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Pergamon

0021-9290(95)00149-2

J. Biomechanics, Vol. 29, No. 7, pp. 955–961, 1996

Printed in Great Britain
0021-9290/96 $15.00 + .00

TECHNICAL NOTE

VALIDATION OF A THREE-DIMENSIONAL MODEL OF THE KNEE

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Abstract—Three-dimensional mathematical models of the tibio-femoral joint require input of the geometry of articulating surfaces and ligament insertions, and the mechanical properties of cartilage and ligaments. This paper describes a validation of a knee model through a direct specimen-related comparison between the knee model and the kinematics of four knee joint specimens from which the geometry data were used as input of the model. The knee model is quasi-static and is based on equilibrium of forces and moments. The stiffness properties of the ligaments and articular cartilage were estimated on the basis of data reported in the literature. The so-called reference strains in the ligament bundles for the joint in extension, were determined by using an optimization procedure, minimizing the difference between the kinematics of the model and the kinematics of experimentally obtained flexion motions with an internally or an externally rotated tibia (±3 Nm load). A reasonable to good agreement between the model and the experimental kinematics could be obtained for internal–external rotation laxity and the coupled translations and varus–valgus rotation. The disparity between model and experiment varied from knee to knee, average deviations ranging from close to zero to 8° internal rotation deviation and from 5 mm posterior to 3 mm anterior position deviation. The average anterior–posterior laxities at both 20° and 90° flexion were within the variations reported in the literature, although for each individual joint with some underestimation or overestimation. It was concluded that the optimization procedure compensated for the lack of menisci and capsular structures by higher prestrains, thereby overestimating the ligament forces. Despite the gross simplifications relative to the complex anatomy of the knee, the present knee model can realistically simulate the passive motion characteristics of the human knee joint. Copyright © 1996 Elsevier Science Ltd.

Keywords: Knee joint, Joint modeling, Joint laxity.

INTRODUCTION

A mathematical model of the tibio-femoral joint can be a versatile tool for parametric analyses of knee ligament function (Blankevoort and Huiskes, 1989), knee prosthetic design (Essinger *et al.*, 1989) and ligament reconstruction procedures (Bach *et al.*, 1992; Gibson *et al.*, 1986). Three-dimensional knee joint models require the geometry of the articular surfaces and the insertion sites of the ligaments, and the mechanical properties of ligaments and cartilage surfaces (Andriacchi *et al.*, 1983; Essinger *et al.*, 1989; Wismans *et al.*, 1980). The geometry of the articular surfaces and the ligament insertion sites of one or more individual joints were usually measured accurately (Essinger *et al.*, 1989; Wismans *et al.*, 1980), whereas the stiffness parameters for the ligaments and articular contact were based on data reported in the literature (Andriacchi *et al.*, 1983; Essinger *et al.*, 1989; Gibson *et al.*, 1986). Sufficient data are available for the nonlinear stress–strain or force–length relationships of the ligaments (e.g. Butler *et al.*, 1986; Trent *et al.*, 1976) but for the reference strain, i.e. the strain in ligaments for the joint in extension (Wismans *et al.*, 1980), and the related zero-load length of the ligaments there are no data available. In knee models, the ligament reference strains were merely estimated, and sometimes adapted by means of trial and error in order to get better agreement with experimental data (Blankevoort *et al.*, 1991) or to prevent the ligaments from being overstrained for particular joint motions (Essinger *et al.*, 1989; Wismans *et al.*, 1980).

Received in final form 13 *September* 1995.

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This study addressed the question how closely a mathematical knee model can approach experimentally obtained passive motion characteristics of a particular knee joint from which the geometry data are used as input. Because the stiffness properties of the articulating surfaces have a small effect on the model characteristics (Blankevoort *et al.*, 1991) and good estimates of the ligament stiffnesses were available, this study was focussed on the unknown parameters, i.e. the reference strains of the ligaments. The ligament reference strains were determined in an optimization procedure based on the minimization of the disparity between the kinematics of the model and those experimentally obtained for a particular knee, for given values of the ligament stiffnesses. After the optimization procedure, the optimized model was then used to stimulate anterior–posterior and varus–valgus laxity tests and the results were compared with data from the literature.

METHODS

The three-dimensional mathematical knee-joint model used in this study, featured anatomically shaped three-dimensional articular surfaces with a thin layer of deformable cartilage and an arbitrary number of nonlinear elastic line elements representing the ligaments. Friction at the articular contact was neglected. The model solved the equilibrium equations of forces and moments from the externally applied loads, the ligament forces, the contact forces and the constraint loads. The constraint forces and moments reacted to the prescribed degrees of freedom. The model accounted for the interaction between the medial collateral ligament and the medial bony edge of the tibia (Blankevoort and Huiskes, 1991). The menisci were not accounted for in the model. Details on the mathematics of the knee model which are

