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# Tactical Capacity Management under Capacity Flexibility in Make-to-Stock Systems

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**Abstract:** In many production systems a certain level of flexibility in the production capacity is either inherent or can be acquired. In that case, system costs may be decreased by managing the capacity and inventory in a joint fashion. In this paper we consider such a make-to-stock production environment with flexible capacity subject to periodic review under non-stationary stochastic demand, where our focus is on the tactical level capacity management. We first develop a simple model to represent this relatively complicated problem. Then we provide the solution to the single period problem and elaborate on the general characteristics of the multi period problem. Finally, we develop some useful managerial insights.

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## 1 Introduction

The issue of capacity management is of vital importance in most production systems, especially under demand volatility. In a make-to-stock system with fixed capacity and volatile demand, elevated levels of inventory and/or significant under-utilization of capacity is unavoidable for meeting the demand timely. Nevertheless, in many production systems a certain level of flexibility in the production capacity is either inherent or can be acquired. In that case, system costs may be decreased by managing the capacity and inventory in a joint fashion. In this paper we consider such a make-to-stock production environment with flexible capacity subject to periodic review under non-stationary stochastic demand, where our focus is on the tactical level capacity management.

Capacity can be defined as the total productive capability of all productive resources utilized, such as workforce and machinery. These productive resources can be permanent or

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contingent. We define permanent capacity as the maximum amount of production possible in regular work time by utilizing internal resources of the company such as existing workforce level on the steady payroll or the machinery owned or leased by the company. Total capacity can be increased temporarily by acquiring contingent resources, which can be internal or external, such as hiring temporary workers from external labor supply agencies, subcontracting, overtime production, renting work stations, and so on. We refer to this additional capacity acquired temporarily as the contingent capacity. Capacity flexibility refers to the ability of adjusting the total production capacity in any period with the option of utilizing contingent resources on top of the permanent ones.

The capacity decisions can be in all hierarchies of decision making: strategic, tactical, and operational. Examples of such decisions would be, determining how many production facilities to operate, determining the permanent capacity of a facility, and making contingent capacity adjustments, respectively. Our focus is on the tactical level. In specific, we consider the problem of determining the permanent capacity of a facility, while the operational level integrated capacity and inventory management is executed in an optimal manner. For the ease of exposition, we refer to the workforce capacity setting in some parts, not to mean that our analysis is confined to that environment. A possible application area of the problem we consider can be an environment where the production is mostly determined by the workforce size. This workforce size is flexible, in the sense that temporary (contingent) workers can be hired in any period in addition to the permanent workforce that is fixed through the planning horizon. Contingent workers are paid only for the periods they work, whereas permanent workers are on a payroll. The firm wishes to find the optimal permanent workforce size as well as their optimal operating policies. We assume that the lead time for acquiring contingent labor is zero. Indeed, it takes as short as 1 or 2 days to acquire temporary workers from the external labor supply agencies for the jobs that do not require high skill levels according to our experience. In some cases temporary labor acquisition is actually practically immediate. In some developing countries, abundant temporary labor that are looking for a job and the companies that are in need of temporary labor gather in some venues early in the morning and the companies hire workers that they are going to make use of that very day.

Changing the level of permanent capacity as a means of coping with demand fluctuations, such as hiring and/or firing permanent workers frequently, is not only very costly in general, but it may also have many negative impacts on the company. In case of labor capacity, the social and motivational effects of frequent hiring and firing makes this tool even less

attractive. Utilizing flexible capacity, such as hiring temporary workers from external labor supply agencies, is a means of overcoming these issues, and we consider this as one of the two main operational tools of coping with fluctuating demand, along with holding inventory. Yet, long-term changes in the state of the world can make permanent capacity changes unavoidable. Consequently, we consider the determination of the permanent capacity level as a tactical decision that is made at the beginning of a finite planning horizon and not changed until the end of the horizon. The capacity-related decisions are the determination of the permanent workforce size to utilize through the planning horizon, and the number of temporary workers to hire in each period. The productivity of temporary workers are allowed to be different than that of permanent workers in our model. Our model also allows for the incorporation of setup costs associated with (i) initiating production in each period (production setup cost) and (ii) ordering contingent capacity.

In this paper, we first develop a simple model to represent this relatively complicated problem and we characterize the solution when the setup costs are negligible. For the case with positive setup costs, we provide the solution to the single period problem and we elaborate on the general characteristics of the solution to the multi period problem. Finally, we develop several useful managerial insights. In specific, we investigate the sensitivity of the optimal solution to the changes in parameters, we study the effects of operating under a suboptimal permanent capacity level, and we explore the parameter settings where capacity flexibility is more valuable.

The rest of the paper is organized as follows. We present a review of relevant literature in Section 2 and present our dynamic programming model in Section 3. The optimal policy for the integrated problem is discussed in Section 4 and our computations that result in managerial insights are presented in Section 5. We conclude the paper in Section 6.

## 2 Related Literature

Capacity management problems received attention of several researchers in the relevant literature at all levels of hierarchical decision making process. Van Mieghem (2003) presents a survey of the literature focusing on strategic decisions whereas Wu et al. (2005) review this literature focusing on tactical and operational level decisions. Capacity management problems include, among others, capacity planning in terms of determination of the capacity levels of productive resources and the timing of the capacity adjustments. Since the problem

of concern in this article is a tactical level capacity planning problem coupled with production/inventory decisions, we mainly review articles in the capacity management literature that attempt to exploit the interactions between capacity planning and production/inventory decisions.

The papers by Bradley and Arntzen (1999), Atamturk and Hochbaum (2001), and Rajagopalan and Swaminathan (2001) are among the examples of research that deal with the joint capacity and inventory management problem at tactical and operational levels under deterministic demand assumption. They provide formulations and solution approaches under different problem settings. However, these approaches do not apply to our problem since we consider stochastic demand in our model.

There are also a number of studies in this stream of research that assume stochastic demand. Zhang et al. (2004) exploit the tradeoff between capacity expansion and lost sales costs in an environment where multiple products and machine types exist, the demand is non-stationary, and inventory holding and backordering are not allowed. The problem is formulated as a max-flow, min-cut stochastic programming problem. Bradley and Glynn (2002) deal with a continuous-review problem where the fixed capacity level of the productive resources are to be determined as well as the production quantities in a single item, infinite horizon environment. It is assumed that the item is replenished according to the base stock policy and that the capacity level is not subject to changes, permanent or temporary. Bradley (2004) extends this model to the case where the capacity level can be increased temporarily with the use of subcontracting, which is similar to the use of contingent capacity in our model. The author assumes that the items are replenished with a suboptimal dual base stock policy, whereas we adopt the optimal replenishment and flexible capacity management policy in our model in a periodic review finite horizon environment. Tan and Gershwin (2004) deal with a similar make-to-stock environment where the demand exceeding the current capacity may be satisfied from one of the subcontractors available, with reduced profits. The demand rate is assumed to have two states as high and low and is dependent on the current backordering level. The decision variables are the production rates of the in-house production and that of the subcontractors. Authors prove that there exist a series of threshold levels in the optimal policy for in house production and for each subcontractor that indicate when there is sufficient surplus and where there is need to use the subcontractors. Cheng et al. (2004) deal with a single item problem without contingent capacity option where the firm determines a fixed capacity level to be used in a medium term planning horizon that cannot be changed

through this horizon with the option of expanding or contracting the capacity when starting the next planning horizon. The authors characterize the optimal capacity management policy. Van Mieghem and Rudi (2002) deal with the problem of determining the optimal capacity and the base stock levels in a single period multi resource problem. The authors extend this problem to the multi period case and show that the myopic policy is optimal when the unmet demand is lost and they also provide the conditions where the myopic policy is still optimal for the backordering case. Angelus and Porteus (2002) deal with the joint capacity and inventory management problem of a short-life-cycle product. The demand is assumed to exhibit a stochastically increasing structure followed by a stochastically decreasing nature. When no inventory carry-over is permitted from one period to the other, authors characterize the optimal capacity management policy which dictates to increase the capacity level to a pre-specified lower capacity limit if the initial capacity level is less than this limit and to decrease it to an upper capacity limit if the initial capacity level is more than this limit. When the inventory carry-overs are permitted the optimal capacity plan is again a target interval policy under certain assumptions, with the critical capacity limits being functions of the inventory levels.

