Some stages in the implementation of Markov decision processes

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Memorandum COSOR 81-13

Some stages in the implementation of Markov Decision Processes

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Eindhoven, October 1981

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Abstract The paper is concerned with applicational aspects of discrete random systems. Such systems appear in a large number of practical areas ranging from purely technical to social and demographic systems. For analyzing such systems the models to be used range from purely descriptive models to optimization models. We will use a simplified example to illustrate how the organizational requirements determine the type of models to be used as well as the way in which they should be incorporated in a system to support the decision making process.

1. Introduction.

The goal of this paper is to discuss some applicational aspects of dynamic systems which show random behaviour. Examples of random systems appear in a large number of practical fields. One may think of typical social or socio-economic phenomena, like a person's family status, his type of housing, his political preference. On the otherhand there are also applications with an industrial, technical or physical nature, where phenomena, like the status of a product in a production process, the inventory level of a certain product, the number of persons waiting in a queue, or the state of an elementary partical in a physical process play an important role.

In this paper we will focus our attention on a class of problems that might arise if one considers discrete random systems that occur in a managerial environment. We will approach these problems from the viewpoint of managers who are interested in the phenomena because analysis might provide valuable information for supporting a decision making process, they are involved in. So, our starting point is the CLASS OF PROBLEMS instead of the mathematical techniques that are used. Consequently, the title of the paper should be DISCRETE RANDOM SYSTEMS instead of refering to Markovian (decision) models. However, it will appear that Markov chains and dynamic programming models appear in a natural way and are practically useful mathematical tools for analyzing reallife discrete random systems.By now, it will be clear that the emphasis in this paper will be on

- model building
- using the models for supporting the integral decision making process.

Of course, the interaction between both subjects will be discussed. We will illustrate our ideas by means of a simple example.

2. The type of support that is needed for several stages in the organizational planning and control cycle.

Planning and control are often separated in literature and by managers for purposes of concentrating on specific aspects. We refer to chapter 17 of (3) for a discussion on this topic. However, it might be clear that these managerial tasks can not really be separated in practice. They both are integral parts of a complete operating cycle for any individual or organization, and their interdependence is evident.

Nevertheless, it might help to make a conceptual separation between different types of decision activities. Anthony (1) distinguishes three types of activities and uses these as a framework for planning and control systems. The three concepts are the following

Strategic planning
Management control
Operational control

These categories often coincide with the three subsystems or levels in an organization which can be characterized as strategic, coordinative and operating. A closely related division is obtained if one classifies the types of decisions according to the time dimension of those decisions. Based on the last criterion the decisons would be:

strategic decisions
tactical decisions
operational decisions.

The "relations" with the hierarchical level is given in figure 1.

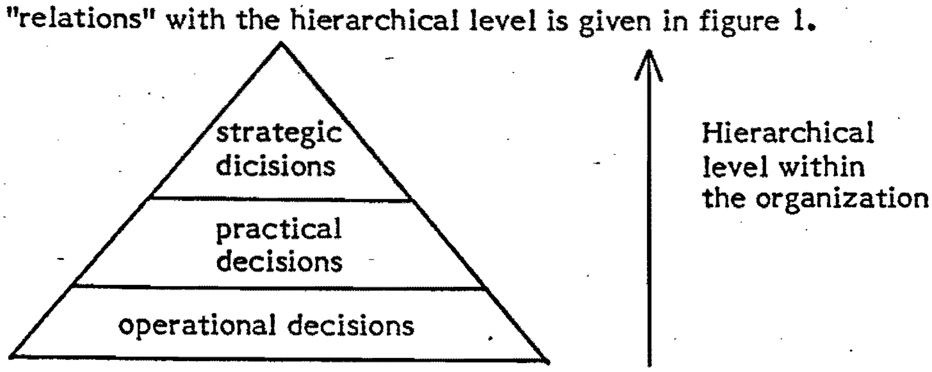

figure 1 Type of decisions versus hierarchical structure of an organization.

Although the above classification is oversimplified and not at all generally applicable, it might quite well help to study the different types of support that is needed for decision making at the different levels. In (1), Anthony gives for each level a number of decision activities which show how the higher level sets boundaries for the lower level. On the other hand, it will be clear that in the planning cycle the possibilities of the lower levels influence the decisions at the higher levels. In fact, strategical decisions might be initiated at the operational level and vice versa.

