Price-based optimal control of electrical power systems

Citation for published version (APA):
Jokic, A. (2007). *Price-based optimal control of electrical power systems*. [Phd Thesis 1 (Research TU/e / Graduation TU/e), Electrical Engineering]. Technische Universiteit Eindhoven. https://doi.org/10.6100/IR629297

DOI:
10.6100/IR629297

Document status and date:
Published: 01/01/2007

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

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Price-based Optimal Control of Electrical Power Systems

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de
Technische Universiteit Eindhoven,
op gezag van de Rector Magnificus, prof.dr.ir. C.J. van Duijn,
voor een commissie aangewezen door het College voor
Promoties in het openbaar te verdedigen op
maandag 10 september 2007 om 16.00 uur

door

Andrej Jokić

geboren te Zagreb, Kroatië

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Price-based optimal control of electrical power systems / by Andrej Jokic.
– Eindhoven : Technische Universiteit Eindhoven, 2007.
Proefschrift. – ISBN 978-90-386-1574-5
NUR 959
Trefw.: elektrische energiesystemen ; regeling / elektriciteitsvoorziening ; economie / gedistribueerde regelsystemen ; optimalisering / energiemarkt.
Subject headings: power system control / power generation economics / distributed control / power markets.

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The Ph.D. work is a part of the Dutch IOP-EMVT program (Innovatiegerichte Onderzoeksprogramma's - Elektromagnetisch Vermogens Techniek) and is supported financially by SenterNovem, an agency of the Dutch Ministry of Economic Affairs.

The Ph.D. work forms a part of the research program of the Dutch Institute of Systems and Control (DISC).

Contents

Summary

Price-based Optimal Control of Electrical Power Systems

During the past decade, electrical power systems have been going through some major restructuring processes. From monopolistic, highly regulated and one utility controlled operation, a system is being restructured to include many parties competing for energy production and consumption, and for provision of many of the ancillary services necessary for system operation. With the emergence of competitive markets as central operational mechanisms, the prime operational objective has shifted from a centralized, utility cost minimization objective to decentralized, profit maximization objectives of competing parties. The market-based (*price-based*) operation is shown to be practically the only approach that is capable to simultaneously provide incentives to hold the prices at marginal costs and to minimize the costs. As a result, such an operational structure inherently tends to maximize the social welfare of the system during its operation, and to accelerate developments and applications of new technologies.

Another major change that is taking place in today's power systems is an increasing integration of small-scale distributed generation (DG) units. Since in future power systems, a large amounts of DG will be based on renewable, intermittent energy sources, e.g. wind and sun, these systems will be characterized by significantly larger uncertainties than those of the present power systems.

Power markets significantly deviate from standard economics since the demand side is largely disconnected from the market, i.e. it is not price responsive, and it exhibits uncertain, stochastic behavior. Furthermore, since electrical energy cannot be efficiently stored in large quantities, production has to meet these rapidly changing demands in *real-time*. In future power systems, efficient real-time power balancing schemes will become crucial and even more challenging due to the significant increase of uncertainties by large-scale integration of renewable sources. Physical and security limits on the maximal power flows in the lines of power transmission networks represent crucial system constraints, which must be satisfied to protect the integrity of the system. Creating an efficient congestion management scheme for dealing with these constraints is one of the toughest problems in the electricity market design, as the line power flows are characterized by complex dependencies on nodal power injections. Efficient congestion control

has to account for those limits by adequately transforming them into market signals, i.e. into electricity prices.

One of the main contributions of this thesis is the development of a novel dynamic, distributed feedback control scheme for optimal real-time update of electricity prices. The developed controller (which is called the *KKT controller* in the thesis) reacts on the network frequency deviation as a measure of power imbalance in the system and on measured violations of line flow limits in a transmission network. The output of the controller is a vector of nodal prices. Each producer/consumer in the system is allowed to autonomously react on the announced price by adjusting its production/consumption level to maximize its own benefit. Under the hypothesis of global asymptotic stability of the closed-loop system, the developed control scheme is proven to continuously balance the system by driving it towards the equilibrium where the transmission power flow constraints are satisfied, and where the total social welfare of the system is maximized. One of the advantageous features of the developed control scheme is that, to achieve this goal, it requires no knowledge of marginal cost/benefit functions of producers/consumers in the system (neither is it based on the estimates of those functions). The *only* system parameters that are explicitly included in the control law are the transmission network parameters, i.e. network topology and line impedances. Furthermore, the developed control law can be implemented in a distributed fashion. More precisely, it can be implemented through a set of nodal controllers, where one nodal controller (NC) is assigned to each node in the network. Each NC acts only on locally available information, i.e. on the measurements from the corresponding node and on the information obtained from NC's of the adjacent nodes. The communication network graph among NC's is therefore the same as the graph of the underlying physical network. Any change is the network topology requires only simple adjustments in NC's that are local to the location of the change.

To impose the hard constraints on the level to which the transmission network lines are overloaded during the transient periods following relatively large power imbalances in the system, a novel price-based hybrid model predictive control (MPC) scheme has been developed. The MPC control action adds corrective signals to the output of the KKT controller, i.e. to the nodal prices, and acts only when the predictions indicate that the imposed hard constraint will be violated. In any other case, output of the MPC controller is zero and only the KKT controller is active. Under certain hypothesis, recursive feasibility and asymptotic stability of the closed-loop system with the hybrid MPC controller are proven.

Next contribution of this thesis is formulation of the *autonomous power* *networks* concept as a multilayered operational structure of future power systems, which allows for efficient large-scale integration of DG and small-scale consumers into power and ancillary service markets, i.e. markets for different classes of reserve capacities. An autonomous power network (AN) is an aggregation of networked producers and consumers, whose operation is coordinated/controlled with one central unit (AN market agent). By performing optimal dispatching and unit commitment services, the main goals of an AN market agent is to efficiently deploy the AN's internal resources by its active involvement in power and ancillary service markets, and to optimally account for the local reliability needs. An autonomous power network is further defined as a major building block of power system operation, which is capable of keeping track of its contribution to the uncertainty in the overall system, and is capable of bearing the responsibility for it. With the introduction of such entities, the conditions are created that allow for the emergence of novel, competitive ancillary service market structures. More precisely, in ANs based power systems, each AN can be both producer and consumer of ancillary services, and ancillary service markets are characterized by double-sided competition, what is in contrast to today's single-sided ancillary service markets. One of the main implications of this novel operational structure in that, by facilitating competition, it creates the strong incentive for ANs to reduce the uncertainties and to increase reliability of the system. On a more technical side, the AN concept is seen as decentralization and modularization approach for dealing with the future, large scale, complex power systems.

As additional contribution of this thesis, motivated by the KKT controller for price-based real-time power balancing and congestion management, the general KKT control paradigm is presented in some detail. The developed control design procedure presents a solution to the problem of regulating a general linear time-invariant dynamical system to a time-varying economically optimal operating point. The system is characterized with a set of exogenous inputs as an abstraction of time-varying loads and disturbances. Economic optimality is defined through a constrained convex optimization problem with a set of system states as decision variables, and with the values of exogenous inputs as parameters in the optimization problem. A KKT controller belongs to a class of dynamic complementarity systems, which has been recently introduced and which has, due to its wide applicability and specific structural properties, gained a significant attention in systems and control community. The results of this thesis add to the list of applications of complementarity systems in control.

Samenvatting

In het laatste decennium hebben elektrische energiesystemen grote structurele veranderingen ondergaan. Was het eens een monopolistisch, centraal aangestuurde organisatie, nu is een systeem ontstaan met vele partijen die onderling concurreren bij de opwekking en het gebruik van energie en voor het verzorgen van de noodzakelijke reservecapaciteit. Door de komst van een transparante energiemarkt is de doelstelling verschoven van een die was gericht op de centrale minimalisatie van de kosten van de opwekkers tot een maximalisatie van de opbrengsten van alle individuele deelnemers. Het marktprincipe (op prijs gebaseerd) is de enige praktische manier om gelijktijdig stimulansen te geven om te leveren/consumeren tegen de marginale kosten en om de kosten te minimaliseren. Een dergelijke markt-gerichte organisatie leidt tot maximalisatie van de maatschappelijke waarde en stimuleert de ontwikkeling en toepassing van nieuwe technologieën.

Een tweede belangrijke verandering in de huidige energiesystemen is een toename van kleinschalige, verspreide opwekking (DG) van energie. Omdat toekomstige energiesystemen met een groot aandeel DG gebaseerd zullen zijn op hernieuwbare energiebronnen, zoals wind and zon, zullen deze systemen worden gekarakteriseerd door een aanmerkelijk grotere onzekerheid dan heden ten dage.

Energiemarkten wijken sterk af van standaard economische modellen omdat de vraag vrijwel volledig is ontkoppeld van de markt: zij is niet gevoelig voor de prijs en vertoont onzeker en onvoorspelbaar gedrag. Omdat elektrische energie niet efficiënt kan worden opgeslagen, moet de productie de vraag direct volgen. Het snel kunnen afstemmen van vraag en aanbod wordt in toekomstige energiesystemen een toenemend kritische factor door de groei van hernieuwbare, minder goed voorspelbare energiebronnen. Fysieke en veiligheidsbegrenzingen van de maximale vermogensstroom in hoogspanningslijnen moeten worden bewaakt om de goede werking van het energiesysteem te allen tijde te kunnen waarborgen. Het ontwerpen van een vermogensverdeler in een net dat overbelasting voorkomt is een van de lastigste problemen bij het ontwerpen van energiemarkten, omdat de vermogensstromen complexe functies zijn van de geïnjecteerde vermogens in de knooppunten. Een efficiënte vermogensverdeler moet deze beperkingen omzetten in marktsignalen: namelijk in marktprijzen.

Een van de belangrijkste bijdragen van dit proefschrift is de ontwikkeling van zo'n vermogensverdeler gerealiseerd als een nieuwe, dynamische, gedistribueerde regelstrategie die deze marktprijzen real-time aanpast. De ontworpen regelaar (die in dit proefschrift KKT regelaar wordt genoemd) reageert op de afwijking van de netfrekwentie als een maat voor de onbalans in het net en op de gemeten overschrijdingen van de maxima in de lijnen van het transmissiesysteem. De uitgang van deze regelaar is een vector met de prijzen voor het vermogen in elk knooppunt. Elke producent/gebruiker kan reageren op deze prijzen door zijn productie/consumptie zo aan te passen dat hij zijn eigen doelstellingen maximaliseert. Onder de aanname dat het totale geregelde systeem stabiel is, is aangetoond dat de ontwikkelde KKT-regelaar het systeem zo aanstuurt dat aan alle beperkingen wordt voldaan en dat de doelfunctie, de maatschappelijke waarde, wordt bereikt. Een van de grote voordelen van deze regelaar is dat deze geen kennis nodig heeft van de marginale kosten van de producenten/consumenten in het energiesysteem, noch is deze regelaar gebaseerd op schattingen van deze kosten. De enige parameters die bekend worden verondersteld zijn de topologie van het netwerk en de lijnimpedanties. Bovendien kan deze regelaar worden geïmplementeerd als gedistribueerde regelaar. Nauwkeuriger uitgedrukt, hij kan worden geïmplementeerd als een knooppuntsregelaar, met op elk knooppunt zo'n regelaar. Elke regelaar werkt alleen met lokaal beschikbare informatie, dat wil zeggen op metingen bij het knooppunt en op informatie van alleen de naburige knooppunten. De communicatie vindt dus met dezelfde knooppunten plaats als die waarmee direct fysieke vermogen wordt uitgewisseld. Iedere verandering in het fysieke netwerk vereist slecht eenzelfde aanpassing in de communicatie.

Om harde grenzen te kunnen stellen aan de overbelasting van de transmissielijnen tijdens redelijke grote verstoringen in het energiesysteem, is een nieuwe, prijs-gebaseerde hybride voorspellende regelaar (MPC) ontwikkeld. Deze MPC regelaar voegt corrigerende signalen toe aan de uitgang van de KKT regelaar, dus aan de knooppuntsprijzen, en deze is alleen actief als de voorspellingen aangeven dat de harde begrenzingen worden overschreden. In elk ander geval is de uitgang van de MPC regelaar nul en is alleen de KKT-regelaar actief. Onder een bepaalde aanname, is de recursieve bestaanbaarheid en asymptotische stabiliteit van het geregelde systeem met hybride MPC regelaar bewezen.

Een andere bijdrage van dit proefschrift is de formulering van het concept autonoom netwerk, dat de groot-schalige integratie van gedistribueerde opwekking en kleine opwekkers toestaat in de energie- en reservecapaciteitsmarkten. Een autonoom netwerk (AN) vertegenwoordigt gekoppelde producenten en consumenten die worden aangestuurd door een centrale organisatie

(AN markt agent). Door het berekenen van de optimale vermogensverdeling en de optimale inzet van productie-eenheden is de hoofdtaak van een AN markt agent het efficiënt benutten van de interne middelen door actief te opereren in de energie- en reservecapaciteitsmarkten en optimaal rekening te houden met de betrouwbaarheid (reservecapaciteit) van het eigen net. Een AN is ook gedefinieerd als een belangrijk bouwblok van het totale energienet dat in staat is rekening te houden met en verantwoordelijkheid te dragen voor zijn eigen reservecapaciteit. Met de introductie van AN's zijn de voorwaarden gecreëerd voor het ontstaan van nieuwe, reservecapaciteitsmarkten. In een op AN's gebaseerd energienet kan elke AN zowel producent als consument zijn van de reservecapaciteit. De reservecapaciteitsmarkt wordt dan gekarakteriseerd door een dubbelzijdige concurrentie, dit in tegenstelling tot de huidige eenzijdige concurrentie. Een van de belangrijkste gevolgen van deze nieuwe operationele structuur is dat, door concurrentie toe te laten, dit een sterke stimulans vormt voor AN's om onzekerheid te reduceren en daarmee de betrouwbaarheid van het energiesysteem te verhogen. Vanuit een meer technisch standpunt gezien is het concept van AN's een modulaire aanpak om om te gaan met de complexiteit van toekomstige energiesystemen.

Als extra bijdrage van dit proefschrift, gemotiveerd door de KKT-regelaar voor prijs-gebaseerd, real-time balanceren van het vermogen en het voorkomen van overbelasting in het transmissienetwerk, wordt de algemene KKT-regelaar nader uitgewerkt. De ontwikkelde ontwerpmethode biedt een oplossing voor het regelen van een algemeen lineair, tijd-invariant dynamisch systeem bij een tijdvariërend economisch optimaal werkpunt. Het systeem wordt gekarakteriseerd met een aantal exogene ingangen als een abstractie van tijdvariërende belastingen en verstoringen. Het economische optimum wordt gedefinieerd door een convex optimalisatieprobleem met begrenzingen met een aantal toestanden als beslisvariabelen en met de waarden van de ingangen als parameters in het optimalisatieprobleem. Een KKT-regelaar behoort tot de klasse van dynamische complementariteitsystemen die recent zijn geïntroduceerd en die, door hun brede toepasbaarheid en specifieke eigenschappen, veel aandacht hebben gekregen in de systeem- en regeltechniek. De resultaten van dit proefschrift vormen een bijdrage aan de lijst toepassingen van complementariteitsproblemen in de regeltechniek.

1

Introduction

Electrification is one of the great engineering achievements of the 20th century. In many studies, the standard of living is directly related to the electricity used per capita. Electrical power systems are also one of the largest and most complex engineering systems ever created. They consist of thousands of generators and substations, and hundreds of millions of consumers, all physically interconnected across circuits of continental scale. This complex engineering system has to cope with technical, economical, business, political, social and environmental aspects. It is therefore a common meeting ground of many different disciplines, a multilayered infrastructure that has evolved over many decades through research efforts in various fields, utilizing new technologies as they have emerged.

Control applications have a long history in electrical power systems. This long history, together with the persistence of a regulated industry have contributed to a perception that the key control problems of electric power systems have been thoroughly solved. Today, however, new questions are being introduced as electrical power systems are undergoing some major paradigm shifts.

The most significant change is a liberalization and a policy shift towards competitive market mechanisms for operation of the system. From a monopolistic, one utility controlled operation, the system is being restructured to include many parties competing for energy production and consumption, and for provision of many of the services necessary for the system operation.

Another crucial change is the encouragement of the large-scale integration of distributed generators (DG), many of which are based on intermittent renewable sources like wind and sun. This integration of DG is already taking place for some time now and, for renewable sources, many countries have posed high targets over ten years horizons. Non-dependence on fossil fuels of many DG technologies, together with environmental issues are the main driving forces towards those targets.

Virtually all aspects, e.g. technical, economical, social, environmental etc., of the power systems are affected by these changes, requiring examination of new control architectures that will enable high operational efficiency and high reliability of tomorrow's novel power systems. As an electrical power system is characterized by the tight and extremely complex interactions of its structural layers, it is necessary to define the system boundaries of the research. That is our goal in this chapter.

1.1 Restructuring of electrical power systems

Throughout most of the twentieth century, electrical utilities, which were often government owned entities, were regional regulated monopolies. As a single business entity, a utility owned and operated the generation, transmission and distribution systems located in a certain geographic area. Following the impressive consequences of the liberalization of the telecommunication and air transport markets during the past two decades, electrical power systems have been going through a similar restructuring processes. Electrical energy is, however, quite a specific commodity, making a design of power markets a very challenging task. There are two main reasons for this:

- Electrical energy cannot be efficiently stored in large quantities, which implies that production has to meet rapidly changing demands immediately in *real-time*.
- Produced energy is transported through a transmission system which has a limited capacity and is characterized by complex dependencies of transmission path flows on nodal power injections (nodal productions and consumptions).

These physical properties of electrical power systems play a prominent role in designing electricity markets and control architectures, and they are responsible for a very tight coupling in between economical and physical/technical layers of an electrical power system. This is also a reason why a straightforward transfer of knowledge and experience from deregulation and restructuring of other sectors to the electric power system sector is often hampered or, even more often, is simply impossible. With its own goals, peculiarities and problems faced in practice over the course of the past two decades, power system restructuring has developed as a research discipline of its own, with a strong interdisciplinary character and with an impressive scientific output.

As an introduction to the presentation of the motivation and the definition of the research scope of this thesis, we continue by giving a short description of the power system operation before restructuring, and the changes occurring during the restructuring process. For a detailed introduction and overview of power system restructuring, we refer to the many books on the subject, e.g. (Schweppe et al., 1988; Ilic et al., 1998; Lai, 2001; Stoft, 2002; Song and Wang, 2003).

1.1.1 Power balance control

One of the main control objectives in an electrical power system is to maintain the balance between power production and demand. However, one must be careful in correctly interpreting the term power balance.

Generally speaking, power balance is a law of physics. At any time instant, the power injected from the generators into the network is equal to the power consumed by the loads plus the losses in the lines. From this point of view, the system is always in balance. However, by considering the *desired consumption* instead of the *actual consumption*, the notion of imbalance becomes well defined, and balancing becomes a control problem. This desired consumption is what we will call a power demand, and is precisely defined as follows: *Power demand is the amount of consumed power if the network frequency and voltage are equal to their target values.*

If at the connection point of a consumer the rest of the power network behaves as a sinusoidal voltage signal of 50Hz and 230 volts rms (target values in Europe), this consumer will withdraw the energy from the network at a desired power rate, e.g. equal to the nominal power of the consuming electrical device. If the frequency and magnitude of the voltage are at their target values for all the consumers in the system, there is a balance between production and demand. *In electrical power systems, the term power balance is used* not *to denote a law of physics, but to denote the balance between power production and demand.*

Since electrical power systems are AC systems, both active and reactive power, i.e. demand for active and demand for reactive power, have to be considered. It turns out that for a large class of problems, active and reactive power can be studied separately, since the flows of active and reactive power in a transmission network are fairly independent of each other and are influenced by different control actions. Frequency and active power are closely related to each other, while reactive power is more closely related to the voltage magnitude.

With the goal of maintaining the frequency to its target value, and therefore to control the active power balance, there exists a number of feedback control loops in the power system that continuously react on its deviations by adequately adjusting power generation. However, the power balance control problem, interpreted in a broader sense, includes much more than "just" controlling the frequency. This will become clear in next subsection, where the basic control structure of a traditional power system is shortly presented. To understand, identify and more precisely formulate the control problems of today's and tomorrow's power system, it is crucial to understand how the same, or similar problems, have been solved traditionally.

1.1.2 Control structure of traditional power systems

A power system control structure can be described as a temporal decomposition based hierarchical structure. Control and decision making occur on different time scales.

Upper and slower layers in the hierarchy are centralized and deal with the *economical and reliability aspects* of the whole system. In traditional power systems utilities own and operate virtually the whole system, i.e. the generation, the transmission and the distribution. All the system-wide decisions and control actions are made at a limited number of utility control centers.

Lower and faster layers are responsible for *real-time power balancing*, and are realized through several feedback loops that continuously react on the network frequency deviations. Some of these control loops are decentralized on the nodal level, e.g. local generator controllers acting on the measurements from the generator bus (node), while some are decentralized on the level of the so-called control areas.

In the following paragraphs we present the main components of the active power balance control in traditional power systems. We start with the fast acting layers and continue towards the slower ones. Figure 1.1 and Figure 1.2 illustrate this time-scale decomposition.

- ***Kinetic energy.*** One of the distinguishing features of a traditional power system is that virtually all power has been generated on large synchronous generators. Synchronous generators are characterized by a large rotating inertia, and therefore present a significant buffer of kinetic energy. Any imbalance occurring in a system will initially be supported by a change in this kinetic energy. For instance, increase in demand will initially be supported by extracting the kinetic energy from a generator, causing the generator to slow down and the network

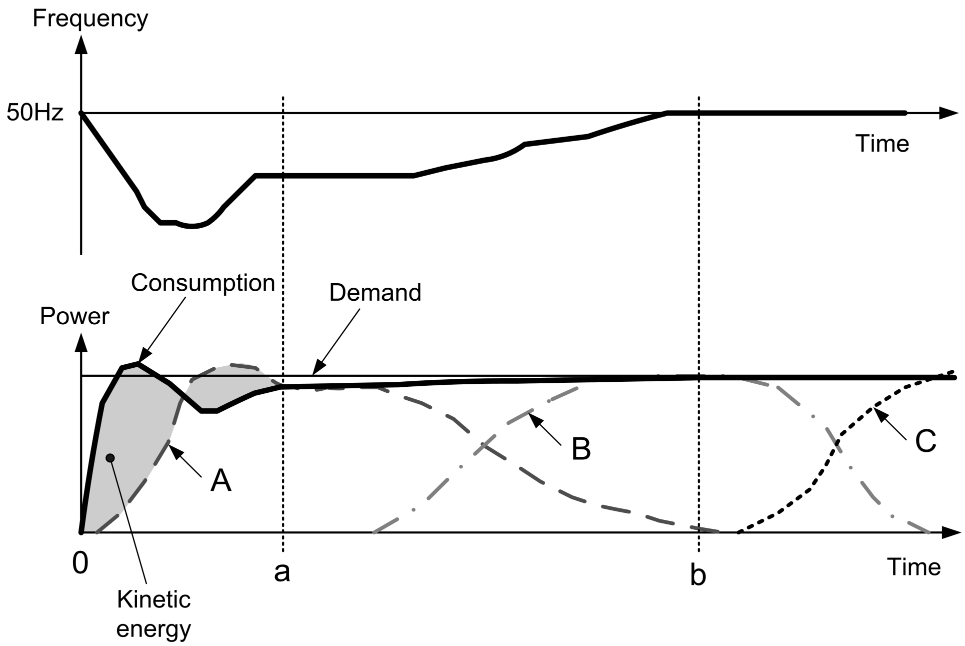

Figure 1.1: Temporal decomposition of a power balance control.

frequency to drop. Figure 1.1 illustrates the systems response following such an increase in the demand. At the time instant zero, the demand increment increased from zero to the value indicated by a thin solid line. The thick solid line labeled "Consumption" represents the actual consumed power as a function of time. The gray shaded area represents the kinetic energy extracted from the rotating inertia to quickly support the demand.

- ***Governor control.*** The fastest feedback loop acting on the frequency deviation is the governor control feedback loop. This is a proportional controller acting on the frequency deviation, which causes the mechanical power that drives the shaft of a generator to increase. This is a local control loop: it acts only on local measurements, i.e. the network frequency measured at the generator bus. The extra power created by the governor control loop is indicated in Figure 1.1 by the line labeled *A*. Not all generators in the system need to be equipped with a governor control, and those that are, supply in addition to the basic service of producing the power, an additional or ***ancillary service***: fast power balance support (also called *primary control action*). In

Figure 1.1, we have indicated the time instant a as a measure of a time-scale characterizing this particular service.

- ***Automatic Generation Control.*** Next, a slower acting control loop in the hierarchy is the automatic generation control (AGC). (The term AGC is more commonly used in USA; another used terms are *load frequency control* or *secondary control action*.) One AGC controller acts on one control area. A control area can correspond to a country (what is often the case in Europe) but it is in general defined as a part of the electric system that is capable and responsible for controlling its own power balance in real-time. In connection to a control area, there is defined an *area control error* (ACE): a signal obtained as a linear combination of the network frequency deviation and the deviation of the power flows in the tie-lines connecting the area with the neighboring areas. The goal of an AGC controller is to drive the ACE to zero. If all areas in the system succeed in driving their ACEs to zero then both the system frequency and the sum of their tie-line exchange powers maintain their target set-points. The output of an AGC controller is a signal representing the total required change of power production in the area. This signal is divided into a set of set-point updates for the generators in the control area. Again, not all the generators in the system are in the AGC loop; only a subset of generators perform the control area balancing ***service***. The power production generated by the AGC loop is indicated in Figure 1.1 by the line labeled B. Note that only with the governor control active, there would remain a steady-state error in the frequency, indicating the sustained power imbalance in the system. The integral action in the AGC drives the frequency deviation to zero, and in Figure 1.1 we have indicated the time instant b as a measure of a time-scale characterizing this particular service. The set-point updates from the AGC controller are send to the generators each 2 to 4 seconds.

- ***Economic dispatch.*** The next hierarchical level belongs to the *economic dispatch* (ED). It is a change in the set-points of the generators with the goal to reduce the systems operating costs by re-dispatching the power production among the generators in a control area. This is the fastest acting hierarchical level dealing explicitly on-line with the utility's economic objectives. For this, and any other slower acting operation level, economic objectives are in the core of any decision making. ED is usually performed each 5 to 15 minutes, and its action is indicated in Figure 1.1 by the line labeled C. In the core of an ED is the *optimal power flow* problem, i.e. the optimization problem where the objective is to minimize the variable production costs, while satisfying the power balance, generators limits, and the network constraints including the line flow limits of the transmission system.

- ***Power scheduling and unit commitment*** are the slowest operating layers, and act on the time horizon of 1 or 2 days ahead. The cost minimizing objective is the same as in ED, but the unit commitment problem is much more difficult since it includes discrete-type decisions, i.e. switching the generators on and off. The set-points for the tie-line flows used in the AGC control are also updated on this time scale. To enforce the system's reliability, at this level the additional amount of stand-by generating capacity is scheduled to be on-line, e.g. the spinning reserve, to provide generating support in case of emergency, e.g. sudden failure of some generating units.

Figure 1.2 illustrates the time-scale decomposition of control actions in a traditional power system. The indicated time scale at the bottom of the figure approximatively characterizes the time characteristics of each block. Each block represents a control, operation or decision making process, and is roughly positioned in the time-scale based either on the frequency of the signal updates in the block, or on the response time of the underlying process. The blocks labeled *exciter, power system stabilizer, maintenance scheduling* and *investments planning* are added to the figure to indicate that the power system operation includes many more crucial ingredients, both on the faster and on the slower time scale relative to those described above. For clarity, many other existing blocks are not included in the figure. For a complete, detailed presentation we refer to the classical power system textbooks, e.g. (Kundur, 1994; Wood and Wollenberg, 1996; Sauer and Pai, 1997).

Over the course of many decades, the traditional power system control structure has proven to be quite reliable. There is no doubt that the traditional power systems have shown an impressive level of robustness, although even now not all dynamic phenomena in a large power system are fully understood. However, this robustness did not come for free. In a monopolistic, regulated, utility-owned traditional system, robustness is achieved through a conservative engineering and operation of the system. The price paid is economical inefficiency, few driving forces for innovations, and in general, a large resistance towards changes. Investments have been made for large,

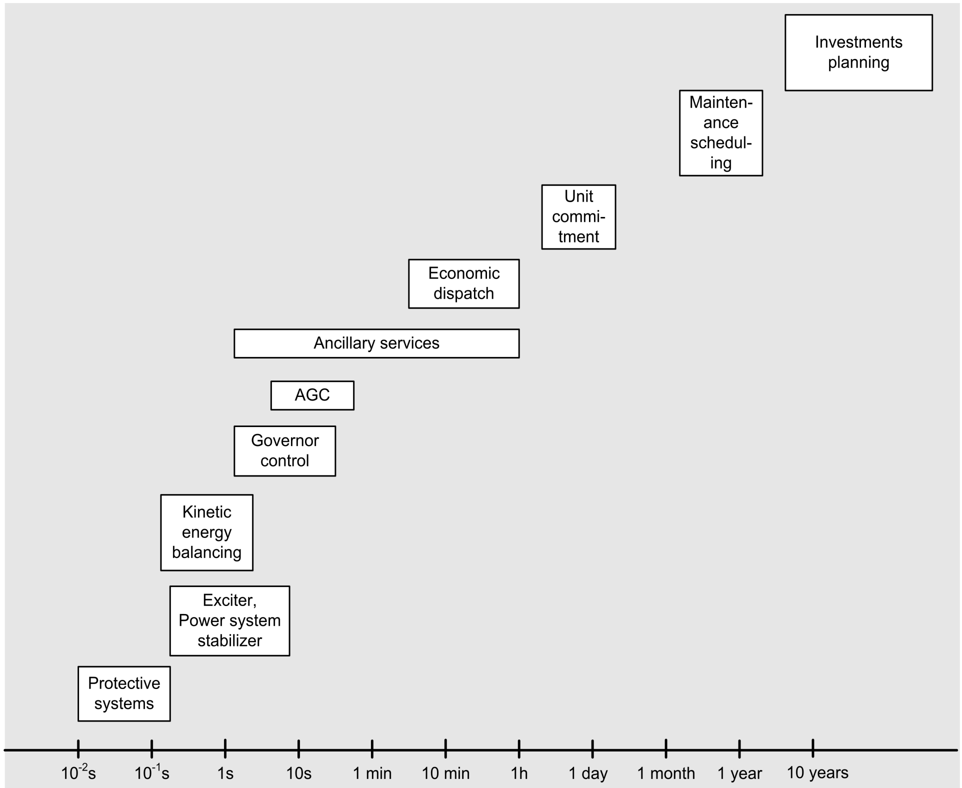

Figure 1.2: Time-scale decomposition of controls and actions in a traditional power system.

long-lasting projects, with large time gaps in between. The government-imposed regulations guaranteed the pay-off by regulating the price of electricity. This regulated price also included the average costs of all the above mentioned services necessary to keep the system running.

Another reason for the reliability of traditional power systems is due to the large synchronous generators. As we have seen, the initial step in the power balance control is pretty much given for free. The synchronous generator as a buffer of kinetic energy has reduced the need for *active control* in covering the continuously occurring fast imbalances. Finally, explanation for the robustness of the 3^{rd} and 4^{th} layer, which are operated in an *open loop* manner, can be found in a highly repetitive pattern of aggregated demand in a traditional power system. From one working day to the other, the variations in the aggregated demand profiles are only a couple of percent. This *high predictability* and *low uncertainty* level, made the open loop operation of higher hierarchal levels possible and, even more, rather successful.

1.1.3 Restructured, market-based power systems

A basic economic principle states that if a competitive market is established for scarce resources, and competition involves equally strong players, then the market will allocate the resources efficiently. A simplified explanation for this is the following: efficient operation of *each player* is essential for its own profit making, and inefficiency of *the system as a whole* is naturally eradicated through competition (Lai, 2001). Furthermore, a competitive market environment accelerates development and application of new technologies, as companies seek to improve their efficiency. Investments and any other changes to eliminate the efficiency bottlenecks are made much faster relative to the traditional, regulated systems. For convincing arguments made in favor of creating competitive markets, and for scientific explanations for their efficiently, we refer to any standard microeconomic textbook, e.g. (Nicholson, 1995; Katz and Rosen, 1998).

In theory, a perfect competitive market will reach an equilibrium which is characterized by maximal *social welfare*. For power systems, social welfare is defined as a combination of the cost of the energy and the benefit of the energy to society as measured by society's willingness to pay for it. A market is perfectly competitive if at least the following three conditions are met: price taking suppliers/consumers, public knowledge of the market price, and well-behaved production costs/consumer benefits (Stoft, 2002).

A *price taking* market player, i.e. supplier or consumer, is a player that optimizations its production as if it does not affect the market price. In other words, even if the player is strong enough to (significantly) influence the market price by altering its production/consumption, he does not use this *market power* to increase its own profit.

One implication of the above stated theoretical result is the following: in a perfectly competitive market, a market price for a commodity is equal to the marginal cost of producing that commodity. We will call this price a *competitive price*, and will use this notion in the following section to define nodal prices.