Tan and Alp (2005), Hu et al. (2004) and Yang et al. (2005) deal with the characterization of the optimal capacity planning and inventory decisions when the capacity levels can be increased temporarily by the use of contingent resources, under different problem settings where they do not deal with the problem of optimization of the initial capacity level. Pinker and Larson (2003) and Kouvelis and Milner (2002) deal with problems where the starting capacity levels are optimized with inventory carry overs not allowed.

### **3 Model Formulation**

In this section, we present a finite horizon dynamic programming model to formulate the problem under consideration. Unmet demand is assumed to be fully backlogged. The relevant costs in our environment are inventory holding and backorder costs, unit cost of permanent and contingent capacity, setup cost of production, and fixed cost of ordering contingent capacity, all of which are non-negative. We assume that there is an infinite supply of contingent workers, raw material is always available, and the lead time of production and acquiring contingent capacity can be neglected. The notation is introduced as need arises, but we summarize our major notation in Table 1 for the ease of reference.

Table 1: Summary of Notation

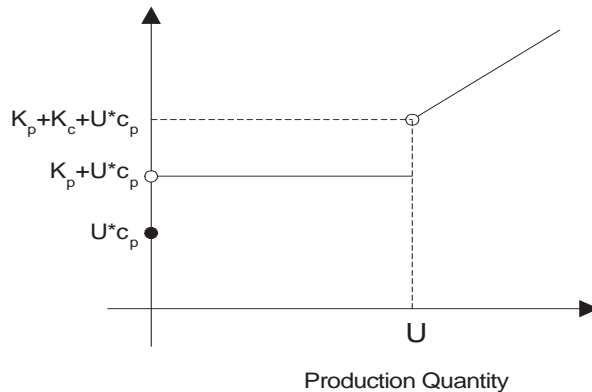
$T$	: Number of periods in the planning horizon
$K_p$	: Production setup cost
$K_c$	: Fixed cost of ordering contingent capacity
$c_p$	: Unit cost of permanent capacity per period
$c_c$	: Unit cost of contingent capacity per period
$h$	: inventory holding cost per unit per period
$b$	: penalty cost per unit of backorder per period
$\alpha$	: discounting factor ( $0 < \alpha \leq 1$ )
$Q_t$	: Number of items produced in period $t$
$W_t$	: Random variable denoting the demand in period $t$
$G_t(w)$	: Distribution function of $W_t$
$U$	: Size of the permanent capacity
$x_t$	: Inventory position at the beginning of period $t$ before ordering
$y_t$	: Inventory position in period $t$ after ordering
$f_t(U, x_t)$	: minimum total expected cost of operating the system in periods $t, t + 1, \dots, T$ , given the system state $(U, x_t)$

We consider a production cost component which is a linear function of permanent capacity in order to represent the costs that do not depend on the production quantity (even when there is no production), such as the salaries of permanent workers. That is, each unit of permanent capacity costs  $c_p$  per period, and the total cost of permanent capacity per period is  $Uc_p$ , for a permanent capacity of size  $U$ , independent of the production quantity. We do not consider material-related costs in our analysis. In order to synchronize the production quantity with the number of workers, we redefine the “unit production” as the number of actual units that an average permanent worker can produce; that is, the production capacity due to  $U$  permanent workers is  $U$  “unit”s per period. We also define the cost of production by temporary workers in the same unit basis, where the cost for flexible workers is related to their productivity. In particular, let  $c'_c$  be the hiring cost of a temporary labor per period, and let  $c''_c$  denote all other relevant variable costs associated with production by temporary workers per period. It is possible that the productivity rates of permanent and temporary workers are different. Let  $\gamma$  be the average productivity rate of temporary workers, relative to the productivity of permanent workers; that is, each temporary worker produces  $\gamma$  units per period. Assuming that this rate remains approximately the same in time, the unit production cost by temporary workers,  $c_c$ , can be written as  $c_c = (c'_c + c''_c)/\gamma$ . It is likely that  $0 < \gamma < 1$ , but the model holds for any  $\gamma > 0$ .

For the sake of generality, we allow for non-negative fixed costs, both for production and contingent capacity ordering. Let  $K_p$  denote the production setup cost and  $K_c$  denote the fixed cost of ordering contingent capacity.  $K_p$  is charged whenever production is initiated, even if the permanent workforce size is zero and all production is due to temporary workers. Therefore, together with the structure of the unit permanent capacity costs, this implies that it is never optimal to order contingent capacity unless permanent capacity is fully utilized. On the other hand,  $K_c$  is charged only when temporary workers are ordered, independent of the amount. Fixed costs of contacting external labor supply agencies and training costs may be among the drivers of  $K_c$ . We ignore the costs that may be associated with acquiring permanent capacity that is incurred at the beginning of the planning horizon.

Under these settings, it turns out that the production quantity of a period,  $Q_t$ , is sufficient to determine the number of temporary workers to be ordered in that period,  $m_t$ , for any level of permanent capacity determined at the beginning of the planning horizon. In specific,  $m_t = [(Q_t - U)^+ / \gamma]$ , ignoring integrality, where  $(\cdot)^+$  denotes the value of the argument inside if it is positive and assumes a value of zero otherwise. Consequently, the problem translates into a production/inventory problem where the level of capacity is a decision variable and the production cost is piecewise linear, which is neither convex nor concave under positive fixed costs. See Figure 1 for an illustration. Note that when  $K_p$  and  $K_c$  are both zero, this function is convex.

Figure 1: Production Cost Function Under Positive Setup Costs



The order of events is as follows. At the beginning of the first period, the permanent capacity level  $U$  is determined. At the beginning of each period  $t$ , the initial inventory level



$x_t$  is observed, the production decision is made and the inventory level is raised to  $y_t$  by utilizing the necessary capacity means; that is, if  $y_t \leq x_t + U$  then only permanent capacity is utilized, otherwise a contingent capacity of size  $m_t = [(y_t - x_t - U)^+/\gamma]$  is hired on top of full permanent capacity usage. At the end of period  $t$ , the demand  $d_t$  is met/backlogged, resulting in  $x_{t+1} = y_t - d_t$ . We denote the random variable corresponding to the demand in period  $t$  as  $W_t$  and its distribution function as  $G_t(w)$ . The state of the system consists of the permanent capacity level and the initial inventory level,  $(U, x_t)$ . Denoting the minimum cost of operating the system from the beginning of period  $t$  until the end of the planning horizon as  $f_t(U, x_t)$ , we use the following dynamic programming formulation to solve the integrated Capacity and Inventory Management Problem (CIMP):

$$f_t(U, x_t) = U c_p + \min_{y_t: x_t \leq y_t} \left\{ K_p \delta(y_t - x_t) + K_c \delta(y_t - x_t - U) + [y_t - x_t - U]^+ c_c + L_t(y_t) + \alpha E [f_{t+1}(U, y_t - W_t)] \right\} \quad \text{for } t = 1, 2, \dots, T$$

where  $L_t(y_t) = h \int_0^{y_t} (y_t - w) dG_t(w) + b \int_{y_t}^{\infty} (w - y_t) dG_t(w)$  is the regular loss function,  $\delta(\cdot)$  is the function that attains the value 1 if its argument is positive, and zero otherwise, and the ending condition is defined as  $f_{T+1}(U, x_{T+1}) = 0$ .

## 4 Analysis

### 4.1 Analysis with no Setup Costs

We first handle the case where the set-up costs are negligible. Tan and Alp (2005) show that the optimal operational policy for any given permanent capacity level is of state-dependent order-up-to type, where the optimal order-up-to level,  $y_t^*(x_t)$ , is

$$y_t^*(x_t) = \begin{cases} y_t^c & \text{if } x_t \leq y_t^c - U \\ x_t + U & \text{if } y_t^c - U \leq x_t \leq y_t^u - U \\ y_t^u & \text{if } y_t^u - U \leq x_t \leq y_t^u \\ x_t & \text{if } y_t^u \leq x_t \end{cases} \quad (1)$$

and  $y_t^u$  and  $y_t^c$  are the minimizers of the functions  $J_t^u(y) = L_t(y) + \alpha E [f_{t+1}(y - W_t)]$  and  $J_t^c(y|x) = J_t^u(y) + c_c(y - x - U)$ , respectively.

