Random dynamics in spatially extended systems

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Abstract

This short contribution describes the scientific programme "Random Dynamics in Spatially Extended Systems" that is supported by the European Science Foundation. In this programme, which runs over the period 2002-2006, 13 European countries participate. The main activities of the programme are listed, and a brief sketch is given of some of the main developments and future challenges in each of the eight research themes the programme is targeting.

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1 Activities

Random Dynamics in Spatially Extended Systems (RDSES) is a scientific programme that is running with the support of the European Science Foundation (2002-2006). The programme centres around *mathematical statistical physics*. Spatially extended systems consist of a large number of components that interact locally but that may nevertheless exhibit a global dependence, resulting in anomalous fluctuation phenomena and phase transitions. The main goal of the programme is to study the random dynamics, acting on the components of such systems, through the application of space-time scaling arguments and probabilistic limiting techniques. The challenge is to give a precise mathematical treatment of the interesting and complex physical phenomena that arise from this random dynamics.

RDSES focusses on the following eight research themes in *equilibrium* and *non-equilibrium* statistical physics:

(a) Gibbsian vs. non-Gibbsian spin systems.

(b) Polymers and self-interacting random processes.

(c) Interfaces and surface phenomena.

(d) Disordered media.

(e) Relaxation to equilibrium and metastability.

(f) Hydrodynamic behaviour of conservative systems.

(g) Entropy production and fluctuations far from equilibrium.

(h) Granular media and sandpile dynamics.

In total 13 European countries are participating in RDSES:

> Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Hungary, Netherlands, Poland, Sweden, Switzerland, United Kingdom.

Each country has a representative on the steering commitee, which oversees the development of the programme. Chair: F. den Hollander.

The main activities of the programme are:

(1) **Visitor exchange: short scientific visits of 1-2 weeks.** A call is sent out 4 times a year to some 120 junior and senior researchers. So far 50 applications have been granted. Special care is taken that the call reaches young people.

(2) **Workshops: 3-5 day meetings on topics selected by the steering committee.** Since the start, 10 workshops have been supported throughout Europe:

- *Constructing Non-Equilibrium Statistical Mechanics*
  (November 2002, Leuven, Belgium);
- *Statistical Mechanics and Probability Theory*
  (March 2003, Marseille, France);
- *Random Walks in Random Environments*
  (August 2003, Cambridge, United Kingdom);
- *Random Matrix Theory*
  (September 2003, Gregynog Hall, Wales, United Kingdom);
- *Gibbs vs. non-Gibbs in Statistical Mechanics and Related Fields*
  (December 2003, Eindhoven, Netherlands);
- *Interacting Particle Systems: New Trends, with Application in Biology and Economy*
  (January 2004, Paris, France);
- *Young European Probabilists I: Conformal Invariance, Scaling Limits and Percolation*
  (April 2004, Eindhoven, Netherlands);
- *Equilibrium and Dynamics of Spin Glasses*
  (April 2004, Ascona, Switzerland);
- *Statistical Mechanics and Interacting Particle Systems*
  (June 2004, Rouen, France);
- *Stochastic-Geometric and Combinatorial Ideas in Statistical Mechanics*
  (June 2004, Gothenburg, Sweden).

Targeted topics for the future are: hydrodynamic scaling, metastability, ageing in disordered systems, and polymers. Two more workshops for *Young European Probabilists* are scheduled for 2005 and 2006, to train junior researchers.

(3) **Summer schools: 2-4 week tutorial programmes for junior researchers.** One summer school on *Mathematical Statistical Mechanics* was organised in Prague, Czech Republic, in July 2003. A follow-up is planned for 2006. A large summer school on *Mathematical Statistical*

Physics will take place in Les Houches, France, in July 2005. There will be lectures on 15 hot topics by top researchers from Europe and from North- and South America. These topics are intended to be a road map for mathematical physics in the next decade.

(4) **Meetings of the steering committee**. The steering committee meets once a year. Thus far, meetings were held in Strasbourg (April 2002), Cambridge (August 2003) and Gothenburg (June 2004). During these meetings the activities of the programme are discussed, the workshop and summer school topics are selected, and strategic discussions take place on the development of the research area in the participating countries and in Europe.

RDSES maintains a *homepage* at the ESF website:

www.esf.org/rdses

This homepage describes the mission, goals and scientific background of the programme, as well as details of the various activities that are being undertaken. Suggestions and comments are welcome.