As we have seen in the previous section, electrical power delivered to a consumer is a bundle of many services. Deregulation of a power system involves "unbundling", which refers to disaggregation of an electric utility service into its basic components and creating a separate market for each component. Furthermore, deregulation involves the separation of ownership

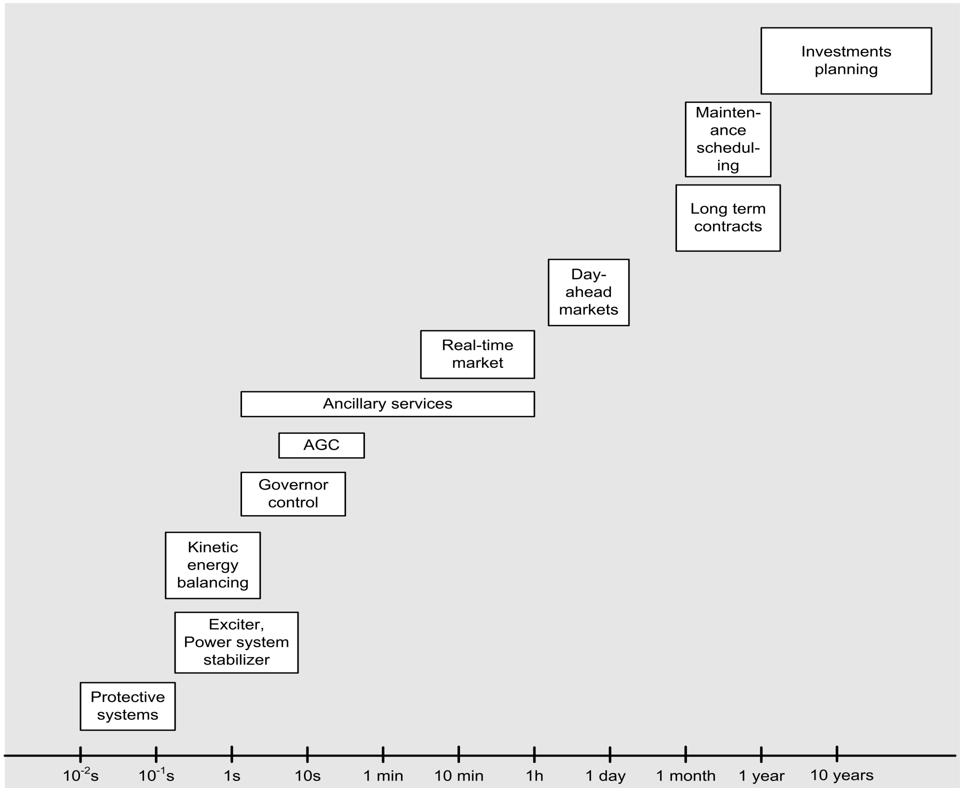

Figure 1.3: Time-scale decomposition of controls and actions in a restructured power system.

and operation.

The term *ancillary service* is commonly used to refer to any service necessary for keeping the power system in operation, e.g. provision of power support in an AGC control loop, and for contributing to its reliability, e.g. provision of spinning reserve. Therefore, in addition to energy markets, there exist several *ancillary service markets*. Although in present power systems the conditions for perfect competition are sometimes violated, in practice these "imperfect" markets are still shown to be efficient.

There is no single, commonly accepted approach or solution to the power system restructuring and markets design. This process is based on national energy strategies and policies, and varies from country to country. An attempt to give a more detailed description of some of the state-of-the-art approaches would not serve the purpose of this introduction. Therefore, we continue by presenting only some of the common features of an operational structure of restructured power systems. At the end of the section, we will explicitly emphasize these characteristics that are relevant for the research presented in this thesis.

In the restructuring process, the scheduling algorithms present in the 3^{rd} and 4^{th} layer of a traditional power system hierarchy, are being replaced by energy and ancillary service markets. Just as within the classical scheduling algorithms, the markets have different time frames, see Figure 1.3.

A forward-time energy market is usually a day-ahead market and is set up by an independent entity, which takes one of a few different forms, e.g. power exchange market or pool market; see (Stoft, 2002) for more details. In the Netherlands a day-ahead energy market is set up and operated by the Amsterdam Power Exchange[1] (APX). Most of the existing day-ahead energy markets can be seen as only one part of the above presented 4^{th} layer of a traditional power system. They are concerned only with the energy production scheduling, and not with the emergency reserve capacity like for instance spinning reserve.

The responsibility for maintaining the systems reliability and security is assigned to another independent entity, usually called the independent system operator (ISO). The system operator buys various classes of capacity reserves, some required for normal operation of the system, and some in case of emergencies. For a normal operation, these reserves include the required generating capacity for support of the governor and the AGC control loops. For a support of the system during emergency situations, like for instance sudden tripping of a large generating unit, there can exist several different classes of reserves (ancillary services). Each class is characterized by the time of response. For example, 1MW reserve of a generic class A denotes a capacity that is available as a power injection into the network within a minutes upon the request. Figure 1.1, which we have used to illustrate the actions during the normal operation of the system, can equally be used to illustrate the actions in the emergency operation. In that case the curves labeled A, B and C in Figure 1.1 correspond to different classes of emergency reserves (ancillary services). The ISO usually purchases the ancillary services in some form of forward-time markets. Then, during the real-time operation, it activates those services to maintain, if required, the real-time power balance.

In some countries, the ISO also runs the *real-time* market, i.e. it uses the real-time updates of electricity price to balance the system. Currently,

[1] For details visit the web address `www.apxgroup.com`.

the fastest operating real-time markets set a new price each five minutes. Other commonly used time intervals are half-hour and one hour. Therefore, in addition to being responsible for accumulating sufficient amount of ancillary services, an ISO acts as an equivalent to the 3^{rd} layer of a traditional power system. An ISO has also the responsibility of keeping the line flows in a transmission system below certain, predefined security limits, i.e. it is responsible for congestion management. In the Netherlands, the function of ISO is performed by TenneT[2].

There are many other important ingredients in the operation of a restructured power system, like for instance existence of bilateral markets and long-term contracts, for which we refer to one of the many books on the subject, e.g. (Stoft, 2002). At the end we have to put emphasis on some of the features of the present market structure, which are of concern in this thesis.

Although today's ancillary service markets are characterized by many parties competing on the supply side, the only entity on the demand side is the ISO. The system operator (ISO) service, which coordinates the ancillary service markets and provides the only demand for them, is therefore a *monopoly service* (Stoft, 2002).

Furthermore, in today's power systems, demand is almost completely unresponsive to price in virtually all power and ancillary service markets. Price fluctuations are usually not passed on to the customers and there does not exist an underlying technical infrastructure that would allow a customer to act upon its benefit maximizing philosophy, i.e. to enable it to act as a price elastic player. For example, there is a lack of an appropriate metering that would take the current real-time price into account in forming the electricity bill. This price inelasticity of customers is possibly *the biggest flaw of the present electricity markets* (Stoft, 2002).

1.1.4 Spatial dimension of power balancing in market-based power systems

Transmission lines have power flow limits which must be enforced during the operation to protect the integrity of the system. The problem of controlling the line power flows to avoid their overloading is usually referred to as a congestion management problem. There are several different phenomena that impose the transfer limits, including thermal limits, voltage limits and stability limits. The most restrictive limit is applied at any time.

[2]For details visit the web address `www.tennet.nl`.

The existence of transfer limits makes a transmission network a scarce resource. Furthermore, a transfer of power over a larger distance in general implies bigger transmission losses. Both existence of the power flow limits and the line losses add a spatial dimension to the problem of power balancing. Dealing with this dimension, e.g. creating an efficient congestion management scheme, is one of the toughest problems in the electricity market design.

Economists have dealt in detail with general transportation systems. However, those systems usually assume a free choice among alternative paths between source and destination nodes and assume that the goods can be stored when they can not be moved (Christie et al., 2000). For an electrical transmission system, none of these assumptions is valid. The flow of power in a transmission network is governed by physical laws characterized by very complex dependencies of line power flows on nodal power injections (nodal productions and consumptions). Changing the amount of power injected in one node of a network will almost instantaneously result in changes of power flows in virtually all lines in the system. For a fixed pattern of nodal power injections, it is possible to influence the line power flows by utilizing the FACTS (Flexible Alternating Current Transmission System) devices (Hingorani and Gyugyi, 1999; Mathur and Varma, 2002). However, this influence is rather limited and can represent only a small part of a congestion management solution. In a restructured power system, a solution to the congestion management problem with economically optimal exploitation of a transmission network is based on converting the system limits to correct market signals. *Nodal prices* represent a notion central to a market-based congestion management (Schweppe et al., 1988).

With an example of a simple three-node network in Figure 1.4, our goal is to illustrate the basic principle of using the electricity prices for congestion management. We assume that all three lines in the network have the same characteristics, i.e. the same impedance, and for simplicity we neglect the line losses. With $C_A(p_A)$ denoting the cost of producing the power p_A at the generator connected in node A, the marginal cost $MC_A(p_A)$ of that generator is defined by $MC_A(p_A) := \frac{\partial C_A(p_A)}{\partial p_A}$. For the considered example, marginal costs of all the generators in the network are given at the top of the figure. Let us assume the conditions of a perfect competitive market, and assume that each generator bids its marginal costs into the market. Furthermore, assume that the market operator knows the demand in each node, and that the demand is not price dependent. In the considered example the demands are equal to 300MW, 600MW and 450MW, and are indicated

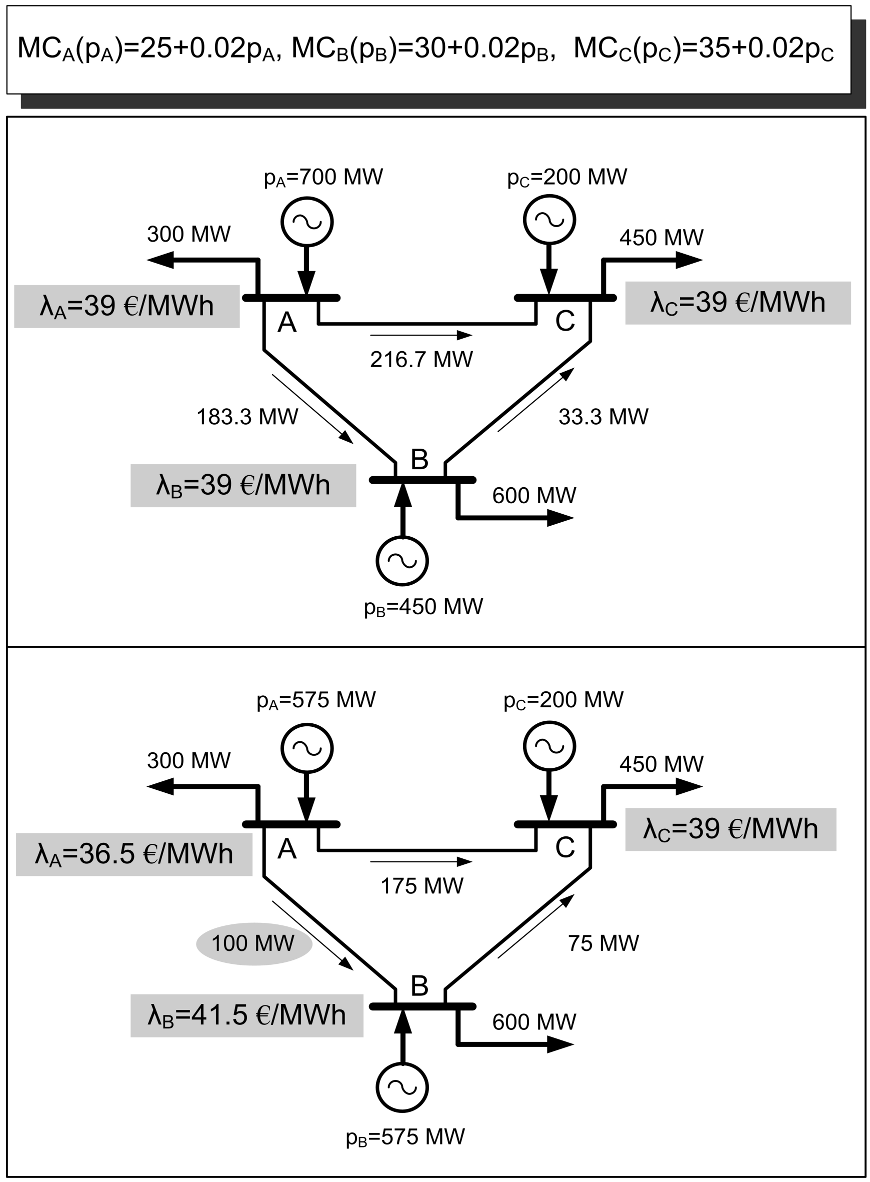

Figure 1.4: Congestion management using nodal prices.

by these numbers in the figure.

The task of a market operator is to determine an electricity price for each node in the network, so that, when each generator adjusts its production according to its bid, the following conditions are satisfied:

i) production equals demand;
ii) power flow in each line in the network is less than or equal to some predefined limit value;
iii) social welfare is maximized.

Due to the price inelasticity of the demand in this example, maximizing the systems welfare is equivalent to minimizing the total cost of production. Let us consider two different situations. In the first case we will assume that the market operator is not responsible for fulfilling the task in (ii) (or simply, assume that there are no line limits), while in the second case it is responsible for this task (there exist line limits).

With no line limits considered, the price that satisfies (i) and (iii) will be the same for all the nodes in the network and each generator will work in an operating point where its marginal cost is equal to the price. The market result for this case is presented in the upper part of Figure 1.4, where $\lambda_A = \lambda_B = \lambda_C =$ 39EUR/MWh denotes the market price. The market price, in a perfectly competitive market, equals the systems marginal cost, i.e. it reflects the cheapest way of supplying an additional increment of power demand. If the network is not congested, and with losses neglected, the minimal costs of supplying an additional increment of power to a node is the same for all nodes in the network. Note that the generator in node A produces power relatively cheaper than the other generators, and with the presented market price it will support a large portion of the total demand, causing the large power transfers from node A to the other nodes.

Now consider the case when the line connecting nodes A and B has a limit on the power flow, which is equal to 100MW. We take the demand to be the same as in the previous case. From the previous case, we see that if the line limit is ignored, there will be a power flow of 183.3MW through this line, i.e. the line will be congested. In order to enforce the constraint, the market operator has a freedom to choose different prices for each node in the network. To fulfil the conditions (i) and (iii) at the same time, it solves a suitably defined optimization problem which includes the model of line power flow dependencies on the nodal power injections. The results, i.e. the *nodal prices* that yield the fulfillment of (i),(ii) and (iii), the corresponding line flows and the generation production levels, are presented in the lower part of Figure 1.4. Note that the nodal price at node A has been decreased, resulting in a reduced power production in that node. On the other side, price in the node B has been increased, causing the corresponding generator to produce more power. Congestion prevents the demand at node B to use cheaper power from a distant location A, and this demand is now in a larger portion supported by a more expensive, local unit B. Note also that the line flows in the remaining two lines have been significantly altered.

Similar as in the first case, the prices obtained in the second case represent the marginal costs of electricity. *The nodal price of electricity at a given instance in time and at a given system location (node) reflects the cheapest way to deliver an increment of power to that particular node from the on-line available generators while respecting all the constraints and system limits.* Behind this definition of nodal prices, it is “hidden” the fact that the nodal prices are those prices that maximize the total social welfare of the system. In more mathematical terms, nodal prices are nothing else than the optimal Lagrange multipliers associated with the nodal power balance equalities of the underlying *optimal power flow* optimization problem.

Based on the nodal prices, the owners of the transmission collect the *congestion rent*, an income that comes as a result from differences in nodal prices. The congestion rents are used to cover the operational costs of the ISO or of a market operator, and to cover fixed and maintenance costs of the transmission system owners. An additional benefit of the nodal prices is that the existence of persistent high prices caused by congestion sends the right signals to investors to build new generators, and to the transmission owners to build the power lines that are needed.

Finally, it is important to note that the nodal prices in a perfectly competitive market are *competitive prices*, and as such do not depend on the specific operational structure of the market (Stoft, 2002). For example, it is possible to establish competitive bilateral market for transmission rights, see (Stoft, 2002) for details, and it can be shown that such markets will produce the same nodal prices as a market that we have just shortly described.

1.1.5 Changes caused by distributed generation

Distributed generation units include micro-turbines, fuel cells, wind turbines, photovoltaic arrays, combined heat and power plants, small hydro-power plants, biomass power plants, geothermal power plants, tidal power plants, etc. Across the set of different DG technologies, there is a huge variety of possible time responses to the power imbalances. DG units are typically not based on synchronous generators. Some units operate asynchronously, coupled to the grid via AC-AC power electric converters. Others are non-rotating, inertia-less sources, such as photovoltaic units and fuel cells. With an absence of the rotating inertia, such sources introduce very different characteristic to the system, and their ability to respond to the fast occurring imbalances will likewise be very different from that of the traditional units.

Furthermore, it is expected that the large portion of the distributed generation units will be based on intermittent renewable sources like wind and sun. The consequence will be a large increase of the uncertainties in any future system state prediction. Large and relatively fast fluctuations in production are likely to become normal operating conditions, standing in contrast to today's operating conditions characterized by highly repetitive, and therefore highly predictable, daily patterns. Note that, as in the traditional power systems, the success of the present power systems heavily relies on high predictability.

With the inherent dynamical characteristics of DG units, and due to the increased uncertainties, we can conclude that *in the future power systems the need for fast acting, power balancing control loops will increase significantly.*

1.2 Scope and outline of the thesis

1.2.1 Motivation

The research presented in this thesis is motivated by the following issue of concern for the operation of future power systems:

> Future power systems will be characterized by significantly increased uncertainties at all time scales and, consequently, their behavior in time will be difficult to predict.

More and more privately owned, distributed generating (DG) units are being integrated in the system. Each DG unit is an autonomous decision maker competing to maximize its own benefit, and is characterized by a badly predictable behavior. Large portion of DG will be based on renewable energy sources, i.e. wind turbines and photovoltaic (PV) systems, and will therefore be practically uncontrollable in their power outputs, introducing large production fluctuations and large uncertainties in any future system state prediction.

The power system liberalization and restructuring processes will continue in the future, and the system reliability and efficiency will increasingly rely on the efficiency and effectiveness of competitive markets for energy and ancillary services. The current market operation significantly relies on the repetitive daily patterns in demand, i.e. on the relatively high predictability of the future system state.

Efficient provision of electrical power and ancillary services in a future power systems will require novel market structures and innovations in supporting the technical infrastructure and control solutions.

1.2.2 Research scope and goals

The research presented in this thesis is exclusively concerned with the power system control and operation in the time-scale window indicated with the gray shaded area in Figure 1.5. With increased uncertainties in future power systems, and for the considered time-scale window, new solutions are needed to anticipate

- efficient and effective real-time power balance and congestion management;
- contribution of large amounts of DG units in competitive, but badly predicable supply of power and ancillary services;
- active involvement of price-sensitive consumers in energy and ancillary services markets.

Our goal is to explore the *technical possibilities* of achieving economic efficiency by investigating price-based control structures for active balancing of both power and ancillary services. Organizational and juridical considerations are outside the scope of this thesis.

In this thesis, we will explore the possibility of real-time, feedback-based nodal pricing. However, we consider nodal prices only as a congestion management tool and not also as a congestion pricing mechanism in transmission system revenue generation.

Figure 1.6 illustrates the price-based control structure of an electrical power system. The figure is with some modifications adapted from (DeMarco, 2001), and serves well in illustrating our goals concerning real-time power balance and congestion control. The goal is to design a price-based feedback controller, indicated in Figure 1.6 by a gray shaded block, which guarantees provision of correct nodal prices in real-time.

1.2.3 Outline of the thesis

The main results of this thesis are presented in four chapters.

In Chapter 2 we will present a novel *explicit, dynamic, distributed* feedback control scheme that utilizes nodal-prices for real-time optimal power balance and network congestion control. The term *explicit* means that the controller is not based on solving an optimization problem on-line. Instead, the nodal prices updates are based on simple, explicitly defined and easily comprehensible rules. We prove that the developed control scheme, which

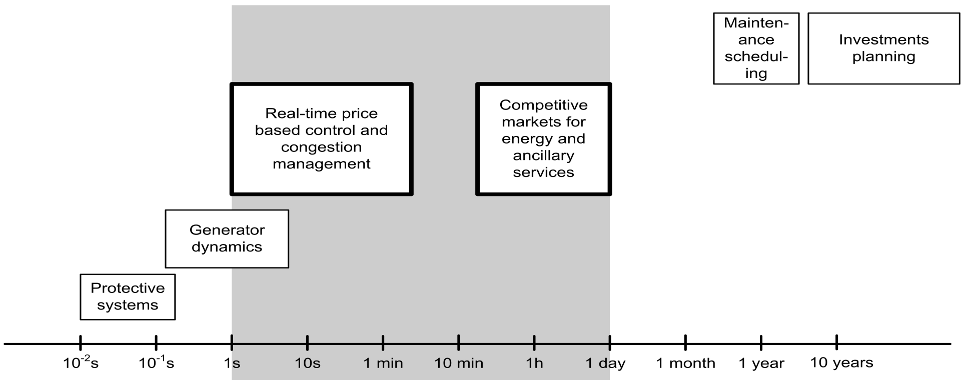

Figure 1.5: Time-scale decomposition of power system operation. The research in this thesis is restricted to the gray-shaded time-scale window.

acts on the measurements from the current state of the system, always provide the correct nodal prices.

In Chapter 3 we will develop a novel, robust, hybrid MPC control scheme for power balance control with hard constraints on line power flows and network frequency deviations. The developed MPC controller acts in parallel with the explicit controller from Chapter 2, and its task is to enforce the constraints during the transient periods following suddenly occurring power imbalances in the system.

In Chapter 4 the concept of autonomous power networks will be presented as a concise formulation to deal with economic, technical and reliability issues in power systems with a large penetration of distributed generating units. With autonomous power networks as new market entities, we propose a novel operational structure of ancillary service markets.

In Chapter 5 we will consider the problem of controlling a general linear time-invariant dynamical system to an economically optimal operating point, which is defined by a multiparametric constrained convex optimization problem related with the steady-state operation of the system. The parameters in the optimization problem are values of the exogenous inputs to the system. The results presented in this chapter present a formalization, generalization and extension of the design methods from Chapter 2.

An overview of the main conclusions from this thesis is given in Chapter 6. Moreover, in this chapter we present several recommendations for future research.

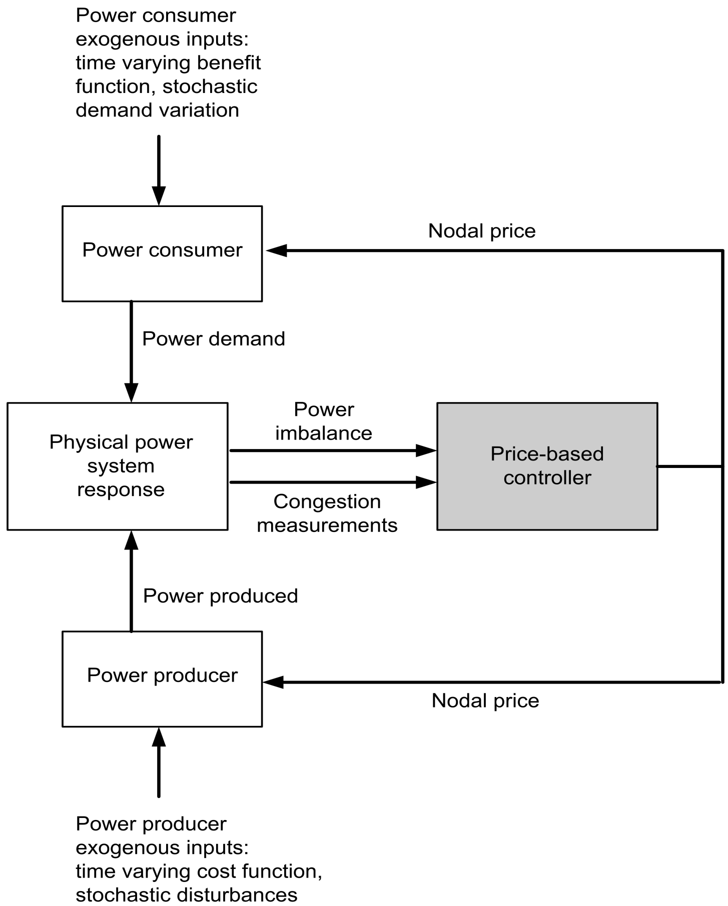

Figure 1.6: Price-Based Control Structure; adapted from (DeMarco, 2001).

1.3 Intelligent power systems research project

The research presented in this thesis was performed within the framework of the "Intelligent Power Systems" project. The project is part of the IOP-EMVT program (Innovation Oriented research Program - Electro-Magnetic Power Technology), which is financially supported by SenterNovem, an agency of the Dutch Ministry of Economical Affairs. The "Intelligent Power Systems" project is initiated by the Electrical Power Systems and Electrical Power Processing groups of the Delft University of Technology and the Electrical Power Systems and Control Systems groups of the Eindhoven University of Technology. In total 10 PhD students, who work closely together,

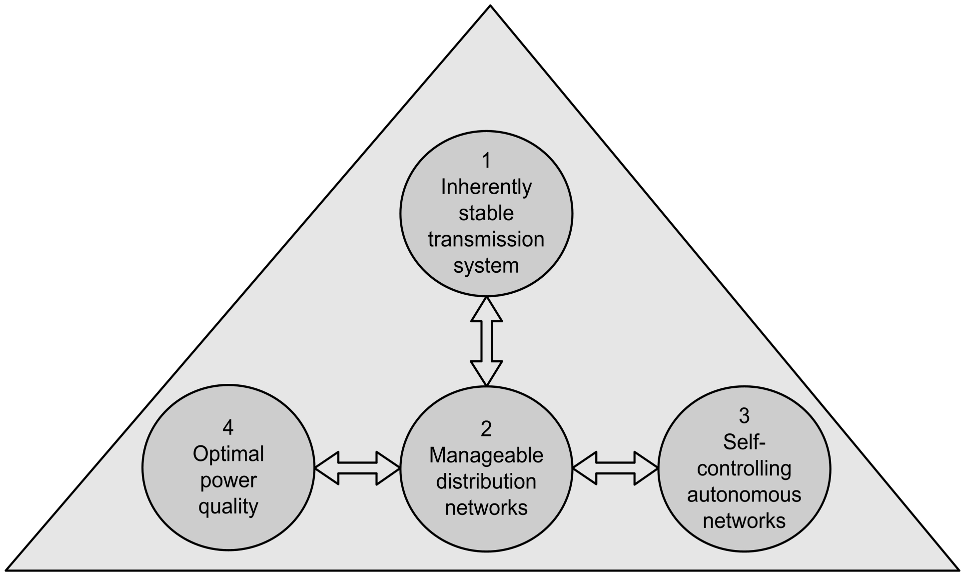

Figure 1.7: The four parts of the intelligent power systems research project.

are involved in the project.

The research mainly focuses on the effects of the structural changes in the generation and consumption which are taking place, like for instance the large-scale introduction of distributed (renewable) generators (Reza et al., 2003).

Such a large-scale implementation of the distributed generators leads to a gradual transition from the current "vertically-operated power system", which is supported mainly by several big centralized generators, into a future "horizontally-operated power system", having also a large number of small to medium-sized distributed (renewable) generators. The project consists of four parts, as illustrated in Figure 1.7.

The first part investigates the influence of decentralized generation without centralized control on the stability of the dynamic behavior of the transmission network. As a consequence of the transition in the generation, fewer centralized plants will be connected to the transmission network as more generation takes place in the distribution networks, close to the loads, or in neighboring systems. Solutions that are investigated include the control of centralized and decentralized generation, the application of power electronic interfaces and monitoring of the system.

The second part focuses on the distribution network, which becomes "active". There is a need for the technologies that can operate the distribution network in different modes and support the operation and robustness of the network. The project investigates how the power electronic converters of decentralized generators or power electronic interfaces between network parts can be used to support the grid. Also, the stability of the distribution network and the effect of the stochastic behavior of decentralized generators on the voltage level are investigated.

In the third part autonomous networks are considered as a realistic concept for enabling the large-scale integration of DG. The project investigates the control functions needed to operate the autonomous networks in an optimal and reliable way. The research presented in this thesis belongs to this part of the project.

The interaction between the grid and the connected appliances has a large influence on the power quality. The last part of the project analyses all aspects of the power quality. The goal is to support the discussion between the polluter and the grid operator who is responsible for compliance with the standards. The realization of the power quality test lab is an integral part of the project.

2

Real-time price-based optimal control of power systems

This chapter is based on the papers (Jokić, Lazar, and Van den Bosch, 2007a; Jokić, Van den Bosch, and Lazar, 2007b)

2.1 Introduction

During the past decade there has been a tremendous amount of research devoted to a market-oriented approach for the electrical power system sector, see e.g. (Schweppe et al., 1988; Ilic et al., 1998; Lai, 2001; Stoft, 2002; Song and Wang, 2003) for an overview. Electrical power systems have some unique properties, which make a market-oriented approach a challenging task. For example, electrical energy cannot be efficiently stored in large amounts, which implies that production has to meet rapidly changing demands in real-time, making electricity a commodity with fast changing production costs. Furthermore, unlike other transportation systems, which assume a free choice among alternative paths between source and destination, the flow of power in electrical energy transmission networks is governed by physical laws and, for some fixed pattern of power injections, it can be influenced only to a certain degree, e.g. by utilizing FACTS devices (Mathur and Varma, 2002). Therefore, physical and security limits on the maximal power flow in the lines of electrical energy transmission networks represent crucial system constraints with a significant impact on power system economics (Christie et al., 2000). Creating efficient congestion management schemes to cope with the transmission constraints is one of the toughest problems in electricity market design (Stoft, 2002).

Publications of Fred Schweppe and his co-workers (Schweppe et al., 1980; Caramanis et al., 1982; Schweppe et al., 1985) are the first studies that systematically investigated the topic of price-based operation of electrical power systems. Many of the results from that period are summarized in the book (Schweppe et al., 1988). Ever since then, the notion of using prices to operate electrical power systems has been used in a number of ways. For an overview of different approaches in price-based congestion management of transmission systems, see e.g. (Hogan, 1992; Wu and Varaiya, 1999; Christie et al., 2000; Rubio-Oderiz and Perez-Arriaga, 2000; Bompard et al., 2003; Alvarado, 2005) and the references therein.

Probably the most closely related to the results presented in this chapter is the work of Alvarado and his co-workers (Glavitsch and Alvarado, 1998; Alvarado, 1999; Alvarado et al., 2001; Alvarado, 2003, 2005).

In (Glavitsch and Alvarado, 1998), the authors have investigated how an independent system operator (ISO) could use electricity prices for congestion management without having an *a priori* knowledge about cost functions of the generators in the system. With an assumption of quadratic, time-invariant cost functions, the authors illustrated how, in principle, a sequence of market observations could be used to estimate the parameters in the cost function of each generator. Based on these estimates, and by solving a suitably defined optimization problem, an ISO could issue the nodal prices causing congestion relieve. Although dealing with an intrinsically dynamical problem, the paper considered all the processes in a static framework. A parameter estimation procedure, which represents the core part of the proposed approach, was based on the hypothesis that at the time of each measurement the system is in steady-state, and was necessarily limited to estimation of time-invariant quadratic costs or quadratic approximations of costs.

In (Alvarado, 2003, 2005) the results of (Glavitsch and Alvarado, 1998) have been extended by addressing possible issues of concern when price-based congestion management is treated as a dynamical process. Furthermore, the papers considered possible problems caused by linear cost functions, time-varying cost functions, as well as possible effects of some units exercising market power.

The usage of price as a *dynamic feedback control* signal for power balance control has been investigated in (Alvarado et al., 2001). There, the effects of interactions of price update dynamics, modeled as integral of network frequency deviation, and the dynamics of an underlying physical system (e.g. generators) on the stability of the overall system have been investigated. However, no congestion constraints have been considered, and therefore only one, scalar valued, price signal was used to balance the power system.

Analysis of real-time market dynamics with the effects of network congestion was performed in (Alvarado, 1999). However, in that paper, network congestion has been modeled as a *static equality constraint* representing the power flow in a congested line.

The work presented in this chapter is an extension of the above mentioned contributions, in the sense that it considers, and solves, *both the price-based congestion management and the power balance problem* in a dynamical framework.

Motivation and overview

Due to the *uncertainty in real-time power demand*, and because of the fast changing variable production costs, there is a general tendency in today's power markets towards increasing the speed with which market prices are updated (DeMarco, 2001). By closing the time gap in between price update dynamics and dynamics of the underlying physical system, it becomes important to consider the two in a unified, dynamical framework (Alvarado et al., 2001).

Furthermore, due to expected large-scale integration of distributed generation with large amount of renewable energy based units, e.g. wind turbines or photovoltaic systems, future power systems will be characterized by significantly larger uncertainties. Success of power balance and congestion management control schemes of present power systems heavily relies on relatively accurate predictions of future systems state, as the vast majority of power production is scheduled in an *open-loop manner*. Presently, only a relatively small amount of on-line capacity is involved in real-time markets and in feedback control balancing loops, e.g. in the automatic generation control (AGC) loop. In contrast to today's power systems, feasibility, reliability and economical efficiency of future power systems will increasingly rely on real-time, price-based feedback control solutions.

In this chapter we present a *price-based, dynamic* controller for real-time optimal power balance and network congestion control. The developed controller is a feedback controller that reacts on the network frequency deviation as a measure of power imbalance in the system and on measured violations of line flow limits in a transmission network. The output of the controller is a vector of nodal prices and the control objective is to continuously drive the system towards an equilibrium where all the transmission network constraints are satisfied, and where the social welfare of the system is maximized.

