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A precursor in waterhammer analysis – rediscovering Johannes von Kries

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ABSTRACT

In 1883 Johannes von Kries published the theory of waterhammer in a study of blood flow in arteries. He derived the "Joukowsky formula" before Joukowsky (1898) and Frizell (1898) did. He considered skin friction in unsteady laminar flow and thus derived formulas for wave attenuation and line pack. The theory was confirmed by experimental results obtained in rubber hoses. In 1892 he published the first textbook describing "classical" waterhammer. It presents formulas for phase-velocity and damping that are frequency-dependent because of skin friction, and in this sense it is the first contribution to the – these days popular – subject of unsteady friction.

Keywords: waterhammer, history, Kries, Joukowsky, Frizell, Allievi, Rankine

INTRODUCTION

Background

The authors have a shared interest in the history of waterhammer. They have read many old papers in English, French, German, Italian and even Russian. Some of these papers are jewels and probably not known to the (younger) members of the waterhammer community. In their previous historical investigations they came across the name *Kries*, e.g. in the extensive reviews by Boulanger (1913, pp. 51, 53), Lambossy (1951, pp. 152, 153, 159, 161), and Stecki and Davis (1986, p. 228), without paying further attention to it.

Another shared interest of the authors is the study of unsteady friction. Tracing back the literature in this area, one ends up with Witzig (1914). Witzig presented two-dimensional solutions for oscillatory laminar flow in tubes with flexible walls. He predicted the annular effect later observed by – and named after – Richardson (1928), Richardson and Tyler (1929). Witzig also presented a one-dimensional wave propagation model with viscous axial dispersion, which he compared with a model by Kries (1892) in which Kries included the much more important laminar skin friction term. This brought us to Kries (1892), an excellent book. In the preface, Kries mentions that it is largely based on a paper presented at the 53rd Meeting of German

Scientists and Medical Doctors held in 1883. In our efforts to get hold of this paper, we found that this had been the 56th meeting. Finally, the paper was found at an oceanographic institute in the Netherlands, in a bound volume, still sealed in its never-opened box.

The missing reference Kries (1883) was there and we were impressed by the quality of its contents. It has never been referred to, as far as we know, except by Kries himself in his book. The waterhammer community is totally ignorant: for example, none of the reviews and textbooks listed at the end of this paper refers to Kries' 1883 paper.

The "Joukowsky equation" for waterhammer

The fundamental equation in waterhammer theory relates pressure changes, ΔP, to velocity changes, ΔV, according to

$$\Delta P = \rho c \Delta V \tag{1}$$

where ρ is the fluid mass density and c is the sonic speed (celerity). Korteweg's (1878) formula defines c for fluid contained in cylindrical pipes of circular cross-section. Relation (1) is commonly known as the "Joukowsky equation", but it is sometimes referred to as either the "Joukowsky-Frizell" or "Allievi" equation. Its first explicit statement in the context of waterhammer is usually attributed to Joukowsky (1898). Frizell (1898) also derived equation (1), but his contribution was criticised and not accepted by his American contemporaries. One of them "saw no reason why this coincidence (with the velocity of sound)" should reassure Frizell regarding the validity of his results (Wood 1970). Allievi (1902), unaware of the achievements by Frizell and Joukowsky, also derived equation (1).

Anderson (2000) notes that Rankine (1870) had already derived equation (1) in a context more general than waterhammer (see our Appendix). Kries (1883, p. 74) also mentions the existence of relation (1) in the theory of shock waves, without a particular reference, but at the same time he states that it had not been validated by experiments, something he would do.

There is a parallel between Joukowsky (1847-1921) and Kries (1853-1928). Both are famous because of their work in other fields: Joukowsky in aerodynamics and Kries in physiology. (E.g., neither Tokaty (1971) or Strizhevsky (1957) mention Joukowsky's work on waterhammer, which was only a small part of his scientific contributions.) Both their investigations of waterhammer impress in clarity and maturity in theory and in experiment. The fast event of waterhammer was difficult to capture in their day. Joukowsky measured in long steel pipes with large wave speeds, Kries in short rubber hoses with small wave speeds, both systems having relatively large L/c times (L is the length of the tube).

