_d + M_{N\varrho}\ddot{\underline{\varrho}}_d + C_{N\theta}\dot{\underline{\theta}}_d + C_{N\varrho}\dot{\underline{\varrho}}_d + K_{N\theta}\underline{\theta}_d + K_{N\varrho}\underline{\varrho}_d \ &+ \\ N^T K_D\dot{\underline{e}} + N^T K_P\underline{e} + \underline{v}_{Nn} + \underline{f}_N \ &= \ \underline{0} \ . \end{aligned} \tag{4.36}$$

As in the previous subsection, it depends on the properties of the matrices in this equation and on the properties of the column $\underline{v}_{Nn} + \underline{f}_N$ whether it is possible to determine a bounded solution for $\underline{\varrho}_d$. For the model of Spong, Eq. (4.36) reduces to

$$M_{N\theta}\ddot{\underline{\theta}}_d + C_{N\theta}\dot{\underline{\theta}}_d + K_{N\theta}\underline{\theta}_d + K_{N\varrho}\underline{\varrho}_d + N^T(K_D\dot{\underline{e}} + K_P\underline{e}) + \underline{v}_{Nn} + \underline{f}_N = \underline{0}\ . \quad (4.37)$$

For manipulators with rigid links and flexible joints, it can be shown that Eq. (4.37) can be easily solved for $\underline{\varrho}_d$. This also holds for manipulators with a mixture of rigid and flexible joints. Because the control law is an extension of passivity-based CTC, it will be called passivity-based CDCTC. Simulations with a similar control approach applied to a flexible-joint translation manipulator were reported by Kuijpers (1993).

4.4.4 Computed Reference Computed Torque Control (CR-CTC).

In Section 3.5, the concept of sliding computed torque control has been discussed. An essential part of that concept is the introduction of a reference trajectory $\underline{q}_r$ that is related to the desired trajectory $\underline{q}_d$ by a relation of the form

$$\dot{\underline{q}}_r = \dot{\underline{q}}_d + \Lambda\underline{e}\ ; \qquad \underline{e} = \underline{q}_d - \underline{q}\ , \quad (4.38)$$

where $\Lambda + \Lambda^T$ is a positive definite matrix. Instead of choosing a discontinuous law, the sliding control term $Gsign(\dot{\underline{e}}_r)$ of Eq. (3.45) can be replaced by the PD-like control term $K_r\dot{\underline{e}}_r$ of Eq. (3.44). For the perfect model of a rigid manipulator, the passivity-based CTC law with reference trajectory (Subsection 3.5.2) then results in the error equation

$$M\ddot{\underline{e}}_r + C\dot{\underline{e}}_r + K_r\dot{\underline{e}}_r = \underline{0}\ . \quad (4.39)$$

To obtain the same error equation for a flexible manipulator, the function $\underline{g} = \underline{g}(\underline{e}, \dot{\underline{e}})$ should be chosen

$$\underline{g}(\underline{e}, \dot{\underline{e}}) = M\Lambda\dot{\underline{e}} + C\dot{\underline{e}}_r + K_r\dot{\underline{e}}_r\ . \quad (4.40)$$

For the model of Spong, however, this choice results in the problems noted earlier, here with respect to the determination of a reference trajectory $\underline{\varrho}_r = \underline{\varrho}_r(t)$ for the flexible coordinates. For that reason, an extra term $K\underline{e}_r$ is added to the right-hand side of Eq. (4.40), i.e.

$$\underline{g}(\underline{e}, \dot{\underline{e}}) = M\Lambda\dot{\underline{e}} + C\dot{\underline{e}}_r + K_r\dot{\underline{e}}_r + K\underline{e}_r\ . \quad (4.41)$$

For the moment, it is assumed that with this choice Eq. (4.18) can result in a bounded solution for $\underline{\varrho}_r = \underline{\varrho}_r(t)$ (and, hence, for $\underline{\varrho}_d = \underline{\varrho}_d(t)$ according to definition (4.38)). The input $\underline{u}$ according to Eq. (4.19) now leads to the following error equation for the closed-loop system:

$$M\ddot{\underline{e}}_r + C\dot{\underline{e}}_r + K_r\dot{\underline{e}}_r + K\underline{e}_r = \underline{0}\ . \quad (4.42)$$

Because the stiffness matrix K is singular, the function V defined by

$$V = \frac{1}{2}\dot{\underline{e}}_r^T M\dot{\underline{e}}_r + \frac{1}{2}\underline{e}_r^T K\underline{e}_r \quad (4.43)$$

is not a "clean" candidate-Lyapunov function (see Appendix A), since it is semi-positive definite and not positive definite. As a consequence, this function cannot be used to analyze the stability of the equilibrium point $\underline{e}_r = \underline{0}$ in the strict sense of Lyapunov. However, the time-derivative of V,

$$\dot{V} = -\dot{\underline{e}}_r^T K_r \dot{\underline{e}}_r \,, \tag{4.44}$$

is negative for all $\dot{\underline{e}}_r \neq \underline{0}$ because the gain matrix K_r must be positive definite. Hence, it can be concluded that $\dot{\underline{e}}_r(t) \to \underline{0}$ for $t \to \infty$. According to definition (4.38) where $\dot{\underline{e}}_r = \dot{\underline{e}} + \Lambda \underline{e}$, this means that $\dot{\underline{e}}(t) \to \underline{0}$ for $t \to \infty$ $\underline{e}(t) \to \underline{0}$ for $t \to \infty$ (see also Subsection 3.6.3). This implies that the control objective is achieved if a bounded solution $\underline{\varrho}_r = \underline{\varrho}_r(t)$ for Eq. (4.18) can be determined.

Substitution of the choice (4.41) in Eq. (4.18) yields, after some reordering,

$$N^T[M\ddot{\underline{q}}_r + C\dot{\underline{q}}_r + K\underline{q}_r + \underline{v}_n + \underline{f} + K_r\dot{\underline{e}}_r] = \underline{0} \,. \tag{4.45}$$

In the same way as Eq. (4.30) and Eq. (4.36), this equation can be written as:

$$\begin{aligned} M_{N\theta}\ddot{\underline{\theta}}_r + M_{N\varrho}\ddot{\underline{\varrho}}_r + C_{N\theta}\dot{\underline{\theta}}_r + C_{N\varrho}\dot{\underline{\varrho}}_r \quad &+ \\ K_{N\theta}\underline{\theta}_r + K_{N\varrho}\underline{\varrho}_r + N^T K_r\dot{\underline{e}}_r + \underline{v}_{Nn} + \underline{f}_N \quad &= \quad \underline{0} \,. \end{aligned} \tag{4.46}$$

As in the previous subsections, it depends on the properties of the matrices in this equation and on the properties of the column $\underline{v}_{Nn} + \underline{f}_N$ whether it is possible to determine a bounded solution $\underline{\varrho}_r = \underline{\varrho}_r(t)$. In the absence of a dynamic coupling between the rigid-body coordinates and the flexible coordinates, i.e. if $M_{N\varrho} = 0$ and $C_{N\varrho} = 0$ as in the model of Spong, Eq. (4.46) simplifies to:

$$M_{N\theta}\ddot{\underline{\theta}}_r + C_{N\theta}\dot{\underline{\theta}}_r + K_{N\theta}\underline{\theta}_r + K_{N\varrho}\underline{\varrho}_r + N^T K_r\dot{\underline{e}}_r + \underline{v}_{Nn} + \underline{f}_N = \underline{0} \,. \tag{4.47}$$

For manipulators with a mixture of rigid and flexible joints, and with no (or neglected) dynamic coupling between rigid-body and flexible coordinates, this equation can be easily solved with respect to $\underline{\varrho}_r$, especially if the gain matrices K_r and Λ are block diagonal. Because of the use of a reference trajectory that is computed on-line, the control concept will be called Computed Reference Computed Torque Control (CRCTC). For a detailed block scheme of the CRCTC approach, see Fig. 4.2. It can be noticed, that it has been considerably extended in comparison with the CTC scheme shown in Chapter 3, but it can be realized reasonably well in practice (see Chapter 5).

4.4.5 Conclusions.

The usefulness of the proposed control concepts for a concrete manipulator depends completely on the availability of an algorithm to determine a bounded solution for the desired trajectory $\underline{\varrho}_d = \underline{\varrho}_d(t)$ from Eq. (4.30) or Eq. (4.36) or for the reference trajectory $\underline{\varrho}_r = \underline{\varrho}_r(t)$ from Eq. (4.46). In general, Eqs. (4.30), (4.36) and (4.46) represent a mixed set of differential and algebraic equations in the unknown $\underline{\varrho}_d = \underline{\varrho}_d(t)$ or $\underline{\varrho}_r = \underline{\varrho}_r(t)$. If all equations are algebraic, as is the case with the model of

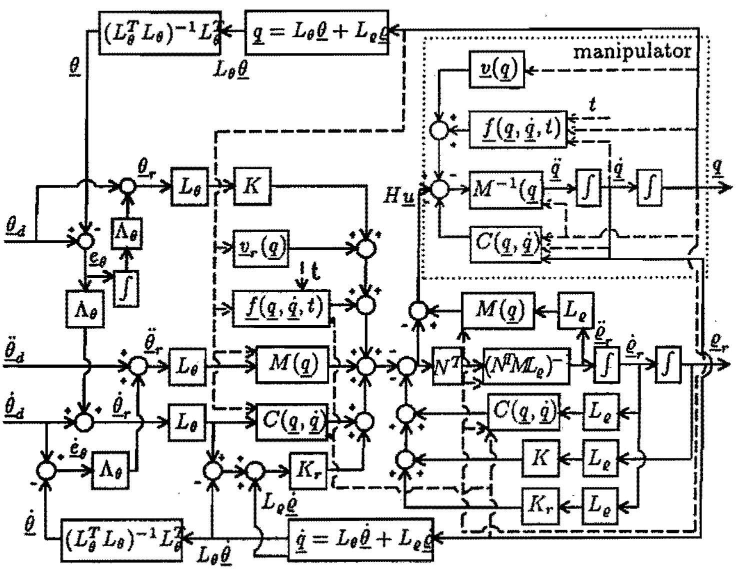


Figure 4.2: Computed reference computed torque control (CRCTC).

Spong, the determination of the (desired or reference) trajectory will not run into great difficulties. If some of the equations are algebraic and the others are first- or second-order differential equations, then well-known numerical time-integration procedures for DAE's (differential algebraic equations) can be used provided that the DAE's with appropriate initial conditions yield a bounded solution for $\underline{\varrho}_d = \underline{\varrho}_d(t)$ or $\underline{\varrho}_r = \underline{\varrho}_r(t)$. Unfortunately, this is not always the case since the differential equations may be unstable. Then, other techniques like multiple shooting (see, for instance, Mattheij and Staarink, 1987) have to be used to find a bounded solution. This will be discussed in more detail for some special cases in the next chapter.

Until now, no attention has been paid to the elaboration of the control law (4.19) for the input torques $\underline{u}$. This generally requires the second derivative of the (desired or reference) trajectory to be available. For the model of Spong, for instance, the second derivative of $\underline{\varrho}_d$ or $\underline{\varrho}_r$ is not computed from Eq. (4.18), since this equation contains only an algebraic relation for the trajectory itself. Let us consider the case of Subsection 4.4.3 in which the unknown desired trajectory $\underline{\varrho}_d$ is solved from Eq. (4.37). Possible ways to yield $\dot{\underline{\varrho}}_d$ and $\ddot{\underline{\varrho}}_d$ are to apply a numerical, (weak) differentiation scheme or to determine them analytically. The last approach needs the first and second time-derivatives of the equation (4.37), which is an algebraic equation for $\underline{\varrho}_d$: $\underline{\varrho}_d = \underline{\varrho}(\underline{q}, \dot{\underline{q}}, \underline{\theta}_d, \dot{\underline{\theta}}_d, \ddot{\underline{\theta}}_d, t)$. To determine the required derivatives of this trajectory analytically, the unknown accelerations $\ddot{\underline{q}}$ and jerks $\underline{q}^{(3)}$ have to be available. Making use of the closed-loop dynamics $M\ddot{\underline{e}} + C\dot{\underline{e}} + K_D\dot{\underline{e}} + K_P\underline{e} + K\underline{q} = \underline{0}$ and of the accessible state variables $\underline{q}$ and $\dot{\underline{q}}$, the required accelerations and jerks can be computed according to

$$\begin{aligned} \ddot{\underline{q}} &= \ddot{\underline{q}}_d + M^{-1}[(C + K_D)\dot{\underline{e}} + (K + K_P)\underline{e}] \ \text{ and} && (4.48) \\ \underline{q}^{(3)} &= \underline{q}_d^{(3)} + M^{-1}[(\dot{M} + C + K_D)\ddot{\underline{e}} + (\dot{C} + K + K_P)\dot{\underline{e}}] \ , && (4.49) \end{aligned}$$

respectively. Similarly, in the CRCTC approach of Subsection 4.4.4 the first and second time-derivatives of the reference trajectory $\underline{\varrho}_r$, computed from Eq. (4.47), can be determined analytically. Because this method is computationally time-consuming (e.g. Doelman, 1993), the unknown (desired or reference) velocities and accelerations in simulations and experiments have mostly been computed on the basis of (weak) differentiation schemes. The results will be discussed in the next chapter.

4.5 Adaptive CRCTC.

The control laws of the preceding section have been based on the assumption that the parameters used in these laws match the parameters $\underline{p}$ of the real system. In practice, however, only an estimate $\hat{\underline{p}}$ for $\underline{p}$ is available. In this section, the CRCTC law of Subsection 4.4.4 is modified in order to account for parametric uncertainty. The resulting adaptive control law for flexible manipulators can be seen as an extension of the adaptive CTC law for rigid manipulators (as discussed in Subsection 3.6.3).

As mentioned in Section 4.2, it is assumed that the structure of the model (4.1) of the manipulator is correct and that the model matrices M, C and K and the column $\underline{v}_n + \underline{f}$ are linear functions of $\underline{p}$, while the distribution matrix H does not depend on $\underline{p}$. The control law for the case with a priori unknown parameters is essentially the same as for the case with known parameters. The only difference is the replacement of $\underline{p}$ in Eq. (4.18) and (4.19) by the available estimate $\hat{\underline{p}}$. Due to the parameter adaptation error $\underline{e}_p = \hat{\underline{p}} - \underline{p}$, the error equation of the closed-loop system becomes

$$M\ddot{\underline{e}}_r + C\dot{\underline{e}}_r + K_r\dot{\underline{e}}_r + K\underline{e}_r = -W_r\underline{e}_p \ , \tag{4.50}$$

where W_r is defined similarly to W in Section 4.2 and can depend on $\underline{q}$, $\dot{\underline{q}}$, $\underline{q}_r$, $\dot{\underline{q}}_r$ and $\ddot{\underline{q}}_r$ (see also Subsection 3.6.3). With the Lyapunov function

$$\begin{gathered} V = \frac{1}{2}\dot{\underline{e}}_r^T M \dot{\underline{e}}_r + \frac{1}{2}\underline{e}_r^T K \underline{e}_r + \frac{1}{2}\underline{e}_p^T \Gamma^{-1} \underline{e}_p \ ; \qquad (4.51) \\ \Gamma = \Gamma^T, \ \ \Gamma > 0, \ \ \Gamma \text{ constant} \ , \end{gathered}$$

and with similar reasoning as in Section 3.6, it can be shown that the adaptation algorithm

$$\dot{\underline{e}}_p = \Gamma W_r^T \dot{\underline{e}}_r \ , \tag{4.52}$$

will result in $\dot{\underline{e}}_r(t) \to \underline{0}$ for $t \to \infty$, and in $\dot{\underline{e}}_p(t) \to \underline{0}$ for $t \to \infty$. Because $\underline{p}$ is assumed to be constant, $\dot{\underline{e}}_p = \dot{\hat{\underline{p}}} - \dot{\underline{p}} = \dot{\hat{\underline{p}}}$ and $\dot{\hat{\underline{p}}} \to \underline{0}$ for $t \to \infty$. However, there is no guarantee that the parameters $\hat{\underline{p}}$ will converge to the real values $\underline{p}$. In general, this is not a problem because it is only required that the tracking error $\underline{e}$ converges to zero.

4.6 CRCTC of the end-effector.

For direct control of the end-effector (see also Section 3.7), a new set of generalized coordinates $\underline{q}^* = [\ \underline{y}^T \ \ \underline{\varrho}^T\ ]^T$ can be used instead of $\underline{q} = [\ \underline{\theta}^T \ \ \underline{\varrho}^T\ ]^T$. In that case, a coordinate transformation has to be performed on the manipulator model (as shown in Subsection 2.3.4) and the CRCTC law for the end-effector can be formulated as folllows:

$$H^*\underline{u} = M^*\ddot{\underline{q}}_r^* + C^*\dot{\underline{q}}_r^* + K^*\underline{q}_r^* + \underline{v}_n^* + \underline{f}^* + K_r\dot{\underline{e}}_r^* \ . \tag{4.53}$$

This method can also be used for CDCTC of the end-effector. The transformed matrices and columns are obtained according to Eq. (2.28). As can be seen from the new set of generalized coordinates, this approach uses feedback of both the end-effector coordinates and the flexible coordinates. Hence, it cannot be referred to as pure "output control" in the sense often used in literature. The most recent development of the software package ASP (Van de Molengraft, 1989-1993) enables to perform simulations of CRCTC on the basis of control scheme (4.53), such that the user only has to give the original model $M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} + \underline{v}_n + \underline{f} = H\underline{u}$, the new set of generalized coordinates $\underline{q}^*$ and its relation to $\underline{q}$, i.e. $\underline{q} = \underline{\psi}(\underline{q}^*)$.

4.7 Conclusions.

In this chapter, some control laws have been designed for flexible manipulators. Summarizing, the *first CDCTC* law (Subsection 4.4.2), based on a modification of the classical CTC scheme, can be given by

$$\begin{aligned} H(\underline{q})\underline{u} &= M(\underline{q},\underline{p})[\ddot{\underline{q}}_d + L_D\dot{\underline{e}} + L_P\underline{e}] + C(\underline{q},\dot{\underline{q}},\underline{p})\dot{\underline{q}} + K(\underline{p})\underline{q} + \\ &+ \underline{v}_n(\underline{q},\underline{p}) + \underline{f}(\underline{q},\dot{\underline{q}},\underline{p},t)\ , \end{aligned} \tag{4.54}$$

Passivity-based CDCTC is a modification of the passivity-based CTC scheme with desired trajectory, and the proposed law has the following form:

$$\begin{aligned} H(\underline{q})\underline{u} &= M(\underline{q},\underline{p})\ddot{\underline{q}}_d + C(\underline{q},\dot{\underline{q}},\underline{p})\dot{\underline{q}}_d + K(\underline{p})\underline{q}_d + \\ &+ \underline{v}_n(\underline{q},\underline{p}) + \underline{f}(\underline{q},\dot{\underline{q}},\underline{p},t) + K_D\dot{\underline{e}} + K_P\underline{e}\ . \end{aligned} \tag{4.55}$$

This is a more useful control law than the first CDCTC law, because for all manipulator configurations it generally results directly in an equation that the unknown desired trajectory $\underline{\varrho}_d(t)$ can be computed from (Subsection 4.4.3). Finally, in Subsection 4.4.4 the *CRCTC* law was proposed as a modification of the passivity-based CTC scheme with reference trajectory:

$$\begin{aligned} H(\underline{q})\underline{u} &= M(\underline{q},\underline{p})\ddot{\underline{q}}_r + C(\underline{q},\dot{\underline{q}},\underline{p})\dot{\underline{q}}_r + K(\underline{p})\underline{q}_r + \\ &+ \underline{v}_n(\underline{q},\underline{p}) + \underline{f}(\underline{q},\dot{\underline{q}},\underline{p},t) + K_r\dot{\underline{e}}_r\ . \end{aligned} \tag{4.56}$$

Global, asymptotic stability of the closed-loop system can be proven via the second method of Lypunov. For a global sketch of the CRCTC approach, see Fig. 4.3. This

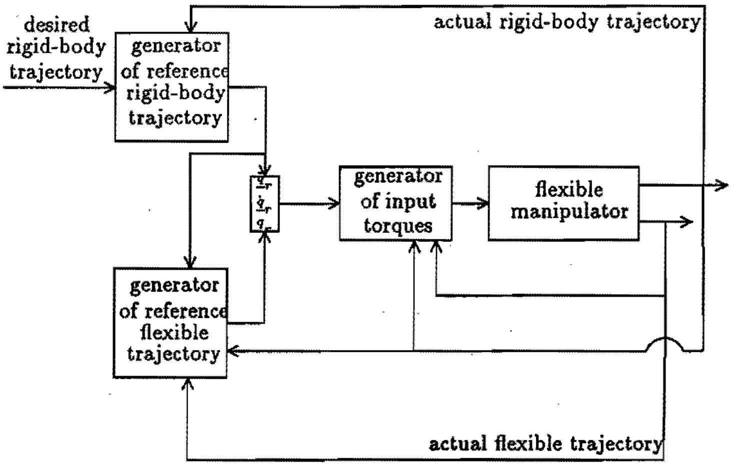


Figure 4.3: A global block scheme of CRCTC.

new control scheme for flexible manipulators can be easily extended to the adaptive control scheme proposed in Section 4.5. The control law then becomes:

$$\begin{aligned} H(\underline{q})\underline{u} &= M(\underline{q},\hat{\underline{p}})\ddot{\underline{q}}_r + C(\underline{q},\dot{\underline{q}},\hat{\underline{p}})\dot{\underline{q}}_r + K(\hat{\underline{p}})\underline{q}_r + \\ &+ \underline{v}_n(\underline{q},\hat{\underline{p}}) + \underline{f}(\underline{q},\dot{\underline{q}},\hat{\underline{p}},t) + K_r\dot{\underline{e}}_r \ , \end{aligned} \tag{4.57}$$

where the model parameters $\hat{\underline{p}}$ are adjusted on-line by the adaptation algorithm (4.52). For a global sketch of the adaptive CRCTC approach, see Fig. 4.4. This block scheme shows some resemblance to the global block scheme in Fig. 3.8 of adaptive CTC for rigid manipulators. Because parametric uncertainty is always present in practice, our research has been focused especially on the CRCTC approach for which an adaptive version can be formulated. Hence, in the next chapter only (adaptive) CRCTC results from simulations and experiments will be shown. A somewhat different approach to CRCTC for flexible manipulators has been discussed in Lammerts et al. (1993). For similar results obtained with CDCTC or CRCTC, we refer to Lammerts (1990), Lammerts et al. (1991), Brevoord (1992), Doelman (1992), Kuijpers (1993), Kerssemakers (1993), Heijman (1993) and Kaats (1993).

In Section 4.6, it was shown that the new control approaches summarized above can — provided that no numerical problems appear — be applied to realize asymptotic tracking of the end-effector directly, while all generalized coordinates remain bounded. This means that the serious drawback of the control concepts discussed in Chapter 3 has been cleared.