In March 2003, an *ESF-brochure* for RDSES was printed, which has been widely distributed. Copies are available upon request from the ESF Administrative Assistant to the programme:

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Mathematical statistical physics is an *eclectic* research area. The aim of the programme is to bring together the various groups that are active in this area within Europe. RDSES also acts as a *forum* for the development of ideas and actions, as witnessed by the start-up of a number a bilateral collaborations that grew out of the RDSES activities.

Mathematical statistical physics is an *interdisciplinary* research area, with interfaces towards physics, chemistry, computer science, the life sciences, engineering, economics and telecommunication. RDSES provides training in the analysis and modelling of complex dynamical processes via the propagation of a common language and the stimulation of international exchange.

2 Research themes

In this section we give a brief sketch of some of the main developments and future challenges in each of the eight research themes that are targeted by

RDSES. The aim is to give the reader a flavour of what is going on and to guide him/her to some of the relevant literature. Obviously, it is not possible to do full justice to the field.

(a) **Gibbsian vs. non-Gibbsian spin systems.** Gibbs theory, which has been successful for almost a century, aims at describing physical systems *in equilibrium*. Such systems consist of countably many interacting components, often referred to as *spins*, that are subject to a *local* interaction, among themselves and/or with an external field. This interaction is given by a *Hamiltonian*, which assigns an energy to each spin configuration. In equilibrium, the probability of a spin configuration is proportional to the negative exponential of its energy, the so-called Boltzmann weight factor. The Hamiltonian typically contains one or more relevant parameters, such as temperature or magnetic field. Depending on the type of Hamiltonian, the system may undergo a *phase transition* along a curve of critical values in the space of parameters. On or close to this curve the system exhibits long-range dependence with *universal* scaling properties. At the extremities of this curve the system is believed to be *scale invariant*.

More recently, it has become clear that Gibbsianness *out of equilibrium* is rare: many physical systems that are subjected to some dynamics do not allow for a Gibbsian description, due to the presence of a *non-local* interaction that cannot be properly described by a Hamiltonian. Examples are spin systems subject to random dynamics, to renormalization transformations or to disorder. For instance, a high-temperature Glauber spin-flip dynamics applied to a low-temperature Ising-spin Gibbs measure may destroy the Gibbs property in finite time and may afterwards restore it.

Currently there is intense research activity to *classify various possible scenarios* for non-Gibbsianness and to investigate how much of classical Gibbs theory can be saved. A particularly important notion, namely that of *weak Gibbsianness* introduced by Dobrushin, gives focus to these efforts. Here the idea is that a Hamiltonian description is still possible for "most" spin configurations, after some "bad" configurations (of measure zero) are discarded. It is still unclear what the full physical consequences of this notion are. Some systems turn out to be weak Gibbs, others not.

One challenge is to find an algorithm that decides non-Gibbsianness. Another challenge is to understand Gibbsianness under conservative dynamics and under non-reversible dynamics.

Georgii [12] is a key monograph for Gibbs theory. A fundamental paper on the issue of Gibbs vs. non-Gibbs is Van Enter, Fernández and Sokal [9]. For a recent overview of the area, see the proceedings of the workshop in

December 2003 that was supported by RDSES, edited by Van Enter, Le Ny and Redig [10]. Key references for interacting particle systems are the monographs by Liggett [22], [23].

(b) **Polymers and self-interacting random processes.** The spatial and temporal behaviour of polymer chains is an exciting area, with applications in the physical, chemical, biological and engineering sciences. Mathematics has been involved since the 1950's, although full immersion is taking place only since 15 years or so. Polymer chains are characterised by an *irregular folding in space* and by a *long-range interaction* (remote parts of the chain meet and interact with each other). As such they are rather different from more classical objects like Brownian motion, percolation or the contact process. There is a host of interesting models: self-repellent polymers, elastic polymers, charged polymers, polymers in a random potential, copolymers near interfaces. Many of these models are still largely unexplored. The self-avoiding walk, which is the archetypical model of a polymer, is described in the monograph by Madras and Slade [24]. For an overview on a variety of different polymer models in a more physical context, see the monograph by Vanderzande [37].