The question is: why is equation (1) not the "Kries equation"? That Kries' paper was on blood flow may not be an excuse, because much of the well known – also to the waterhammer community (e.g. Jouguet 1914) – 19th century work was also on blood flow, starting with Young's obscure paper dated 1808, where the described phenomenon was the same: one-dimensional linear wave propagation. It has probably more to do with dissemination. Joukowsky's paper (presented in 1898, published in Russian in 1899, published in German in 1900) has been (partly) translated into English by Simin (1904) and (partly) into French by Goupil (1907). Furthermore, the standard work telling the History of Hydraulics by Rouse and Ince (1957) declares Joukowsky as the founder of the waterhammer discipline. Kries' work on

blood flow has never been translated. Our synopsis in English herein should put this right.

The analogy with longitudinal waves in solid bars has not been noticed by the early investigators of waterhammer. Young (1807, pp. 135, 144) already found the solid-bar equivalent of Eq (1) (with pressure replaced by axial stress). Bergeron (1950; 1961, pp. 194-233) is probably the first to apply – the other way around – waterhammer theory to the axial vibration of solid bars.

Outline

Historical papers highlighting one (or two) scientists are interesting but not yet commonplace in our community. Examples are known to the authors: Joukowsky portrayed by Strizhevsky (1957), Gariel (and waterhammer research prior to 1914) reviewed by Réméniéras (1961), Ménabréa translated by Anderson (1976), Michaud commemorated by Betâmio de Almeida (1979), Allievi commemorated by Franke (1992) and Ceccarelli (1999), and Schnyder and Jaeger celebrated by Hager (2001). This paper on Johannes von Kries is a further such contribution. Four sections describe his life, his work on blood flow, his 1883 paper and his 1892 book.
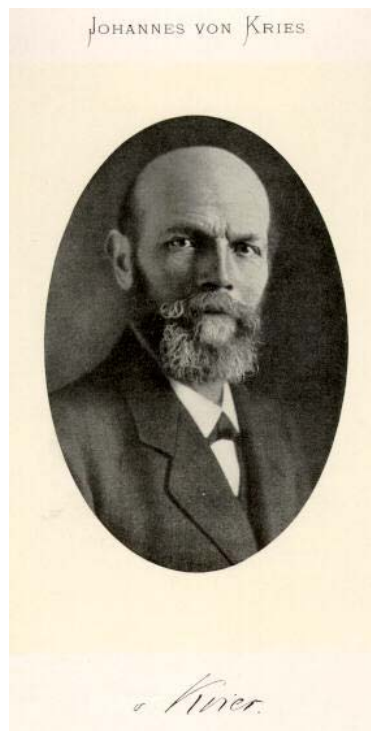

Fig. 1 Johannes von Kries (1853-1928)

HIS LIFE

Johannes von Kries (Fig. 1) died 75 years ago – on 30 December 1928 – at the age of 75. His life and his scientific achievements have been extensively described in dissertations by Lorenz (1996) and Oser (1983). The philosophical side of Kries is brought to the attention of the public in a paper by – his successor in Freiburg – Hoffmann (1957). Kries' view of his own work can be found in Grote (1925). Kries was one of the big names of the late 19th / early 20th centuries. He contributed to the areas of physiology, psychology, philosophy, mathematics and law. He is best

known for his physiological work, in particular for his studies of the sense of vision. The sense of hearing, nerves and muscle mechanics, haemodynamics, the theory of probability, and more, are subjects in his 121 publications.