The following result is useful in determining the optimal level of the permanent capacity.

**Theorem 1**  $f_t(U, x_t)$  is jointly convex in  $U$  and  $x_t$ .

**Proof :** By induction. Let  $J_t(U, y_t) = c_c[y_t - x_t - U]^+ + L_t(y_t) + \alpha E[f_{t+1}(U, y_t - W_t)]$ . We first note that, for any pair of  $(U^1, x_t^1)$ ,  $(U^2, x_t^2)$ , where  $U^1 \geq 0$  and  $U^2 \geq 0$ , and every scalar  $\lambda \in [0, 1]$ , defining  $(U, x_t) \equiv \lambda(U^1, x_t^1) + (1 - \lambda)(U^2, x_t^2)$ , we have  $[y_t - x_t - U]^+ = [\lambda[y_t - x_t^1 - U^1] + (1 - \lambda)[y_t - x_t^2 - U^2]]^+ \leq \lambda[y_t - x_t^1 - U^1]^+ + (1 - \lambda)[y_t - x_t^2 - U^2]^+$ , which shows that  $[y_t - x_t - U]^+$  is convex. Then, following standard induction arguments and noting that convexity is preserved through the minimization operation, one can show that  $J$ , and consequently  $f$  are convex.  $\square$

Consequently, one can search for the optimal permanent capacity level using this convexity result for any starting inventory level. We next consider the single period problem and provide the solution explicitly, which is of newsboy-type. While the optimal tactical level capacity determination problem implies a multi period setting in the problem environment we have discussed, there are some useful insights that can be gained from the analytical solution that the single period problem brings. We suppress the time subscript in the analysis of the single period problem.

**Theorem 2** *The optimal permanent capacity level of the single period problem is given by*

$$U^* = \begin{cases} 0 & \text{if } c_p \geq c_c \\ (G^{-1}(\frac{b-c_p}{h+b}) - x)^+ & \text{if } c_p < c_c \end{cases}$$

**Proof :** The optimal replenishment policy stated in (1) implies that

$$\begin{aligned} f(U, x) &= U c_p + \min_{y: x \leq y} \{ (y - x - U)^+ c_c + L(y) \} \\ &= U c_p + \begin{cases} c_c(y^c - x - U) + L(y^c) & \text{if } U \leq y^c - x \\ L(x + U) & \text{if } y^c - x \leq U \leq y^u - x \\ L(y^u) & \text{if } 0 \leq y^u - x \leq U \\ L(x) & \text{if } y^u - x \leq 0 \leq U \end{cases} \\ &= U c_p + \begin{cases} c_c(y^c - x - U) + L(y^c) & \text{if } U \leq y^c - x \\ L(x + U) & \text{if } y^c - x \leq U \leq y^u - x \\ L(\max\{y^u, x\}) & \text{if } y^u - x \leq U. \end{cases} \end{aligned}$$

Then,

$$\frac{\partial f(U, x)}{\partial U} = c_p + \begin{cases} -c_c & \text{if } U \leq y^c - x \\ (h + b)G(x + U) - b & \text{if } y^c - x \leq U \leq y^u - x \\ 0 & \text{if } y^u - x \leq U. \end{cases}$$

When  $c_p \geq c_c$ ,  $\frac{\partial f(U, x)}{\partial U}$  is positive in the first and third regions, and because of convexity it must also be positive in the second region. Therefore, if  $c_p \geq c_c$  then  $U^* = 0$ .

When  $c_p < c_c$ ,  $\frac{\partial f(U,x)}{\partial U}$  is negative in the first region and positive in the third region. Therefore, the sign must switch from negative to positive at a particular point in the second region due to convexity. Consequently, noting also that  $U$  must be nonnegative,

$$U^* = \left( G^{-1} \left( \frac{b - c_p}{h + b} \right) - x \right)^+ . \quad \square$$

Note that  $U^*$  is independent of  $c_c$  as long as  $c_p < c_c$ , because no contingent capacity would be used in the single period problem in that case. Note also that  $U^*$  is decreasing in  $c_p$ , which means that expensive permanent resources result in a smaller permanent capacity. If  $c_p < c_c$ , then  $y^c - x \leq U^* \leq y^u - x$ , where  $y^u = G^{-1} \left( \frac{b}{h+b} \right)$  and  $y^c = G^{-1} \left( \frac{b-c_c}{h+b} \right)$ . Consequently,  $y^*(x) = x + U^*$ , which implies that in a single period problem the optimal policy is first to install a permanent capacity of  $U^*$  and then to produce in full terms without hiring any contingent capacity. If the unit cost of permanent capacity exceeds that of the contingent one, then it is optimal to hold no permanent capacity at all and to utilize only the contingent resources to produce up to  $y^c$ .

Finally in this section, we analyze the behavior of the optimal permanent capacity level as a function of the number of periods in the planning horizon. We show in Table 2, by the use of a stationary problem instance, that there exists no monotonic relation between the two. In this particular example, the optimal capacity level first increases and then decreases as the length of the planning horizon increases. We also observe that there are some other problem instances where  $U^*$  either monotonically increases or decreases where such a relation depends on the problem parameters.

Table 2: Optimal Capacity Levels vs Length of the Horizon when  $h = 1, b = 7, c_p = 1.5, c_c = 3, \alpha = 0.99$ , and  $W_t$  is Poisson with  $E[W_t] = 10$  for all  $t$

$T$	1	2	3	4	5	6	7	8	9	10
$U^*$	11	12	12	11	11	10	10	10	10	10

## 4.2 Analysis with Setup Costs

In this section, we analyze the problem when both setup costs are positive. First we present our results on the optimal capacity level of a single period problem. Upon solving the single period problem we elaborate on the structural properties of the optimal solution in the multi period problem.

Let  $Q^p$  denote the production that is conducted solely by making use of permanent capacity,  $Q^c$  denote the production that is conducted solely by making use of contingent capacity, and let  $Q = Q^p + Q^c$  denote the total production. Then,

**Lemma 1** *In the optimal solution to the single period problem,  $U = Q^p$ .*

**Proof :** First note that  $Q^p \leq U$  by the definition of  $Q^p$ . The total cost of the capacity installed,  $Uc_p$ , does not depend on the production quantity and the production cost increases by  $c_p$  per each unit of capacity that is installed but not used. Hence,  $U = Q^p$ .  $\square$

Lemma 1 states that the permanent capacity to be installed, if any, must be equal to the production that is wanted to be conducted with that capacity. That is, there should be no under-utilization in the optimal solution.

The following complementarity result is useful in solving the single period problem.

**Lemma 2** *In the optimal solution to the single period problem,  $Q^p Q^c = 0$ .*

**Proof :** By contradiction. Let  $\hat{Q} = \hat{Q}^p + \hat{Q}^c$  be an optimal solution, where  $\hat{Q}^p > 0$  and  $\hat{Q}^c > 0$ . Then, the production cost associated with  $\hat{Q}$  is  $PC(\hat{Q}) = K_p + K_c + \hat{Q}^p c_p + \hat{Q}^c c_c$ , since  $U = Q^p$  from Lemma 1. Now consider two alternative ways of producing the same quantity:  $\dot{Q} = \dot{Q}^p + \dot{Q}^c = \hat{Q}$  with  $\dot{Q}^c = 0$  and  $\ddot{Q} = \ddot{Q}^p + \ddot{Q}^c = \hat{Q}$  with  $\ddot{Q}^p = 0$ . Then, the production cost associated with  $\dot{Q}$  is  $PC(\dot{Q}) = K_p + \dot{Q}^p c_p$ , and that with  $\ddot{Q}$  is  $PC(\ddot{Q}) = K_p + K_c + \ddot{Q}^c c_c$ . If  $c_p \leq c_c$ , then  $PC(\dot{Q}) < PC(\hat{Q})$ , since  $K_c > 0$ . If  $c_p > c_c$  then  $PC(\ddot{Q}) < PC(\hat{Q})$ . Consequently,  $\hat{Q}^p > 0$  and  $\hat{Q}^c > 0$  cannot be an optimal solution. Note also that if  $c_p > c_c$  and  $K_c > (c_p - c_c)\hat{Q}^c$ , then  $PC(\dot{Q}) < PC(\hat{Q})$ . Hence,  $Q^p Q^c = 0$ .  $\square$

Lemma 2 states that the production will be due to one type of capacity only, either permanent or contingent. That is, the optimal solution is either to set the permanent capacity to the level of desired production and not to utilize any contingent capacity, or to set the permanent capacity level to zero and conduct all production with contingent capacity, depending on the cost parameters.

