On infinitely differentiable and Gevrey vectors for representations

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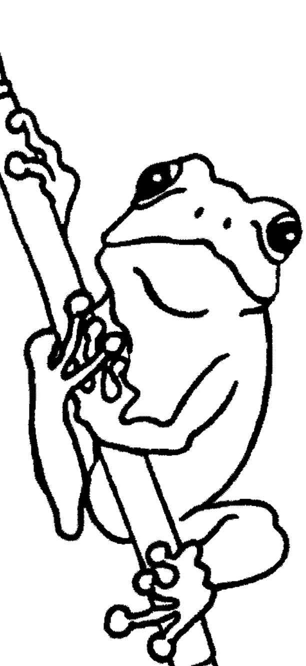

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Abstract

In the present paper we give a condition in order that the set of infinitely differentiable vectors for a representation π in a Banach space is equal to the set of all infinitely differentiable vectors for the restriction of π to a subgroup. Similar results for Gevrey vectors and analytic vectors are proved for unitary representations.

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1 Introduction and notations

Let π be a continuous representation of a d-dimensional real Lie group G in a Banach space E. For each $u \in E$ define $\widetilde{u} : G \to E$ by $\widetilde{u}(x) := \pi_x u$ $(x \in G)$. A vector $u \in E$ is said to be *infinitely differentiable, analytic* resp. a *Gevrey vector of order* λ for π, $\lambda \geq 1$, if the map $\widetilde{u}$ is infinitely differentiable, (real) analytic resp. a Gevrey function of order λ for π. (Cf. [Går], [Nel] and [GW] respectively.) Let $D^\infty(\pi)$, $D^\omega(\pi)$ and $G_\lambda(\pi)$ denote the space of all infinitely differentiable vectors, of all analytic vectors and of all Gevrey vectors of order λ for π respectively. Note that $D^\omega(\pi) = G_1(\pi)$. For each X in the Lie algebra $\mathfrak{g}$ of G let $d\pi(X)$ denote the infinitesimal generator of the one-parameter group $t \mapsto \pi_{\exp tX}$ and let $\partial\pi(X)$ denote the restriction of $d\pi(X)$ to $D^\infty(\pi)$. The map $X \mapsto \partial\pi(X)$ extends uniquely to an associative algebra homomorphism from the *complex universal enveloping algebra* $U(\mathfrak{g})$ of $\mathfrak{g}$ into the set of all linear operators from $D^\infty(\pi)$ into $D^\infty(\pi)$. The extension is denoted by $\partial\pi$ also.

There exist infinitesimal characterizations for the spaces $D^\infty(\pi)$, $D^\omega(\pi)$ and $G_\lambda(\pi)$. Therefore, let $\mathcal{A}$ be a set of (possibly unbounded) operators in E. Define the *joint C^∞-domain* $D^\infty(\mathcal{A})$ of the set $\mathcal{A}$ by

$$D^\infty(\mathcal{A}) := \bigcap_{n \in \mathbb{N}_0} \bigcap_{A_1,\ldots,A_n \in \mathcal{A}} D(A_1 \circ \ldots \circ A_n).$$

Here $D(A_1 \circ \ldots \circ A_n)$ denotes the domain of the operator $A_1 \circ \ldots \circ A_n$. For $\lambda \geq 1$ define the *Gevrey space $S_\lambda(\mathcal{A})$ of order λ relative to $\mathcal{A}$* by

$$S_\lambda(\mathcal{A}) := \{u \in D^\infty(\mathcal{A}) : \exists_{c,t>0} \forall_{n \in \mathbb{N}_0} \forall_{A_1,\ldots,A_n \in \mathcal{A}} \left[\|A_1 \circ \ldots \circ A_n u\| \leq c t^n n!^\lambda\right]\}.$$

(Cf. [GW, Section 1].) Now Goodman and Wallach have proved the following infinitesimal characterization of the spaces $D^\infty(\pi)$ and $G_\lambda(\pi)$.