Copolymers are polymer chains consisting of a random concatenation of monomers of two (or more) types, e.g. hydrophobic and hydrophilic. In the presence of an interface separating two immiscible fluids, e.g. oil and water, the copolymer may or may not localise near the interface. Which of these two scenarios it chooses depends on the Hamiltonian of the interaction, which favours one type of monomer in one type of fluid and vice versa. A phase transition between the two scenarios depends on the parameters in the Hamiltonian and on the shape of the interface. The techniques to study this phase transition rely on the *theory of large deviations*. For an introduction to large deviation theory, see the monograph by den Hollander [16]. For an overview on the behaviour of random copolymers, see Soteros and Whittington [32].

Branched polymers, consisting of a network of polymer chains appropriately tied together, turn out to scale to *super-Brownian motion* in high dimensions. The same type of scaling occurs in a variety of models that are (or turn out to be) close to branching random walk, such as critical percolation, lattice trees and the critical contact process. The key technique to prove this scaling is the *lace expansion*, a diagrammatic perturbation technique that is able to deal with complex interactions in high dimensions. A key reference for percolation is the monograph by Grimmett [13]. For an overview on the lace expansion and its applications, see Slade [31]. For related aspects, see the contribution by T. Luczak elsewhere in this volume.

In dimension two, *conformal invariance* and the *Schramm-Löwner evolution* are central to a whole range of models at criticality. This theory, which combines ideas from stochastic analysis and conformal map theory, has led to a spectacular development, providing the identification of scaling limits and of associated critical exponents (the latter describe the behaviour close to criticality). Overviews are given in Werner [38] and in Kager and Nienhuis [18]. The candidate scaling limits of a variety of discrete critical models have been identified, but it remains a challenge to prove that these scaling limits actually exist and are conformally invariant. This has so far been achieved for only a few models, like critical site percolation on the triangular lattice, loop erased random walk, uniform spanning trees, and the harmonic explorer. Still open are the self-avoiding walk and the Potts model. See the contributions by O. Schramm and W. Werner elsewhere in this volume.

(c) **Interfaces and surface phenomena.** Interfaces in spatially extended systems arise from geometric constraints or from inhomogeneous initial conditions in combination with conservation laws. Examples are *wetting phenomena* (droplets interacting with a wall) and *metastable phenomena* (droplets acting as the energy barrier for a crossover between different phases).

Wulff droplets have recently been the object of intense investigation. For a system in equilibrium at a first-order phase transition, a large droplet of one phase inside another phase assumes the so-called Wulff shape. On the *macroscopic* scale, the shape is *deterministic* and is the solution of a variational problem involving the surface tension associated with the interface between the two phases. Examples occur in Ising and Potts models and in solid-on-solid models. For an overview, see Bodineau, Ioffe and Velenik [5].

An open question is to identify the shape of large droplets subject to a random dynamics, such as large critical droplets for metastable transitions between different phases. For Ising spins subject to a Glauber spin-flip dynamics, it was shown by Schonmann and Shlosman [30] that, close to the phase transition line, the critical droplet has the Wulff shape, i.e., the dynamics manages to keep the droplet close to (quasi-)equilibrium while it is growing, shrinking and moving. It is a challenge to extend this result to the lattice gas subject to a Kawasaki hopping dynamics. Here, particle conservation turns out to be a serious obstacle, since it causes long-range dependence and depletion of the gas around growing droplets. Anisotropic dynamics are expected not to preserve the Wulff shape.

On the *mesoscopic* scale, the interface of droplets typically shows *anomalous fluctuations*. Remarkably, these fluctuations exhibit a high degree of universality. In dimension two considerable progress has been made, with

Wigner's semi-circle law and the Tracy-Widom distribution appearing as *universal attractors* for the scaling. A unification is envisioned for a whole range of different models, all in some way related to the behaviour of *spectra of large random matrices*. Here a new world is opening up, linking geometry and analysis. A key reference is Baik, Deift, Johansson [2]. See also the contribution by A. Guionnet elsewhere in this volume.

Simulations indicate that limiting shapes are delicate objects, which typically retain part of the information of the underlying lattice structure.

(d) **Disordered media.** This has been a very active area for several decades already, with applications to amorphous materials, neural networks, chemical catalysis and biomolecules. Percolation, the random field Ising model, the Hopfield model, the random energy model, and random walk in random environment are by now classical. Exciting recent developments concern *spin glasses* ("random magnetic alloys"), in particular, the Sherrington-Kirkpatrick model and the Edwards-Anderson model. Here, new types of phase transitions are expected to occur due to a competition of interactions ("frustration"), causing a highly complex energy landscape given by a random Hamiltonian. For the Sherrington-Kirkpatrick model, which is a mean-field model with a long-range interaction, Parisi predicted the occurrence of *replica symmetry breaking*. After many years of hard effort, this prediction has recently been proved to be correct by Guerra [14] and Talagrand [35]. (See the contribution by F. Guerra elsewhere in this volume.) In Parisi's solution, a key concept is the *ultrametric structure* of the ground states. The role of this ultrametric structure has been elucidated through the work of Aizenmann, Sims and Starr [1]. The techniques that are developed in this area find application in a range of different areas, including coding and hard combinatorial optimisation. An overview of spin glass theory can be found in the monograph by Talagrand [34].

