Magneto-elastic buckling of superconducting structural systems

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OF SUPERCONDUCTING
STRUCTURAL SYSTEMS
by

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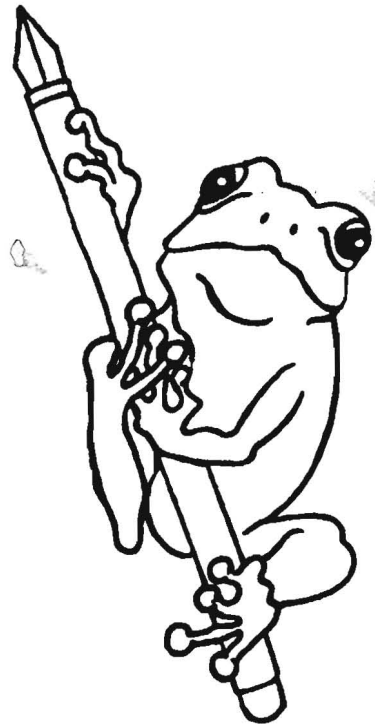
ABSTRACT

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The stability (buckling) of systems of superconducting elastic slender bodies under prescribed current is investigated.

Two methods for the calculation of the buckling current are presented.

The first method, based upon a variational principle, yields an explicit expression for the buckling current. For the evaluation of this expression the magnetic fields pertinent to the deformed superconductor must be calculated.

The second method employs a formula for the Lorentz force on one conductor in interaction with a second conductor, which follows from the law of Biot and Savart.

Applications of both methods are presented for sets of straight parallel rods and for pairs of (concentric or coaxial) rings. The respective buckling currents differ a constant factor, which turns out to be the ratio of the elastic energies.

The differences in results of both methods are small as long as the rods or rings are not too nearby.

SUMMARY

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The stability of superconducting structural systems can be investigated on the basis of a variational principle (cf. [1] or [2]). In this presentation we consider systems consisting of superconducting slender bodies (rods or rings) under prescribed total current I_0. What we are looking for is that value of I_0 (called the buckling value I_{0c}) for which the natural state of the body becomes unstable (buckles). The method described in [1] and [2] results in an explicit expression for the buckling current I_{0c} as a quotient of two terms (both referring to the buckled state of the system)

-   the elastic energy W of the deformed system;
-   an integral K over the surface of the body; the integrand of K contains the magnetic fields and the displacements of the body (see [3] or [4]).

The further procedure consists of

-   a choice for the displacement field $\underline{u}$, specific for the slender body under consideration;
-   calculation of the elastic energy W;
-   determination of the magnetic fields (by solution of the equations obtained by the variational principle) and, finally, calculation of K.

In this way an explicit value of I_{0c} is obtained.

An alternative method is based upon a formula for the force on a curved current carrier (wire) derived from a generalization of the law of Biot and Savart, as given by F.C. Moon (cf. [5], Sect. 2.6, Eq. 2-6.4). In this method, which is less rigorous than the preceding one, the wires are considered as one-dimensional curves. For two curves L_1 and L_2 the force on L_1 is calculated as the Lorentz force due to the current through L_1 times the magnetic field caused by L_2. The buckling value is then obtained in the classical way by solution of beam or ring equations, under the assumptions

-   the displacements are small;
-   the systems are slender (see below).

Applications of both methods will be presented for:

1. A set of two straight parallel rods (infinitely long but periodically supported, support length l).

2. A pair of two concentric rings in one plane (radii b_1 and b_2).

3. A pair of two (identical) coaxial rings (radius b).

4. A set of an infinite number of parallel rods (of the type 1.) in one plane.

The cross-sections of the rods or rings are circular (radius R) and the distance between two members of a system is always $2a$. The systems are called slender if $R \leq a \ll L$, where L can be l, b_1 (or b_2) or b. Moreover, the currents are identical in each rod or ring.

The following results are obtained (here I_0 is the buckling current, E Young's modulus, μ_0 the permeability in vacuum and Q a numerical factor dependent on a/R only (cf. [3])):

	Variational method	Biot–Savart method
1.	$I_0 = \dfrac{\pi^3 R^3}{l^2} \sqrt{\dfrac{E}{\mu_0 Q}}$	$I_0 = \dfrac{\pi^3 a R^2}{l^2} \sqrt{\dfrac{E}{\mu_0}}$
2.	$I_0 = \dfrac{3\pi R^3}{b^2} \sqrt{\dfrac{E}{\mu_0 Q}}$ $(b = \tfrac{1}{2}(b_1 + b_2))$	$I_0 = \dfrac{3\pi a R^2}{b^2} \sqrt{\dfrac{E}{\mu_0}}$
3.	$I_0 = \dfrac{6\pi R^3}{\sqrt{5+v}\, b^2} \sqrt{\dfrac{E}{\mu_0 Q}}$	$I_0 = \dfrac{6\pi a R^2}{\sqrt{5+v}\, b^2} \sqrt{\dfrac{E}{\mu_0}}$
4.	in preparation	$I_0 = \dfrac{\pi^2 a R^2}{l^2} \sqrt{\dfrac{E}{\mu_0}}$

Conclusions

i) The results for 1., 2. and 3. only differ in a constant factor. This factor is solely due to the different elastic energies of the systems; the integral K takes for all these systems the same value ([3], [4]).

ii) The results of the variational and the Biot-Savart method differ from each other only in a factor

$$\frac{a}{R} \sqrt{Q} \ .$$

Hence the results should be in agreement if

$$\frac{1}{\sqrt{Q}} = \frac{a}{R} \ .$$

It turns out ([3]) that for (a/R) not too close to 1 the difference between $Q^{-1/2}$ and (a/R) is small and decreases with increasing (a/R) (e.g. for $a/R \geq 4$, the relative difference is

less than 5%, whereas this difference is at most 80% for $a/R \rightarrow 1$).

iii) Comparing 1. and 4. we conclude that the buckling current for an infinite set of rods is a factor π less than that of a pair of rods.

References

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