For an interpretation of the control laws proposed in this chapter for flexible

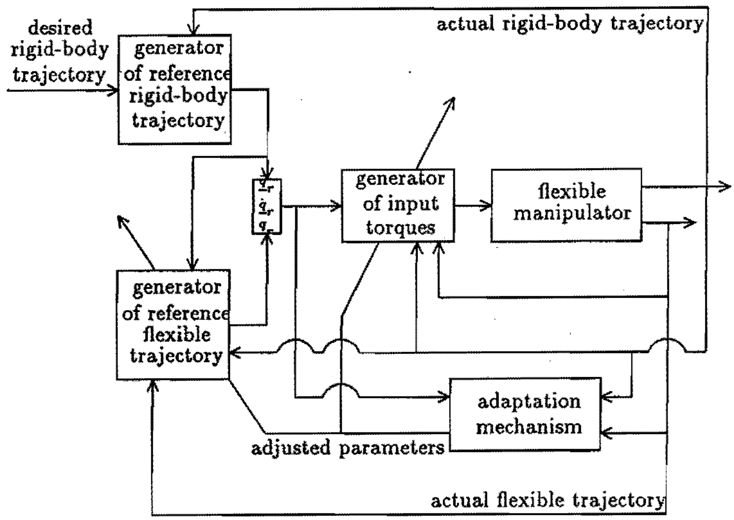


Figure 4.4: A global block scheme of adaptive CRCTC.

manipulators, consider the selected column of generalized coordinates $\underline{q}^T = [\ \underline{\theta}^T\ \underline{\varepsilon}^T\ ]^T$ for the model of a manipulator with flexible joints and rigid links (Subsection 2.5.3), and $\underline{q}^T = [\ \underline{\theta}_v^T\ \underline{\beta}^T\ ]^T$ for the model of a manipulator with rigid joints and flexible links (Subsection 2.5.4), with the column $\underline{\varepsilon}$ or $\underline{\beta}$ that contains the elastic deformations. The corresponding stiffness matrix is $K = \begin{bmatrix} 0 & 0 \\ 0 & K_d \end{bmatrix}$ and $K = \begin{bmatrix} 0 & 0 \\ 0 & K_\beta \end{bmatrix}$, respectively. If the flexibility is negligibly small, i.e. if the largest element in K_d^{-1} or K_β^{-1} is very small, the second set of equations of the model that contains the elastic torques $K_d\underline{\varepsilon}$ or $K_\beta\underline{\beta}$ reduces to $\underline{\varepsilon} \simeq \underline{0}$ or $\underline{\beta} \simeq \underline{0}$. Similarly, for a CDCTC or CRCTC law the equivalent set of equations reduces to a condition for the unknown desired trajectory, i.e. $\underline{\varepsilon}_d \simeq \underline{0}$ or $\underline{\beta}_d \simeq \underline{0}$, or for the unknown reference trajectory, i.e. $\underline{\varepsilon}_r \simeq \underline{0}$ or $\underline{\beta}_r \simeq \underline{0}$. Hence, these conditions cause the set of equations of the CDCTC or CRCTC law that do not contain the elastic torques to be simplified to a CTC law for the equivalent rigid manipulator (with the inertia matrix $M = M_l + M_s + R^T M_{sr} + M_{sr} R + R^T M_r R$ or $M = M_{vv}$). This means that each of the control schemes is composed of a computed torque controller for the equivalent rigid manipulator and a corrective control term to compensate for the effects of flexibility. In Chapter 5, it will be shown that the contribution of this corrective control term becomes more significant as flexibility increases. In Lammerts et al. (1991), the composite structure was shown for a manipulator with a mixture of flexible and rigid joints.

Chapter 5

Simulations and experiments.

5.1 Introduction.

In this chapter, some simulation and experimental results with (adaptive) CRCTC applied to flexible manipulators are presented. As an introduction, first a brief description of some recent results from our control laboratory is given. They mostly deal with control laws based on rigid manipulator models, as discussed in Chapter 3.

De Jager (1992a, 1992c) has investigated the robustness of various control techniques such as adaptive CTC, sliding control and acceleration-based control in comparison with PD-control and CTC. Simulations has been performed on a rotation-translation (RT) manipulator, a horizontal translation-translation (TT) manipulator, and a translation (T) manipulator. Furthermore, experiments have been carried out on the TT-manipulator. This so-called XY-table has some properties that are usually undesired but unavoidable in practice, such as friction and flexibility. This flexibility makes the XY-table a suitable set-up for the investigation of (adaptive) CRCTC.

The tracking performance of a rigid manipulator can be considerably improved by applying adaptive control instead of non-adaptive control (e.g. De Jager, 1992a, 1992c; Doelman, 1992). Vijverstra (1992) has performed simulations with adaptive CTC on a translation-rotation (TR) manipulator, and he has made a comprehensive comparison of direct, indirect and composite adaptive control techniques by means of simulations and experiments on the XY-table. He has investigated also the robustness of these techniques for unmodeled dynamics. Vijverstra (1992) and De Jager (1992c) both concluded that adaptive CTC is often advantageous for the trajectory tracking of mechanical systems with uncertain parameters, provided that the number of degrees of freedom equals the number of control inputs. Because of the ease of implementation, direct adaptive control (Section 3.6) is to be preferred above indirect or composite adaptive control.

Adaptive CTC results in smaller tracking errors than PD-control (De Jager, 1992c). However, adaptive and non-adaptive CTC appeared to be more sensitive to unmodeled dynamics than a PD-controller (De Jager, 1992c). The robustness can be improved by addition of a sliding control term (e.g. De Jager, 1992b, 1992c).

However, because robust controllers cannot compensate completely for model uncertainty, De Jager (1992c) and Vijverstra (1992) recommended the use of a more detailed model in control design. For the XY-table, it was necessary to extend the model with relevant dynamics, such as Coulomb friction and flexibility of the drive spindle. De Jager (1993) has studied adaptive friction compensation and he has developed a detailed friction model for the XY-table that incorporates Coulomb, viscous and harmonic friction terms.

In Chapter 4, some control laws for flexible manipulators were proposed. Brevoord (1992) has tested non-adaptive CRCTC by means of simulations and experiments on the XY-table. Satisfying results have been obtained using the model identified by Van de Molengraft (1990). Because some parameters, especially the friction parameters, are uncertain, Kaats (1993) has implemented adaptive CRCTC on the flexible XY-table.

Extensive simulations with CRCTC have been carried out. Some of the results for a flexible TR-manipulator will be presented in Section 5.2. This manipulator has significant nonlinearities due to Coriolis and centrifugal effects and can have a flexible joint and a flexible link. The ideal case will be considered in which the structure of the model is exactly known. Simulation results with CRCTC will be discussed for the *rigid* TR-manipulator (Subsection 5.2.2), for the TR-manipulator with a *flexible joint* and a rigid link (Subsection 5.2.3), for the TR-manipulator with a rigid joint and a *flexible link* (Subsection 5.2.4), and for the TR-manipulator with a *flexible joint and a flexible link* (Subsection 5.2.5). In Subsection 5.2.6, some of the simulation results obtained with adaptive CRCTC of the flexible TR-manipulator will be shown. Subsection 5.2.7 will consider explicitly CRCTC of the end-effector of the flexible TR-manipulator. The simulations have been performed with ASP (Van de Molengraft, 1989-1993).

To investigate the practical performance of (adaptive) CRCTC in the presence of structure uncertainty, it has been implemented on two experimental set-ups, i.e. a flexible rotation (R) manipulator and the flexible XY-table. Some experimental results for the flexible R-manipulator (see also Kerssemakers, 1993) will be given in Subsection 5.3.1, and some experimental results for the flexible XY-table (see also Kaats, 1993) will be presented in Subsection 5.3.2. Computer programs in C++ have been used for the implementation of the control laws (Banens, 1992-1993), whereas the data of the experiments have been analyzed in MATLAB.

5.2 Simulations.

5.2.1 A flexible translation-rotation (TR) manipulator.

The model.

Here, the system used for the simulations with the (adaptive) CRCTC law is a flexible translation-rotation manipulator (see Fig. 5.1) with four degrees of freedom ($n = 4$) and two inputs ($m = 2$). The carriage (mass m_1) can move only in horizontal

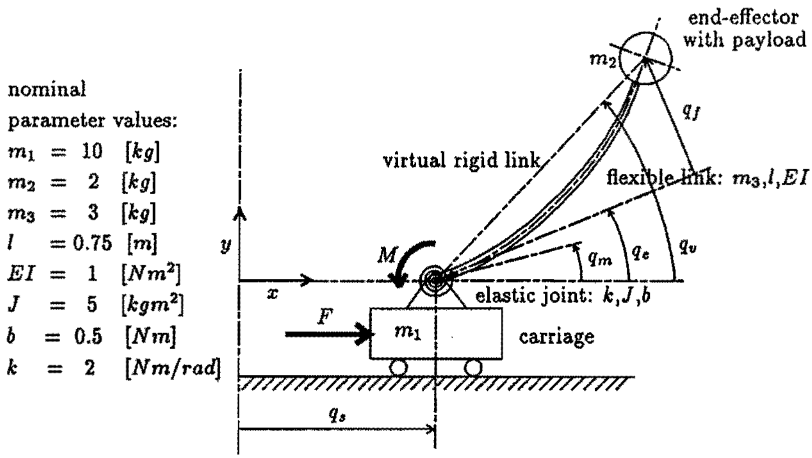


Figure 5.1: TR-manipulator with flexible joint and flexible link.

direction. It is driven by the actuator force F, and its position is characterized by the coordinate q_s. A flexible link (a cylindrical bar with mass m_3, length l and bending stiffness EI) with a concentrated payload (mass m_2) is attached at the revolute joint. The link is driven via a flexible transmission (ratio 1), which is modeled as a linear elastic, massless, torsional spring (stiffness k). The motor (relevant rotation inertia J) generates a torque M at the link. Dissipation in the drive is modeled by means of a linear, viscous, torsional damper (damping coefficient b) between the rotor and the carriage. The rotation of the rotor and of the link at the revolute joint are denoted by q_m and q_e, respectively. The deformation of the link is approximated with a displacement field ω with one extra degree of freedom q_f. The chosen field $\omega(\xi, t)$ is a third-order polynomial in the axial coordinate ξ, such that the boundary conditions $\omega(0,t) = 0$ and $\omega(l,t) = q_f$ are satisfied for all t:

$$\omega(\xi, t) = \frac{1}{2}(3 - \frac{\xi}{l})(\frac{\xi^2}{l^2})q_f(t) \ . \tag{5.1}$$

Axial deformation of the link is not taken into account and it is assumed that the deflection q_f is rather small in comparison with the length l. With regard to most of the analyses in this section, it is advantageous to use the virtual link rotation q_v (see Fig. 5.1) instead of the deflection q_f to describe the deformation of the link. Since $| q_f | \ll l$, it is seen that

$$q_f \approx l(q_v - q_e) \ . \tag{5.2}$$

For the sake of simplicity, it is assumed that the manipulator moves in a horizontal plane, i.e. that gravity can be ignored. Furthermore, no Coulomb friction or viscous damping other than the damping in the drive of the link is taken into account.

The displacement q_s and the rotations q_v, q_e and q_m are the elements of the column $\underline{q}$ of generalized coordinates. The force F and the torque M are the elements of the column $\underline{u}$ of inputs, i.e.

$$\underline{q}^T = [\ q_s \quad q_v \quad q_e \quad q_m\ ]\ ; \qquad \underline{u}^T = [\ F \quad M\ ]\ . \tag{5.3}$$

If the mathematical model of the manipulator is derived according to the procedure from Chapter 2, the standard form results:

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} + \underline{v}_r + \underline{f} = H\underline{u}\ , \tag{5.4}$$

where the symmetric, inertia matrix M is given by

$$M = \begin{bmatrix} A_{11} & -A_{12}\sin q_e & -A_{13}\sin q_e - A_{12}(q_v - q_e)\cos q_e & 0 \\ \star & A_{22} & A_{23} & 0 \\ \star & \star & A_{33} + A_{22}(q_v - q_e)^2 & 0 \\ \star & \star & \star & A_{44} \end{bmatrix} . \tag{5.5}$$

The constants $A_{11}, A_{12}, \ldots, A_{44}$ in this relation depend only on the inertia quantities m_1, m_2, m_3 and J and on the length of the link:

$$\begin{aligned} A_{11} &= m_1 + m_2 + m_3\ ; \quad A_{12} = \frac{1}{8}m_3 l\ ; \quad A_{13} = (m_2 + \frac{3}{8}m_3)l\ ; \\ A_{22} &= (m_2 + \frac{33}{140}m_3)l^2\ ; \quad A_{23} = \frac{11}{280}m_3 l^2\ ; \quad A_{33} = \frac{2}{105}m_3 l^2\ ; \quad A_{44} = J\ . \end{aligned} \tag{5.6}$$

The matrix C, such that $\dot{M} - 2C$ is skew-symmetric, is found to be

$$C = \begin{bmatrix} 0 & -A_{12}\cos q_e \dot{q}_e & C_{13} & 0 \\ 0 & 0 & -A_{22}(q_v - q_e)\dot{q}_e & 0 \\ 0 & A_{22}(q_v - q_e)\dot{q}_e & A_{22}(q_v - q_e)(\dot{q}_v - \dot{q}_e) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} , \tag{5.7}$$

where the element C_{13} is given by
$C_{13} = -A_{13}\dot{q}_e \cos q_e - A_{12}[(\dot{q}_v - \dot{q}_e)\cos q_e - (q_v - q_e)\dot{q}_e \sin q_e]$.
The matrices K and H and the columns $\underline{v}_n$ and $\underline{f}$ are given by

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_f & -k_f & 0 \\ 0 & -k_f & k + k_f & -k \\ 0 & 0 & -k & k \end{bmatrix} ; \ \underline{v}_n = \underline{0}\ ; \ \underline{f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b\dot{q}_m \end{bmatrix} ; \ H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} . \tag{5.8}$$

Here, k_f is the relevant stiffness of the link, associated with the assumed displacement field (5.1):

$$k_f = 3\frac{EI}{l}\ . \tag{5.9}$$

In the case of parametric uncertainty, suitable parameters for adaptation are the inertia-type parameters $A_{11}, A_{12}, \ldots, A_{44}$, the stiffnesses k and k_f and the damping coefficient b.

Actuator dynamics are not taken into account. However, it is assumed that the absolute value of the torque M, which can be generated by the actuator in the revolute joint, is limited to a maximum of 10 $[Nm]$.

For cases in which the link and/or the joint are rigid, the given four-degree-of-freedom model is used as a starting-point for the derivation of reduced models. If the link is rigid then $q_e = q_v$, whereas if the joint is rigid $q_e = q_m$. Let $\underline{q}'$ be a column with some of the independent elements of $\underline{q}$. Now, there is such a constant matrix Ω of maximum rank that

$$\underline{q} = \Omega \underline{q}' \ . \tag{5.10}$$

It is easily seen that the reduced model is given by

$$M'\ddot{\underline{q}}' + C'\dot{\underline{q}}' + K'\underline{q}' + \underline{v}_n' + \underline{f}' = H'\underline{u} \ , \tag{5.11}$$

where M', C', etc. are related to M, C, etc. by

$$\begin{aligned} M' &= \Omega^T M \Omega \ ; \quad C' = \Omega^T C \Omega \ ; \quad K' = \Omega^T K \Omega \ ; \\ \underline{v}_n' &= \Omega^T \underline{v}_n \ ; \quad \underline{f}' = \Omega^T \underline{f} \ ; \quad H' = \Omega^T H \ . \end{aligned} \tag{5.12}$$

For the rigid model, where $q_e = q_m$ and $q_e = q_v$, the column $\underline{q}'$ and the associated reduction matrix Ω are given by

$$\underline{q}' = \begin{bmatrix} q_s \\ q_v \end{bmatrix} \ ; \qquad \Omega^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \ . \tag{5.13}$$

With Eq. (5.12), this results in:

$$\begin{aligned} M' &= \begin{bmatrix} A_{11} & -A_{12}' \sin q_v \\ -A_{12}' \sin q_v & A_{22}' + A_{44} \end{bmatrix} \ ; \quad C' = \begin{bmatrix} 0 & -A_{12}' \cos q_v \dot{q}_v \\ 0 & 0 \end{bmatrix} \ ; \\ K' &= 0 \ ; \quad \underline{v}_n' = \underline{0} \ ; \quad \underline{f}' = \begin{bmatrix} 0 \\ b\dot{q}_v \end{bmatrix} \ ; \quad H' = I \ , \end{aligned} \tag{5.14}$$

where $A_{12}' = (m_2 + \frac{1}{2}m_3)l$ and $A_{22}' = (m_2 + \frac{1}{3}m_3)l^2$.

For the model with flexible joint but rigid link ($q_e = q_v$), it is seen that

$$\underline{q}' = \begin{bmatrix} q_s \\ q_v \\ q_m \end{bmatrix} \ ; \qquad \Omega^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ , \tag{5.15}$$

and that the matrices of the reduced model are

$$\begin{aligned} M' &= \begin{bmatrix} A_{11} & -A_{12}' \sin q_v & 0 \\ -A_{12}' \sin q_v & A_{22}' & 0 \\ 0 & 0 & A_{44} \end{bmatrix} \ ; \quad C' = \begin{bmatrix} 0 & -A_{12}' \cos q_v \dot{q}_v & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ ; \\ K' &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & k & -k \\ 0 & -k & k \end{bmatrix} \ ; \quad \underline{v}_n' = \underline{0} \ ; \quad \underline{f}' = \begin{bmatrix} 0 \\ 0 \\ b\dot{q}_m \end{bmatrix} \ ; \quad H' = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \ , \end{aligned} \tag{5.16}$$

with the same inertia parameters A_{11}, A'_{12}, A'_{22} and A_{44} as before.

Finally, for the model with flexible link but rigid joint ($q_e = q_m$) it follows that

$$\underline{q}' = \begin{bmatrix} q_s \\ q_v \\ q_e \end{bmatrix} \; ; \qquad \Omega^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} . \tag{5.17}$$

The inertia matrix M of the reduced model is given by

$$M' = \begin{bmatrix} A_{11} & -A_{12}\sin q_e & -A_{13}\sin q_e - A_{12}(q_v - q_e)\cos q_e \\ \star & A_{22} & A_{23} \\ \star & \star & A_{33} + A_{22}(q_v - q_e)^2 + A_{44} \end{bmatrix} , \tag{5.18}$$

and the reduced C' is given by Eq. (5.7) if row and column four are removed. The other matrices of the reduced model are

$$K' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_f & -k_f \\ 0 & -k_f & k_f \end{bmatrix} \; ; \; \underline{v}'_n = \underline{0} \; ; \; \underline{f}' = \begin{bmatrix} 0 \\ 0 \\ b\dot{q}_e \end{bmatrix} \; ; \; H' = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} , \tag{5.19}$$

Because of the assumption that $| q_f | \ll l$, the quadratic term $(q_v - q_e)^2$ in M can be neglected, as well as the corresponding terms in C.

The control objective.

The control objective is to let the end-effector (with payload) track a periodical, smooth trajectory in the (x, y)-plane (see Fig. 5.1). The x- and y-coordinate of the end-effector are the elements of the output $\underline{y}$ and are related to the generalized coordinates by

$$\underline{y} = \begin{bmatrix} q_s + l\cos q_v \\ l\sin q_v \end{bmatrix} . \tag{5.20}$$

In accordance with Chapters 2 and 4, this means that q_s and q_v are the elements of the column $\underline{\theta}$ of rigid-body coordinates. The remaining generalized coordinates are the elements of the column $\underline{\varrho}$ of flexible coordinates. The number of flexible coordinates is equal to 0 if both the link and the joint are rigid, it is equal to 1 if either the link or the joint are flexible, and it is equal to 2 if both the link and the joint are flexible.

The desired trajectory of the end-effector is a circle with radius $r = \frac{7}{15}l$ and center O at position $(x_O, y_O) = (0, \frac{1}{2}l)$. The circle has to be followed with a constant angular velocity $\omega = 1$ $[rad/sec]$. So, from the relation

$$\underline{y}_d(t) = \begin{bmatrix} q_{sd}(t) + l\cos q_{vd}(t) \\ l\sin q_{vd}(t) \end{bmatrix} , \tag{5.21}$$

the desired trajectories $q_{sd} = q_{sd}(t)$ and $q_{vd} = q_{vd}(t)$ for the rigid-body coordinates q_s and q_v can be determined (see Fig. 5.2).

All simulation results in this section are based on the CRCTC approach, i.e. the control law is as follows (see Subsection 4.4.4):

$$H\underline{u} = M\underline{\ddot{q}}_r + C\underline{\dot{q}}_r + K\underline{q}_r + \underline{v}_r + \underline{f} + K_r\underline{\dot{e}}_r , \tag{5.22}$$

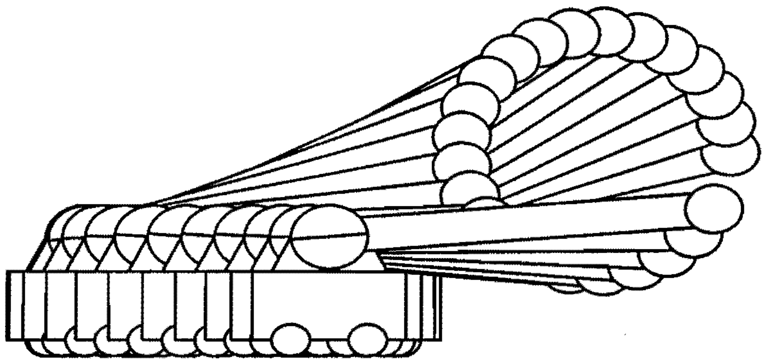


Figure 5.2: Desired end-effector trajectory.

where K_r is a diagonal matrix with equal diagonal elements. Unless stated otherwise, these elements are equal to 10:

$$K_r = k_r I \; ; \qquad k_r = 10 \; . \tag{5.23}$$

Furthermore, the reference trajectory $\underline{\theta}_r = \underline{\theta}_r(t)$ of the rigid-body coordinates is given by

$$\dot{\underline{\theta}}_r = \dot{\underline{\theta}}_d + \Lambda_\theta \underline{e}_\theta \; ; \qquad \underline{e}_\theta = \underline{\theta}_d - \underline{\theta} \; , \tag{5.24}$$

where Λ_θ is a diagonal matrix with identical diagonal elements λ_θ. Unless stated otherwise, λ_θ is equal to 10:

$$\Lambda_\theta = \lambda_\theta I \; ; \qquad \lambda_\theta = 10 \; . \tag{5.25}$$

Simulation results with other control laws are not reported here, but can be found in, for instance, Lammerts (1990), Lammerts et al. (1991) and Kuijpers (1993). The control laws with a desired trajectory instead of the reference trajectory (CDCTC as discussed in Subsections 4.4.2 and 4.4.3) result in a similar tracking performance as CRCTC.

In the following subsections, the CRCTC law (5.22) will be used in combination with four different models of the TR-manipulator. In the first model, the joint and the link are rigid (see also Vijverstra, 1992), in the second model the joint is flexible but the link is still rigid (see also Doelman, 1992), in the third model the joint is rigid but the link is flexible. Finally, in the fourth model both the joint and the link are flexible.

5.2.2 CRCTC of the rigid TR-manipulator.

The number of flexible coordinates is equal to zero if both the joint and the link are rigid. Now, $\underline{q} \equiv \underline{\theta}$ and $\underline{q}_r \equiv \underline{\theta}_r$, and the input $\underline{u}$ follows directly from Eq. (5.22).