In 1869 – at the age of sixteen – he started studies of medicine at the University of Halle (under Richard von Volkmann) and at Leipzig and Zurich, finishing as a Doctor of Medicine (at Leipzig) in 1875. After one year of military service, he became a voluntary (i.e. unpaid) "postdoc" at the Institute of Physics of the University of Berlin (under Hermann von Helmholtz) (1876-1877). It is possibly only coincidence, but Helmholtz had interests in both acoustics and haemodynamics and was credited by both Korteweg (1878) and Joukowsky (1898) with first suggesting that the disturbance wave speed in pipes is influenced by both fluid and pipe wall elasticities. In 1877 he became assistant to Carl Ludwig at Leipzig; 1878 "Habilitation" in physiology; 1878-1880 private lecturer at Leipzig; 1880 associate professor of physiology at Freiburg im Breisgau (in succession to Otto Funke); 1882 professor of physiology and director of the physiological institute; retirement in 1924. Kries was co-founder of the *Zeitschrift für Psychologie* with Ebbinghaus and one of its first editors. He received the German order *Pour le mérite* (1918), and three honorary doctorates.

HIS WORK ON BLOOD FLOW

Kries' first publication on haemodynamics appeared in 1878. It described and theorised the manometer measurement of the average blood pressure. In 1883 his memorable paper summarised in the next section saw daylight. Two papers in 1887(ab) presented an improvement of existing techniques to measure the pulse in human bodies. The measuring device is sketched in Fig. 2. A person's forearm (or foot) is to be enclosed in a narrow container filled with air (or petrol vapour for better results). The in- and out-flow of blood to the forearm makes its volume change. Air, thus driven in and out of the container, feeds the flame. The time-varying flow in the arteries typically let a 3 cm flame increase to 4 to 10 cm height. The flow variations were directly related to pressure variations via equation (1), already derived in 1883. In this way Kries obtained nice photographic records (novelty) of the pulse, which he called "tachograms". A third paper in 1887(c) recognised the fact that in laminar pipe flow the maximum velocity (at the central axis) is twice the average velocity. To verify this theoretical result, Kries carried out accurate tests with water and with milk. The experiment involved the measurement of length and volume, but not of time. The issue was of importance in estimating blood circulation times in arterial systems. All his previous work on blood flow was incorporated in his book (1892) to be described later. His last published contribution on the human pulse is dated 1911.
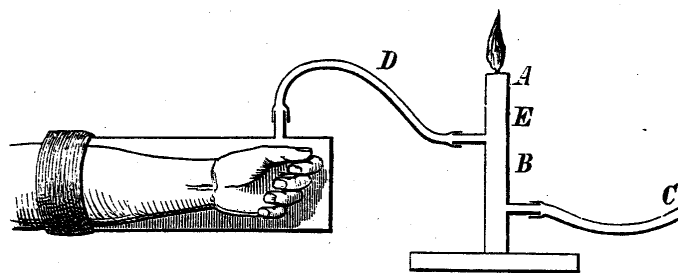

Fig 2 "Flammentachometer" (flame velocity meter), (Kries 1887a, Figure 3).

HIS 1883 PAPER

The paper has four sections and an introduction.

In the introduction Kries describes the state of knowledge of the pulse in 1883. Much experimental data exists, but a proper theoretical background is missing. He is aware of previous work by Young (1808), Weber (1866), Résal (1876), Korteweg (1878), and others, but he feels that all these studies are not of much interest to physiologists because they focus just on one aspect: a theoretical value for the wave propagation speed. Kries wants to go beyond that. He mentions the equivalence of incompressible fluid in an elastic tube (pulse) and compressible fluid in a rigid tube (waterhammer). His one-dimensional model for linear wave propagation can describe both.

The first part of Section I presents the basic theory of waterhammer including the "Joukowsky formula" on page 74. Kries makes the right assumptions: uniform pressure in radial direction, cross-sectional averages of velocities, hoop stress proportional to pressure, negligible influence of convective terms, Moens-Korteweg wave propagation speed (noting that there might be a dependency on pressure in flexible rubber hoses). The continuity equation and the equation of motion are combined into the second-order wave equation, which has D'Alembert travelling-wave solutions for pressure and velocity. The derivation of the pressure-velocity relation (1) follows then from basic principles. As already noted in our introduction, Kries stated that an analogue of this relation was already known – but not validated (too difficult) – in the theory of sound waves (in air).