Table 1. The ligament parameters in the knee model

			Knee specimen 1		2		3		4	
Ligament	Ligament bundle	k (N)	ε_r opt. int/ext.	ε_r opt. int/ext. ant/post.	ε_r opt. int/ext.	ε_r opt. int/ext. ant/post.	ε_r opt. int/ext.	ε_r opt. int/ext. ant/post.	ε_r opt. int/ext.	ε_r opt. int/ext. ant/post.
Anterior Cruciate	a	5000	0.015	−0.004	0.067	−0.006	0.031	−0.014	0.049	−0.030
	p	5000	−0.061	0.005	0.100	0.007	0.040	0.072	−0.033	−0.001
Posterior Cruciate	a	9000	−0.053	−0.211	−0.230	−0.248	−0.214	−0.231	−0.138	−0.145
	p	9000	−0.023	−0.249*	−0.024	0.009	−0.030	0.004	0.017	0.044
Lateral Collateral	a	2000	0.080	−0.004	−0.263	−0.217	−0.216*	−0.123*	0.077	0.100
	s	2000	−0.059	−0.131	−0.059	−0.061	−0.029	−0.121*	0.004	−0.010
	p	2000	−0.040*	0.064	0.100	0.076	0.005	0.027	0.039	0.029
Medial Collateral	a	2750	−0.113*	−0.029	0.024	−0.076	−0.161*	−0.083*	0.049	−0.006
	i	2750	0.070	0.100	−0.024	0.100	−0.049*	0.100	−0.010	0.100
	p	2750	−0.008	0.050	−0.150*	0.051	−0.043*	−0.170*	−0.009	0.100
Medial Capsule	a	1000	−0.203	−0.115	−0.144	−0.161	0.099	0.079	0.100	0.090
	p	1000	−0.074	0.100	−0.012	0.035	0.060	0.094	0.051	−0.006

k is the linear stiffness and ε_r is the reference strain. ε_r is given for all four knees after the optimization (opt.) procedure to match the models of the four knee joints with internal–external rotation only (int/ext.) and to both internal–external rotation and anterior–posterior translation (int/ext., ant/post.) for flexion motions along the envelope of passive knee motion. Ligament bundles which are slack for both the internal and external motion pathways are marked with *. (Bundle identifications: a = anterior, p = posterior, s = superior, i = inferior.)

not given below can be found in Blankevoort *et al.* (1991b) and Blankevoort and Huiskes (1991).

The femur was assumed to move relative to the tibia. The displacements were expressed as the translations of the origin of the femoral coordinate system, which was located 15 mm proximal relative to the posterior insertion site of the anterior cruciate ligament on the tibia when the joint was extended (Blankevoort *et al.*, 1988). The rotation convention was similar to the one proposed by Grood and Suntay (1983) in the sense that joint rotations were expressed as rotations of the tibia relative to the femur.

Each ligament was represented by multiple line elements connecting the femur and the tibia. The magnitude of the ligament force f in a line element was related to the ligament strain ε by (Wismans, 1980):

$$f(\varepsilon) = \tfrac{1}{4} k \varepsilon^2/\varepsilon_1 \quad \text{when } 0 \leqslant \varepsilon \leqslant 2\varepsilon_1,$$

$$f(\varepsilon) = k(\varepsilon - \varepsilon_1) \quad \text{when } \varepsilon > 2\varepsilon_1,$$

$$f(\varepsilon) = 0 \quad \text{when } \varepsilon < 0, \tag{1}$$

where ε_1 is a strain constant and k is a stiffness constant. The actual strain ε was determined from the actual length L of the line element and its zero-load length L_0, by

$$\varepsilon = (L - L_0)/L_0. \tag{2}$$

The actual length L followed directly from the translations and rotations and the insertion locations (Blankevoort and Huiskes, 1991). At the reference extension position of the joint, the initial strain in each line element was denoted by the parameter ε_r, the "reference strain", given by

$$\varepsilon_r = (L_r - L_0)/L_0, \tag{3}$$

where L_r is the reference length.

The geometric data of four knee specimens from the previous studies (Blankevoort *et al.*, 1988, 1991a) were used as input for the knee model. The locations of the ligament insertions were obtained by Roentgen-stereophotogrammetry (Blankevoort *et al.*, 1991a). The geometry of the articular surfaces was measured by a stereophotogrammetric technique from Meijer *et al.* (1989) which was based on the close-range stereophotogrammetric method described by Huiskes *et al.* (1985). The surfaces were approximated by polynomial functions with an accuracy varying between 0.14 and 0.39 mm (root mean square error). In all knee models, the same stiffnesses were chosen for the ligaments (Table 1) (Blankevoort *et al.*, 1991b). The initial reference strains for each knee were estimated on the basis of the ligament length and recruitment patterns of each knee (Blankevoort *et al.*, 1991a). A representation of the model of knee specimen 2 is shown in Fig. 1.