With the results of the preceding subsection it is easily seen that

$$F = A_{11}\ddot{q}_{sr} - A'_{12}\sin q_v \ddot{q}_{vr} - A'_{12}\cos q_v \dot{q}_v \dot{q}_{vr} + k_r \dot{e}_{sr} \; ; \tag{5.26}$$
$$M = -A'_{12}\sin q_v \ddot{q}_{sr} + (A'_{22} + A_{44})\ddot{q}_{vr} + b\dot{q}_v + k_r \dot{e}_{vr} \; . \tag{5.27}$$

The left graph of Fig. 5.3 shows the actual and the desired trajectory for the displacement of the carriage (q_s and q_{sd}) and for the rotation of the link (q_v and q_{vd}), starting from zero initial conditions for q_s, $\dot{q}_s$, q_v and $\dot{q}_v$. The right graph of Fig. 5.3 shows the applied force F and torque M as a function of time t. For small values

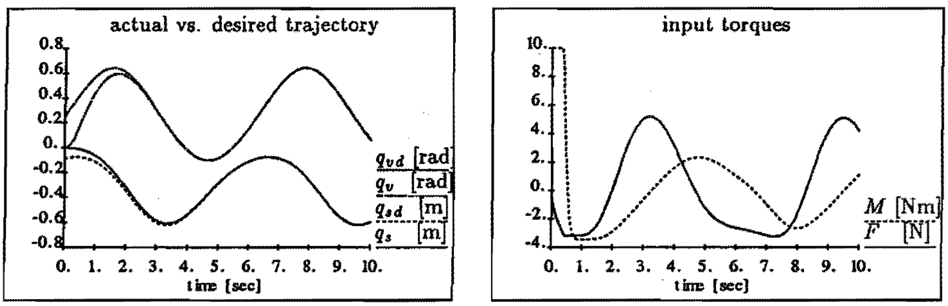


Figure 5.3: CRCTC of the rigid TR-manipulator.

of t, the value of M as determined from Eq. (5.26) exceeds the maximum torque of 10 $[Nm]$ that can be generated by the actuator. Therefore, $M = 10$ $[Nm]$ is used in the simulations for this range of t.

5.2.3 CRCTC of the TR-manipulator with a flexible joint.

For the TR-manipulator with a rigid link and a flexible joint, there is one flexible coordinate being the rotation q_m of the actuator rotor. Hence, $\underline{\varrho} = [\; q_m \;]$ and $\underline{\theta} = [\; q_s \;\; q_v \;]$, where $q_v = q_e$. Following the procedure of Section 4.3 and Subsection 4.4.4, Eq. (5.22) can be written as

$$0 = -A'_{12}\sin q_v \ddot{q}_{sr} + A'_{22}\ddot{q}_{vr} + kq_{vr} - kq_{mr} + k_r \dot{e}_{vr} \; ; \tag{5.28}$$
$$F = A_{11}\ddot{q}_{sr} - A'_{12}\sin q_v \ddot{q}_{vr} - A'_{12}\cos q_v \dot{q}_v \dot{q}_{vr} + k_r \dot{e}_{sr} \; ; \tag{5.29}$$
$$M = A_{44}\ddot{q}_{mr} - kq_{vr} + kq_{mr} + b\dot{q}_m + k_r \dot{e}_{mr} \; . \tag{5.30}$$

The required reference value $q_{mr}(t)$ for the flexible coordinate is solved on-line from Eq. (5.28). The computation of the input F from Eq. (5.29) is straightforward, because the first and second time-derivatives of the desired trajectory for the rigid-body coordinates are known beforehand. However, the computation of the input

M from (5.30) gives rise to difficulties, because it requires that the first and second time-derivatives of the reference trajectory q_{mr} are available. A possible solution is to differentiate the Eq. (5.28) twice and to eliminate the required accelerations $\ddot{\underline{\theta}}$ and jerks $\underline{\theta}^{(3)}$ of the rigid-body coordinates on the basis of the known closed-loop dynamics, as discussed in Subsection 4.4.5. However, this results in rather complex relations for $\dot{q}_{mr}$ and $\ddot{q}_{mr}$, and it is usually impractical or impossible to reconstruct or measure the quantities $\ddot{\underline{\theta}}$ and jerks $\underline{\theta}^{(3)}$. A more practical solution has appeared to be the use of weak differentiators in order to determine $\dot{q}_{mr}$ and $\ddot{q}_{mr}$. The main drawback of this alternative is that the initial values of $\dot{q}_{mr}$ and especially of $\ddot{q}_{mr}$ can be rather unrealistic. As a consequence, the computed torque M can be very large for small values of t. A third alternative, which is closely related to weak differentiation, is to replace the algebraic equation for q_{mr} by a second-order differential equation:

$$\Delta m \ddot{q}_{mr} + \Delta b \dot{q}_{mr} + k q_{mr} = -A'_{12} \sin q_v \ddot{q}_{sr} + A'_{22} \ddot{q}_{vr} + k q_{vr} + k_r \dot{e}_{vr} \ , \tag{5.31}$$

where Δm and Δb must be chosen very small. The best results with this approach were obtained for critical damping, i.e. for $\Delta b = 2\sqrt{k \Delta m}$. Then, after a short initial transient the computed values for q_{mr}, $\dot{q}_{mr}$ and $\ddot{q}_{mr}$ are quite acceptable. For most of the simulations, the approach of weak differentiation has been used. Because we are interested especially in the tracking of the periodical trajectory, for the moment the initial transient is disregarded here and results are reported only for larger values of t.

The left graph of Fig. 5.4 shows the inputs F and M for the time-interval from $t = 0.5$ [sec] to $t = 20$ [sec]. The right graph shows the actual rotation q_m of the

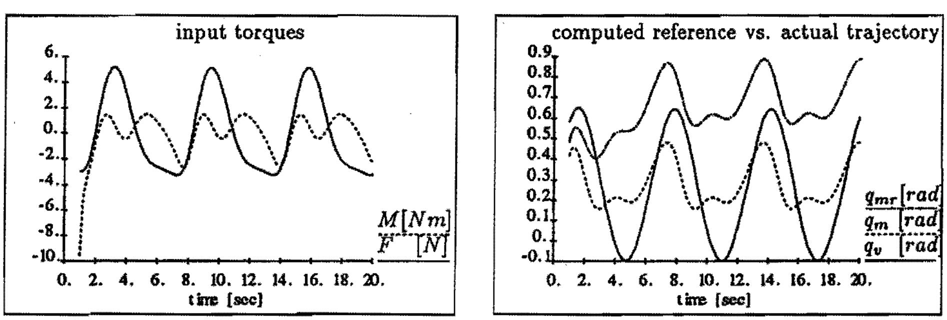


Figure 5.4: CRCTC of the TR-manipulator with flexible joint: $k = 2$ [Nm/rad].

actuator rotor, the computed reference q_{mr} of this rotation and the actual rotation q_v of the rigid link. The difference $e_{mr}(t) = q_{mr}(t) - q_m(t)$ does not converge to zero but to a constant value. This is in accordance with the control objective, because it is required that $\dot{e}_{mr}$ converges to zero and not that $e_{mr}(t) \to 0$ for $t \to \infty$. The deformation $q_m - q_v$ fluctuates with a bounded amplitude and with a frequency that strongly depend on the stiffness k.

The torque M fluctuates with larger magnitudes for a more flexible joint. To illustrate this, we refer to Fig. 5.5, where the same quantities are plotted as in Fig. 5.4, but now for $k = 0.2$ $[Nm/rad]$ instead of $k = 2$ $[Nm/rad]$. The results in Fig. 5.6 for

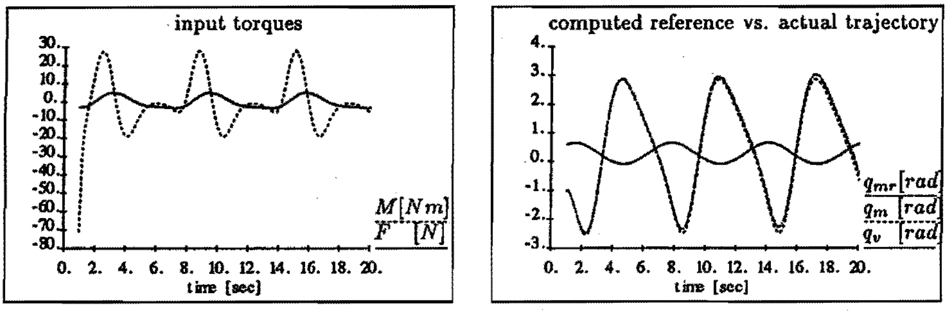


Figure 5.5: CRCTC of the TR-manipulator with flexible joint: $k = 0.2$ $[Nm/rad]$.

$k = 20$ $[Nm/rad]$ show that the joint now is practically rigid. As expected, CRCTC

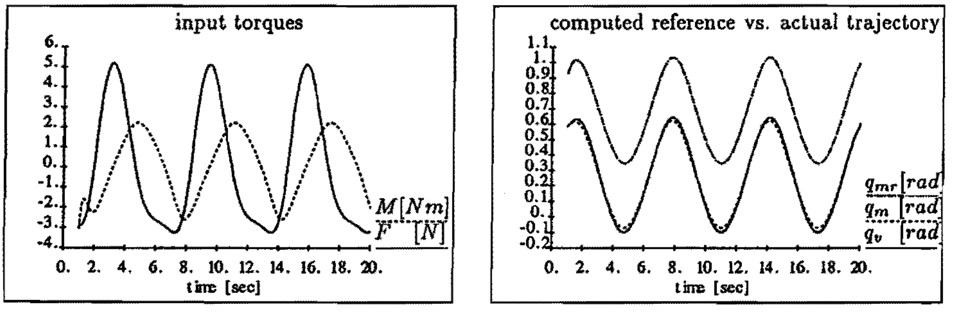


Figure 5.6: CRCTC of the TR-manipulator with flexible joint: $k = 20$ $[Nm/rad]$.

for a rather stiff joint hardly differs from CRCTC for the rigid TR-manipulator (see, for instance, the input torques F and M in the preceding subsection).

5.2.4 CRCTC of the TR-manipulator with a flexible link.

The mathematical model for the TR-manipulator with a rigid joint and a flexible link was given in Subsection 5.2.1. It contains one flexible coordinate, being the rotation q_e of the link at the revolute joint. With $N^T = [\ 0\ \ 1\ \ 0\ ]$, the procedure of Subsection 4.4.4 yields two relations for the input quantities F and M and one relation for the reference trajectory of the flexible coordinate. The relations for F and M are not given here, as they are not relevant to the discussion. The relation for the flexible coordinate is given by

$$-A_{12}\sin q_e \ddot{q}_{sr} + A_{22}\ddot{q}_{vr} + A_{23}\ddot{q}_{er} - A_{22}(q_v - q_e)\dot{q}_e\dot{q}_{er} + k_f q_{vr} - k_f q_{er} + k_r \dot{e}_{vr} = 0\ . \quad (5.32)$$

For realistic manipulators the deformation $q_v - q_e$ is very small, which means that the term $A_{22}(q_v - q_e)\dot{q}_e\dot{q}_{er}$ in Eq. (5.32) can be neglected. Now, this equation becomes

$$A_{23}\ddot{q}_{er} - k_f q_{er} = A_{12}\sin q_e \ddot{q}_{sr} - A_{22}\ddot{q}_{vr} - k_f q_{vr} - k_r \dot{e}_{vr}\ . \quad (5.33)$$

Numerical determination of a bounded solution for q_{er} by forward integration is impossible, because the above second-order differential equation in q_{er} is unstable. This will be discussed in more detail in Subsection 5.2.7. To avoid this problem, the roles of q_e and q_v are exchanged here: it is assumed that the output equation (5.20) can be used to determine the desired trajectory $q_{ed} = q_{ed}(t)$, simply by replacing q_{vd} in Eq. (5.21) q_{ed}. This makes sense only if the deformation of the link is small enough, i.e. if the link is sufficiently stiff. Now, a reference trajectory q_{vr} for q_v can be solved from Eq. (5.33). For this purpose, this equation is written as

$$A_{22}\ddot{q}_{vr} + k_r \dot{q}_{vr} + k_f q_{vr} = A_{12}\sin q_e \ddot{q}_{sr} - A_{23}\ddot{q}_{er} + k_r \dot{q}_v + k_f q_{er}\ , \quad (5.34)$$

and no numerical problems are to be expected in the computation of q_{vr}. It is noted that, unlike the situation in the previous subsection, numerical (weak) differentiation of the computed reference trajectory q_{vr} is not necessary.

Some simulation results for different values of the stiffness $k_f = 3EI/l$ are presented in Fig. 5.7 for $EI = 1\ [Nm^2]$, in Fig. 5.8 for $EI = 10\ [Nm^2]$, and in Fig. 5.9 for $EI = 100\ [Nm^2]$. The left graphs show the reference velocity vectors $\dot{e}_{sr}$, $\dot{e}_{er}$ and $\dot{e}_{fr}$ as a function of time. Because these errors converge to zero, the actual trajectory of the coordinates will converge to their desired trajectory, irrespective of the value of the stiffness of the link. However, this does not mean that the actual trajectory of the end-effector converges to the desired trajectory. The tracking error of the end-effector is determined by $l(q_{ed} - q_v)$. The graphs at the right of the Figs. 5.7, 5.8 and 5.9 represent the deformation $q_f = l(q_v - q_e)$ and the corresponding reference $q_{fr} = l(q_{vr} - q_{er})$ for this deformation as functions of time. It is seen that both the amplitude and the frequency of the deformation strongly depends on the stiffness k_f. Higher stiffness values of the link do not lead to higher input torques but do lead to higher-frequency input torques (see Fig. 5.10).

5.2.5 CRCTC of the TR-manipulator with flexible joint and flexible link.

If both the joint and the link are flexible, there are two flexible coordinates, namely the rotation q_m of the rotor and the rotation q_e of the link at the revolute joint.

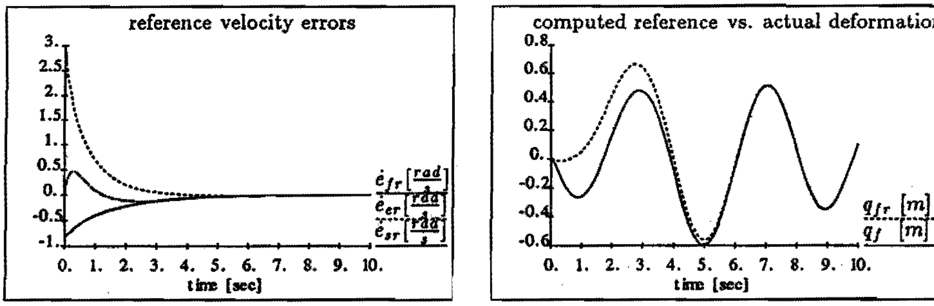


Figure 5.7: CRCTC of the TR-manipulator with flexible link: $EI = 1$ $[Nm^2]$.

Hence, $\underline{\varrho} = [\ q_e \quad q_m\ ]$ and $\underline{\theta} = [\ q_s \quad q_v\ ]$. With the matrix N, given by

$$N^T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{5.35}$$

and with the mathematical model of Subsection 5.2.1, two relations for the input quantities F and M plus two relations for the reference trajectory of the flexible coordinates can be derived. For the discussion, again, the relations for F and M are omitted here. The other two relations are given by:

$$-A_{12}\sin q_e\ddot{q}_{sr} + A_{22}\ddot{q}_{vr} + A_{23}\ddot{q}_{er} - A_{22}(q_v - q_e)\dot{q}_e\dot{q}_{er} + k_f q_{vr} - k_f q_{er} + k_r\dot{e}_{vr} = 0 \tag{5.36}$$

$$\begin{aligned} &-[A_{13}\sin q_e + A_{12}(q_v - q_e)\cos q_e]\ddot{q}_{sr} + A_{23}\ddot{q}_{vr} + [A_{33} + A_{22}(q_v - q_e)^2]\ddot{q}_{er} + \\ &+A_{22}(q_v - q_e)[\dot{q}_e\dot{q}_{vr} + (\dot{q}_v - \dot{q}_e)\dot{q}_{er}] - k_f q_{vr} + (k + k_f)q_{er} - kq_{mr} + k_r\dot{e}_{er} = 0 \end{aligned} \tag{5.37}$$

As in the previous subsection, it is assumed that the deformation $q_v - q_e$ of the link is small enough to neglect all terms with $q_v - q_e$ in these equations. This results in

$$-A_{12}\sin q_e\ddot{q}_{sr} + A_{22}\ddot{q}_{vr} + A_{23}\ddot{q}_{er} + k_f q_{vr} - k_f q_{er} + k_r\dot{e}_{vr} = 0\ ; \tag{5.38}$$

$$-A_{13}\sin q_e\ddot{q}_{sr} + A_{23}\ddot{q}_{vr} + A_{33}\ddot{q}_{er} - k_f q_{vr} + (k + k_f)q_{er} + k_r\dot{e}_{er} = kq_{mr}\ . \tag{5.39}$$

The last relation is an algebraic equation in q_{mr}. To obtain the time-derivatives $\dot{q}_{mr}$ and $\ddot{q}_{mr}$, which have to be available for the computation of the input quantities F and M, the same procedure as in Subsection 5.2.3 can be followed. However, computation of q_{er} by forward integration of Eq. (5.38) is impossible for the same reason as mentioned in the previous subsection. To overcome this difficulty, again the roles of q_e and q_v are exchanged by replacing q_{ed} and q_{er} by q_{vd} and q_{vr}, respectively. Hence, q_{vr} is regarded as the reference trajectory that has to be determined by Eq. (5.38).

Some simulation results for different values of the stiffnesses k and $k_f = 3EI/l$ are shown in Fig. 5.11, where $k = 2$ $[Nm/rad]$ and $EI = 1$ $[Nm^2]$, and in Fig. 5.12,

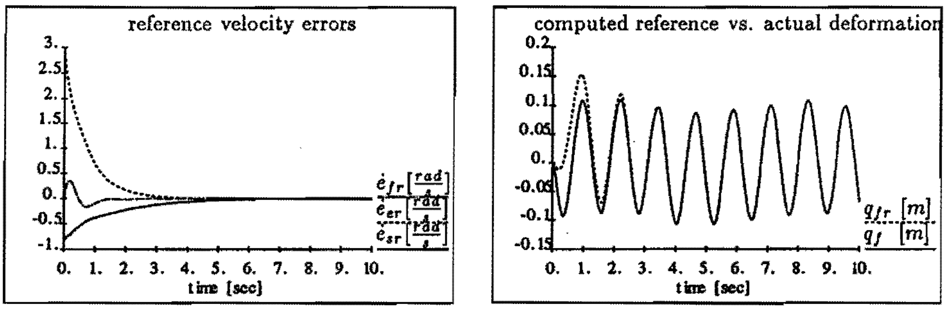


Figure 5.8: CRCTC of the TR-manipulator with flexible link: $EI = 10$ $[Nm^2]$.

where $k = 20$ $[Nm/rad]$ and $EI = 10$ $[Nm^2]$. Although both joint and link flexibility are considered now, the results are quite similar to the results in the last two subsections of either joint and link flexibility. Therefore, they need no further discussion here.

5.2.6 Adaptive CRCTC of the flexible TR-manipulator.

In case of parametric uncertainty, adaptive CRCTC may be required to obtain trajectory tracking. Here, the adaptive CRCTC law as proposed in Section 4.5 is applied to the flexible TR-manipulator of the previous subsection. As an example, uncertainties in the masses m_1, m_2 and m_3 are considered. The three model parameters A_{11}, A'_{12} and A'_{22} are selected to be adjusted on-line on the basis of the adaptation algorithm (see Section 4.5). From the adjusted parameters $\hat{A}_{11} = \hat{m}_1 + \hat{m}_2 + \hat{m}_3$, $\hat{A}'_{12} = (\hat{m}_2 + \frac{1}{2}\hat{m}_3)l$ and $\hat{A}'_{22} = (\hat{m}_2 + \frac{1}{3}\hat{m}_3)l^2$, the mass estimates $\hat{m}_1$, $\hat{m}_2$ and $\hat{m}_3$ can be determined, so that estimates for the parameters $\hat{A}_{11}, \hat{A}_{12}, \ldots, \hat{A}_{33}$ can also be computed on-line. In these simulations, it is assumed that the state of the system at time $t = 0$ corresponds to the desired state.

Fig. 5.13 shows some results of the simulations in which the initial estimates $\hat{A}_{11}, \hat{A}'_{12}, \hat{A}'_{22}$ differ approximately 25% from the real values of A_{11}, A'_{12}, A'_{22}. The adaptation matrix Γ (see Section 4.5) is a diagonal matrix with $\Gamma_{11} = 1000$, $\Gamma_{22} = 100$ and $\Gamma_{33} = 100$ as diagonal elements. The stiffnesses are $k = 2$ $[Nm/rad]$ and $k_f = 3EI/l$ with $EI = 10$ $[Nm^2]$. The right graph shows the three parameters that are adjusted by the adaptation algorithm as a function of time. They converge to constant values, which happen to be the real values because the specified desired trajectory of the end-effector is sufficiently rich for this adaptation. The other parameters are determined on-line on the basis of these adjusted parameters. With respect to the trajectory tracking, the left graph of Fig. 5.13 shows that, again, the reference velocity errors $\dot{\underline{e}}_{sr}$ and $\dot{\underline{e}}_{er}$ (with initial value zero) become zero after some time,

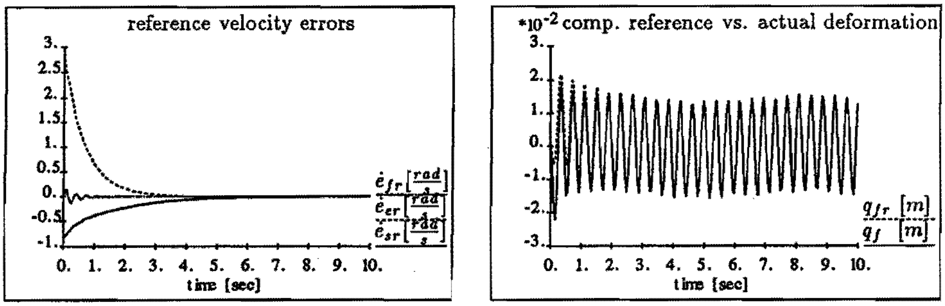


Figure 5.9: CRCTC of the TR-manipulator with flexible link: $EI = 100$ $[Nm^2]$.

which is needed for the parameter adaptation. Varying the values of the diagonal elements of the adaptation matrix Γ shows that lower values result in a more smooth behavior of the parameter adjustments and in a slower tracking error convergence, whereas higher values do result in a less smooth behavior of the estimates but do not always yield a faster tracking error convergence. Furthermore, if the stiffnesses k and k_f are varied, similar tracking results are obtained and the actual trajectories q_s, q_v, q_e and q_m and the input torques F and M resemble the corresponding quantities in the non-adaptive case (after the initial adaptation process).

5.2.7 CRCTC of the end-effector of the flexible TR-manipulator.

Control of the end-effector trajectory for the TR-manipulator with a flexible link boils down to control of the rigid coordinates q_s and q_v. This means that the rotation q_e of the link at the revolute joint is a flexible coordinate and that the reference trajectory q_{er} for this coordinate has to be determined on-line by solving Eq. (5.33) in case of a rigid joint or Eq. (5.38) in case of a flexible joint. This, however, introduces an essential problem as these second-order differential equations behave *unstable* in forward numerical-time integration. In Subsections 5.2.4 and 5.2.5, this problem is circumvented by exchanging the roles of q_v and q_e. From a theoretical point of view — and also from a practical point of view in case of small or moderate link stiffnesses — this is unsatisfactory.