Let  $y^p = G^{-1}\left(\frac{b-c_p}{h+b}\right)$  and recall that  $y^c = G^{-1}\left(\frac{b-c_c}{h+b}\right)$ , and  $y^u = G^{-1}\left(\frac{b}{h+b}\right)$ . Define two auxiliary functions as

$$\begin{aligned} s^c(x) &= \min\{s : L(s) = K_p + K_c + L(y^c) + c_c(y_c - x)^+\} \text{ and} \\ s^p(x) &= \min\{s : L(s) = K_p + c_p(y^p - x)^+ + L(y^p)\}. \end{aligned}$$

The following theorem characterizes the optimal capacity level and the production quantity of a single period problem.

**Theorem 3** Let  $\tilde{U} = y^p - x$  for a starting inventory level of  $x$ . Then, the optimal capacity and order up to levels,  $(U^*, y^*)$ , of a single period problem can be characterized as follows

**Case 1.**  $c_p \leq c_c$ :

$$(U^*, y^*) = \begin{cases} (\tilde{U}, y^p) & \text{if } x \leq s^p(x) \\ (0, x) & \text{otherwise} \end{cases}$$

**Case 2.**  $c_p > c_c$ :

$$(U^*, y^*) = \begin{cases} (0, y^c) & \text{if } x \leq s^c(x) \text{ and } s^p(x) \leq s^c(x) \\ (\tilde{U}, y^p) & \text{if } x \leq s^p(x) \text{ and } s^c(x) \leq s^p(x) \\ (0, x) & \text{otherwise} \end{cases}$$

**Proof :** Due to Lemmas 1 and 2, we have either one of the following in the optimal solution:

- (i) Production with only permanent capacity:  $U > 0, Q^p = U$ , and  $Q^c = 0$
- (ii) Production with only contingent capacity:  $U = 0, Q^p = 0$ , and  $Q^c > 0$
- (iii) No production:  $U = Q^p = Q^c = 0$

By comparing the costs incurring in each of the above situations, we prove the optimality for each case. If the optimal solution has the form of (i) then the cost function is  $f^{(i)}(U, x) = K_p + U c_p + L(x + U)$  since  $y^* = x + Q^p = x + U$ . The optimal  $U$  value that minimizes this function is given by  $\tilde{U} = G^{-1} \left( \frac{b - c_p}{h + b} \right) - x = y^p - x$ . By noting that  $U > 0$ , the optimal cost function can be rewritten as  $f^{(i)}(\tilde{U}, x) = K_p + \tilde{U} c_p + L(x + \tilde{U}) = K_p + c_p (y^p - x)^+ + L(y^p)$ . In this case, we have  $(U^*, y^*) = (\tilde{U}, x + \tilde{U}) = (\tilde{U}, y^p)$ . If the optimal solution has the form of (ii) then the cost function is  $f^{(ii)}(0, x) = \min_{y: y \geq x} \{K_p + K_c + c_c(y - x) + L(y)\}$ . It can be shown that  $y^c = G^{-1} \left( \frac{b - c_c}{h + b} \right)$  is the unconstrained minimizer of the function inside the minimization. Hence, provided that  $Q^c > 0$ , we have  $f^{(ii)}(0, x) = K_p + K_c + c_c(y^c - x) + L(y^c)$  and  $(U^*, y^*) = (0, y^c)$ . If the optimal solution has the form of (iii) then  $f^{(iii)}(0, x) = L(x)$  and  $(U^*, y^*) = (0, x)$ .

*Case 1.*  $c_p \leq c_c$ : In such a case only (i) and (iii) are viable options. Since  $L(y)$  is convex,  $y^p \leq y^u$ , and  $y^u$  is the minimizer of  $L(y)$ , if  $x \leq s^p(x) \leq y^p$  then  $f^{(iii)}(0, x) = L(x) \geq L(s^p(x)) = K_p + c_p(y^p - x)^+ + L(y^p) = f^{(i)}(\tilde{U}, x)$ . Hence  $(U^*, y^*) = (\tilde{U}, y^p)$ . The result of the other condition ( $x > s^p(x)$ ) can be shown in the same manner.

*Case 2.*  $c_p > c_c$ : In such a case all three options (i), (ii), and (iii) are viable. In this case we have  $y^p \leq y^c \leq y^u$ . Note also that  $s^c(x) \leq y^u$  by definition. If  $x \leq s^c(x)$  and  $s^p(x) \leq s^c(x)$  then  $L(s^p(x)) = K_p + c_p(y^p - x)^+ + L(y^p) = f^{(i)}(\tilde{U}, x) \geq L(s^c(x)) = K_p + K_c +$

$L(y^c) + c_c(y^c - x)^+ = f^{(ii)}(0, x)$ . Similarly,  $L(x) = f^{(iii)}(0, x) \geq L(s^c(x)) = f^{(ii)}(0, x)$ . Hence  $(U^*, y^*) = (0, y^c)$ . The results of the other conditions can be shown in a similar way.  $\square$

Theorem 3 suggests that

- (i) If the unit cost of permanent capacity is less than that of contingent capacity, then no contingent capacity should be utilized.
- (ii) The optimal permanent capacity level is either  $\tilde{U}$  or 0.
- (iii) Contingent capacity will only be utilized if its unit cost is cheaper than that of the permanent capacity and the required production quantity is high enough to compensate for the extra set-up cost that will be incurred for utilizing contingent capacity, in which case no permanent capacity will be installed. In that case the inventory level after production will be higher than that with the alternative option of producing with the permanent capacity.

The following result is an implementation of Theorem 3 when the starting inventory level is zero.

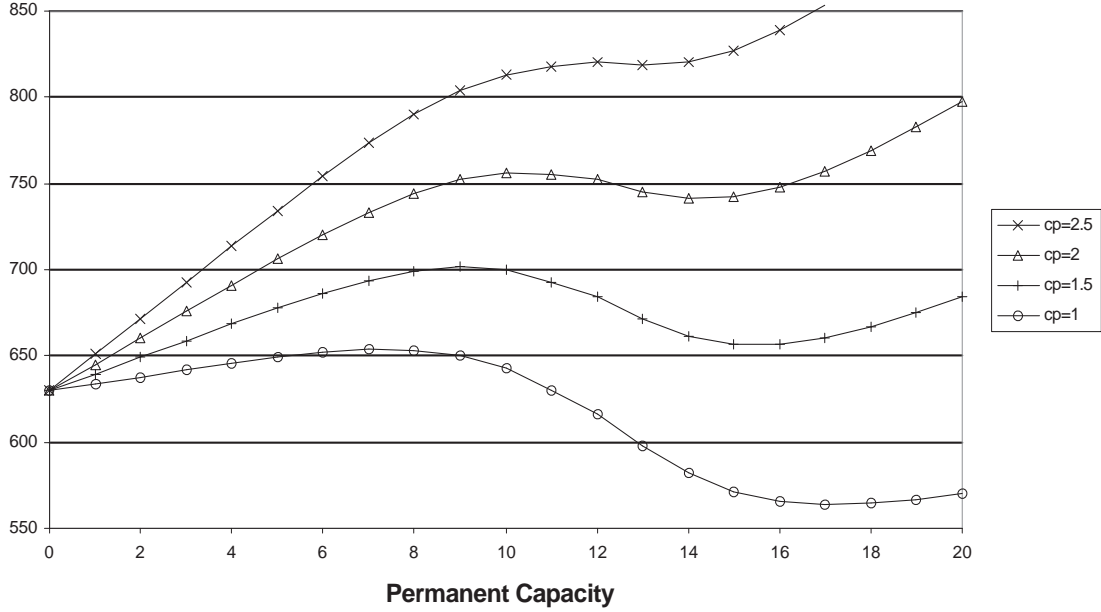
**Corollary 1** *When  $c_p \leq c_c$  and the starting inventory level is zero then*

$$U^* = \begin{cases} y^p & \text{if } E[W] \geq \frac{c_p y^p + K_p + L(y^p)}{b} \\ 0 & \text{otherwise.} \end{cases}.$$

When the starting inventory level is zero, it is optimal to make production and install permanent capacity if the expected demand is greater than a pre-specified value given by the problem parameters and distribution of demand. Otherwise production is not economical.