However, a simplified structure as given in figure 1 can help in clearifying some of our points of view. We will use an oversimplified car insurance problem to illustrate what kind of models is needed to support the decissions at the different levels, as well as to show how these models should be used.

Let us first describe the vehicle insurance problem. At the basic of all decisions to be taken, we have the process of the individual car owner who can decide, whenever a damage occurs, whether he claims or not. Normally, his decision will depend on the size of the damage, the no-claim discount he is currently receiving, the point of time in the year, etc.

Let us for simplicity reasons assume that the discount system that a vehicle insurance company "Simplesure Inc." is currently offering, is as follows. A new client has to pay the full premium. If a client makes no claim during a year, a discount of 20% is granted. If he does not claim for 2 or more years in succession, he receives a discount of 30%. If a client makes a claim, the premium for the next year will be up again to 100% of the full premium, independent of his current discount rate.

Now, we can consider for each level some of the managerial questions that arise. At the operational level, we encountered questions which were closely related to the kind of questions an individual car owner would be interested in.

<u>Some questions at the Operational level.</u>

- What is the optimal claiming policy for the individual car owner?
- What is the average premium to be paid?
- What is the difference in cost between claiming only if the damage is above the no-claim discount and the optimal policy?
- How does the claim limit depend on the point of time in the (insurance) year?
- How sensitive are such advices for changes in the accident rate?
- How do these rates depend on age, area, etc.?
- Should a specific client accept a certain own-risk option?
- Etc.

Of course, there is a large number of questions, in a different area, which are not indicated in the above list. However, we will concentrate on those problems which are closely connected with the claiming process.

At the tactical level decisions of which the impact last for a longer time have to be taken.

Some questions at the tactical level.

- How should premiums be adapted if the accident rate changes?
- What to do if repair costs increase by e.g. 7%, especially, since this will affect the claiming behaviour of clients?
- For which policies, and where, and how should one advertise for new clients?
- Which own-risk options should be adapted if changes in specific costs occur?
- What might be the consequences of publications of consumer organizations on the client claiming behaviour?
- How many new clients should be attracted to fill up the deficiencies that occur by clients that are leaving the company?
- etc.

At the strategical level several long lasting decisions have to be taken.

Some questions at the strategical level.

- Which kind of no-claim discount policies should be used in the future?
- Which kind of own-risk options should be offered?
- Should Simplesure Inc. offer insurance policies depending on age or area?
- Which types of clients should they focus on in the long run?
- etc.

If we consider the kind of questions for the different types of decisions (levels), we will see that there are a number of closely related problems. Nevertheless, some tendencies become clear.

In figure 2 a few of these tendencies are reported as far as they have influence on the models that are required and the way in which these models have to be used to support decision making with respect to the above questions.

	strategic	tactical	operational
horizon	long term planning	mid term planning and control	short term planning and control
models required	models which enable the comparison of different alternative insurance policies	models which enable the analysis group behaviour	models which enable the determination of the individual optimal policy
type of sensitivity analysis required	sensitivity of models	sensitivity of models and parameters	sensitivity of parameters
required aggregation level	high		low
required decomposition level	low		high

figure 2 Some tendencies in the type of models required.

In the next section we will discuss the type of models that can be used.

3. Some comments of the models to be used.

At the operational level a Markov decision model can be used to analyze some of the questions which are relevant for individual clients. Such models are described in more detail by De Leve (5) and Hastings (3). We will give a simplified discrete model.
Suppose we consider the system every 6 months. At these point in time a client can be indentified as being in one of the states 0P100, 0P80, 0P70, 1P100, 1P80 and 1P70. The states 0P100, 0P80 and 0P70 represent the premium level to be paid just before premium is due.
While, the states 1P100, 1P80 and 1P70, represent the premium to be paid next year if no damage is claimed in the coming 6 months. So, a possible realization of an individual process could be as described in figure 3.

The kind of possible decision of the individual client are to decide, depending on the premium level and the period, the amount of damage k above which a claim is made. We suppose k to be an element of a number of claim limits, $k \in \{ 1, 2, 3, \ldots, K \}$.
As a consequence of a decision k, the transition probabilities p^k_{ij} can be defined. A transition will take place from state i tot state j, if the claim limit is chosen to be k, with probability p^k_{ij}. As a consequence of such a decision, the client incurs one stage expected costs r(i,k) = Pr(i) + C(k) which are composed of the premium Pr(i) to be paid at discount level i if the state is 0P100, 0P80 or 0P70, and the expected costs for damages.

