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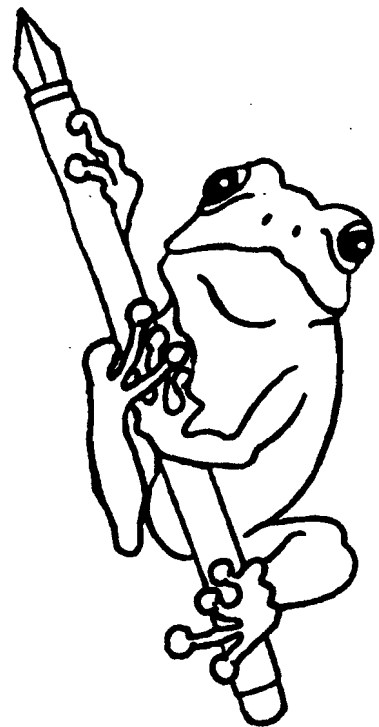
Jacobi-Davidson Methods and Preconditioning with Applications in Pole-zero Analysis

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Summary. The application of Jacobi-Davidson style methods in electric circuit simulation will be discussed in comparison with other iterative methods (Arnoldi) and direct methods (QR, QZ). Preconditioning of the correction equation is used to improve the Jacobi-Davidson process, but also reveals some problems in the correction equation.

1 Introduction

Pole-zero analysis is used in electrical engineering to analyze the stability of electric circuits [6, 9]. For example, if a circuit is designed to be an oscillator, pole-zero analysis is one of the ways to verify that the circuit indeed oscillates. Because the complexity of the circuits designed nowadays grows, there is need for faster algorithms, not neglecting the accuracy. In this paper, Sect. 2 introduces the pole-zero problem. Section 3 gives an overview of conventional methods used in pole-zero analysis, and describes the Jacobi-Davidson style methods as an alternative. In Sect. 4, the methods will be compared by numerical results, concluding with some future research topics.

2 Pole-zero Analysis in Circuit Simulation

The Kirchhoff Current Law and the Kirchhoff Voltage Law describe the topology of an electric circuit. Together with the Branch Constitutive Relations, which reflect the electrical properties of the branches, the two Kirchhoff Laws result in a system of differential algebraic equations [6]:

$$\frac{d}{dt}\mathbf{q}(t,\mathbf{x}) + \mathbf{j}(t,\mathbf{x}) = 0, \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ contains the circuit state and $\mathbf{q},\mathbf{j} : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ are functions representing the reactive and resistive behaviour, respectively. The way (1) is solved depends on the kind of analysis (DC-analysis, AC-analysis, transient analysis, pole-zero

analysis). In every analysis, the capacitance matrix $C \in \mathbb{R}^{n \times n}$ and the conductance matrix $G \in \mathbb{R}^{n \times n}$ appear:

$$C(t,\mathbf{x}) = \frac{\partial \mathbf{q}(t,\mathbf{x})}{\partial \mathbf{x}}, \qquad G(t,\mathbf{x}) = \frac{\partial \mathbf{j}(t,\mathbf{x})}{\partial \mathbf{x}}.$$

Both matrices are real, non-symmetric and sparse.

Starting from a linearization round the DC-operating point, the time-domain formulation is as follows:

$$\begin{cases} C\frac{d\mathbf{x}(t)}{dt} + G\mathbf{x}(t) = \mathbf{e}(t) \\ \mathbf{x}(0) = 0, \end{cases} \tag{2}$$

where $\mathbf{e}(t)$ models the excitation. Because not all properties can be computed in the time domain, the problem is transformed to the frequency domain by applying a Laplace transform:

$$(sC + G)\mathcal{X}(s) = \mathcal{E}(s), \tag{3}$$

where $\mathcal{X}, \mathcal{E}$ are the Laplace-transforms of the variables $\mathbf{x}, \mathbf{e}$ and s is the variable in the frequency domain. The response of the circuit to a variation of the excitation is given by the transfer function

$$\mathcal{H}(s) = (sC + G)^{-1}. \tag{4}$$

The elementary response of circuit variable $\mathcal{X}_o$ to excitation $\mathcal{E}_i$ is given by

$$\mathcal{H}_{oi}(s) = \mathbf{e}_o^T (sC + G)^{-1} \mathbf{e}_i. \tag{5}$$

The poles are the values $p_k \in \mathbb{C}$ that satisfy $\det(p_k C + G) = 0$, hence $(G + p_k C)\mathbf{x} = 0$ for some $\mathbf{x} \neq 0$, which leads to a generalized eigenproblem ($\lambda = -p_k$):

$$G\mathbf{x} = \lambda C\mathbf{x}, \qquad \mathbf{x} \neq 0. \tag{6}$$

Because the problem of computing the zeroes is similar to the problem of computing the poles, the rest of this paper will consider the problem of computing the poles.

