Numerical calculations for the vibrations of a kettledrum shell

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REPORT IWDE 89-11

NUMERICAL CALCULATIONS FOR THE
VIBRATIONS OF A KETTLEDRUM SHELL

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October 1989

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Arbeitsgruppe Technomathematik)

October 1989

CONTENTS

I. SUMMARY

In this report we will give a short description of a kettledrum and of its mechanical vibrations. By reducing it to the basic parts we develop a model, so that we can make use of some FE-packages we have access to. These FE-packages yield results which contain the displacements of the occuring modes and its eigenfrequencies. Since we know something about the properties of our model, we can transfer the results to our problem, the vibrations of a kettledrum.
This investigations and some simple examples of FE-program applications are contents of this report.

II. INTRODUCTION

We describe now some attempts to analyse the acoustical and vibrational properties of a kettledrum by means of available finite element computer packages. The work is part of a bigger project, currently going on, for 'Adams BV Paukenfabriek, Thorn'.

The acoustics of a kettledrum is a rather complex problem (as a matter of fact not even completely understood), and by no means solvable in the two months available for the present work. We think, however, that we were able to explore a few possible approaches with already useful results.

In the following we will describe: a) the physics of the vibrations of the wall and influences from outside; b) our experience with SYSNOISE (a package providing the completely coupled description of mechanical vibrations and sound); c) the exercises with DYNOPT (a FE program dedicated to analyze vibrations of a bell shaped cylindrically symmetric elastic object without airloading); d) a listing and discussion of the results.

In the APPENDICES examples of input and output files are provided for possible future use.

III. ACOUSTICAL AND VIBRATIONAL PROPERTIES OF A KETTLEDRUM

III.1. Description Of A Kettledrum

First we want to give a short description of the kettledrum, i.e., the basic constituting parts, the geometry and the dimensions, to get a picture what we will talk about. To describe the whole musical instrument in detail is not only too complicated but it is for the moment also not necessary because there are only few parts of the instrument which are really of importance in our following investigations.

There are mainly two things one needs to get a drum: a vibrating surface and a resonance volume. The resonance volume of the kettledrum is a big kettle made of thin copper with a shape which can be described by a combination of a truncated cone shell with a truncated sphere shell put together. A typical height is about 0.482m, the maximal radius 0.645m. At the edge there is fixed a steel ring giving the conical part of the kettle more stiffness and increasing the height to a total of 0.524m. Over that ring the membrane, usually made of calfskin, is stretched within a larger outer ring. Its tension and so the tone can be changed with the help of a pedal.
This is the 'active part' of the kettledrum and has to be supported by a frame which will not be considered further in this report. But it should be mentioned that it may very well affect the quality of the sound. This is known from tests made in the ADAMS-factory.

III.2. Mechanical Vibrations

In this chapter we will describe heuristically the free mechanical vibrations occuring in the copper shell of the kettle. Also the influences of air and of the chosen boundary conditions shall be considered.

Since the kettle is made of very thin copper, shell theorie can be used, where the curvature is mainly responsible for the stiffness and so for the vibrating possibilities in that direction. The differences of curvature and the conclusions in the two connected parts of the kettle are shown in the following table:

		conical part	spherical part
curvature parallel to z-axis		0	const. > 0
	=> stiffness	low	high
allowing axial	=> frequency	low	high
modes with	=> displacements	large	small
curvature perpendicular to z-axis		proportional to 1/z	const. > 0
	=> stiffness	medium	high
allowing circum-	=> frequency	medium	high
ferential modes	=> displacements	medium	small

Since we have a curved shape (shell), no distinction between transversal and longitudinal waves can be made. For example, longitudinal type vibrations at the edge propagate to the bottom where they become out of plane and therefore of transversal wave type. This makes it hard to order the possible modes other than by their circumferential periodicity. There are two kinds of nodal lines which we can recognize and name very easily:

a) parallel to the z-axis; they are nodes of the circumferential part of the mode in the r,phi-plane
b) along a meridian, i.e., nodes in the r,z-plane

Due to the axialsymmetry the number of circumferential nodes is just the circumferential mode number, and independent of any problem parameter. Circumferentially, the modes are orthogonal. On the other hand, this is not the case in the z-direction. Per circumferential mode number, we may count a mode by its number of z-nodes, but this is not a fixed number when parameters are varied.