To elucidate the problem, first, the situation is considered in which the reference acceleration $\ddot{q}_{sr}$ of the carriage is identically zero. Then, Eq. (5.38) becomes a linear, constant, second-order differential equation:

$$A_{23}\ddot{q}_{er} - k_f q_{er} = -[A_{22}\ddot{q}_{vr} + k_f q_{vr} + k_r \dot{e}_{vr}] \;, \tag{5.40}$$

which is clearly seen to be unstable in forward numerical-time integration. One

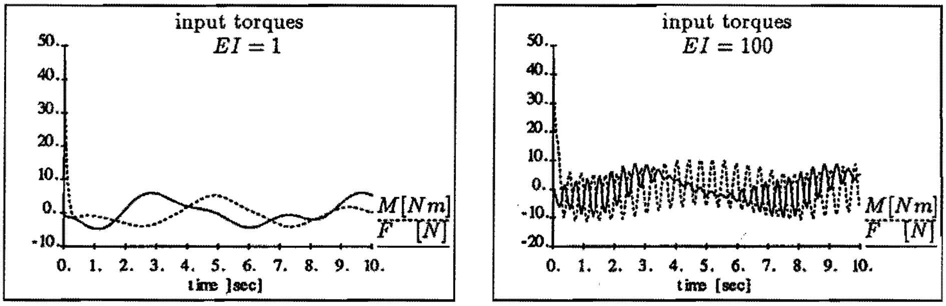


Figure 5.10: CRCTC of the TR-manipulator with flexible link: $EI = 1$ $[Nm^2]$ and $EI = 100$ $[Nm^2]$.

solution may be to analytically derive a particular solution for q_{er} to this second-order differential equation (such as, for instance, $q_{er} = B\cos\omega t$, Veldpaus, 1993b; see also Heijman, 1993, for a flexible link that is modeled with three extra degrees of freedom, instead of one). At least, this shows that bounded particular solutions exist to this 'unstable' differential equation. However, this is a rather laborious way to solve the problem, and nonlinear and/or time-varying second-order differential equations can hardly or not be solved in this way. Therefore, the above second-order differential equation is rewritten as a set of two linear, first-order differential equations:

$$\dot{\eta} + \omega_o \eta = -A_{23}^{-1}[A_{22}\ddot{q}_{vr} + k_f q_{vr} + k_r \dot{e}_{vr}] \; ; \qquad (5.41)$$

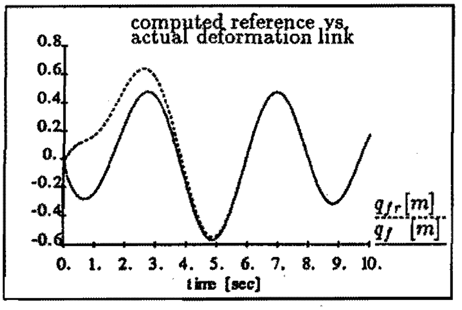
$$\dot{q}_{er} - \omega_o q_{er} = \eta \; , \qquad (5.42)$$

where $\omega_o = \sqrt{k_f/A_{23}}$. With a given initial condition for $\eta(t)$, the first equation can be solved by *forward time integration* over the time-interval $[t, t_e]$. After that, the second equation can be solved by *backward time integration* from t_e to t, at least if a value for $q_{er}(t_e)$ is specified.

There are two essential drawbacks in this approach. Firstly, the right-hand side of Eq. (5.41) has to be known for all t in the interval $[t, t_e]$. However, this side contains terms like $\dot{e}_{vr} = \dot{q}_{vr} - \dot{q}_v$, and, so, the real, future values of q_v in the interval $[t, t_e]$ have to be known in this approach. This is certainly not the case in practice. A possible solution to this problem is to replace the reference quantities q_{vr} and $\ddot{q}_{vr}$ by the desired quantities q_{vd} and $\ddot{q}_{vd}$, respectively, and to neglect the influence of the error term in Eq. (5.41). This means that Eq. (5.41) is replaced by

$$\dot{\eta} + \omega_o \eta = -A_{23}^{-1}[A_{22}\ddot{q}_{vd} + k_f q_{vd}] \; ; \qquad (5.43)$$

Secondly, the given approach requires a specification of the value of q_{er} at time t_e (see Eq. (5.42)). For the CRCTC law, only q_{er} and the first and second time-derivative at



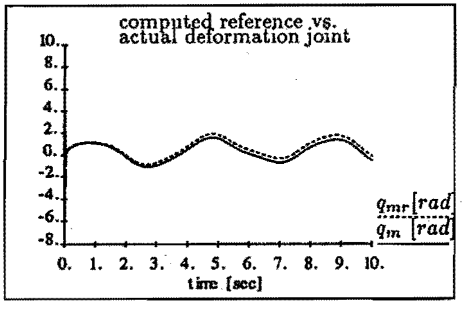


Figure 5.11: CRCTC of the TR-manipulator with flexible joint and flexible link: $k = 2$ $[Nm/rad]$ and $EI = 1$ $[Nm^2]$.

time t are relevant. Hence, a possibly wrong condition for $q_{er}(t_e)$ is of no importance if $t_e - t$ is large enough, i.e. if $\omega_o(t_e - t) \gg 1$, because in that case the influence of $q_{er}(t_e)$ on the current $q_{er}(t)$, $\dot{q}_{er}(t)$ and $\ddot{q}_{er}(t)$ for $t \ll t_e$ is negligible. Furthermore, if the desired end-effector trajectory is periodical with period time T, it is obvious to choose $t_e = t + zT$ (with z being a positive integer) and $q_{er}(t_e) = q_{er}(t)$. In general, the specification of the end condition for q_{er} is not expected to cause insuperable problems, which relaxes the second drawback in this approach.

Recently, the approach outlined above has also been applied by Heijman (1993) to a flexible link that is modeled with three extra degrees of freedom. However, it is not usable for the original problem (with a translating carriage) as given by the non-linear differential equation (5.38), because it is not possible to decompose this second-order equation in one first-order equation that is stable in forward time integration plus one first-order equation that is stable in backward time integration. Fortunately, various numerical algorithms are available to compute a bounded solution to Eq. (5.38) if it is formulated as a two point boundary value problem, i.e. if an appropriate initial condition for time t and an appropriate end condition for time $t_e > t$ are given. Algorithms of this kind are implemented in, for instance, MUSN (Mattheij and Staarink, 1987; Ascher et al., 1988) where a multiple shooting technique is used, in COLSYS (Ascher et al., 1979) where a collocation technique is used, and in the NAG library (1987) where a finite difference technique is used.

If one of these algorithms is used to solve q_{er} from Eq. (5.38), all other quantities in that equation have to be known for each time in the interval $[t, t_e]$. As mentioned before, this is certainly not the case in practice. Therefore, the reference quantities are replaced by the desired quantities and the error terms are neglected, resulting in

$$A_{23}\ddot{q}_{ed} - k_f q_{ed} - A_{12} \sin q_{ed} \ddot{q}_{sd} = -[A_{22}\ddot{q}_{vd} + k_f q_{vd}] \tag{5.44}$$

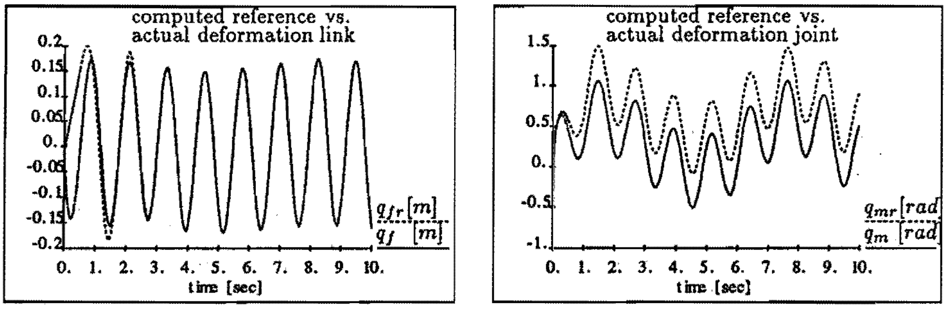


Figure 5.12: CRCTC of the TR-manipulator with flexible joint and flexible link: $k = 20$ $[Nm/rad]$ and $EI = 10$ $[Nm^2]$.

As for the linear case discussed earlier, it turns out that the end condition at time t_e is irrelevant to the solution in the neighborhood of time t if t_e is much larger than t.

In the simulations shown here only the COLSYS algorithm is used. Because the desired trajectory of the end-effector is periodical with period time $T = 2\pi/\omega$ (see Subsection 5.2.1), we want to compute a periodic solution to Eq. (5.44). This can be achieved with COLSYS, by taking the length $t_e - t$ of the integration interval equal to a multiple of the period time and requiring that the end conditions at time t_e are equal to the initial conditions at time t. The periodic solution for q_{ed}, obtained in this way, is used in exactly the same way as q_{sd} and q_{vd} in the on-line computation of the input quantities.

It is noted that in Eq. (5.44) neither the actual generalized coordinates nor the actual generalized velocities occur. Hence, the computation of the (periodic) solution of Eq. (5.44) can be done off-line. However, this solution is only an approximate solution to the original differential equation.

Fig. 5.14 shows some of the simulation results obtained with the CRCTC law, using the off-line computed periodic desired trajectory for the flexible coordinate (Van de Molengraft, 1993). For the sake of simplicity, this CRCTC law will be called the *modified CRCTC* law. Trajectory tracking of the end-effector (see the first two graphs in Fig. 5.14) is not realized completely and is slower than the trajectory tracking of q_s and q_e in Fig. 5.8 for the same link stiffness ($EI = 10$ $[Nm^2]$). However, the actual end-effector coordinates x and y track their desired trajectory quite well for sufficiently large values of t. Hence, it can be concluded that the modified CRCTC law performs well.

The CRCTC law as discussed in Section 5.2.4 and the modified CRCTC law result in a different deformation of the link, especially for small values of t. This can be seen by comparison of the right graph in Fig. 5.8 with the lower-left graph in Fig. 5.14. The reason is that the desired trajectory q_{ed}, which is off-line computed on the basis

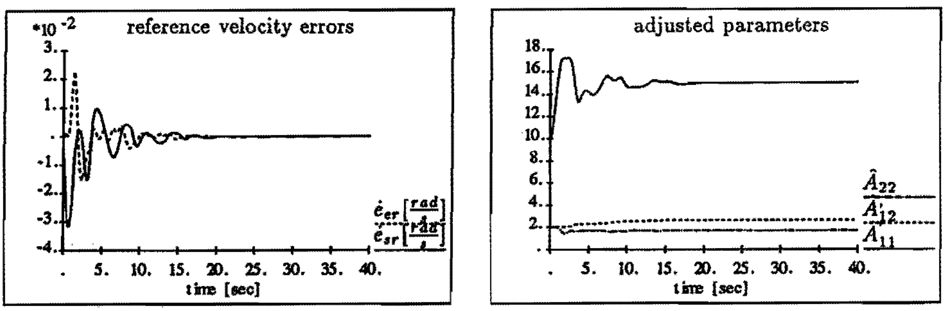


Figure 5.13: Adaptive CRCTC of the TR-manipulator with flexible joint and flexible link: $k = 2$ $[Nm/rad]$ and $EI = 10$ $[Nm^2]$.

of the modified law, is just a periodic solution with the same period time $T = 2\pi/\omega$ as the desired trajectory of the end-effector. Each mismatch between the actual and the desired initial state will result in an excitation of the link and will introduce natural vibrations in the link. As can be seen from Fig. 5.14, these vibrations are damped out by the modified CRCTC law such that, after an initial transient, the actual deformation of the link tracks the desired flexible trajectory with the same period time as the desired trajectory of the end-effector. In the lower-right graph of the same figure, the computed torques F and M are shown.

5.2.8 Conclusions.

The CRCTC law has been tested in a number of simulations for a rather large range from moderate to large flexibilities of the joint and/or the link. In each of these simulations, this law performs well and is able to yield trajectory tracking of the end-effector while the elastic deformations that occur are kept bounded. However, in case of a flexible link the CRCTC law is hampered by the necessity to compute a bounded solution of an unstable differential equation. Although a possible solution to this problem was suggested and applied in the form of the modified CRCTC law, it has not been investigated in detail and seems to be an important topic for future research. The main drawback of the modified CRCTC scheme is that, in the off-line computation of the unknown reference trajectory, all reference quantities have to be replaced by the corresponding desired quantities and that all errors have to be neglected. Hence, the stability proof given in Chapter 4 for the CRCTC approach does not apply to the modified CRCTC law. In the simulations no stability problems occur, but it is nevertheless worthwhile to investigate the stability of the closed-loop system if a modified CRCTC law is applied.

A topic in the research on control of nonlinear systems is feedback linearization

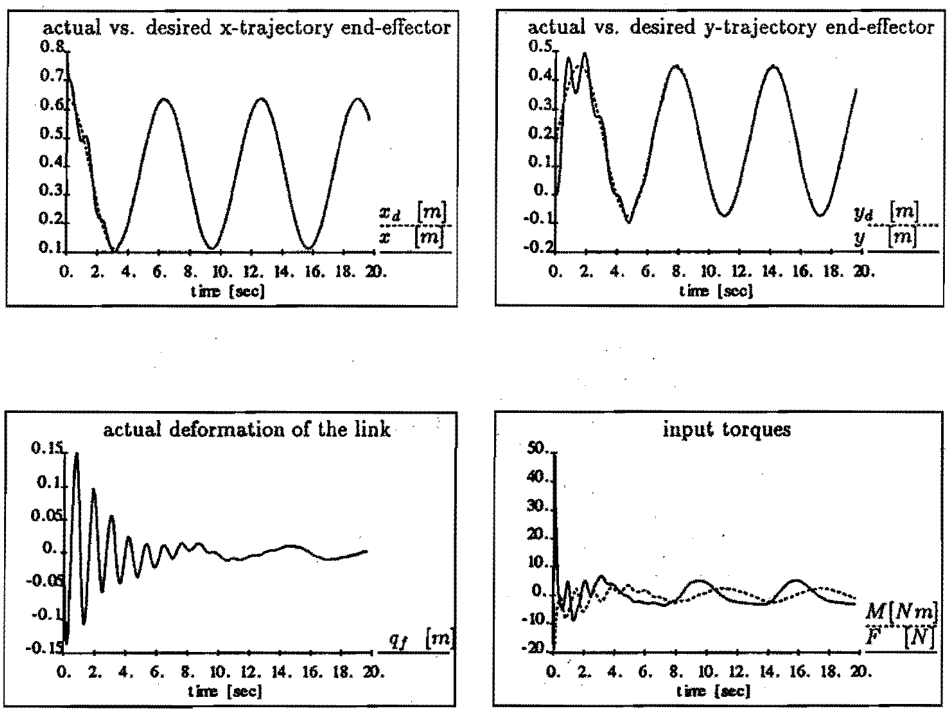


Figure 5.14: Modified CRCTC of the TR-manipulator with flexible link: $EI = 10$ $[Nm^2]$.

(Slotine and Li, 1992; Nijmeijer and Van der Schaft, 1990). One version of feedback linearization, the so-called input-output linearization, decomposes the mathematical model of the system in a set of equations that relate (derivatives of) the output to the input, and a set of equations that describe the so-called internal dynamics of the system. If the output is kept zero for all t, the internal dynamics reduce to the zero dynamics of the system. The unstable differential equation for the flexible coordinate q_e probably has a lot to do with unstable zero dynamics. It is worthwhile to investigate this in more detail.

5.3 Experiments.

5.3.1 Adaptive CRCTC of a flexible rotation manipulator.

The experimental set-up and the control model.

Section 5.2 has shown, by means of simulations, that CRCTC is a suitable tool to achieve trajectory tracking of the end-effector. The practical use of CRCTC has been verified by means of experiments on, for instance, the flexible R-manipulator of Fig. 5.15. A block scheme representation of this manipulator is given in Fig. (5.16).

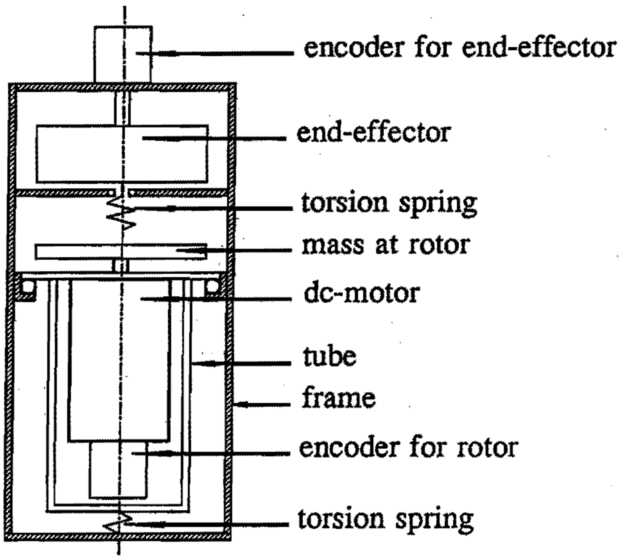


Figure 5.15: The experimental set-up of the flexible R-manipulator.

The end-effector is a cylindrical body (rotation inertia m_e), which can rotate around the vertical axis (angle of rotation q_e). It is connected to the actuator rotor by a torsional spring (linear, elastic, stiffness k) and a damper (linear, viscous, damping coefficient b). The rotor (rotation inertia m_r) can rotate around the vertical axis (angle of rotation q_m). The connection between the stator and the frame is flexible, but the stiffness of this spring is made large for the present experiments. Energy dissipation in the bearings is modeled by a combination of Coulomb and viscous friction and results in a torque f_e on the end-effector and a torque f_m on the rotor. Preliminary experiments learned that the magnitude of the Coulomb friction depends on the direction of rotation. Therefore, f_e and f_m are modeled by

$$f_e = F_e^+ + A_e^+ \dot{q}_e \quad \text{if } \dot{q}_e > 0 \; ; \qquad f_e = F_e^- + A_e^- \dot{q}_e \quad \text{if } \dot{q}_e < 0 \; ; \tag{5.45}$$

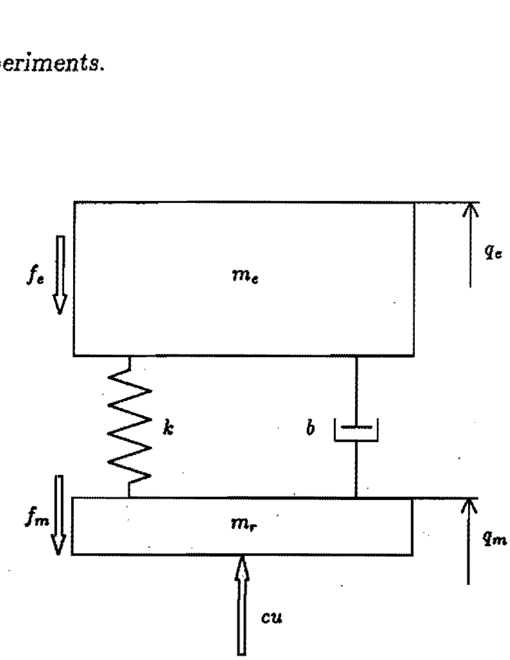


Figure 5.16: Model of the flexible R-manipulator.

$$f_m = F_m^+ + A_m^+ \dot{q}_m \quad \text{if } \dot{q}_m > 0 \; ; \qquad f_m = F_m^- + A_m^- \dot{q}_m \quad \text{if } \dot{q}_m < 0 \; . \tag{5.46}$$

The actuator is a DC-motor, which generates the torque $M = cu$ between the rotor and stator. Here, u is the input of the manipulator and c is a constant. The rotation q_e is the output of the system, i.e. $\underline{y} = q_e$. For a detailed description of this experimental set-up and of the controller hardware, see Kerssemakers (1993).

If the deformation of the spring between the stator and the frame is neglected, the dynamic model of this flexible manipulator is given by

$$m_e\ddot{q}_e + b(\dot{q}_e - \dot{q}_m) + k(q_e - q_m) + f_e = 0 \; ; \tag{5.47}$$

$$m_r\ddot{q}_m - b(\dot{q}_e - \dot{q}_m) - k(q_e - q_m) + f_e = cu \; . \tag{5.48}$$

The model parameters are uncertain and are to be adapted on-line. Some of the parameters have first been estimated off-line (relative to c: $m_e/c = 4.7$ $[kgm/N]$, $m_r/c = 0.39$ $[kgm/Nm]$, $b/c = 2.7$ $[sec/rad]$, $k/c = 978$ $[rad^{-1}]$; Kerssemakers, 1993). Subsequently, these values have been used as the initial estimates for adaptive control. The initial estimates for the Coulomb friction parameters are set to values between 3 and 8, and the viscous friction parameters are set to 0.

The control objective and some control strategies.

The control objective is to let the end-effector track a sinusoidal trajectory with amplitude A and frequency f, so $q_{ed} = Asin(2\pi ft)$. For the results presented here,

$A = 0.2$ $[rad]$ and $f = 1$ $[Hz]$. Apart from (adaptive) CRCTC, a number of other control techniques has been implemented, such as PD-control, fuzzy control and optimal control (Kerssemakers, 1993). For the PD-controller, the desired rotation $q_{md} = q_{md}(t)$ of the rotor was taken equal to the desired rotation $q_{ed} = q_{ed}(t)$ of the end-effector. For the fuzzy-like PD-controller, the desired trajectory of the rotor was determined on-line with fuzzy logic rules that express the intuitive feeling of how the motor rotor should rotate to obtain reasonably well trajectory tracking of the end-effector as well as to maintain stability of the closed-loop system by avoiding large elastic deformations. The optimal controller does not need the desired trajectory q_{md} explicitly. The (adaptive) CRCTC law incorporates explicitly the flexibility between the (mass at the) rotor and the end-effector. For the sake of simplicity, the control gain matrices K_r and Λ and the adaptation gain matrix Γ (see Section 4.5) are chosen to be diagonal matrices. For the experimental results shown here, an integral-like control term $K_i \underline{e}_r$ with $K_i = diag[\ k_{ei}\ \ k_{mi}\ ]$ has been added to the adaptive CRCTC law (4.57) in order to avoid asymmetry of the 'steady-state' errors, which are caused by unmodeled dynamics and which result in a computed reference q_{mr} that slightly drifts away. In theory, stability can still be proven with a similar Lyapunov function as used in Section 4.5, but the positive definite matrix K_i is supplemented to the non-negative definite stiffness matrix K (i.e. $V = \frac{1}{2}\dot{\underline{e}}_r M \dot{\underline{e}}_r + \frac{1}{2}\underline{e}_r[K + K_i]\underline{e}_r + \frac{1}{2}\underline{e}_p \Gamma^{-1} \underline{e}_p$).

The reference trajectory for the flexible coordinate is determined from the algebraic equation

$$q_{mr} = [m_e \ddot{q}_{er} + b(\dot{q}_e - \dot{q}_m) + f_e + k_{er}(\dot{q}_{er} - \dot{q}_e) + k_{ei}(q_{er} - q_e)]/k + q_{er}\ , \qquad (5.49)$$

and the input u is computed from

$$cu = m_r \ddot{q}_{mr} + b(\dot{q}_m - \dot{q}_e) + k(q_{mr} - q_{er}) + f_m + k_{mr}(\dot{q}_{mr} - \dot{q}_m) + k_{mi}(q_{mr} - q_m)\ . \quad (5.50)$$

Implementation of each of the control techniques mentioned above requires the availability of the rotations q_e and q_m and their time-derivatives for all t. Standard incremental encoders are used for the measurement of the rotations. However, no tachometers are used, so the angular velocities have to be reconstructed on-line, for instance, with a linear observer (e.g. Berghuis, 1993). Here, a Kalman filter is used. It predicts q_e, q_m, $\dot{q}_e$ and $\dot{q}_m$ one step ahead, in order to compensate for the computational time-delay (Kerssemakers, 1993).