In the proposed solution, the real-time update of nodal prices, i.e. the controller, is explicitly described as a dynamical *linear complementarity system* (van der Schaft and Schumacher, 1996, 1998; Heemels et al., 2000a). As it will be shown in this chapter, the dynamic complementarity framework naturally arises in network congestion control problems, since the optimal nodal prices necessarily have to fulfill certain complementarity slackness conditions.

In addition to providing the correct nodal prices in real-time, the developed controller has the following two advantageous properties:
- it requires no knowledge of cost/benefit functions of producers/consumers (neither is it based on their on-line estimates);
- the transmission network structure is preserved in the controller, allowing for its distributed implementation.

Nomenclature

The field of real numbers is denoted by $\mathbb{R}$, while $\mathbb{R}^{m\times n}$ denotes m by n matrices with elements in $\mathbb{R}$. For a matrix $A \in \mathbb{R}^{m\times n}$, $[A]_{ij}$ denotes the element in the i-th row and j-th column of A. For a vector $x \in \mathbb{R}^n$, $[x]_i$ denotes the i-th element of x. The transpose of a matrix A is denoted by $A^\top$. All inequalities are interpreted elementwise. For $u, v \in \mathbb{R}^k$ we write $u \perp v$ if $u^\top v = 0$. We use the compact notational form $0 \leq u \perp v \geq 0$ to denote the complementarity conditions $u \geq 0$, $v \geq 0$, $u \perp v$. $\operatorname{Ker} A$ and $\operatorname{Im} A$ denote the kernel and the image space of A, respectively. We use I_n and $\mathbf{1}_n$ to denote an identity matrix of dimension $n \times n$ and a column vector with n elements all being equal to 1, respectively. The operator $\operatorname{col}(\cdot, \ldots, \cdot)$ stacks its operands into a column vector, and $\operatorname{diag}(\cdot, \ldots, \cdot)$ denotes a square matrix with its operands on the main diagonal and zeros elsewhere. The nonnegative orthant of $\mathbb{R}^n$ is defined by $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n \mid \ x \geq 0\}$. $\mathcal{L}^n_1(\mathbb{R}_+)$ denotes the space of all measurable functions $g : \mathbb{R}_+ \to \mathbb{R}^n$ which satisfy $\int_0^\infty \|g(t)\|_1 dt < \infty$, where for $x \in \mathbb{R}^n$, $\|x\|_1 := \sum_{i=1}^n |[x]_i|$.

For a scalar-valued differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, $\nabla f(x)$ denotes its gradient at $x = \operatorname{col}(x_1, \ldots, x_n)$ and is defined as a *column vector*[1], i.e. $\nabla f(x) \in \mathbb{R}^n$, $[\nabla f(x)]_i = \frac{\partial f}{\partial x_i}$. For a vector-valued differentiable function $f : \mathbb{R}^n \to \mathbb{R}^m$, $f(x) = \operatorname{col}(f_1(x), \ldots, f_m(x))$, the Jacobian at $x = \operatorname{col}(x_1, \ldots, x_n)$ is the matrix $Df(x) \in \mathbb{R}^{m\times n}$ and is given by $[Df(x)]_{ij} = \frac{\partial f_i(x)}{\partial x_j}$. For a vector valued function $f : \mathbb{R}^n \to \mathbb{R}^m$, we will use $\nabla f(x)$ to denote the transpose

[1]Different definitions, i.e. $\nabla f(x)$ as row vector vs. $\nabla f(x)$ as column vector, are often found in literature.

of the Jacobian, i.e. $\nabla f(x) \in \mathbb{R}^{n \times m}$, $\nabla f(x) \triangleq Df(x)^\top$, which is consistent with the notation when f is a scalar-valued function.

We will use graph-theoretic terminology, see e.g. (Bollobas, 2002), to represent power networks.

For the practical reason of easily relating *variables in a static optimization problem* defined for a steady-state operation of a dynamical system, with the corresponding *signals* in that dynamical system, we will make the following abuse of notation: *we will use the same symbol to denote a signal, i.e. a function of time, as well as possible values that the signal may take at any time instant.*

2.2 Steady-state optimal control problem

Consider a connected undirected graph $G = (V, E, A)$ as an abstraction of an electrical power network. $V = \{v_1, \ldots, v_n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of undirected edges, and A is a weighted adjacency matrix. Undirected edges are denoted as $e_{ij} = (v_i, v_j)$, and the adjacency matrix $A \in \mathbb{R}^{n \times n}$ satisfies $[A]_{ij} \neq 0 \Leftrightarrow e_{ij} \in E$ and $[A]_{ij} = 0 \Leftrightarrow e_{ij} \notin E$. No self-connecting edges are allowed, i.e. $e_{ii} \notin E$. We associate the edges with the power lines of the electrical network and, for convenience, we set the weights in the adjacency matrix as follows: $[A]_{ij} = -\frac{1}{z_{ij}} = -b_{ij}$, where z_{ij} is the inductive reactance of a line, i.e. the imaginary part of the line impedance, and b_{ij} is the line susceptance, see (Christie et al., 2000) for details. Note that the matrix A has zeros on its main diagonal and $A = A^\top$. The set of neighbors of a node v_i is defined as $N_i \triangleq \{v_j \in V \mid (v_i, v_j) \in E\}$. Often we will use the index i to refer the node v_i. We define $I(N_i)$ as the set of indices corresponding to the neighbors of node i, i.e. $I(N_i) \triangleq \{j \mid v_j \in N_i\}$. We associate the nodes with the buses in the electrical energy transmission network.

Optimal power flow problem

To define the steady-state related optimization problem, which reflects economic objectives of a power system, with each node v_i we associate a singlet $\hat{p}_i$ and a quadruplet $(p_i, \underline{p}_i, \overline{p}_i, J_i)$, where $p_i, \underline{p}_i, \overline{p}_i, \hat{p}_i \in \mathbb{R}$, $\underline{p}_i < \overline{p}_i$ and $J_i : \mathbb{R} \to \mathbb{R}$ is a strictly convex, continuously differentiable function. The values p_i and $\hat{p}_i$ denote the reference values for node power injections into the network. Positive values correspond to a flow of power into the network (production), while negative values denote power extracted from the network (consumption). Both p_i and $\hat{p}_i$ can take positive as well as negative values, and the only difference is that, in contrast to $\hat{p}_i$, the value p_i has an associated objective function J_i and a constraint $\underline{p}_i \leq p_i \leq \overline{p}_i$. In the case of a positive p_i, the function J_i represents the variable costs of production, while for negative values of p_i, it denotes the negated benefit function of a consumer. We will refer to p_i as the power from a price-elastic producer/consumer (or simply, power from a price-elastic unit), and to $\hat{p}_i$ as the power from a price-inelastic producer/consumer (price-inelastic unit). Our choice for the use of terms *price-elastic* and *price-inelastic* will become clear further in the text.

Remark 2.2.1 The assumption that one price-elastic unit and one price-inelastic unit are associated with each node is made to simplify the presentation. However, the results of this chapter are directly applicable to the case where some nodes in the network have several (many) units of each kind (price-elastic or price-inelastic), or when at some nodes there are no only units of one kind, or when at some nodes there are no units at all. All combinations are allowed. □

We use a "DC power flow" model (Christie et al., 2000) to determine the power flows in the network for given values of node power injections. This model is a linear approximation of a complex "AC power flow" model, and is often used in practice. For a study comparing the AC and DC power flow models of nodal price calculations, we refer to (Overbye et al., 2004).

With δ_i denoting a voltage phase angle at the node v_i, the power flow in a line $e_{ij} \in E$ is given by $p_{ij} = b_{ij}(\delta_i - \delta_j) = -p_{ji}$. If $p_{ij} > 0$, power in the line e_{ij} flows from node v_i to node v_j. The power balance in a node yields $p_i + \hat{p}_i = \sum_{j \in I(N_i)} p_{ij}$. With the abbreviations $p = \text{col}(p_1, \ldots, p_n)$, $\hat{p} = \text{col}(\hat{p}_1, \ldots, \hat{p}_n)$, $\delta = \text{col}(\delta_1, \ldots, \delta_n)$ the overall network balance condition is

$$p + \hat{p} = B\delta,$$

where the matrix B is given by

$$B = A - \text{diag}(A\mathbf{1}_n). \tag{2.1}$$

We define the optimal power flow problem as follows.

Problem 2.2.2 *Optimal power flow (OPF) problem.*
For any constant value of $\hat{p}$,

$$\min_{p,\delta} J(p) \triangleq \min_{p,\delta} \sum_{i=1}^{n} J_i(p_i) \tag{2.2a}$$

subject to

$$p - B\delta + \hat{p} = 0, \tag{2.2b}$$

$$\underline{p} \leq p \leq \overline{p}, \tag{2.2c}$$

$$b_{ij}(\delta_i - \delta_j) \leq \overline{p}_{ij}, \ \forall (i, j \in I(N_i)), \tag{2.2d}$$

where $\underline{p} = \text{col}(\underline{p}_1, \ldots, \underline{p}_n)$, $\overline{p} = \text{col}(\overline{p}_1, \ldots, \overline{p}_n)$, and $\overline{p}_{ij} = \overline{p}_{ji}$ is the maximal allowed power flow in the line e_{ij}. □

We will refer to a vector p that solves the OPF problem as a *vector of optimal power injections*. For an appropriately defined matrix L and a suitably defined vector of power line limits $\overline{p}_L$, the set of constraints in (2.2d) can be written in a more compact form as follows:

$$L\delta \leq \overline{p}_L. \tag{2.3}$$

Note that in Problem 2.2.2 we have included δ explicitly as a decision variable, which will be crucial in the control design later in this chapter. Another possibility, common in the literature, is to introduce a "slack bus" with zero voltage phase angle and to solve the equations for the line flows, completely eliminating δ from the problem formulation (Christie et al., 2000; Schweppe et al., 1988). However, in that case a specific structure, i.e. sparsity, of the power flow equations is lost. As we will see later in this chapter, preserving this sparsity will show to be beneficial for *distributed* controller implementation.

Ensuring that the operating point of the system coincides with the solution of the OPF problem is one of the major operational goals of a power system. In traditional power system structures, where the production units are owned by one utility and there are little or no price elastic consumers, adjusting the production according to the solution of the OPF problem is one of the major operational goals of a utility. In such a system, the OPF problem is directly solved at a utility dispatch center, and the optimal reference values p are sent to the production units. In this paper we are concerned with a liberalized, market-based power system, where the OPF problem is important due to its relation to the optimal nodal price problem, which is defined next.

Optimal nodal prices problem

In a liberalized, market-oriented power system, different units are owned by separate parties and each of them acts autonomously to maximize its own benefit. In other words, when a price-elastic unit at node i receives the current nodal price λ_i, it adjusts its production level to be equal to p_i, where $p_i = \arg\min_{\tilde{p}_i} \{J_i(\tilde{p}_i) - \lambda_i \tilde{p}_i \mid \underline{p}_i \leq \tilde{p}_i \leq \overline{p}_i\}$. Since J_i is a strictly convex, continuously differentiable function, this relation defines a unique mapping from λ_i to p_i for any $\lambda_i \in \mathbb{R}$. For convenience, we denote this mapping with $\Upsilon_i : \lambda_i \to p_i$, i.e.

$$p_i = \Upsilon_i(\lambda_i) \triangleq \arg\min_{\tilde{p}_i} \{J_i(\tilde{p}_i) - \lambda_i \tilde{p}_i \mid \underline{p}_i \leq \tilde{p}_i \leq \overline{p}_i\}, \tag{2.4}$$

and define $\Upsilon(\lambda) \triangleq \mathrm{col}(\Upsilon_1(\lambda_1), \ldots, \Upsilon_n(\lambda_n))$. The operational goal in a liberalized power system is to determine the nodal price λ_i for each node i in the network, in such a way that the total benefit of the system is maximized, while all system's constraints are fulfilled. Formally, we define the optimal nodal price problem as follows.

Problem 2.2.3 *Optimal nodal prices (ONP) problem.*
For any constant value of $\hat{p}$,

$$\min_{\lambda,\delta} \sum_{i=1}^{n} J_i(\Upsilon_i(\lambda_i)) \tag{2.5a}$$

subject to

$$\Upsilon(\lambda) - B\delta + \hat{p} = 0, \tag{2.5b}$$

$$L\delta \leq \overline{p}_L, \tag{2.5c}$$

where $\lambda = \mathrm{col}(\lambda_1, \ldots, \lambda_n)$ is a vector of nodal prices. □

A vector λ that solves the ONP problem is the *vector of optimal nodal prices*. The OPF and ONP problems are related through Lagrange duality, as it will be shown later in this chapter. The ONP problem is employed next to define the optimal power balance and congestion control problem.

Real-time power balance and congestion control problem

Consider a power network where each unit, i.e. producer/consumer, is a dynamical system, and assign to each such unit an appropriate model of its dynamics. Let G_i and $\hat{G}_i$ denote respectively a dynamical model of price-elastic and price-inelastic unit at node i. (Recall that the assumption of one price-elastic and one price-inelastic unit at each node is made only to simplify the presentation, see Remark 2.2.1.)

Let G_i be an LTI system given by its state-space realization

$$\dot{x}_i = A_i x_i + F_i p_i^A + B_i p_i = A_i x_i + F_i p_i^A + B_i \Upsilon_i(\lambda_i), \quad \forall i, \tag{2.6}$$

where x_i is the state vector, p_i^A denotes the *actual* node power injection from the system G_i into the network, and the input $p_i = \Upsilon_i(\lambda_i)$ denotes a price-dependent *reference* signal for power injection, i.e. $p_i = \Upsilon_i(\lambda_i)$ represents desired production/consumption as a function of price.

Similarly, let the dynamical model $\hat{G}_i$ of a price-inelastic unit at node i be given by

$$\dot{z}_i = \hat{A}_i z_i + \hat{F}_i \hat{p}_i^A + \hat{B}_i \hat{p}_i, \quad \forall i, \tag{2.7}$$

where z_i denotes the state vector, $\hat{p}_i^A$ denotes the *actual* node power injection from the system $\hat{G}_i$ into the network, and $\hat{p}_i$ is a *reference* value for the power injection, i.e. desired production/consumption. The desired production/consumption $\hat{p}_i$ of a price-inelastic unit is a function of exogenous inputs, i.e. it does not depend on the electricity price λ_i, neither on any other signal from the power system. All residential and a large part of the industrial consumers represent the largest portion, measured in on-line power at any moment, of price-inelastic units in today's power systems. The aggregated behavior of those consumers is most often modeled by static relations, as opposed to the dynamical model (2.7). For an aggregated price-inelastic consumption at node i, this model is given by

$$\hat{p}_i^A = \hat{p}_i - D_i \Delta f_i, \qquad \forall i, \tag{2.8}$$

where $\hat{p}_i^A$ is the actually consumed power, $\hat{p}_i$ is desired consumption, $\Delta f_i = f_i - f_{\text{ref}}$ is the network frequency deviation, and D_i denotes a load-damping constant, see (Kundur, 1994) for more details. Here, $f_i = \frac{1}{2\pi}\dot{\delta}_i$ is the network frequency [Hz], and f_{ref} denotes the frequency reference value, e.g. $f_{\text{ref}} =$ 50Hz in Europe.

Note that (2.2b) is always fulfilled when p and $\hat{p}$ are replaced with $p^A = \text{col}(p_1^A, \ldots, p_n^A)$ and $\hat{p}^A = \text{col}(\hat{p}_1^A, \ldots, \hat{p}_n^A)$, since in this case (2.2b) represents the conservation law, i.e.

$$p^A - B\delta + \hat{p}^A = 0. \tag{2.9}$$

The complete dynamical model of a power system (physical layer of a power system) is described with the set of differential algebraic equations (2.6),(2.7),(2.8),(2.9), with p and $\hat{p}$ as inputs. For a detailed presentation of power system modeling for real-time power balance control problems, i.e. for classical dynamical models used in AGC studies, we refer to (Kundur, 1994), Chapter 11, or (Saadat, 1999), Chapter 12.

As opposed to the *actual* power injections, which are always in balance (2.9), keeping the balance in *reference* values (2.2b), i.e. balance in *desired* production and consumption, is a control problem. For future reference, we will always use the term *power balance* to refer to the power balance in sense of (2.2b), and not to the physical law (2.9).

To solve the power balance control problem, a measure of imbalance has to be available. The network frequency serves that purpose. Let $\Delta f \triangleq \mathrm{col}(\Delta f_1, \ldots, \Delta f_n)$ denote the vector of nodal frequency deviations. In steady-state the network frequency is equal for all nodes in the system and the system is in balance if the network frequency is equal to its reference value, i.e. if $\Delta f = 0$. More precisely, if a system is in a steady-state with $\Delta f = 0$, then for each node (2.6) implies $p_i = p_i^A$, while (2.7) and (2.8) imply $\hat{p}_i = \hat{p}_i^A$, and therefore (2.9) implies (2.2b). If for some steady-state conditions the network frequency deviation is positive, the total desired production in a network exceeds the total desired consumption.

The desired production/consumption of price-elastic units is a function of current nodal prices λ. Therefore, nodal prices can be effectively used as a feedback signal for power balance control. Each price-elastic system G_i receives a price signal λ_i and, based on its benefit maximization objective (2.4), it maps the signal into a reference p_i. We will assume this mapping to be instantaneous, although the model can easily be extended with dynamics, time delays, threshold based rules, etc.

In addition to controlling the power balance, nodal prices are used for congestion control, i.e. for fulfilment of the inequality constraints (2.3). For convenience we will define the vector of line overflows as $\Delta p_L \triangleq L\delta - \overline{p}_L$.

From (2.6)-(2.9), by eliminating the algebraic equations (2.9), the complete dynamical model of a power system with λ as input and $\mathrm{col}(\Delta f, \Delta p_L)$ as output, is given by

$$\dot{x} = Ax + \hat{H}\hat{p} + H\Upsilon(\lambda), \tag{2.10a}$$

$$\begin{pmatrix} \Delta f \\ \Delta p_L \end{pmatrix} = Cx + \begin{pmatrix} 0 \\ -\overline{p}_L \end{pmatrix}, \tag{2.10b}$$

where x is the complete system state vector containing all subsystem's states, i.e $x = \mathrm{col}(x_1, \ldots, x_n, z_1, \ldots, z_n)$, and A, $\hat{H}$, H, C are suitably defined constant matrices.

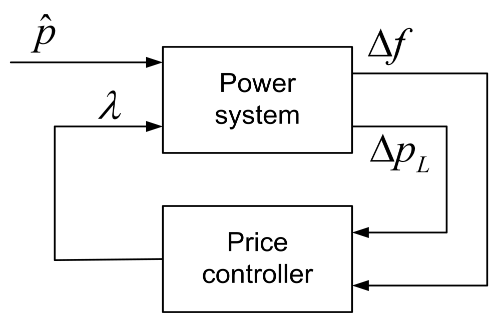

Figure 2.1: Price-based control scheme.

Note that, with the model (2.10), the steady-state power balance condition is expressed as follows

$$\left.\begin{array}{c} 0 = Ax + \hat{H}\hat{p} + H\Upsilon(\lambda) \\ 0 = \Delta f \end{array}\right\} \quad \Leftrightarrow \quad p - B\delta + \hat{p} = 0. \tag{2.11}$$

Furthermore, note that the mapping $\Upsilon : \lambda \rightarrow p$, defined by (2.4), is linear if and only if for each price elastic unit J_i is a quadratic function and $\underline{p}_i = -\infty$, $\overline{p}_i = \infty$. In practice, however, it will always be a nonlinear mapping and therefore, the model (2.10) with the nodal prices λ as inputs, is nonlinear. Finally, we are able to define the control problem considered in this chapter.

Problem 2.2.4 *Optimal power balance and congestion control problem.* For a power system of the form (2.10), design a feedback controller that has the network frequency deviation vector Δf and the vector of line overflows Δp_L as inputs, and the nodal prices λ as output (see Figure 2.1), such that the following objective is met: for any constant value of $\hat{p}$ such that the ONP problem is feasible, the state of the closed-loop system converges to an equilibrium point where the nodal prices are the *optimal nodal prices* as defined in Problem 2.2.3. □

2.3 Price-based optimal controller

In this subsection we employ the relation between the solutions of the OPF and the ONP problems to obtain a solution to Problem 2.2.4. We start by presenting the following basic result from power system economics.

Proposition 2.3.1 *The optimal dual variable (Lagrange multiplier) associated with the power balance constraint* (2.2b) *in the Lagrange dual problem of OPF, is the vector of optimal nodal prices for the corresponding ONP problem.* □

Proof. Consider some constant value $\hat{p}$ such that the OPF and ONP problems are feasible. The OPF problem is a convex problem which satisfies Slater's constraint qualification (Boyd and Vandenberghe, 2004). This implies that strong duality holds and that first-order Karush-Kuhn-Tucker (KKT) conditions are *necessary and sufficient* conditions for optimality. For the OPF problem, the Lagrangian is given by

$$\begin{aligned}\mathcal{L}(p, \delta, \nu^+, \nu^-, \lambda, \mu) = J(p) - \lambda^\top (p - B\delta + \hat{p}) + \\ (\nu^-)^\top (\underline{p} - p) + (\nu^+)^\top (p - \overline{p}) + \mu^\top (L\delta - \overline{p}_L)\end{aligned} \tag{2.12}$$

and the KKT conditions are given by:

$$p - B\delta + \hat{p} = 0, \tag{2.13a}$$

$$B\lambda + L^\top \mu = 0, \tag{2.13b}$$

$$\nabla J(p) - \lambda + \nu^+ - \nu^- = 0, \tag{2.13c}$$

$$0 \leq \ (-L\delta + \overline{p}_L) \ \perp \ \mu \ \geq 0, \tag{2.13d}$$

$$0 \leq \ (-p + \overline{p}) \ \perp \ \nu^+ \ \geq 0, \tag{2.13e}$$

$$0 \leq \ (p + \underline{p}) \ \perp \ \nu^- \ \geq 0, \tag{2.13f}$$

where λ, ν^+, ν^- and μ are (*vector*) Lagrange multipliers. The multiplier λ is associated with the equality constraint (2.13a) and is therefore not sign restricted. At the optimum, the value of the objective function and the vector of optimal power injections in the OPF problem are equal to the value of the objective function and the vector of power injections in the ONP problem, i.e. $p = \Upsilon(\lambda)$. Assume that the optimal Lagrange multiplier λ from the OPF dual problem is taken to be the vector of nodal prices. In this case the conditions (2.13c),(2.13e) and (2.13f) correspond to the KKT conditions for the constrained minimization problem in (2.4) for all price-elastic units in the network. Therefore, for this particular λ, the vector of power injections $p = \Upsilon(\lambda)$ in the ONP problem corresponds to the vector of *optimal power injections* in the OPF problem, which concludes the proof. □

Remark 2.3.2 The matrix B, defined in (2.1), is a singular matrix with rank deficiency one and with the kernel space spanned by the vector $\mathbf{1}_n$. Physically, this reflects the fact that only the relative voltage phase angles determine the power flow. Note also that $\mathbf{1}_n \notin \operatorname{Im} B$, since $B = B^\top$ implies $\mathbf{1}_n^\top B = 0$, i.e. $\mathbf{1}_n$ is orthogonal to each column of B. Finally, analyzing the structure of L (see (2.2d) and (2.3)) one can easily observe that $\mathbf{1}_n^\top L^\top = 0$ and therefore, $\mathbf{1}_n \notin \operatorname{Im} L^\top$. These properties of B and L will be used later in this section to prove Theorem 2.3.4 and Theorem 2.3.7. □

Consider the OPF problem solution for some constant value $\hat{p}$ such that the problem is feasible. We denote the minimizers of OPF with $\tilde{p}$, $\tilde{\delta}$, and with $\tilde{\lambda}$ the value of the corresponding Lagrange multiplier. Strict convexity of each J_i implies that at the optimum $\tilde{p}$ is unique. On the other hand, due to singularity of B, if $\tilde{\delta}$ is a minimizer so is $\tilde{\delta} + \mathbf{1}_n c$ where $c \in \mathbb{R}$ is an arbitrary constant. However, note that for all minimizers the set of active constraints is uniquely determined. Furthermore, we denote with $\breve{\mu}$ and $\tilde{\mu}$ the Lagrange multipliers corresponding to inactive and active line power flow constraints, respectively. Analogously, we define $\breve{\nu}^+$, $\tilde{\nu}^+$ and $\breve{\nu}^-$, $\tilde{\nu}^-$. For inactive constraints $\operatorname{col}(\breve{\mu}, \breve{\nu}^+, \breve{\nu}^-) = 0$. The equality (2.13b) yields $B\tilde{\lambda} = -L^\top \tilde{\mu}$. This condition implies that $\tilde{\lambda} \in \operatorname{Ker} B$ in the case that no lines are congested. This further implies $\tilde{\lambda} = \mathbf{1}_n \lambda^\star$, $\lambda^\star \in \mathbb{R}$, i.e. at the optimum, there is one price in the network for all nodes. *In case at least one line in the system is congested, it follows that the optimal nodal prices will in general be different for each node in the system.*

Next, we present the explicit dynamic controller that solves Problem 2.2.4.

Let K_λ, K_f and K_p be positive definite diagonal matrices, such that $K_f = \alpha K_\lambda, \alpha \in \mathbb{R}$ and $\alpha > 0$. Consider the following dynamic linear complementarity controller:

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L \end{pmatrix} + \begin{pmatrix} 0 \\ w \end{pmatrix}, \tag{2.14a}$$

$$0 \leq \; w \;\perp\; x_\mu \;\geq\; 0, \tag{2.14b}$$

$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix}, \tag{2.14c}$$

$$x_\mu(0) \geq 0, \tag{2.14d}$$

where x_λ and x_μ denote the controller states, $\operatorname{col}(\Delta f, \Delta p_L)$ and w denote inputs to the controller, while λ denotes the output. The matrices K_λ, K_f and K_p represent the controller gains. The input $\operatorname{col}(\Delta f, \Delta p_L)$, which collects the nodal frequency and line overflow vectors, is an exogenous input to the controller, while the input w is required to be a solution to the finite dimensional complementarity problem (2.14b). The output λ is a vector of nodal prices.

The initialization constraint (2.14d) is required as a necessary condition for well-posedness, in conformity with the inequality in the complementarity condition (2.14b).

Assumption 2.3.3 The closed-loop system resulting from the interconnection of the controller (2.14) with the power system (2.10) is globally asymptotically stable for any constant value of $\hat{p}$ (i.e. with respect to the corresponding steady-state) such that the ONP problem is feasible. □

Theorem 2.3.4 *Suppose that Assumption 2.3.3 holds. Then the dynamic controller* (2.14) *solves the optimal power balance and congestion control problem, as defined in Problem 2.2.4.* □

Proof. To prove Theorem 2.3.4, it suffices to show that in steady-state, the vector of nodal prices λ in (2.14) coincides with the Lagrange multiplier λ in (2.13), and therefore, by Proposition 2.3.1, is a vector of optimal nodal prices. Let $\hat{p}$ be such that the ONP problem is feasible. In steady-state, the values of the closed-loop system state vectors x, x_λ and x_μ are a solution of the following complementarity problem

$$0 = Ax + \hat{H}\hat{p} + H\Upsilon(x_\lambda), \tag{2.15a}$$

$$0 = Bx_\lambda + L^\top x_\mu + \alpha\Delta f, \tag{2.15b}$$

$$0 = K_p \Delta p_L + w, \tag{2.15c}$$

$$0 \leq w \perp x_\mu \geq 0, \tag{2.15d}$$

where Δf and Δp_L are functions of the state vector x as defined in (2.10b). In steady-state, the frequency is equal for all nodes in the network, i.e. $\Delta f = \mathbf{1}_n \Delta f^\star$, $\Delta f^\star \in \mathbb{R}$. Furthermore, from (2.15), with $\lambda = x_\lambda$ and $\mu := x_\mu$, the following holds:

(i) from (2.15b) it follows that $B\lambda + L^\top\mu + \mathbf{1}_n\alpha\Delta f^\star = 0$, which, together with $\mathbf{1}_n \notin \operatorname{Im}\begin{pmatrix} B & L^\top \end{pmatrix}$ (see Remark 2.3.2), implies that $\Delta f^\star = 0$ and $B\lambda + L^\top\mu = 0$, i.e. the optimality condition (2.13b) is satisfied;

(ii) from (2.15a) and $\Delta f^\star = 0$, with (2.11), the power balance condition (2.13a) is satisfied;

(iii) from (2.15c) and (2.15d), and since K_p is a positive definite diagonal matrix, the complementarity condition (2.13d) is satisfied;

(iv) conditions (2.13c), (2.13e) and (2.13f) are satisfied since they correspond to the KKT conditions for optimization problem in (2.4).

To summarize, implications (i)-(iv) prove that in steady-state, the KKT conditions (2.13) will necessarily be satisfied, with the vector of nodal prices λ from the controller (2.14) acting as the corresponding Lagrange multiplier in (2.13), which proves Theorem 2.3.4. □

Remark 2.3.5 The *only* system parameters that are explicitly included in the controller (2.14) are the transmission network parameters, i.e. the network topology and line impedances, which define the matrices B and L. To provide the correct nodal prices, the controller requires no knowledge of cost/benefit functions J and of power injection limits $(\underline{p}, \overline{p})$ of producers/consumers in the system (neither is it based on their estimates). In practice, often only a relatively small subset of all lines is critical concerning congestion, and for the controller (2.14) it suffices to include only these critical lines by appropriately choosing Δp_L and L. □

Remark 2.3.6 In the proof of Theorem 2.3.4 we have used the positive definiteness of K_p, but not also the positive definiteness of K_λ and K_f (we have only exploited nonsingularity of those matrices to obtain (2.15b)). The positive definiteness of K_λ and K_f is required for stability of the closed-loop system. Being positive, these gains in (2.14) ensure that the prices will be updated in the "correct direction", e.g. if the total production exceeds the total demand, prices will *decrease* causing decrease in production and increase in demand. □

The controller (2.14) has a structure of the *saturation-based KKT controller*. To solve Problem 2.2.4 the *max-based KKT controller* structure can also be used. Both controller structures are presented in detail in Chapter 5 of this thesis. In Chapter 5 we also discuss the problem of checking asymptotic stability of the closed-loop system with a KKT controller, i.e. we discuss methods to verify Assumption 2.3.3.

The property $\mathbf{1}_n \notin \operatorname{Im}\begin{pmatrix} B & L^\top \end{pmatrix}$, which ensures zero frequency deviation in steady-state ($\Delta f = 0$), is not robust with respect to possible errors in the controller implementation, i.e. to perturbations of matrices B and L in (2.14a). To overcome this drawback and to robustly ensure $\Delta f = 0$ in steady-state, the controller (2.14) can easily be modified to explicitly include integral action on the network frequency deviation. This modified controller is given by the following dynamical complementarity system

$$\begin{pmatrix} \dot{x}_{\lambda_0} \\ \dot{x}_{\Delta\lambda} \\ \dot{x}_{\mu} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -K_\Delta B_\Delta & -K_\Delta L_\Delta^\top \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda_0} \\ x_{\Delta\lambda} \\ x_{\mu} \end{pmatrix} + \begin{pmatrix} -k_f \mathbf{1}_n^\top & 0 \\ 0 & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}, \tag{2.16a}$$

$$0 \leq \ w \ \perp \ x_\mu \ \geq \ 0, \tag{2.16b}$$

$$\lambda = \begin{pmatrix} 1 & 0 & 0 \\ \mathbf{1}_{n-1} & I_{n-1} & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda_0} \\ x_{\Delta\lambda} \\ x_{\mu} \end{pmatrix}, \tag{2.16c}$$

$$x_\mu(0) \geq 0, \tag{2.16d}$$

where the diagonal and positive definite matrices K_Δ ($K_\Delta \in \mathbb{R}^{(n-1)\times(n-1)}$) and K_p, and the positive scalar k_f represent controller gains. B_Δ is a submatrix of B obtained by removing the first column and the first row of B, and L_Δ is a submatrix of L, obtained by removing the first column of L.

In the controller (2.16), nodal prices λ are composed of two parts (see (2.16c)): the *scalar* valued x_{λ_0} and the *vector* valued $x_{\Delta\lambda}$. The main idea for this decomposition is the following: x_{λ_0} represents a *reference price* and reacts solely on frequency deviations Δf (see first row in (2.16a)), while the elements of the vector $x_{\Delta\lambda}$ represent *congestion prices*, and are defined as the differences between the nodal prices and the reference price. The reference price update robustly ensures zero frequency deviation in a steady-state, while the congestion prices ensure that the optimality condition (2.13b) is satisfied in steady-state.