And every of these modes, and so its eigenfrequency, do not only depend on the geometry, but on the three following things, too:

- The material of the kettle : it is determined by Young's modulus E, Poisson's ratio v and mass density rho

- Influence of air : two effects are here worth to mention:
  - effect of airloading: The inertial of the air that sloshes incompressibly back and forth lowers the frequencies of the vibration modes Internal friction of the air gives rise to damping
  - effect of radiation : Depending on mode shape and frequency the vibrations radiate more or less efficiently sound. This loss of energy is effectively a kind of damping

- Influence of boundary conditions : The high stiffness of the steelring effectively fixes the edge of the kettle, i.e., prevents it from vibrating in any direction. This increases the frequencies of the vibrations.

III.3. Description Of The Model:

The present paragraph introduces a model to describe, predict and understand the vibrations of the kettle considered above. For this model it is obviously important to include all these aspects which are essential to calculate anything qualitative and then to try to include additional aspects to get useful quantitative results. As said before, we were here interested in the mechanical vibrations of the kettle, so we started to ignore the membrane (which has a rather loose coupling with the kettle) and to ignore the air effects. These air effects are important in their reducing the frequency and radiating energy as sound, but since the material (copper plate) is relatively heavy compared to air, these effects are probably only of secondary importance.

As a first step we handled the kettle - in view of its thinness and its shape - as a semi-sphere shell with given dimensions and material data. Then there are already a lot of assumptions inclu-ded like shelltheory, axisymmetry, uniform material properties etc. Then we improved our model by replacing the semi-sphere shell by a sphere-cone shell combination, which describes the real shape better. The steelring which has very high stiffness allows no displacements in r,phi and z-direction, what we take into account by setting boundary conditions at the edge.

This model is now hopefully good enough to get results which are not very far away from reality, and is yet simple and easy to adapt to similar axisymmetric bodies.

III.4. Numerical Calculation Of Frequencies With FEM-Programs

In this chapter we will introduce different FEM-packages which allow the numerical calculation of frequencies of some models. Each of the following programs is developed to handle special geometries - each in an own manner - and so they are of different use for us:

- The SYSNOISE package is at time the only commercially available program providing the completely coupled description of mechanical vibrations and sound. Elements may consist here of zero thickness.

- The SEPRAN program is a FE-package containing a set of modules which makes it very flexible, effective and user friendly. The approach is, however, very general. No use can be made of axisymmetry. Infinite domains (to include airloading) are difficult to portray.

- The DYNOPT program is dedicated to analyze vibrations of a bell shaped cylindrically symmetric elastic object of finite thickness. Symmetry is utilized by using elements consisting of circumferential Fourier components.

It is of main importance to know how a program works and what kind of modules are implemented, so that one knows what may be expected and what not. For this reason a short description of the 'Run' and how we used the possibilities of each program follows.

III.4.1. SYSNOISE

In SYSNOISE the BEM (= Boundary Element Method) and the FEM (= Finite Element Method) are implemented.
The BEM is used to model both interior and exterior acoustic problems, and variational and direct collocational BEM's have been employed.
The FEM is used for interior acoustic problems as well as for structural modelling applications.
When we had the opportunity to test SYSNOISE, already the testrun of our most simple model broke down by a mathematical error in the modal analysis part (air was said to be modelled as incompressible), and it was impossible for us to repair it. So we can't use SYSNOISE's modal analysis option. A possible alternative could be via a frequency scan of 'direct response' calculations, but this appeared to be very inefficient and time consuming.
But in the future, when this error is repaired, SYSNOISE may yield useful results. Furthermore, presentation and input/output handling are very easy and impressive.
In case of just a vibrating membrane fixed at the edge SYSNOISE yields right results and offers colorful pictures which are easy to interpret.

III.4.2. SEPRAN

SEPRAN has the possibility to calculate mechanical vibrations, but without airloading effects. However, an approximate air-loading by effective mass increase may in some cases be good enough.

SEPRAN has already been used to calculate eigenfrequencies of membranes with an inhomogenity. These are only 2-D objects built up of 2-D elements, but it is possible to apply SEPRAN to 3-D objects with 3-D elements, too. We tested the possibilities of SEPRAN by calculating the eigenfrequencies and eigenvectors of a membrane with an inhomogenity out of the center. The input file for this problem, the description of the geometry, is rather simple (see APPENDICES); the calculation time for the FE-discretization and for getting the eigenvectors is in the order of minutes on the VAX.

III.4.3. DYNOPT

The DYNOPT program developed at the Mechanical Engineering department of TU Eindhoven is used already for some years to calculate and optimize eigenfrequencies of bells, so there is a lot of experience with this program. This was rather helpful for us, so that we could apply DYNOPT to our problem rather quickly.