Especially for large circuits ($n > 10^4$), robust, iterative methods for the generalized eigenvalue problem (6) with sufficient accuracy and acceptable computational costs are needed. Furthermore, all right half-plane poles and no false right half-plane poles are desired. The dominant poles and zeroes must be accurate enough to produce correct Bode-plots for the frequency range of interest.

Two kinds of pole-zero methods are known in literature [6]: combined and separate pole-zero computation. This paper focuses on separate pole-zero computation.

3 Conventional vs Jacobi-Davidson Style Methods

Given the eigenproblem (6), a choice has to be made for the kind of solver to use. Despite the fact that the dimensions of the problem can be very large ($n > 10^4$), the full-space QR-method is still a popular choice, because of its robustness and accuracy. As problems become even larger, the costs of $O(n^3)$ of the QR-method become unacceptable. Besides that, the transformation to an ordinary eigenproblem, by inverting G, introduces some other problems. Firstly, an LU-decomposition of G

has to be made, which causes inaccuracies and destroys the sparsity of G. Secondly, the transformation may cause numerical instabilities. Thirdly, the transformation distracts the focus from the original problem. The generalized alternative, the QZ-method, does not suffer from these three problems and can detect instabilities where the QR-method cannot. Nevertheless, the QZ-method is used less often than the QR-method, because of its costs, which are three times larger than the costs of the QR-method.

Concerning iterative methods, the implicitly restarted Arnoldi method is often used. The same problems as for the full-space methods hold with respect to the transformation of the generalized problem. Furthermore, the Arnoldi method is designed to compute only a few $(m \ll n)$ eigenvalues and has no advantage if all eigenvalues are desired.

The Jacobi-Davidson method [8], on the other hand, is designed to converge fast to a few selected eigenvalues. Based on the Jacobi-Davidson method, the JDQR-method [5], which computes a partial Schur form, and the JDQZ-method [5], which computes a partial generalized Schur form, are developed. Without going into much detail, the basic idea behind the Jacobi-Davidson methods is as follows. For the problem $Ax = \lambda x$, given the eigenpair approximation $(\theta_k, \mathbf{u}_k)$:

- Search a correction $\mathbf{v} \in \mathbf{u}_k^\perp$ for $\mathbf{u}_k$ such that

$$A(\mathbf{u}_k + \mathbf{v}) = \lambda(\mathbf{u}_k + \mathbf{v}).$$

- Solve $\mathbf{v}$ from the *correction equation*, with $\mathbf{r}_k = A\mathbf{u}_k - \theta_k\mathbf{u}_k$:

$$(I - \mathbf{u}_k\mathbf{u}_k^*)(A - \theta_k I)(I - \mathbf{u}_k\mathbf{u}_k^*)\mathbf{v} = -\mathbf{r}_k.$$

- Orthogonally expand the current basis V with $\mathbf{v}$.

The Ritz-vector $\mathbf{u}_k = V\mathbf{s}$ is obtained by applying a full-space method, for instance the QR-method, to the projected matrix V^*AV, resulting in the eigenpair $(\theta_k, \mathbf{s})$. The Jacobi-Davidson method satisfies a Ritz-Galerkin condition [8].

The correction equation needs more attention. For the JDQR-method, it is

$$(I - QQ^*)(A - \theta_k I)(I - QQ^*)\mathbf{v} = -\mathbf{r}_k, \tag{7}$$

where $Q \in \mathbb{R}^{n \times k}$. If the correction equation is solved exactly, the convergence of the Jacobi-Davidson method is quadratic [8]. Besides solving the correction equation exactly, one can use linear iterative methods, like GMRES, with or without preconditioning. Because exact solvers are often not feasible in practice, the focus is on iterative methods with preconditioning. Using a preconditioner, however, is not as easy as it seems. Consider a preconditioner $K \approx A - \theta_k I$. There are three major issues. Firstly, the preconditioner is projected afterwards $(\tilde{K} = (I - QQ^*)K(I - QQ^*))$. Secondly, $A - \theta_k I$ becomes more and more ill conditioned as the approximations θ_k become near the eigenvalue λ. Thirdly, this θ_k changes every iteration, and so does $A - \theta_k I$. These problems, and the severity of these problems, will be illustrated in the following section by some practical examples.

