# Portplan : decision support system for port terminals

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EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Mathematics and Computing Science

PORTPLAN, DECISION SUPPORT SYSTEM FOR

PORT TERMINALS

by

K.M. VAN HEE

B. HUITINK and

D.K. LEEGWATER

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## 1. INTRODUCTION

A decision support system is a computerized system that fulfils three
types of functions for decision makers:

a. computing of effects of decisions proposed by the decision maker;
b. generating of decisions that are optimal with respect to a criterion
   specified by the decision maker;
c. sensitivity analysis of the decisions by computing the effects of
   changes in parameters.

In general a decision process is too complicated to describe with one
mathematical model. Therefore we develop several mathematical models each
describing some aspects of the decision process. The integrated implemen-
tation of these models is the kernel of a decision support system.

PORTPLAN is a decision support system to assist decision makers in harbours.
The system is built from several models developed by us during the last
five years, for special problems in the harbour. PORTPLAN is meant for
planning problems with a horizon varying from a month to several years, so
it cannot be used for the day-to-day planning. Hence the system may be
used for strategic and tactical planning purposes. The system is used by
three categories of decision makers or decision analysists: managers of
stevedoring companies, port authorities and the management of a pool of
dockers. Although the system is only used for breakbulk terminals we
expect that it will be useable for the container sector and the bulk
sector after some modifications (cf. [van Hee and Wijbrands (1986)]).
The type of decisions one can support with the system involve:

a. capacities of immovables such as sheds and quays,
b. capacity of human resources and equipment,
c. periodically allocation of human resources.

One of the basic inputs of the system is a forecast of the total tonnages per commodity that will pass through the harbour during a period. The impact of each such forecast on the utilisation of harbour facilities and human resources is one of the main applications of the system. Another important application is the optimization of the distribution of the dockers over the stevedoring companies in Rotterdam (about 15) and the pool of dockers.

The mathematical models in the system can be split up logically into two parts:

a. Models for the transformation from expected tons to the work-load distributions for the human resources and equipment, and the occupation distributions of immovables.

b. Models to allocate the human resources capacities under several criterions for larger periods.

The second part consists in fact of two models:

b1. Simulation model to calculate the characteristic quantities given a policy for allocation of the human resources. This model takes into account the rules of the day-to-day assignment of human resources given the policy.

b2. Optimization model that simplifies the day-to-day assignment but that can be manipulated analytically to determine an optimal policy for allocation.

The models b1 and b2 are used in an iterative way.

The iteration stops when the input variables of the optimization model deliver a policy that gives in the simulation model output variable values that do not differ too much from the input variables. The first part (a) can be skipped and the system can start directly with part b, if the workload distributions of the individual stevedoring companies are given. The system has integrated several different techniques in a tractable and useful way. In the system several heuristics and approximations are implemented. Most of them are tested against real data. Nevertheless we made also a general simulation model to check the model (a). In the meantime we

have about 5 years of experience and the users have confidence in the system. PORTPLAN is implemented in APL and in Basic.
In section 2 the problem field is surveyed, in section 3 model (a) is treated, in section 4 model (b) and finally in section 5 some comments on the decision support system are made.


## 2. PORT TERMINALS

Port terminals fulfil two different functions in the transport chain:

a. Transhipment from seagoing vessels into inland vessels, coasters, trains or trucks and vice versa.

b. Warehousing, first of all to solve the inevitable discrepancies in arrival times of seagoing ships on the one hand and the other means of transport on the other hand. Secondly the warehouse is also a buffer to absorb discrepancies between demand and supply of the stored goods.

Several statistical investigations on a bulk of data gathered at stevedoring companies as well as the port authorities in Rotterdam did not give information to reject the hypothesis that ships arrive at terminals according to a Poisson process. One reason for this behaviour are the random interruptions during voyages of ships, due to the weather and waiting times in other harbours. Another reason is found in Palm's result (cf. the theorem of Grigelious in [Barlow and Prochan 1976]), saying that the superposition of renewal processes tends to a Poisson process. Hence even if a stevedore has several independent lines with a very punctual schedule then the Poisson character of the arrivals will occur.
Although the Poisson behaviour is nice for mathematical modeling it causes bad utilisation of resources at the terminals. The coefficient of variation of the workload for human resources is large. Ships do not like to wait at terminals, and therefore the stevedore has to have available enough manpower and equipment to handle the ships. Ships for which there is no free berth usually go to another stevedore for discharging and may be considered as 'lost calls'. The large variation in the production speed of gangs of dockers forms a special difficulty for the operational management of

stevedoring companies. Variations of 50 percent for one type of commodity is not unusual. Causes for these variations are: the way ships are stowed, the composition of the gang and traffic congestion at the terminal.
Another factor that complicates the daily planning of terminals is the weather. In principle ships are served at maximal speed, this means that at the heavy holds (i.e. the hold that contains the most work) always a gang and a crane are working in the working hours. If the terminal is very busy then a small delay is acceptable.
To avoid idleness of human resources the stevedores in many harbours created pools of dockers. The Rotterdam pool operates in the following way. Each stevedore that participates in the pool guarantees the cost of a portion of dockers in the pool. The whole pool is guaranteed by the stevedores. For each day- or nightshift the stevedores demand for a number of dockers from the pool (a shift is a period of eight hours). The Dutch government supports the pool by subsidizing the idleness (upto 50%) in the pool. The assignment of dockers from the pool to the companies is rules by a set of formulas. These rules are also applicable to other pool systems (cf. section 4).
For fork lift trucks one may organize a pool as well. In Rotterdam this is not done. Here private companies are hiring out these trucks. For other production factors as the immovables and the cranes, it is impossible to exchange capacities from time to time. Here it is only possible to share workload.