Since DYNOPT gave up to now the best results, we will describe here, in the interest of future use, its use in detail.
So how does DYNOPT work ? After starting the program, first the names of input and output file are required. The input file contains a complete description of the problem. Then the assembling of the elements must be done, i.e., the mass- and stiffness matrices for the actual mode are built. The problem to be solved then is to find solutions of the following homogenous differential equation:

$$M \, \ddot{\underline{q}}_m + K_m \, \underline{q}_m = \underline{0} \qquad , \; K_m \, , \; M \text{ matrices}$$

substituting $\underline{q}_m := \underline{u}_m e^{i(w_m t - m\theta)}$ yields to the EVP

$$(w_m^2 M + K_m) \, \underline{u}_m = \underline{0} \; .$$

So the solutions we are interested in is the null- space of the operator $(w_m^2 M + K_m)$. Usually, its dimension is equal to the size of these matrices. Most interesting are the first few w_m . This equation can be solved and leads to a series of corres-sponding results $(w_m, \underline{u}_m)$. This procedure, even the building of the stiffness matrix, must be repeated for every new m, because the DE to solve varies with m (3).

In spite of the great efficiency obtained by splitting up the problem in m components, still all this needs a lot of calculation time. The DYNOPT program, which is installed on the APPOLLO-computer system, has the following options:

- optimization of frequencies
- experimental design
- tone curve computation

Meanwhile there exists an extended version of DYNOPT on the ALLIANT-computer system which decreases the calculation time by a factor of about 60. This version requires an extended input file and can then calculate also stresses on the vibrating object.

In an example of an input file and of an output file one sees what is necessary to know. This data can be changed and calculated easily by a driver program, and if the whole description of the shape is included in the declaration part, it is easy to adapt the driver program and so the DYNOPT program to different shapes.

III.5. Results And Discussion

We want to sum up now the results we obtained from the FE packages and try to explain how they are to understand. In case of the model of our kettledrum the results we obtained from the DYNOPT program help us to understand the physical interactions occuring in a vibrating shell.

As mentioned before, the SYSNOISE program was not very useful for us and did not yield any result, because it broke down by an error.

The results we obtained from the testrun of the problem mentioned in III.4.2. with the SEPRAN program is shown in Fig 1. One can see that the modes with at least one diametric nodal line split up into two different modes with frequencies close together. One mode has the inhomogenity on a nodal line, at the other it is vibrating up and down. These are two different motions with different eigenfrequencies. In case of a homogeneous membrane, they coalesce into one.

With the DYNOPT program we had two different testruns:

- We modelled the kettle as a semisphere with free edge by 8 elements (= 43 nodes). The result was, that to every m-mode there belongs a very low first frequency (12 and 14 Hz), which is due to the free edge, which has a very low stiffness in r-direction and so can easily vibrate that way. On the other hand, the second frequency is very high (680 and 1560 Hz), which indicates a high stretching stiffness of the shell in r- and z-direction near the bottom.

  Pictures of the first six modes and the original shape can be seen in Fig. 2. The displacements are added to the original coordinates with an amplitude large enough to see what happens. The input and output files are listed in the APPENDICES, so an other user may find some helpful remarks.