Theorem 2.3.7 *Suppose that Assumption 2.3.3 holds. Then the dynamic controller* (2.16) *solves the optimal power balance and congestion control problem, as defined in Problem 2.2.4.* □

Proof. The dynamics of the scalar valued state x_{λ_0} acts as an integral action for network frequency deviations Δf, and therefore $\Delta f = 0$ is *robustly* ensured in steady-state. With (2.11), this implies that, in steady-state, the condition (2.13a) is satisfied. With λ and $\mu := x_\mu$, fulfilment of the conditions (2.13c)-(2.13f) in a steady-state follows analogously as in the proof of Theorem 2.3.4 (see items (iii) and (iv)). Finally, to complete the proof, it remains to show that in steady-state the values of λ and $\mu := x_\mu$ from (2.16) satisfy the optimality condition (2.13b), i.e. that $B\lambda + L^\top \mu = 0$. To show this, we first represent the matrices B and L with B_Δ and L_Δ as submatrices as follows:

$$B = \begin{pmatrix} b_1 & b_2^\top \\ b_2 & B_\Delta \end{pmatrix}, \qquad L = \begin{pmatrix} l_1 & L_\Delta \end{pmatrix}. \tag{2.17}$$

Since $\mathbf{1}_n^\top \begin{pmatrix} B & L^\top \end{pmatrix} = 0$ (see Remark 2.3.2), it follows that $b_2^\top = -\mathbf{1}_{n-1}^\top B_\Delta$ and $l_1^\top = -\mathbf{1}_{n-1}^\top L_\Delta^\top$. With $B\mathbf{1}_n = 0$ (see Remark 2.3.2) and $\lambda = \mathbf{1}_n x_{\lambda_0} + \begin{pmatrix} 0 \\ x_{\Delta_\lambda} \end{pmatrix}$ (see (2.16c)) we have

$$\begin{aligned} B\lambda + L^\top \mu &= B \left[\mathbf{1}_n x_{\lambda_0} + \begin{pmatrix} 0 \\ x_{\Delta\lambda} \end{pmatrix} \right] + L^\top \mu \\ &= B \begin{pmatrix} 0 \\ x_{\Delta\lambda} \end{pmatrix} + L^\top \mu \\ &= \begin{pmatrix} b_2^\top \\ B_\Delta \end{pmatrix} x_{\Delta\lambda} + \begin{pmatrix} l_1^\top \\ L_\Delta^\top \end{pmatrix} \mu \\ &= \begin{pmatrix} -\mathbf{1}_{n-1}^\top \\ I_{n-1} \end{pmatrix} (B_\Delta x_{\Delta\lambda} + L_\Delta^\top \mu). \end{aligned} \tag{2.18}$$

Finally, from (2.16a) and from non-singularity of K_Δ it follows that in steady-state $B_\Delta x_{\Delta\lambda} + L_\Delta^\top \mu = 0$, which, with (2.18), implies $B\lambda + L^\top \mu = 0$. □

2.3.1 Well-posedness of the closed-loop system

The term *well-posedness* is in general used to denote the property of *existence and uniqueness of solutions*, and is a fundamental issue for any class of dynamical systems. A mathematical model of a dynamical system is said to be well-posed if, given initial conditions, it has a unique solution.

Checking well-posedness of a closed-loop system with the price-based controller (2.14),(2.16) is a nontrivial task, since this system is characterized by a discontinuous right hand side and therefore classical techniques based on Lipschitz continuity are not applicable. As one of the fundamental questions in the theory of hybrid systems, the problem of well-posedness of dynamical systems with discontinuous right hand side has recently gained an increasing interest. Especially, well-posedness of complementarity systems has been successfully studied, see (van der Schaft and Schumacher, 1996, 1998; Heemels et al., 1999, 2000a; Schumacher, 2004), and the references therein, for many of the available results on the topic.

Power systems in closed-loop with price-based controller belong to a specific class of *gradient-type complementarity systems* (GTCS) (Heemels et al., 2000b), for which well-posedness conditions have already been presented in (Heemels et al., 2000b; Brogliato et al., 2006). In Section 5.4 of Chapter 5 in this thesis, conditions for well-posedness of the GTCS class of systems are shortly summarized from (Brogliato et al., 2006; Heemels et al., 2000b). In particular, based on Theorem 5.4.5 and Proposition 5.4.6, it can be verified that for the power system (2.10) with the price-based controller (2.14),(2.16), the following proposition holds.

Proposition 2.3.8 *Suppose that the function* $\Upsilon_i : \mathbb{R} \to \mathbb{R}$, *defined by* (2.4), *is globally Lipschitz for all* $i \in \{1, \dots, n\}$, *and suppose that the exogenous input* $\hat{p}$ *is a measurable function of time* t *and* $\hat{p}(t) \in \mathcal{L}_1^n(\mathbb{R}_+)$. *Then the closed-loop system, i.e. system* (2.10) *interconnected with the controller* (2.14)/(2.16) *in a feedback loop, is well-posed, i.e. for any admissible initial condition, the closed-loop system state has a unique solution over the time interval* $[0, \infty)$. □

Proof. The proposition is a direct consequence of Theorem 5.4.5 and Proposition 5.4.6, which are given in Section 5.4 of Chapter 5 in this thesis. □

In Proposition 2.3.8 by the term *admissible initial condition*, we mean fulfillment of the condition (2.14d),(2.16d).

2.4 Distributed control implementation

Matrices B and L in (2.14) are highly structured and related in such a way that this structure can be effectively utilized for *distributed* implementation of the proposed explicit controller. For simplicity and clarity, we will present this distributed implementation by considering an example of a simple power network. In the next subsection the efficiency of the developed methodology will be demonstrated on the IEEE 39-bus New England test network.

Example. Consider a simple network depicted in Figure 2.2 and assume that at the optimum the lines e_{12} and e_{13} are congested so that $p_{12} = \overline{p}_{12}$ and $p_{13} = \overline{p}_{13}$. With μ_{12} and μ_{13} denoting the corresponding Lagrange multipliers from (2.13d), the optimality condition (2.13b) relates the optimal nodal prices with the following equality:

$$\left(\begin{array}{cccc|cc} b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} & b_{13} \\ -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} & 0 \\ -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 & -b_{13} \\ 0 & 0 & -b_{34} & b_{34} & 0 & 0 \end{array}\right) \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \hline \mu_{12} \\ \mu_{13} \end{pmatrix} = 0, \quad (2.19)$$

where $b_{12,13} = b_{12} + b_{13}$ and so on. Each row in (2.19) represents an equality related to the corresponding node in the network, i.e. the first row is related to the first node etc. Note that the i-th row directly relates the nodal price λ_i only with the nodal prices of its neighboring nodes, i.e. with λ_j, $j \in I(N_i)$. Similarly, only the nodal prices in the nodes corresponding to the congested line e_{ij} are directly related to the corresponding Lagrange multiplier μ_{ij}. Note that in practice B is usually sparse; the number of neighbors for most of the nodes is small, e.g. two to four. These highly structured relations from the optimality conditions (2.13) are as well present in the proposed controller (2.14) and (2.16), allowing for its *distributed* implementation. This means that the control law (2.14) can be implemented through a set of *nodal controllers*, where a nodal controller (NC) is assigned to each node in the network, and each NC communicates only with the NC's of the neighboring nodes. From (2.14) and (2.19) it is easy to derive that the NC corresponding to node 1 in the network depicted in Figure 2.2 is given by:

$$\begin{pmatrix} \dot{x}_{\lambda_1} \\ \dot{x}_{\mu_{12}} \\ \dot{x}_{\mu_{13}} \end{pmatrix} = \begin{pmatrix} -k_{\lambda_1} b_{12,13} & k_{\lambda_1} b_{12} & k_{\lambda_1} b_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda_1} \\ x_{\mu_{12}} \\ x_{\mu_{13}} \end{pmatrix} +$$

$$\begin{pmatrix} k_{\lambda_1} b_{12} & k_{\lambda_1} b_{13} & -k_{f_1} & 0 & 0 \\ 0 & 0 & 0 & k_{p_{12}} & 0 \\ 0 & 0 & 0 & 0 & k_{p_{13}} \end{pmatrix} \begin{pmatrix} x_{\lambda_2} \\ x_{\lambda_3} \\ \Delta f_1 \\ \Delta p_{12} \\ \Delta p_{13} \end{pmatrix} + \begin{pmatrix} 0 \\ w_{12} \\ w_{13} \end{pmatrix}, \quad (2.20a)$$

$$0 \leq \begin{pmatrix} x_{\mu_{12}} \\ x_{\mu_{13}} \end{pmatrix} \perp \begin{pmatrix} w_{12} \\ w_{13} \end{pmatrix} \geq 0, \quad (2.20b)$$

$$\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda_1} \\ x_{\mu_{12}} \\ x_{\mu_{13}} \end{pmatrix}, \quad (2.20c)$$

where $k_{\lambda_1} = [K_\lambda]_{11}$, $k_{f_1} = [K_f]_{11}$, and $k_{p_{12}}$, $k_{p_{12}}$ are the corresponding elements from the gain matrix K_p in (2.14c).

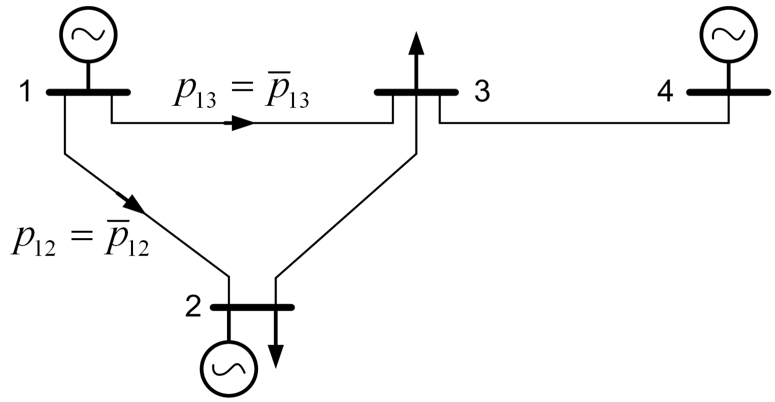

Figure 2.2: An example of a simple congested network.

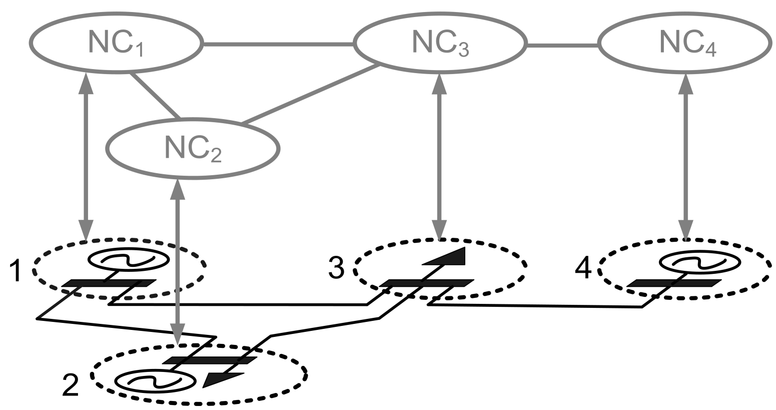

Figure 2.3: Distributed control scheme for power balance and congestion control.

Note that the state $x_{\mu_{ij}}$ is present only in one of the adjacent nodal controllers, i.e. in node i or in node j, and is communicated to the NC in the other node. □

The distributed implementation of the developed controller is graphically illustrated in Figure 2.3. The communication network graph among NC's is the same as the graph of the underlying physical network. Any change in the network topology requires only simple adjustments in NC's at the location of the change. A distributed control structure is specially advantageous taking into account the large-scale of electrical power systems.

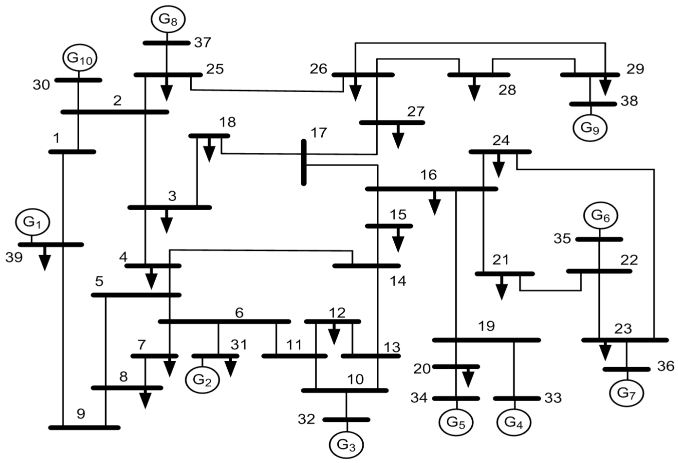

Figure 2.4: IEEE 39-bus New England test system.

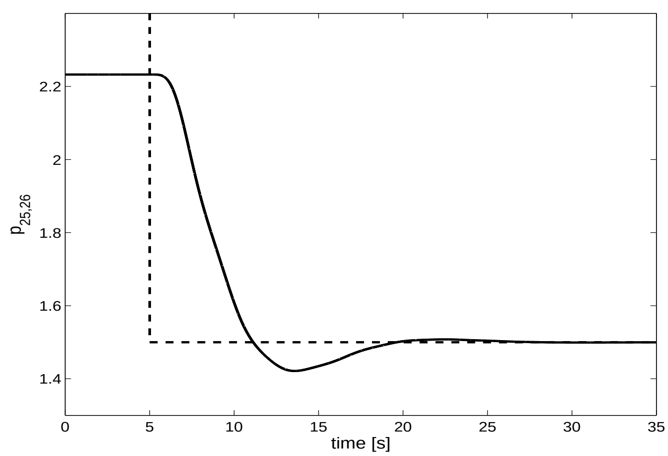

Figure 2.5: Power flow in the line connecting buses 25 and 26.

2.4.1 Application case study

To illustrate the potential of the developed methodology for practical application we consider the widely used IEEE 39-bus New England test network. The network topology, generators and loads are depicted in Figure 2.4. The complete network data, including reactance of each line and load values can be found in (Pai, 1989). All generators in the system are modeled using a third order model consisting of governor, turbine and rotor dynamics. This is a standard model used in “automatic generation control” studies (Kundur,

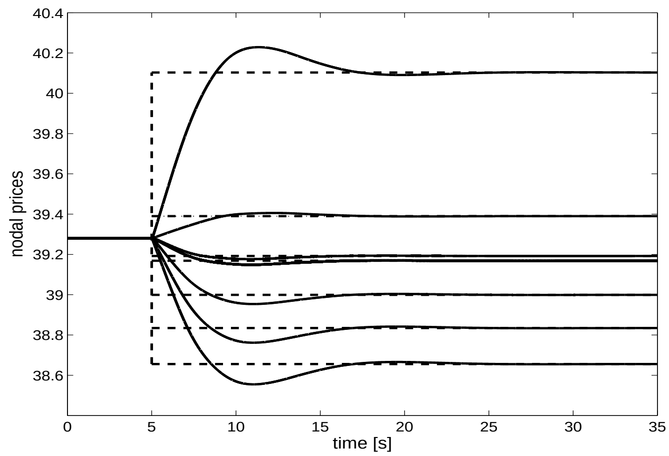

Figure 2.6: Trajectories of nodal prices for generator buses 30-39.

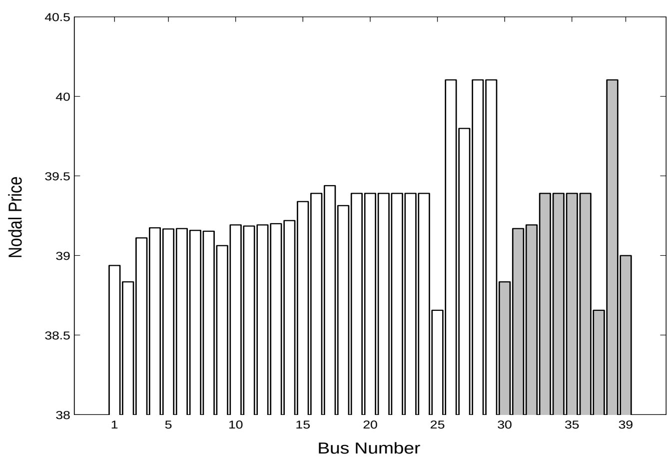

Figure 2.7: Nodal prices in the case of congestion.

1994). The parameter values, in per units, are taken to be in the ±20% interval from the values given in (Saadat, 1999), pp. 545. Each generator is taken to be equipped with a proportional feedback controller for frequency control with the gain in the interval [18, 24]. We have used quadratic functions to represent the variable production costs, i.e.

$$J_i(p_i) = \frac{1}{2}c_{g,i}p_i^2 + b_{g,i}p_i,$$

with the values of parameters $c_{g,i}$, $b_{g,i}$, for $i = 1, ..., 10$ as listed in Table 5 in (Alvarado et al., 2001). The lower saturation limit and the upper saturation limit for each generator was set to 0 and 10, respectively. All loads are taken to be price-inelastic, with the values from (Pai, 1989).

The proposed distributed controller (2.14) was implemented with the following values of the gain matrices: $K_\lambda = 3I_{39}$, $K_f = 8I_{39}$. For simplicity of exposition, the line power flow limit was assigned only for the line connecting nodes 25 and 26, and the corresponding gain K_p in the controller was set equal to 1.

The simulation results are presented in Figure 2.5 and Figure 2.6. In the beginning of the simulation, the line flow limit $\overline{p}_{25,26}$ was set to infinity, and the corresponding steady-state operating point is characterized by the unique price of 39.28 for all nodes. At time instant 5s, the line limit constraint $\overline{p}_{25,26} = 1.5$ was imposed. The solid line in Figure 2.5 represents the simulated trajectory of the line power flow $p_{25,26}$. In the same figure, the dashed line indicates the limits on the power flow $\overline{p}_{25,26}$. The solid lines in Figure 2.6 are simulated trajectories of nodal prices for the generator buses, i.e. for buses 30 to 39, which is where the generators are connected. In the same figure, dashed lines indicate the off-line calculated values of the corresponding steady-state optimal nodal prices. For clarity, the trajectories of the remaining 29 nodal prices were not plotted. In the simulation, all these trajectories converge to the corresponding optimal values of nodal prices as well. The optimal nodal prices for all buses are presented in Figure 2.7. In this figure, the nodal prices corresponding to generator buses 30-39 are emphasized with the gray shaded bars. The obtained simulation results clearly illustrate the efficiency of the proposed distributed control scheme.

2.5 Conclusions

In this chapter we have considered the problem of real-time, price-based, economically optimal power balance control and congestion management. We have designed a dynamic feedback controller for an optimal real-time update of electricity prices. Under the hypothesis of global asymptotic stability of the closed-loop system, we have proven that the developed controller will continuously drive the system towards the equilibrium where all the network constraints are satisfied, and where the total economical benefit of the system is maximized. In other words, we have proven that the controller will, based on the measurements from the current state of the system, always provide the correct, optimal nodal prices. Furthermore, the proposed control structure is characterized by certain properties which make it especially suitable for practical applications. Summarized, these properties are:

- The *only* system parameters that are explicitly included in the control law are the transmission network parameters, i.e. network topology and line impedances. To provide the correct nodal prices, the controller requires no knowledge of marginal cost/benefit functions of producers/consumers in the system (neither is it based on the estimates of these functions).
- The controller is given in an explicit form, i.e. it is not based on solving an optimization problem on-line. The nodal price updates are based on simple, explicitly defined and easily comprehensible rules.
- The transmission network structure is preserved in the controller, allowing for its distributed implementation.

The effectiveness of the proposed distributed control scheme has been illustrated on the IEEE 39-bus New England test system with 10 price elastic generating units.

3

Hybrid model predictive control of power systems

This chapter is based on the paper (Jokić, Lazar, and Van den Bosch, 2007a)

3.1 Introduction

In Chapter 2 we have developed an *explicit, price-based, dynamic* controller for real-time optimal power balancing and congestion management. The proposed explicit controller guarantees that, following any admissible change in power consumption/production, the power system will settle in the corresponding economically optimal steady-state point with all line flow constraints satisfied. Due to the inequality constraints representing the line flow limits, the optimal controller was developed in a dynamical complementarity framework (van der Schaft and Schumacher, 1996, 1998; Heemels et al., 2000a), and can be seen as an appropriate dynamical extension of the Karush-Kuhn-Tucker (KKT) conditions for economic optimality of the power system. For brevity, in this chapter we will refer to the price-based controller developed in Chapter 2 with the name *KKT controller*.

Although steady-state optimal, the KKT controller does not guarantee that during the transients following sudden imbalances in the system, some of the power lines will not become overloaded to such an extent that it will threaten the safety of the system's operation. To solve this issue, in this chapter we complement the KKT controller with a hybrid model predictive controller (MPC), i.e. a MPC controller that uses a piecewise affine model for predictions. The MPC control action amends the KKT control law so that constraints are met during the transients following load changes, and it converges to zero in steady-state. As a result, the response of the system is optimized and the constraints are satisfied in the transient period as well, while the advantageous steady-state properties of the KKT controller are preserved.

The results of this chapter add the optimal power flow problem with congestion constraints to a list of previously considered problems in electrical energy transmission systems, where the benefits of utilizing hybrid MPC control schemes were already illustrated. The interested reader is referred to (Geyer et al., 2003; Beccuti et al., 2005), where hybrid MPC was utilized for efficient emergency voltage control, and to (Ferrari-Trecate et al., 2004), where hybrid MPC was used for optimal control of co-generation power plants. Recently, in (Bemporad et al., 2006), hybrid MPC was employed for solving the problem of multi-period investments for maintenance and upgrade of electrical energy distribution networks.

3.2 Model predictive control

Model predictive control (also referred to as receding horizon control) is a control strategy that offers attractive solutions for the regulation of constrained linear or nonlinear systems and, more recently, also for the regulation of hybrid systems. Within a relatively short time, MPC has reached a certain maturity due to the continuously increasing interest shown for this distinctive part of control theory. This is illustrated by its successful implementation in industry and by many excellent articles and books as well. For a detailed overview of MPC see, for example, (Garcia et al., 1989; Mayne et al., 2000; Qin and Badgwell, 2003; Findeisen et al., 2003; Camacho and Bordons, 2004) and the references therein.

One of the reasons for the fruitful achievements of MPC algorithms consists in the intuitive way of addressing the control problem. In comparison with conventional control, which often uses a pre-computed state or output feedback control law, predictive control uses a discrete-time model of the system to obtain an estimate (prediction) of its future behavior. This is done by applying a set of input sequences to a model, with the measured state/ouput as initial condition, while taking into account constraints. An optimization problem built around a performance oriented cost function is then solved to choose an optimal sequence of controls from all feasible sequences. The feedback control law is then obtained in a receding horizon manner by applying to the system only the first element of the computed sequence of optimal controls, and repeating the whole procedure at the next discrete-time step. Summarizing the above discussion, one can conclude that

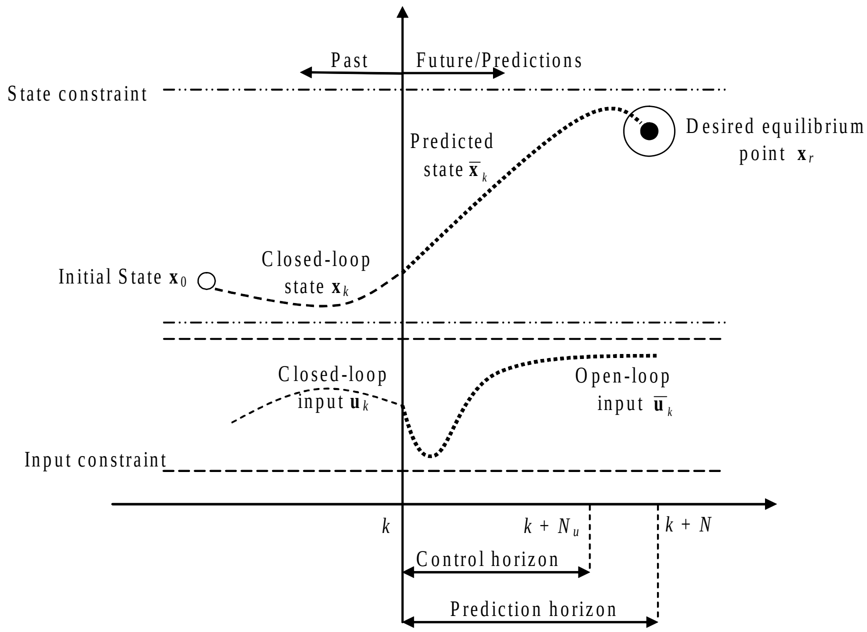

Figure 3.1: A graphical illustration of model predictive control.

MPC is built around the following key principles:

- The explicit use of a process model for calculating predictions of the future plant behavior;
- The optimization of an objective function subject to constraints, which yields an optimal sequence of controls;
- The receding horizon strategy, according to which only the first element of the optimal sequence of controls is applied on-line.

The MPC methodology involves solving on-line an open-loop finite horizon optimal control problem subject to input, state and/or output constraints.

A graphical illustration of this concept is depicted in Figure 3.1. At each discrete-time instant k, the measured variables and the process model (linear, nonlinear or hybrid) are used to (predict) calculate the future behavior of the controlled plant over a specified time horizon, which is usually called the prediction horizon and is denoted by N. This is achieved by considering a future control scenario as the input sequence applied to the process model, which must be calculated such that certain desired constraints and objectives are fulfilled. To do that, a cost function is minimized subject to constraints, yielding an optimal sequence of controls over a specified time horizon, which is usually called control horizon and is denoted by N_u. According to the receding horizon control strategy, only the first element of the computed optimal sequence of controls is then applied to the plant and this sequence of steps is repeated at the next discrete-time instant, for the updated state.

3.3 Hybrid MPC scheme

The first step in the design of an MPC controller is to obtain an appropriate dynamical model for calculating predictions of the future plant behavior. Since our goal is to combine the MPC controller with the KKT controller developed in Chapter 2, we first need an appropriate model of the power system interconnected with the KKT controller in a feedback loop, as this closed-loop system represents the plant for the MPC scheme. Here, the term *appropriate* model has a twofold meaning. Firstly, some modeling frameworks are more suitable for MPC purposes, and therefore it is desirable to represent the model in one of such frameworks. Secondly, the model has to be a discrete-time model. Concerning the first issue, in this chapter we will work in a piecewise-affine framework (Sontag, 1981), for which efficient MPC design methods have already been developed, see e.g. (Lazar, 2006) and the references therein.

3.3.1 Piecewise affine model of the power system

In the previous chapter, in Section 2.2, we have in general terms presented the basic ingredients of a power system: price-elastic and price-inelastic units as dynamical subsystems, all coupled by means of algebraic power flow equations. In a compact form, an appropriate model of a power system (physical layer of the power system), is given by (Chapter 2, equation (2.10)):

$$\dot{x} = Ax + \hat{H}\hat{p} + H\Upsilon(\lambda), \tag{3.1a}$$

$$\begin{pmatrix} \Delta f \\ \Delta p_L \end{pmatrix} = Cx + \begin{pmatrix} 0 \\ -\overline{p}_L \end{pmatrix}, \tag{3.1b}$$

where A, $\hat{H}$, H, C are suitably defined constant matrices, x denotes the system's state vector, exogenous input $\hat{p}$ denotes the desired power injections from price-inelastic units, control input λ denotes the vector of nodal prices, the output Δf denotes the network frequency deviations from the reference value (50Hz in Europe), and the output vector Δp_L is a vector of line overflows, which is defined as the vector of line flows minus the corresponding vector of line flow limits $\overline{p}_L$. The vector valued function $\Upsilon(\cdot)$, which is defined by (2.4), describes the benefit maximization behavior of price-elastic units. In practice, $\Upsilon(\cdot)$ is always a nonlinear function and therefore, the model (3.1) with the nodal prices λ as inputs, is nonlinear. With the assumption of strictly convex/concave and continuously differentiable cost/benefit functions of price-elastic units, the mapping $\Upsilon_i : \lambda_i \rightarrow p_i$ can be arbitrarily well approximated with a continuous piecewise affine function. Therefore, one can always obtain a piecewise affine model that approximates (3.1) arbitrarily well. We will assume for the remainder of this chapter that $\Upsilon(\cdot)$ is a piecewise affine function and thus, *the power system model* (3.1) *is piecewise affine (PWA) with respect to* λ *as input*.

Next, we present a PWA representation of the KKT controller, which was in Chapter 2 presented as a linear complementarity system (2.14). For simplicity, we only consider the controller given by (2.14), although the controller (2.16) can be considered equivalently.

With positive definite diagonal matrices K_λ, K_f and K_p representing the controller gains, and with Γ denoting a diagonal matrix of the same size as K_p, the controller (2.14) in a piecewise affine form is given by:

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & \Gamma \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L \end{pmatrix}, \tag{3.2a}$$

$$\lambda^{\text{KKT}} = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix}, \tag{3.2b}$$

$$\begin{cases} [\Gamma]_{ii} = [K_p]_{ii} & \text{if} \quad [x_\mu]_i \geq 0 \text{ and } [\Delta p_L]_i \geq 0 \\ [\Gamma]_{ii} = [K_p]_{ii} & \text{if} \quad [x_\mu]_i > 0 \text{ and } [\Delta p_L]_i < 0 \\ [\Gamma]_{ii} = 0 & \text{if} \quad [x_\mu]_i = 0 \text{ and } [\Delta p_L]_i < 0, \end{cases} \tag{3.2c}$$

$$x_\mu(0) \geq 0. \tag{3.2d}$$

To summarize, we have shown that both the power system and the KKT controller can be represented using a PWA model. The basic result on interconnection of PWA systems states that the feedback (as well as series and parallel) interconnections of PWA systems are themselves PWA systems (Johansson, 1999). Therefore, the closed-loop system, i.e. the system obtained by interconnecting the power system (3.1) with the KKT controller (3.2) in the feedback loop, is a PWA system.

3.3.2 MPC control problem

Following a large disturbance acting on the system, for example, a relatively large, sudden change in $\hat{p}$, it is desirable that the closed-loop system response is such that the power lines overload Δp_L and the frequency deviations Δf are limited during the transient period, i.e.

$$\Delta p_L \leq \Delta p_L^{\max}, \tag{3.3a}$$
$$-\Delta f^{\max} \leq \Delta f \leq \Delta f^{\max}, \tag{3.3b}$$

at all times, where $\Delta p_L^{\max} > 0$, $\Delta f^{\max} > 0$ are some predefined values. Since the lines can be overloaded only for a short period of time, satisfying the constraints imposed on Δp_L and Δf, as well as fast convergence to the new steady-state, is crucial.

This is even more emphasized for future power systems that are expected to rely on a large amount of uncontrollable, price-inelastic renewable energy sources, like wind turbines or photovoltaic systems. Therefore, they will be characterized by an increased uncertainty and faster changes in $\hat{p}$.

Furthermore, the ability to guarantee (3.3a) for a vector $\Delta p_L^{\max}$ with small elements has a positive impact on power system economics, since in this case the elements in the steady-state line limits vector $\overline{p}_L$ can safely take larger values, increasing the feasible region in the ONP problem, i.e. Problem 2.2.2. The constraint (3.3b) is equally important, since large, prolonged frequency excursions outside the imposed range can cause underfrequency protective relays or load-shedding schemes to activate, resulting in interruptions of power supply for some consumers (Kundur, 1994).

To guarantee the fulfillment of the constraints (3.3) during the transient period, following a change in $\hat{p}$, we design a hybrid MPC algorithm whose purpose is to complement the output λ^{KKT} of the KKT controller (3.2) such that inequalities (3.3) are satisfied at all times. The hybrid control scheme is depicted in Figure 3.2. Let G denote the continuous-time model of the power system (3.1) in closed-loop with the KKT controller (3.2), as shown in Figure 3.2. To define the MPC algorithm, we first consider the following discrete-time approximation of G in closed-loop with the MPC controller:

$$G_D : \begin{cases} x_{k+1} = A_j x_k + B_j \lambda_k^{\text{MPC}} + a_j(\hat{p}_k) \\ y_k = C_j x_k = \begin{pmatrix} \Delta p_{L\,k} \\ \Delta f_k \\ -\Delta f_k \end{pmatrix} \end{cases} \quad \text{if } H_j x_k \leq h_j, \quad k \geq 0, \tag{3.4}$$

where $A_j, B_j, a_j(\hat{p}_k), C_j, H_j, h_j$ are matrices and vectors of appropriate dimensions for all $j \in \mathcal{S} = \{1, \ldots, s\}$, with $\mathcal{S}$ a finite set of indices. The

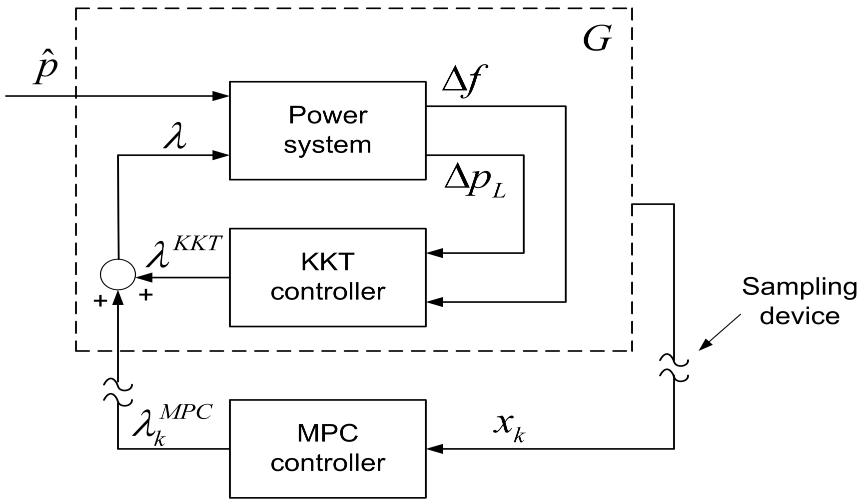

Figure 3.2: Hybrid control scheme.

state vector x incorporates now both the states of the power system model and the states of the KKT controller. As G is a piecewise affine system, an *equivalent* discrete-time piecewise affine counterpart cannot be obtained. However, one can obtain a discrete-time piecewise affine *approximation* of G by discretizing each continuous-time affine sub-system of G and taking into account physical insights when deriving the switching hyperplanes of the discrete-time model, see, for example, (Geyer et al., 2003; Beccuti et al., 2005).