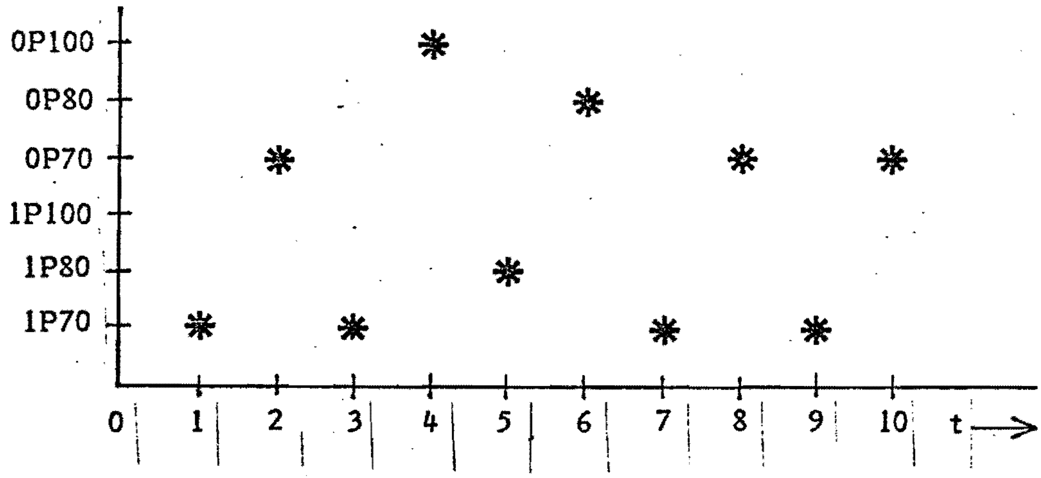

figure 3. A possible individual process realization

Within a model of the above type we can determine the optimal claiming policy for individuals. The average cost for premiums as well as damages can be computed. Moreover, the consequences of changes in the parameters like accident rate, reparation costs, own risk options can be computed. One could study the influence of the time point in the (incurance) year on the claim by comparing the result for more detailed models with a period length of e.g. 1 month instead of 6. So, models with different levels of aggregation might be necessary. Note that at this level one is also interested in comparing the costs of alternative individual policies like, what difference in costs is there between optimal indiviual policies and e.g. the policy of only claiming if the damage is larger then the discount.

Althought the individual should take his decision such that futural savings are taken into account, in an actual decision for claiming a damage or not, the outcomes for Simplesure Inc. are mainly of a short term effect. Systems that can be described by Markov decision models, as indicated above, where the possibility to take actions depending on the actual state, is built into the model, might be classified as controlled systems.

For tactical decisions one needs, together with the above models, models which enable the analyzes of the behaviour of groups of clients. For these goals the cohort models which are based on Markov chains can give valuable support. Let us give an oversimplified example.
Let us, from now on, work with a model where the time unit is one year, instead of 6 months and with state P100, P80 an P70 indicating the premium level in the current year.
Let $N_i(t)$ be the expected number of clients in state i at time t. Then the expected number of clients in state j at time t+1 can be given by (see also Bartholomew (2))

$$N_j(t+1) = R_j(t+1) + \sum_j N_i(t) P_{ij}$$

With P_{ij} the fraction of clients in state i that makes a transition from state i to state j. The quantity $R_j(t+1)$ is the number of new clients which is recruited in state j in the last period.

Cohort type models can be used to analyze e.g. expected profit in year t which equal

$$\sum_i N_i(t) \times (Pr(i) - f_i)$$

with f_i the expected costs for the damages to be paid by the insurance company. So, consequences of changes in Pr(i) and f_i can be analyzed. Moreover, one can study changes in $R_i(t)$. This disaggregation into more categories of clients might be required for the analyzes of changes in the client behaviour (P_{ij}), but also for gathering insight in where and what kind of advertisement should be done. In systems that can be described by the above cohort models, one can see that some control is possible. For example, the recruitment $R_i(t)$ can be influenced. On the other hand, one supposes the clients to behave according to the autonomous transition law determined by P_{ij}. Changes of the P_{ij}'s have to be studied by comparing separate models. In this sense one could speak of controlled autonomous systems.