## 3. MODELS FOR CAPACITY UTILISATION AND WORKLOAD

A model is described by a set of scenario or input variables and a set of transformations on these variables.
Each function from the set of variables to a set of values is called a scenario and determines a model-instance or model state.
First of all the set of scenario (or input) variables is described. Then, in a data flow diagram, the usage of these data and transformations (or computations) are presented. Finally, in section 3.3, the transformations are treated in detail.

## 3.1 Variables and transformations

Scenario variables:

S1    Total tonnage per commodity type, expected to flow in the planning
period through the harbor, divided into import and export.

S2    Fractions of these flows that will pass through each terminal.

S3    Fractions of tons per type of manipulation per terminal (e.g. from
ship to shed, from ship to railway, from ship to truck, from ship to
coaster).

S4    Production speed per commodity, i.e. time a gang needs to move one
ton, depending on the type of commodity and the type of manipulation.
These speeds are random variables and the mean and variance are
required.

S5    Number of dockers in a gang per commodity-type.

S6    Residence time distribution of cargo on the terminal, per commodity
and per direction (import or export).

S7    Tonnage per commodity-type per ship per stevedore. From these random
variables the mean and the covariance matrix are required.

S8    Fraction of tons in the heavy hold per ship, i.e. the hold containing
the most work to load or unload. From this random variable $F$ the mean
and variance are required and also the mean and variance of $1/F$.

S9    Number of berths per terminal.

S10    Storage capacity per terminal.

S11    Acceptable ships delay, as a percentage of the mean service time of
ships, per terminal.

The variables S1, S2, S3 characterize the cargo flow, the variables S4,
S5, S6 characterize the handling of cargo on the terminal, S7, S8 charac-
terize the ships arriving at the terminals, S9, S10 and S11 characterize
the terminal.
Most of the values of the scenario variables can be obtained from recording
of activities in the past. The terminals and the port authorities are
assumed to maintain databases in which they record the cargo movements and
the amount of work per ship. If, for a future period changes of the varia-
bles are expected, then the registered values from the past may be used to
produce forecasts.

Transformations:

T1  Expected number of ships per terminal.

T2  Mean and variance of service time of the ship if it is serviced at maximal speed. From these values the parameters of the service distribution function are derived.

T3  Number of gangs simultaneously at work per ship.

T4  Berth utilisation distributions, and expected number of lost calls, in two situations: ships are served at maximal speed, or ships delay is at maximal acceptable level.

T5  Utilisation distributions of sheds and excess probability of the storage capacity (for both situations of T4).

T6  Workload distribution of dockers (for both situations of T4).

Report variables:

R1  Berth utilisation and expected number of lost calls.

R2  Utilisation of sheds.

R3  Workload distribution for dockers.

## 3.2  Data flow diagram

We adopt the data flow diagram notation to express the relationships between the models that form the first part of the decision support system.

| | |
|---|---|
| S1 | Total tonnage expected per commodity per harbour |
| S2 | Fraction per terminal of total tonnage per commodity |

| | |
|---|---|
| S3 | Fractions of manipulation |

Calculation of tonnage per commodity

Calculation of tonnage per commodity per manipulation

| | |
|---|---|
| S7 | Tonnage per commodity per ship |

| | |
|---|---|
| S4 | Production speed |

| | |
|---|---|
| S8 | Fraction of tons in heavy hold |

**T1**
Calculation of expected number of ships

**T2**
Calculation of mean and variance of service time

**T2**
Determination of service time distribution

| | |
|---|---|
| S9 | Number of berths |

| | |
|---|---|
| S11 | Acceptable ships delay |

**T4 (R1)**
Calculation of berth utilisation and lost calls

**T3**
Calculation of number of gangs simultaneously working at one ship

| | |
|---|---|
| S6 | Storage norms per commodity |

| | |
|---|---|
| S10 | Storage capacity |

| | |
|---|---|
| S5 | Number of dockers per gang |

**T5 (R2)**
Calculation of storage utilisation and probability of excess

**T6 (R3)**
Calculation of workload distribution of dockers

Note: S1, S2 and S3 are given for the whole harbour, all other values and calculations are per stevedore.

## 3.3 Models underlying the data transformations

In this section the transformations are treated in some detail and special attention is paid to the model assumptions.

All random variables are indicated by capitals, other variables usually by undercasts. Letter $\mathbb{E}$ is reserved for the mathematical expectation and $\sigma^2$ for the variance of a random variable.

### 3.3.1 T1: Computation of the expected number of ships per terminal

Let for some terminal:

$TT_j$ = the total tonnage of commodity type j during the planning period.

$T_{ij}$ = the tonnage of commodity j in ship i (note that $T_{ij}$, i = 1,2,...,, are i.d.d. random variables)

N = the number of ships destinated to this terminal.

Then the following relation should hold:

$$TT_j = \sum_{i=1}^{N} T_{ij}$$

and therefore

$$\mathbb{E}\,TT_j = \mathbb{E}\,N \cdot \mathbb{E}\,T_{ij}$$

It is assumed that the random variables $TT_j$ are observable (scenario variable S1). This assumption says that the total tonnages per commodity type that will pass through the harbour in the future period are predictable. Of course the prediction of such an aggregate is more precise than the forecasts of the tonnages per commodity per terminal. Further an estimate for $\mathbb{E}\,T_{ij}$ is known from the past (collected in S7).
Note that

$$\mathbb{E}\,N = \lambda t$$

where $\lambda$ is the Poisson arrival rate of this terminal and t the planning period.

The parameter $\lambda$ is estimated by minimization of

$$\sum_j (TT_j - \lambda t \cdot \mathbb{E} T_{ij})^2$$

Hence the estimate $\hat{\lambda}$ becomes:

$$\hat{\lambda} = \frac{1}{t} \sum_j TT_j \; \mathbb{E} T_{ij} \; / \sum_j (\mathbb{E} T_{ij})^2 \;.$$

Of course the estimator is unbiased. It is not difficult to compute also the variance of the estimator if the covariance matrix of $TT_j$ is available.