The second part of Section I describes the experimental validation. A constant-head reservoir supplied water to a 4 to 5 m long, thin-walled, rubber hose of 5 mm diameter. The steady mass flow was measured. Rapid valve-closure caused the pressure rise, measured with a spring-manometer, shown in Fig. 3. The valve closed at time "s" (in Fig. 3) and the reflection from the reservoir arrived at time "r" (in Fig. 3). The value of the wave speed, c, was estimated from the reflection time $2L/c$. Measured pressure rises (in mmHg) for three different flow rates were: 31.1, 50.0 and 72.0; the corresponding values according to equation (1) were: 29.9, 47.6 and 71.6. Experiments in a tube with a more rigid wall gave values: $\Delta P = 69.0$ mmHg and $\rho c \Delta V = 70.0$ mmHg. Happy with this validation, Kries considered periodic velocity-excitation of an infinitely long tube (no reflections), giving the classical square wave in Fig. 4.
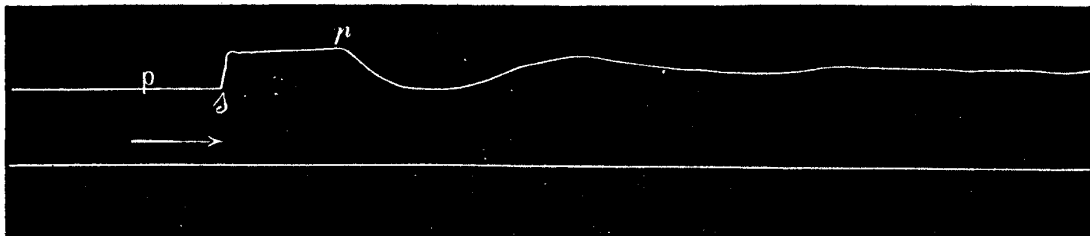

Fig 3   Measured "Joukowsky" pressure in large-diameter hose,
(p = pressure, s = closure, r = reflection), ($r – s \approx 0.6$ seconds), (Kries 1883, Fig. 8).
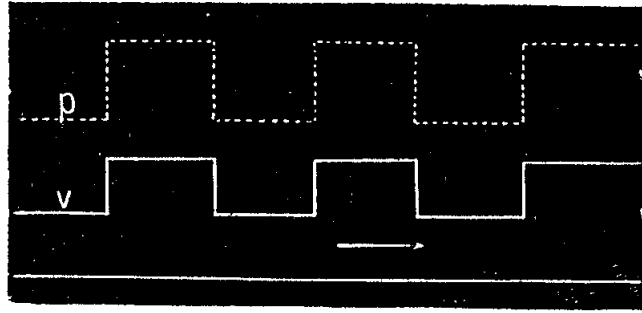
Fig 4 Pressure and described velocity changes in an infinitely long tube (without friction), (p = pressure, v = velocity), (Kries 1883, Fig. 9).

The first part of Section II develops theory for waterhammer with linear friction. The friction term, added to the equation of motion, is taken proportional to the flow velocity. Kries mentions that the constant of proportionality, η, depends on fluid properties and tube diameter, but he does not specify its value – for laminar pipe flow given by Hagen (1839) and Poiseuille (1840). It is noted that in the same year 1883, Gromeka modelled the same friction term, thus making his more advanced equations too difficult to solve.

Kries ends up with the telegrapher's equation, which he subjects to a harmonic analysis. For small friction terms he derives constant values for wave damping and phase velocity (the Moens-Korteweg wave speed). The friction term causes small differences in phase and amplitude between velocity and pressure (note that pressure and velocity are in phase in the frictionless case shown in Fig. 4). These differences are not constant, but frequency dependent. The changed, due to friction, amplitude of the pressure in the harmonic solution is used to predict line pack. After some manipulation, line pack at the valve was estimated from:

$$\frac{\mathrm{d}P_{\text{linepack}}}{\mathrm{d}t} \approx \tfrac{1}{4}\eta\rho c\,V \tag{2}$$

where according to Hagen-Poiseuille theory $\eta = 8v/R^2$ (v is the kinematic viscosity of the fluid and R is the tube radius). However, the present authors are not entirely convinced by Kries' derivation of formula (2). A better explanation, in terms of the initial pressure gradient, and more examples are given in his book (1892). It is noted that Joukowsky (1898, Section 11) also recognised that line pack is the consequence of an initial (steady-state) pressure gradient and he explained this nicely in terms of a step-wise increasing initial pressure (Joukowsky 1898, Fig. 20). Joukowsky (1898, Fig. 19) observed line pack in steel pipes and he proposed a formula similar to equation (2).