The outputs of system (3.4) are given by Δp_{Lk} - the power flow deviation in the lines of the transmission network and Δf_k - the frequency deviation in the nodes of the transmission network ($-\Delta f_k$ is considered an output just to easily specify the constraints (3.3) in the MPC algorithm).

3.3.3 MPC algorithm

The MPC control action λ_k^{MPC} acts additively on the output of the KKT controller (3.2) such that the constraints (3.3) are fulfilled at all times $k \geq 0$. In steady-state, λ_k^{MPC} converges to zero so that the network is only controlled by the output of the KKT controller, which then guarantees optimality and constraint satisfaction.

Problem 3.3.1 *MPC optimization problem.* Let $N \geq 1$ be given and let $x_k =: x_{0|k}$ denote the measured state at time $k \geq 0$. Let $\mathbf{c}_k := (c_{0|k}, \ldots, c_{N|k})$

denote a sequence of optimization variables and let Ψ be a positive definite and symmetric matrix. Minimize the cost:

$$J(\mathbf{c}_k) \triangleq \sum_{i=0}^{N-1} c_{i|k}^{\top} \Psi c_{i|k}, \tag{3.5}$$

subject to the constraints

$$\begin{cases} x_{i+1|k} = A_j x_{i|k} + B_j c_{i|k} + a_j(\hat{p}_{i|k}) \\ y_{i|k} = C_j x_{i|k} = \begin{pmatrix} \Delta p_{i|k} \\ \Delta f_{i|k} \\ -\Delta f_{i|k} \end{pmatrix} \end{cases} \quad \text{if } H_j x_{i|k} \leq h_j, \quad \forall i = 0, \ldots, N,$$

$$y_{N+1|k} = C_j x_{N+1|k} = \begin{pmatrix} \Delta p_{N+1|k} \\ \Delta f_{N+1|k} \\ -\Delta f_{N+1|k} \end{pmatrix} \quad \text{if } H_j x_{N+1|k} \leq h_j, \tag{3.6a}$$

$$y_{i|k} \leq \begin{pmatrix} \Delta p_L{}^{\max} \\ \Delta f^{\max} \\ \Delta f^{\max} \end{pmatrix}, \quad \forall i = 1, \ldots, N+1, \tag{3.6b}$$

$$c_{N|k} = 0. \tag{3.6c}$$

□

Let $\mathbf{c}_k^* := (c_{0|k}^*, \ldots, c_{N-1|k}^*, 0)$ denote an optimal sequence of variables obtained by solving Problem 3.3.1. Then, the MPC control law is defined as follows:

$$\lambda_k^{\mathrm{MPC}} := c_{0|k}^*; \quad k \geq 0. \tag{3.7}$$

The value of $\hat{p}_k$, which is employed in (3.6a), can be estimated from the measured values of the system state. With the current estimated value $\hat{p}_k$ available, to obtain the future outputs in (3.6a), we assume $\hat{p}_{i|k} = \hat{p}_k$ for all $i = 0, \ldots, N$.

Assumption 3.3.2 The closed-loop system resulting from the interconnection of the KKT controller (3.2) with the power system (3.1) is globally asymptotically stable for any constant value of $\hat{p}$ (i.e. with respect to the corresponding steady-state) such that the optimal nodal prices (ONP) problem, i.e. Problem 2.2.3, is feasible. □

Assumption 3.3.3 For any constant value of $\hat{p}_k$, the prediction horizon N is long enough such that the predicted state $x_{N|k}$ lies in a subset of the state-space that is invariant for the closed-loop system (3.4) with $\lambda_k^{\mathrm{MPC}} = 0$ for all $k \geq 0$, and where the constraints (3.6b) are satisfied. Furthermore, the global optimum is attained in Problem 3.3.1 for any x_k and all $k \geq 0$. □

Theorem 3.3.4 (i) *Suppose that Assumption 3.3.3 holds. If Problem 3.3.1 is feasible at time $k \geq 0$ for the measured state $x_k = x_{0|k}$, then Problem 3.3.1 remains feasible at time $k+1$ for state $x_{k+1} = A_j x_k + B_j \lambda_k^{MPC} + a_j(\hat{p}_k)$ if $H_j x_k \leq h_j$.*

(ii) *Suppose that Assumption 3.3.2 holds for G_D with $\lambda_k^{MPC} = 0$ for all $k \geq 0$ and for any constant value of $\hat{p}_k$ such that the ONP problem is feasible. Furthermore, suppose that Assumption 3.3.3 holds. Then the discrete-time closed-loop system (3.4) with the input λ_k^{MPC} defined as in (3.7) is asymptotically stable for any constant value of $\hat{p}_k$ such that the ONP problem is feasible.* □

Proof. (i) The first statement follows, as done classically, by observing that the shifted sequence of optimization variables $\tilde{\mathbf{c}}_{k+1} := (c^*_{1|k}, \ldots, c^*_{N-1|k}, 0, 0)$ is feasible with respect to Problem 3.3.1 at time $k+1$. This is due to feasibility of $c^*_{1|k}, \ldots, c^*_{N-1|k}, 0$, while feasibility of the last zero element follows from Assumption 3.3.3.

(ii) First we prove that $\lim_{k\to\infty} \lambda_k^{\text{MPC}} = 0$. Let $V_k := V(x_k) = J(\mathbf{c}_k^*)$ and let

$$\widetilde{V}_{k+1} := \widetilde{V}(x_{k+1}) = J(\tilde{\mathbf{c}}_{k+1}) = V_k - (c^*_{0|k})^\top \Psi(c^*_{0|k}) = V_k - (\lambda_k^{\text{MPC}})^\top \Psi(\lambda_k^{\text{MPC}})$$

for any $k \geq 0$. Then, by optimality, it follows that

$$\Delta V_k := V_{k+1} - V_k \leq \widetilde{V}_{k+1} - V_k = -(\lambda_k^{\text{MPC}})^\top \Psi(\lambda_k^{\text{MPC}}) \leq 0.$$

Since $\Delta V_k \leq 0$ and V_k is lower bounded by zero for any $k \geq 0$, it follows that $\lim_{k\to\infty} V_k = V_L \geq 0$ exists. Then, $\lim_{k\to\infty} \Delta V_k = V_L - V_L = 0$. Since $0 \leq (\lambda_k^{\text{MPC}})^\top \Psi(\lambda_k^{\text{MPC}}) \leq -\Delta V_k$, and Ψ is positive definite it follows that

$$\lim_{k\to\infty} (\lambda_k^{\text{MPC}})^\top \Psi(\lambda_k^{\text{MPC}}) = 0 \Rightarrow \lim_{k\to\infty} \lambda_k^{\text{MPC}} = 0. \tag{3.8}$$

Then, the statement (ii) readily follows since by Assumption 3.3.2 the closed-loop system (3.4) with $\lambda_k^{\text{MPC}} = 0$ is globally asymptotically stable for any constant value of $\hat{p}_k$, which is such that the ONP problem is feasible. □

Fulfilment of the first part of Assumption 3.3.3 can be a priori guaranteed by adding a terminal[1] equality or inequality constraint to Problem 3.3.1, see, for example, (Lazar, 2006).

[1] By this we mean a constraint on the predicted state $x_{N|k}$.

Remark 3.3.5 The proposed hybrid MPC scheme is characterized by the following advantageous features: (i) it is less sensitive to model mismatches, as the model is used only to predict the constraints violations during transients and, if the system is not characterized by persistent and relatively large disturbances, the MPC control action will be zero for the most of the time; (ii) if the MPC controller fails to provide input to the system, the KKT controller guarantees that the system will be driven to a state where no constraints are violated; (iii) the fact that only the control input, and not also the complete system's state, is penalized in the MPC cost function reduces the MPC computational time. □

3.4 Illustrative example

The proposed hybrid control scheme was implemented for a three nodes triangular network with a synchronous generator at each node. Each generator is modeled by a third order model, which is standardly used in AGC studies (Kundur, 1994; Saadat, 1999). The generator model is taken from (Saadat, 1999), pp. 543-545. We have used the following parameter values: $\tau_g \in \{0.2, 0.25, 0.2\}$, $\tau_T \in \{0.5, 0.45, 0.5\}$, $H \in \{5, 5, 5\}$, $D \in \{0.7, 0.7, 0.8\}$, $R \in \{\frac{1}{22}, \frac{1}{24}, \frac{1}{20}\}$, where the first element in each set denotes the parameter value for a generator at the first node, etc., and the symbols are the ones from (Saadat, 1999). We use quadratic functions to represent the variable production costs J_i, with quadratic terms $\{1.2, 1.24, 1.4\}$ and linear terms $\{35.9, 36.1, 36\}$, respectively. For simplicity, no saturation limits were considered for the generators, i.e. $\underline{p}_i = -\infty$, $\overline{p}_i = \infty$, $\forall i$. Furthermore, we use the following reactance values for each line: $z_{12} = 0.1$, $z_{13} = 0.09$, $z_{23} = 0.13$. Appropriate values for the gains of the KKT controller were chosen as $K_\lambda = \text{diag}(2, 2, 2)$, $K_f = \text{diag}(7, 7, 7)$. For the line connecting nodes 1 and 2, the *steady-state* power flow limit was set equal to 1.2, i.e. $\overline{p}_{12} = 1.2$, and the maximal allowed violation of this constraint in a transient period was set to 0.1, i.e. the corresponding element in $\Delta p_L^{\max}$ from (3.3) is $\Delta p_{L_{12}}^{\max} = 0.1$. For simplicity, no line flow limits were considered in the remaining lines, and the KKT controller is implemented to act only on Δp_{12} with gain $K_p = 2$. No frequency deviation constraints were imposed during transients.

The resulting closed-loop system G is a continuous-time piecewise affine model with 15 states and 3 affine sub-systems. The (robust) asymptotic stability of G for any values of $\hat{p}$ in the interval indicated below was established via a quadratic Lyapunov function, which validates Assumption 3.3.2. We

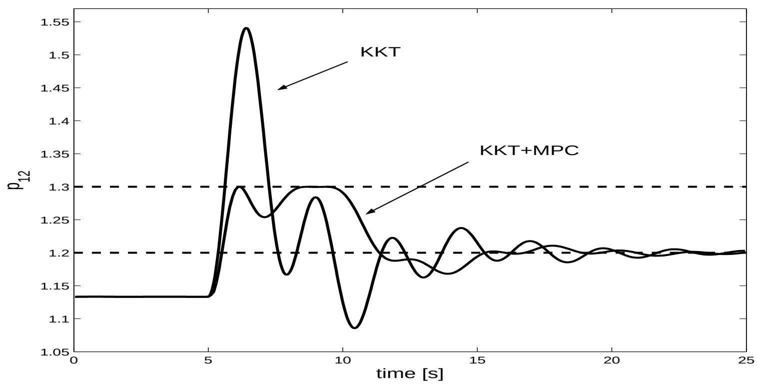

Figure 3.3: Line power flow p_{12}. Dashed lines represent power flow restriction: 1.2 in steady-state; 1.3 in transient.

obtained a discrete-time model G_D by discretizing each affine sub-system of G with a sampling period of $0.1s$ and keeping the same switching hyperplanes as the ones of the continuous-time model. The simulations showed that G_D is a good (i.e. for control purposes) approximation. The asymptotic stability test was successfully carried out for the discrete-time model G_D with $\lambda_k^{\text{MPC}} = 0$ for all $k \geq 0$ as well, which validates Assumption 3.3.2 for G_D. For the MPC controller, the following tuning parameters were used: $N = 12$, $\Psi = \text{diag}(0.1, 0.1, 0.1)$. For simplicity, we have used the following estimate for the value of $a_j(\hat{p}_k)$:

$$a_j(\hat{p}_k) = a_j(\hat{p}_{k-1}) = x_k - (A_j x_{k-1} + B_j \lambda_{k-1}^{\text{MPC}}) \qquad \text{if} \quad H_j x_{k-1} \leq h_j.$$

A price-inelastic load was connected to each node. In the simulations, in nodes 1 and 3, the corresponding loads were set equal to constant values: $\hat{p}_1(t) = 0$, $\hat{p}_3(t) = 2$. The load at node 2 was taken to be a time varying signal given by $\hat{p}_2(t) = 3$ for $t < 5s$ and $\hat{p}_2(t) = 4$ for $t \geq 5s$.

The system's response to the change in the load is presented in Figures 3.3 - 3.6. At time instant $t = 0$ the system is in a steady-state. In Figure 3.3, solid lines represent simulated trajectories of power flow p_{12} in line e_{12}. One trajectory (labeled "KKT") corresponds to the system in closed-loop with the KKT controller alone, while the other trajectory (labeled "KKT + MPC") corresponds to the MPC closed-loop system described in Figure 3.2. In Figure 3.3, dashed lines represent the steady-state line flow limit $\overline{p}_{12} = 1.2$ and the transient line flow limit $\overline{p}_{12} + \Delta p_{L_{12}}^{\max} = 1.3$, respectively. Figure 3.4 presents the values of the nodal prices λ as a function of time (solid lines). These trajectories correspond to the summation of the outputs from the KKT

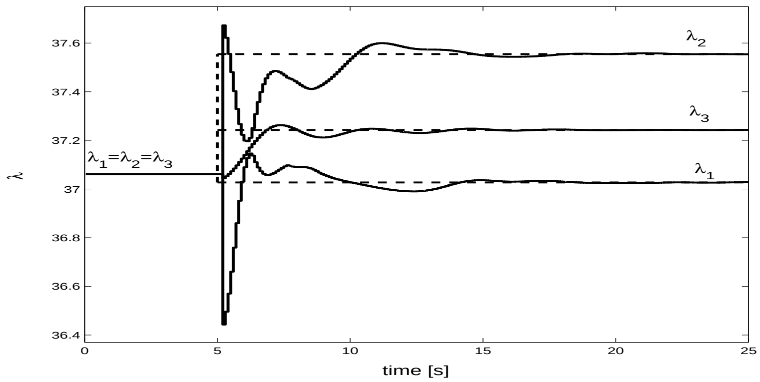

Figure 3.4: Nodal prices λ as sum of λ^{KKT} and λ^{MPC}. Dashed lines represent steady-state optimal nodal prices.

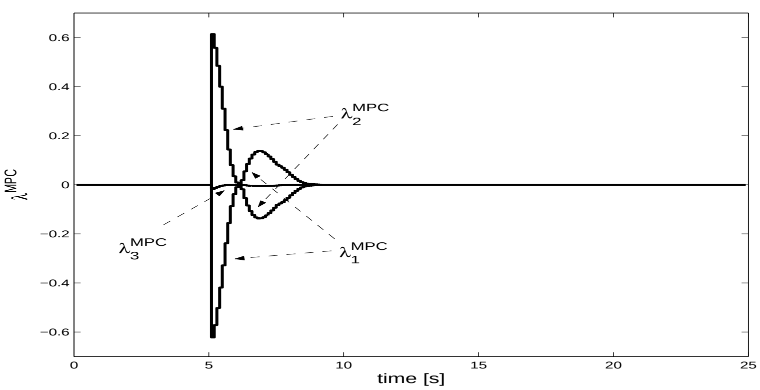

Figure 3.5: The MPC control actions.

controller and the MPC controller. In the same figure, dashed lines represent the off-line calculated values for the steady-state optimal nodal prices, which depend on the value of $\hat{p}_2(t)$. The MPC control actions are presented in Figure 3.5. The simulated trajectories of the node frequency deviations are presented in Figure 3.6. Note that, before the change in the load $\hat{p}_2$, the line is not congested and all nodal prices are equal for the steady-state optimal operating point. After the step increase in the load, in the optimal steady-state operating point the line is congested and, as a consequence, the optimal nodal prices have different values. The simulation results clearly illustrate the effectiveness of the proposed hybrid control scheme for both steady-state and transient behavior of the closed-loop system.

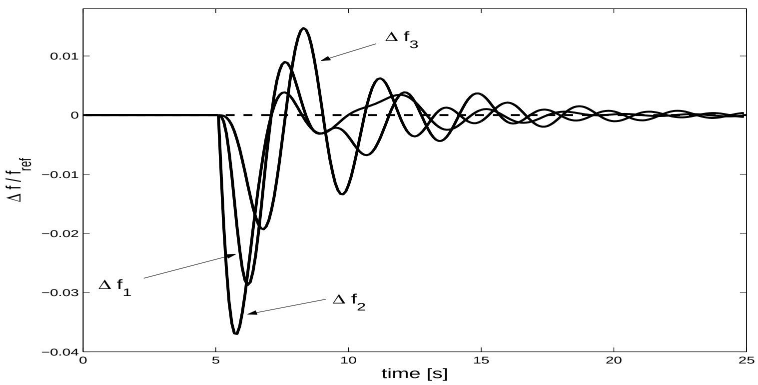

Figure 3.6: Relative frequency deviations $\frac{\Delta f}{f_{ref}}$.

3.5 Conclusions

In this chapter we have developed a novel hybrid MPC scheme for price-based network frequency and congestion control. With the developed control scheme, we have shown the potential of using price signals to control of fast occurring imbalances. The developed control scheme is shown to be efficient in limiting large frequency excursions and extensive line overloads.

The line flow constraints in a transmission system are specified for steady-state operation of the system, and we have shown that the price-based optimal controller developed in Chapter 2 guarantees fulfilment of these constraints in steady-state. In other words, in Chapter 2 these constraints were treated as *soft constraints*, i.e. their temporary violations were allowed during transient periods. By combining the price-based KKT controller developed in Chapter 2 with a suitably defined MPC controller, we could impose *hard constraints* on the maximal transient violations of the steady-state related line flow limits. Furthermore, we could impose *hard constraints* on the maximal network frequency deviations.

The MPC controller serves only to add corrective signals to the output of the KKT controller, i.e. to the nodal prices, and acts only when the predictions indicate that the imposed hard constraints will be violated. In any other case, the output of the MPC controller is always zero and only the basic price-based KKT controller is active.

Under certain assumptions, we have proven asymptotic stability of the complete closed-loop system, i.e. of the closed-loop system with both KKT controller and MPC controller, while performed simulations illustrated the effectiveness of the proposed control scheme.

As we have shown in Chapter 2, the KKT controller allows for its distributed implementation. Therefore, it would be desirable if a hybrid MPC scheme could also be implemented in a distributed fashion. Recently, a topic of distributed model predictive control (MPC) has gained a significant interest in control system community. For recent results on this topic see (Venkat et al., 2006a,b; Alessio and Bemporad, 2007) and the references therein. For an application of distributed MPC control strategies in automatic generation control of power systems see (Venkat et al., 2006a,b).

4

Autonomous power networks

This chapter is based on the papers (Jokić and Van den Bosch, 2006; Jokić, Wittebol, and Van den Bosch, 2006; Wittebol, Jokić, and Van den Bosch, 2005; Agović, Jokić, and Van den Bosch, 2005).

4.1 Introduction

Power systems are going through significant changes in many aspects. Central to the many of the changes are two major paradigm shifts occurring in the structure and operation of power systems. Major structural changes are caused by large-scale integration of privately owned Distributed Generators (DG) in all levels of traditional, vertically structured power system. In the operational sense, there is a shift towards the use of competitive markets as mechanisms for both balancing power production and consumption and for ensuring the system's reliability. Indeed, these two major changes are coupled and one is supporting the other. Introduction of de-regulated, open-access markets should encourage investment in DG by creating new business opportunities. On the other hand, non-dependence on fossil fuels and environmental issues supporting renewable based DG together with high efficiency of some DG units, like for instance combined heat and power units, are introducing novel players into the system. This has an impact on power system economics and creates a rich playground for existence of competitive markets.

All this as well results in large changes on the more technical side of power systems. The network, as a dynamical system, is changing its characteristics on all time scales. The introduction of large amounts of DG is a change towards a system with striking characteristics as large time and space heterogeneity, low inertia, extremely large uncertainties, and an extreme increase of the number of generators in comparison to traditional power systems. A large variety of DG technologies (micro-turbines, fuel cells, wind turbines, photovoltaic (PV) arrays, to name a few) makes the system more heterogeneous with respect to the traditional power system that was completely relying on large-scale synchronous generators. Across the set of these different DG technologies, there is a huge variety of possible time responses to the changes in reference values for power outputs. Furthermore, many of the technologies, like fuel cells or PV, are inertia-less, and all the renewables, like wind turbines and PV, are practically uncontrollable in their power outputs and are introducing large production fluctuations and large uncertainties in any future system state prediction.

Managing a system of such an underlying technical complexity is a major challenge, moreover since the solution has to enhance the main driving force causing the change: existence of unbundled, competitive markets as central mechanisms for system operation and reliability. It is also evident that a solution necessarily needs to be based on a large amount of players capable of competing in all the markets.

Microgrids

One of the novel paradigms for defining the operation of distributed generation, which has gained significant attention in the power system society, is the concept of microgrids (Lasseter, 2002; Venkataramanan and Illinadala, 2002). There is no simple or rather complete definition for a microgrid that is able to present in a concise way its major characteristics and objectives. The papers (Lasseter, 2002) and (Venkataramanan and Illinadala, 2002), which are among the first references on the topic, define a microgrid as a cluster of loads and microsources (DG units) that operates as a single controllable system, which provides both power and heat to its local area. To the utility a microgrid can be thought of as a controlled cell of a power system, while to the customer inside the microgrid, it can be designed to meet customers special needs; such as, enhance local reliability, reduce feeder losses, support local voltages, provide increased efficiency through use of waste heat, or provide uninterruptible power supply functions. As in these two papers, in most of the references dealing with microgrids, see e.g. (Lasseter and Piagi, 2004; Kueck et al., 2003) and the references therein, the major emphasis is on the microgrid's internal objectives, problems and their solutions. One of these objectives that has been widely studied (Pecas Lopes et al., 2003) is the possibility of a microgrid to efficiently operate in an island mode, i.e. disconnected from the rest of the power system.

Much less attention has been paid to the relationship between the microgrid and the local utility, and the main feature of this relationship is often summarized in the statement that a microgrid is a well behaved, "good citizen" or "model citizen" (Lasseter et al., 2002) in the overall power system. By "good" behavior, it is in most cases thought of a low impact, or more importantly on absence of a negative impact, that a microgrid has on the rest of electricity network, despite a potentially significant level of generation by intermittent renewable sources (Abu-Sharkh et al., 2004). This low impact is attained by a good match between generation and load inside microgrids, even for faster time scale (seconds).

What is however less clear is the reward that would encourage a microgrid to behave in this way, especially in the case when the outside system is rather strong and the microgrid's total installed capacity is relatively small so that even in the worst case, sudden internal imbalances would have a rather low impact on the outside system. In the case of a significant number of microgrids in the system, their mutual and overall system impacts will indeed become significant, and "good citizen" behavior would not only be desirable but is also becoming a necessity. To the best of our knowledge, some more detailed elaboration of the overall system's operation in such a microgrids-based system has not yet been presented. Still, several references are addressing this topic. In (Dimeas and Hatziargyriou, 2005) the operation of a multiagent system for control of microgrids in a market environment is presented. Only the real power market has been considered, and the emphasis of the paper is on the details of an auction algorithm and the operation of microgrids market agents. In (Abu-Sharkh et al., 2004) the possibility of creating local ancillary services markets has been addressed as an important issue. Similarly (Kueck et al., 2003) refers to provision of ancillary services from microgrids as a future research need.

The goal of this chapter is to take the microgrid concept one step further. It presents the concept of autonomous power networks as a realistic approach to deal with increased complexity and uncertainty of the future power systems, while enabling markets-based operation for both dispatch (economic) and ancillary services (reliability).

For an autonomous power network internal control action, with the emphasis on voltage magnitude control, we refer to (Provoost et al., 2005b,a, 2007).

4.2 Autonomous power networks

An autonomous power network (AN) is an aggregation of networked producers and consumers in a relatively small area with respect to the overall power system, whose operation is coordinated/controlled with one central unit (AN market agent) acting as an interface in between internal producers/consumers and the rest of the power system, see Figure 4.1 and Figure 4.2. The goal of an AN market agent is efficient deployment of AN's internal resources and its active involvement in overall system competitive markets where it reflects the preferences of its owners, i.e. of its internal producers/consumers. In the physical as well as in the economical layer, i.e. in power and ancillary service markets, each AN is presented as one producer/consumer.

Although it shares many of the objectives and characteristics with the microgrid, an AN has an additional property: it is a major building block of a power system in all of its layers, i.e. physical, economic and reliability. The idea of the overall system being a network of ANs is the central idea of the AN concept. It is the requirement for active involvement of an AN in all of the layers of the system, that defines an AN as a major building block of the system. For power balance and reliability issues (ancillary services) this implies that AN is obliged to provide, in an appropriate form, the information of its own actions and to take the responsibility for these actions. Note that the "good citizen" behavior, if defined as within the microgrid concept, implies high uniformity of microgids in some of their characteristics, e.g. each microgrid has well controlled power exchange with the rest of the network, even for the time scale of seconds. In the AN concept, there are no such *a priori* characterizations (constraints), and different ANs can have significantly different characteristics. However, it is crucial that all these, possibly different and time-varying characteristics are taken into account on the global level, so that the overall system will operate efficiently and reliably.

Good citizen behavior for an AN is defined by the requirement for its active and responsible involvement in the energy and ancillary service markets, as well as for following a set of pre-defined rules in provision of the commodities sold in these markets. Energy and ancillary services markets provide global coordination and time synchronization of ANs actions. These markets have a central role in keeping the power balance between ANs and in ensuring the overall system's reliability by accumulating sufficient levels of ancillary services, e.g. regulation capacity, spinning, non-spinning or operational reserves, etc. As presented in the next section, ANs can be both

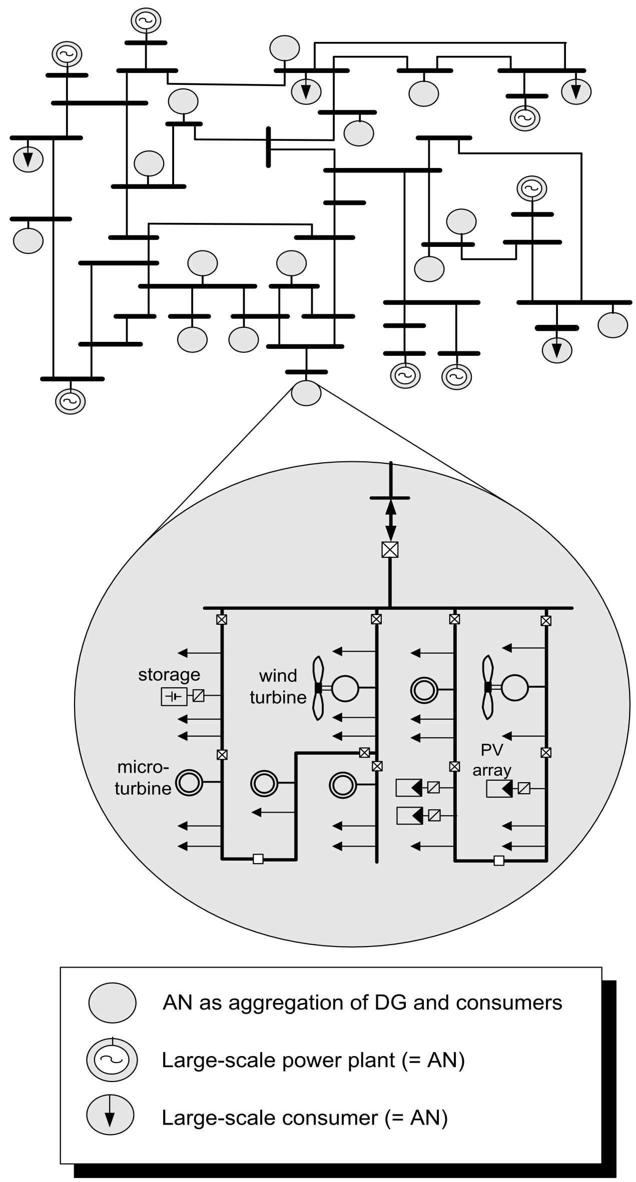

Figure 4.1: Autonomous power networks.

producers and consumers of ancillary services. This is a novel and unique feature, and is in line with the driving forces for the power system restructuring, since it introduces a large amount of well-defined players in ancillary service markets. In AN based power systems, spare capacities for ancilla-

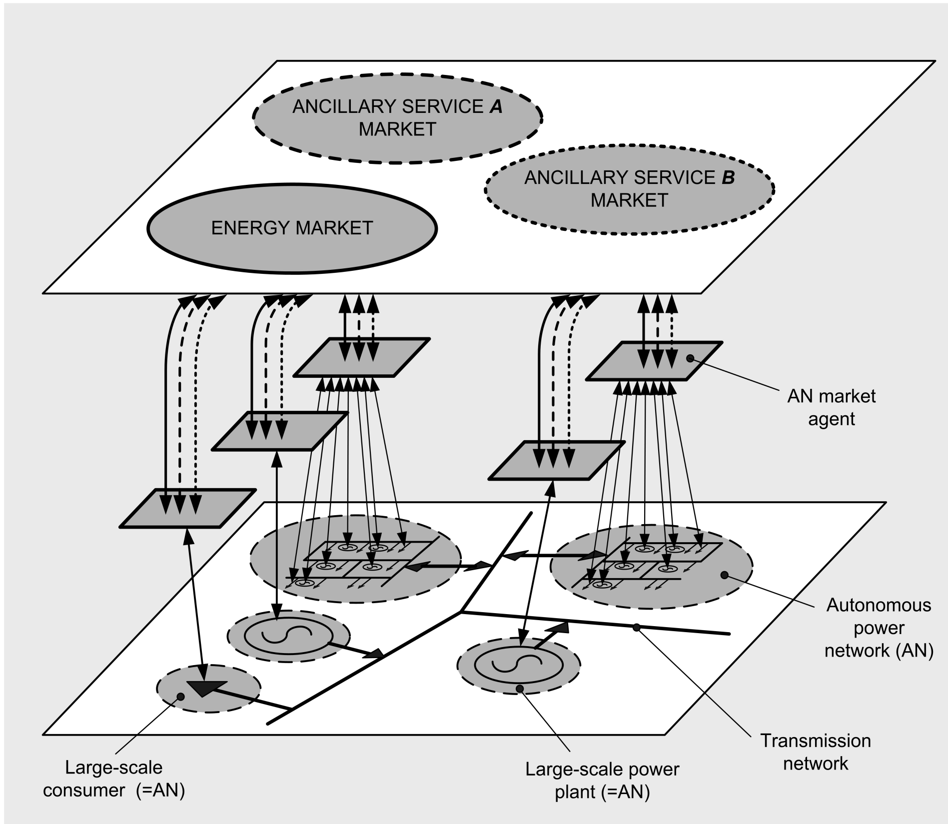

Figure 4.2: Operational structure of a power system based on autonomous networks.

ry services are treated in an equivalent way as energy is treated in energy markets. As a result, the operational structure of ancillary service markets becomes analogous to energy market. With the increase of uncertainties in the future power systems, e.g. due to the increased penetration of renewable sources, it is expected that the value of transactions in these markets will increase as well.

To summarize, an AN is seen as an intrinsically local co-operative venture. Each of its internal members accepts the AN, presented by the AN market agent, as its operational authority and in return shares the benefits of this co-operation. The AN market agent is therefore the highest authority for these members. Each unit (producer/consumer) can also act as an isolated player, but it is then required to take responsibilities in all layers of the system, i.e. it is presented in the overall system as an AN. This especially holds for large-scale synchronous generator based producers or large-scale consumers, e.g. large factories, that are by themselves capable of efficient involvement in markets. For price inelastic consumer inside some AN (e.g. residential loads), AN becomes a market agent acting on their behalf. Furthermore, for those consumers, the AN offers a possibility of their efficient involvement in ancillary service markets if they agree on a certain level of interruptions in power supply.

In contrast to the micrograms, which are usually associated with a "small community", like for instance typical housing estate, isolated rural communities, academic or public communities such as universities and schools, commercial areas or trading estates, what implicitly defines a microgrid as a very small cell of a power system, we do not restrict to any particular sizes of networks candidates for AN. Possible different sizes of coexisting ANs are a result of their efficiency in a competitive market environment. For existence of efficient overall markets, it is important that large amounts of ANs are mutually interconnected. A well-meshed topology of transmission networks fits well for this purpose. In this network topology the candidate areas for ANs are all medium and low voltage networks with aggregated DG units and corresponding loads. An illustration of New England system as an AN-based system is presented in Figure 4.1.

4.3 Power balance and reliability

In this section we formulate the optimal power balance and reliability problem of a power system as a constrained optimization problem. The concept of autonomous power networks is formally presented as a decomposition based method for solving this optimization problem. The optimization problem is decomposed into a set of subproblems where each subproblem is assigned to one autonomous network. Coordination and time synchronization among subproblems, i.e. among ANs, is performed through power and ancillary services markets.

Ancillary services and reliability

By treating ancillary services, e.g. regulation capacity, spinning, non-spinning or operational reserves, etc., as market commodities, the objectives of the corresponding markets are to ensure a required reliability level of the power system. This required level of reliability is defined by the required total accumulated amount of each commodity. For instance, hourly spinning reserve requirements are usually defined to be greater than a fixed percentage of total forecast demand or the largest on-line unit.