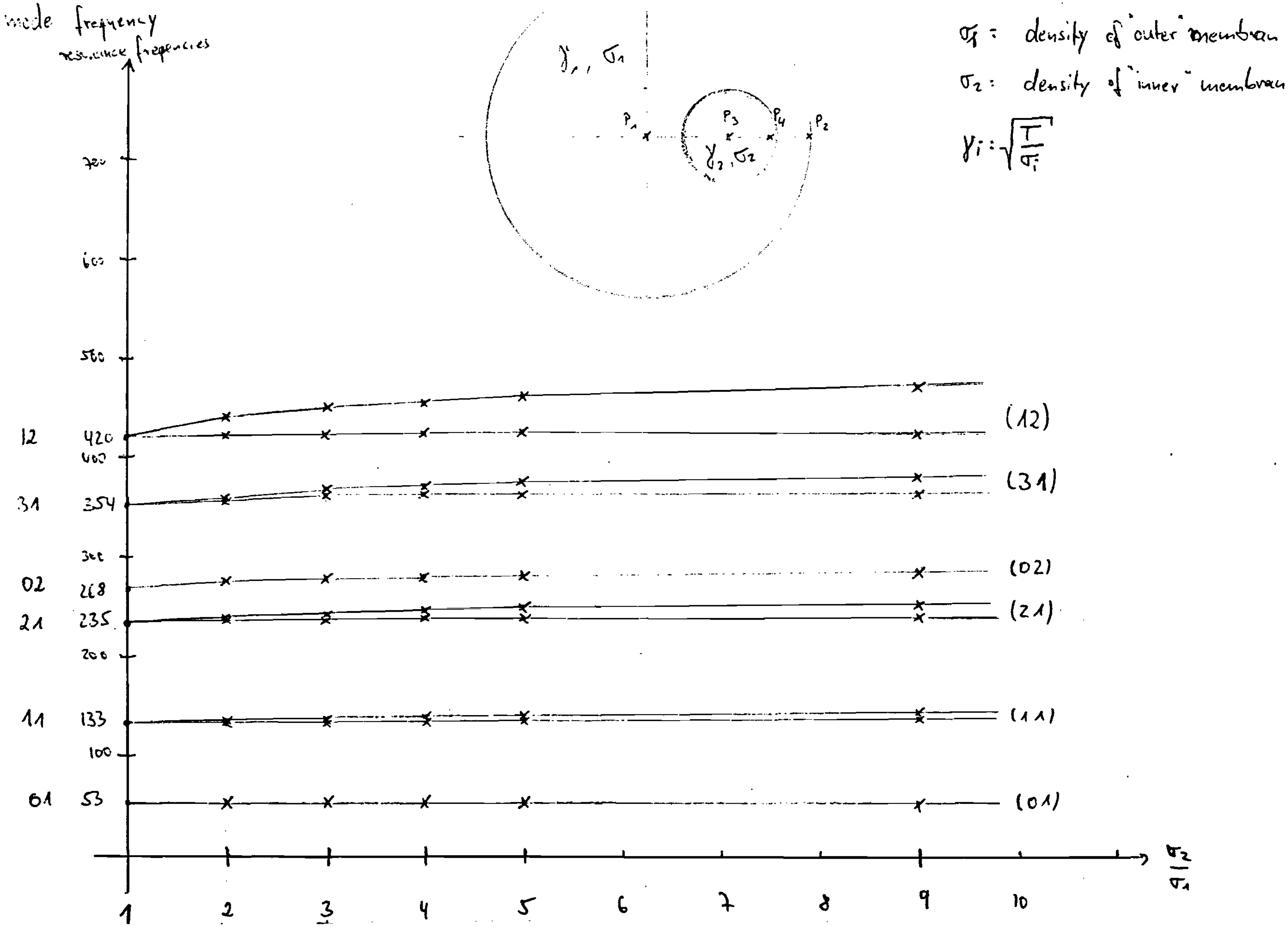
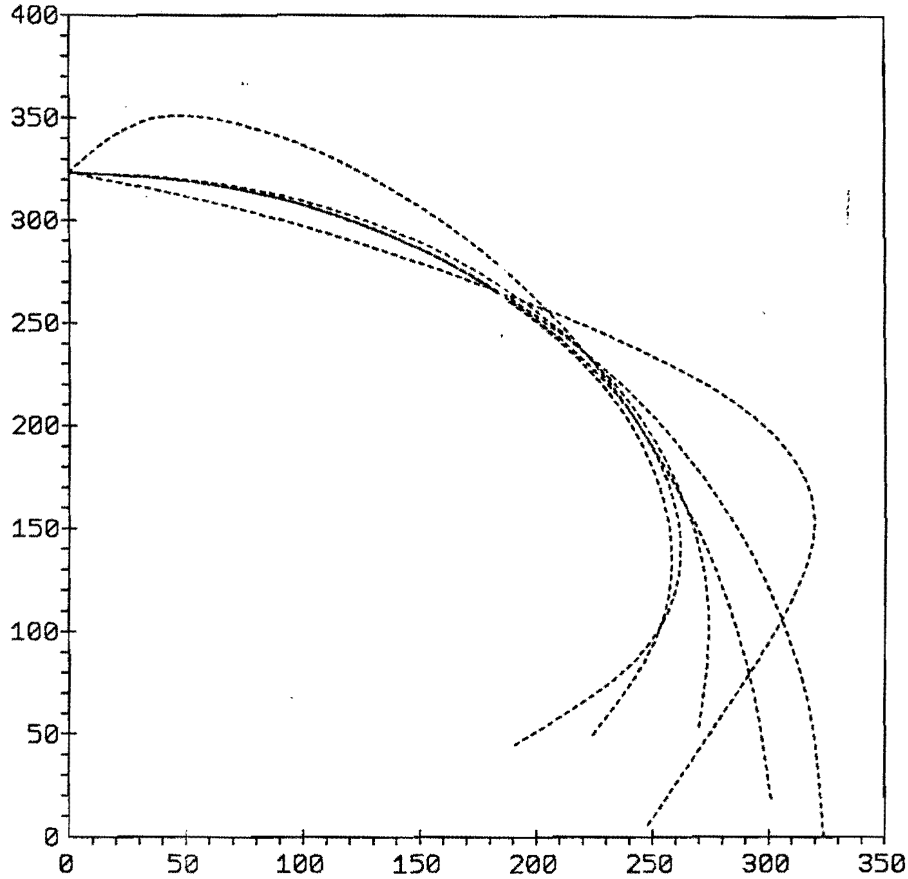