Theorem 1 *Let π be a representation of a Lie group G in a Banach space E. Let $X_1, \ldots, X_d$ be any basis in the Lie algebra $\mathfrak{g}$ of G. Let $\lambda \geq 1$. Then*

$$D^\infty(\pi) = D^\infty(\{d\pi(X_1), \ldots, d\pi(X_d)\})$$

and

$$G_\lambda(\pi) = S_\lambda(\{d\pi(X_1), \ldots, d\pi(X_d)\}).$$

Proof. See [Goo, Proposition 1.1] and [GW, Proposition 1.5]. □

Let $d_1 \in \{1, \ldots, d-1\}$, where $d = \dim \mathfrak{g}$. Then clearly for any basis $X_1, \ldots, X_d$ in $\mathfrak{g}$:

$$D^\infty(\pi) = D^\infty(\{d\pi(X_1), \ldots, d\pi(X_d)\}) \subset D^\infty(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\}).$$

In the present paper we give conditions on the Lie algebra $\mathfrak{g}$ and the representation π in order that $D^\infty(\pi) = D^\infty(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\})$ for suitable $X_1, \ldots, X_{d_1}$ in $\mathfrak{g}$. Also, in case $\mathfrak{k} := \mathrm{span}\{d\pi(X_1), \ldots, d\pi(X_{d_1})\}$ is a subalgebra of $\mathfrak{g}$, there exists a subgroup K of G with Lie algebra $\mathfrak{k}$ and we obtain

$$D^\infty(\pi) = D^\infty(\pi|_K).$$

For a semisimple Lie group G these conditions are satisfied if for $\mathfrak{k}$ we take the subalgebra $\mathfrak{k}$ in the Cartan decomposition $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ of $\mathfrak{g}$ and for the representation π a completely irreducible one or a principal series representation.

For unitary representations we prove similar theorems for the set of Gevrey vectors.

2 Infinitely differentiable vectors for restrictions to subgroups

In this section we prove the following theorem.

Theorem 2 *Let π be a representation of a Lie group G in a Banach space E. Let $X_1, \ldots, X_d$ be a basis in the Lie algebra $\mathfrak{g}$ of G. Let $d_1 \in \{1, \ldots, d-1\}$ and let*

$$C := X_1^2 + \ldots + X_{d_1}^2 - X_{d_1+1}^2 - \ldots - X_d^2 \in U(\mathfrak{g}).$$

Suppose C belongs to the center of $U(\mathfrak{g})$ and suppose there exists $\tau \in \mathbb{C}$ such that

$$\partial\pi(C) = \tau I.$$

Then

$$D^\infty(\pi) = D^\infty(\{d\pi(X_1), \ldots, d\pi(X_d)\}) = D^\infty(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\}).$$

Proof. Without loss of generality, we may assume that G is connected. Let

$$\begin{aligned}
\Delta &:= X_1^2 + \ldots + X_d^2 \in U(\mathfrak{g}), \\
\Delta_1 &:= X_1^2 + \ldots + X_{d_1}^2 \in U(\mathfrak{g}), \\
\widetilde{\Delta} &:= \widetilde{X}_1^2 + \ldots + \widetilde{X}_d^2, \\
\widetilde{\Delta}_1 &:= \widetilde{X}_1^2 + \ldots + \widetilde{X}_{d_1}^2.
\end{aligned}$$

Here $\widetilde{X}$ denotes the left invariant differential operator on G which corresponds to X. Let $u \in D^\infty(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\})$ be fixed. We have to prove that the function $\widetilde{u}$ from G into E is infinitely differentiable. By [Pou] it is enough to prove that $\widetilde{u}$ is weakly infinitely differentiable, i.e. the function $f \circ \widetilde{u}$ from G into $\mathbb{C}$ is infinitely differentiable for all $f \in E'$. We shall show that for all $f \in E'$ and all $m \in \mathbb{N}$ there exists a continuous function g on G such that $f \circ \widetilde{u}$ is a weak solution of the equation $\widetilde{\Delta}^m F = g$ and then by using regularity theory for elliptic differential operators the regularity of $f \circ \widetilde{u}$ follows.