The experiments have shown that adaptive CRCTC, in general, leads to smaller tracking errors than the other implemented control techniques. Besides, (adaptive) CRCTC results in a more smooth initial transient of the system. In all experiments, the system was initially at rest.

Experimental results with PD-control and adaptive CRCTC.

The results obtained with adaptive CRCTC are compared here with the results that are obtained with a PD-controller. Fig. 5.17 shows a characteristic tracking result obtained with PD-control. The left graph shows both the actual trajectory q_e and the desired trajectory q_{ed}. In the right graph, the tracking error $e_e = q_{ed} - q_e$ $[rad]$ is

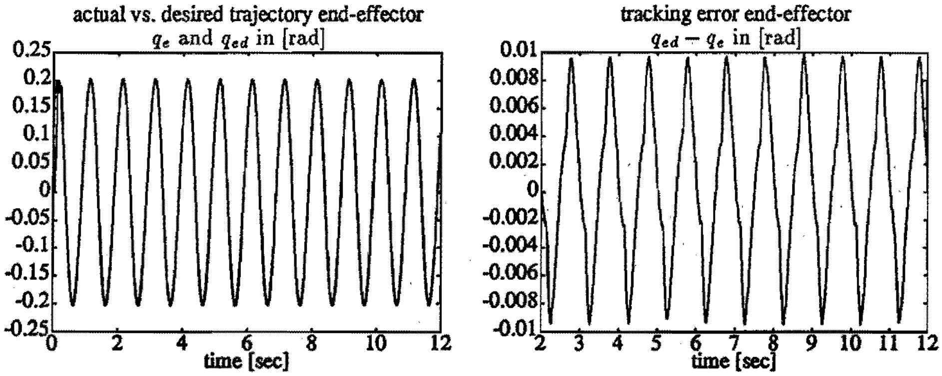


Figure 5.17: PD-control of the flexible R-manipulator.

shown for $t = 2$ [*sec*] to $t = 12$ [*sec*]. The input u is based on state feedback, i.e. $cu = k_{pl}(q_{ed} - q_e) + k_{pm}(q_{md} - q_m) + k_{dl}(\dot{q}_{dl} - \dot{q}_e) + k_{dm}(\dot{q}_{md} - \dot{q}_m)$, where the gains $k_{pl} = 16859$, $k_{pm} = 1176$, $k_{dl} = 4668$ and $k_{dm} = 69$ are determined with optimal pole-placement. Fig. 5.18 shows results for this PD-controller, but now with adaptive compensation of Coulomb and viscous friction. The right graph shows that the tracking accuracy has somewhat been improved (the amplitude of the tracking error reduces from 5% to 4% of the amplitude of the desired trajectory). The left graph, where the actuator torque is shown with the solid line, illustrates the 'chattering' effect of actuator saturation during the first transient. The contribution of the adaptive friction compensation to u is given in the same graph with a dashed line.

Fig. 5.19 shows a characteristic performance obtained with adaptive CRCTC for the experimental flexible R-manipulator. It can be seen in the first two graphs, that after 9 seconds the amplitude of the tracking error is smaller than 1% of the amplitude of the desired trajectory. In comparison with PD-control, adaptive CRCTC results in a much better tracking accuracy. The resulting deformation that occurs for adaptive CRCTC is shown in the lower-left graph of Fig. 5.19.

In Section 4.5, the stability proof of the closed-loop system with adaptive CRCTC was given for the continuous-time case, whereas its implementation in the experimental set-up discussed here is in discrete-time. As a result of limited computing speed, the sampling rate is 200 [*Hz*]. Furthermore, there are also measurement noise, nonlinear motor and amplifier dynamics, nonlinear behavior of the spring for large deformations, and imperfections in the mechanical construction (such as the connection between the stator and the frame, which is not perfectly rigid). These phenomena prevent the parameter adjustments to converge to constant values and, hence, the tracking errors do not become zero in the experiments. In order to avoid drift in the parameter estimates, the parametric adaptation is stopped at $t = 7$ [*sec*]. At that

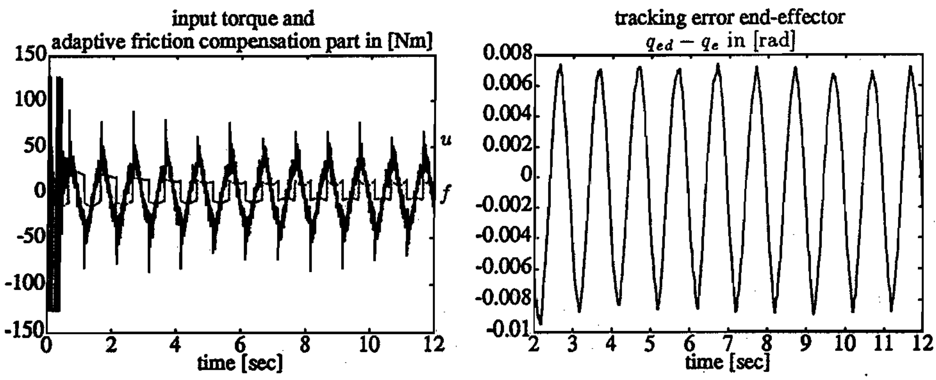


Figure 5.18: PD-control with adaptive friction compensation.

time, the parameter estimates have become nearly constant as can be seen in the lower-right graph of Fig. 5.19.

The performance of the closed-loop system strongly depends on the control and adaptation parameters. Increasing these parameters leads to higher control activity, quicker convergence of the parameter estimates and a better tracking accuracy. However, increasing them too much led to instability, because in that case unmodeled high-frequency dynamics are excited too much. To avoid stability problems, a compromise with respect to the parameter choice has been made between tracking accuracy and robustness (for measurement noise, disturbances and unmodeled dynamics. Experiments with adaptive CRCTC applied to the R-manipulator where also the connection between the stator and the frame is flexible (Fig. 5.16) are planned for the near future.

5.3.2 Adaptive CRCTC of a flexible translation-translation manipulator.

The experimental set-up.

A large number of experiments has been carried out on the XY-table (see Fig. 5.20). This translation-translation (TT) manipulator serves as an experimental set-up to test nonlinear control and identification concepts (see Section 5.1). Fig. 5.21 gives a schematic representation of the system; for a detailed description we refer to Heeren (1989).

The objective is to let the end-effector track a desired path in the horizontal xy-plane. Within the limitations of the construction (circa $1 \times 1\ m$), this can be performed by means of three slides: two slides (each with mass m_s) in x-direction and one slide with the end-effector (with mass m_e) in y-direction. The motors (relevant

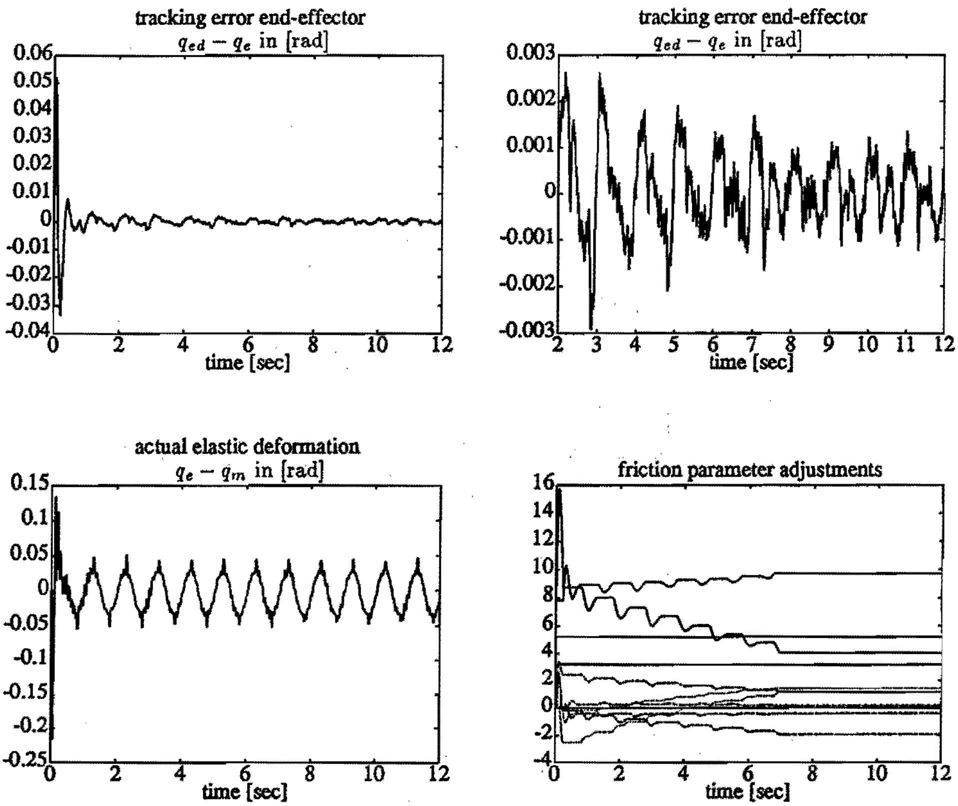


Figure 5.19: Adaptive CRCTC of the experimental flexible R-manipulator.

rotation inertias J_1 and J_2) generate the torques u_1 and u_2 for the x- and y-translation, respectively. These torques are assumed to be proportional to the motor currents i_1 and i_2, i.e. $u_1 = g_1.i_1$ and $u_2 = g_2.i_2$. The belt wheels of the two slideways in the x-direction are connected to a spindle that is driven by motor 1. Along the transverse slideway (mass m_t), the y-slide is belt driven by motor 2. The distance between the parallel slideways is $d = 1.004$ $[m]$, the length of the transverse slideway is $l = 1.23$ $[m]$, and the radius of the belt wheels is $r = 0.0105$ $[m]$ (Kaats, 1993).

The stiffness k of the spindle between the x-slideways can be easily varied in a broad range. For a rigid spindle, the XY-table has two degrees of freedom (q_1 and q_2) and can be easily controlled with the two motors. If the spindle is flexible, its elastic deformation introduces one extra degree of freedom (q_3). The end-effector position can be determined from incremental optical encoder measurements of the rotations q_1 and q_2 at the output shafts of the motors, and of the rotation q_3 at the non-driven

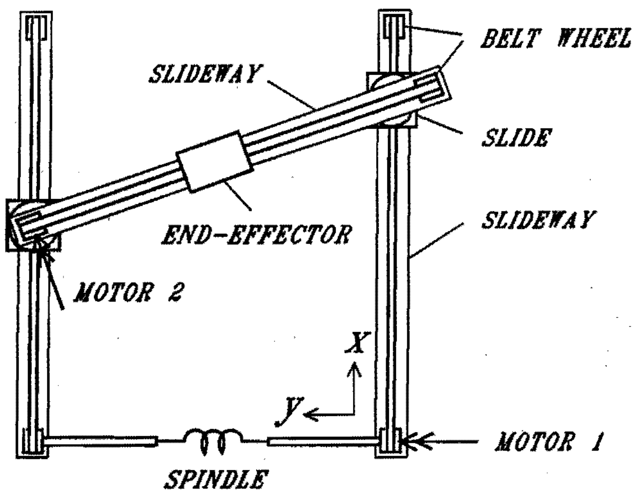


Figure 5.20: The experimental set-up.

end of the flexible spindle.

Experiments have shown that substantial *nonlinear* phenomena are caused by Coulomb friction (e.g. Van de Molengraft, 1990). The corresponding friction parameters w_1, w_2 and w_3 are uncertain, due to their slowly time-varying nature. Furthermore, the axis of the y-slideway does not have to coincide with the y-axis if the spindle is flexible. This results in nonlinear terms in the model due to Coriolis and centrifugal effects. These terms are small and are, therefore, neglected in the Kalman filter for the reconstruction of the unknown velocities $\dot{q}_1$, $\dot{q}_2$ and $\dot{q}_3$.

The control model and the control strategy.

Many control laws have been implemented on the flexible XY-table. Until recently, these laws were based on a model of the rigid XY-table (e.g. De Jager, 1992; Vijverstra, 1992). The objective is to let the end-effector of the XY-table follow a desired trajectory (x_d, y_d). For the rigid model this can be readily translated into a desired trajectory (q_{1d}, q_{2d}) for the motor coordinates. Brevoord (1992) has investigated the properties of CRCTC by experiments, based on a model that accounts explicitly for the flexibility of the spindle. This model, derived and identified by Van de Molengraft (1990), will not be given here in detail. However, the structure of the model matrices

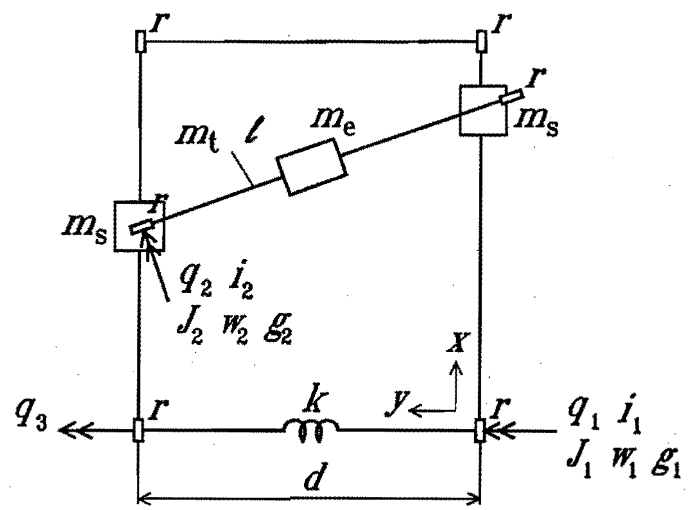


Figure 5.21: A scheme of the flexible TT-manipulator.

is given by

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \; ; \; \underline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \; ; \; M = \begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} \; ; \; C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & 0 & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \; ;$$

$$K = \begin{bmatrix} k & 0 & -k \\ 0 & 0 & 0 \\ -k & 0 & k \end{bmatrix} \; ; \; \underline{v}_n = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; \underline{f} = \begin{bmatrix} w_1 sign(\dot{q}_1) \\ w_2 sign(\dot{q}_2) \\ w_3 sign(\dot{q}_3 \end{bmatrix} \; ; \; H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} ,$$

The model parameters estimated off-line are $J_1 = 0.0022$ $[kgm^2]$, $J_2 = 0.000158$ $[kgm^2]$, $m_s = 2.3$ $[kg]$, $m_t = 8.5$ $[kg]$, $m_e = 2.3$ $[kg]$, $k = 0.46$ $[Nm/rad]$, $w_1 = 0.447$ $[Nm]$, $w_2 = 0.12$ $[Nm]$, $w_3 = 0.1$ $[Nm]$, $g_1 = 0.258$ $[Nm/A]$, and $g_2 = 0.056$ $[Nm/A]$.

CRCTC, based on this model, appears to be quite attractive to cope with flexibility of the spindle and shows an improved tracking performance compared to the control laws applied earlier. Actuator saturation occurs for large control parameters if the real initial conditions do not coincide with the desired ones. As we already experienced in simulations and other experiments, stability seems hardly to be affected by this saturation. Unfortunately, the tracking accuracy is limited by the limited sampling rate, the measurement noise, the velocity reconstruction errors, the unmodeled dynamics and the parametric uncertainty. To cope with parametric uncertainty, Kaats (1993) has investigated the properties of adaptive CRCTC. Some of his results with adaptive and non-adaptive CRCTC will be discussed here. For non-adaptive CRCTC, the model parameters are chosen to be 70% of the real values. The same values are used as the initial parameter estimates for adaptive CRCTC. All experiments have been done with a sampling frequency of 200 $[Hz]$. Furthermore, the control gains are the same for CRCTC and adaptive CRCTC. Suitable values are $\Lambda = diag[\ 5\ \ 5\ ]$ and $K_r = diag[\ 0.1\ \ 0.01\ \ 0.01\ ]$. An extra, integral-like control term $K_i \underline{e}_r$ has been used (as in the previous subsection) to omit the asymmetry in the tracking errors (which caused the computed reference to slightly drift away), where $K_i = diag[\ 0.3\ \ 0.1\ \ 0.001]$.

Experimental results with non-adaptive and adaptive CRCTC.

Fig. 5.22 shows some characteristic results that have been obtained with non-adaptive CRCTC. In the first two graphs, the solid lines represent the desired trajectories $q_{1d} = -25cos(\pi t)$ and $q_{2d} = 25sin(\pi t)$. They are determined from the desire to let the end-effector track a circle with a radius of 0.25 $[m]$ and an angular speed of π $[rad/sec]$. The dashed curves in the same graph represent the actual coordinates q_1 and q_2. The corresponding tracking errors $e_1 = q_{1d} - q_1$ and $e_2 = q_{2d} - q_2$ and the required actuator torques u_1 and u_2 are shown in the two lower graphs of this figure. As shown in Fig. 5.23, the tracking errors are considerably reduced when adaptive CRCTC is used. Here, nine parameters have been adapted, being the inertias $J_1 + m_s r^2$ and J_2, the masses m_s, m_t and m_e, the spindle stiffness k, and the Coulomb friction coefficients w_1, w_2 and w_3 (with adaptation parameters of 3.10^{-6}, 2.10^{-7}, 0.6, 0.8, 0.1, 2.10^{-4}, 0.07, 0.002 and 0.006, respectively). The parameter estimates, globally shown in the lower-right graph of Fig. 5.23, quickly converge to nearly constant values. Usually, these values differ from the "real" values.

The dashed line in the upper-right graph of Fig. 5.23 represents the reference trajectory q_{3r}, which has been computed on-line. It is followed at a small, constant distance by the actual trajectory q_3. The elastic deformation $q_3 - q_1$ of the spindle is shown in the lower-left graph of Fig. 5.23.

Conclusions.

The experiments have shown that the flexible XY-table can be appropriately controlled with adaptive CRCTC. In comparison with non-adaptive CRCTC, a more accurate trajectory tracking for the motor coordinates q_1 and q_2 can be achieved, while the deformation $q_3 - q_1$ of the spindle remains bounded. However, during the experiments, non-zero 'steady-state' errors occurred. They arise from unmodeled phenomena, such as nonlinear motor and amplifier dynamics, backlash, imperfections in the construction, measurement noise, quantization errors in the encoder measurements, prediction errors in the Kalman filter, and the discrete-time implementation of the control law. Fortunately, these phenomena do not endanger the stability of the closed-loop system, provided that the control and adaptation gain matrices are kept small enough. The tracking errors e_1 and e_2 can be made smaller by replacing the term $K_r \underline{\dot{e}}_r$ in the CRCTC law by a saturation term (Kaats, 1993). Finally, it

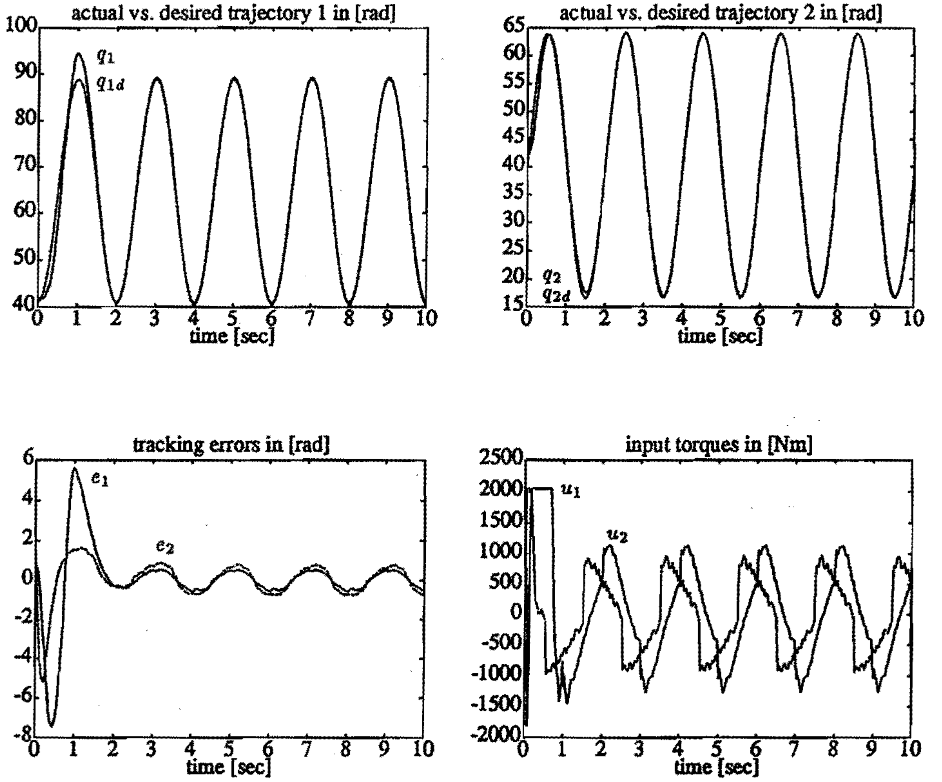


Figure 5.22: Non-adaptive CRCTC of the experimental flexible TT-manipulator.

must be noted that trajectory tracking of the end-effector position, which depends on all three coordinates q_1, q_2 and q_3, is disturbed by the deformation $q_3 - q_1$ of the spindle. Simulations and experiments with the (modified) CRCTC law and with direct measurement of the end-effector position for the benefit of end-effector trajectory tracking are planned for the near future.

5.4 Conclusions.

In this chapter, the effectiveness of (adaptive) CRCTC for flexible manipulators has been demonstrated by means of simulations and experiments. It has been shown that it is feasible to use a control law such as (adaptive) CRCTC, which realizes the motion control objective while an acceptable limitation of elastic deformations and vibrations is obtained. Because flexibility and parametric uncertainty are incorpo-

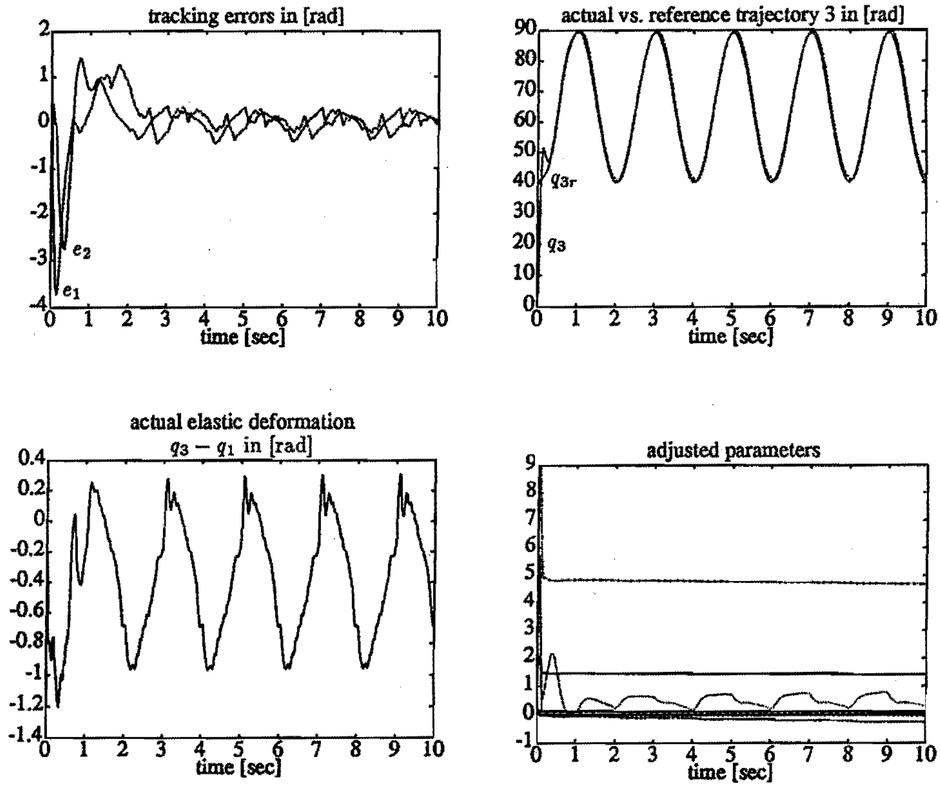


Figure 5.23: Adaptive CRCTC of the experimental flexible TT-manipulator.

rated in the control design explicitly, these do not give rise to tracking errors and do not endanger the stability of the closed-loop system. As a consequence, the stiffness of manipulators can be decreased and the accelerations can be increased without loss of tracking accuracy.