mode frequency
resonance frequencies
σ1 : density of "outer" membran
σ2 : density of "inner" membran
$\gamma_i = \sqrt{\frac{T}{\sigma_i}}$
γ_1, σ_1
γ_2, σ_2
P1
P2
P3
P4
(12)
(31)
(02)
(21)
(11)
(01)
12
31
02
21
11
01
420
354
268
235
133
53
$\frac{\sigma_2}{\sigma_1}$

Fig. 7

- In the results of the vibrating sphere-cone shell combination model, which we simulated by 25 elements, the following observation is interesting:
With increasing mode number m, i.e., with increasing number of circumferential nodes, we get a decreasing value of frequency. How do we have to interpret this heuristically ? As said before, the conical part can vibrate easily and the spherical part is rather stiff. For low m we have large wavelengths and so the typical size of a mode is comparable to the radius of curvature and the dimensions of the object. So these modes "feel" more of the shell. The stiff spherical part then acts as a boundary constraint enforcing relatively high frequencies. With increasing m we have more circumferential nodes, shorter wavelengths, and so we get a decoupling of cone and sphere. Now the cone can vibrate easily while the sphere is nearly at rest (low displacements here). The cone can vibrate as if there were no spherical part, and the frequencies of these modes are not as high as the ones for low m modes. Fig. 3 shows the modes 1-1 up to 6-2, and in the table below one can see the frequencies belonging to each mode.

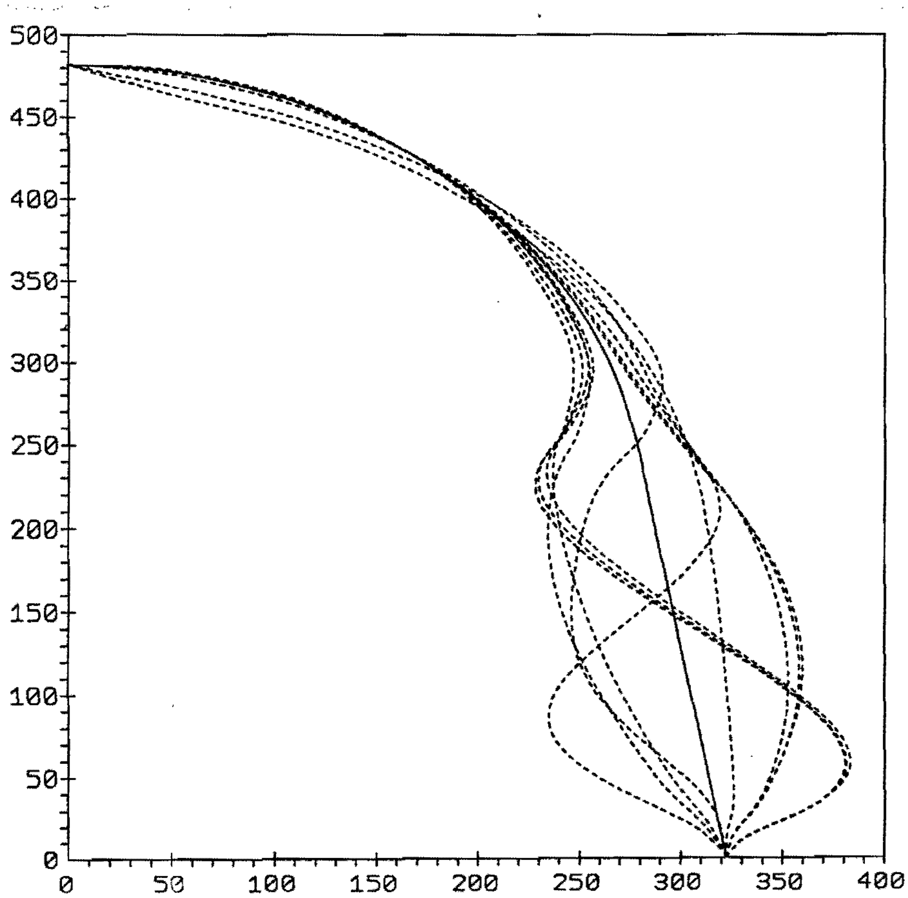
NUMBER	NAME	COMPUTED VALUE(HZ)
1	1-1	942.5752
2	1-2	1608.931
3	2-1	988.5887
4	2-2	1542.448
5	3-1	795.3348
6	3-2	1401.714
7	4-1	642.9033
8	4-2	1244.226
9	5-1	531.3008
10	5-2	1098.339
11	6-1	451.1039
12	6-2	972.5787