The Edwards-Anderson model has a short-range interaction. Its relation to the Sherrington-Kirkpatrick model remains unclear: replica symmetry breaking may not occur in short-range models (see Newman [28] for an alternative scenario).

Caricatures of spin glasses, such as the Hopfield model and the random energy model, are by now well understood. They shed light on the universality of ultrametricity in mean-field models. See Bovier and Kurkova [6] for an overview on the developments around random energy models.

Ageing in disordered media is a new challenge on the horizon. Here one studies the evolution of systems that go through a cascade of metastable equilibria. This results in a correlation structure of the system that evolves

with time. The behaviour of spin glasses subject to a random dynamics is still largely open.

Random walk in random envrionment has recently gone through major developments, especially in higher dimensions, where now some of the hard questions are finally reaching a solution. See Zeitouni [40] for an overview. *Catalytic branching models*, describing a reactant evolving in the presence of a catalyst, are models of disorder with random dynamics. This is an area that is growing fast, with applications in population dynamics. See the overviews by Dawson and Fleischmann [7] and by Klenke [21].

(e) **Relaxation to equilibrium and metastability.** A physical system out of equilibrium tends to relax towards equilibrium. This relaxation may, however, be extremely slow, a phenomenon that is called *metastability*. Consider, for instance, a system in equilibrium with parameters on one side of a first-order phase transition curve. Suppose that the parameters are suddenly changed to values corresponding to the opposite side of this curve. Then the system wants to relax from the old phase to the new phase, but in order to do so it has to overcome an *energy barrier*. Before crossing this energy barrier, the system persists for a long time in what is called a *metastable state*, which is characterised by many unsuccessful attempts to cross the barrier. The crossover is typically achieved after the system creates a *critical droplet* of the new phase inside the old phase.

Several models are of interest, such as Ising spins under a Glauber spin-flip dynamics or the lattice gas under a Kawasaki hopping dynamics. The challenge is to give a detailed description of the crossover time and of the typical trajectories followed by the system prior to the crossover. The theory either relies on large deviation theory for the trajectories of the system ("pathwise approach") or on a close analogy between metastable transition times and capacities in electric networks ("potential-theoretic approach").

In two dimensions substantial progress has been made and key questions have been settled for a variety of different models. In three dimensions the geometry of critical droplets is rather complex and progress has only been partial. Describing metastable behaviour under a conservative dynamics is a hard challenge.

For an overview of the history and the developments in metastability, see the monograph by Olivieri and Vares [29]. For a critical comparison between Glauber and Kawasaki, as well as for mathematical references to droplet growth in metastability, see den Hollander [17].

(f) **Hydrodynamic behaviour of conservative systems.** One of the basic problems of non-equilibrium statistical mechanics is the derivation of

hydrodynamic equations. On the proper macroscopic space-time scales, interacting particle systems develop autonomous behaviour for a collection of *locally conserved quantities*, such as density, momentum and energy. The evolution of these quantities is given by a set of coupled partial differential equations. For *deterministic* microscopic dynamics only mild progress has been made, with even issues like ergodicity and mixing being still largely open. For *random* microscopic dynamics (i.e., in the presence of noise), progress has been fast over the past decade, especially for those systems whose quasi-equilibria conditioned on the locally conserved quantities are well understood. The type of pde depends on the scaling that is chosen. Eulerian scaling (space scales like time) leads to *hyperbolic* pde's, diffusive scaling (space scales like square root of time) leads to *parabolic* pde's. Diffusive systems are generally well understood, hyperbolic systems are much less so, since they may develop "shocks" in finite time. For an overview on hydrodynamic scaling, see the monographs by Spohn [33] and by Kipnis and Landim [19].