The CRCTC law has first been tested in a number of simulations for a range from moderate to large flexibilities of the joint and/or link of the TR-manipulator. In each of these simulations, CRCTC performed well and yielded stable trajectory tracking of the end-effector. This has been realized by means of the on-line computation of a reference trajectory for the flexible coordinate(s) and of the input torques, a manipulator model being used that incorporates the flexibility explicitly. However, for the TR-manipulator with a flexible link this has been approximated by the off-line computation of a desired trajectory for the flexible coordinate. This is due to an unstable differential equation for this coordinate, which is probably due to unstable zero-dynamics. It is worthwhile to investigate this in more detail and to search for a numerical procedure to compute the reference trajectory on-line. The latter is important, because an adaptive version of CRCTC is required to obtain trajectory tracking of the end-effector in case of parametric uncertainty.

Theoretically, the tracking errors of adaptive CRCTC converge asymptotically to zero, and the parameter adjustments become constant. For future research, it is suggested to investigate cases in which flexibility is modeled with more than one extra degree of freedom (e.g. Heijman, 1993) and to consider a simulation model that is more detailed than the control model in order to study the robustness properties of the adaptive CRCTC scheme.

The practical extension of CRCTC to an adaptive version can be easily performed, but complicates the implementation somewhat because the control and adaptation parameters have to be properly tuned. Just as for adaptive CTC of rigid manipulators, no systematic approach is known to determine a suitable set of control and adaptation gains. To avoid stability problems in practice, these gains are to be chosen by means of a trade-off between tracking accuracy and robustness for measurement noise, disturbances and unmodeled dynamics.

Although the prove of stability is not valid in the presence of unmodeled dynamics, simulations and experiments (e.g. Doelman, 1992; Brevoord, 1992; Kuijpers, 1993; Kerssemakers, 1993; Kaats, 1993) have shown that the (adaptive) CRCTC approach is rather robust for actuator saturation, slowly time-varying manipulator parameters, a limited sampling rate, measurement noise and for the discrete-time implementation of the control law. In the experiments performed on the flexible R-manipulator and on the flexible XY-table, the 'steady-state' errors actually stayed within the range of a few percent of the desired trajectory values. Whenever the 'steady-state' errors are not small enough, significant dynamics that have not been modeled have to be included in the control model. Whereas flexibility can be taken into account explicitly in the approach of adaptive CRCTC, the experiments have shown that unmodeled friction was one of the main sources of tracking inaccuracy. An improved tracking performance has been obtained by means of adaptive compensation based on a detailed friction model.

The simulations and experiments reported here have been performed using a PC-386. Faster control hardware enables the use of a more detailed control model (for instance, with nonlinear motor and amplifier dynamics) and of a more accurate velocity observer than the Kalman filter used here. Furthermore, it offers the opportunity to use a higher sampling rate and, thus, the opportunity to increase the control and adaptation gains. Improved controller hardware is expected to yield improved tracking performance. Moreover, end-effector control of the XY-table is expected to ask for powerful hardware, as the control algorithm will be quite complex.

Chapter 6

Conclusions and recommendations.

In this thesis, a new control technique for flexible manipulators has been presented. The idea of computed torque control for rigid manipulators has been used to derive the *Computed Reference Computed Torque Control* (CRCTC) law, which is based on a model that takes concentrated joint flexibility and/or distributed link flexibility into account. An adaptive version of CRCTC has been designed to cope with parametric uncertainty. Simulations and experiments with (adaptive) CRCTC have shown that the inclusion of flexibilities in the control model improves the tracking accuracy in comparison with a controller that is based on a rigid model. Although the number of degrees of freedom exceeds the number of control inputs, global, asymptotic stability of the closed-loop system is theoretically guaranteed for all values of the stiffness of joints and links. For those degrees of freedom for which no desired trajectory is known beforehand, a reference (or desired) trajectory is computed on-line. With (adaptive) CRCTC, such reference trajectory and, subsequently, such input torques are computed that asymptotic tracking of the desired end-effector trajectory is achieved *with bounded deformations*. Hence, the CRCTC technique contributes to the solution of the control problem for flexible manipulators.

In addition to the equations for the input quantities, the CRCTC scheme includes a set of algebraic and/or differential equations for the a priori unknown reference trajectories. If the relation for the unknown trajectory is an algebraic equation, the unknown reference trajectory can be easily computed on-line, and its first and second time-derivative can be computed analytically or numerically by means of a (weak) differentiation scheme. This is the case, for instance, for a flexible joint that is modeled according to Spong (1987). If the relation for the unknown trajectory is a stable second-order differential equation, the unknown reference trajectory and its first and second time-derivative can be directly computed on-line. Problems arise, if the relation for the unknown trajectory is an unstable differential equation, e.g. in case of end-effector control (which is possibly caused by unstable zero dynamics; Slotine and Li, 1991; Nijmeijer and Van der Schaft, 1990). It was shown that, if a numerical problem as mentioned above occurs, a desired trajectory for the flexible coordinate(s) can be computed off-line (see also Heijman, 1993). The use of such a trajectory in the CRCTC scheme for the flexible-link TR-manipulator has led to satisfying tracking results of the end-effector.

The proposed controller can be regarded as a conventional computed torque controller (CTC) plus a corrective term that accounts for the effects of flexibility (see also Lammerts, 1990; Lammerts et al., 1991). The contribution of this corrective control term increases with increasing flexibility. Thus, for a rather stiff manipulator the input resembles the input according to CTC for the equivalent rigid manipulator plus a small signal, which becomes zero for a perfectly stiff construction. For increasing stiffness of a link — and probably also of a joint if it is not modeled according to Spong — , however, unacceptable high-frequency signals may occur in the motor inputs. Then, one should decide to use a control law that is based on a model with this link (or joint) assumed to be rigid. But, whenever fast and precise trajectory tracking requires that even small flexibilities are taken into consideration, the computed inputs can be filtered to eliminate the very high-frequency signals. It should be investigated then whether stability is seriously endangered.

In CRCTC, the manipulator does not need to be rather stiff as must be in the singular perturbation approach. Furthermore, unlike many of the approaches presented in literature, CRCTC is not limited to a restricted class of flexible manipulators, because both concentrated joint flexibility and distributed link flexibility can be taken into account. To cope with parametric uncertainty, an adaptive version of CRCTC can be used. In theory, asymptotic trajectory tracking can be realized then for a model structure that exactly matches the real system, while all signals in the closed-loop system remain bounded. In this thesis, unmodeled dynamics has been encountered only in experiments. In that case, a trade-off had to be made between tracking accuracy and robustness for unmodeled dynamics, for measurement noise, and for disturbances.

For practical implementation of (adaptive) CRCTC, it is required that all the generalized coordinates and velocities are available for feedback. Hence, in addition to the usual sensors also sensors for the elastic deformations are required. These sensors can, for instance, be strain gauges. If not all state variables are measured, a suitable observer must be designed to estimate the unknown deformations and unknown velocities. Observers for flexible-joint manipulators have been designed by, for instance, Nicosia and Tomei (1990). They proposed a model-based observer that requires knowledge of the link positions only. Berghuis (1993) investigated linear velocity observers in combination with (adaptive) CTC for rigid manipulators. It is worthwhile to investigate whether his approach can be extended to flexible manipulators. This recommendation is based on the fact that CTC and CRCTC showed similar characteristics and robustness properties during simulations and experiments. Moreover, both approaches can be extended to an adaptive version using direct, indirect or composite parameter adaptation. Investigations of these adaptive control methods by means of simulations and experiments have been carried out by, for instance, Vijverstra (1992, for adaptive CTC) and Kaats (1993, for adaptive CRCTC).

A topic for future research, especially with regard to flexible-joint manipulators, is to compare CRCTC with the input-output feedback linearization approach and the method of singular perturbation. Furthermore, it is suggested to investigate the robustness properties of CRCTC by using a simple control design model and a more detailed simulation model. It is worthwhile, also, to investigate the relation between the numerical problems that can occur for the end-effector control of certain flexible manipulators and the (unstable) zero-dynamics of these manipulators (e.g. Slotine and Li, 1991; Nijmeijer and Van der Schaft, 1989). In literature, stable end-effector control of flexible manipulators is considered to be a fundamental problem that is still a hot issue in control research. The CRCTC approach reduces this problem to the search for a numerical procedure that can compute all the a priori unknown references on-line. The latter is important for stability reasons and also because adaptive CRCTC can only be used with reference trajectories that are computed on the basis of the on-line adjusted parameters. Furthermore, it is worthwhile to investigate the dynamic terms in the manipulator model that cause the differential equations to be unstable, and whether numerical problems can be avoided by neglecting these terms. This is interesting, also because the model of Spong (1987) for flexible-joint manipulators, which does not have terms that represent a dynamic coupling between the rigid-body coordinates and the flexible coordinates, does not give rise to any problems if it is used for (adaptive) CRCTC. Hence, especially with regard to flexible-link manipulators, it should be investigated which dynamic coupling terms can be neglected in control design to enable the on-line computation of all a priori unknown references. For the on-line computation of the input torques, it is, in fact, not necessary to neglect these terms.

In this thesis, a distinction has been made between rigid-body coordinates and flexible coordinates. The desired trajectory for the rigid-body coordinates can be determined off-line from the desired end-effector trajectory. The desired or reference trajectory for the flexible coordinates are computed on-line and, subsequently, used to compute the control inputs. In Lammerts et al. (1993), this approach has been generalized. There, a subset of generalized coordinates, for which the desired trajectory is specified explicitly, and a subset of generalized coordinates, for which the desired trajectory is beforehand unknown, are introduced. In this approach, the end-effector position can depend on all generalized coordinates and the flexible manipulator does not have to be modeled according to Chapter 2. This offers the opportunity to incorporate other control objectives than only the end-effector trajectory tracking, provided that these objectives can be related to the generalized coordinates. This may, for instance, be helpful with regard to redundant (flexible) manipulators, where extra requirements may reflect the desire to minimize the energy consumption of the servo motors or to minimize the maximum torques of the motors. Another advantage of the approach in Lammerts et al. is that end-effector control can be achieved indirectly, i.e. by feedback of the measured joint positions and elastic deformations, while the on-line computation of all desired or reference trajectories for these generalized coordinates can be based on the inverse kinematics and the desired end-effector trajectory. Lammerts et al. indicate that CRCTC is a promising control technique for nonlinear mechanical systems with more degrees of freedom than input signals.

Other sources of extra degrees of freedom than flexibility, such as backlash, can be investigated in the future.

Appendix A

Stability of a control system.

A.1 Definitions of stability.

Stability is a fundamental requirement of any control system. Much work has been done on the stability analysis of non-adaptive and adaptive control systems (e.g. Landau, 1979; Åstrom and Wittenmark, 1989; Narendra and Annaswamy, 1989; Slotine and Li,1991). In this appendix, we briefly discuss the stability of non-adaptive control systems. The first step is to consider the *error model*,

$$\dot{\underline{e}} = \underline{f}(\underline{e}, t) \ , \qquad \underline{e}(t_0) = \underline{e}_0 \ , \tag{A.1}$$

where $\underline{e}$ represents the difference between the desired and the actual trajectory. The function $\underline{f}$ may be a nonlinear function of $\underline{e}$ and t, but has to satisfy the condition $\underline{f}(\underline{0}, t) = \underline{0}$ for all t[1]. The solution $\underline{e} = \underline{0}$ of $\dot{\underline{e}} = \underline{f}(\underline{e}, t)$ is called an *equilibrium point.* The equilibrium point $\underline{e} = \underline{0}$ is *stable* if for all $t_0 \geq 0$ and $\epsilon > 0$ there is such a positive number $\sigma(t_0, \epsilon)$ that

$$\mid \underline{e}_0 \mid < \sigma(t_0, \epsilon) \ \rightarrow \ \mid \underline{e}(t) \mid < \epsilon \ , \qquad \forall \, t \geq t_0 \ . \tag{A.2}$$

If σ in definition (A.2) is independent of t_0, the equilibrium point $\underline{e} = \underline{0}$ is said to be uniformly stable. Global stability of the equilibrium point $\underline{e} = \underline{0}$ means that $\underline{e} = \underline{0}$ is stable for each $\underline{e}_0$.

Stability is a <u>mild</u> requirement, because it does not require that the actual trajectory tends to the desired trajectory asymptotically. The equilibrium point $\underline{e} = \underline{0}$ is *asymptotically stable* if it is stable and attractive, i.e. if the actual trajectory tends towards the desired one as time increases. *Global, asymptotic stability* of the equilibrium point $\underline{e} = \underline{0}$ means that $\underline{e} = \underline{0}$ is asymptotically stable for each $\underline{e}_0$.

The definition of asymptotic stability of the equilibrium point $\underline{e} = \underline{0}$ does not involve any requirement on the speed of convergence of $\underline{e}$ to $\underline{0}$. If the convergence to $\underline{0}$ is required to be at least exponential, the error system (A.1) must be exponentially stable, which means that there are such constants $a > 0$ and $b > 0$ that

$$\mid \underline{e}(t) \mid \leq a \exp^{-b(t-t_0)} \mid \underline{e}_0 \mid \ , \qquad \text{for all } \ t \geq t_0 \ . \tag{A.3}$$

[1]In fact, it is only required that the relation $\underline{f}(\underline{e}, t) = \underline{0}$ for all t has a constant solution $\underline{e}_s$ for $\underline{e}$. Now, by the transformation $\underline{e}^* = \underline{e} - \underline{e}_s$ it is always possible to write the error model for the new variable $\underline{e}^*$ in the form of Eq. (A.1).

Exponential convergence of the tracking error of a controlled system is attractive, because it favors robustness (e.g. Slotine and Li, 1991). Exponential stability always implies that the system is asymptotically stable. A linear system that is asymptotically stable is also exponentially stable.

In Appendix B, we will focus on stability for an adaptive control system. The best known stability method for non-linear systems is the second method of Lyapunov, which will be introduced in the next section.

A.2 The second method of Lyapunov.

A.2.1 Stability in the sense of Lyapunov.

The *second* or *direct method of Lyapunov* for a nonlinear system starts from the nonlinear error equations, rather than from a linearization around the equilibrium point. Therefore, instead of local stability in a small region around that equilibrium point, global stability can be analyzed. With the second method of Lyapunov, stability can be studied without determining explicitly the solution of the error equations (A.1). As before, in the sequel it is assumed that $\underline{e} = \underline{0}$ is an equilibrium point of the error system.

The basic idea behind the second method of Lyapunov is to try to find such a positive definite function $V = V(\underline{e}, t)$ that its time-derivative $\dot{V} = \dot{V}(\underline{e}, t)$ is non-positive (Vidyasagar, 1978; Narendra and Annaswamy, 1989; Slotine and Li, 1991). If this so-called Lyapunov function exists, the equilibrium point is stable. By imposing extra requirements on the function V and its time-derivative $\dot{V}$, conclusions about more specific kinds of stability (uniform, asymptotic, global, etc.) can be drawn. Here, global, asymptotic stability is of special interest. According to Slotine and Li (1991), the equilibrium point $\underline{e} = \underline{0}$ is globally, asymptotically stable if there is such a function $V = V(\underline{e}, t)$ that

1. $V(\underline{0}, t) = 0$ for all t;
2. $V(\underline{e}, t)$ has continuous first partial derivatives with respect to $\underline{e}$ and t;
3. $V(\underline{e}, t) \rightarrow \infty$ for $\| \underline{e} \| \rightarrow \infty$ and for all t;
4. $V(\underline{e}, t) > 0$ for $\underline{e} \neq \underline{0}$ and for all t;
5. $\dot{V}(\underline{e}, t) < 0$ for $\underline{e} \neq \underline{0}$ and for all t.

For the manipulator control systems in this thesis, the error system does not depend explicitly on time t, i.e. $\dot{\underline{e}} = \underline{f}(\underline{e})$. Hence, global, asymptotic stability implies uniform, global, asymptotic stability.

A.2.2 Finding an appropriate Lyapunov function.

An important objective in the design of a control system is to assure global, asymptotic stability. However, there is no general procedure to determine a suitable Lyapunov function for any nonlinear error system. For a linear, time-invariant error system

$$\dot{\underline{e}} = A\underline{e} \ , \tag{A.4}$$

the choice of the function

$$V(\underline{e}) = \underline{e}^T P \underline{e} \tag{A.5}$$

results in the time-derivative $\dot{V} = \underline{e}^T(PA + A^T P)\underline{e} = -\underline{e}^T Q \underline{e}$. The function V is positive definite and its derivative $\dot{V}$ is negative definite, if there exists such a constant, symmetric, positive definite matrix P that the matrix Q defined by

$$PA + A^T P = -Q \tag{A.6}$$

is positive definite. If so, then V is a *Lyapunov function* for the system (A.4) and the equilibrium point $\underline{e} = \underline{0}$ is globally, asymptotically stable. If there is such a constant, symmetric, positive definite matrix P that $Q = \gamma P$ with positive scalar γ, then $\dot{V} = -\underline{e}^T Q \underline{e} = -\gamma \underline{e}^T P \underline{e} = -\gamma V$ and the error system is exponentially stable.

A.3 The hyperstability theory of Popov.

A.3.1 Introduction.

In the 1960's and 1970's, when many of the significant investigations in (adaptive) system theory took place, Popov (1973) developed the so-called hyperstability theory for autonomous systems with a nonlinearity in the feedback loop. This theory originated from the concept of absolute stability (e.g. Narendra and Annaswamy, 1989) and is quite suitable for dealing with adaptive control problems. Especially in the design of MRAC systems (e.g. Landau, 1979) and of manipulator control systems (e.g. Dubowsky and DesForges, 1979; Horowitz and Tomizuka, 1980; Nicosia and Tomei, 1984), this theory has been extensively used. Although recent adaptive manipulator control schemes often rely on the second method of Lyapunov (e.g. Craig et al., 1986; Slotine and Li, 1987), the hyperstability theory can lead to more flexibility in non-adaptive and adaptive control design, and it seems to be of interest to investigate its relationship with the second method of Lyapunov.

A.3.2 Standard feedback system.

In order to find a relation between Lyapunov's second method and the hyperstability theory of Popov, the error system

$$\dot{\underline{e}} = A\underline{e} + I\underline{z} \ , \tag{A.7}$$

with constant matrix A and input $\underline{z}$ is considered. To apply the hyperstability theory of Popov, this error system is reformulated in the form of a "standard feedback system", that consist of a *linear time-invariant feedforward block*

$$\dot{\underline{e}} = A\underline{e} + I\underline{z} \quad ; \qquad \underline{\epsilon} = D\underline{e} \ , \tag{A.8}$$

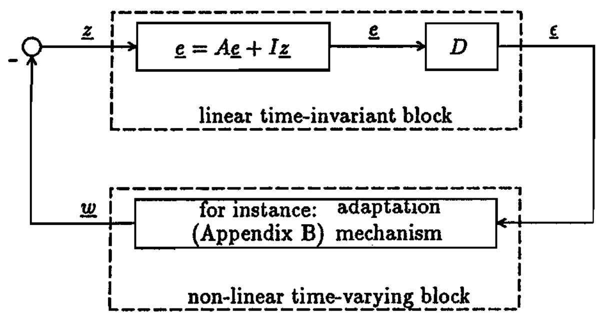


Figure A.1: "Standard feedback system".

with input $\underline{z}$ and output $\underline{\epsilon}$, and a *nonlinear time-varying feedback block* with input $\underline{\epsilon}$ and output $\underline{w} = -\underline{z}$. A block scheme of this system is given in Fig. A.1. The output ϵ of the feedforward block is generated by a linear compensator D, which has to meet certain requirements according to the hyperstability theory of Popov. The introduction of this compensator gives some extra freedom in the design of stable non-adaptive and adaptive control schemes. In the hyperstability theory, a characteristic property of the standard feedback system is used: if the standard feedback system is a parallel combination of two asymptotically stable blocks, it is also asymptotically stable (Popov, 1973). Therefore, an asymptotically stable system is ensured if both the feedforward and the feedback block satisfy certain conditions. According to Popov (1973), the system is asymptotically stable if

1) the linear compensator D is such that the transfer matrix of the feedforward block, i.e. $Z(s) = D(sI - A)^{-1}$, is strictly positive real (e.g. Landau, 1979),
2) the feedback block is a passive mapping of the input $\underline{\epsilon}$ to the output $\underline{w}$.

In Landau (1979), these conditions are investigated in detail for MRAC systems. In the next two parts of this subsection, these conditions are discussed in correspondence with the second method of Lyapunov.

A.3.3 A positivity condition for the feedforward block.

The Kalman-Yakubovitch lemma plays an important role in establishing the relationship between the existence of a Lyapunov function (and, hence, global, asymptotic stability) and the earlier-mentioned frequency domain conditions for the linear time-invariant feedforward block (e.g. Narendra and Annaswamy, 1989). This lemma states that, if the conditions of the hyperstability theory are satisfied, a Lyapunov function exists. The candidate Lyapunov function is composed now of a quadratic term in the output tracking error $\underline{e}$ and of an integral function:

$$V = \underline{e}^T P \underline{e} - 2 \int_0^t \underline{\epsilon}^T \underline{z} d\tau \ , \tag{A.9}$$

where P is a constant, symmetric, positive definite matrix and the integral has to satisfy the so-called passivity condition for the feedback block. This condition is given in the next subsection. To investigate the relationship between Lyapunov's second method and the hyperstability theory of Popov, the time-derivative of V is considered:

$$\begin{aligned} \dot{V} &= 2\underline{e}^T P \underline{\dot{e}} - 2\underline{\epsilon}^T \underline{z} = \\ &= \underline{e}^T (PA + A^T P) \underline{e} + 2\underline{z}^T (P - D) \underline{e} \ . \end{aligned} \tag{A.10}$$

The term $\underline{e}^T(PA + A^T P)\underline{e}$ is negative definite if P satisfies the Lyapunov matrix equation (A.6), where Q is a symmetric, positive definite matrix. The term $2\underline{z}^T(P - D)\underline{e}$ of $\dot{V}$ is equal to 0, if the linear compensator D is equal to P. Resuming, we obtain a negative definite derivative $\dot{V} = -\underline{e}^T Q \underline{e}$, if the so-called ***positivity condition for the feedforward block*** is satisfied:

$$PA + A^T P = -Q \ ; \quad Q = Q^T \ , \ Q > 0; \quad D = P \ . \tag{A.11}$$

A.3.4 A passivity condition for the feedback block.