Fig. 2



400
350
300
250
200
150
100
50
0
0
50
100
150
200
250
300
350

Fig. 3



500
450
400
350
300
250
200
150
100
50
0
0
50
100
150
200
250
300
350
400

IV. CONCLUSIONS

One sees that the calculation of eigenmodes and eigenfrequencies of a kettledrum - although it is a rather complex musical instrument - is possible if one has a model which contains the basic constituting parts which affects the vibration.
We assumed the instrument to vibrate in vacuum because the only FE-package which allows to include airloading and radiation into air - SYSNOISE - failed because of an internal error. But the air has a main influence on the whole vibration and the coupling of vibrations, so it should be mentioned here again that we had to neglect air influences in this report. But we yield some results which seem to be right: the increasing of frequencies in case of a vibrating semi-sphere with free edge, a low first and a high second, and a decreasing of frequencies for the modes of a sphere-cone shell combination because there is a kind of de-coupling of the vibrations of the two parts of the shell. So we think that the FE-packages DYNOPT - and later SYSNOISE - are very useful tools in solving such kinds of investigations, and maybe they can even be adapted to a lot of further problems to yield then remarkable results, too.

V. APPENDIX 1

```
********************************************************************************
*                                                                              *
*          S E P R A N   O U T P U T   F I L E                                 *
*                                                                              *
********************************************************************************


         ****************************************
         *                                      *
         *              sepran                  *
         *      finite element package          *
         *                                      *
         *                                      *
         *       copyright (c) 1982             *
         *       ingenieursbureau sepra         *
         *                                      *
         *                                      *
         ****************************************

   ***************************************************************
   *                                                             *
   *          S E P R A N   I N P U T   F I L E                  *
   *                                                             *
   ***************************************************************

mesh2d                          *   plots a 2-D picture of the object     *
coarse(unit=0.1)                *   coarseness of subdivision             *
points                          *       >  points defining the object     *
p1=(0,0)                        *      /                                  *
p2=(1,0)                        *     /                                   *
p3=(0.25,0)                     *    /                                    *
p4=(0.75,0)                     *   /                                     *
curves                          *      >   curves bounding the object     *
c1=carc1(p2,p2,p1)              *     /                                   *
c2=carc1(p4,p4,-p3)             *    /                                    *
c3=cline1(p2,p4)                *   /                                     *
surfaces                        *     > connect curves to a closed curve  *
s1=general3(c1,c3,c2,-c3)       *    /                                    *
s2=general3(-c2)                *   /                                     *
meshsurf                        *     >   submeshes                       *
selm 1=(s1)                     *    /                                    *
selm 2=(s2)                     *   /                                     *
plot(plotfm=10d0,yfract=1d0,jmark=5)
end
problem
types
elgrp1=(type=100)
elgrp2=(type=100)
essbouncond
curves0(c1)
end


*********  warnings in subroutine                    mesh  (msh054)     ******


**********  warning nr. 180    **********

user point is not a nodal point

         user point    1
```

```
**********  warning nr. 180      **********

user point is not a nodal point

         user point     3


number of nodal points      347
number of elements      628
problem
types
elgrp1=(type=100)
elgrp2=(type=100)
essbouncond
curves0(c1)
end
input for subroutine probdf:
maximal number of degrees of freedom in a nodal point   1
number of arrays of special structure   1
number of different types per standard element   1

 type numbers of standard elements:
    1:          100
    2:          100

number of degrees of freedom in nodal points of standard elements of
array of special structure   1
    1      1:    2      2:    2      3:    2
    2      1:    2      2:    2      3:    2

number of degrees of freedom =     347
number of points with prescribed boundary conditions
              (including double points) =      63
number of prescribed degrees of freedom =      63
number of periodical boundary conditions =      0


output from eigval:



output scan results after    10 steps
     0 intervals are found


output scan results after    65 steps
convergence at lower side
convergence at upper side

     1 intervals at lower end
     7 intervals are found

 nr        lowerbound                upperbound

  1         0.4975572122505E+01      0.4975572122505E+01
  2         0.6902564879663E+03      0.6902564879663E+03
  3         0.7423702728410E+03      0.7423702728410E+03
  4         0.7518207251965E+03      0.7518207251965E+03
  5         0.8209683625055E+03      0.8209683625055E+03
  6         0.8278021008205E+03      0.8278021008205E+03
  7         0.9771783087518E+03      0.9771783087518E+03


output scan results after   120 steps
convergence at lower side
convergence at upper side
```