### 3.3.2 *T2: Service time distributions by maximal speed*

Let for some terminal

$\quad$ F $\quad$ = fraction of tonnage in the heavy hold(s)

$\quad$ $m_k$ $\quad$ = fraction of tonnage of manipulation type k
$\qquad\quad$ (assumed to have no variance)

$\quad$ $S_{jk}$ = production speed per commodity j and manipulation type k.

Time a ship i needs to be served:

$$F \sum_j T_{ij} \cdot \sum_k m_k S_{jk} \;.$$

The random variables F, $T_{ij}$ and $S_{jk}$ are assumed to be mutually independent. This assumption is motivated by statistical inferences. Further, $cov(S_{jk}, S_{j'k'}) = 0$ for $j \neq j'$ and $k \neq k'$, however, $cov(T_{ij}, T_{i'j'}) \neq 0$. The mean time needed for service is:

$$a = \mathbb{E} F \sum_j (\mathbb{E} T_{ij} \cdot \sum_k m_k \mathbb{E} S_{jk})$$

and it is straightforward to express the variance:

$$b = \sigma^2(F)\sigma^2(P) + \sigma^2(F)(\mathbb{E} P)^2 + \sigma^2(P)(\mathbb{E} F)^2$$

where

$$P = \sum_j \sum_k T_{ij} \, m_k \, S_{jk}$$

and

$$\mathbb{E} P = \sum_j \sum_k \mathbb{E} T_{ij} \, m_k \, \mathbb{E} S_{jk}$$

and

$$\sigma^2(P) = \sum_{\substack{j,j' \\ j \neq j'}} \sum_{\substack{k,k' \\ k \neq k'}} m_k \, m_{k'} \, cov(T_{ij}, T_{i'j'}) \, E(S_{jk} \, S_{j'k'})$$

$$+ \sum_j \sum_k m_k^2 \{ (\sigma^2(T_{ij}) + (E \, T_{ij})^2)(\sigma^2(S_{jk}) + (E \, S_{jk})^2)$$

$$- (E \, T_{ij})^2 (E \, S_{jk})^2 \} \ .$$

All these quantities can be obtained from the scenario variables as indicated in section 3.2.

The service of ships, lower than maximal speed is treated by the description of T4.

In T4 we consider a queuing model, where the service time distribution is needed. Up to now we only have the mean and variance of this distribution. For reasons of mathematical tractability we only consider phase-type distributions with maximal two phases: in parallel or serial. This assumption is better than the usual assumption of an exponential distribution, and we can compute the berth occupation and lost calls with a Markov process having a two dimensional state space (see section 3.3.4).

The coefficient of variation of the service time, $v_s$, determines the structure of the phase-type distribution. Note that $v_s = \sqrt{b}/a$.

1.  If $v_s^2 > 1$ then it is assumed that the service distribution is hyper-exponential with two parallel exponentially phases with mean values:

$$\frac{1}{\mu} = a + \sqrt{\frac{1-\alpha}{\alpha}} \ \sqrt{\tfrac{1}{2}(b-a^2)}$$

$$\frac{1}{\nu} = a - \sqrt{\frac{\alpha}{1-\alpha}} \ \sqrt{\tfrac{1}{2}(b-a^2)}$$

where $\alpha$ free and a, b are the mean, respectively variance, of the time needed to handle the heavy hold.

We choose $\alpha = \tfrac{1}{2}$ to get a minimal difference between $1/\mu$ and $1/\nu$.

2.  If $\tfrac{1}{2} \leq v_s^2 < 1$ then the service distribution is assumed to consist of two exponential phases in series with mean values:

$$\frac{1}{\mu} = \frac{a}{2} \pm \frac{1}{2}\sqrt{2b-a^2}$$

$$\frac{1}{\nu} = a - \frac{1}{\mu}.$$

If $v_s^2 < \frac{1}{2}$ we choose: $1/\mu = 1/\nu = a/2$, which choice gives the minimal $v_s^2$ in the class of two phase distributions.

3.  If $v_s^2 = 1$ then the service distribution is assumed to consist of only one exponentially phase with mean value a.

The interpretation of the two phases is the following: if stevedores have a hyperexponential service distribution then they are assumed to have two types of vessels (large ones and small ones), so with probability $\alpha$ a ship is of type one and with probability $1-\alpha$ it is of the other type. If the service distribution consists of two phases in series then the first phase may be considered to be the import cargo handling and the second one the export cargo handling.

### 3.3.3  T3: Number of dockers simultaneously working on a ship

In the models we need to know the distribution of gangs working simultaneously on a ship. In practice this will depend on all kinds of influence, such as the availability of pool dockers and the other vessels at the quay. However, we choose in the models an assignment rule that guarantees that

> "the maximal number of gangs working simultaneously is minimal under the condition that the berth of the ship time is minimal".

Before we will consider the rule in more detail, we note that in several terminals the planners are using this rule to compute the needed capacity in the short run.

The assignment rule is defined by:

-   let w be the total number of gang-shifts to work on the ship;
-   let y be the total number of gang-shifts to work in (one of) the heavy hold(s);
-   assume w and y are integers and $w = ay + k$, $0 \leq k \leq y-1$;
-   the rule is:

> assign during the first y-k shifts a gangs and during the next k shifts a+1 gangs, and assign at least to all heavy holds a gang.

Note that the total amount of work done by this rule is w. We will show that this rule is always applicable.