Let $\check{\pi}$ be the contragredient representation of π on the Banach space $\check{E}$ in the sense of Bruhat. So $\check{E}$ consists of all $f \in E'$ for which the map $x \mapsto (\pi_{x^{-1}})^* f$ from G into E' is (strongly) continuous. (Here $(\ )^*$ denotes the dual operator in the dual space.) Then for all $x \in G$ the operator $\check{\pi}_x$ is defined by $\check{\pi}_x := (\pi_{x^{-1}})^*|_{\check{E}}$. So $x \mapsto \check{\pi}_x$ is a continuous representation of G in the Banach space $\check{E}$. (See [Bru, §I.2.2].) We first consider infinitely differentiable vectors for $\check{\pi}$. Let $f \in D^\infty(\check{\pi})$. Then $f \circ \widetilde{u}(x) = f(\pi_x u) = [\check{\pi}_{x^{-1}} f](u)$ for all $x \in G$, so $f \circ \widetilde{u}$ is an infinitely differentiable function from G into $\mathbb{C}$. Let $m \in \mathbb{N}$. Let

$$w_m := \left(2 \sum_{k=1}^{d_1} [d\pi(X_k)]^2 - \tau I \right)^m u.$$

Recall that $u \in D^\infty(\{d\pi(X_1), \dots, d\pi(X_{d_1})\})$.

Assertion 1. For all $f \in D^\infty(\check{\pi})$ and all $x \in G$ we have

$$\left[\widetilde{\Delta}^m (f \circ \widetilde{u})\right](x) = [f \circ \widetilde{w}_m](x).$$

Proof of Assertion 1. Let $f \in D^\infty(\check{\pi})$. Let $n \in \mathbb{N}$ and let $j_1, \dots, j_n \in \{1, \dots, d\}$. Then for all $x \in G$:

$$\begin{aligned}
&[\widetilde{X}_{j_1} \circ \dots \circ \widetilde{X}_{j_n}(f \circ \widetilde{u})](x) = \\
&= \left.\frac{\partial}{\partial t_1}\right|_0 \cdots \left.\frac{\partial}{\partial t_n}\right|_0 f(\pi_x \pi_{\exp(t_1 X_{j_1})} \circ \dots \circ \pi_{\exp(t_n X_{j_n})} u) \\
&= \left.\frac{\partial}{\partial t_1}\right|_0 \cdots \left.\frac{\partial}{\partial t_n}\right|_0 f(\pi_{\exp(t_1 \mathrm{Ad}(x) X_{j_1})} \circ \dots \circ \pi_{\exp(t_n \mathrm{Ad}(x) X_{j_n})} \pi_x u) \\
&= \left.\frac{\partial}{\partial t_1}\right|_0 \cdots \left.\frac{\partial}{\partial t_n}\right|_0 \left[\check{\pi}_{\exp(-t_n \mathrm{Ad}(x) X_{j_n})} \circ \dots \circ \check{\pi}_{\exp(-t_1 \mathrm{Ad}(x) X_{j_1})} f\right](\pi_x u) \\
&= (-1)^n \left[\partial\check{\pi}(\mathrm{Ad}(x)(X_{j_n} \dots X_{j_1})) f\right](\pi_x u) \\
&= \left[\partial\check{\pi}(\mathrm{Ad}(x)(X_{j_1} \dots X_{j_n})^*) f\right](\pi_x u).
\end{aligned}$$

Here $L \mapsto L^*$ denotes the usual antiautomorphism from $U(\mathfrak{g})$ onto $U(\mathfrak{g})$. Let $Y \in \mathfrak{g}$. Then $\mathrm{Ad}(\exp Y)(C) = e^{\mathrm{ad} Y}(C) = C$, because C belongs to the center of $U(\mathfrak{g})$. Since G is connected, $\mathrm{Ad}(x)(C) = C$ for all $x \in G$. Moreover, for all $v \in D^\infty(\pi)$ we have

$$[\partial\check{\pi}(C) f](v) = f(\partial\pi(C^*)v) = f(\partial\pi(C)v) = \tau f(v).$$

Since $D^\infty(\pi)$ is dense in E, by [Går], $\partial\check{\pi}(C) f = \tau f$, by continuity. Note that $\Delta = 2\Delta_1 - C$. So we obtain for all $x \in G$:

$$\begin{aligned}
\left[\widetilde{\Delta}^m (f \circ \widetilde{u})\right](x) &= \left[\partial\check{\pi}(\mathrm{Ad}(x)(\Delta^m)) f\right](\pi_x u) \\
&= \sum_{k=0}^{m} (-1)^k \binom{m}{k} \left[\{\partial\check{\pi}(\mathrm{Ad}(x)(C))\}^k \partial\check{\pi}(\mathrm{Ad}(x)(2\Delta_1)^{m-k}) f\right](\pi_x u) \\
&= \sum_{k=0}^{m} (-1)^k \binom{m}{k} \left[\tau^k \partial\check{\pi}(\mathrm{Ad}(x)(2\Delta_1)^{m-k}) f\right](\pi_x u) \\
&= \left[\partial\check{\pi}(\mathrm{Ad}(x)((2\Delta_1 - \tau)^m)) f\right](\pi_x u) \\
&= \left[(2\widetilde{\Delta}_1 - \tau)^m (f \circ \widetilde{u})\right](x) \\
&= [(f \circ \widetilde{w}_m)](x).
\end{aligned}$$

This proves Assertion 1.