To simplify mathematical formulations we will consider only one ancillary service and denote it with A. However, extension to include more different types of ancillary services is straightforward. Generally, with A we denote a capacity that is available as a power injection (positive or negative) in the system within some specified time interval. The system's required reliability level at some time t ($t \geq 0$) is given by

$$\sum_i A_i(t) \geq A^{req}(t), \tag{4.1}$$

where $A_i(t)$ is the available capacity from unit (producer/consumer) i, and $A^{req}(t)$ is the required capacity that has to account for an uncertainty in power production/demand, and can be different for a different t.

Suppose that for a certain autonomous network n it is required to have the amount $A_n^{req}(t)$ of the ancillary service capacity to ensure reliable operation of the AN. Then, the reliability condition of an autonomous network n is given by

$$\sum_{i=1}^{m_n} A_{n,i}(t) - A_n^{ex}(t) \geq A_n^{req}(t), \tag{4.2}$$

where $A_{n,i}(t)$ is the capacity [W] available from unit i inside AN, m_n is the total number of units inside the autonomous network n, $\sum_{i=1}^{m_n} A_{n,i}(t)$ is the total capacity available in the AN, and $A_n^{ex}(t)$ is the capacity that the AN sells to the outside system, i.e. to other ANs.

For simplicity, further in the text we will leave out time dependence in the notation, although we consider all the quantities being time dependent.

Note that if A_n^{ex} is negative, the AN is buying this amount of the ancillary service capacity from the outside system. Suppose that the overall power system consists of N autonomous networks. Then, from (4.2), we can write

$$\sum_{n=1}^{N}\sum_{i=1}^{m_n} A_{n,i} - \sum_{n=1}^{N} A_n^{ex} \geq \sum_{n=1}^{N} A_n^{req}. \tag{4.3}$$

Since the overall system is a network of ANs, i.e. each consumer or producer is necessarily within some AN, we can take $A^{req} = \sum_n A_n^{req}$, i.e. required overall system capacity is sum of required capacity levels for each AN. From

(4.3), to ensure the reliability of the overall system, the task of the corresponding ancillary service market is to achieve the balance of ancillary service capacities among ANs, i.e. to ensure the following equality

$$\sum_{n=1}^{N} A_n^{ex} = 0. \tag{4.4}$$

Note that with the above formulation, capacity from ancillary services is traded in an equivalent way as electrical energy is traded in today's markets. All ANs determine their own required level of spare capacity, i.e. A_n^{req}, all ANs are producers and *consumers* of spare capacity, and the overall balance has to be achieved. This is in contrast with today's power systems where the required capacity for system's reliability is determined by an independent system operator (ISO). An ISO therefore presents the only demand for ancillary service capacities, i.e. in today's power systems ISO is the only "consumer" of ancillary services.

In AN-based power system, attaining the required overall system reliability level becomes a decentralized decision and each AN contributes. This decentralization is desirable due to the overall system's complexity and its large scale, as for an AN it is easier to assess its internal required capacity level based on local predictions and on generally better insight into its internal situation.

Power balance

The power balance within an autonomous power network n is given by

$$\sum_{i=1}^{m_n} P_{n,i} - P_n^{ex} = L_n, \tag{4.5}$$

where $P_{n,i}$ is the power injection from unit i in an autonomous power network n, $\sum_{i=1}^{m_n} P_{n,i}$ is the total power produced inside the AN, L_n is the total internal load of the AN, and P_n^{ex} is power that the AN sells to the outside system. If P_n^{ex} has negative value, the AN buys power from the outside system. Obviously, from (4.5), the goal of the real power market is to ensure the following equality

$$\sum_{n=1}^{N} P_n^{ex} = 0, \tag{4.6}$$

i.e. to ensure the power balance among ANs.

Coupling between power and ancillary services

Since power and ancillary services are provided by the same set of units, there is a coupling in between these commodities. Naturally, if a generating unit operates on its maximal power output, it has no capacity left for additional (positive) power injection into the network, i.e. it can not perform the (up-regulating) ancillary service.

More precisely, let $P_{n,i}$ and $A_{n,i}$ denote the power and ancillary service capacity of a generating unit i in an autonomous network n, and let $P_{n,i}^{min}$ and $P_{n,i}^{max}$ denote the lower and upper production limit of the unit, respectively. Furthermore, suppose that the ancillary service A is defined as the capacity for both up and down regulation in power production, i.e. as the capacity which is available as both positive and negative power injection within a predefined time interval upon the request. Then the following constraints hold

$$P_{n,i}^{min} \leq P_{n,i} \leq P_{n,i}^{max}, \qquad i = 1, \ldots, m_n, \tag{4.7a}$$

$$0 \leq A_{n,i} \leq \min(A_{n,i}^{max}, P_{n,i} - P_{n,i}^{min}, P_{n,i}^{max} - P_{n,i}), \quad i = 1, \ldots, m_n, \tag{4.7b}$$

where $A_{n,i}^{max}$ is the absolute value of the increment in power production that the corresponding unit can achieve within the predefined time interval.

The constraints (4.7b), which ensure that the working point of a production unit is sufficiently distanced from the saturation limits of the unit, couple the power and ancillary service capacity.

Social welfare maximization problem

In addition to the fulfilment of the power balance and reliability conditions, the operational goal of a power system is to perform this task in an optimal way. Here, optimality is defined through the total social welfare of the system.

For simplicity of the presentation, we will not consider the benefit functions of consumers, i.e. we will assume that the social welfare maximization problem is equivalent to the total cost minimization problem. This is without any loss of generality and inducing the consumption side in the problem is straightforward.

With the abbreviations $P_n = \mathrm{col}(P_{n,1}, \ldots, P_{n,m_n})$, $A_n = \mathrm{col}(A_{n,1}, \ldots, A_{n,m_n})$, $i = 1, \ldots, N$ and $P = \mathrm{col}(P_1, \ldots, P_N)$, $A = \mathrm{col}(A_1, \ldots, A_N)$, $P^{ex} = \mathrm{col}(P_1^{ex}, \ldots, P_N^{ex})$, $A^{ex} = \mathrm{col}(A_1^{ex}, \ldots, A_N^{ex})$, the cost problem for the overall power system is defined as follows:

$$\min_{P,A,P^{ex},A^{ex}} \quad \sum_{n=1}^{N} f_n(P_n, A_n) \tag{4.8a}$$

subject to

$$\sum_{i=1}^{m_n} P_{n,i} - P_n^{ex} = L_n, \qquad n = 1, \ldots, N \tag{4.8b}$$

$$\sum_{i=1}^{m_n} A_{n,i} - A_n^{ex} \geq A_n^{req}, \quad n = 1, \ldots, N \tag{4.8c}$$

$$g_n(P_n, A_n) \leq 0, \qquad n = 1, \ldots, N \tag{4.8d}$$

$$\sum_{n=1}^{N} A_n^{ex} = 0, \tag{4.8e}$$

$$\sum_{n=1}^{N} P_n^{ex} = 0, \tag{4.8f}$$

where $f_n(P_n, A_n) := \sum_{i=1}^{m_n} f_{n,i}(P_{n,i}, A_{n,i})$ denotes the aggregated costs of an autonomous network n for producing P_n and A_n ($f_{n,i}(P_{n,i}, A_{n,i})$ denotes the cost of unit i for producing $P_{n,i}$ and $A_{n,i}$). For compactness, the constraints (4.7) are represented in (4.8) by the generic constraints (4.8d). More precisely, for each $n \in \{1, \ldots, N\}$ the constraint $g_n(P_n, A_n) \leq 0$ stands for the set of constraints (4.7). Note that the constraints (4.8b), (4.8c) and (4.8d) are defined on the AN level, i.e. they are decoupled for each AN, while (4.8e) and (4.8f) are the coupling constraints among ANs.

For all the considerations in this section, we assume that the optimization problem (4.8) is convex.

The solution of the optimization problem (4.8) defines the optimal operating point for the overal power system and corresponds to the operating point that would be achieved with efficiently designed power and ancillary services markets under the assumption of perfect competition. So, the above formulated optimization problem implicitly defines the *ideal* system operation that the market-based system strives for.

Dual decomposition

To present a market-based solution to the social welfare maximization problem (4.8), we use the Lagrangian relaxation and we dualize only the coupling constraints (4.8e) and (4.8f). The partial Lagrangian is then given by

$$\mathcal{L}(P, A, P^{ex}, A^{ex}, \lambda^P, \lambda^A) = \sum_{n=1}^{N} f_n(P_n, A_n) - \lambda^P \sum_{n=1}^{N} P_n^{ex} - \lambda^A \sum_{n=1}^{N} A_n^{ex}, \quad (4.9)$$

where λ^P and λ^A denote the Lagrange multipliers, and the Lagrange dual problem is given by

$$\max_{\lambda^P, \lambda^A} \ell(\lambda^P, \lambda^A), \quad (4.10)$$

where $\ell(\lambda^P, \lambda^A)$ is the dual objective function, which is defined as follows

$$\ell(\lambda^P, \lambda^A) := \min_{P, A, P^{ex}, A^{ex}} \Big\{ \mathcal{L}(P, A, P^{ex}, A^{ex}, \lambda^P, \lambda^A) \quad \text{subject to} \quad (4.8\text{b}), (4.8\text{c}), (4.8\text{d}) \Big\}. \quad (4.11)$$

Note that for fixed λ^P and λ^A, the minimization problem in (4.11) can be decomposed in N subproblems, where each subproblem is assigned to one AN only. For an autonomous network n, this subproblem is given by

$$\min_{P_n, A_n, P_n^{ex}, A_n^{ex}} \quad f_n(P_n, A_n) - \lambda^P P_n^{ex} - \lambda^A A_n^{ex} \quad (4.12\text{a})$$

subject to

$$\sum_{i=1}^{m_n} P_{n,i} - P_n^{ex} = L_n, \quad (4.12\text{b})$$

$$\sum_{i=1}^{m_n} A_{n,i} - A_n^{ex} \geq A_n^{req}, \quad (4.12\text{c})$$

$$g_n(P_n, A_n) \leq 0, \quad (4.12\text{d})$$

and it represents the benefit maximization problem of the AN in a market-based system with λ^P and λ^A corresponding to the current market prices for power and ancillary service.

For some given values of λ^P and λ^A, the minimizers $\tilde{P}_n^{ex}$ and $\tilde{A}_n^{ex}$ of the optimization problem (4.12) represent the optimal values of power and ancillary service capacity for exchange with the rest of the power system, i.e. with other ANs. In other words, with the given market prices λ^P and λ^A, the autonomous network n will gain a maximal profit if it buys (sells) the amount of power equal to $\tilde{P}_n^{ex}$ and if, at the same time, it buys (sells) the amount of ancillary service capacity equal to $\tilde{A}_n^{ex}$.

The maximizers $\tilde{\lambda}^P$ and $\tilde{\lambda}^A$ of the dual problem (4.10) correspond to the optimal market prices for power and ancillary service. For the optimal market prices it holds that $\sum_{n=1}^{N} \tilde{P}_n^{ex} = 0$ and $\sum_{n=1}^{N} \tilde{A}_n^{ex} = 0$, i.e. that the overall power system is in balance, and that the total social welfare of the system is maximized.

To summarize, market-based operation of an AN-based power system corresponds to the dual decomposition of the social welfare maximization problem (4.8). Each AN solves its own benefit maximization problem (4.12) as a function of spot prices λ^P and λ^A, and in summation all ANs define the dual objective function (4.11). The goal of the power and ancillary service markets is to solve the dual problem (4.10) by finding the optimal spot prices $\tilde{\lambda}^P$ and $\tilde{\lambda}^A$.

Before continuing with a more detailed presentation of market operation of AN-based power systems, note that in a more precise formulation the power balance constraint between ANs, i.e. the constraint (4.8f), is replaced by power flow equations of the transmission network which connects ANs. Furthermore, transmission line power flow limits are added as inequality constraints. In this case, the partial Lagrangian (4.9) and the dual problem (4.11) include the Lagrange multipliers corresponding to the power balance equalities in each node of the transmission network, as well as the Lagrange multipliers for power flow limit constraints in the lines of the transmission network. The result of such a formulation is that, at the optimum, each node in the transmission network, and therefore each AN, in general can have a different *nodal price* for electrical power. For more details on nodal pricing see Chapter 2 of this thesis.

It is also possible to account for transmission network limits in the pricing of ancillary services. The result is the introduction of nodal prices for ancillary services. For more details on nodal pricing of ancillary services we refer to (Chen et al., 2003; Alvarado, 2006).

4.3.1 Market operation

In this subsection we consider the operation of power and ancillary service markets of AN-based power systems in some more detail. We present two possible market operation structures: *bids-based markets* and *iterative markets*. Our main emphasis is on the trade-offs that each AN is facing in its desire to maximize its own benefit, which are induced by the coupling between power and ancillary services. Finally we address the need and the possibility of creating the markets that are operating in a receding horizon.

Bids-based markets

In present power systems, forward time power markets, e.g. day-ahead power markets, are bids-based markets. In such a market, producers submit power-price supply bids, while the consumers (or retailers) submit power-price demand bids. The market operator aggregates all the supply bids into the supply curve and all the demand bids into the demand curve. Intersection of the aggregated supply and demand curves defines the market price for electricity λ^P.

In AN-based power systems, bids-based operation of a power market is analogous to the operation of present power markets. The only difference is that the market operator does not make a distinction between supply and demand bids. More precisely, an autonomous network n submits a power-price bid $\bar{\lambda}_n^P(P_n^{ex})$ that generally ranges from negative (demand offer) to positive values (supply offer) of the power exchange P_n^{ex}, as each AN can be both producer and consumer of electrical energy. Each point (P_n^{ex}, λ^p) on a bid curve $\bar{\lambda}_n^P(P_n^{ex})$, see Figure 4.3, defines how much power P_n^{ex} the AN will to trade with the rest of the power system, if the market price is equal to $\lambda^p = \bar{\lambda}_n^P(P_n^{ex})$. The market operator aggregates the bids from all ANs into one curve and determines the market price $\tilde{\lambda}^P$ as the price for which the total aggregated curve fulfils the constraint (4.8f), i.e. as the price which clears the market by establishing the power balance among ANs. The bids-based operation of an AN-based power market is illustrated in Figure 4.3.

A novelty introduced by AN-based power systems is a unique and novel treatment of ancillary services. Each AN submits a capacity-price bid $\bar{\lambda}_n^A(A_n^{ex})$, which generally ranges from negative (demand offer) to positive values (supply offer) of the capacity exchange A_n^{ex}, as each AN can be both *producer and consumer* of ancillary service capacity. The market operator aggregates all the capacity bids into one curve, and determines the market price λ^A as the price that results in the capacity balance among ANs.

Note that in the bids-based markets, each AN is obliged to submit an *independent* bid to each of the markets, i.e. to the power and the ancillary service markets. Therefore, due to the coupling between power and ancillary services, an AN has to take the responsibility of determining its own trade-off between the prices and quantities of power and ancillary service capacities in the submitted bids. To be more precise on this issue, let us consider the AN's benefit maximization problem (4.12), where the set of inequality constraints (4.12d) includes the coupling constraints between power and ancillary service capacity, e.g. the constraints (4.7). We will say that the pair (P_n^{ex}, A_n^{ex}) is feasible if for those values the constraints (4.12b),(4.12c),(4.12d) are fea-

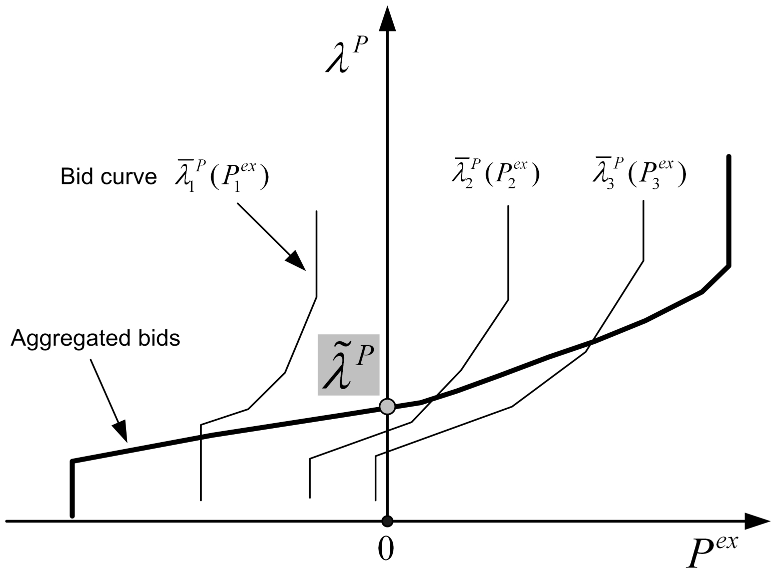

Figure 4.3: Operation of bids-based markets in an autonomous networks based system.

sible in P_n and A_n. We make the following definition in connection with the optimization problem (4.12):

$$J_n(P_n^{ex}, A_n^{ex}) := \min_{P_n, A_n} \big\{ f_n(P_n, A_n) \\ \text{subject to} \quad (4.12\text{b}), (4.12\text{c}), (4.12\text{d}) \big\}, \quad (4.13)$$

where we take $J_n(P_n^{ex}, A_n^{ex}) := \infty$ if (P_n^{ex}, A_n^{ex}) is not a feasible pair. Furthermore, we define the following mappings:

$$\tilde{\lambda}_n^P(P_n^{ex}, A_n^{ex}) := \frac{\partial J_n(P_n^{ex}, A_n^{ex})}{\partial P_n^{ex}}, \quad (4.14\text{a})$$

$$\tilde{\lambda}_n^A(P_n^{ex}, A_n^{ex}) := \frac{\partial J_n(P_n^{ex}, A_n^{ex})}{\partial A_n^{ex}}, \quad (4.14\text{b})$$

where for a fixed A_n^{ex}, $\frac{\partial J_n(P_n^{ex}, A_n^{ex})}{\partial P_n^{ex}}$ denotes a subgradient of J_n at P_n^{ex}, and the right hand side in (4.14b) is defined analogously. Note that $\tilde{\lambda}_n^P(P_n^{ex}, A_n^{ex})$ and $\tilde{\lambda}_n^P(P_n^{ex}, A_n^{ex})$ are defined only for a feasible pair (P_n^{ex}, A_n^{ex}). The value of $\tilde{\lambda}_n^P(P_n^{ex}, A_n^{ex})$ represents the incremental cost of the AN for a change in P_n^{ex}, while the value of $\tilde{\lambda}_n^A(P_n^{ex}, A_n^{ex})$ represents the incremental cost of the AN for a change in A_n^{ex}.

In perfectly competitive markets, an AN would bid its incremental costs to the power and ancillary service markets. However, the power incremental cost $\tilde{\lambda}_n^P(P_n^{ex}, A_n^{ex})$ depends on the value of A_n^{ex}, and to make a valid bid $\bar{\lambda}_n^P(P_n^{ex})$ to the power market, the AN market agent has to predict A_n^{ex} and, based on this prediction, to determine $\bar{\lambda}_n^P(P_n^{ex})$ from $\tilde{\lambda}_n^P(P_n^{ex}, A_n^{ex})$. Analogously, the AN market agent has to approximate the ancillary service incremental costs $\tilde{\lambda}_n^A(P_n^{ex}, A_n^{ex})$ to define a valid bid $\bar{\lambda}_n^A(A_n^{ex})$ for the ancillary service market. Even in the conditions of perfect competition, due to the necessary approximations, the bids-based markets in general do not guarantee the maximization of the total social welfare.

Iterative markets

It is possible do design power and ancillary service markets that, under the conditions of perfect competition, guarantee that the maximal social welfare is attained. Such markets are more complex, as they are based on iterative exchange of information between markets operators and market players. For completeness of the presentation we shortly address the possibility of this operational structure.

The operation of iterative markets corresponds directly to solving the social welfare optimization problem (4.8) by a dual decomposition method. For details of decomposition methods in optimization we refer to Chapter 6 of (Bertsekas, 1999). For an application of such an approach in power markets, see (Conejo and Aguado, 1998; Aguado and Quintana, 2001).

Suppose that at the beginning of the iterative process, a market operator announces the prices λ^P and λ^A. Then the operation of the iterative markets is based on iterations beween the following two steps:
1) Based on the announced prices λ^P and λ^A, each AN solves its benefit maximization problem and communicates the values of the corresponding minimizers $\tilde{P}_n^{ex}$ and $\tilde{A}_n^{ex}$ to the market operator.
2) The values $(-\sum_{n=1}^{N} \tilde{P}_n^{ex})$ and $(-\sum_{n=1}^{N} \tilde{A}_n^{ex})$ define a sub-gradient for the dual problem (4.10). Knowing this sub-gradient, the market operator makes an update of the prices λ^P and λ^A, using a subgradient, a cutting-plane, or some other method.

For a convex social welfare maximization problem (4.8), the above iterative algorithm converges to the optimum, i.e. optimal market prices are attained, which maximizes the social welfare of the system.

Receding horizon markets

The above presented general operational formulation of AN-based power systems is valid for any forward time markets. Usually forward time markets have a horizon of one day and result in a unit commitment solution for the overall system. For the AN-based power systems, we can still expect the existence of markets with day-ahead horizon, since for large-scale units this long horizon is necessary for solution of unit commitment. However, DG units have much shorter up and down minimum times, and can quickly be brought back on-line once they where shut down. Furthermore, increased uncertainties (renewables) make the future AN's state badly predictable. Therefore it might be beneficial, or even necessary, that an AN performs a dispatching and unit commitment in a receding horizon manner with a much shorter "sampling time", which is properly adjusted with respect to the AN's internal characteristics. To accommodate such operation of ANs, future time markets should operate in a receding horizon as well.

For illustration, suppose that all the markets operate in a receding horizon with the trading intervals of 30 minutes, and with the horizon of one day. Receding horizon operation means that all the markets are cleared each 30 min. Here, clearing the markets mean that the prices for power and all ancillary services are determined for each trading period in the horizon, i.e. for each half-hour interval, looking one day ahead from the current moment.

Now, suppose that some AN has a large amount of small DG units. To optimally exploit these internal resources, this AN can internally operate with a shorter "sampling time" than the sampling time of the markets, i.e. it can re-dispatch its resources, for example, each 10 minutes. However, since AN has to represent itself to the markets with the bids for power and ancillary services over the time intervals of 30 min, in performing the dispatch (and unit commitment), the AN market agent has to include additional set of equality constraints in its optimization problem. These equality constraints couple the blocks of three subsequent (*AN's internal*) sampling periods by requiring that P^{ex}, A^{ex}, B^{ex}, etc., are constant over these periods.

Note also that different ANs can have significantly different internal operational structure. For example, some ANs might operate based on *internal markets* (with different, internal sampling time) for power and ancillary services. In that case AN market agent acts as an ISO for its internal producers/consumers, and as a single player in the system-wide markets. On the other side, some ANs can have complete knowledge of the cost/benefit functions of their internal members and act (only) as a players in the system-wide markets.

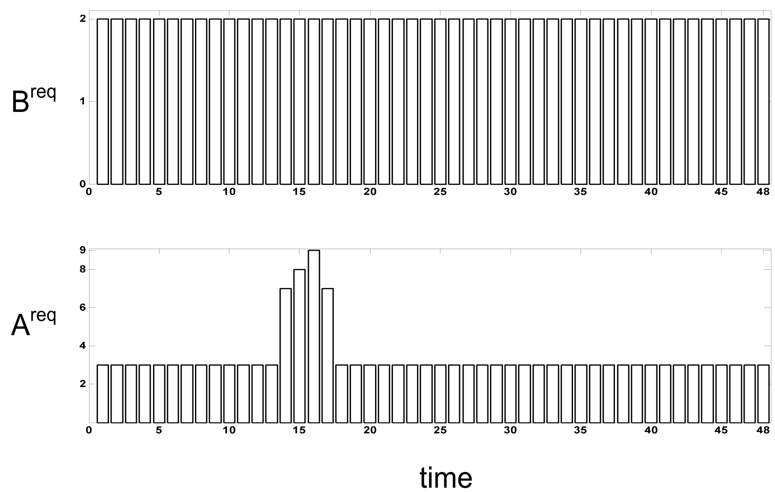

Figure 4.4: Required capacities B^{req} and A^{req} for AN's reliability.

To summarize, each AN can optimally determine the parameters, e.g. sampling time, of its internal operation, but in its operation it is responsible to comply with the system-wide integration "protocols", i.e. it has to submit the bids with the sampling time of the markets. This allows for co-existence of ANs that have significantly different internal operational characteristics and structure.

4.4 Example

The simulation of optimal AN dispatching in the day ahead markets, based on predicted spot prices for real power, and two ancillary services, named A and B, has been performed. The simulated AN consists of 8 controllable generating units. The considered trading period was half an hour. The required levels (B^{req}, A^{req}) of capacities for ancillary services are presented in Figure 4.4. Predicted spot prices of real power P and ancillary services A and B are presented in Figure 4.5, while Figure 4.6 presents the corresponding optimal values of P^{ex}, A^{ex} and B^{ex}.

The set of constraints that accounts for the coupling between real power and ancillary service capacity (i.e. corresponding to (4.12d) in the general

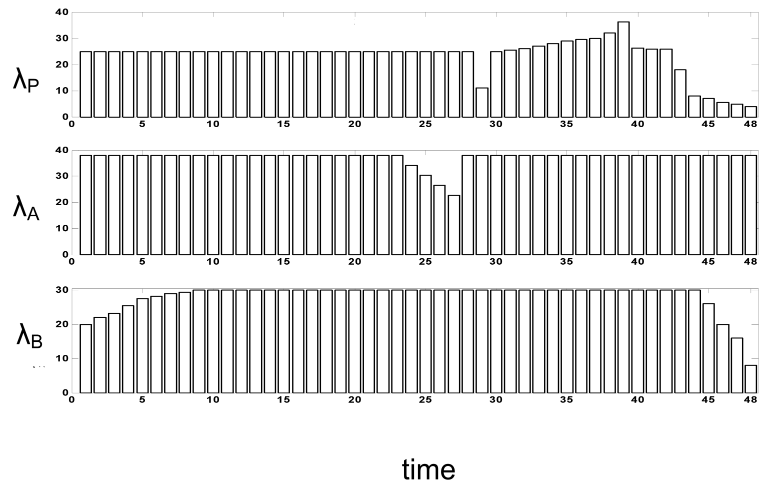

Figure 4.5: Market prices λ_P, λ_A and λ_B.

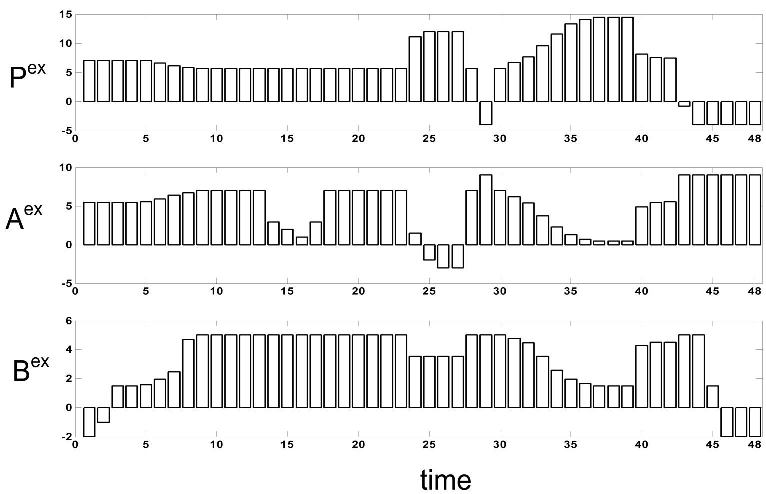

Figure 4.6: Optimal values of P^{ex}, A^{ex} and B^{ex}.

formulation), is taken in the following form:

$$0 \leq A_i \leq \min(A_i^{max}, P_i - P_i^{min}, P_i^{max} - P_i), \quad i = 1, \ldots, 8, \tag{4.15a}$$
$$0 \leq B_i \leq \min(B_i^{max}, P_i^{max} - P_i), \quad i = 1, \ldots, 8, \tag{4.15b}$$

i.e. the ancillary service A denotes the capacity available for both up and down movements in power production, while the ancillary service B denotes the (slower) available capacity for up movements in power production (power injection). The values of prices, internal AN's loads, and production from uncontrollable generators, e.g. wind turbines, are chosen in such a way that the trade-offs induced by coupling of commodities (P, A, and B) are easily observed, rather than taken to have some realistic daily profile. Internal loads, as well as the production from uncontrollable generators, are taken to be constant. Costs for power production of controllable units are taken to be quadratic functions of produced power.

From the obtained simulations, we can easily observe the coupling between the commodities. For example, the increase of the price λ_B during the trading periods $1 - 9$ resulted in the increase of B^{ex}, ranging from the negative to the positive values of B^{ex}. With the increase of B^{ex}, P^{ex} is decreased. Loosely speaking, this is because of the following reason: to allocate more spare capacity B, according to (4.15b), the generating units necessarily need to operate in a restricted operating range, what puts the limits on the maximal value of P^{ex}. By restricting their operating range according to (4.15b), the same generating units now as well have an opportunity to sell more capacity A, see (4.15a), and as a result we can observe the increase of A^{ex}, although the price λ_A is constant over these trading periods.

Similar effects of the coupling between the commodities can as well be observed in the remaining $(10 - 48)$ trading periods.

4.5 The autonomous power networks concept: benefits and challenges

In the previous sections we have introduced the autonomous power networks as main building blocks of the *operational* structure of a power system. The ANs were represented as an *intermediate* layer in the hierarchy of the system's operational structure, see Figure 4.2. More precisely, the "ANs layer" stands between the lowest layer, represented by *atomic* producers/consumers, e.g. single power/capacity units, and the highest layer, which corresponds to the power and ancillary service markets. One may argue about the necessity and the added value of introducing this intermediate layer in the operational structure of a power system. Furthermore, one may argue that the ANs are already present in today's power systems as control areas (which in Europe often correspond to national borders) and the corresponding system operators. We have left the discussion of these issues for the final section in the chapter, as the subjects presented in the previous sections form the basis for such discussions.

In the following subsections we discuss the benefits of the proposed AN concept with respect to its implications on dealing with increased uncertainties, the large-scale and complexity of the power system, and on its support of competitive markets.

4.5.1 Uncertainties and reliability

In today's power system, the independent system operator (ISO) determines the required amounts of ancillary services for each trading period. For each ancillary service, this task is performed based on the corresponding reliability standards and regulations. For example, the standards require that the amount of spinning reserve has to be greater than the largest committed unit, as it has to account for its possible loss. This reserve requirement is known as the $(N-1)$ reliability criterion and is based on the rule that all loads have to be served at all times without interruption. As another example, the required capacity for automatic generation control is correlated to the aggregated uncertainty and/or predicted rates of change in power demand.

In contrast to this, in the ANs based power systems each AN determines its own required amount of ancillary service capacities over each time period. Such decentralization of (parts of) ISO services carries both advantageous and disadvantageous implications for the system operation. Our emphasis is therefore on the need of finding the optimal trade-offs in between the two.

It is well known that the uncertainty in the prediction of power production/consumption of an aggregated set of producers/consumers is smaller as the considered set is larger. This is the effect of aggregation of partially or fully uncorrelated effects. Having in mind this positive effect of aggregation, it is preferable to rely on a highly centralized operational structure with one central authority, e.g. ISO, responsible for determining the required amounts of ancillary services for the complete power system. With access to the system-wide information, the central authority can adequately comply to the $(N-1)$ reliability criterion. Therefore, a crucial question is the following: what are the driving forces to base operation of the system on the ANs, which correspond to the aggregated *subsets* of consumers/producers and are therefore characterized by larger relative uncertainties than those on the overall system level?

Spatial dimension of reliability

One part of the answer to the above question is in the spatial character of a power system and its reliability. Due to finite transmission network capacity and possible outages of transmission lines, to ensure and to increase reliability of power supply, power production and ancillary services need to be distributed across the system. If the operation of the system would be based on one central authority, this authority would therefore necessarily need to keep track of *spatial distribution of uncertainties* and consequently of the spatial distribution of ancillary services. It is this spatial dimension of uncertainties that puts the limits on the positive effect that one can gain from the aggregation of uncertainties. For example, there is a little benefit of having less uncertain predictions for the aggregated European power system, if one has to account for its spatial dimension. Consequently, there is a little benefit of having one European ISO, in comparison with a set of *efficiently cooperating* ISOs, each acting upon "local" predictions. The latter describes the actual operational structure of today's European system, while increase of the cooperation efficiency is one of the central research topics for many years now.

We can conclude that the ANs are already present in today's power systems in the form of control areas (which in Europe often correspond to national borders) and the corresponding system operators. They are an *intermediate*, decomposition/coordination based layer of the operational structure of European power system. Loosely speaking, we can see the concept of ANs as further refinements (decomposition) of this already present structure. Introduction of large amounts of distributed generation (DG) is taking place in virtually all geographical ares and not only on the transmission network level, but also on-side to the loads in the medium and low voltage (distribution) networks. The system is going through the transformation from vertically structured into the horizontally structured system. There is no longer a clear division between "top" production end of the system, and the "bottom" consumption end. Further refinement of already present ANs (control areas) can be justified by a tendency to reach the operational structure that will in the most practical way facilitate the optimal exploitation of mixed distribution of consumers and producers in a horizontal power system, and to facilitate the *local character* of DG by enhancing *local* reliability.