When we have more than one period in the planning horizon, it is not possible to obtain exact expressions for the optimal capacity levels. It is shown in Section 4.1 that the expected total costs of the system is convex in the permanent capacity levels under the lack of setup costs. This result enables us to determine the optimal permanent capacity level easily. Yet, one could expect a similar behavior of the expected total cost function under the existence of positive setup costs, since extremely high and low levels of capacity would still be more costly than an intermediate level. This intuition turns out to be incorrect, as we demonstrate in Figure 2. This figure denotes the expected total costs of the system for varying permanent capacity levels with problem parameters of  $K_p = 40$ ,  $K_c = 20$ ,  $N = 12$ ,  $b = 5$ ,  $h = 1$ ,  $c_c = 2.5$ ,  $\alpha = 0.99$  and a seasonal Poisson demand pattern with a cycle of four periods with expected demand values of 10, 15, 10, and 5, respectively.

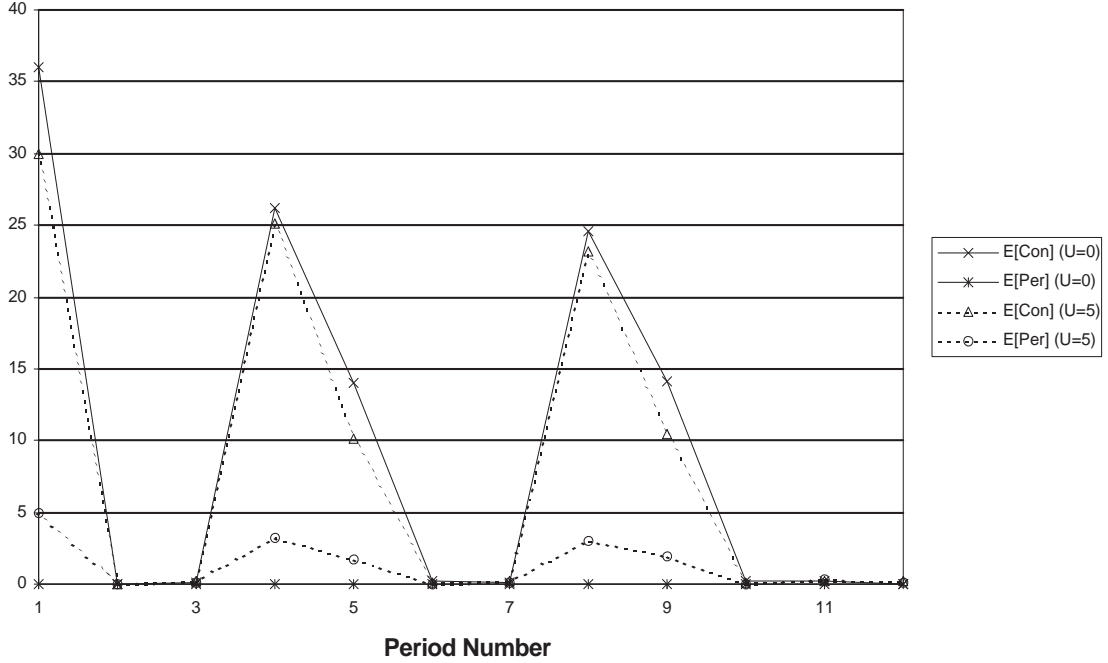
Figure 2: Expected Total Costs vs Permanent Capacity



The reason why convex structure does not hold anymore is as follows. If the system is working under a very low or zero permanent capacity, then the only way of avoiding both of the set-up costs in every period -other than cumulative backlogging of the demand- is to operate with elevated inventory levels by making use of contingent capacity in large amounts every time production is initiated, followed by a number of periods with no production. In that case, a marginal increase in permanent capacity may increase the system costs despite the decreased production costs in the periods where there is positive production, because that capacity will be paid in the periods with no production as well. However, as the permanent capacity level becomes sufficiently high, the total costs may decrease due to decreased production costs, since permanent capacity would be utilized most of the time. Finally, as the permanent capacity becomes excessively high, the system costs will increase due to low utilization. Consequently, we conjecture that the expected total cost function is (no more complex than) quasi-concave/-convex<sup>1</sup> in the permanent capacity level for a given starting inventory level.

<sup>1</sup>Although Figure 2 gives the impression that the expected total cost function is (strictly) concave/convex, we encountered some cases in our computations where there only exists a quasi-concave/-convex structure due to the complex dynamics brought by positive setup costs.

Figure 3: Expected Production under Low Permanent Capacity



We illustrate the reason of the cost behavior that we discussed above for low permanent capacity levels in Figure 3. This figure depicts the expected production quantities under two different values of low permanent capacity, namely  $U = 0$  and 5, where the other parameters are the same as those we reported for Figure 2, with  $c_p = 1$ .  $E[Per]$  denotes the expected production by permanent resources, and  $E[Con]$  denotes that by contingent resources. Expected production quantities are found by simulating the system. Observe that no production takes place in the majority of the periods in both cases.

We note that there is one local maxima on each graph of Figure 2 where cheaper resources of permanent capacity start paying off the setup costs and two local minima, one at  $U = 0$  and the other at the point where accumulation of ample permanent capacity starts being too costly. Naturally, there are some parameter values where the quasi-concave part of this quasi-concave/-convex behavior would not be observed. In specific, a quasi-convex structure would hold for sufficiently low values of set-up costs, where the limiting case is the one with no set-up costs discussed in Section 4.1. Similarly, there would not be a local maxima for sufficiently high values of  $c_p$ , since the permanent capacity costs would then dominate all other costs.



Establishing the exact form of the expected total cost function requires full characterization of the optimal ordering policies, which is difficult even for some special cases. Therefore, we apply explicit enumeration in our computations. Yet, provided that the structure mentioned above holds (which is the case in the numerical tests we conducted), the optimal permanent capacity level should be either zero or it corresponds to the local minima. Consequently, one could find the local minima by a suitable search method and then the expected total costs at this level and at zero level of permanent capacity can be compared to select the optimal size.

In case of positive setup costs, the non-monotonic relation between the optimal capacity levels and the length of the planning horizon naturally persists with even more erratic structure as a result of the setup costs. Table 3 exhibits such an example on a stationary problem instance. The optimal capacity level of a single period problem is 12 which is slightly higher than the expected demand whereas it jumps to 20 for a two period problem. In this case, keeping a larger permanent capacity level but initiating production only once is preferred to keeping a lower permanent capacity level and initiating production twice in the optimal solution as a result of the tradeoffs between the cost components of this specific problem instance.