Let λ be a **right** Haar measure on G.

Assertion 2. For all $\varphi \in C_c^\infty(G)$ and all $f \in E'$ we have

$$\int_G \left[\widetilde{\Delta}^m \varphi\right](x) \, [f \circ \widetilde{u}](x) d\lambda(x) = \int_G \varphi(g) \, [f \circ \widetilde{w}_m](x) d\lambda(x)$$

Proof of Assertion 2. Let T_K be the polar topology for E' of uniform convergence on the compact subsets of E. Since E is complete, the topology T_K is compatible with the dual pair (E', E) by [Wil] Example 9-2-10 and Theorem 9-2-12. Now it follows by the same arguments as in [Bru, page 113] that $\check{E}$ is not only w^*-dense in E', but $\check{E}$ is even dense in (E', T_K).

Since $D^\infty(\check{\pi})$ is dense in the Banach space $\check{E}$ and since $\check{E}$ is dense in (E', T_K), it follows that $D^\infty(\check{\pi})$ is dense in (E', T_K). Now let $f \in E'$ and $\varphi \in C_c^\infty(G)$. Let $\varepsilon > 0$, let $K := \{\pi_x u : x \in \text{supp}\,\varphi\} \cup \{\pi_x w_m : x \in \text{supp}\,\varphi\}$ and $M := 1 + \int_G |\widetilde{\Delta}^m \varphi(x)| d\lambda(x) + \int_G |\varphi(x)| d\lambda(x)$. There exists $g \in D^\infty(\check{\pi})$ such that for all $a \in K$: $|f(a) - g(a)| \leq \varepsilon M^{-1}$. Then by Assertion 1:

$$\begin{aligned}\int_G \left[\widetilde{\Delta}^m \varphi\right](x)\ [g \circ \widetilde{u}](x) d\lambda(x) &= \int_G \varphi(x) \left[\widetilde{\Delta}^m (g \circ \widetilde{u})\right](x) d\lambda(x) \\ &= \int_G \varphi(x) \left[g \circ \widetilde{w}_m\right](x) d\lambda(x).\end{aligned}$$

So

$$\begin{aligned}&\left| \int_G \left[\widetilde{\Delta}^m \varphi\right](x)\ [f \circ \widetilde{u}](x) d\lambda(x) - \int_G \varphi(x) [f \circ \widetilde{w}_m](x) d\lambda(x) \right| \leq \\ &\leq \int_G \left| \left[\widetilde{\Delta}^m \varphi\right](x)\ (f(\pi_x u) - g(\pi_x u)) \right| d\lambda(x) + \\ &\qquad + \left| \int_G \left[\widetilde{\Delta}^m \varphi\right](x)\ [g \circ \widetilde{u}](x) d\lambda(x) - \int_G \varphi(x) [g \circ \widetilde{w}_m](x) d\lambda(x) \right| + \\ &\qquad + \int_G |\varphi(x) (g(\pi_x w_m) - f(\pi_x w_m))| d\lambda(x) \\ &\leq \varepsilon M^{-1} \left(\int_G |\widetilde{\Delta}^m \varphi(x)| d\lambda(x) + \int_G |\varphi(x)| d\lambda(x) \right) \\ &\leq \varepsilon.\end{aligned}$$

This proves Assertion 2.