The optimization procedure, as applied to each knee model, minimized the differences in kinematic parameters between the knee model and the experiment by variation of the reference strains in the line-elements. The optimization function was represented by the error vector

$$\mathbf{E}(\boldsymbol{\varepsilon}_r) = \mathbf{P}_m(\boldsymbol{\varepsilon}_r) - \mathbf{P}_e \tag{4}$$

in which the vector $\mathbf{P}_e$ contained the experimental observations and $\mathbf{P}_m(\boldsymbol{\varepsilon}_r)$ contained the results of the equivalent model calculations for a given set of reference strains contained in the vector $\boldsymbol{\varepsilon}_r$. The flexion angle was prescribed in the experiments and the remaining five degrees of freedom were unconstrained. Internal–external (I–E) rotation and anterior–posterior (A–P) translation were chosen as the variables to be optimized relative to the experimental data. The varus–valgus (V–V) rotation, proximal–distal (P–D) and medial–lateral (M–L) translations were assumed to be coupled motions, which are mainly dependent on the articular geometry and not very sensitive to variations of the reference strains in the ligaments. The minimization was performed through a modification of the Levenberg-Marquardt algorithm by using the general least-squares solver LMDIF (from MINPACK (Moré *et al.*, 1980), Argonne National Laboratory, Argonne, Illinois, U.S.A.). A limit of 0.1 was set to the reference strain ε_r in order to prevent the occurrence of unrealistic reference strains.

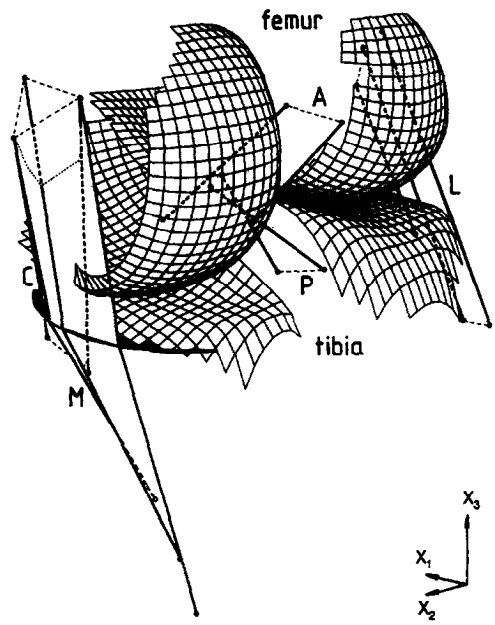

Fig. 1. Representation of the geometry of the model of knee 2, including the femoral condyles, the tibial surfaces and the line elements which model the four major ligaments. The joint position shown is extension: A = anterior cruciate ligament; P = posterior cruciate ligament; M = medial collateral ligament; L = lateral collateral ligament.

The motions simulated in the optimization procedures were flexion along the envelope of passive motion (Blankevoort *et al.*, 1988). These flexion motions were defined by two loading conditions, the first with an internally applied axial torque (M_3) of 3 Nm and the second with an externally applied torque (M_3) of 3 Nm, in absence of other external loads. The axial torque was applied about the long axis (x_3) of the tibia, independent of the femoral motions. The choice for these motion pathways was based on the rationale that a limited number of experimental positions were required for the optimization procedure (limitation of computing time) and that all ligaments needed to be strained at least once. The I–E rotations and A–P translations for these two motion pathways, each simulated by 7 joint positions, defined the goal of the optimization procedure. For each model two optimizations were performed. In the first optimization the goal parameter was I–E rotation as functions of flexion only, while in the second optimization the goal parameters were both I–E rotation and A–P translation as functions of flexion. Because the values of the I–E rotation as expressed in degrees were generally higher than those of the A–P translation expressed in mm, a weight factor of 3 was used for the A–P translation.

The final analysis concerned a comparison of the laxity characteristics of the optimized knee-models with data reported in the literature. For this purpose the A–P laxity at ± 100 N at 20 and 90° flexion and the V–V laxity data at ± 20 Nm at extension and 20° flexion were chosen from the *in vitro* and *in vivo* studies of Markolf *et al.* (1976, 1978, 1984). These studies were consistent and considered representative of other knee laxity studies (Hirokawa, 1993). Each model was used to simulate the A–P laxity tests and the V–V laxity tests while the flexion angle and the I–E rotation were constrained.

RESULTS

As illustrated for one knee specimen (Fig. 2), the similarity between the model and the experimental data was not very close. A considerable improvement of the I–E rotations was obtained after optimization of the I–E rotation only at the cost of an