Table 3: Optimal Capacity Levels vs Length of the Horizon when  $K_p = 50, K_c = 10, h = 1, b = 10, c_p = 1.5, c_c = 3, \alpha = 0.99$ , and  $W_t$  is Poisson with  $E[W_t] = 10$  for all  $t$

$T$	1	2	3	4	5	6	7	8	9	10
$U^*$	12	20	15	0	16	16	0	0	16	0

For planning horizons with more than three periods, we observe an interesting structure where the optimal capacity level alternates between 0 and 16. In these situations, the expected total cost of the local minima (see Figure 2)  $U = 0$  and  $U = 16$  are very close to each other and they exhibit structurally different operating characteristics. In Table 4, the expected production by permanent and contingent resources in each period are presented for the same problem instance with  $T = 5$ . When  $U = 0$ , all production is due to contingent resources and in order to alleviate the effect of large fixed costs in the system, there is only one major production setup scheduled in the first period followed by occasional production instances towards the end of the horizon. As a matter of fact, the optimal operating policy is holding a permanent capacity of size 16 and making more frequent production with permanent resources by almost neglecting the use of contingent capacity.

Table 4: Expected Production by Permanent and Contingent Resources for a 5-Period Problem

	$U = 0$		$U = 16$	
t	E[Perm Prod] <sup>a</sup>	E[Cont Prod] <sup>b</sup>	E[Perm Prod]	E[Cont Prod]
1	0	45	16	0
2	0	0	13.91	0
3	0	0.01	6.49	0.01
4	0	1.72	11.18	0.09
5	0	1.26	3.56	0

<sup>a</sup>Expected Production with Permanent Resources

<sup>b</sup>Expected Production with Contingent Resources

As this example points out, it is very difficult to characterize the optimal structure of the capacity levels in problems with setup costs as there are many alternative potential ways of coping with the demand uncertainty with the help of flexibility inherent in the system.

## 5 Computations

In this section we present the results of our computational study that is conducted to gain insights on the characteristics of the problem. In our computations, we used the parameter set presented in Table 5. A seasonal demand stream having a cycle of 4 with expected demand values of 10, 15, 10, and 5 is used. The demand distribution is assumed to be Poisson, Normal, and Gamma. We used three different values of coefficient of variation (CV) for the Normal distribution : 0.1, 0.2, and 0.3. Similarly, we used three different values of coefficient of variation for the Gamma distribution : 0.5, 1, and 1.5. While investigating the effect of a change in one parameter, we kept the rest of the parameters unchanged. In order to avoid trivialities, we assume that the starting inventory level is zero. We provide intuitive explanations to all of our results below and our findings are verified through several numerical studies. However, one should be careful in generalizing them, as for any experimental result, especially for extreme values of problem parameters. In the results that we present, we use the term “increasing” (“decreasing”) in the weak sense to mean “non-decreasing” (“non-increasing”).

Table 5: The Set of Parameters Used in Computation

$K_p$	0, 10, 20, 30, 40, 50, 60
$K_c$	0, 10, 20, 30, 40
$h$	1
$b$	3, 4, 5
$c_p$	1.5, 2.5, 3.5
$c_c$	1.5, 2.5, 3.5
$\alpha$	0.99
$T$	12

## 5.1 Optimal Capacity Level

In this section we investigate the sensitivity of the optimal permanent capacity level as the problem parameters change. As the fixed cost of production increases, we observe that the optimal permanent capacity level also increases up to a certain threshold point. After this threshold, production is initiated only a few times due to high setup cost and all production becomes due to contingent resources. In this region, the optimal permanent capacity level becomes zero because paying for the permanent resources also in the non-productive periods becomes too costly. When the unit cost of permanent capacity increases, the optimal capacity level decreases since the utilization of contingent resources becomes more critical, as discussed in Section 5.3. On the other hand, as the costs of contingent resources decrease, the proportion of the production that is conducted by the contingent resources increase and hence the optimal permanent capacity levels decrease.

One might expect that the optimal permanent capacity level increases as the variability of demand increases. However, an interesting observation that we make in our computations is that the relation between the coefficient of variation of the demand and the optimal capacity levels exhibit different behaviors for different parameter settings. As shown in Table 6, the direction of the change in the optimal capacity level (if any) may vary as the variability of demand increases.

## 5.2 Effect of Operating at Suboptimal Permanent Capacity Levels

We also evaluate the effect of operating at suboptimal permanent capacity levels under different problem parameters. We measure this effect by the percentage penalty of installing a suboptimal capacity which is defined as  $\%PSU = (f_1(U, 0) - f_1(U^*, 0)) / f_1(U^*, 0)$  where  $U^*$

Table 6: Optimal Capacity Levels for Different Coefficient of Variation values

CV	Prob. 1 <sup>a</sup>	Prob. 2 <sup>b</sup>	Prob. 3 <sup>c</sup>	Prob. 4 <sup>d</sup>
0.1	9	10	11	10
0.2	9	9	12	11
0.3	9	9	12	10

---

<sup>a</sup> $K_p = 0, K_c = 0, b = 5, c_p = 1.5, c_c = 1.5$

<sup>b</sup> $K_p = 0, K_c = 10, b = 5, c_p = 1.5, c_c = 1.5$

<sup>c</sup> $K_p = 0, K_c = 10, b = 5, c_p = 1.5, c_c = 3.5$

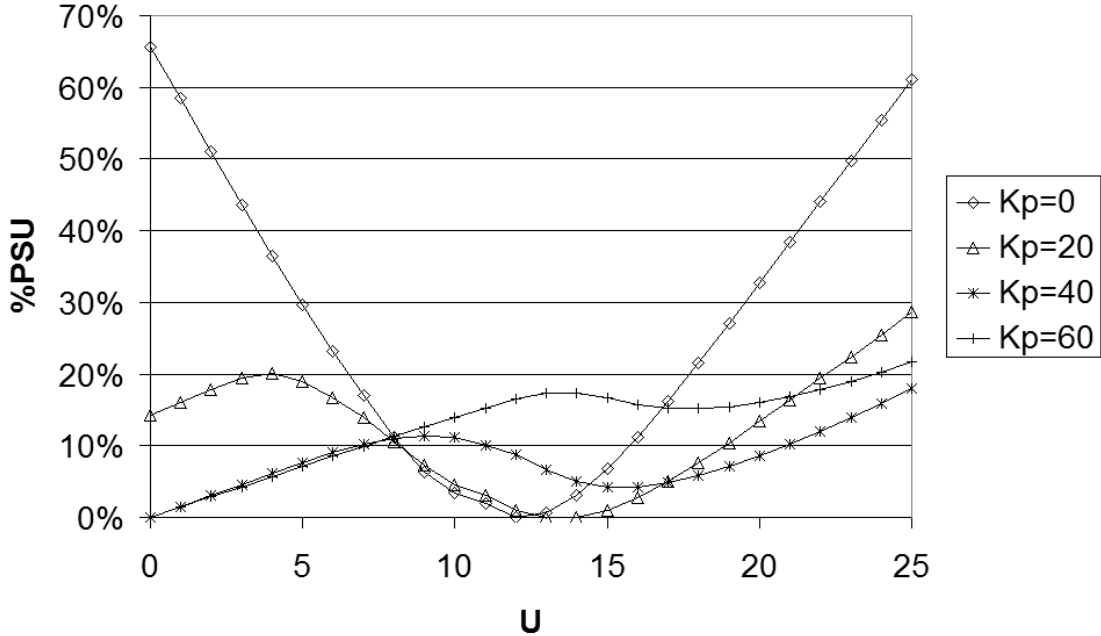
<sup>d</sup> $K_p = 0, K_c = 30, b = 4, c_p = 3.5, c_c = 3.5$

is the optimal capacity level. By definition, the structure of %PSU function is expected to be very similar to the structure of the expected total cost function (see Figure 2). Possible and typical behaviors are presented in Figure 4 for different values of  $K_p$ . We observe similar characteristics for other problem parameters.

When deciding on the values of operating parameters in a typical production/inventory environment, an intuitive solution would be to base operating decisions on expectations of the demand together with its variability. However, such a solution would incur significantly higher costs when the value of capacity flexibility is underestimated. For example, for situations where the value of flexibility is high and the optimal permanent capacity is 0, such an approach results with high percentage penalty values, as can be observed in Figure 4. When  $K_p = 60$  for example, the percentage penalty of operating with any permanent capacity level between 10 and 15 (recall that the expected demand is 10) results in percentage penalty values around 15%. We note that for some other problem instances this penalty may even be more severe in the same range and is observed up to 40% in our computational study.