Define:

$$w_n = w - na \qquad \text{if } n = 0,1,\ldots,y-k$$
$$= w + y - k - n(a+1) \quad \text{if } n = y-k+1,\ldots,y$$

and

$$y_n = y - n .$$

First we note that $w_0 = w$, $y_0 = y$ and for $n < y-k$:

$$w_n \underline{\text{div}} \; y_n = a$$

and for $n \geq y-k$:

$$w_n = y_n(a+1) .$$

THEOREM. Under the rule defined above we have for $n = 0,1,2,\ldots,y$: at the end of shift n:

(i)   the total amount of work is $w_n$;

(ii)   the work in a heavy hold is $y_n$;

(iii)   there are at least a (if $n < y-k$) or a+1 (if $n \geq y-k$) holds with work;

(iv)   the number of heavy holds is not greater than a (if $n < y-k$) or a+1 (if $n \geq y-k$).

PROOF. For $n = 0$ (i) and (ii) are obvious. Assume the number of heavy holds $\geq a+1$, then $w \geq (a+1)y$. However, $w = ay+k < (a+1)y$. Hence (iv) is true. Assume (iii) does not hold. Then the number of holds with work $< a$ and then $w < ay$. However, $w = ay+k$. Hence (iii) is true.

Assume (i) - (iv) are true for n. Then the rule is applicable in period n+1 so (i) and (ii) are true.

Suppose (iii) does not hold at the end of period n+1. If $n+1 < y-k$ then the number of holds with work $< a$. Hence the total amount of work $w_{n+1} \leq (a-1)y_{n+1}$. However, $w_{n+1} = ay_{n+1} + k$.

If $n+1 \geq y-k$ then the number of holds with work $< a+1$. Hence the total amount of work $w_{n+1} \leq ay_{n+1}$. However, $w_{n+1} = (a+1)y_{n+1}$. Therefore (iii) is true for n+1.

Suppose (iv) does not hold at the end of period $n+1$. If $n+1 < y-k$ then the number of heavy holds $> a$. Hence $w_{n+1} \geq (a+1)y_{n+1}$. However, $w_{n+1} = ay_{n+1} + k < (a+1)y_{n+1}$.

If $n+1 \geq y-k$ then the number of heavy holds $> a+1$. Hence $w_{n+1} \geq (a+2)y_{n+1}$. However, $w_{n+1} = (a+1)y_{n+1}$. Therefore (iv) is true. This proves the induction hypothesis. □

The random variable $1/F$ equals $w/y$. Hence this is a good approximation of the number of gangs working simultaneously.

### 3.3.4  Berth utilisation and lost calls

We assume that the arrival process for each terminal is a nonhomogeneous Poisson process with arrival rate $\lambda(t)$ at time $t$. The service distribution, described in section 3.3.2, is a hyperexponential distribution or a serial-two-phase distribution. The average amount of work in a heavy hold has two exponentially distributed phases with averages $1/\mu$ and $1/\nu$, respectively. (For the mathematical treatment of Markov processes we refer to [Kleinrock (1970)].)

In case of the hyperexponential distribution we also have to indicate the probability of each phase $\alpha$ for the first, $1-\alpha$ for the second one.

Ships are assumed to be served at maximal speed or to wait until a service unit is available. A service unit consists of a number of gangs; each gang has equipment at its disposal (for instance a crane). The number of gangs is determined by $1/F$ (cf. section 3.3.3).

Here we are only interested in the berth occupation distribution, however, in section 3.3.6 we consider the workload distribution expressed in dockers and equipment.

Let $A(t)$ be the number of service units available at time $t$. We may model the process of the number of ships at the terminal as an inhomogeneous, continuous time Markov chain with state space:

$$S = \{(k,\ell) \mid k,\ell \in \mathbb{N} \wedge k+\ell \leq c\}$$

where $k,\ell$ are the number of ships in phase 1, respectively phase 2, and $c$ is the berth capacity. The infinitesimal generator at time $t$, $Q_t$, of this Markov chain is given by three transitions in case of the serial-two-phases.

| Transition | Probability rate | |
|---|---|---|
| $(k,\ell)$ to $(k+1,\ell)$ | $\lambda(t)$ | (a) |
| to $(k-1,\ell+1)$ | $k \cdot \mu \cdot s(t,k,\ell)$ | (b) |
| to $(k,\ell-1)$ | $\ell \cdot \nu \cdot s(t,k,\ell)$ | (c) |

where

$$s(t,k,\ell) := \min(1,\frac{A(t)}{k+\ell}) \ .$$

Note that these formulas hold provided that the states remain in the state space. In case $k = c$, (a) has to be replaced by 0, if $k = 0$, (b) becomes 0 and if $\ell = 0$ then (c) becomes 0.

In case of a hyperexponential distribution $Q_t$ is determined by:

| Transition | Probability rate | |
|---|---|---|
| $(k,\ell)$ to $(k+1,\ell)$ | $\alpha\lambda(t)$ | (a) |
| to $(k,\ell+1)$ | $(1-\alpha)\lambda(t)$ | (b) |
| to $(k-1,\ell)$ | $k \cdot \mu \cdot s(t,k,\ell)$ | (c) |
| to $(k,\ell-1)$ | $\ell \cdot \nu \cdot s(t,k,\ell)$ | (d) |

provided both states remain in the state space. In case $k+\ell = c$, (a) and (b) become 0, if $k = 0$ then (c) becomes 0, if $\ell = 0$ then (d) becomes 0.

It is assumed that $\lambda$ and A are periodically. Usually the periodicity is one week. This implies that we may model the process as a homogeneous Markov process with state space

$$\{(k,\ell,t) \mid k,\ell \in \mathbb{N} \wedge k+\ell \leq c \wedge t \in [0,T)\} \ ,$$

where T is the periodicity.