Now we prove the theorem. Let $f \in E'$. By Assertion 2 the function $f \circ \widetilde{u}$ is a weak solution of the equation $\widetilde{\Delta}^m F = f \circ \widetilde{w}_m$. Since $f \circ \widetilde{w}_m$ is a continuous function and $\widetilde{\Delta}^m$ is an elliptic operator of order $2m$, it follows from the local regularity theorem for elliptic operators that $f \circ \widetilde{u}$ has locally L^2 derivatives of order $\leq 2m$. (See [Fol, Theorem 6.30].) Hence by [Fol, Lemma 6.9] (the Sobolev lemma), the function $f \circ \widetilde{u}$ is $2m - d$ times continuously differentiable. Therefore $f \circ \widetilde{u}$ is infinitely differentiable for all $f \in E'$ and hence the function $\widetilde{u}$ is infinitely differentiable. Thus $u \in D^\infty(\pi)$. □

Corollary 3 *Let G be a semisimple Lie group with Lie algebra $\mathfrak{g}$. Let π be a representation of G in a Banach space. Let $C \in U(\mathfrak{g})$ be the Casimir element. Suppose there exists $\tau \in \mathbb{C}$ such that $\partial\pi(C) = \tau I$. Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ be a Cartan decomposition of $\mathfrak{g}$ and let K be a subgroup of G with Lie algebra $\mathfrak{k}$. Then*

$$D^\infty(\pi) = D^\infty(\pi|_K).$$

Proof. Let B denote the Killing form of $\mathfrak{g}$. Let $X_1, \ldots, X_{d_1}$ be a basis in $\mathfrak{k}$ and $X_{d_1+1}, \ldots, X_d$ be a basis in $\mathfrak{p}$ such that $B(X_i, X_j) = -\delta_{i,j}$ for all $1 \leq i,j \leq d_1$ and $B(X_i, X_j) = \delta_{i,j}$ for all $d_1 < i,j \leq d$. Then $C = \sum_{k=d_1+1}^{d} X_k^2 - \sum_{k=1}^{d_1} X_k^2$. So by Theorems 2 and 1 we obtain that

$$D^\infty(\pi) = D^\infty(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\}) = D^\infty(\pi|_K).$$

□

Remark. Note that there are no conditions on the center of G in the previous corollary.

Corollary 4 *Let G be a connected semisimple Lie group with finite center. Let K be a maximal compact subgroup. Let π be a principal series representation of G. Then $D^\infty(\pi) = D^\infty(\pi|_K)$.*

Corollary 5 *Let π be a completely irreducible representation of a Lie group G in a Banach space. Let $X_1, \ldots, X_d$ be a basis in the Lie algebra $\mathfrak{g}$ of G. Let $d_1 \in \{1, \ldots, d-1\}$. Let*

$$C := X_1^2 + \ldots + X_{d_1}^2 - X_{d_1+1}^2 - \ldots - X_d^2 \in U(\mathfrak{g}).$$

Suppose C belongs to the center of $U(\mathfrak{g})$. Then $D^\infty(\pi) = D^\infty(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\})$.

Proof. Since π is completely irreducible, by [Tay, Proposition 0.4.5], there exists $\tau \in \mathbb{C}$ such that $\partial\pi(C) = \tau I$. □

3 Gevrey vectors for restrictions to subgroups

In this section we prove a similar theorem as in the previous section, but now for Gevrey vectors instead of infinitely differentiable vectors. However, in this section we only consider unitary representations. We immediately formulate the main theorem of this section.

Theorem 6 *Let π be a unitary representation of G. Let $X_1, \ldots, X_d$ be a basis in the Lie algebra $\mathfrak{g}$ of G. Let $d_1 \in \{1, \ldots, d-1\}$. Let*

$$C := X_1^2 + \ldots + X_{d_1}^2 - X_{d_1+1}^2 - \ldots - X_d^2 \in U(\mathfrak{g}).$$

Suppose C belongs to the center of $U(\mathfrak{g})$ and suppose there exists $\tau \in \mathbb{R}$ such that

$$\partial\pi(C) = \tau I.$$

Let $\lambda \geq 1$. Then

$$G_\lambda(\pi) = S_\lambda(\{d\pi(X_1), \ldots, d\pi(X_d)\}) = S_\lambda(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\}).$$

In particular,

$$D^\omega(\pi) = S_1(\{d\pi(X_1), \ldots, d\pi(X_{d_1})\}).$$

Proof. First we prove that $S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_d)\}) = S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_{d_1})\})$ Let $\Delta := X_1^2+\ldots+X_d^2 \in U(\mathfrak{g})$ and $\Delta_1 := X_1^2+\ldots+X_{d_1}^2 \in U(\mathfrak{g})$ be as in the proof of Theorem 2. Let $u \in S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_{d_1})\})$. By an elementary counting argument it easily follows that $u \in S_{2\lambda}(\{\partial\pi(\Delta_1)\})$. Let $C, t > 0$ be such that