For strategical decisions design problems play an important role. At this level the way of decision-making is often such that choices have to be made between alternative options that are possible. The criteria for chosing a certain option are often partly qualitative. Nevertheless, quantitative analysis might deliver valuable support. For example, Simplesure Inc. might be interested in comparing the advantages and disadvantages of the following two autonomous systems, represented by their transition diagram.

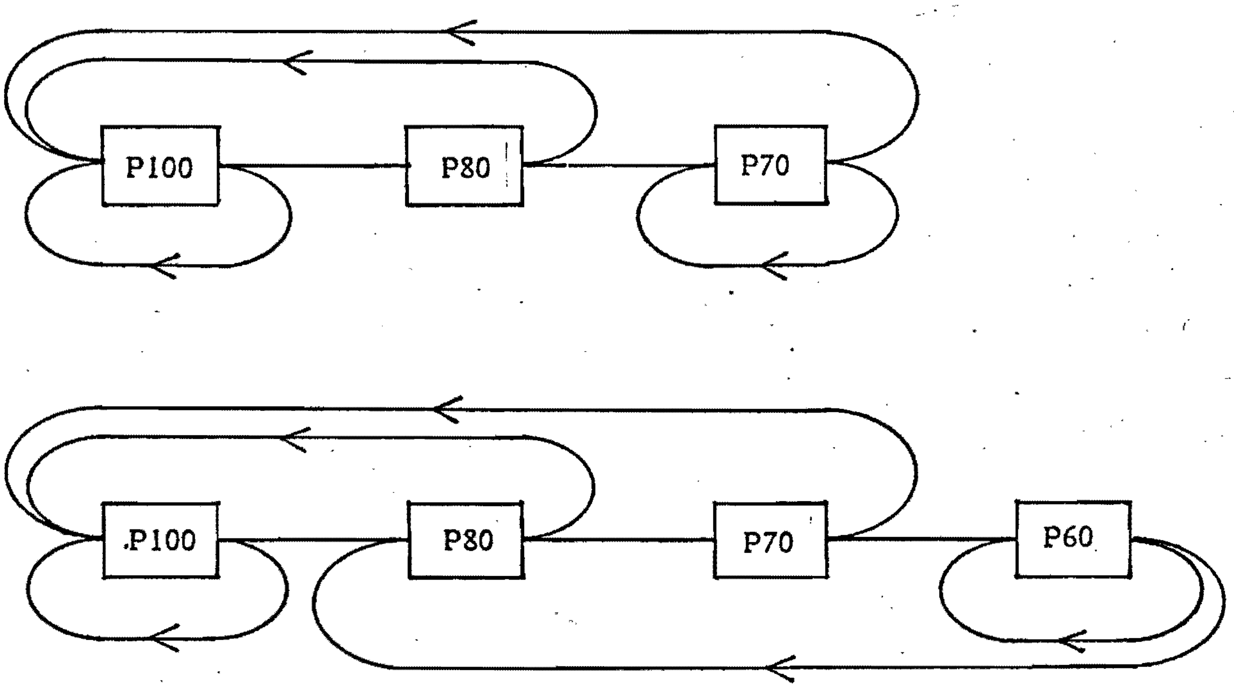

figure 4. Transition diagram for two different discount systems.

In the above (simple) system the different discount levels are indicated. P80, for example means premium level of 80% of the full premium. The arrows indicate the possible transitions.

For comparison of two different premium strategies each model (Markov chain or cohort model) itself has to be evaluated completely. Nevertheless the level of aggregation might be high.

4. Some final comments.

It will be clear that in a practical application several mixtures of controlled systems and autonomous systems might occur for the different types of decision or at the different levels. In fact we indicated examples of mixtures of the required models for each of the levels. However, the apporach described above can be usefull in practical situation since it relates the different types of models with the different types of decisions or with levels in the organization. Especially in situations where one is first involved in only part of the problem, for example those arising at the "tactical level", the approach enables the embedding in the overall decision process, which involves as well "autonomous systems" as "controlled autonomous systems" and "controlled systems".

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