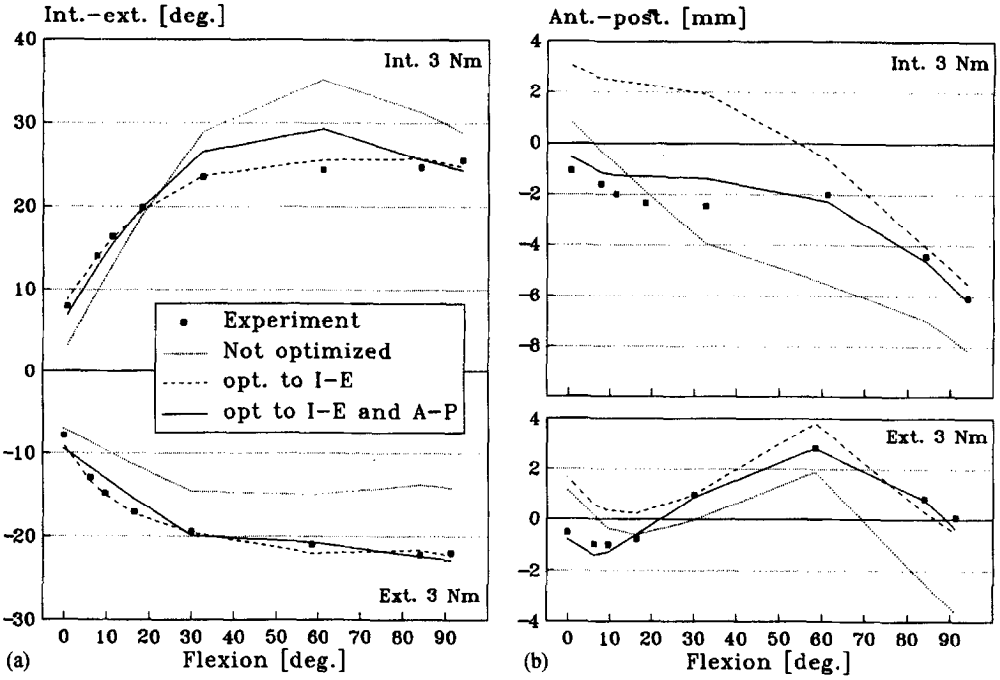

Fig. 2. Internal (+) external (−) rotations (a) and anterior (+) posterior (−) translations (b) as functions of flexion for the internal and external motion pathway with a torque of 3 Nm. The curves are shown for the nonoptimized model, for the model optimized to internal–external rotation only and for the model optimized to both internal–external rotation and anterior–posterior translation relative to the experimental data. The results are from knee specimen 2.