To compute the stationary distribution of this process we will consider an approximation of the original process, obtained by discretizing time. We take the time unit so small that the probability of making two or more transitions within this time unit is neglectable. In practice the time unit is about one hour. Now we may consider the transition rates multiplied by the time unit h as transition probabilities of a homogeneous Markov chain with discrete time and state space:

$$\{(k,\ell,t) \mid k,\ell,t \in \mathbb{N} \wedge k+\ell \leq c \wedge t < T'\}$$

where $T' = \lceil T/h \rceil$, and transition probabilities:

$$P((k,\ell,t),(k',\ell',t')) = Q_{th}((k,\ell),(k',\ell')) \cdot h$$

for $t' = (t+1) \bmod T'$, Q defined above and h the time unit.

For this transition matrix P we may determine the stationary distribution $\pi$ by solving $\pi = \pi P$ by some standard technique. However, the state space becomes very large if the time unit is decreased and the computation must be carried out frequently on a micro computer in an interactive session with a planner. Therefore we have chosen an other solution method.

We consider the inhomogeneous Markov chain with the original state space

$$\{(k,\ell) \mid k,\ell \in \mathbb{N} \wedge k+\ell \le c\}$$

and transition matrix $P_t$ obtained from P by fixing parameter t $(0 \le t < T')$. Let $\pi_0$ be an estimate of the stationary distribution and define

$$\pi_{t+1} = \pi_t \, P_{t \bmod T'} \, , \qquad t = 0,1,2,\ldots$$

We stop the iteration at $t = nT'$ if

$$\max \{ |\pi_t(k,\ell) - \pi_{t+T'}(k,\ell)| \mid (k,\ell) \in S \wedge (n-1)T' \le t < nT'\}$$

is sufficiently small.

In practice this criterion is reached after a few weeks because the influence of the ship handling in past weeks on the handling in an actual week is small.

We call this distribution the stationary distribution and we denote it by $\tilde{\pi}$. So $\tilde{\pi}_t(k,\ell)$ is the probability to have k ships in phase 1 and $\ell$ in phase 2 at time point t in the period $\{0,1,\ldots,T'-1\}$.

We will aggregate this distribution to obtain the distributions per shift. There are N shifts per week. In practice N = 21. This is done by taking the average:

$$ps_n(k,\ell) := \frac{N}{T'} \sum_{t \in [\frac{nT'}{N}, \frac{(n+1)T'}{N})} \tilde{\pi}_t(k,\ell) \, ,$$

where N is the number of shifts per period, n = 1,2,...,N.

In practice we only obtain data for A(t) and $\lambda(t)$ per shift, therefore these functions are constant per shift.

We define:

- $$\lambda := \frac{T}{N^2} \sum_{n=1}^{N} \lambda(n) \qquad \text{(average arrival rate)}$$

- $$p(m) := \frac{1}{N} \sum_{n=1}^{N} \sum_{k+\ell=m} ps_n(k,\ell) \qquad \text{(the probability of m ships at the quay)}$$

- $$L := \sum_{m=0}^{c} mp(m) \qquad \text{(average quay occupation)}$$

- $$Se := \frac{1}{\mu} + \frac{1}{\nu} \qquad \text{(average net service time per ship)}$$

- $$W := \frac{L}{\lambda} \qquad \text{(average sojourn per ship, Little's formula)}.$$

Now we can express the number of lost calls:

- per shift:

$$Lc(n) := \sum_{k+\ell=c} ps_n(k,\ell)\lambda(n)\frac{T}{N} \; ;$$

- on average:

$$Lc := \frac{1}{N} \sum_{n=1}^{N} Lc(n) \; .$$

It is obvious that Lc, L, W depend on A. In the model we assume:

$$A(t) = \beta f(t) \; ,$$

where f is determined by a labour scheme that cannot be influenced by the terminal planner. So the only control variable is $\beta$. Therefore we write $Lc_\beta$, $L_\beta$, $W_\beta$. The minimal service time $\widetilde{W}$ is the minimum of $W_\beta$ over $\beta$. Note that $\beta \in \{0,1,\ldots,c\}$.
The planner will consider for each choice of $\beta$:

- $W_\beta$
- $Lc_\beta$
- cost of having handling capacity $\beta$ (cf. sections 3.3.6 and 4).

He will compare $\widetilde{W}$ with Se and $W_\beta$ with $\widetilde{W}$.
Note that $W_\beta$ and $Lc_\beta$ are descending if $\beta$ grows and that the cost is ascending.

*3.3.5  T5: Storage utilisation*

The process of storing shiploads into sheds and on yards is considered in order to compute the capacity needs. A shipload is defined as the total amount of cargo to be stored for the ship. We may distinguish import and export separately and we will consider import first.
Although the arrival process of ships is Poisson, due to the lost calls, the arrival process of import shiploads is not Poisson anymore. However, we approximate this process with a Poisson proces with parameter (cf. section 3.3.4):

$$\tilde{\lambda} := \lambda - Lc \frac{N}{T} \; .$$

We consider a shipload to consist of units of the same size. The number of units per ship is an independent random variable V from which we only use the mean and variance. Further we need the residence time of the cargo units. We assume they have a distribution G, so G(x) is the probability that the residence will not be longer than x time units. Both, the moments of V and G have to be estimated from the scenario data (cf. section 3.1). We will compute the mean and variance of the total amount of units in storage at an arbitrary moment. From these numbers one may, using the normal approximation, calculate the probability of overflow of a given storage capacity. The next theorem is treated in a different way in [Holman (1983)].

THEOREM.  The mean and variance of the required storage capacity are, respectively:

$$\tilde{\lambda} \; \mathbb{E} V \; \mu_G$$

$$\tilde{\lambda} \; \mathbb{E} V \int_0^\infty (1-G(x))G(x)dx + \lambda \; \mathbb{E} V^2 \int_0^\infty (1-G(x))^2 \, dx \; ,$$

where $\mu_G$ = the mean of G, i.e. $\int_0^\infty (1-G(x))dx$.