According to Popov, the standard feedback system defined in Subsection A.3.2 is asymptotically stable if the feedforward block satisfies the positivity condition (A.11) and the feedback block satisfies a passivity condition, which guarantees that the function V as defined in Eq. (A.9) is non-negative. Therefore, the last term of V in Eq. (A.9) has to satisfy

$$\int_{t_0}^t \underline{\epsilon}^T \underline{w} d\tau > -\gamma_o^2 \ ; \quad \gamma_o \text{ constant} \ , \tag{A.12}$$

and this implies that the feedback block has to represent a passive mapping of the input ϵ to the output $\underline{w} = -\underline{z}$. This ***Popov integral inequality*** is called the passivity requirement for the feedback block. Especially for stable adaptive control design, it offers more flexibility in the choice of an adaptation algorithm.

Appendix B

Adaptive control.

B.1 Introduction.

Non-adaptive controllers, i.e. controllers with a fixed structure and fixed parameters, cannot always provide acceptable system performance. Especially if the plant (i.e. system to be controlled) has unknown or time-varying parameters, a fixed setting of the controller parameters will generally not result in an acceptable closed-loop behavior. In the early 1960's, this observation led to the introduction of adaptive control, especially in the field of flight control systems. However, there was a delay before this topic was widely studied in control engineering (e.g. Landau, 1979; Åstrom and Wittenmark, 1989; Slotine and Li, 1991; Narendra and Annaswamy, 1989) and the current number of successful applications in industry is not large. Nevertheless, adaptive control has been successfully applied in, for instance, ships and airplanes, power systems, and, last but not least, robots and manipulators (e.g. Van Amerongen et al., 1984; Åstrom and Wittenmark, 1989). For a review of adaptive control schemes for rigid manipulators, see e.g. Landau, 1985; Hsia, 1986; Ortega and Spong, 1988. Although there are many other versions of adaptive control (see in the references above), the main idea behind the version of adaptive control to be considered <u>here</u> is to adjust the controller parameters on-line, using measurements of the actual states of the plant. The purpose of this appendix is to present a short review of some of the developments that have contributed to the current state of the art in these kind of adaptive control techniques (see also De Jager et al., 1991).

Adaptive control is one of the techniques to control uncertain systems. Model uncertainty may consist of parametric uncertainty and unmodeled dynamics. According to Webster's Ninth New Collegiate (1991), to 'adapt' means *a modification according to changing circumstances.* Then, an adaptive control system may be defined as a feedback system that automatically modifies its characteristics if the environment changes. More specifically, a control system is said to be *parameter adaptive* if it changes one or more parameters in order to force the response of the resulting closed-loop system towards a desired one despite unknown or time-varying plant parameters. This means, that (a part of) the procedure of modeling, identification and control design is performed on-line, based on system measurements.

B.2 Feedback control with open-loop adaptation.

One of the earliest and most simple approaches to adaptive control is *gain scheduling*, which is a form of feedback control with open-loop adaptation. It was introduced in the early 1960's with the development of automatic ship and flight control devices (autopilots), long before adaptive control theory was well developed. Later, it was also successfully used in, for instance, process control. This technique can be applied if the poorly known plant dynamics depend in a well-known way on auxiliary variables that can be measured but are not used for feedback (like, for instance, air pressure for airplanes). Then, the controller parameters can be tuned (see Fig. B.1). Quite

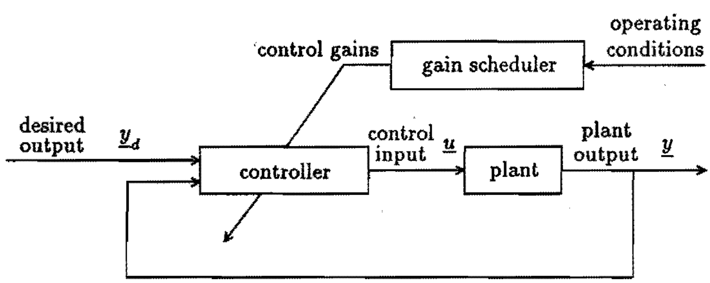


Figure B.1: Feedback control with open-loop adaptation.

often, suitable controller parameter values are computed off-line as a function of the auxiliary signals and stored in a look-up table. However, for complex plants these tables can be quite large.

Gain scheduling is also applied in the area of process control. In general, process models are quite complex, nonlinear and uncertain. They may change considerably due to various disturbances (for instance, due to variations in the raw materials that enter the process) and due to unmeasured process variables (such as temperature, pressure, etc.). These uncertainties may require to continuously tune hundreds of parameters to keep the system close to its optimal operating point. Automatic tuning is then unavoidable and quite attractive.

The main disadvantages of gain scheduling are that the auxiliary variables have to be measured and that the parameters are changed in open loop. Therefore, it is questionable whether gain scheduling can be considered as an adaptive control technique or not.

B.3 Feedback control with closed-loop adaptation.

B.3.1 Introduction.

If (some of) the plant parameters are unknown and/or time-varying and the controller parameters cannot be appropriately tuned, it is obvious to perform adaptation in closed-loop. Then, the controlled system (see Fig. B.2) consists of an inner loop with the plant and the feedback controller, and an outer loop with an adaptation mechanism that adjusts the controller parameters, using the measured output $\underline{y}$ and the desired output $\underline{y}_d$. In this section, some parameter-adaptive control schemes are

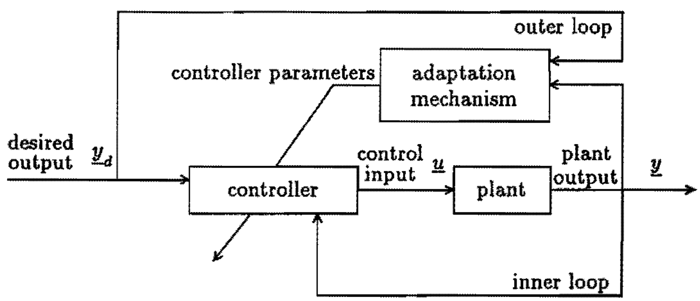


Figure B.2: Feedback control with closed-loop adaptation.

briefly discussed.

B.3.2 Feedback control with a signal generator.

Often, the desired output trajectory $\underline{y}_d = \underline{y}_d(t)$ is generated off-line and specified explicitly. Another possibility is to generate this trajectory in a signal generator with a known reference input $\underline{r}$ (see Fig. B.3). Let $\underline{e}_y$ be the output error, i.e. the difference between the plant output $\underline{y}$ and the desired output $\underline{y}_d$:

$$\underline{e}_y = \underline{y}_d - \underline{y} \, . \tag{B.1}$$

The controller now has to realize asymptotic output trajectory tracking, i.e.:

$$\lim_{t \to \infty} \underline{e}_y(t) = \underline{0} \, . \tag{B.2}$$

The signal generator may represent a stable, so-called *reference model* of the considered system. Then, Fig (B.3) represents a *model-following* control scheme, which can be extended to an adaptive control scheme if parameters are unknown

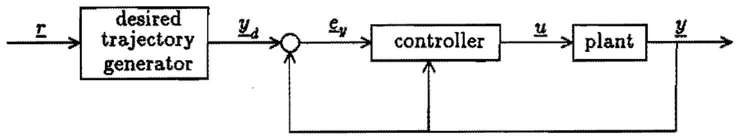


Figure B.3: Feedback control with a desired trajectory generator.

(to be compared with series MRAC, see Landau, 1979). If this signal generator is not in series with the plant but parallel (see Fig. B.4), the stable reference model is often chosen such that the output $\underline{y}_m$ of this model tracks the desired output $\underline{y}_d$ asymptotically, i.e.

$$\lim_{t\to\infty}[\underline{y}_d(t) - \underline{y}_m(t)] = \underline{0} \ , \tag{B.3}$$

instead of $\underline{y}_m = \underline{y}_d$. The adavantage of this approach is that the reference model can express the desired performance of the controlled plant. Then, the adaptation mechanism of MRAC compares the actual performance of the closed-loop system, i.e. $\underline{y}$, with the desired one, i.e. $\underline{y}_m$, and with this information it modifies the controller parameters such that

$$\lim_{t\to\infty}(\underline{y}_m - \underline{y}) = \underline{0} \ . \tag{B.4}$$

This approach is also referred to as *model-following control.* With regard to mechanical systems, it is often used if the desired system performance can be specified in terms of overshoot, required damping, etc. To cope with model uncertainty, it is extended to a parallel MRAC scheme (Landau, 1979), which will be discussed in Section B.3.5. This approach is also discussed in e.g. De Jager et al. (1991). For the conditions that the reference input signal $\underline{r}$ and the reference model have to satisfy, as well as for more details of this approach, see also Landau (1979).

Another possibility for the signal generator is to generate a reference trajectory $\underline{y}_r$, not based on a reference model with a reference input $\underline{r}$ but on the desired trajectory $\underline{y}_d$ and the tracking error $\underline{e}_y = \underline{y}_d - \underline{y}$ (see Fig. B.5). For example, $\underline{y}_r$ can be defined by

$$\dot{\underline{y}}_r = \dot{\underline{y}}_d + \Lambda_y \underline{e}_y \quad ; \qquad \Lambda_y + \Lambda_y^T > 0 \ . \tag{B.5}$$

The control objective now is to let $\dot{\underline{y}}$ track $\dot{\underline{y}}_r$ asymptotically, i.e. to realize $\dot{\underline{e}}_{yr}(t) \to \underline{0}$ for $t \to \infty$ where $\underline{e}_{yr} = \underline{y}_r - \underline{y}$. Then, from $\dot{\underline{e}}_{yr} = \dot{\underline{e}}_y + \Lambda_y \underline{e}_y$ and $\Lambda_y + \Lambda_y^T > 0$, it follows that $\underline{e}_y(t) \to \underline{0}$ for $t \to \infty$, i.e. that the original tracking objective is achieved. This definition of the reference trajectory is proposed by, e.g., Slotine and Li (1986). Other definitions are possible (e.g. Sadegh and Horowitz, 1987).

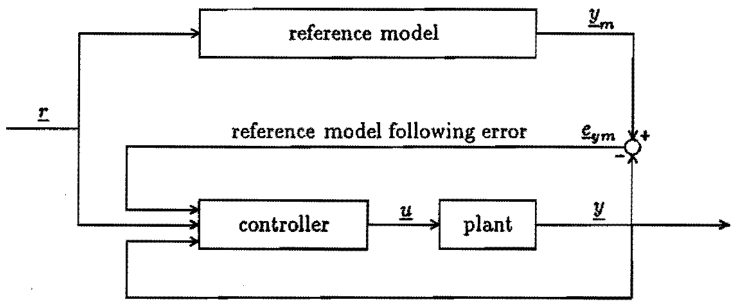


Figure B.4: Feedback control with a reference model.

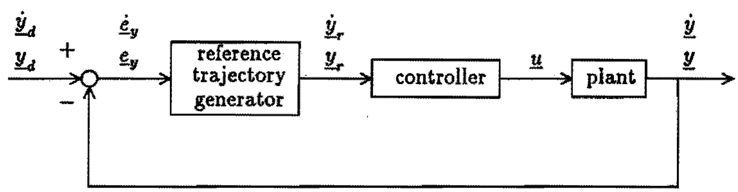


Figure B.5: Feedback control with a reference trajectory generator.

B.3.3 Indirect and direct adaptive control.

In this subsection, attention is focused on an adaptive control scheme, with the controller and the plant (in its inner loop) and an adaptation mechanism to adjust the controller parameters (in its outer loop). In this so-called *parametric adaptation*[1], a parameter adaptation algorithm tunes the control gains and, furthermore, adjusts the plant model parameters on-line if the control law is based explicitly on a plant model. In general, this implies that the adaptive control system has to solve two problems that are closely related but different in approach:

1) the *estimation* or *identification* problem: based on feedback information, the unknown parameters of the plant model must be estimated.

[1]Another form of adaptation is *signal synthesis adaptation*, where an auxiliary input signal is generated to adapt for unknown plant dynamics. Parameter adaptive control schemes can be transformed in control schemes with signal synthesis adaptation, such that the auxiliary input signal combined with the control input of a non-adaptive controller may have the same result as the control input of a parameter adaptive controller.

2) the *control* problem: based on feedback information and the adjusted plant parameters, control actions have to be determined.

These problems are typical for adaptive control and lead to two different approaches:

- Identification and control are considered as separate problems. Firstly, the unknown plant parameters are estimated on-line, using an observer that is based on a known structure of the plant model. Secondly, the controller parameters are adjusted using the current parameter estimates. Examples of this *indirect adaptive control* via *explicit identification* are self-tuning regulator (STR) with adaptive pole placement and STR with adaptive least squares.

- The controller parameter adaptation is performed in one step. The unknown plant parameters are not estimated explicitly by this *direct adaptive control* scheme. Instead, the plant model is parameterized in terms of the controller parameters and the latter are adjusted directly using the measured output. Examples of this control technique, also referred to as *implicit identification*, are the model reference adaptive control (MRAC) and the self-tuning minimum variance regulator.

Sometimes, an appropriate re-parameterization of the plant model in an indirect adaptive control scheme may transform this scheme in a direct adaptive control scheme. An example is the self-tuning minimum variance regulator (Landau, 1979).

The objective of adaptive control is to cope with bounded model uncertainty. However, no general directives are known to choose for an indirect or a direct approach. For chemical processes with complex, poorly known and slowly changing dynamics, indirect adaptive control can be effectively used. On the other hand, for manipulators with unknown parameters, variable payloads and rapidly changing dynamics, the direct adaptive control approach is more appropriate.

In the sequel of this appendix, only two adaptive control methods, notably the *self-tuning regulator* (STR) and the *model reference adaptive control* (MRAC), are considered. Historically, STR is associated with indirect and MRAC with direct adaptive control. Sometimes, MRAC can be seen as a special case of STR if a transformation from identifier parameters to controller parameters is introduced. In both schemes, the inner control loop may be the same, whereas the outer adaptation loop contains either an identification model and/or a reference model.

B.3.4 Self-tuning regulator (STR).

The self-tuning regulator (STR) is a controller which tunes its parameters, using a model that is identified on-line. It was proposed by Kalman (1958) and elaborated by, for instance, Åstrom and Wittenmark (1989). Originally, it was developed for the stochastic minimum variance regulator problem.

Mostly, STR is an indirect adaptive control (see Fig. B.6), consisting of an inner loop with the controller and the plant, and an outer loop with the identifier and the adaptation mechanism. For conditions under which this scheme can be used, we refer to Åstrom and Wittenmark (1989). The adaptation mechanism adjusts the controller

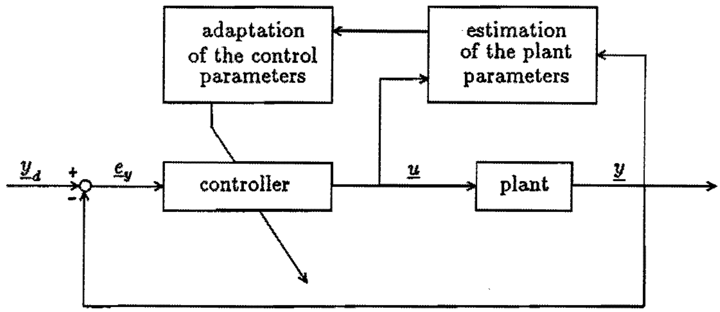


Figure B.6: Self-tuning regulator (STR).

parameters. The approach of STR is to identify first the plant model parameters recursively and then to use these estimates to modify the controller parameters, in order to minimize a criterion function in terms of the output error and the control effort. Various possibilities exist for the estimation method (for instance, least-squares, extended Kalman filtering and maximum likelihood) and for the control technique (for instance, gain-phase margin, minimum variance, LQG and pole placement). Due to this flexibility and due to its ease of understanding and implementation, over the years STR has been investigated in great detail and enjoyed a wide popularity. However, the literature on STR is mostly concerned with stochastic systems and plants operating in discrete time.

B.3.5 Model reference adaptive control (MRAC).

Model reference adaptive control (MRAC) is mostly presented as a direct adaptive control scheme. Most of the early research on MRAC was done in the framework of deterministic, continuous-time servomechanisms. Later, the concept of MRAC is extended to discrete-time systems and to systems with stochastic disturbances. Fig. B.7 gives the basic scheme of a MRAC system, which is referred to as the *parallel* MRAC scheme in order to differentiate it from other possible configurations like series MRAC (e.g. Landau, 1979; Van Amerongen et al., 1984; Åstrom and Wittenmark, 1989).

MRAC can be used for problems in which the desired performance of the closed-loop system is specified in terms of a so-called *reference model.* This model specifies how the plant output should ideally respond to the reference signal $\underline{r}$. The actual performance of the closed-loop system is compared with this desired response by comparing the output $\underline{y}$ of the plant with the output $\underline{y}_m$ of this reference model, both with the same reference input. The error $\underline{e}_{ym} = \underline{y}_m - \underline{y}$ is used to modify

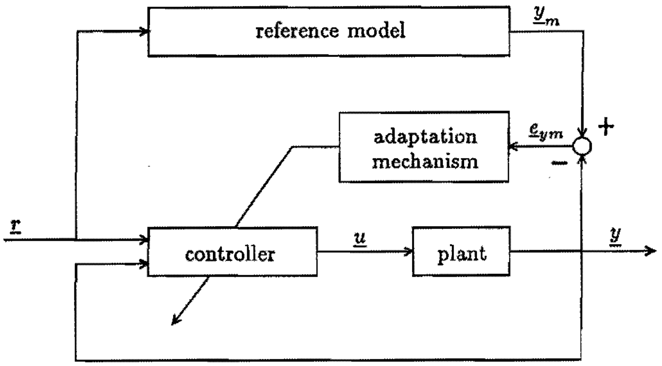


Figure B.7: Model reference adaptive control (MRAC).

the controller parameters directly. Because this modification is based on the output error and because no effort is made for on-line identification, the speed of adaptation can be quite high. However, a key problem in the design of an adaptive controller of this kind is the choice of a suitable adaptation algorithm to adjust the controller parameters.

MRAC applied to rigid manipulators is discussed by e.g. Dubowsky and DesForges, 1979; Landau, 1985; Horowitz and Tomizuka, 1980; Nicosia and Tomei, 1984; Tomizuka et al., 1988 (for a comparison of the proposed MRAC schemes, see ,e.g., Lammerts, 1989). MRAC applied to flexible manipulators is discussed by e.g. Meldrum and Balas, 1986; Siciliano et al., 1986.

B.4 Design of the adaptation algorithm.

B.4.1 Introduction.

The adaptive control system in this thesis attempts to make the states of the manipulator match the desired trajectory, despite partially unknown model parameters $\underline{p}$. An adaptation algorithm in an outer loop adjusts the parameter estimates $\hat{\underline{p}}$, such that the inner control loop can drive the tracking error to zero. The key problem in adaptive control design is to find a suitable adaptation algorithm for $\hat{\underline{p}}$. The first approach in this direction is the *gradient* or *sensitivity approach* (e.g. Åstrom and Wittenmark, 1989; Bulter, 1990). It is based on local minimization techniques, using sensitivity functions to update the controller parameters. This approach heavily relies on the assumption that the unknown model parameters change slowly in comparison with the state variables. This admits a quasi-stationary treatment of the adjusted controller parameters and is essential for the computation of the sensitivity functions. The main disadvantage of this approach is that very little can be said about the resulting closed-loop stability. This inspired the search for systematic procedures to design stable adaptive control systems. Because of the nonlinear, time-varying character of adaptive control systems, linear stability analysis tools cannot be applied, but stability tools for nonlinear dynamic systems like the hyperstability theory of Popov and the second method of Lyapunov have to be used. These methods rely on the error model of the adaptive closed-loop system.

B.4.2 The error model of an adaptive control system.

Let $\underline{x}$ be the state of the considered system and let $\underline{e} = \underline{x}_d - \underline{x}$ be the difference between the desired state $\underline{x}_d = \underline{x}_d(t)$ and the actual state. Furthermore, let $\underline{e}_p = \hat{\underline{p}} - \underline{p}$ be the difference between the available estimate $\hat{\underline{p}}$ and the real value $\underline{p}$ of the parameters. It is assumed that such a control law is determined that, for the case in which the estimates $\hat{\underline{p}}$ coincide with $\underline{p}$, the error equation for the closed-loop system is given by

$$\dot{\underline{e}} = \underline{f}(\underline{e}, t) \quad ; \qquad \underline{f}(\underline{0}, t) = \underline{0} \, , \tag{B.6}$$

where $\underline{e} = \underline{0}$ is a uniformly, globally, asymptotically stable equilibrium point. Furthermore, it is assumed that $V_c = V_c(\underline{e}, t)$ is a Lyapunov function for this error system, i.e. that

$$V_c(\underline{0}, t) = \underline{0} \;; \quad V_c = V_c(\underline{e}, t) \text{ positive definite ,} \tag{B.7}$$

$$\begin{aligned} \dot{V}_c(\underline{e}, t) &= \frac{\partial V_c}{\partial e}\frac{\partial e}{\partial t} + \frac{\partial V_c}{\partial t} = \\ &= \underline{F}^T(\underline{e}, t)\underline{f}(\underline{e}, t) + \frac{\partial V_c(\underline{e}, t)}{\partial t} \text{ negative definite .} \end{aligned} \tag{B.8}$$

Here, $\underline{F} = \underline{F}(\underline{e}, t)$ is the partial derivative of $V_c = V_c(\underline{e}, t)$ with respect to $\underline{e}$.

In general, the parameters will occur in the control law. In the presence of parametric uncertainty, only estimates $\hat{\underline{p}}$ are available and the error equation given earlier will no longer be valid but has to be adjusted with a correction term $\underline{v}$ that can depend on $\underline{x}$, $\underline{x}_d$, $\dot{\underline{x}}_d$, $\underline{p}$ and $\hat{\underline{p}}$, and possibly on t. For a suitable choice of the parameters, it is often possible to achieve that the correction term $\underline{v}$ is proportional to the parameter error $\underline{e}_p$. The error equation for the closed-loop system is now given by

$$\dot{\underline{e}} = \underline{f}(\underline{e}, t) - W(\underline{x}, \underline{x}_d, \dot{\underline{x}}_d, t)\underline{e}_p \, , \tag{B.9}$$

with the same function $\underline{f} = \underline{f}(\underline{e}, t)$ as before.

For the control law that yields the given error equation, now a suitable adaptation algorithm has to be found. The adaptation problem can be stated as follows:

> find such an algorithm for the adjustment of the parameter estimates $\hat{\underline{p}}$ that, with the control law, $\underline{e} = \underline{0}$ is a uniformly, globally, asymptotically stable equilibrium point.

Hence, it is not required that the parameter estimates $\underline{\hat{p}}$ converge to the real parameter values $\underline{p}$.

B.4.3 Design of an adaptation algorithm according to Lyapunov.