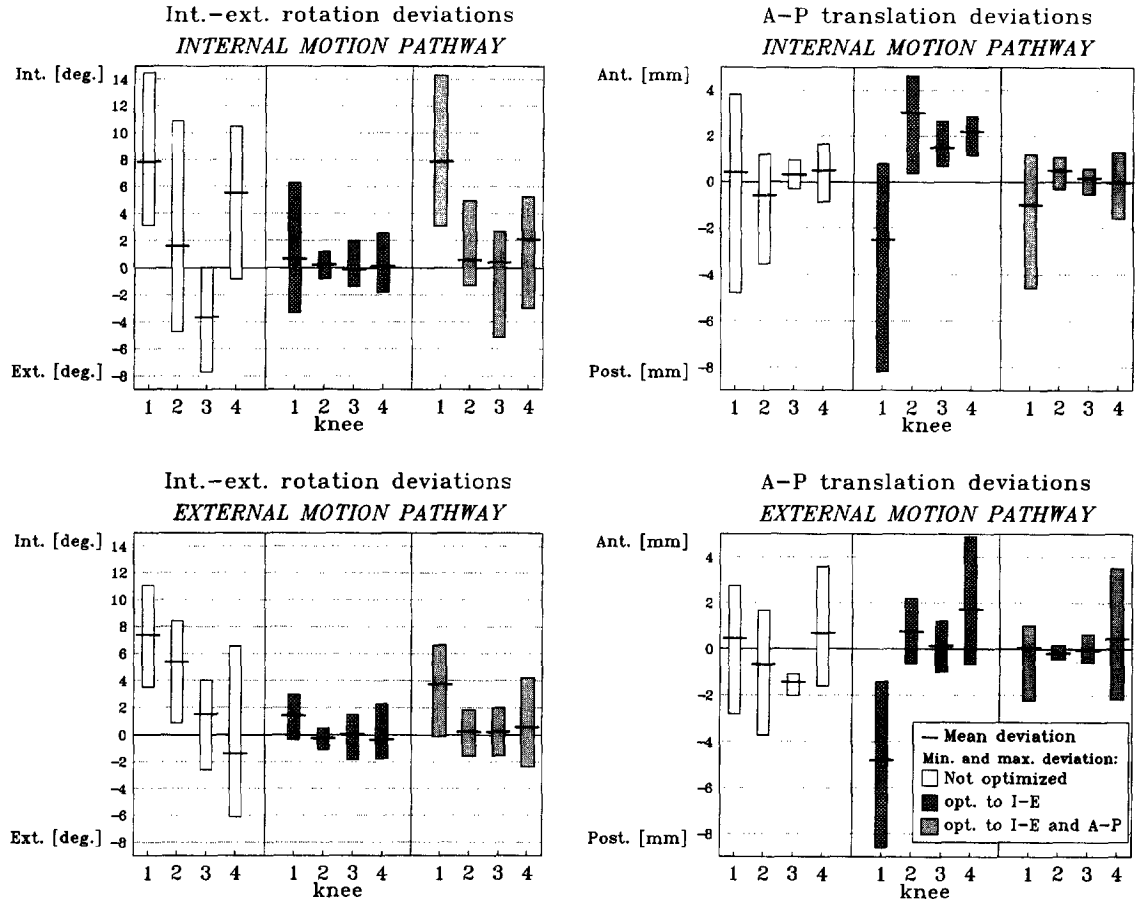

Fig. 3. The minimum, mean and maximum deviations of the four optimized knee models relative to the experimental data for the internal (−) external (+) rotations and anterior (+) posterior (−) translations of the internal (top) and external (bottom) motion pathways. The data are given for the nonoptimized models, for the models optimized to internal–external rotation only and for the model optimized to both internal–external rotation and anterior–posterior translation relative to the experimental data.

anterior deviation for the internal motion pathway exceeding 4 mm at one point. Optimization of both the I–E rotation and A–P translation brought the A–P translation of the model within 1 mm of the experimental data with a slight increase in the I–E rotation deviation.

In the four knee models, the resulting reference strains differed considerably between the two optimization procedures, to I–E only and to both I–E and A–P (Table 1). In some cases, line elements had reference strains that were so low, that they remained slack for both the internal and external motion pathways. These line elements were not required in the knee model to obtain the optimal match with the experimental data in the cases concerned. When starting the optimization procedures with different initial reference strains, the models converged in nearly all situations to the same solution. Only in those cases where the starting values were extremely high or extremely low, did the solutions differ. In these (physically unrealistic) cases, the optimal solution showed less agreement with the experimental values. A reduction of the reference strain limit from 0.1 (10%) to 0.05 (5%) affected the reference strains in the solution only marginally and hardly affected the quality of fit with the experimental data, showing slight increases in the internal or external rotations. The reference strain limit of 0.05 did, however, reduce the strains in those line-elements which originally had reference strains between 0.05 and 0.1.

The optimization procedures brought the motion characteristics of all four knee models closer to those obtained experimentally, as illustrated by the comparison of the mean deviations and the extreme deviations before and after optimization (Fig. 3). For all specimens, the external motion pathway was found to match the experiments the best, whereas the results for the internal motion pathway varied considerably among the four joints. The optimized models of the knee specimens 2 and 3 showed the best results, whereas the model of knee specimen 1 was the worst. The model of specimen 1 optimized with respect to both I–E and A–P showed an increased deviation of internal rotaion of more than 10°, with flexion angles higher than 30°. Two of the models of specimen 4, the one before optimization and the one optimized with respect to I–E only, luxated for internal rotation at flexion angles above 70°. A luxation was characterized by one of the condyles sliding off the tibial plateau, which resulted in a mechanically unstable position. In the overall comparison, the data of the internal motion pathway above 70° of flexion was omitted for this specimen. When the model of specimen 4 was optimized with respect to I–E and A–P, it did not luxate but showed an increased internal rotation relative to the experimental values of the same order of magnitude as found in specimen 1 for the internal motion pathway above 70° flexion.

The other motion components, i.e. varus–valgus (V–V) rotation, medial–lateral (M–L) translation and proximal–distal (P–D) translation, were assumed to be coupled motions. They were hardly affected by the optimization process. The mean deviations for the coupled V–V and M–L motions were the highest for specimen 2, 1.6° varus and 1.5 mm medial, and the

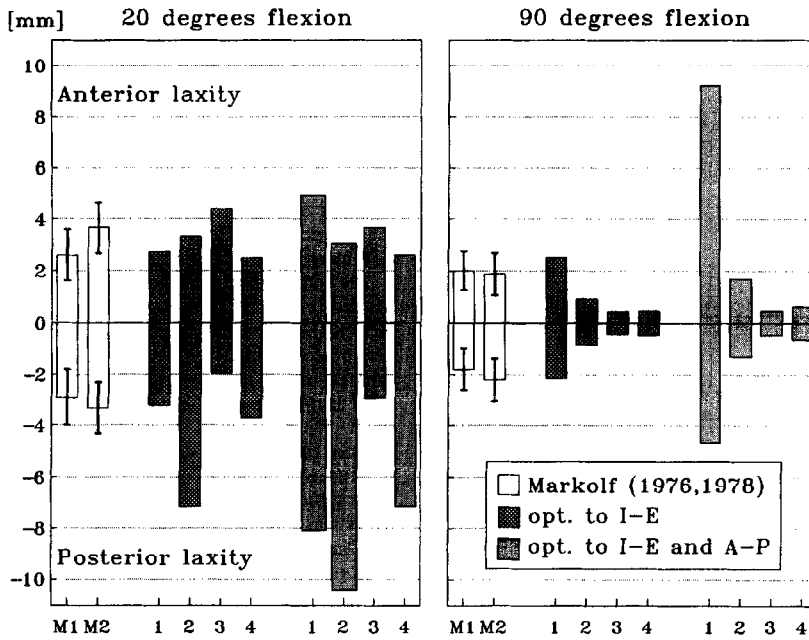

Fig. 4. The anterior and posterior laxities of the four optimized knee models at 20 and 90° flexion for anterior–posterior forces of 100 N, with axial rotation constrained, as compared with (M1) the *in vitro* data of Markolf *et al.* (1976; cf. 1978) and (M2) the *in vivo* data of Markolf *et al.* (1978).

lowest for specimen 2, 0.7° valgus and 0.2 mm anterior. The deviation in the P–D direction was about − 2 mm for all models, which means that the femur was about 2 mm closer to the tibia as in the experiment.