6 intervals at lower end
15 intervals are found

nr	lowerbound	upperbound	
1	0.4975572122505E+01	0.4975572122505E+01	These are the inter-
2	0.1119495277545E+02	0.1119495277545E+02	esting intervals,
3	0.1177117212203E+02	0.1177117212203E+02	i.e.,the calculated
4	0.2104742323581E+02	0.2104742323581E+02	eigenfrequencies of
5	0.2115566559265E+02	0.2115566559265E+02	the object defined in
6	0.2116856746940E+02	0.2116856746940E+02	the input file !
7	0.6601433197028E+03	0.6601433197030E+03	
8	0.6626318934068E+03	0.6626318934068E+03	
9	0.6821782453506E+03	0.6821782453506E+03	
10	0.6902564879663E+03	0.6902564879663E+03	
11	0.7423702728410E+03	0.7423702728410E+03	
12	0.7518207251965E+03	0.7518207251965E+03	
13	0.8209683625055E+03	0.8209683625055E+03	
14	0.8278021008205E+03	0.8278021008205E+03	
15	0.9771783087518E+03	0.9771783087518E+03	

```
interval nr: 1  multiplicity: 1  smallest errornorm: 0.1875525051381E-10
interval nr: 2  multiplicity: 1  smallest errornorm: 0.9203342590387E-11
interval nr: 3  multiplicity: 1  smallest errornorm: 0.3374860838791E-10
interval nr: 4  multiplicity: 1  smallest errornorm: 0.5110815196750E-05
interval nr: 5  multiplicity: 1  smallest errornorm: 0.2909504694934E-04
```