The second part of Section II concerns the first observation of line pack and the verification of equation (2). Figure 5 shows the result of an experiment in a narrow tube. The line-pack effect makes the pressure slowly rise after the valve's rapid closure at time "s". For tubes with sufficient friction Kries measured line pack values of 18.3, 40.0 and 72.9 mmHg/s. The corresponding values calculated from equation (2) were: 19.6, 38.3 and 73.1 mmHg/s. Theoretical results for an infinitely long tube, excited by periodic velocity-pulses at one point, are sketched in Fig. 6.
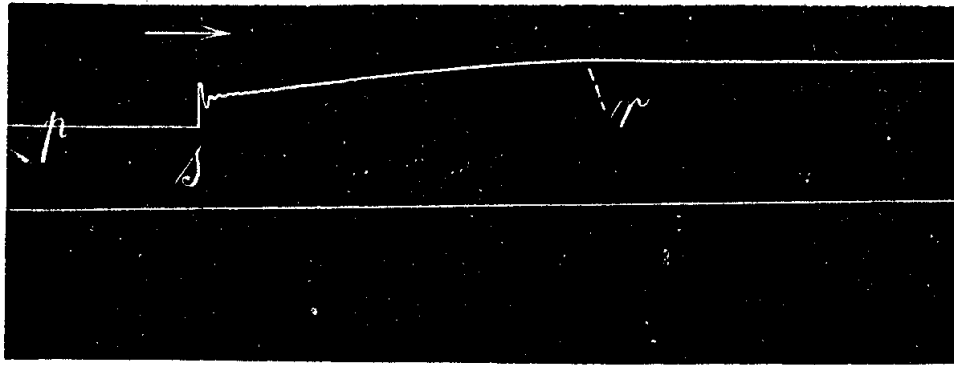
Fig 5 Measured "Joukowsky" pressure and line pack in small-diameter hose,
(p = pressure, s = closure, r = reflection), ($r - s \approx 1.5$ seconds), (Kries 1883, Fig. 10).
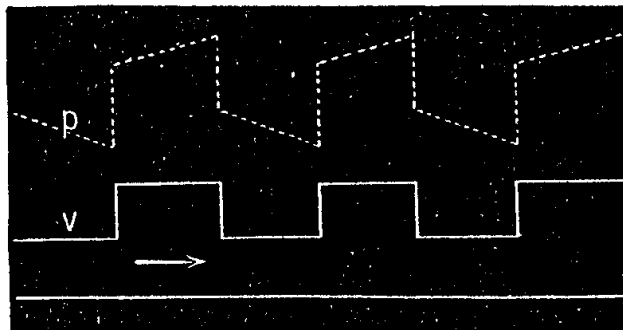

Fig 6 Pressure and described velocity changes in an infinitely long tube (with friction),
(p = pressure, v = velocity), (Kries 1883, Fig. 11).

Section III discusses pressure pulse and blood flow in the aorta. Backed up by his experiments, Kries states that friction is unimportant in the aorta. The beating heart induces flow velocity changes that directly relate to the pressure pulse through equation (1). He also considers the longitudinal stretching of the aorta wall, and explains the possibility of a secondary pressure rise, shown in Figure 7, because of axial motion of the closed heart-valve. This is one of the first examples of fluid-structure interaction (junction coupling). Kries concludes with the remark that reflections (from ends that are neither open nor closed) and many other secondary effects exist in the vascular system, for which the theoretical background is absent.
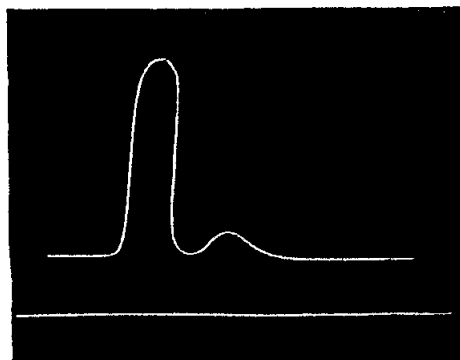

Fig 7 Sketch of pressure pulse in stretching aorta (with FSI), (Kries 1883, Fig. 15).