The anterior and posterior laxities at a load level of 100 N of the optimized models compare relatively well with the experimental values of Markolf *et al.* (1976, 1978) (Fig. 4). At 20° flexion, the models optimized with respect to I–E only, compare better with the values of Markolf *et al.* (1976, 1978) in particular for posterior laxity. At 90° flexion, the difference between the two optimization strategies is small, except for the model of knee specimen 1, which also showed the worst results after optimization to the internal and external motion pathways. The anterior and posterior laxities at 90° flexion of the models of specimens 3 and 4 were low relative to the average experimental data, but taking into account the 95% confidence interval (approximately mean ± two times standard deviation), the values are not unrealistic.

The V–V laxities of the four models at extension and 20° flexion of the four optimized models compare very well with the average data of Markolf *et al.* (1976, 1978, 1984) (Fig. 5). The magnitudes of the V–V laxities of the models optimized with respect to both I–E and A–P are within the range of the reported standard deviations. The magnitudes of the V–V laxities for the models optimized with respect to I–E only were not much different, less than 10% of the magnitudes obtained with the models optimized with respect to both I–E and A–P.

DISCUSSION

In this study the geometric data on the ligament insertions and the articular geometry were obtained from each knee specimen of which also the passive motion characteristics were measured. The geometric data were used as input of a three-dimensional mathematical model whereby the motion characteristics were used to determine the ligament reference strains by mathematical optimization of the match between the experiments and the model, for a given set of mechanical properties of the ligaments and the articular cartilage.

The process of determining the reference strains is based on the assumption that, for a given joint position, the sum of the forces in the ligaments and in the articular contacts balance the externally applied loads. The ligament forces were changed by altering the reference strains. For a fixed joint position this is similar, to some extent, to changing the ligament stiffness whereby the load balance is influenced (Blankevoort *et al.*, 1987). The ligament forces, in turn, will affect the contact forces (Blankevoort and Huiskes, 1989). Implicitly, it is assumed that the configuration of multiple line-elements representing the ligaments, is adequate for the function of all capsular and ligamentous structures of the knee. The structures which are loaded in the real knee, but not represented in the model, have to be compensated for by introducing additional loads on the structures which are represented in the model. When there was a redundancy in the ligament configuration within the knee model, some of the line elements were eliminated in the optimization process by adopting a very low reference strain. This points to a weakness of present knee-joint models. The reference strains as determined in the optimization process, do not represent precisely the reference strains in the real joint because of the compensation mechanism. It can be assumed that the reference strains in the model were overestimated, given the fact that an increase of the reference strain will lead to a higher ligament force. Thus also the ligament strains and ligament forces are overestimated in the model. On the other hand, the optimization technique resolves issues bothering one, whether or not a chosen model configuration is valid for simulating the load balance across the knee. The optimization process will not lead to a satisfactory match with the motion characteristics when those structures are discarded, whose function cannot be compensated for by the structures included in the model.

The suitability of the optimization method to determine the best model was clearly illustrated. Of the four knee models in the present study, those of specimens 2 and 3 performed excellently when optimized to both I–E and A–P. The other two showed poor behaviour with respect to internal rotation at the higher flexion angles. The insufficient compensation of the absence of the menisci by the ligaments in the model could well be the reason for this, although the differences between the intact knees