4 Numerical Results and Conclusions

The data for the test problems was generated by the in-house analog electric circuit simulator Pstar of Philips Research [9]. Both full-space and iterative methods have

been used to solve the ordinary and generalized eigenproblem. A small selection of the results presented in [6] has been made to identify the problems which are typical for the different approaches. Implementations of the Jacobi-Davidson methods are based on the algorithms in [2]. Experiments have been done in Matlab 5.3 [1].

The transformation of the generalized eigenproblem to the ordinary eigenproblem may introduce inaccuracies, as has been mentioned before. Bode-plot (a) in Fig. 1 shows an example of this. The solution computed by QR differs significantly on two points from the exact solution, which is computed by using (5) for several frequencies s. The two notches are caused by non-cancelling poles and zeroes, which do cancel in the original problem. It is conceivable that this is caused by the inversion of G. The iterative methods Arnoldi and JDQR suffer even more from inaccuracies. Bode-plot (b) in Fig. 1 shows the computed solutions for the generalized eigenpoblem. In this case, the QZ-method nearly resembles the exact solution, while the iterative schemes still suffer from inaccuracies. The fact that QZ performs better than QR, while both methods in theory compute the same eigenvalues, strengthens the argument that the inversion of G introduces critical inaccuracies. A general remark
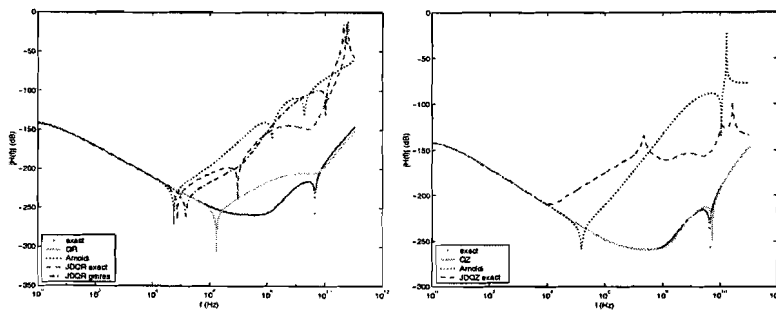

Fig. 1. (a) Bode-plot computed from the ordinary eigenproblem; (b) Bode-plot computed from the generalized eigenproblem.

can be made about the interpretation of Bode-plots. It is not clear how accurate the original data of the circuit is. As a consequence, one may argue that the resulting Bode-plots are only representative up to a certain frequency.

Using preconditioners when solving the correction equation of the JDQR method does indeed improve the speed of convergence, as can be seen in Fig. 2, where graph (a) shows the convergence history when using GMRES as solver, and graph (b) when using GMRES with an ILUT preconditioner ($t = 10^{-8}$). The quality of the improvement strongly depends on the accuracy of the preconditioner. When using an ILUT [7] preconditioner, a drop-tolerance of maximal $t = 10^{-6}$ is acceptable. This shows also one of the difficulties: the preconditioner has to be rather accurate, and in the case of ILU based preconditioners this means in general high costs. Apart from that, the ILU based preconditioners experience problems for singular matrices, and the matrix $A - \theta_k I$ becomes more and more singular. This last problem has appeared to be more severe for the JDQZ method. The fact that the preconditioner is projected afterwards has not a significant influence on the quality.
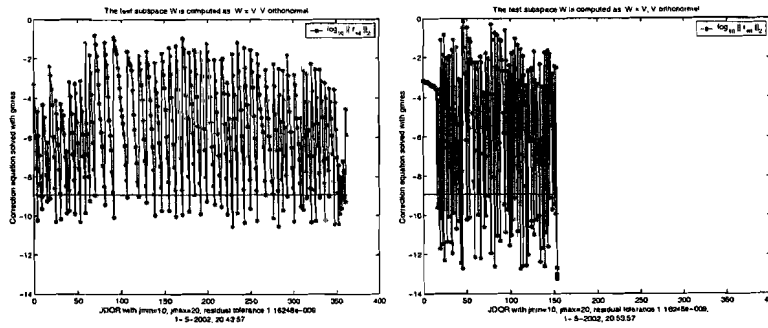
Fig. 2. (a) Convergence history for JDQR with GMRES; (b) Convergence history for JDQR with ILUT-precontioned GMRES ($t = 10^{-8}$). A convergence history plots the residual against the Jacobi-Davidson iteration number; each drop below the tolerance means an accepted eigenvalue.

The observations launch ideas for future work. One can think of efficient updates for preconditioners [3], model reduction techniques and reduction techniques for the generalized eigenproblem, and combinations of Jacobi-Davidson with other iterative methods like Arnoldi or combined pole-zero methods, such as Padé via Lanczos [4].

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