To summarize, *one of the driving forces for introduction of ANs is to enhance local reliability of power supply by optimally exploiting distributed generation*. Note that this is also one of the central ideas in the microgrid concept (Lasseter, 2002; Venkataramanan and Illinadala, 2002). However, as already argued in the introduction of this chapter, a microgrid is a rather extreme step in this direction, with an unclear formulation from the overall system point of view. In the ANs concept, ANs represent building blocks of an operational structure, which is tailored to optimally account for the trade-offs between local security and reliability of supply on one side and complete reliance on the benefits of large-scale networking on the other side.

Note that in future power systems with a large amount of relatively small producers, there might be a need for revision of requirements for $(N-1)$ reliability criterion. Suppose that at a certain time instant, there are no large-scale production units committed, and virtually complete demand is balanced by DG. Apparently, it such a situation it is a challenging task to fined a well-grounded rules for defining the (equivalent to) $(N-1)$ reliability criterion.

Double-sided competition for ancillary services

The next important driving force for introduction of ANs is the desire to impose the the following principle: the system operation is based on such rules (protocols) that create the incentive for reducing the uncertainties in the system. A first step towards the possibility of having such operational protocols is that the basic building elements of the system are capable of keeping track of their contribution to the uncertainty in the overall system, and are capable of bearing the responsibility for it. The protocols then have to adequately define those responsibilities for an element in the system. Here "adequately" means that the protocols will indeed result in the desired incentives.

For example, consider the operational structure where renewable energy sources, e.g. wind turbines, pay the ISO certain fixed amount to account for the uncertainties in their production over some relatively long time horizon, e.g. month or year. Here, by *relatively* long time horizon we mean *relative* to the time scale present in the definition of each particular ancillary service. Furthermore, suppose that these wind turbines are not required to define and announce their day-ahead production profile, but the ISO is responsible to account for this profile by adding to the uncertainty in predictions and by adequately increasing the required amount of ancillary services. Obviously, this operational structure provides no incentive to reduce the uncertainties. Providers of ancillary services only have to satisfy the demand, but have no influence on it, while the wind turbines bear no further responsibility for their actions. It is important to note that the wind turbines have a technical possibility (although limited) of reducing its uncertainties and even of participating in frequency control or acting as a spinning reserve, see (Kristoffersen, 2005; Verhaegen et al., 2006) for more details.

Now suppose that a set of wind turbines acts as an autonomous power network, and is therefore required to comply to the set of predefined system integration protocols. This set of turbines is then required to keep track of its own uncertainty by estimating its own $A^{req}(t), B^{req}(t)$, etc., and it is responsible to ensure that at each time instant these amounts of ancillary services will be allocated in the system. By accepting this responsibility (and the accompanying penalties), this AN has an incentive to estimate $A^{req}(t), B^{req}(t)$, etc., as accurately as possible. Furthermore, depending on the current market situation, by taking appropriate internal actions, this AN now has an incentive to reduce its own uncertainties on the point of interconnection with the reset of the power system.

The benefit of the ANs concept is the introduction of the double-sided competitive markets for ancillary services, i.e. markets which are characterized with large amount of players on both supply and demand side. Competition and responsibility of delivering the treated commodities, create the incentive for reduction of uncertainties in the system. The ANs are well defined players in such a system, as it is reasonable to demand that they bear the responsibility for of their own actions. Note that for the latter issue, the size of an ANs plays a crucial role. It is not reasonable to expect that each small, single "atomic" consumer, e.g. a household, complies to all the integration protocols and is held responsible for all its action. This is so for the following two reasons. Firstly, such requirements would put serious constraints on the behavior of the consumer, which would then no longer enjoy simplicity of the "plug and play" principle that today's power systems provide. Secondly, if a consumer desire to use the network with plug and play simplicity, it would be required to buy the ancillary services to account for almost its complete installed capacity. In other words, if a small consumer is held responsible to ensure its own reliability, such a system would almost completely disregard the benefits of aggregation.

To summarize, in forming the ANs as the building blocks of the system, there are different opposing forces concerning the sizes of the coexisting ANs. On one side, to exploit the benefits of aggregation of uncertainties, one desires an AN to be as large as possible subsets of the overall power system. However, to cope with the spatial dimension of the system, there is a limit from the benefits that one can obtain from this aggregation by increasing the size of ANs. On the other side, to create the incentives for dealing with the uncertainties in the system by facilitating competition, one would desire a large amount of players capable for competing in all markets. Obviously, there is a trade-off between those opposing forces. In the optimal scenario ANs should be characterized as both competitive and thrust worth parters.

Finally, there is an additional crucial driving force for introduction of ANs and that is the necessity of operational decomposition to deal with the large-scale and complexity of the overall system. This is especially important for the unit commitment problem, which is not scalable. It is impossible to solve this problem for a large amount of units in some reasonable time period. An AN offers the unit commitment service for its internal members. With the coordination and time synchronization of ANs through the system-wide markets, the unit commitment problem is efficiently solved for large systems.

4.6 Conclusions

Due to increased uncertainties, reliable and economically optimal provision of ancillary services will become increasingly important in future power systems. Price inelasticity of customers is one of the biggest flaws of the present electricity markets. Active involvement of consumers in both energy and ancillary service markets, is an issue with the largest, yet unexploited, potential for increasing efficiency of electrical power systems.

In this chapter we have presented the concept of *autonomous power networks* as a concise formulation to deal with economic, technical and reliability issues in future power systems characterized by a large amount of distributed generation units. The autonomous power network (AN) was defined as an aggregated set of producers and consumers, which is capable and responsible for complying to the set of integration protocols required for efficient and reliable operation of the overall power system. In other words, the AN was presented as a major building block of a power system in all of its layers, i.e. physical, economic and reliability.

Specifically, we have introduced the AN as a new market entity that enables creation of competitive markets for ancillary services which are characterized by large amount of players on both supply and consumption side. Each AN is presented to the rest of the system as both potential *producer* and *consumer* of electrical energy and ancillary services. This is in contrast with the present power systems, where the independent system operator coordinates the ancillary service markets and at the same time provides the only demand for them. In AN based power systems, the independent system operator only acts as a coordinator of markets.

Furthermore, ANs enable active involvement of virtually all consumers in energy and ancillary service markets. Each dealing with a limited set of consumers, ANs present both technical infrastructure and market support for a small consumer, e.g. residential loads, in its integration to the markets.

5

Constrained steady-state optimal control

5.1 Introduction

In many production facilities, the optimization problem reflecting economical benefits of production is associated with a *steady-state operation* of the system. The control action is then required to maintain the production in an optimal regime in spite of various disturbances, and to efficiently and rapidly respond to changes in demand, while settling the system in a new steady-state that is optimal for novel conditions. The vast majority of control literature is focused on regulation and tracking with respect to known setpoints or trajectories, while coping with different types of uncertainties and disturbances in both plant and its environment. Typically, setpoints are determined off-line by solving an appropriate optimization problem, and they are updated in an open-loop manner. The optimization problem typically reflects variable costs of production and economical benefits under the current market conditions, e.g. fuel or electricity prices, and accounts for physical and security limits of the plant. In this chapter we reformulate the problem of *real-time economic optimization* as a *feedback control problem*.

If a production system is required to follow a time-varying demand in real-time, e.g. if produced commodities cannot be efficiently stored in large amounts, it becomes crucial to perform economic optimization on-line. Furthermore, in such systems, increase of the frequency with which the economically optimal setpoints are updated can result in a significant increase of economic benefits accumulated in time. If the time-scale on which economic optimization is performed approaches the time-scale of the underlying physical system, i.e. of the plant dynamics, dynamic interaction in between the two has to be considered. Economic optimization then becomes a challenging control problem, even more since it has to cope with inequality constraints that reflect the physical and security limits of the plant.

In this chapter we present a feedback control design procedure as a solution to the problem of regulating a general linear time-invariant dynamical system to a time-varying economically optimal operating point. The considered dynamical system is characterized with a set of exogenous inputs as an abstraction of time-varying loads and disturbances acting on the system. Economic optimality is defined through a convex constrained optimization problem with a set of system states as decision variables, and with the values of exogenous inputs as parameters in the optimization problem.

The results of this chapter present a formalization, generalization and extension of the methods we have used to design a price-based power balance and congestion management controller in Chapter 2.

Nomenclature

The field of real numbers is denoted by $\mathbb{R}$ and $\mathbb{R}^{m\times n}$ denotes m by n matrices with elements in $\mathbb{R}$. For a matrix $A \in \mathbb{R}^{m\times n}$, $[A]_{ij}$ denotes the element in the i-th row and j-th column of A. For a vector $x \in \mathbb{R}^n$, $[x]_i$ denotes the i-th element of x. Given $A \in \mathbb{R}^{m\times n}$ and two sets $I \subseteq \{1,\dots,m\}$ and $J \subseteq \{1,\dots,n\}$, we write $[A]_{I\bullet}$ to denote a submatrix of A formed by rows I of A, and with $[A]_{\bullet J}$ we denote a submatrix of A formed by columns J of A. The transpose of a matrix A is denoted by $A^\top$. $\operatorname{Ker} A$ and $\operatorname{Im} A$ denote the kernel and the image space of A, respectively. We use I_n to denote an identity matrix of dimension n, and when the dimension is clear from the context we often use I. A vector $x \in \mathbb{R}^n$ is said to be nonnegative (nonpositive) if $[x]_i \geq 0$ ($[x]_i \leq 0$) for all $i \in \{1,\dots n\}$, and in that case we write $x \geq 0$ ($x \leq 0$). The nonnegative orthant of $\mathbb{R}^n$ is defined by $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n \mid x \geq 0\}$. $\mathcal{L}^n_1(\mathbb{R}_+)$ denotes the space of all measurable functions $g : \mathbb{R}_+ \to \mathbb{R}^n$ which satisfy $\int_0^\infty \|g(t)\|_1 dt < \infty$, where for $x \in \mathbb{R}^n$, $\|x\|_1 := \sum_{i=1}^n |[x]_i|$. The operator $\operatorname{col}(\cdot,\dots,\cdot)$ stacks its operands into a column vector, and $\operatorname{diag}(\cdot,\dots,\cdot)$ denotes a square matrix with its operands on the main diagonal and zeros elsewhere. In the Euclidian space $\mathbb{R}^k$ the standard inner product is denoted by $\langle\cdot,\cdot\rangle$ and the associated norm is denoted by $\|\cdot\|$. For $u, v \in \mathbb{R}^k$ we write $u \perp v$ if $\langle u, v\rangle = u^\top v = 0$. We use the compact notational form $0 \leq u \perp v \geq 0$ to denote the complementarity conditions $u \geq 0$, $v \geq 0$, $u \perp v$. The matrix inequalities $A \succ B$ and $A \succeq B$ mean A and B are Hermitian and $A - B$ is positive definite and positive semi-definite, respectively.

For a scalar-valued differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, $\nabla f(x)$ denotes its gradient at $x = \mathrm{col}(x_1, \ldots, x_n)$ and is defined as a *column vector*[1], i.e. $\nabla f(x) \in \mathbb{R}^n$, $[\nabla f(x)]_i = \frac{\partial f}{\partial x_i}$. For a vector-valued differentiable function $f : \mathbb{R}^n \to \mathbb{R}^m$, $f(x) = \mathrm{col}(f_1(x), \ldots, f_m(x))$, the Jacobian at $x = \mathrm{col}(x_1, \ldots, x_n)$ is the matrix $Df(x) \in \mathbb{R}^{m\times n}$ and is defined by $[Df(x)]_{ij} = \frac{\partial f_i(x)}{\partial x_j}$. For a vector valued function $f : \mathbb{R}^n \to \mathbb{R}^m$, we will use $\nabla f(x)$ to denote the transpose of the Jacobian, i.e. $\nabla f(x) \in \mathbb{R}^{n\times m}$, $\nabla f(x) \triangleq Df(x)^\top$, what is consistent with the gradient notation ∇f when f is a scalar-valued function.

With a slight abuse of notation we will often use the same symbol to denote a signal, i.e. a function of time, as well as possible values that the signal may take at any time instant.

5.2 Problem formulation

In this subsection we formally present the constrained steady-state optimal control problem considered in this chapter. Furthermore, we list several standing assumptions, which will be instrumental in the subsequent sections.

Consider an LTI system Σ described by a state-space realization

$$\begin{aligned}\dot{x} = \begin{pmatrix}\dot{x}_p \\ \dot{x}_q\end{pmatrix} &= \begin{pmatrix}A_{pp} & A_{pq} \\ A_{qp} & A_{qq}\end{pmatrix}\begin{pmatrix}x_p \\ x_q\end{pmatrix} + \begin{pmatrix}F_p \\ F_q\end{pmatrix} w + \begin{pmatrix}B_p \\ B_q\end{pmatrix} u, \\ &\triangleq Ax + Fw + Bu, \end{aligned} \tag{5.1a}$$

$$y = \begin{pmatrix}I & 0\end{pmatrix}\begin{pmatrix}x_p \\ x_q\end{pmatrix} \triangleq Cx, \tag{5.1b}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^{n_w}$ is an exogenous input and $y(t) \in \mathbb{R}^m$ is the measured output. The state x is partitioned into $x_p \in \mathbb{R}^m$ and $x_q \in \mathbb{R}^{n-m}$, inducing the corresponding partitioning of the matrices $A \in \mathbb{R}^{n\times n}$, $F \in \mathbb{R}^{n\times n_w}$, $B \in \mathbb{R}^{n\times m}$ as indicated in (5.1a).

With $W \subset \mathbb{R}^{n_w}$ denoting a known bounded set and for a constant $w \in W$, consider the following convex optimization problem associated with the

[1]Both possible definitions, i.e. $\nabla f(x)$ as a row vector or $\nabla f(x)$ as a column vector, are commonly found in literature.

partial state vector x_p of the dynamical system (5.1):

$$\min_{x_p} \quad J(x_p) \tag{5.2a}$$

subject to

$$Lx_p = h(w), \tag{5.2b}$$

$$q_i(x_p) \leq r_i(w), \quad i = 1, \ldots, k, \tag{5.2c}$$

where $J : \mathbb{R}^m \to \mathbb{R}$ is a strictly convex and continuously differentiable function, $L \in \mathbb{R}^{l \times m}$ is a constant matrix, $h : \mathbb{R}^{n_w} \to \mathbb{R}^l$ and $r_i : \mathbb{R}^{n_w} \to \mathbb{R}$, $i = 1, \ldots, k$ are continuous functions, while $q_i : \mathbb{R}^m \to \mathbb{R}$, $i = 1, \ldots, k$ are convex, continuously differentiable functions. For the matrix L we require $\operatorname{rank} L = l < m$.

For a constant exogenous signal $w(t) = w \in W$, the optimization problem (5.2) reflects the corresponding optimal steady-state operating point for the state x_p in (5.1). A typical example is the case when Σ represents some production unit and the corresponding optimization problem reflects the variable production costs, i.e. it reflects the economic objectives of the plant under the current market conditions. For instance, with (5.1) describing the relevant dynamics of an electrical power plant, (5.2a) reflects the variable production costs of producing electrical power. In this case, the exogenous signal w represents the demand for the commodity (demand for electrical power) and the equality constrains (5.2b) include the production-demand balance constraints (electrical power balance). The inequality constraints (5.2c) represent the security-type "soft" constraints for which some degree of transient violation may be accepted, but whose feasibility is required for steady-state operation (power flow levels in the transmission lines of the electrical power system). The state vector x_p collects only the states which appear explicitly in (5.2). Note that in general not all of the elements of x_p that appear in the constraints (5.2b),(5.2c) need to appear in the objective function J, and *vice versa*. The objective of the control input u is to drive the state x_p to its optimal, constrained steady-state operating point. We continue by listing the set of assumptions concerning the dynamics (5.1) and the optimization problem (5.2).

Let $\mathcal{I}_l$ denote the set of indices i for which the function q_i in (5.2c) is a linear function, and let $\mathcal{I}_n$ denote the set of indices corresponding to nonlinear q_i. We make the following assumption:

Assumption 5.2.1 For each $w \in W$ the set

$$\{x_p \mid Lx_p = h(w),\ \ q_i(x_p) < r_i(w) \text{ for } i \in \mathcal{I}_n,\ \ q_i(x_p) \leq r_i(w) \text{ for } i \in \mathcal{I}_l\}$$

is nonempty. □

Assumption 5.2.1 states that the convex optimization problem (5.2) satisfies Slater's constraint qualification (Boyd and Vandenberghe, 2004) for each $w \in W$, implying that strong duality holds for the considered problem. Note also that due to strict convexity of the objective function in (5.2), the optimization problem has an unique minimizer $\tilde{x}_p$ for each $w \in W$. For all the considerations in this chapter, we assume the following to hold:

Assumption 5.2.2 For each $w \in W$, in the optimization problem (5.2) the minimum is attained. □

To emphasize the hypothesis in Assumption 5.2.2, in (5.2) we use *minimum* (min) instead of *infimum* (inf). Furthermore, in connection with the dynamical system (5.1) we make the following simplifying assumption:

Assumption 5.2.3 The matrix A is Hurwitz. The sub-matrix of $A^{-1}B$ formed by taking the first m rows of $A^{-1}B$ has full rank. (Note that this is a square matrix.) □

Remark 5.2.4 Assumption 5.2.3 guarantees that for all constant $w(t) = w \in W$, the partial state vector x_p can be driven to an arbitrary steady-state point, which is then characterized by a unique, constant value of the input signal u. □

In other words, Assumption 5.2.3 implies that the steady-state relations from (5.1) do not pose any additional constraints to the optimization problem (5.2). Although seemingly restrictive, this assumption is in practice almost always fulfilled since x_p represents the states which directly appear in the economical objective of the plant, e.g. the optimization problem (5.2). Well designed systems allow complete steady-state control (in the sense of the above stated assumption) of these "economically relevant" states. However, it is also possible to relax this assumption. Then, any constraint on the steady state imposed by (5.1) should be included in (5.2b). The assumption that A is Hurwitz is also reasonable. Thinking of u as a setpoint signal for steady-state operation, (5.1) represents the plant which already includes a stabilizing controller.

Assumption 5.2.5 In (5.1) the output matrix C in (5.1b) is given by $C = \begin{pmatrix} I & 0 \end{pmatrix}$, i.e. the state vector x_p can be measured. Furthermore, all components of w that appear in (5.2) are known at all time instants. □

From Assumption 5.2.5 it follows that violations of the constraints (5.2b) and (5.2c) are available for control. In practice, and in contrast to the above assumption, violations of the constraints are often directly measurable, and not only indirectly through x_p and w. To illustrate this, consider again an example of a electrical power system. Demand for electrical power, which corresponds to the exogenous signal w, is never explicitly known. However, the network frequency serves as a measure of production-demand imbalance, i.e. as a measure of the violation of an equality constraint in (5.2b). Furthermore, if the power flow in a tie-line should not exceed a certain constant value, the preferred solution in practice is to directly measure the violation of this inequality constraint, and not to determine it indirectly by solving on-line the power flow equations. Assumption 5.2.5 is made only for the purpose of simplifying the presentation in the next subsection, and it does not result in a significant loss of generality.

With the definitions and assumptions made so far, we are now ready to formally state the control problem considered in this chapter.

Problem 5.2.6 *Steady-state optimal control problem.*
For a dynamical system Σ given by (5.1), design a feedback controller that has y as input signal and u as output signal, such that the following objective is met for any constant-valued exogenous signal $w(t) = w \in W$: the state of the closed-loop system globally converges to an equilibrium point with $x_p = \tilde{x}_p(w)$, where $\tilde{x}_p(w)$ denotes the minimizer of the optimization problem (5.2) for some $w \in W$. □

In connection to Problem 5.2.6 we define the following two subproblems:
i) ensuring existence of a closed-loop system equilibrium characterized by $x_p = \tilde{x}_p(w)$;
ii) ensuring global convergence to an equilibrium with $x_p = \tilde{x}_p(w)$, for all $w \in W$.
Solving the first subproblem is not a very difficult task and a detailed presentation of a solution to the subproblem (*i*) is given in the following section. However, for a general dynamical system (5.1) and a general optimization problem (5.2), solving the subproblem (*ii*) is very difficult task. Several important issues on this subject are discussed in Section 5.6.

5.3 KKT Controllers

In this section we present several controller structures that guarantee the existence of an equilibrium point with $x_p = \tilde{x}_p(w)$ as described in Problem 5.2.6.

Assumption 5.2.1 implies that for each $w \in W$, the first order Karush-Kuhn-Tucker (KKT) conditions are *necessary and sufficient* conditions for optimality. For the optimization problem (5.2) these conditions are given by the following set of equalities and inequalities:

$$\nabla J(x_p) + L^\top \lambda + \nabla q(x_p)\mu = 0, \tag{5.3a}$$
$$Lx_p - h(w) = 0, \tag{5.3b}$$
$$0 \leq -q(x_p) + r(w) \perp \mu \geq 0, \tag{5.3c}$$

where we use the abbreviations $q(x_p) = \mathrm{col}(q_1(x_p), \ldots, q_k(x_p))$, $r(w) = \mathrm{col}(r_1(w), \ldots, r_k(x_p))$ and $\lambda \in \mathbb{R}^l$, $\mu \in \mathbb{R}^k$ are Lagrange multipliers. Since the above conditions are necessary and sufficient conditions for optimality, it is apparent that the existence of an equilibrium point with $x_p = \tilde{x}_p(w)$ is implied if for each $w \in W$ the controller guarantees the existence of the vectors λ and μ, such that that the conditions (5.3) are fulfilled in a steady-state of the closed-loop system. In what follows, we present two different controller structures for which we prove, in Theorem 5.3.1, that they achieve this goal.

Max-based KKT controller. Let $K_\lambda \in \mathbb{R}^{l\times l}$, $K_\mu \in \mathbb{R}^{k\times k}$, $K_c \in \mathbb{R}^{m\times m}$ and $K_o \in \mathbb{R}^{k\times k}$ be diagonal matrices with non-zero elements on the main diagonal and $K_o \succ 0$. Consider a dynamic controller with the following structure:

$$\dot{x}_\lambda = K_\lambda(Lx_p - h(w)), \tag{5.4a}$$
$$\dot{x}_\mu = K_\mu(q(x_p) - r(w) + v), \tag{5.4b}$$
$$\dot{x}_c = K_c(L^\top x_\lambda + \nabla q(x_p)x_\mu + \nabla J(x_p)), \tag{5.4c}$$
$$0 \leq v \perp K_o x_\mu + q(x_p) - r(w) + v \geq 0, \tag{5.4d}$$
$$u = x_c, \tag{5.4e}$$

where x_λ, x_μ and x_c denote the controller states and the matrices K_λ, K_μ, K_c and K_o represent the controller gains. Note that the input vector $v(t) \in \mathbb{R}^k$ in (5.4b) is at any time instant required to be a solution to a finite-dimensional linear complementarity problem (5.4d). □

Saturation-based KKT controller. Let $K_\lambda \in \mathbb{R}^{l\times l}$, $K_\mu \in \mathbb{R}^{k\times k}$ and $K_c \in \mathbb{R}^{m\times m}$ be diagonal matrices with non-zero elements on the main diagonal and $K_\mu \succ 0$. Consider a dynamic controller with the following structure:

$$\begin{aligned}
\dot{x}_\lambda &= K_\lambda(Lx_p - h(w)), && (5.5a)\\
\dot{x}_\mu &= K_\mu(q(x_p) - r(w)) + v, && (5.5b)\\
\dot{x}_c &= K_c(L^\top x_\lambda + \nabla q(x_p)x_\mu + \nabla J(x_p)), && (5.5c)\\
0 \leq v \;&\perp\; x_\mu \geq 0, && (5.5d)\\
u &= x_c, && (5.5e)\\
x_\mu(0) &\geq 0, && (5.5f)
\end{aligned}$$

where x_λ, x_μ and x_c denote the controller states and the matrices K_λ, K_μ and K_c represent the controller gains. Note that the input vector $v(t) \in \mathbb{R}^k$ in (5.5b) is at any time instant required to be a solution to a finite-dimensional linear complementarity problem (5.5d). The initialization constraint (5.5f) is required as a necessary condition for well-posedness, conform with the inequality in the complementarity condition (5.5d). □

The choice of names *max-based KKT controller* and *saturation-based KKT controller* will become clear later in this section. Both controllers belong to the class of dynamic complementarity systems (van der Schaft and Schumacher, 1996, 1998). Although other closely related modeling frameworks, such as projected dynamical systems, see (Brogliato et al., 2006; Heemels et al., 2000b) for more details, could equivalently have been used to describe the same dynamical system, the complementarity framework arises as a natural framework for the topic considered here. In particular, this modeling framework is seen as a natural dynamical extension of the algebraic complementarity in the KKT conditions (5.3).

Theorem 5.3.1 *Let $w(t) = w \in W$ be a constant-valued signal, and suppose that Assumption 5.2.1 and Assumption 5.2.3 hold. Then the closed-loop system, i.e. the system obtained from the system* (5.1) *connected with the controller* (5.4)/(5.5) *in a feedback loop, has an equilibrium point with $x_p = \tilde{x}_p(w)$, where $\tilde{x}_p(w)$ denotes the minimizer of the optimization problem* (5.2) *for some $w \in W$.* □

Proof. We first consider the closed-loop system with max-based KKT controller, i.e. controller (5.4). By setting the time derivatives of the closed-loop system states to zero and by exploiting the non-singularity of the matrices

K_λ, K_μ and K_c, we obtain the following complementarity problem:

$$0 = A \begin{pmatrix} x_p \\ x_q \end{pmatrix} + Bx_c + Fw, \tag{5.6a}$$

$$0 = Lx_p - h(w), \tag{5.6b}$$

$$0 = q(x_p) - r(w) + v, \tag{5.6c}$$

$$0 = L^\top x_\lambda + \nabla q(x_p) x_\mu + \nabla J(x_p), \tag{5.6d}$$

$$0 \le v \ \perp \ K_o x_\mu + q(x_p) - r(w) + v \ge 0, \tag{5.6e}$$

with the closed-loop system state vector $x_{cl} := \mathrm{col}(x_p, x_q, x_\lambda, x_\mu, x_c)$ and the vector v as variables. Any solution x_{cl} to (5.6) is an equilibrium point of the closed-loop system. By substituting $v = -q(x_p) + r(w)$ from (5.6c) and utilizing $K_o \succ 0$, the complementarity condition (5.6e) reads as $0 \le -q(x_p) + r(w) \ \perp \ x_\mu \ge 0$. With $\lambda := x_\lambda$ and $\mu := x_\mu$, the conditions (5.6b),(5.6c),(5.6d),(5.6e) therefore correspond to the KKT conditions (5.3) and, under Assumption 5.2.1, they necessarily have a solution. Furthermore, for any solution $(x_p, x_\lambda, x_\mu, v)$ to (5.6b),(5.6c),(5.6d),(5.6e), it necessarily holds that $x_p = \tilde{x}_p(w)$. It remains to show that (5.6a) admits a solution in (x_q, x_c) for $x_p = \tilde{x}_p(w)$. This, however, readily follows from Assumption 5.2.3. Moreover, Assumption 5.2.3 implies uniqueness of x_q and x_c in an equilibrium.

Now, consider the closed-loop system with saturation-based KKT controller, i.e. controller (5.5). The difference in this case comes only through (5.5b) and (5.5d), and for a proof it is therefore sufficient to show that the conditions

$$0 = K_\mu(q(x_p) - r(w)) + v, \tag{5.7a}$$

$$0 \le v \ \perp \ x_\mu \ge 0, \tag{5.7b}$$

imply $0 \le -q(x_p) + r(w) \perp x_\mu \ge 0$, similarly as implied by the conditions (5.6c) and (5.6e). The rest of the proof then follows as above. The desired implication directly follows from (5.7), since $K_\mu \succ 0$. □

Note that Theorem 5.3.1 only ensures the existence of an equilibrium point with $x_p = \tilde{x}_p(w)$, and says nothing about its uniqueness. However, from the proof of the theorem it follows that for *any* equilibrium it necessarily holds that $x_p = \tilde{x}_p(w)$, and that any equilibrium is characterized by the same, unique values of the state vectors x_q and x_c. Conditions for uniqueness of the equilibrium, i.e. uniqueness of the remaining states (x_λ, x_μ), are important for the question of global asymptotic stability of the closed-loop system. These conditions are presented later in this chapter.

Remark 5.3.2 This remark concerns Assumption 5.2.5. As already mentioned, violations of the constraints are often directly measurable, and not only indirectly through x_p and w, as stated in Assumption 5.2.5. Assume that the output y of the system (5.1), instead of being given by (5.1b), has the form $y = \mathrm{col}(\alpha(x,w), \beta(x,w), x_p)$, where $\alpha : \mathbb{R}^n \times \mathbb{R}^{n_w} \to \mathbb{R}^l$ and $\beta : \mathbb{R}^n \times \mathbb{R}^{n_w} \to \mathbb{R}^k$ are continuous functions, characterized by the following *steady-state-related* properties:

$$[\alpha(x,w)]_i = 0 \quad \Leftrightarrow \quad [Lx_p - h(w)]_i = 0, \quad i = 1, \dots, l, \tag{5.8a}$$
$$[\beta(x,w)]_j \leq 0 \quad \Leftrightarrow \quad [q(x_p) - r(w)]_j \leq 0, \quad j = 1, \dots, k. \tag{5.8b}$$

Here, by *steady-state-related* we mean that the above proporties hold when $w \in W$ is a given constant signal and the system is in a steady-state. The values of α and β therefore carry the information of the violation of the constraints, and as such they can be directly used for control. From (5.8) it follows that by replacing $Lx_p - h(w)$ by $\alpha(x,w)$ and $q(x_p) - r(w)$ by $\beta(x,w)$ in (5.4a),(5.4b),(5.4d),(5.5a), and (5.5b), the statement of Theorem 5.3.1 still holds. □

Remark 5.3.3 Assumption 5.2.3 implies the steady-state relation $x_p = Ru + Pw$ for any constant-valued signal w, where R and P are constant matrices and R is square and nonsingular. Using this relation, it is possible to eliminate the state vector x_c from the controllers (5.4)/(5.5) by replacing the differential equations (5.4c)/(5.5c) with appropriate algebraic relations. For the controllers obtained via this modification Theorem 5.3.1 still holds. Here we present the modification for controller (5.4). Let $g(x_\lambda, x_\mu, w)$ be a function such that

$$u = g(x_\lambda, x_\mu, w) \quad \Leftrightarrow \quad L^\top x_\lambda + \nabla q(Ru + Pw)x_\mu + \nabla J(Ru + Pw) = 0.$$

Then the closed-loop system with the controller of the following structure:

$$\dot{x}_\lambda = K_\lambda \, \alpha(x,w), \tag{5.9a}$$
$$\dot{x}_\mu = K_\mu(\beta(x,w) + v), \tag{5.9b}$$
$$0 \leq v \;\perp\; K_o \, x_\mu + \beta(x,w) + v \geq 0, \tag{5.9c}$$
$$u = g(x_\lambda, x_\mu, w), \tag{5.9d}$$

has an equilibrium point with $x_p = \tilde{x}_p(w)$. Here $\alpha(x,w)$ and $\beta(x,w)$ are the signals satisfying the conditions from Remark 5.3.2. The proof for the above statement is the same as the proof of Theorem 5.3.1: in a steady-state, the closed-loop system relations necessarily include the KKT conditions (5.3).

Furthermore, utilizing the steady-state relation $\alpha(x,w)=0$, in some cases it is possible to eliminate w in $g(x_\lambda,x_\mu,w)$ and to obtain $u=g(x_\lambda,x_\mu,x)$ instead of (5.9d). □

Remark 5.3.4 Consider the case when the optimal steady-state-related optimization problem is given by $\min_u\{J(u) \mid Lx_p=h(w),\, q(x_p)\le r(w)\}$ instead of (5.2). This is often the case in practice, i.e. economic optimality is often explicitly expressed as a function of the set-point signals (inputs) to the system. For example, this is the case in optimal control of electrical power systems, which was considered in Chapter 2 of this thesis. Using the steady-state relation $x_p=Ru+Pw$, as in Remark 5.3.3, the controller (5.9) with $g(x_\lambda,x_\mu,w)$ defined by

$$u=g(x_\lambda,x_\mu,w) \quad \Leftrightarrow \quad R^\top L^\top x_\lambda + R^\top \nabla q(Ru+Pw)x_\mu + \nabla J(u)=0,$$

will guarantee the existence of an equilibrium point with $u=\tilde{u}(w)$, where $\tilde{u}(w)$ is a unique minimizer of the steady-state related optimization problem. □

Up to now, all the considerations in this chapter dealt only with the steady-state behavior of the closed-loop system. Utilizing integral action and/or algebraic relations, it is possible to "shape" this behavior in such a way that the closed-loop system will be guaranteed to have an equilibrium for which the subset of system states coincides with the solution of a given constrained optimization problem. We have seen that this desired *static* behavior of the system can be obtained in several different ways. However, the main idea was always the same: utilize the integral action and/or algebraic relations to "built in" the KKT conditions in the closed-loop system steady-state relations.