As the  $CV$  of demand increases, the expected total costs of the system increases, both for the optimal and suboptimal permanent capacity levels, as may be expected. Nevertheless, there is not much that can be deducted for the %PSU as a function of the increase in demand variability. In specific, we encountered some sets of parameters where %PSU increases in  $CV$  for some values of  $U$ , and some other sets of parameters where %PSU decreases in  $CV$  for the same -or different- values of  $U$ , under the same expected demands. The same arguments hold for %PSU as a function of  $b$ .

Figure 4: Operating at Suboptimal Capacity Levels under Different Setup Costs of Production when  $K_c = 20, h = 1, b = 5, c_p = 1.5, c_c = 2.5$ , and Normal demand with  $CV=0.1$

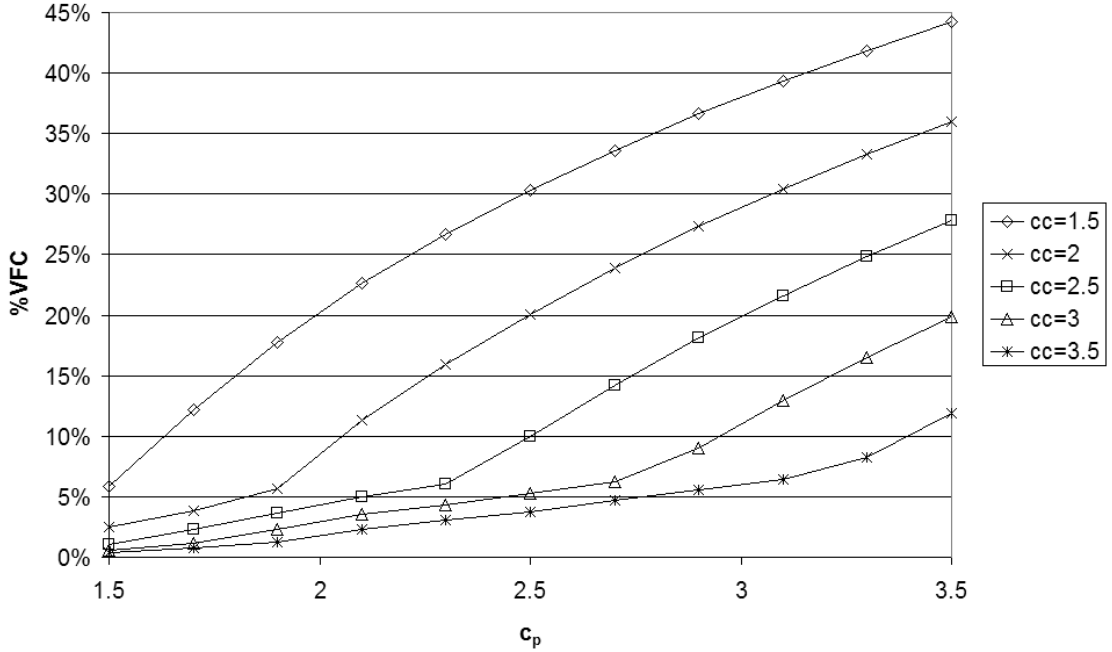


### 5.3 Value of Utilizing Flexible Capacity

In this section, our aim is to investigate the general behavior of the value of flexible capacity ( $VFC$ ) under different problem parameters. We define  $VFC$  as  $ETC_{IC} - ETC_{FC}$  where  $ETC_{IC}$  and  $ETC_{FC}$  are the expected total costs of operating in an inflexible environment (where no contingent resources are available) and in a flexible environment, under the respective optimal permanent capacity levels. Similarly, the percentage value of utilizing flexible capacity is defined as  $\%VFC = VFC/ETC_{IC}$ .

First we analyze the effect of unit costs of permanent and contingent capacity on  $VFC$ . In all problem instances that we solved, we observe that  $VFC$  has an increasing structure as the unit cost of keeping permanent capacity ( $c_p$ ) increases (Figure 5). The average  $\%VFC$  is observed as 7.02%, 18.87%, and 30.33% in all problem instances with Normal demand when  $c_p$  is 1.5, 2.5, and 3.5, respectively. We can also observe from Figure 5 that the value of flexibility increases as the unit cost of contingent resources decreases. This indicates that one should search for more possibilities of using contingent resources as their relative costs decrease.

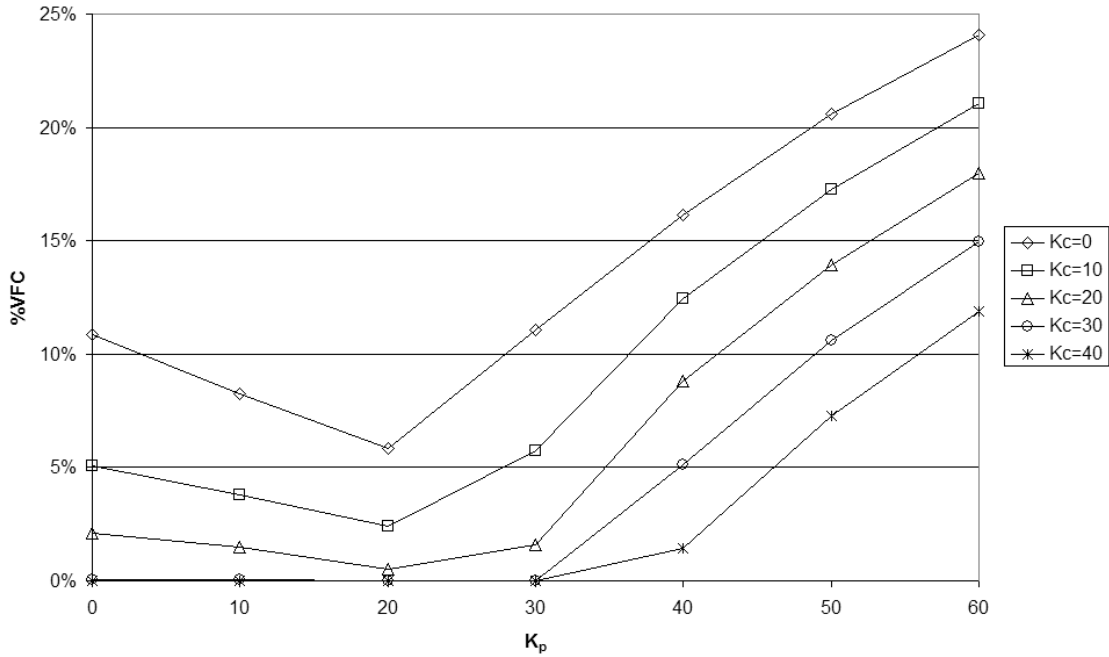
Figure 5: Effect of Unit Permanent Capacity Cost on the Value of Flexible Capacity under Normal demand when  $K_p = 10, K_c = 10, h = 1, b = 5$ .



As to the effect of the fixed costs of production on the value of flexibility (see Figure 6), we first note that  $VFC$  (as well as  $\%VFC$ ) exhibits a unimodal behavior after some particular value of  $K_p$ . Because, for moderately large values of  $K_p$ , the inflexible system starts to prefer complete backordering whereas the flexible system may still be better off by using contingent resources. Similarly, for very large values of  $K_p$  both systems are better off by complete backordering with zero permanent capacity and hence they converge to each other so that  $VFC$  becomes zero. For the rest, we do not necessarily observe a steady behavior. In almost all of the cases, we observe either a monotonic increase or a monotonic decrease followed by a monotonic increase, prior to the unimodal behavior explained above, such as the example provided in Figure 6. The decrease in that specific example is caused by  $ETC_{FC}$  increasing with a higher rate than  $ETC_{IC}$  for an increase in  $K_p$  while  $K_p$  is low, since the optimal permanent capacity levels do not change (at least significantly). Similarly, the increase is caused by the inflexible system's inability to react to increased production setup cost, which results in either under-utilization of a large permanent capacity, or incurring the production setup cost frequently, before complete backordering. Nevertheless, we encountered some

problem instances where  $VFC$  (as well as  $\%VFC$ ) fluctuates as the value of  $K_p$  increases. Finally we note that  $VFC$  increases as the fixed cost of contingent resources decreases as shown in Figure 6.