PROOF.  First we consider a new, fictive process, namely the process of shiploads in the storage. We further assume that a shipload remains in the system for a fixed period of time d. According to the assumption of

Poisson arrivals we know that the distribution of shiploads in storage is again Poisson (cf. [Kleinrock (1976)]) with parameter $\widetilde{\lambda}d$. Let $V_i$ be the remaining number of units in shipload i at time t. The r-th moment of $V_i$ can be computed by conditioning on the arrival time, the group size and the number N of groups in the system at t:

$$\mathbb{E}\,[V_i^r \mid N = n] = \frac{1}{d} \int\limits_0^d \sum_{j=0}^{\infty} p_j \sum_{v=0}^{j} v^r \binom{j}{v} (1-G(x))^v \, G(x)^{j-v} \, dx \qquad (a)$$

where $p_j$ is the probability that group size is j. To verify this formula note that the arrival times of shiploads in storage at some moment t, given the number of shiploads, are uniformly distributed over [t-d,t] (cf. [Kleinrock (1976)]). This gives the other integral and the factor 1/d. If the shipload arrived at x, the probability that there are still v units of this load at t is binomially distributed, with parameters j and (1-G(x)). From (a) we easily derive

$$A := \mathbb{E}\,[V_i \mid N = n] = \frac{\mathbb{E}\,V}{d} \int\limits_0^d (1-G(x))dx$$

and

$$B := \mathbb{E}\,[V_i^2 \mid N = n] = \frac{\mathbb{E}\,V}{d} \left\{ \int\limits_0^d (1-G(x))dx - \int\limits_0^d (1-G(x))^2 \, dx \right\} +$$

$$+ \frac{\mathbb{E}\,V^2}{d} \int\limits_0^d (1-G(x))^2 \, dx \; .$$

Hence:

$$\mathbb{E}\left[ \sum_{i=1}^{N} V_i \right] = \mathbb{E}\,N \cdot A = \widetilde{\lambda}d \cdot A = \widetilde{\lambda} \, \mathbb{E}\,V \int\limits_0^d (1-G(x))dx$$

and by Wald's lemma

$$\mathbb{E}\left[ \left( \sum_{i=1}^{N} V_i \right)^2 \right] = \mathbb{E}\left[ \sum_{i \neq j}^{N} V_i \, V_j + \sum_{i=1}^{N} V_i^2 \right] = \mathbb{E}\,[N^2 - N]A^2 + \mathbb{E}\,N \cdot B \; .$$

Letting d tend to infinity gives the result after some straightforward manipulations.                                                                     □

We have considered the important case above. In that case the residence intervals of all units belonging to one shipload have a common left bound. In case of export cargo the units are delivered at arbitrary points and they stay till the departure time. So if we reverse the time axis we cover also the export case.

### 3.3.6 *T6: Calculation of workload distribution*

The distribution of the workload for dockers and equipment is rather easy now.

From T4 we have the distribution of ships at the quay per shift. Call this random variable $K(n)$ for shift n. Recall from section 3.3.4 that

$$\mathbb{P}\,[K(n) = m] = \sum_{k+\ell=m} ps_n(k,\ell)\ .$$

The number of gangs working simultaneously on one ship is the random variable $1/F$. So the total number of gangs working simultaneously in shift n is:

$$K(n)\ \frac{1}{F} \tag{a}$$

To compute the number of dockers per gang we have to compute the average over the commodity types:

$$\frac{m_j\ d_j}{\sum\limits_j m_j} \tag{b}$$

where $m_j$ is the average tonnage of commodity type j in a ship and $d_j$ is the gang size of commodity j (data belonging to the scenario variables S7 and S5). Hence multiplying (a) and (b) gives the wanted quantity. If we are only interested in the mean and variance of the workload distribution we get respectively:

$$\frac{m_j\ d_j}{\sum\limits_j m_j}\ \mathbb{E}\,K(n)\cdot\mathbb{E}(\tfrac{1}{F})\ ,$$

$$\left(\frac{m_j\ d_j}{\sum\limits_j m_j}\right)^2 \{(\sigma^2(K(n)) + (\mathbb{E}\,K(n))^2)(\sigma^2(\tfrac{1}{F}) + (\mathbb{E}\tfrac{1}{F})^2) - (\mathbb{E}\,K(n))^2(\mathbb{E}\tfrac{1}{F})^2\}\ .$$

Note that we assume $K(n)$ and $1/F$ to be independent.

For equipment, like fork lift trucks, we may apply the same calculation. Only $d_j$ is different in that case.

The workload distribution is used in section 4.

## 4. MODELS FOR ALLOCATION OF HUMAN RESOURCES

In section 3 we only considered one terminal. Here we will consider all terminals in one harbour and a pool of dockers. From section 3 we only use the workload distribution per terminal. Note that this distribution is one of the final results of all the foregoing computations.

The aim of the models we will describe here is to determine the optimal quantities of own dockers and the guarantee of pool-dockers per terminal. Although the real decision support system works on basis of different shifts we assume here all shifts are the same. An important assumption in this section is that the workloads of the terminals are mutually independent random variables. This assumption is tested empirically and found to be acceptable.

We will consider the rules for daily assignment of the pool-dockers in more detail.

For an arbitrary shift we define

$$d_i = \text{demand for dockers from the pool by company } i$$

$$g_i = \text{guaranteed portion of company } i$$

$$x_i = (d_i - g_i)^+$$

$$x = \sum_i x_i$$

$$y_i = (g_i - d_i)^+$$

$$y = \sum_i y_i$$

$$a_i = \text{assignment of dockers to company } i$$

(Note that $x^+ = x$ if $x \geq 0$ and $x^+ = 0$ otherwise.)