APPENDIX II

```
*********************************************************************
*                                                                   *
*        D Y N O P T   I N P U T   F I L E                          *
*                                                                   *
*********************************************************************

*TITLE
pauk
*ELEMENTS
8
*NODPOINTS
43
*NBOUNDCOND
9
*ELTYPE
1                    *  number of elements  (here: isoparametric   *
2                    *  8-node axisymmetric Fourier-element          *
*NFOURIER
1                    *  number of node-modes                         *
*NOPTFREQ
2  0                 *  number of frequencies desired ( in Hz )     *
*OPTIMAFLAG          *  unimportant                                 *
*OUTPUTOPTIONS
0  0  1              *  output to screen                            *
*END PARA
*BOUNDCON
 41  1               *    >  r-coordinate fixed                     *
 42  1               *   /                                          *
 43  1               *  /                                           *
 41  2               *    >  phi-coordinate fixed                   *
 42  2               *   /                                          *
 43  2               *  /                                           *
 41  3               *    >  z-coordinate fixed                     *
 42  3               *   /                                          *
 43  3               *  /                                           *
*OPTIMIZA
  1  2  1   1.0   1000.0    *  modenumber = 2 ( 2nd column )        *
  2  2  2   1.0   1200.0    *  1st and 2nd frequency to calculate   *
*MATDATA
0.1012E09  0.340  0.8830E-05       * E     v     rho                *
*CONNECTIVITY
* #elt eltype  counterclockwise numeration of each element          *
    1    2    1    2    3    5    8    7    6    4
    2    2    6    7    8   10   13   12   11    9
    3    2   11   12   13   15   18   17   16   14
    4    2   16   17   18   20   23   22   21   19
    5    2   21   22   23   25   28   27   26   24
    6    2   26   27   28   30   33   32   31   29
    7    2   31   32   33   35   38   37   36   34
    8    2   36   37   38   40   43   42   41   39
*COORDINATES
      43             *  # of nodes                                  *
*  #nod      r-          z-      coordinate                         *
   1  323.000000  0.000000
   2  323.500000  0.000000
   3  324.000000  0.000000
   4  321.444667  31.659536
   5  322.439851  31.757553
   6  316.793646  63.014174
   7  317.284038  63.111719
   8  317.774431  63.209264
   9  309.091728  93.761951
  10  310.048669  94.052235
  11  298.413089  123.606749
  12  298.875029  123.798090
```

```
13   299.336969   123.989432
14   284.860568   152.261146
15   285.742490   152.732543
16   268.564685   179.449185
17   268.980420   179.726970
18   269.396155   180.004755
19   249.682377   204.909031
20   250.455387   205.543424
21   228.395491   228.395490
22   228.749044   228.749044
23   229.102597   229.102597
24   204.909031   249.682376
25   205.543424   250.455387
26   179.449186   268.564685
27   179.726971   268.980419
28   180.004756   269.396154
29   152.261146   284.860568
30   152.732543   285.742489
31   123.606749   298.413089
32   123.798091   298.875029
33   123.989432   299.336968
34   93.761951   309.091728
35   94.052236   310.048669
36   63.014175   316.793645
37   63.111720   317.284038
38   63.209265   317.774431
39   31.659537   321.444667
40   31.757554   322.439851
41   0.000000   323.000000
42   0.000000   323.500000
43   0.000000   324.000000
*ITERATION                          *  only in the optimization process   *
0  1.0E-4  1.0E-5  1.0E-6  1.0E-8  5.0   *  here: only 0 important *
*END DATA
```

```
***********************************************************************
*                                                                     *
*         D Y N O P T   O U T P U T   F I L E                         *
*                                                                     *
***********************************************************************


 TIMOUT,AT LABEL :  START

 **********************
 * ECHO OF INPUT FILE *
 **********************
      .
      .
      .

 **************
 * MODEL DATA *
 **************

 PARAMETERS SIZING ELEMENT MODEL AND OPTIMIZATION
 ************************************************
      .
      .

 MATERIAL CHARACTERISTICS
 ************************

 YOUNG`S MODULUS:   .. (*1000 N/MM2)
 POISSON`S RATIO:   ..
 MASS DENSITY   :   .. (KG/MM3)


 CONTROL VALUES FOR ITERATION
 ****************************
      .
      .

 SWITCHES
 ********
      .
      .

 CONNECTIVITY AND ELEMENTTYPES
 *****************************
      .
      .

 BOUNDARY CONDITIONS
 *******************
      .
      .

 DESIGN VARIABLES
 ****************
    NONE

 FREQUENCIES TO BE OPTIMIZED
 ***************************
      .
      .

 OPTIMIZATION OF FD-PARAMETER
 ****************************
    NONE
```

```
COMBINATIONS TO BE ANALYSED
***************************
   NONE


*****************
* START PROGRAM *
*****************
TIMOUT,AT LABEL :  START

STORAGE
*******
    .
    .

BANDWIDTH OF STRUCTURAL MATRICES
********************************
BANDWIDTH : ..


TIMOUT,AT LABEL : INIT END

DESIGN VARIABLES      ITERATION   0
****************
   NONE


TIMOUT,AT LABEL : ASSEM IN
TIMOUT,AT LABEL : EIGSOLIN
TIMOUT,AT LABEL : EIGSOLEX

FREQUENCIES      ITERATION   0
***********

***********************************************************
*                                           COMPUTED     *       OPTIMUM
*   NUMBER   NAME                          VALUE(HZ)     *      VALUE(HZ)
*     .        .                               .         *          .
*     .        .                               .         *          .
***********************************************************
TIMOUT,AT LABEL : DERIVAIN
TIMOUT,AT LABEL : DERIVAEX

OPTIMIZATION FUNCTION      ITERATION   0
*********************
    .
    .

DERIVATIVES OF FREQUENCIES      ITERATION   0
**************************
   NONE

COORDINATES      ITERATION   0
***********
    .
    .

EIGENVECTORS      ITERATION   0
************

EIGENVECTOR :    ..
FREQUENCY   :    ..   HZ
NAME        :    ..
NODE MODE   :    ..
CONSTR. MODE:    ..
    .
    .
    .
```

VI. REFERENCES

(1) H.SAUNDERS & P.PASLAY : Inextensional Vibrations Of A Sphere-Cone Shell Combination. The Journal Of The Acoustical Society Of America; Vol. 31, #5. May 1959

(2) T.D.ROSSING : The Physics Of Kettledrums. Scientific American, Nov. 1982

(3) WFW 84.012 : Report Of The Department Of Mechanical Engineering, TU Eindhoven. 1984

```
END OF ITERATION PROCESS
************************
     .
     .

TIMOUT,AT LABEL :   END
CPU-TIME FOR THIS JOB IN SECONDS : 0.000E+00 .
```