The large deviation techniques developed by Kipnis, Olla and Varadhan [20], and the relative entropy method of Yau [39], yield a derivation of the hydrodynamic equations in a rather broad context of models. Both Eulerian and diffusive scaling can be handled. In the former, the shocks and their microscopic counterpart have been the subject of intense research. Typically, the methods that are employed only give the hydrodynamic equation until the first time when a shock appears.

A majour breakthrough in the understanding of hydrodynamics with shocks is made in recent work by Fritz and Tóth [11], where, with the help of the analytic theory of conservation laws, the validity of the hydrodynamic equation is obtained beyond shocks. This promises to open up a new line of research. Particularly challenging is the analysis of *multi-component* hyperbolic systems, where attractiveness typically fails, causing trouble with uniqueness issues. Important progress has been achieved in the recent paper by Tóth and Válko [36].

(g) **Entropy production and fluctuations far from equilibrium.** For systems in a *non-equilibrium steady state*, such as a gas flowing through a pipe or a fluid in contact with two heat reservoirs at different temperatures, it is no longer possible to use considerations that are valid for systems in or close to equilibrium. Especially when driven by large external fields, the system is beyond the regime where linear response theory can be applied. Therefore it is of key importance to search for general principles in non-equilibrium, in particular, symmetry relations between the transport coefficients. A non-equilibrium steady state is non-reversible, and so it produces entropy. The

study of *entropy production* and its fluctuations is therefore a central issue. For a discussion, see Maes, Redig and Van Moffaert [26].

The *Gallavotti-Cohen fluctuation theorem* expresses a symmetry property for the large deviations of the entropy production that holds in complete generality. Close to equilibrium, this symmetry reduces to the classical Onsager reciprocity relations for the response coefficients. In a recent approach, put forward by Maes [25], a non-equilibrium steady state is viewed as a Gibbs measure on space-time trajectories. In this setting, the Gallavotti-Cohen fluctuation theorem immediately follows from the Dobrushin-Lanford-Ruelle conditions on the space-time Gibbs measure. The entropy production is precisely the time-reversal antisymmetric part of the Hamiltonian of the space-time Gibbs measure. The Gallavotti-Cohen fluctuation theorem can thus be viewed as similar to the Ward identities in quantum field theory.

The area is witnessing the slow emergence of a microscopic theory, from which not only the thermodynamics of irreversible processes close to equilibrium can be derived, but which promises to go far beyond the linear regime.

Further challenges in the study of non-equilibrium systems are recent efforts to derive Fourier's law (relating macroscopic flow with external field) and to construct non-equilibrium fluctuation symmetries for quantum systems.

(h) **Granular media and sandpile dynamics.** Granular media are systems whose components have a physical shape, rather than being idealised point particles. Examples are powder, sand, grains or rocks. The question is how this shape affects the microscopic, mesoscopic and macroscopic behaviour. Inelastic collisions between the components and internal degrees of freedom play an important role. For the proceedings of a recent workshop in this area, see Helbing, Hermann, Schreckenberg and Wolf [15]. Mathematically, the area is largely undeveloped.

In sandpile dynamics, grains of sand topple and cause *avalanches*, i.e., a motion involving a large number of components at the same time. Since these avalanches are highly non-local, it is hard to even define the dynamics properly.

The concept of *self-organised criticality* (SOC), originally proposed by Bak, Tang and Wiesenfeld [3], has become central to a variety of physical, chemical and biological systems. SOC means that the system is "dynamically tuned towards criticality", even though it has no parameter to tune. In other words, the system exhibits "power-law decay of avalanche sizes" (power law decay of correlations being typical for systems at criticality). Experiments on granular media, such as sandpiles, have confirmed the presence of these

power laws.

One outstanding paradigm of SOC is the so-called *abelian sandpile model*, which allows for a mathematical treatment because of an underlying abelian group structure, originally revealed by Dhar [8]. This model has strong connections with fundamental objects in graph theory, such as the discrete Laplacian, wired spanning forests, and two-component spanning trees. In two dimensions, physicists predict a conformal field theory in the continuum limit.

The abelian sandpile model also appears in algebraic combinatorics, in discrete potential theory, in group theory, and in computer science (see Biggs [4]). From the perspective of mathematical physics, the limit of infinite graphs is important, corresponding to what is called the thermodynamic limit in statistical physics. The first results in this direction have been obtained by Maes, Redig and Saada [27] for the abelian sandpile model on an infinite tree. By now, much progress has been made in a global understanding of the ergodic theory of this system, and its relation to random walks on compact groups.

A challenge is to understand the basic features of abelian sandpile models in high dimensions.

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