$$\|[\partial\pi(\Delta_1)]^n u\| \le C t^n n!^{2\lambda}$$

for all $n \in \mathbb{N}_0$. Since $\partial\pi(\Delta) = 2\partial\pi(\Delta_1) - \partial\pi(C) = 2\partial\pi(\Delta_1) - \tau I$, we obtain for all $n \in \mathbb{N}_0$:

$$\begin{aligned}\|[\partial\pi(\Delta)]^n u\| &\le \sum_{k=0}^{n}\binom{n}{k} 2^k|\tau|^{n-k}\|[\partial\pi(\Delta_1)]^k u\| \\ &\le C\sum_{k=0}^{n}\binom{n}{k} 2^k t^k|\tau|^{n-k}k!^{2\lambda} \\ &\le Cn!^{2\lambda}\sum_{k=0}^{n}\binom{n}{k} 2^k t^k|\tau|^{n-k} \\ &= C(2t+|\tau|)^n n!^{2\lambda}.\end{aligned}$$

So $u \in S_{2\lambda}(\{\partial\pi(\Delta)\})$.

Now by [GW] Example folowing Theorem 1.7, we obtain that $u \in S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_d)\})$. (Here we used that π is a unitary representation.) So

$$S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_{d_1})\}) \subset S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_d)\}).$$

Thus

$$S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_{d_1})\}) = S_\lambda(\{\partial\pi(X_1),\ldots,\partial\pi(X_d)\}).$$

By Theorem 2 we have the equality of the joint C^∞-domains

$$D^\infty(\{d\pi(X_1),\ldots,d\pi(X_{d_1})\} = D^\infty(\{d\pi(X_1),\ldots,d\pi(X_d)\}).$$

So

$$S_\lambda(\{d\pi(X_1),\ldots,d\pi(X_{d_1})\} = S_\lambda(\{d\pi(X_1),\ldots,d\pi(X_d)\}).$$

This proves the theorem. □

Remark. Another proof of this theorem has been presented in [tE, page 102].

Now for the Gevrey vectors for unitary representations we can state the same type of corollaries as in Section 2, for example:

Corollary 7 *Let G be a semisimple Lie group with Lie algebra $\mathfrak{g}$. Let π be a unitary representation of G. Let $C \in U(\mathfrak{g})$ be the Casimir element. Suppose there exists $\tau \in \mathbb{C}$ such that $\partial\pi(C) = \tau I$. (For example, π is irreducible.) Let $\mathfrak{g} = \mathfrak{k}+\mathfrak{p}$ be a Cartan decomposition of $\mathfrak{g}$ and let K be a subgroup of G with Lie algebra $\mathfrak{k}$. Then*

$$G_\lambda(\pi) = G_\lambda(\pi|_K).$$

In particular

$$D^\omega(\pi) = D^\omega(\pi|K).$$

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References

[Bru] BRUHAT, F., Sur les représentations induites des groupes de Lie. *Bull. Soc. Math. France* **84** (1956), 97-205.

[tE] ELST, A.F.M. TER, Gevrey spaces related to Lie algebras of operators. Thesis, Eindhoven University of Technology, Eindhoven, The Netherlands, November 1989.

[Fol] FOLLAND, G.B., Introduction to partial differential equations. Princeton University Press, Princeton, New Jersey, 1976.

[Går] GÅRDING, L., Note on continuous representations of Lie groups. *Proc. Nat. Acad. Sc. U.S.A.* **33** (1947), 331-332.

[Goo] GOODMAN, R., One-parameter groups generated by operators in an enveloping algebra. *J. Funct. Anal.* **6** (1970), 218-236.

[GW] GOODMAN, R. AND N.R. WALLACH, Whittaker vectors and conical vectors. *J. Funct. Anal.* **39** (1980), 199-279.

[Nel] NELSON, E., Analytic vectors. *Ann. Math.* **70** (1959), 572-615.

[Pou] POULSEN, N.S., On C^∞-vectors and intertwining bilinear forms for representations of Lie groups. *J. Funct. Anal.* **9** (1972), 87-120.

[Tay] TAYLOR, M.E., Noncommutative harmonic analysis. Math. Surv. and Monographs 22, Amer. Math. Soc., Providence etc., 1986.

[Wil] WILANSKY, A., Modern methods in topological vector spaces. McGraw-Hill, New York etc., 1978.