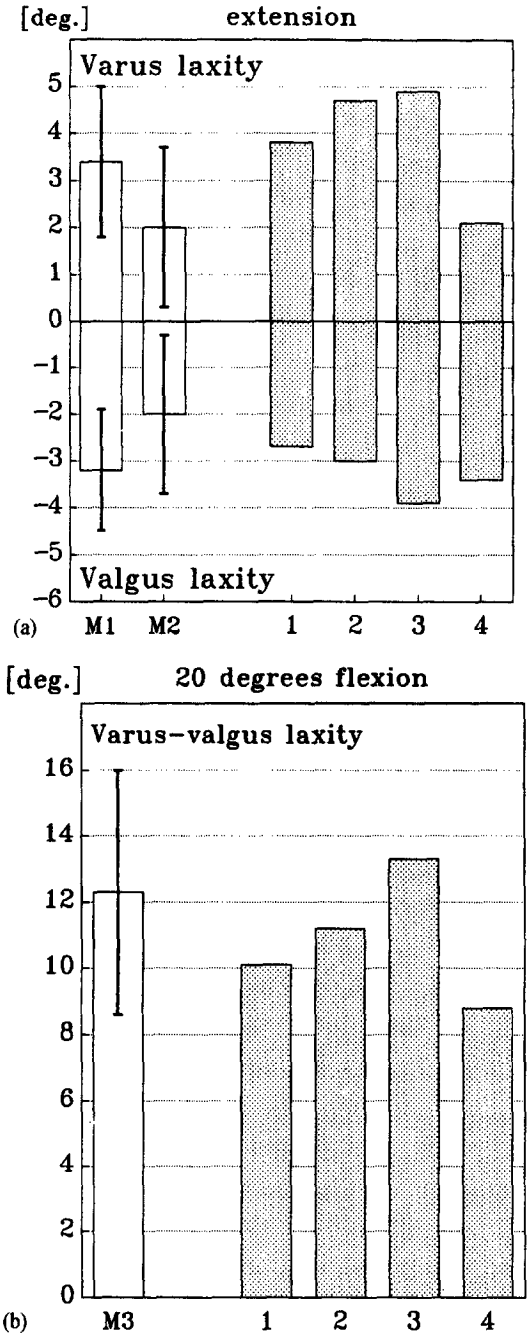

Fig. 5. (a) The varus and valgus laxities of the four optimized models at extension for varus–valgus moments of 20 Nm, with axial rotation constrained, as compared with (M1) the *in vitro* data of Markolf *et al.* (1976; cf. 1978) and (M2) the *in vivo* data of Markolf *et al.* (1978). (b) The total varus–valgus laxity of the four optimized models at 20° flexion for varus–valgus moments of 20 Nm as compared with the (M3) *in vivo* data of Markolf *et al.* (1984).

and meniscectomized knees with regard to I–E rotation and A–P translation in the experiments (Blankevoort *et al.*, 1984) were not similar to the remaining deviations in the optimized knee models. However, the combination of absence of the capsular structures, causing a more mobile medial collateral ligament as compared to the real knees, and the insufficient ligament representation could have resulted in a poor internal rotation restraint.

Representing each ligament by only two or three ligaments and disregarding the menisci and most of the capsular structures is a weakness of the present knee models. It may lead to a nonstable knee joint model for certain joint positions and loading conditions. The question then is whether a dissected knee joint, in which only those structures which are also contained in the knee model are left, will be unstable. In the experimental situation, no compensation for the dissected ligamentous structures can be achieved by increasing the initial tensions in the remaining ligaments. The model analyses do show that the compensation mechanism may not always lead to satisfactory results.

The comparison with the anterior and posterior laxity data of Markolf *et al.* (1976, 1978), showed that for nearly all optimized models the anterior laxities were realistic for 20 and 90° flexion. The posterior laxity value at 20° flexion seemed to be overestimated in the models which are optimized to both I–E and A–P. It must be noted here that during the optimization process the A–P position of the femur relative to the tibia was optimized and not the A–P laxity. At 90° flexion some overconstraint was present in two of the four models. It was concluded that when forcing the knee model to a correct A–P position, the posterior laxity was affected unfavorably. This was due to the simplified two-line model of the posterior cruciate ligament. Such a discretization of the posterior cruciate ligament may have had a considerable effect on the posterior laxity when omitting a possibly important load-bearing part of the ligament which is located in between the most antero-lateral and most postero-medial bundles. In the knee models optimized to both I–E and A–P, the postero-medial part of the posterior cruciate ligament is taut only near extension while the anterolateral part becomes taut for the higher flexion angles. Both bundles are slack and thus not functional at 20° flexion where an intermediate bundle, not included in the model, may have been functional. This problem does not occur with the anterior cruciate ligament because its anterior bundle should remain taut throughout the whole flexion range.

The lateral and medial collateral ligaments functioned sufficiently to restrain external moments and varus–valgus moments. The external rotation as functions of flexion were close to the experimental values and the varus–valgus laxity data were close to the experimental findings of Markolf *et al.* (1976, 1978, 1984). Here the lack of capsular structures and menisci were well compensated for.

The optimization technique proved to be a powerful tool in optimizing the motion characteristics of the knee model relative to the corresponding experimental data when the values of one or more of the parameters of the model are unknown or of an uncertain nature. It provided an excellent guide for future enhancements of the knee model and the subsequent validations. Similar to other previously reported models (Andriacchi *et al.*, 1983; Essinger *et al.*, 1989; Wismans *et al.*, 1980), the knee model in the present study was of a crude nature relative to the complex anatomy of the knee, e.g., the line element representation of the ligaments and the absence of some of the capsular structures and absence of the menisci. However, an optimized knee model did simulate the passive motion characteristics of the knee more realistically than a nonoptimized knee model. In particular, the simulation of the force balance across the knee can give valuable information on the function and the functional mechanisms of the supporting structures.