Section IV deals with the (im)possibilities of measuring pressure pulse and volume flow in peripheral arteries. Theoretically, reflected waves can be distinguished in a signal if pressure and velocity are measured at the same location. Such a simultaneous measurement could not be done with sufficient reliability in 1883.

HIS 1892 BOOK

Kries' book is well written and a pleasure to read. It is based on his 1883 paper, but the material is improved, extended and presented in a more structured way. He uses the theory developed in Chapter I to explain the pulse in the remaining chapters. Chapter II considers the form of the pulse and the dicrotic wave (secondary pulse), Chapter III deals with the aortic bifurcation, and Chapter IV discusses various aspects (gravity, temperature) that affect the pulse. The Appendices give the mathematics behind his theory. The book displays many experimental results: manometer measurements in rubber hoses and accurate pulse records with the "Flammentachometer" (Fig. 2).

The fundamental Chapter I gives the general theory of travelling waves, standing waves, attenuation, reflection, forced oscillation, tube breathing and ovalization, etc. The theory is applied to rubber hoses and in this respect Kries observed, from careful tests, visco-elastic retardation of the wall material. He understood the phenomenon; it explained the fact that Weber (1866) measured wave speeds that were 12% larger than the theoretical predictions. The internal pressure may influence the wave speed, because it changes the tube diameter and the wall properties (like stiffness). The latter is the case for tubes made from intestine membranes. He validated equation (1), now through tests with fluid injection instead of valve closure. His treatment of wave attenuation due to skin friction is an elaboration of his 1883 work. He finds exponential damping of the waveform and, for systems with much friction, waveform distortion because of frequency-dependent wave speeds. Slow pressure variations have a low phase velocity and low damping; fast pressure variations have a high phase velocity (about the Moens-Korteweg wave speed) and high damping. Line pack is correctly explained, but equation (2) is absent in the book. Appendix IV, giving formulas for frequency-dependent damping and phase velocity, can be seen as the first step in the investigation of unsteady friction. The frequency-dependent phase velocity, $c(\omega)$, follows from:

$$\frac{1}{c(\omega)^2} = \frac{1}{2c^2}\left(1 + \sqrt{1 + \frac{\eta^2}{\omega^2}}\right) \qquad (3)$$

where ω is the circular frequency, $\eta = 8v/R^2$ and c is the Moens-Korteweg wave speed. His theoretical study of reflections from open ends, closed ends, branches, tapered sections and abrupt changes of friction were backed up by experimental results. Kries wondered how the pulse, travelling from the aorta into the many branches of the arterial system, could occur without reflections. As a result he derived the condition for reflection-free branches (boundaries). The fundamental waterhammer periods $2L/c$ and $4L/c$ appear in his section on travelling and standing waves in finite-length tubes. In an explanation of experimental results by Moens (1878) he produced the mid-point pressure history shown in Fig. 8, which is typical for waterhammer in a single tube. Kries discussed the possible evidence of wall vibration in measured pressure histories. First, he considered "hoop" vibration where the wall remains circular. He found that the corresponding "ring" period, $\pi D/c_{wall}$, was much too small to be measured. Hoop vibration changes the *size* of the cross-sectional area and thus causes axial flow and axial wave propagation in fluid and wall. Second, he considered wall vibration changing the *form* of the cross-sectional