The assignment to company $i$ is $a_i$ and it is defined by:

$$\text{if } x = y \text{ then } a_i = d_i$$

$$\text{if } x > y \text{ and } d_i > g_i \text{ then } a_i = g_i + s_i$$

if $x > y$ and $d_i \leq g_i$ then $a_i = d_i$

if $x < y$ and $d_i \leq g_i$ then $a_i = g_i - t_i$

if $x < y$ and $d_i > g_i$ then $a_i = d_i$

where $s_i$ and $t_i$ are determined by:

$$s_i = \min \{x_i, g_i \cdot c\} \quad \text{with } c \text{ determined by } \sum_i s_i = y$$

and

$$t_i = \min \{y_i, g_i \cdot c\} \quad \text{with } c \text{ determined by } \sum_i t_i = x .$$

A nice feature of this assignment rule is that if, for instance, $x > y$ then exaggeration of the demand of a company does not influence the assignment. The assignment is based as much as possible by the guaranteed portion. Companies for which $a_i < d_i$, have to go to the free labour market to try to hire dockers. Companies for which $a_i > d_i$ have $(a_i - d_i)$ people idle. From time to time the stevedores revise the distribution of guaranteed portions because their capacity needs are varying. These variations are caused by mutual competition, and the supply of season dependent products. There are several systems to do this (cf. [van Hee (1984)] or [Leegwater (1983)]). In the moment this is done in the same way as the day-to-day assignment of dockers. If there is a shortage of dockers $(x > y)$ then the pool-board may decide to enlarge the pool by recruitment or it may try to fire dockers.

Note that the demand of company i is a random variable $D_i$ determined by subtracting the number of own dockers $b_i$ from the workload $W_i$ derived in T6: $D_i = (W_i - b_i)^+$. Hence all variables $d_i$, $a_i$, $x_i$, $y_i$, $x$ and $y$ are random variables and therefore we will write from now on capitals for them.

The exploitation cost for one arbitrary shift of company i (we omit i) can be expressed by:

(a) $\quad bk + pg + q \, \mathbb{E}(D-A)^+ - \{p \, \mathbb{E}(g-A)^+ + \gamma p \, \mathbb{E}(A-V)^+\}$

where

$k$ = internal cost of a docker per shift,

$p$ = pool price of a docker per shift ($p \geq k$),

$q$ = price of a docker per shift outside the pool ($q \geq p$),

$\gamma$ = fraction of governmental subsidy.

Because the assignments A are very complicated random variables that can only be expressed by an algorithm, we cannot manipulate these exact exploitation cost to derive optimal values for b and g.
Therefore consider an approximation for the exploitation cost:

(b) 
$$bk + pg + \{p\beta + q(1-\beta)\}\, \mathbb{E}(D-g)^{+}$$
$$- \{p\gamma\alpha + p(1-\alpha)\}\, \mathbb{E}(g-V)^{+}$$

where

$\alpha$ is the chance a superfluous docker of the guarantee is not assigned to another company and therefore stays on the account of the guaranteeing company,

$\beta$ is the chance a docker wanted above the guaranteed portion can be obtained from the pool.

The approximation (b) has the advantage that it is easy to compute the optimal values $b_i$ and $g_i$ for all companies under several constraints. The constraints, which represent several policies from the pool board and the companies, are

1. $b_i$ and $g_i$ both free,

2. $\lambda b_i + \mu g_i$ fixed ($\lambda$ and $\mu$ constants),
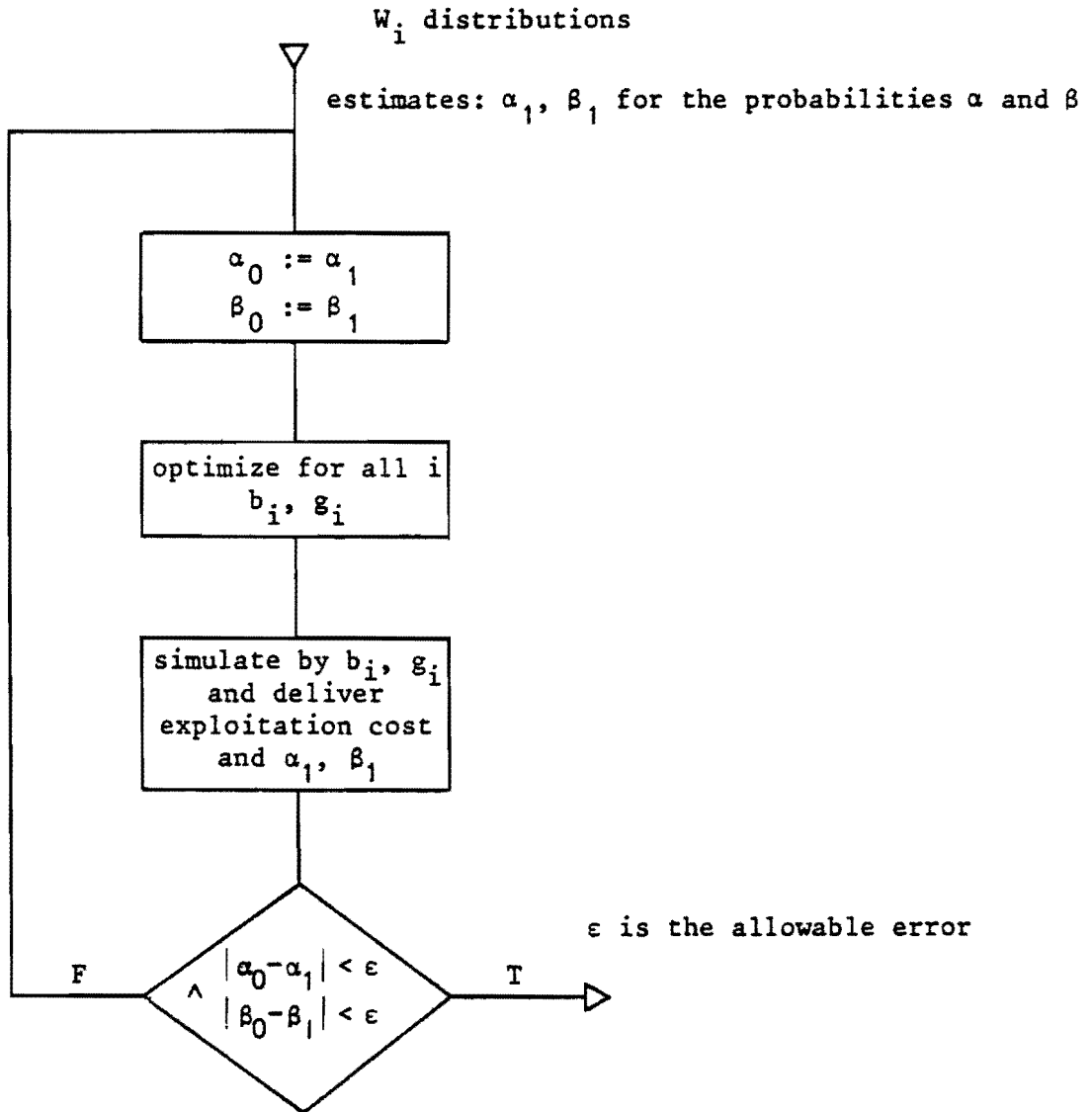
3. $b_i/g_i$ fixed.