Figure 6: Effect of Setup Costs on the Value of Flexible Capacity when  $h = 1, b = 5, c_p = 2.5, c_c = 3.5$ , and Normal demand with  $CV=0.1$



For a given level of permanent capacity, Tan and Alp (2005) demonstrate some monotonicity results for the value of flexibility as a function of  $CV$  or  $b$ . Nevertheless, similar to the case in Section 5.2, we observe that there are no monotonicity results for  $VFC$  (and  $\%VFC$ ) as  $CV$  or  $b$  increases. For example, there are some problem instances where the value of flexibility decreases (and there are some others where it increases) as the variability in the system increases, even when both of the setup costs are zero. The reason for the non-monotonicity is the system's ability to adapt itself to changes in  $CV$  or  $b$  by optimizing the permanent capacity level accordingly.

## 6 Conclusions

In this paper the problem of determining the permanent capacity level in a make-to-stock environment under non-stationary stochastic demand with the option of temporary increase

of capacity via contingent resources is considered. A dynamic programming model is built to represent this problem, where the possibility of incurring distinct set-up costs for initiating production and for ordering contingent capacity is also incorporated.

We provided the solution to the single period problem. We note that the single period model with  $c_c < c_p$  fits also to a case where “contingent capacity” refers to the alternative of conducting the production in a developing country with cheaper production costs. In that case,  $K_p$  mimics the investment that is required independent of the country of investment (such as the specific machinery that needs to be procured),  $K_c$  mimics the additional costs that would be undertaken for initiating production in that developing country (such as the costs of the additional research required for investing there, the additional risks taken, etc.), and the single period production quantity mimics the total production that will be materialized. In that case, such an investment should only be undertaken if the required production amount is large enough to recoup the additional investment costs. If it is the case, our solution suggests that all the production should be materialized there, which brings the inventory to a level that is higher than that of producing with the alternative option of more expensive local production. We also note that the multi period model also fits to this scenario for some specific parameter settings, such as  $K_p$  being sufficiently high and/or  $c_c \ll c_p$ , which result in the optimal permanent capacity level to be zero.

For the multi period problem when the set-up costs are negligible, we showed that the expected total costs of the system is jointly convex in the permanent capacity level and the starting inventory, using which the optimal permanent capacity level for any starting inventory level can be searched. The convexity result is intuitive, since too low levels of permanent capacity would result in elevated production and/or backorder costs and too high levels of permanent capacity would result in low utilization of capacity. Nevertheless, this is not necessarily true for the case with positive set-up costs. If the system is working under too low a permanent capacity, then a marginal increase in permanent capacity may increase the system costs, because that capacity will be paid in the periods with no production as well, which may occur in order to avoid incurring set-up costs in every period. Yet, we conjecture that the expected total cost function is (no more complex than) quasi-concave/-convex in the permanent capacity level for a given starting inventory level, which is not violated in any of the problem instances that we have solved.

Our computational analyses pointed out some useful managerial insights. In specific, our computations revealed that the optimal permanent capacity



- decreases as the costs of the contingent resources decrease,
- increases as the fixed cost of production increases until a threshold level, after which it is economically better to conduct all of the production with contingent resources,
- decreases as the unit cost of permanent capacity increases.

Consequently, the optimal permanent capacity level may be equal to, greater than, or less than the expected average demand, with the possibility of zero as well. Indeed, we observe many problem instances with reasonable parameter settings where the optimal permanent capacity level is zero. In such instances, all productive activities rely on contingent resources even though they may be more expensive or less productive. Production takes place at most a few times and the demand is mainly satisfied from stocks. If this optimal course of action cannot be perceived by the decision maker and a positive permanent capacity level is installed then the consequences of taking such a suboptimal action may be very costly.

One might expect that the optimal permanent capacity level increases as the variability of demand increases. However, we show in our computations that the optimal permanent capacity level does not necessarily exhibit a monotonic behavior as the variability of the demand increases and the direction of the change in the optimal capacity level (if any) may vary as the variability of demand increases.

There exists relative values of problem parameters where introducing flexibility reduces the costs of the system significantly, even when the corresponding inflexible system is operated with an optimal capacity level: (i) lower costs of contingent capacity, (ii) higher set-up cost of production, and (iii) higher unit cost of keeping permanent capacity. Finally, no monotonicity results can be deduced for the value of flexibility as backorder costs and demand variability change, due to the system's ability to adapt itself by optimizing the permanent capacity level accordingly.

## References

- Angelus, A. and Porteus, E. 2002. *Simultaneous Capacity and Production Management of Short-Life-Cycle, Produce-to-Stock Goods Under Stochastic Demand*. Management Science 48(3). 399–413
- Atamturk, A. and Hochbaum, D. 2001. *Capacity Acquisition, Subcontracting, and Lot Sizing*. Management Science 47(8). 1081–1100

- Bradley, J. 2004. *A Brownian Approximation of a Production-Inventory System with a Manufacturer that Subcontracts*. *Operations Research* 52(5). 765–784
- Bradley, J. and Arntzen, B. 1999. *The Simultaneous Planning of Production, Capacity, and Inventory in Seasonal Demand Environments*. *Operations Research* 47(6). 795–806
- Bradley, J. and Glynn, P. 2002. *Managing Capacity and Inventory Jointly in Manufacturing Systems*. *Management Science* 48(2). 273–288
- Cheng, L., Subrahmanian, E., and Westerberg, A. 2004. *Multi-objective Decisions on Capacity Planning and Production-Inventory Control under Uncertainty*. *Industrial & Engineering Chemistry Research* 43. 2192–2208
- Hu, J., Vakili, P., and Huang, L. 2004. *Capacity and Production Management in a Single Product Manufacturing System*. *Annals of Operations Research* 125. 191–204
- Kouvelis, P. and Milner, J. 2002. *Supply Chain Capacity and Outsourcing Decisions: the Dynamic Interplay of Demand and Supply Uncertainty*. *IIE Transactions* 34(8). 717–728
- Pinker, E. and Larson, R. 2003. *Optimizing the Use of Contingent Labor When Demand is Uncertain*. *European Journal of Operational Research* 144. 39–55
- Rajagopalan, S. and Swaminathan, J. 2001. *A Coordinated Production Planning Model With Capacity Expansion And Inventory Management*. *Management Science* 47(11). 1562–1580
- Tan, B. and Gershwin, S. 2004. *Production and Subcontracting Strategies for Manufacturers with Limited Capacity and Volatile Demand*. *Annals of Operations Research* 125. 205–232
- Tan, T. and Alp, O. 2005. *An Integrated Approach to Inventory and Flexible Capacity Management under Non-stationary Stochastic Demand and Setup Costs*. Technical Report WP 132. Technische Universiteit Eindhoven
- van Mieghem, J. 2003. *Capacity Management, Investment, and Hedging: Review and Recent Developments*. *Manufacturing & Service Operations Management* 5(4). 269–302
- van Mieghem, J. and Rudi, N. 2002. *News vendor Networks: Inventory Management and Capacity Investment with Discretionary Activities*. *Manufacturing & Service Operations Management* 4(4). 313–335
- Wu, S., Erkoc, M., and Karabuk, S. 2005. *Managing Capacity in the High Tech Industry: A Review of Literature*. *The Engineering Economist* 50. 125–158
- Yang, J., Qi, X., and Xia, Y. 2005. *A Production-Inventory System with Markovian Capacity and Outsourcing Option*. *Operations Research* 53(2). 328–349
- Zhang, J., Roundy, R., Cakanyildirim, M., and Huh, W. 2004. *Optimal Capacity Expansion*

*For Multi-Product, Multi-Machine Manufacturing Systems With Stochastic Demand.* IIE  
Transactions 36(1). 23–36