area, but preserving its size. Axial interaction is less in this case and the fundamental periods are larger. The deformation (flattening) – due to internal hydrostatic pressure – of flexible tubes laid on a flat floor was studied theoretically and the associated oscillation was examined in tests with bouncing tubes. The period of oscillation of an ovalizing tube was estimated from a formula originally derived by Rayleigh for the capillary oscillation of free jets. The theoretical predictions were confirmed by experimental results obtained in free-hanging tubes (in Appendix VIII).
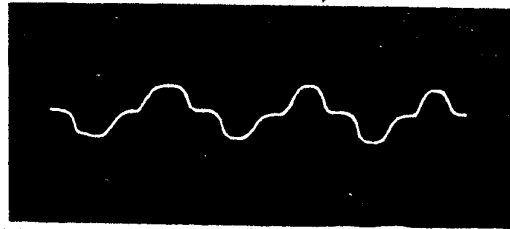

Fig. 8   Sketch of theoretical waterhammer pressure at midpoint of tube (Kries 1892, Fig. 16).

CONCLUSIONS

Not only has this historical investigation shown that it is increasingly difficult to be original, it has also shown the value of pursuing the references that are available in textbooks and review papers. In addition, it demonstrates the importance of being aware of related developments in other disciplines. In comparison with physiological flows, engineering waterhammer analysts have been relatively slow to (re-)discover the importance in certain situations of fluid-structure interaction, unsteady friction and visco-elastic pipe materials, all topics addressed by Papers at this Conference. This other, "hidden" literature is important not just because it is earlier, but because it may suggest to us apparently novel alternative experimental and analytical approaches to investigating these phenomena. Despite his lack of historical impact compared with Joukowsky, Allievi and others, Kries has much to offer us even now, and was clearly an innovator whose contributions deserve to be recognised by us in future.

ACKNOWLEDGEMENT

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APPENDIX   Rankine (1870) and the "Joukowsky equation"

Rankine opened this paper by writing that: "The object of the present investigation is to determine the relations which must exist between the laws of the elasticity of any substance, whether gaseous, liquid or solid, and those of the wave-like propagation of a finite longitudinal disturbance in that substance."

He set up a simple model of a wave front of invariable length for a wave of uniform type in a uniform prismatic pipe. Continuity across this gives (where m is "the mass of matter through which a disturbance is propagated in a unit of time while advancing along a prism of the sectional area unity"):

$$m = \rho_1 \left(V_1 - c \right) = \rho_2 \left(V_2 - c \right) \tag{A1}$$

where c is the uniform wave propagation velocity, ρ denotes fluid density and V denotes fluid particle velocity, with the velocity change $\left(V_1 - V_2 \right)$ occurring across the wave front. He called this constant quantity m the "mass velocity" of the wave, pointing out in the Supplement added to his paper that "the method of investigation in the present paper, by the aid of mass-velocity to express the speed of advance of a wave, is new, so far as I know; and it seems to me to have great advantages in point of simplicity, enabling results to be demonstrated in a very elementary manner".

Next, he immediately states that "Then in each unit of time the difference of pressure, $(P_1 - P_2)$, impresses on the mass m the acceleration $(V_1 - V_2)$, and consequently, by the second law of motion, we have the following value for the difference of pressure":

$$(P_1 - P_2) = m(V_2 - V_1) \tag{A2}$$

Now Rankine, with his interests in gas dynamics and shock waves, was considering the case with "the velocity of the particles being so great that it is not to be neglected in comparison with the velocity of propagation", but for the case with $V \ll c$ then the above give simply:

$$(P_1 - P_2) = -\rho_1 c(V_2 - V_1)(1 - V_1/c) \approx -\rho_1 c(V_2 - V_1) \tag{A3}$$

which is the classic "Joukowsky equation" for this special case likely to occur with liquids with negligible density change. Rankine therefore not only predates Joukowsky, but also has a more general formulation, though he did not express it in the modern form, but with his "mass-velocity" m.

It is interesting to note that, according to Tokaty (1971), Joukowsky delivered a lecture on "the speed of sound in fluids" at Moscow University in 1886 and he was certainly familiar with the literature of gas dynamics (Strizhevsky 1957). However, he did not refer to Rankine in his 1898 paper on waterhammer, even though he seems to have been scrupulous in referring to previous European and North American studies, so there is no reason to suppose that he had seen this result previously.