The partial derivatives of the exploitation cost (b) can be expressed in terms of the constants and excess probabilities of the form

$$\mathbb{P}\,[W_i \leq c_1 b_i + c_2 g_i] \qquad (c_1 \text{ and } c_2 \text{ constants}).$$

If we approximate the distribution of $W_i$ by the normal distribution, which is allowed on basis of empirical tests, it is easy to compute $b_i$ and $p_i$ per company, just by solving for each company a pair of nonlinear equations with only two variables. We omit the details of these equations here

because it requires some tedious, straightforward calculations. In these calculations we assumed $\alpha$ and $\beta$ as constants and known. These assumptions are not true.

We combine these optimization calculations with a simulation process in which for each shift, for each company a value of $W_i$ is drawn. Then the exact exploitation costs (a) are computed. For the simulation process it is required that the values $b_i$ and $g_i$ are known. Therefore we use the optimization and simulation processes in an iterative way.

$W_i$ distributions

estimates: $\alpha_1$, $\beta_1$ for the probabilities $\alpha$ and $\beta$

$\varepsilon$ is the allowable error



It seems to be impossible to proof convergence for this iteration, however in practice we always found values within the error bound rather quickly.

As mentioned before, the simulation model is not suitable for optimization of $b_i$ and $g_i$ because there are about 30 of them. In principle the same method can be used for pools for equipment like fork lift trucks.

The pool price (p) must balance the exploitation cost of the pool and the income from selling gangs per shift and the governmental subsidy.

The simulation model is also used to determine the equilibrium price by iterating p in the same way as $\alpha$ and $\beta$. In fact we are looking for a fixed point here.

The model described in this section is used by the individual port terminals and also by the pool board.

In fact the pool organization gets all the information from the terminals and keeps it secret for the other ones. So the pool organization is the only one who can compute the factors $\alpha$ and $\beta$. They can deliver these values to the individual terminals who can use it for their own optimization purposes. Note that the pool board tries to satisfy the wishes of the terminals, so they will do their best to assign the guaranteed portions according to the needs of the terminals.

However, the shifting of these guaranteed portions is a difficult problem itself (cf. [van Hee (1984)], [Leegwater (1983)]). In practice the pool organization may use the model of section 4 separately from the model of section 3. One approach is that the pool only gets $D_i$-distributions that are computed by the terminals themselves. Another approach is based on historical data of $D_i$. The latter approach is not meant for assignment decisions but for evaluation of decisions from the past.


## 5.  COMMENTS ON THE DECISION SUPPORT SYSTEM

In the foregoing sections we paid attention to the mathematical models underlying the system. The system has a lot of scenario variables, some of them with complicated data structures. The planner or decision maker has sufficient facilities to manipulate all these variables in a very user friendly way. In principle it is possible to get historical data from the registrative information system of the company in a direct way. (In only one case this has been realized.)

Each scenario variable can be considered as a decision variable in principle. However, in the part described in section 3 the user is playing with the number of service units per shift (the function A or the control variable $\beta$, cf. section 3.3.4). The planner will judge this decision on basis of the following characteristics:

- quay occupation (mean and variance),
- expected number of lost calls,
- average sojourn time compared to the minimal service time,
- cost of the operation by fixed or free $b_i$ and $g_i$,
- needed capacity on human resources and equipment to realize the number of service units.

Of course this is a multicriteria decision problem, so the decision maker will consider several alternatives.
The second type of decision is about the division of human resources over own dockers and pool dockers (model of section 4). Here several constraints may be considered.

PORTPLAN is a decision support system that is used by different categories of decision makers. It is also used by consultants who prepare decisions. A lot of decision support systems in practice are in fact tools for consultants more than tools for decision makers. However, PORTPLAN is used by decision makers at port terminals.
As noted in the introduction a decision support system fulfils three kinds of functions. In PORTPLAN the generation of decisions is not very complicated because the optimizations are quite simple. Only in the models for allocation of manpower a complicated optimization model occurs; however this model was approximated by a number of simple ones.
The number of decision variables for the decision maker is also rather small. However the computation of the effect of a decision is complicated and involves a lot of models. Hence the complexity of this decision support system is concentrated in the first function-type of decision support systems.

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