Acknowledgement—This research program was sponsored in part by Grant 90-90 from the Netherlands Organization for Research (NWO/MEDIGON).

REFERENCES

Andriacchi, T. P., Mikosz, R. P., Hampton, S. J. and Galante, J. O. (1983) Model studies of the stiffness characteristics of the human keen joint. *J. Biomechanics* **16**, 23–29.

Bach, B. R., Daluga, D. J., Mikosz, R., Andriacchi, T. P. and Seidl, R. (1992) Force displacement characteristics of the posterior cruciate ligament. *Am. J. Sports Med.* **20**, 67–71.

Blankevoort, L. and Huiskes, R. (1987) Mathematical simulations of passive knee joint motions. In *Biomechanics: Basic and Applied Research* (Edited by Bergmann, G., Kölbel, R. and Rohlmann, A.). pp 285–290. Martinus Nijhoff, Dordrecht.

Blankevoort, L. and Huiskes, R (1989) An alternative rotation restraint in the knee joint. *Orthop. Trans.* **13,** 257.

Blankevoort, L. and Huiskes, R. (1991) Ligament bone interaction in a three dimensional model of the knee. *J. biomech. Engng* **113,** 263–269.

Blankevoort, L., Huiskes, R. and de Lange, A. (1984) An *in vitro* study of the passive kinematic behavior of the human knee-joint. In 1984 *Advances in Bioengineering* (Edited by Spilker, R. L.). pp 57–58. The American Society of Mechanical Engineers, New York.

Blankevoort, L., Huiskes, R. and de Lange, A. (1988) The envelope of passive knee joint motion. *J. Biomechanics* **21,** 705–720.

Blankevoort, L., Huiskes, R. and de Lange, A. (1991a) Recruitment of knee joint ligaments. *J. biomech. Engng* **113,** 94–103.

Blankevoort, L., Kuiper, J. H., Huiskes, R. and Grootenboer, H. J. (1991b) Articular contact in a three dimensional model of the knee. *J. Biomechanics* **24,** 1019–1031.

Butler, D. L., Kay, M. D. and Stouffer, D. C. (1986) Comparison of material properties in fascicle-bone units from human patellar tendon and knee ligaments. *J. Biomechanics* **19,** 425–432.

Essinger, J. R., Leyvraz, P. F., Heegard, J. H. and Robertson, D. D. (1989) A mathematical model for the evaluation of the behaviour during flexion of condylar-type knee prostheses. *J. Biomechanics* **22,** 1229–1241.

Gibson, M., Mikosz, R., Reider, B. and Andriacchi, T. (1986) Analysis of the Muller anterolateral femorotibial ligament reconstruction using a computerized knee model. *Am. J. Sports Med.* **14,** 371–375.

Grood, E. S. and Suntay, W. J. (1983) A joint coordinate system for the clinical description of three dimensional motions: application to the knee. *J. biomech. Engng* **105,** 136–144.

Hirokawa, S. (1993) Biomechanics of the knee joint: a critical review. *Critical Rev. Biomed. Engng* **21,** 79–135.

Huiskes, R., Kremers, J., de Lange, A., Woltring, H. J., Selvik, G. and van Rens, Th. J. G. (1985) Analytical stereophotogrammetric determination of three-dimensional knee-joint geometry. *J. Biomechanics* **18,** 559–570.

Markolf, K. L., Bargar, W. L., Shoemaker, S. C. and Amstutz, H. C. (1981) The role of joint load in knee stability. *J. Bone Jt Surg.* **63A,** 570–585.

Markolf, K. L., Graff-Radford, A. and Amstutz, H. C. (1978) *In vivo* knee stability. A quantitative assessment using an instrumented clinical testing apparatus. *J. Bone Jt Surg.* **60A,** 664–674.

Markolf, K. L., Kochan, A. and Amstutz, H. C. (1984) Measurement of knee stiffness and laxity in patients with documented absence of the anterior cruciate ligament. *J. Bone Jt Surg.* **66A,** 242–252.

Markolf, K. L., Mensch, J. S. and Amstutz, H. C. (1976) Stiffness and laxity of the knee—the contribution of the supporting structures. A quantitative in vitro study. *J. Bone Jt Surg.* **58A,** 583–594.

Meijer, R. C. M. B., Huiskes, R. and Kauer, J. M. G. (1989) A stereophotogrammetric method for measurements of ligament structure. *J. Biomechanics* **22,** 177–184.

Moré J., Garbow, B. and Hillstrom, K. (1980) User guide for MINPACK-1, Argonne National Labs Report ANL-80-74, Argonne, Ills.

Trent, P. S., Walker P. S. and Wolf, B. (1976) Ligament length patterns, strength, and rotational axes of the knee joint. *Clin. Orthop.* **117,** 263–270.

Wismans, J., Veldpaus, F., Janssen, J., Huson, A. and Struben, P. (1980) A three-dimensional mathematical model of the knee-joint. *J. Biomechanics* **13,** 677–685.