The second method of Lyapunov can be used as an expedient to solve the adaptation problem stated above. For this purpose, a Lyapunov function candidate V is introduced, consisting of the function V_c (B.7) plus a positive definite, quadratic term in the parameter error $\underline{e}_p$:

$$V(\underline{e}, \underline{e}_p, t) = V_c(\underline{e}, t) + \frac{1}{2}\underline{e}_p^T \Gamma^{-1} \underline{e}_p \ . \tag{B.10}$$

Here, Γ^{-1} is a constant, symmetric, positive definite matrix. As mentioned, it is assumed that such a suitable control law has been determined that the error equation $\underline{\dot{e}} = \underline{f}(\underline{e}, t)$ is uniformly, globally, asymptotically stable (with the Lyapunov function $V_c = V_c(\underline{e}, t)$). In order to find a suitable adaptation algorithm, again the second method of Lyapunov is used. Therefore, with regard to the error equation $\underline{\dot{e}} = \underline{f}(\underline{e}, t) - W\underline{e}_p$ due to parametric uncertainty, the time-derivative of V is now given by:

$$\begin{aligned} \dot{V} &= \frac{\partial V_c}{\partial e}\frac{\partial e}{\partial t} + \frac{\partial V_c}{\partial t} + \frac{1}{2}\underline{e}_p^T \Gamma^{-1} \underline{\dot{e}}_p + \frac{1}{2}\underline{\dot{e}}_p^T \Gamma^{-1} \underline{e}_p + = \\ &= \underline{F}^T[\underline{f} - W\underline{e}_p] + \frac{\partial V_c}{\partial t} + \underline{e}_p^T \Gamma^{-1} \underline{\dot{e}}_p) = \\ &= \underline{F}^T \underline{f} + \frac{\partial V_c}{\partial t} + \underline{e}_p^T(-W^T \underline{F} + \Gamma^{-1}\underline{\dot{e}}_p) \ . \end{aligned} \tag{B.11}$$

The first two terms on the right-hand side are negative for all $\underline{e} \neq \underline{0}$, because $V_c = V_c(\underline{e}, t)$ is a Lyapunov function for the error system (B.6). To solve the adaptation problem, it is sufficient to cancel the third term for all values of $\underline{e}_p$. This yields the adaptation rule:

$$\underline{\dot{e}}_p = \Gamma W^T \underline{F} \ . \tag{B.12}$$

Therefore, the matrix Γ influences the speed and the behavior of the parameter adaptation.

The resulting relation for $\dot{V}$ with this adaptation rule is given by:

$$\dot{V} = \underline{F}^T \underline{f} + \frac{\partial V_c}{\partial t} \ , \tag{B.13}$$

and hence, $\dot{V}$ is non-positive for all values of $\underline{e}$ and $\underline{e}_p$. However, whereas V_c in (B.8) is negative definite for all values of $\underline{e}$, $\dot{V}$ is not negative definite for all values of $\underline{e}$ and $\underline{e}_p$ because $\dot{V} = 0$ for all $\underline{e}_p$ if $\underline{e} = \underline{0}$. It can be shown in this case (e.g. LaSalle and Lefschetz, 1961; Slotine and Li, 1991), that both $\underline{e}$ and $\underline{e}_p$ are bounded, that $\underline{e}(t)$ converges to $\underline{0}$ for $t \to \infty$, and that the parameter error converges to a constant value $\underline{e}_{p\infty}$ for $t \to \infty$. Convergence of $\underline{e}_p(t)$ to $\underline{0}$ for $t \to \infty$, and hence of the parameter estimates $\underline{\hat{p}}(t)$ to their true values $\underline{p}$, will occur if the input signals are persistently exciting (e.g. Narendra and Annaswamy, 1989; Slotine and Li, 1991).

B.4.4 A link with the hyperstability theory of Popov.

Popov's hyperstability theory has been extensively used in the design of MRAC systems (e.g. Landau, 1979) and for manipulator control (e.g. Horowitz and Tomizuka, 1980). Landau (1979) showed that this stability theory leads to more flexibility in adaptive control design than the second method of Lyapunov, because the problem to find a suitable Lyapunov function can be replaced by two conditions, namely the positivity and passivity condition, to be satisfied separately.

In Appendix A, it is shown for non-adaptive control design, that the positivity condition for the linear compensator D of the feedforward block (A.8) corresponds with the condition (A.6) for the matrix P of the Lyapunov function (A.5). However, this is sufficient for a stability proof only if the input $\underline{z} = \underline{0}$. In case of parametric uncertainty, $\underline{z} \neq \underline{0}$ and the Lyapunov function (A.9) consists complementary of an integral that has to satisfy the Popov integral inequality (A.12). This means that the adaptation algorithm has to be designed such that the second term in the Lyapunov function represents a passive mapping from the input $\underline{\epsilon}$ to the output $\underline{w} = -\underline{z}$ of the feedback block of the "standard error system". In contradiction to the quadratic expressions commonly chosen for the Lyapunov functions (such as in Eq (B.10)), the Popov integral inequality in combination with the positivity condition leads, in fact, to an infinite number of solutions, which probably justifies the pronouncement of "hyper"stability in the theory of Popov. Hence, the hyperstability theory of Popov results in a greater flexibility in the choice of adaptation algorithms (e.g. Landau, 1979; Horowitz and Tomizuka, 1980) rather than applying the second method of Lyapunov. Unfortunately, Landau and others do not clearly discuss the different possible adaptation algorithms in comparison with each other, and the problem to make an optimal choice between them is not in the scope of this work. It can easily be shown, that the adaptation algorithm (B.12), obtained via Lyapunov's second method, is one of the possible solutions to the Popov integral inequality (A.12). Although the Popov integral inequality in fact offers more possible solutions for an adaptation algorithm, the adaptation rule derived according to a quadratic-like Lyapunov expression is commonly used in practice and is used in this thesis, too. For more details about the second method of Lyapunov as well many other stability methods, we refer to Narendra and Annaswamy (1989) and especially to Slotine and Li (1991) in considering nonlinear control schemes for manipulators.

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Summary.

This thesis presents a new control law for manipulators with joint and link flexibility. An adaptive version of this law can be applied in case some of the manipulator parameters are unknown.

A manipulator can be modeled as an open chain of links, interconnected by joints. This mechanism, which is driven by actuators at the joints, has to move an object at the end-effector along a prescribed trajectory in space. Until now, manipulator control has been based on the assumption that the drives are stiff and that the links behave as rigid bodies. Most of today's manipulators are indeed stiff (and heavy) to minimize undesirable elastic deflections at the expense of low operational accelerations and high energy consumption. For higher operational accelerations, lightweight manipulators are required to reduce the driving torques. However, the links of a lightweight manipulator may deform elastically, while elasticity may also exist in the transmissions between actuators and links (joint elasticity). The elastic deformations and vibrations during rapid movements decrease the tracking accuracy and can cause instability if the controller is designed under the assumption of perfect rigidity. In order to improve the performance of the manipulator, a more complex control design model that accounts for the flexibilities is required. The control law must guarantee global, asymptotic stability for the closed-loop system. Because parametric uncertainty (such as a variable payload, poorly known inertias and friction parameters that vary slowly in time) is one of the main sources of model uncertainty that endanger stability, the overall objective in this research project is to design a control scheme for manipulators that takes into account flexibility in both joints and links as well as parametric uncertainty.

The main problems in controlling a flexible manipulator are that the number of actuators is smaller than the number of degrees of freedom, and that the desired trajectories of the generalized coordinates cannot all be determined uniquely from the desired end-effector trajectory. Consequently, in comparison with a rigid manipulator, the control objective with regard to a flexible manipulator must be reformulated. The same motion objective must be achieved, i.e. asymptotic trajectory tracking of the end-effector, and the vibrations in the drives and the links must be restricted to an acceptable level. Because of the complexity of the equations of motion for manipulators with flexibility in both joints and links, usually either joint or link flexibility is considered in literature, or, alternatively, simple configurations, such as single-link manipulators, are studied. Many different control approaches were proposed for flexible manipulators. In many cases, they attempt to realize trajectory tracking by actively damping the vibrations as well as possible and to control the resulting rigid-like manipulator dynamics. However, this always implies a compromise between tracking accuracy and damping of the deflections, and will probably never result in exact end-effector trajectory tracking. Two essentially different approaches

in control theory for flexible manipulators are the input-output linearization method and the singular perturbation method. The input-output linearization method is computationally expensive and requires an accurate model of the manipulator. The singular perturbation method can cope with parametric uncertainty by means of parameter adaptation, but is limited to manipulators with small flexibilities only. Although various approaches have been presented in literature to solve (a part of) the control problem for flexible manipulators in the presence of parametric uncertainty, this problem is still a challenge.

A well-known approach to control rigid manipulators is computed torque control (CTC), which compensates for nonlinearities and guarantees a desired closed-loop behavior. If manipulator parameters are unknown, the CTC scheme can be modified to adapt the unknown model parameters on-line. However, this method assumes that the number of actuator inputs equals the number of degrees of freedom. If flexibilities are significant, (adaptive) CTC results in a loss of tracking accuracy and can even cause instability. Therefore, if a high performance is required, a modification is needed to account for flexibility and to guarantee stability as well as boundedness of the elastic deformations.

Starting from the idea of CTC for rigid manipulators, a new control scheme for flexible manipulators is designed in this thesis. This so-called *computed reference computed torque control* (CRCTC) scheme is based on a model that takes concentrated joint flexibility and/or distributed link flexibility into account. Because the desired trajectories of the generalized coordinates cannot all be determined uniquely from the desired end-effector trajectory, a desired or reference trajectory is determined on-line for those generalized coordinates that no desired trajectory is known for beforehand. Such inputs are computed that asymptotic tracking of the end-effector trajectory is realized while deformations remain bounded. Simulations and experiments have shown that joint-level control of flexible manipulators leads to satisfying results. For direct control of the end-effector, numerical problems arise if the differential equation from which the unknown reference trajectory has to be solved on-line is unstable. This will have to be studied in more detail in future. For the moment, an approximate solution has been found by in the off-line computation of a desired trajectory for the flexible coordinate(s). It is important to search for a numerical procedure that can compute the unknown reference trajectory on-line, as this would enable the use of adaptive CRCTC in the case of parametric uncertainty. In cases where the a priori unknown references can be computed on-line, adaptive CRCTC leads to an improved tracking performance of the manipulator in comparison with non-adaptive CRCTC and (adaptive) CTC. Because the full state must be available for feedback purposes, the manipulator must be equipped with additional sensors for the end-effector position and/or elastic deformations. In the experiments, a one-step prediction-type Kalman filter has been used to reconstruct the unknown velocities. However, it is advisable to design a more accurate velocity observer for future research.

Simulations and experiments have shown that the inclusion of flexibilities in the control model improves the tracking performance in comparison with a controller that is based on a rigid model. Although the number of degrees of freedom exceeds the number of control inputs, global, asymptotic stability of the closed-loop system is ensured in theory for any stiffness values of both joints and links. For this case in which the extra degrees of freedom are the result of flexibilities, the CRCTC law is composed of a computed torque control part for the equivalent rigid manipulator and a corrective term to compensate for the effect of flexibilities. This corrective term stabilizes the elastic deflections around the computed references and becomes small in case of small flexibilities. However, in case of small flexibility, the corrective term can be a high-frequency signal, which is in fact unacceptable for practical implementation. Then, either that flexibility has to be neglected in control design or the very high-frequency signals in the CRCTC inputs have to be eliminated by means of filtering. One should investigate, for which magnitudes of the stiffnesses this has to be carried out in practice and whether this is an acceptable solution.

Although some numerical problems have been encountered in the implementation of adaptive CRCTC, suitable numerical procedures were found to solve them and satisfying results have been obtained in simulations and experiments. In conclusion, adaptive CRCTC is found to be a promising control technique for mechanical systems with more degrees of freedom than inputs and with uncertain parameters.

Samenvatting.

In dit proefschrift wordt een nieuwe regelwet beschreven voor manipulatoren met flexibiliteit in de aandrijvingen en de schakels. Indien sommige parameters van de manipulator onbekend zijn kan deze regelwet in adaptieve vorm worden toegepast.

Een manipulator kan worden gemodelleerd als een open keten van schakels met aandrijvingen in de verbindingen. Het doel is om met de grijper van de manipulator een object langs een voorgeschreven baan in de ruimte te bewegen. Tot dusver werd de regeling van manipulatoren meestal gebaseerd op de veronderstelling dat de aandrijvingen onvervormbaar zijn en dat de schakels ten opzichte van elkaar bewegen als starre lichamen. Inderdaad zijn de hedendaagse manipulatoren veelal stijf (en zwaar) gecontrueerd om ongewenste vervormingen en trillingen te minimaliseren met als gevolg lage versnellingen en een hoog energieverbruik. Om met evengrote of kleinere stuurkrachten te komen tot hogere versnellingen zijn lichtgewicht manipulatoren vereist. Vooral tijdens versnelde bewegingen kunnen de schakels van lichtgewicht manipulatoren doorbuigen en kunnen er vervormingen optreden in de aandrijvingen. Als de regelaar is ontworpen in de veronderstelling dat de manipulator stijf is, kan dit leiden tot een verminderde volgnauwkeurigheid en zelfs tot instabiliteit. Voor een betere prestatie van de manipulator is een complex regelmodel benodigd dat rekening houdt met de flexibiliteiten. De regelwet dient zodanig ontworpen te worden dat globale, asymptotische stabiliteit van het geregelde systeem gegarandeerd is. Onzekerheid in de parameters van de manipulator is één van de belangrijkste modelonzekerheden die de stabiliteit van het geregelde systeem in gevaar brengen. Voorbeelden van parametrische onzekerheid zijn een variabele massa van het object dat door de grijper verplaatst wordt, traagheden die slecht bekend zijn en wrijvingsparameters die langzaam veranderen in de tijd. Het hoofddoel van dit onderzoeksproject is het ontwerp van een regelwet die zowel flexibiliteit in de manipulator als parametrische onzekerheid in rekening brengt.

De voornaamste problemen bij het regelen van flexibele manipulatoren zijn toe te schrijven aan het feit dat het aantal motoren kleiner is dan het aantal vrijheidsgraden en aan het feit dat het niet mogelijk is om op voorhand uit de gewenste baan voor de grijper een gewenste trajectorie te bepalen voor elk van de gegeneraliseerde coördinaten. Daarom moet het regeldoel voor flexibele manipulatoren in vergelijking tot stijve manipulatoren opnieuw worden geformuleerd: er dient eenzelfde volggedrag te worden gerealiseerd, namelijk het asymptotisch volgen van de gewenste baan door de grijper, terwijl bovendien de vervormingen en trillingen begrensd moeten blijven. Daar de bewegingsvergelijkingen voor manipulatoren met flexibiliteit in zowel aandrijvingen als schakels complex zijn, worden in de literatuur vaak alleen flexibiliteiten in ofwel aandrijvingen ofwel schakels beschouwd. De bestaande regelconcepten voor flexibele manipulatoren zijn divers en meestal toegespitst op eenvoudige configuraties (bijvoorbeeld één schakel). Veelal wordt gestreefd naar een regeling die de defor-

maties zoveel mogelijk tracht te elimineren, zodat de manipulator als star mechanisme beschouwd kan worden. Dit betekent dat er een compromis gezocht wordt tussen de volgnauwkeurigheid en het elimineren van trillingen met als gevolg dat de grijper de gewenste baan niet exact zal volgen. Twee wezenlijk verschillende methoden zijn de 'input-output feedback linearization' methode en de 'singular perturbation' methode. De eerste methode is rekenintensief en vereist een zeer nauwkeurig model van de manipulator. De tweede methode kan wel overweg met modelonzekerheid o.a. door middel van parameteradaptatie, maar kan slechts worden toegepast op manipulatoren met kleine vervormingen.

Een bekend regelconcept voor stijve manipulatoren is 'computed torque control' (CTC), waarmee niet-lineariteiten gecompenseerd worden en het gewenste systeemgedrag gegarandeerd wordt. Indien er onzekerheid ten aanzien van modelparameters in rekening moet worden gebracht, kan deze regeling in adaptieve vorm de onbekende parameters on-line aanpassen. Helaas is deze methode gebaseerd op de veronderstelling dat het aantal motoren gelijk is aan het aantal vrijheidsgraden. Bij aanzienlijke vervormingen resulteert de (adaptieve) CTC-methode in een verlies aan volgnauwkeurigheid en kan zij instabiliteit tot gevolg hebben. Voor een betere prestatie dient het regelschema zodanig veranderd te worden dat flexibiliteiten mede in beschouwing worden genomen teneinde stabiliteit te kunnen garanderen en de vervormingen begrensd te houden.

Uitgaande van het idee van CTC voor stijve manipulatoren wordt in dit proefschrift een nieuw regelschema ontworpen voor flexibele manipulatoren. Dit zogenaamde 'computed reference computed torque control' (CRCTC) schema is gebaseerd op een model van de manipulator met daarin expliciet verdisconteerd de flexibiliteit in aandrijvingen en/of schakels. Daar het niet mogelijk is om uit de gewenste baan voor de grijper een gewenste trajectorie te bepalen voor elk van de gegeneraliseerde coördinaten, wordt er voor die gegeneraliseerde coördinaten waarvoor op voorhand geen gewenste trajectorie bekend is on-line een gewenste of *referentie*-trajectorie bepaald. De regelingangen worden zodanig berekend dat de grijper asymptotisch de gewenste baan volgt terwijl de deformaties begrensd blijven. Simulaties en experimenten met deze regelwet hebben geleid tot bevredigende resultaten. Indien de regelwet expliciet beoogt om de grijper de gewenste baan te laten doorlopen, treden er numerieke problemen op wanneer de differentiaalvergelijking waaruit de onbekende referentietrajectorie on-line moet worden opgelost instabiel is. Dit fenomeen zal in de toekomst nader onderzocht moeten worden. Vooralsnog is een oplossing gevonden waarbij voor de flexibele coördinaten een gewenste trajectorie off-line berekend wordt. Gezocht moet nog worden naar een numerieke procedure die de onbekende trajectorie on-line kan berekenen, daar dit het gebruik van adaptieve CRCTC ingeval van parametrische onzekerheid mogelijk zou maken. In die gevallen waarin de onbekende referenties wel on-line kunnen worden bepaald, leidt adaptieve CRCTC tot een betere prestatie dan niet-adaptieve CRCTC en (adaptieve) CTC. Daar alle toestandsvariabelen beschikbaar moeten zijn voor de terugkoppeling, dient de manipulator met extra sensoren te worden uitgerust voor het meten van de positie van de grijper en/of van de deformaties. Tijdens de experimenten is een Kalman-filter gebruikt voor het schatten van de snelheden, doch het verdient aanbeveling in de toekomst een nauwkeuriger reconstructie-algoritme te ontwerpen.

Simulaties en experimenten hebben aangetoond dat het opnemen van de flexibiliteiten in de regeling de prestaties verbetert in vergelijking tot een regeling die uitgaat van een stijve manipulator. Ondanks het feit dat het aantal vrijheidgraden groter is dan het aantal regelingangen, is globale asymptotische stabiliteit in theorie gewaarborgd voor willekeurige waarden van de stijfheid van aandrijvingen en schakels. In dit geval waarin de extra vrijheidsgraden het gevolg zijn van flexibiliteiten, bestaat CRCTC uit een CTC-term voor de overeenkomstige stijve manipulator en een correctieterm die het effect van de flexibiliteiten compenseert. Deze correctieterm stabiliseert de vervormingen rondom de berekende referenties en wordt klein in geval van kleine vervormingen. Helaas kan deze correctieterm bij een hoge stijfheid hoogfrequent zijn en daardoor in feite niet geschikt voor praktische implementatie. In dat geval zal ófwel die flexibiliteit verwaarloosd moeten worden bij het ontwerp van de regeling ófwel een filter de hoog-frequente signalen in de CRCTC-ingangen moeten elimineren. Onderzocht moet worden bij welke stijfheden deze actie noodzakelijk is en of hiermee een bevredigend resultaat verkregen wordt.

Ondanks het optreden van enkele numerieke problemen bij het implementeren van adaptieve CRCTC, zijn er met geschikte numerieke procedures bevredigende resultaten behaald in simulaties en experimenten. Tot besluit kan worden geconcludeerd dat adaptieve CRCTC goede perspectieven biedt voor het regelen van mechanische systemen met meer graden van vrijheid dan ingangen en met onbekende parameters.

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Ivonne

Curriculum vitae.

Ivonne Lammerts was born on July 17th, 1966 in Geldrop, the Netherlands. In her childhood, she has lived successively in Istanbul, Turkey, and in Addis Abeba, Ethiopia. Back in the Netherlands, she settled with parents, brother and sister in Veldhoven, nearby Eindhoven. Between 1977 and 1984, she went to the grammar school Anton van Duinkerkencollege, which is situated next to the congress center Koningshof in Veldhoven. At that time, of course, she did not know that she would later return to that place attending meetings as an engineer... In September 1984, she started her study in Mechanical Engineering at Eindhoven University of Technology. In June 1989, she received her M.Sc. degree in Mechanical Engineering on a project on adaptive control of rigid manipulators. She gratefully accepted the opportunity to continue the research within the scope of a Ph.D. project. The results of this project on the adaptive control of flexible manipulators are reported on in this thesis.

STELLINGEN

behorende bij het proefschrift

Adaptive Computed Reference Computed Torque Control of Flexible Manipulators

1. Het besturen van mechanische systemen door open sturing of door terugkoppeling van de motorcoördinaten vereist een stijf geconstrueerd systeem. Het gevolg hiervan is veelal een nodeloos lage verhouding van nuttige en totale last.

2. De toenemende mogelijkheden van soft- en hardware zijn mede aanzet tot onderzoek naar regelconcepten die rekening houden met ongewenste deformaties in de constructie van mechanische systemen. Toepassing van dergelijke concepten zal in de toekomst leiden tot lichter geconstrueerde en daarmee tot goedkopere en snellere machines en manipulatoren *(dit proefschrift)*.

3. Het opnemen van de te regelen uitgangsgrootheden als vrijheidsgraden in het regelmodel van flexibele manipulatoren vereenvoudigt de implementatie van de regeling *(dit proefschrift)*.

4. Het veel gebruikte model van Spong voor manipulatoren met flexibele overbrengingen is hierop niet algemeen van toepassing *(dit proefschrift)*.

5. De ontwikkeling van geavanceerde regelconcepten voor niet-lineaire mechanische systemen heeft de regeltechniek op een hoger plan gebracht, waarvan andere disciplines nu dankbaar gebruik kunnen maken.

6. Het in de lineaire systeemtheorie nuttig gebleken begrip "bandbreedte" wordt bij gebrek aan beter ook toegepast op niet-lineaire systemen. Daaruit blijkt de behoefte aan een met bandbreedte equivalent begrip voor niet-lineaire systemen.

7. De benaming "fuzzy control" is suggestief en in feite onjuist.

8. De positieve rol van techniek in de maatschappij is als vanzelfsprekend doch veelal onzichtbaar. De maatschappelijke appreciatie van techniek kan worden vergroot door enthousiasme te creëren voor de creatieve en praktische aspecten van de techniek en door inzicht te geven in mechanismen en processen opdat de techniek geen "black box" meer is.

9. De bewering dat de malaise en puinhoop in Afrika uitvloeisels zijn van het koloniale tijdperk is achterhaald, o.a. door het feit dat vele Afrikaanse landen, tientallen jaren nadat ze onafhankelijk zijn geworden, er in economisch opzicht slechter aan toe zijn dan toen.

10. De ontwikkeling van de taal voltrekt zich goeddeels langs de lijn van fouten, die gaandeweg als correct Nederlands geaccepteerd worden *(uit "Schrijven is schrappen" van Godfried Bomans, De Boekerij bv, Amsterdam, 1988).*

11. Een korte termijn voordeel weegt veelal zwaarder dan een lange termijn nadeel.

Ivonne Lammerts
Eindhoven, 28 september 1993