5.3.1 Complementarity integrators

The main distinguishing feature between the max-based KKT controller (5.4) (and its variations) and the saturation-based KKT controller (5.5) (and its variations) is in the way the steady-state complementarity slackness condition (5.3c) is enforced. Although characterized by the same steady-state relations, the two controllers, and therefore the corresponding closed-loop systems, have some significantly different *dynamical* features which will be discussed further in this section. In the following two paragraphs our attention is on the equations (5.4b),(5.4d) and (5.5b),(5.5d), and the goal is to show the following:

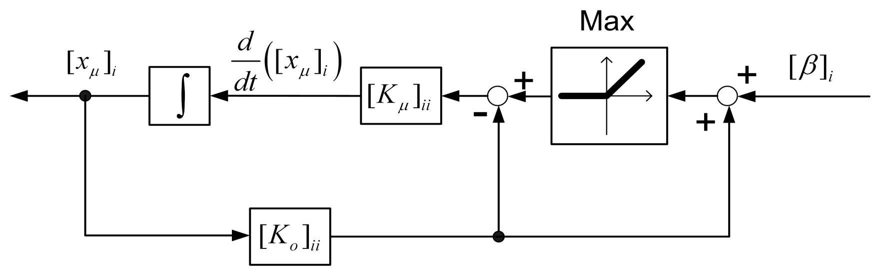

Figure 5.1: Max-based complementarity integrator.

• The *max-based KKT controller*, i.e. the controller (5.4), can be represented as a dynamical system in which certain variables are coupled by means of static, *continuous*, piecewise linear characteristics;
• The *saturation-based KKT controller*, i.e. the controller (5.5), can be represented as a dynamical system with state saturations.

Max-based complementarity integrator. Let a, b and c be real scalars related through a complementarity condition $0 \leq c \perp a{+}b{+}c \geq 0$. It is easily verified, e.g. by checking all possible combinations, that this complementarity condition can equivalently be written as $b+c = \max(a+b, 0) - a$. With $\max(\cdot,\cdot)$ defined for vectors as an elementwise maximum, i.e. for $v, w \in \mathbb{R}^n$, $(z = \max(v, w)) \Leftrightarrow ([z]_i = \max([v]_i, [w]_i), i = 1, \ldots, n)$, the above equivalence holds as well for a, b, c being vectors of the same dimension. Now, by taking $c = v$, $a = K_o x_\mu$ and $b = q(x_p) - r(w)$, it follows that (5.4b) and (5.4d) can be equivalently described by

$$\dot{x}_\mu = K_\mu(\max(K_o\, x_\mu + q(x_p) - r(w), 0) - K_o\, x_\mu). \tag{5.10}$$

With $\beta := q(x_p) - r(w)$, Figure 5.1 presents a block diagram representation of the i-th row in (5.10). The block labeled "Max" in the figure, represents a scalar max relation as a static piecewise linear characteristics. With $[K_o]_{ii} > 0$, it is easy to verify that if the system in Figure 5.1 is in steady-state, than the value of its input signal $[\beta]_i$ and the value of its output signal $[x_\mu]_i$ necessarily satisfy the complementarity condition $[x_\mu]_i \geq 0$, $[\beta]_i \leq 0$, $([x_\mu]_i\ [\beta]_i) = 0$. □

Saturation-based complementarity integrator. The differential algebraic equations (5.5b), (5.5d) restrict the state vector x_μ to the nonnegative orthant $\mathbb{R}^k_+$. For $x_\mu > 0$, i.e. when x_μ is in the interior of $\mathbb{R}^k_+$, its dynamics is described by the equation $\dot{x}_\mu = K_\mu \beta(x, w)$, where $\beta(x, w) := q(x_p) - r(w)$.

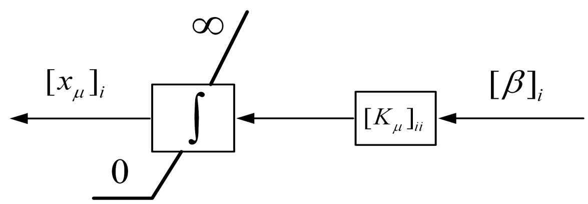

Figure 5.2: Saturation-based complementarity integrator.

However, on the boundary this dynamics is modified to prevent the solution from leaving $\mathbb{R}^k_+$. Precisely, the dynamics of the i-th element in x_μ is given by

$$[\dot{x}_\mu]_i = \begin{cases} 0 & \text{if} \quad [x_\mu]_i = 0 \quad \text{and} \quad [K_\mu]_{ii}[\beta]_i < 0, \\ [K_\mu]_{ii}[\beta]_i & \text{otherwise.} \end{cases} \tag{5.11}$$

Figure 5.2 presents a block diagram representation of (5.11), which is a saturated integrator with the lower saturation point equal to zero. The equivalence of the dynamics (5.5b), (5.5d) and the saturated integrators defined by (5.11) is shown in the next section.

With $[K_\mu]_{ii} > 0$, it is easy to verify that in steady-state the value of the input signal $[\beta]_i$ and the value of the output signal $[x_\mu]_i$ necessarily satisfy the complementarity condition $[x_\mu]_i \geq 0$, $[\beta]_i \leq 0$, $([x_\mu]_i\ [\beta]_i) = 0$. □

The above discussions present us with the basic building blocks for imposing steady-state complementarity conditions. We will use the term *max-based complementarity integrator* to refer to a system with the structure as depicted in Figure 5.1, and the term *saturation-based complementarity integrator* for the system in Figure 5.2. Together with a pure integrator, complementarity integrators form the basic building block of a KKT controller.

5.4 Well-posedness of the closed-loop system

The term *well-posedness* is in general used to denote the property of *existence and uniqueness of solutions*, and is a fundamental issue for any class of dynamical systems. A mathematical model of a dynamical system is said to be well-posed if, given initial conditions, it has a unique solution.

In this section we present sufficient conditions for well-posedness of the closed-loop system, i.e. of the system (5.1) interconnected with a KKT controller in a feedback loop. It turns out that, due to the continuity of the right hand side, checking well-posedness of the closed-loop system with the max-based KKT controller can be performed based on the classical Lipschitz continuity conditions. For the completeness of the presentation, we recall these conditions in the following subsection. However, the saturation-based KKT controller is characterized by discontinuous dynamics and for well-posedness conditions one has to reside on the theory of complementarity systems.

5.4.1 Max-based KKT controller

We have seen that (5.4b) and (5.4d), i.e. complementarity conditions in the max-based KKT controller, can equivalently be represented using the piecewise linear max-relation (5.10). The closed-loop system with max-based KKT controller (5.4) is therefore given by

$$\begin{pmatrix} \dot{x}_p \\ \dot{x}_q \end{pmatrix} = A \begin{pmatrix} x_q \\ x_p \end{pmatrix} + Bx_c + Fw, \tag{5.12a}$$

$$\dot{x}_\lambda = K_\lambda(Lx_p - h(w)), \tag{5.12b}$$

$$\dot{x}_\mu = K_\mu(\max(K_o\, x_\mu + q(x_p) - r(w), 0) - K_o\, x_\mu), \tag{5.12c}$$

$$\dot{x}_c = K_c(L^\top x_\lambda + \nabla q(x_p)x_\mu + \nabla J(x_p)). \tag{5.12d}$$

We will use the abbreviation $x = \mathrm{col}(x_q, x_p, x_\lambda, x_\mu, x_c)$ for the closed-loop state vector, and denote its dimension by n_{cl}, i.e. $x(t) \in \mathbb{R}^{n_{cl}}$. Note that, since J and q are continuously differentiable functions, the right hand side of (5.12) is continuous in x. Furthermore, since h and r are continuous functions, if $w(t)$ is a piecewise continuous function of t, so is the right hand side of (5.12). In a compact form (5.12) is then represented by

$$\dot{x} = f(x, t), \tag{5.13}$$

where $f(x, t)$ is piecewise continuous in t and continuous in x. The following standard notion of solution is used in connection to the system (5.13):

Definition 5.4.1 A function $x : [t_0, t_1] \mapsto \mathbb{R}^{n_{cl}}$ is called a solution to the differential equation (5.13) with initial state x_0 at time t_0, if x is continuous, x is differentiable on (t_0, t_1) (i.e. $\dot{x}(t)$ exists for all $t \in (t_0, t_1)$), $\dot{x}(t) = f(x(t), t)$ for all $t \in (t_0, t_1)$ and $x(t_0) = x_0$. □

A sufficient condition for *global* existence and uniqueness of solutions to (5.13) is given by the following well-known theorem.

Theorem 5.4.2 *(Khalil (2002), Theorem 3.2) Suppose that $f(x,t)$ is piecewise continuous in t and satisfies the* Lipschitz continuity *condition*

$$\|f(x,t) - f(y,t)\| \le c\|x-y\| \tag{5.14}$$

for all $x, y \in \mathbb{R}^{n_{cl}}$ and for all $t \in [t_0, t_1]$. Then, the equation $\dot{x} = f(x,t)$, with $x(t_0) = x_0$, has a unique solution over $[t_0, t_1]$. □

The term *globally Lipschitz* is commonly used to indicate that the condition (5.14) holds on whole $\mathbb{R}^{n_{cl}}$. A function $f(x,t)$ is said to be *locally Lipschitz* on a domain (open and connected set) $\Omega \subset \mathbb{R}^{n_{cl}}$ if each point in Ω has a neighborhood Ω_0 such that f satisfies condition (5.14) for all points in Ω_0 and for all $t \in [t_0, t_1]$ with some *Lipschitz constant* c.

Since the function $\max(\cdot, 0)$ is globally Lipschitz continuous, and since sums, products and compositions of Lipschitz continuous functions are Lipschitz continuous, it is obvious that the closed-loop system with max-based KKT controller is globally (locally) well-posed if the functions q, ∇J, and all entries in ∇q are globally (locally) Lipschitz.

5.4.2 Saturation-based KKT controller

A saturation-based KKT controller, as well as the closed-loop system with saturation-based KKT controller, belongs to a specific class of complementarity systems. In (Heemels et al., 2000b) this class of systems is named *gradient-type complementarity systems* (GTCS), and it is shown that, under certain mild assumptions, the class of GTCS is equivalent to the class of *projected dynamical systems* (Dupuis and Nagurney, 1993). The results from (Heemels et al., 2000b) are further extended in (Brogliato et al., 2006) by showing that class of GTCS is, under certain conditions, equivalent to a specific class of differential inclusions. In both references, sufficient conditions for well-posedness have been presented.

In this subsection we shortly present and summarize some of the results from (Brogliato et al., 2006; Heemels et al., 2000b). These results can be used to perform a well-posedness analysis of the the closed-loop system with saturation-based KKT controller. For all further details and for all the proofs we refer to (Brogliato et al., 2006; Heemels et al., 2000b).

Furthermore, based on the equivalence of the GTCS and the projected dynamical systems, in this subsection we finally show that the saturation-based KKT controller is a dynamical system with *state saturations*, which is the distinguishing property we emphasized by choosing the names "saturation-based KKT controller" and "saturation-based complementarity integrator".

Gradient-type complementarity systems

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous function, let $g : \mathbb{R}_+ \to \mathbb{R}^n$ be a measurable function, and let $s : \mathbb{R}^n \to \mathbb{R}^k$ be a *concave* continuously differentiable function, such that the set

$$S := \{x \in \mathbb{R}^n \mid -s(x) \leq 0\} \tag{5.15}$$

has a nonempty interior. Note that S is a closed convex set. The gradient-type complementarity system is given by the equations

$$\dot{x}(t) = -f(x(t)) - g(t) + \nabla s(x(t))v(t), \tag{5.16a}$$

$$z(t) = s(x(t)), \tag{5.16b}$$

$$0 \leq z(t) \perp v(t) \geq 0, \tag{5.16c}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, while $v(t) \in \mathbb{R}^k$ and $z(t) \in \mathbb{R}^k$ denote the input and the output, which are coupled by the complementarity condition (5.16c).

In (Heemels et al., 2000b; Brogliato et al., 2006), the following notion of solution to GTCS system (5.16) was presented.

Definition 5.4.3 An absolutely continuous function $x : [0,T] \to S$ is a *solution* to the system (5.16) on $[0,T]$ with initial state $x_0 \in S$ if $x(0) = x_0$ and (5.16) holds almost everywhere in $[0,T]$. □

Note that, due to (5.16c), and according to Definition 5.4.3, any solution $x(t)$, $t \in [0,T]$ of the system (5.16) must necessarily lie in S for almost every $t \in [0,T]$.

For $x \in S$ we define the *active index set* $\mathcal{I}(x)$ as

$$\mathcal{I}(x) := \{i \in \{1,\ldots,k\} \mid [s(x)]_i = 0\}, \tag{5.17}$$

and make the following assumption.

Assumption 5.4.4 The matrix $[\nabla s(x)]_{\bullet\mathcal{I}(x)}$ has full column rank for all $x \in S$. □

Theorem 5.4.5 *(Brogliato et al. (2006), Theorem 3.2) Suppose that Assumption 5.4.4 holds and suppose that $g \in \mathcal{L}_1^n(\mathbb{R}_+)$. Furthermore, suppose that f is continuous over $\mathbb{R}^n$ and* hypermonotone, *i.e. that there exists $c \geq 0$ such that*

$$\langle f(x) - f(y), x - y\rangle \geq -c\|x-y\|^2 \qquad \textit{for all} \quad x, y \in \mathbb{R}^n. \tag{5.18}$$

Then, for any initial condition $x(0) = x_0 \in S$, the GTCS (5.16) has a unique solution [2] $x(t)$, over the whole $\mathbb{R}_+$. □

Proposition 5.4.6 *If $f : \mathbb{R}^n \to \mathbb{R}^n$ is globally Lipschitz, then it is hypermonotone, as defined in (5.18).* □

Proof. For all $x, y \in \mathbb{R}^n$ it necessarily holds that

$$-\langle f(x) - f(y), x - y\rangle \;\leq\; |\langle f(x) - f(y), x - y\rangle| \;\leq\; \|f(x) - f(y)\| \cdot \|x - y\|, \tag{5.19}$$

were the second inequality is the so-called Cauchy-Schwartz inequality. Since f is globally Lipschitz, by definition there exists $c \in \mathbb{R}$ such that

$$\|f(x) - f(y)\| \leq c\|x - y\| \tag{5.20}$$

holds for all $x, y \in \mathbb{R}^n$. The inequality (5.18) follows directly from (5.19) and (5.20). □

Remark 5.4.7 In (Heemels et al., 2000b), the functions f and s in (5.16) are assumed to be real-analytic and $g(t) \equiv 0$. With these assumptions, additional information about the solutions to GTCS (5.16) has been provided. Loosely speaking, the notion of solution in Definition 5.4.3 has been further refined by excluding the so-called left accumulations of discrete event times. For details, see (Heemels et al., 2000b). For definitions of different solution concepts used for non-smooth systems see (Çamlibel et al., 2002). □

Saturation-based KKT controller as GTCS

The closed-loop system with saturation-based KKT controller (5.5) can be written in the following suitable form

$$\begin{pmatrix} \dot{x}_a \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -f_a(x_a, x_\mu) \\ -f_\mu(x_a) \end{pmatrix} + \begin{pmatrix} -g_a(w) \\ -g_\mu(w) \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} v, \tag{5.21a}$$

$$z = \begin{pmatrix} 0 & I \end{pmatrix} \begin{pmatrix} x_a \\ x_\mu \end{pmatrix}, \tag{5.21b}$$

$$0 \leq z \perp v \geq 0, \tag{5.21c}$$

[2] Here, the notion of solution is according to Definition 5.4.3

where $x_a := \mathrm{col}(x_p, x_q, x_\lambda, x_c)$ and

$$\left(\frac{-f_a(x_a, x_\mu)}{-f_\mu(x_a)}\right) = \left(\begin{array}{c} A\,\mathrm{col}(x_p, x_q) + Bx_c \\ K_\lambda L x_p \\ K_c(L^\top x_\lambda + \nabla q(x_p) x_\mu + \nabla J(x_p)) \\ \hline K_\mu q(x_p) \end{array}\right), \tag{5.22a}$$

$$\left(\frac{-g_a(w)}{-g_\mu(w)}\right) = \left(\begin{array}{c} Fw \\ -K_\lambda h(w) \\ 0 \\ \hline -K_\mu r(w) \end{array}\right). \tag{5.22b}$$

Obviously (5.21) belongs to a GTCS class of systems (5.16). For the system (5.22), i.e. for a closed-loop system with saturation-based KKT controller, ∇s is given by $\begin{pmatrix} 0 & I \end{pmatrix}^\top$, see (5.21a) and (5.16a), and therefore Assumption 5.4.4 is trivially fulfilled. Furthermore, since J and q are continuously differentiable functions, f_a and f_μ in (5.21) are continuous functions. Therefore, according to Theorem 5.4.5, with restriction of the input signals w, $h(w)$ and $r(w)$ to the corresponding $\mathcal{L}_1(\mathbb{R}_+)$ spaces, the system (5.21) is well-posed if the function $\mathrm{col}(f_a, f_\mu)$ defined by (5.22a) is hypermonotone, i.e. if it fulfills the condition (5.18).

Projected dynamical systems

As before, in the definition of gradient-type complementarity systems, let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous function, let $g : \mathbb{R}_+ \to \mathbb{R}^n$ be a measurable function, and let $s : \mathbb{R}^n \to \mathbb{R}^k$ be a *concave* continuously differentiable function, such that the set

$$S := \{x \in \mathbb{R}^n \mid -s(x) \leq 0\} \tag{5.23}$$

has a nonempty interior. We first recall the definition of projected dynamical systems (Dupuis and Nagurney, 1993) and for that purpose we make the following definitions.

The cone of inward normals at $x \in S$ is defined by

$$n(x) = \{\gamma \mid \langle \gamma, x - z \rangle \leq 0 \quad \text{for all} \quad z \in S\}. \tag{5.24}$$

Given $x \in S$ and $v \in \mathbb{R}^n$, the projection of the vector v at x with respect to S is defined by

$$\Pi_S(x, v) = v - \langle v, n^*(x) \rangle n^*(x), \tag{5.25}$$

where

$$n^*(x) \in \arg \max_{n \in n(x), \|n\| \leq 1} \langle v, -n \rangle. \tag{5.26}$$

Note that $\Pi_S(x, v)$ is well-defined even though $n^*(x)$ may not be uniquely specified by (5.26). Finally, the *projected dynamical system* (PDS) is defined by

$$\dot{x}(t) = \Pi_S(x(t), -f(x(t)) - g(t)), \qquad x(0) = x_0 \in S. \tag{5.27}$$

In (Brogliato et al. (2006), Theorem 3.2) it is shown that under the conditions stated in Theorem 5.4.5, the PDS (5.27) is well-posed, with the notion of solution defined by Definition 5.4.3. Moreover, it is shown that in that case the solution of the PDS (5.27) and the solution of the GTCS (5.16) coincide.

If the closed-loop system (5.21) is well-posed, by rewriting it as a PDS of the form (5.27), it is easy to verify (from the definition of the mapping Π_S) that the dynamics of the i-th element in x_μ is described by

$$[\dot{x}_\mu]_i = \begin{cases} 0 & \text{if} \quad [x_\mu]_i = 0 \quad \text{and} \quad [-f_\mu(x_a) - g_\mu(w)]_i < 0, \\ [-f_\mu(x_a) - g_\mu(w)]_i & \text{otherwise}, \end{cases} \tag{5.28}$$

which defines the dynamics of a saturated integrator with the lower saturation point equal to zero and the input $[-f_\mu(x_a) - g_\mu(w)]_i$.

To summarize, equivalence of the dynamics (5.5b), (5.5d) and the saturated integrators defined by (5.11) directly follows from the equivalence of GTCS ((5.5b), (5.5d) belong to GTCS class) and projected dynamical systems ((5.11) belongs to PDS class).

5.5 Example

Consider a third-order system of the form (5.1) with (5.1a) given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \hline \dot{x}_3 \end{pmatrix} = \left(\begin{array}{cc|c} -2.5 & 0 & -5 \\ 0 & -5 & -15 \\ \hline 0.1 & 0.1 & -0.2 \end{array}\right) \begin{pmatrix} x_1 \\ x_2 \\ \hline x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \hline -0.1 \end{pmatrix} w + \begin{pmatrix} 2.5 & 0 \\ 0 & 5 \\ \hline 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \tag{5.29}$$

where $x_p = \mathrm{col}(x_1, x_2)$, $x_q = x_3$ and $u = \mathrm{col}(u_1, u_2)$. The associated steady-state related optimization problem is defined as follows:

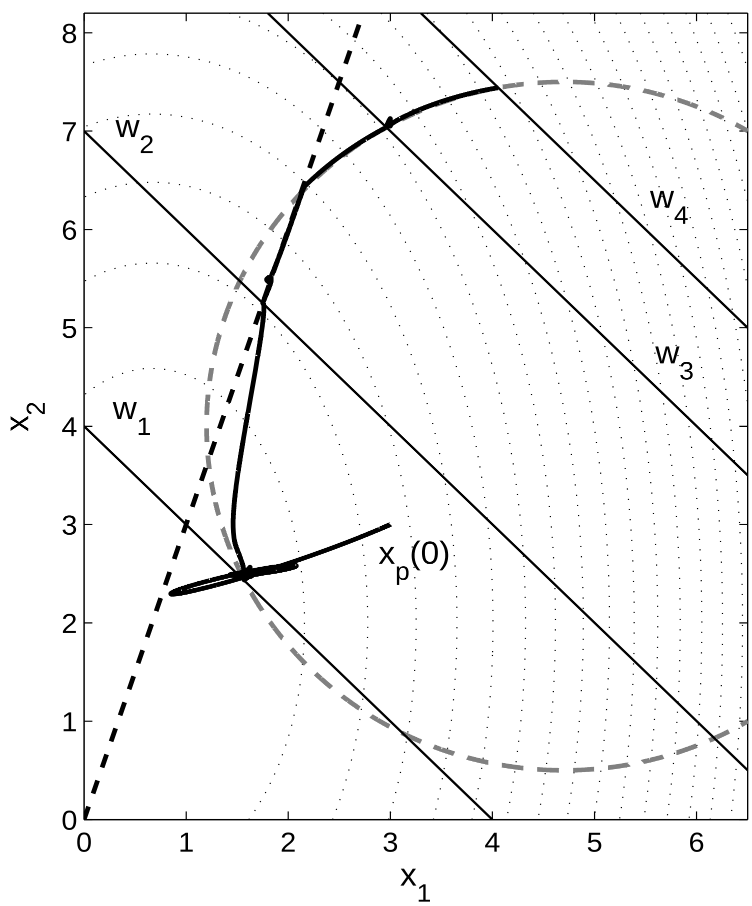

Figure 5.3: Simulated trajectory of the state vector x_p for the close-loop system with max-based KKT controller.

$$\min_{x_p} \quad \frac{1}{2} x_p^\top H x_p + a^\top x_p \tag{5.30a}$$

subject to

$$x_1 + x_2 = w, \tag{5.30b}$$

$$(x_1 - 4.7)^2 + (x_2 - 4)^2 \leq 3.5^2, \tag{5.30c}$$

where $H = \text{diag}(6, 2)$, $a = \text{col}(-4, -4)$, and the value of the exogenous signal w is limited to be in the interval $W = [4, 11.5]$. It can be verified that for this set W and the constraints (5.30b) and (5.30c), the Assumption 5.2.1

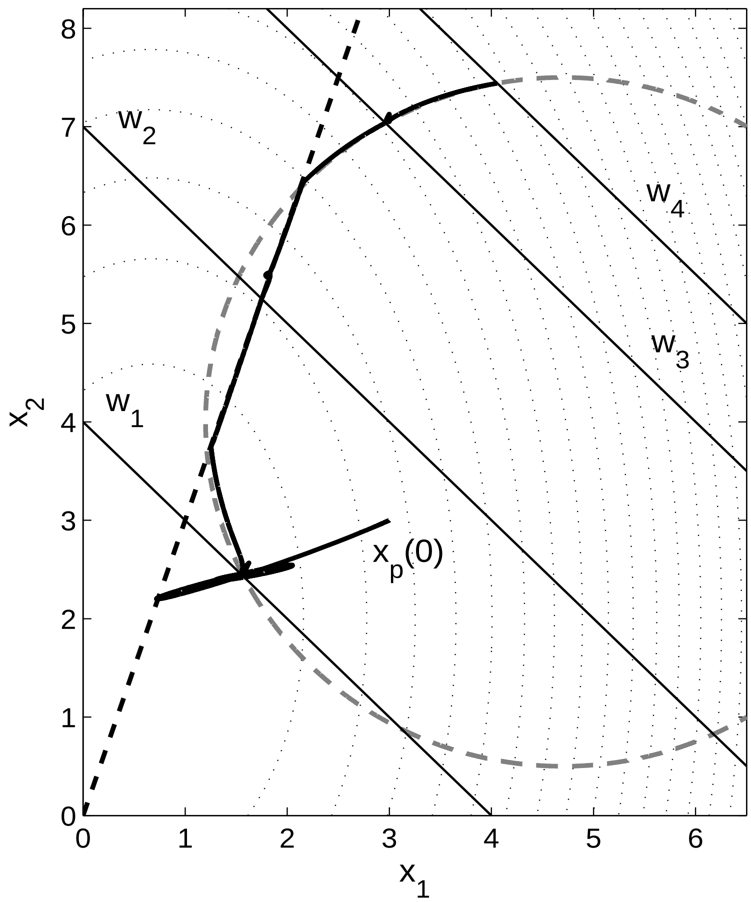

Figure 5.4: Simulated trajectory of the state vector x_p for the close-loop system with saturation-based KKT controller.

holds true. Furthermore, since $A^{-1}B = \frac{1}{7}\begin{pmatrix} -5 & 2 \\ 3 & -4 \\ \hline -1 & -1 \end{pmatrix}$, the condition in the Assumption 5.2.3 is fulfilled. We assume that the complete state vector is available for control, i.e. that the output equation (5.1b) is given by $y = x$. From the dynamics of the state x_3, it follows that in steady-state the equality $x_1 + x_2 - 2x_3 = w$ holds. Therefore, in steady-state, $x_3 = 0$ implies fulfilment of the constraint (5.30b). This implies that for control we can directly use the value of the state x_3 as a measure for violation of this constraint (see Remark 5.3.2).

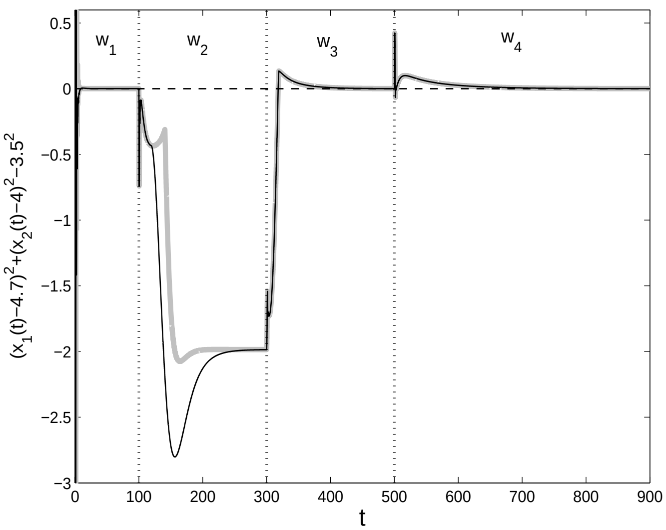

Figure 5.5: Violation of the inequality constraint as a function of time.

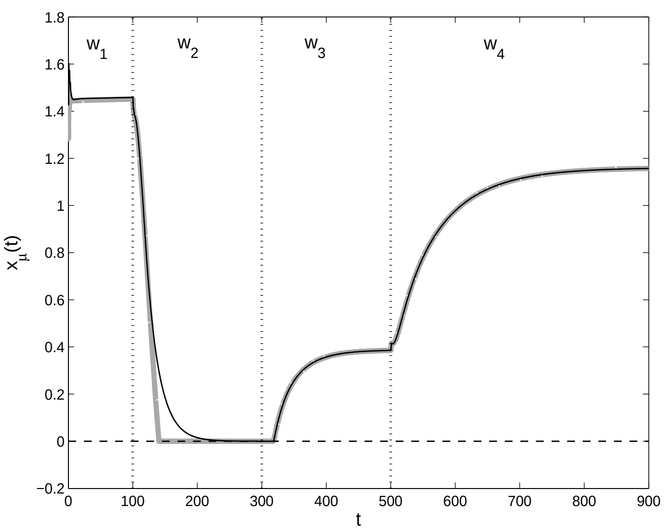

Figure 5.6: Simulated trajectory of the controller state x_μ.

A max-based KKT controller is given by

$$\begin{aligned}
\dot{x}_\lambda &= K_\lambda\, x_3,\\
\dot{x}_\mu &= K_\mu((x_1-4.7)^2+(x_2-4)^2-3.5^2+v),\\
\dot{x}_c &= K_c\left(\begin{pmatrix}1\\1\end{pmatrix} x_\lambda + \begin{pmatrix}2x_1-9.4\\2x_2-8\end{pmatrix} x_\mu + \begin{pmatrix}6&0\\0&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} + \begin{pmatrix}-4\\-4\end{pmatrix}\right),\\
0 \le v &\perp (K_o x_\mu + (x_1-4.7)^2+(x_2-4)^2-3.5^2+v) \ge 0,\\
u &= x_c,
\end{aligned}$$

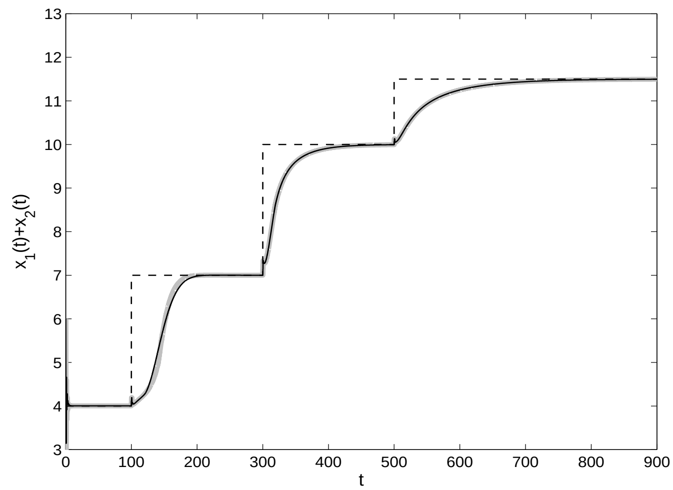

Figure 5.7: The value $x_1 + x_2$ as a function of time.

while a saturation-based KKT controller is given by the same set of differential equations except that the complementarity condition is replaced by $0 \leq v \perp x_\mu \geq 0$.

Thick solid lines in Figure 5.3 and Figure 5.4 represent the trajectories of the state vector x_p which were obtained from simulations of the closed-loop system controlled with the max-based KKT controller (Figure 5.3) and with the saturation-based KKT controller (Figure 5.4). Both controllers were implemented with the gains $K_\lambda = 0.15$, $K_\mu = 0.1$, $K_c = -0.7I_2$, and the gain K_o in the max-based controller was set to 0.5. In both figures, thin solid lines labeled w_i, $i = 1, \ldots, 4$, represent the equality constraint $x_1 + x_2 = w_i$ where $w_i \in \{4, 7, 10, 11.5\}$. At the beginning of the simulation, the value of the exogenous signal w is set to $w_1 = 4$ and the initial vale of the state vector x_p indicated in the figures with $x_p(0)$. At time instances $t = 100$, $t = 300$ and $t = 500$ the value of exogenous signal w is stepwise changed to $w_2 = 7$, $w_3 = 10$ and $w_4 = 11.5$, respectively. In both figures, the tick, dashed circle represents the inequality constraint (5.30c), i.e. the steady-state feasible region for x_p is within this circle. Thin dotted lines represent the contour lines of the objective function (5.30a), while the straight dashed line represents the locus of the optimal point $\tilde{x}_p(w)$ for the whole range of values w in the case when the inequality constraint (5.30c) would be left out from the optimization problem.

The obtained simulation results clearly illustrate the action of the KKT controllers, which guarantee that, following any change in w, the system will settle in the optimal steady-state point with all the constraints satisfied.

Figures 5.5 and 5.6 present the violation of the inequality constraint (5.30c) as a function of time and the trajectory of the state vector x_μ, respectively. Vertical dotted lines in these figures indicate the time instances at which the signal w changes its value, and the labels w_i in between these lines indicate the value of this signal. In both Figures 5.5 and Figure 5.6, the thick gray line corresponds to the trajectory of the system with the saturation-based KKT controller, while thin black line corresponds to the trajectory when the max-based KKT controller is used. Note that the trajectories in Figures 5.5 represent $(x_1(t) - 4.7)^2 + (x_2(t) - 4)^2 - 3.5^2$ and not the minimal Euclidian distance from each point in the trajectory of $x_p(t)$ to the circle in Figure 5.3 and Figure 5.4.

The solid lines in Figure 5.7 represent the trajectory of $x_1(t) + x_2(t)$, i.e. the left hand side of the equality constraint (5.30b) as a function of time. The thick gray line corresponds to the trajectory with the saturation-based KKT controller, while thin black line corresponds to the trajectory when the max-based KKT controller is used. The dashed line in the same figure represents $w(t)$, i.e. the right hand side of the equality constraint (5.30b) as a function of time.

5.6 Stability analysis of the closed-loop system

In this section we address the problem of stability analysis of the closed-loop system, i.e. of the system (5.1) interconnected with a KKT controller in a feedback loop.

For global asymptotic stability, the equilibrium point of the closed-loop system necessarily needs to be unique. Therefore, we start by presenting conditions for uniqueness of the closed-loop equilibrium.

5.6.1 Uniqueness of the equilibrium point

Theorem (5.3.1) states that for any admissible constant-valued exogenous signal $w(t)$, the closed-loop system necessarily has an equilibrium. Furthermore, from the proof of this theorem it follows that for *all* corresponding equilibrium points the values of the state vectors x_p, x_q and x_c are *unique*. It remains to present the conditions which further imply uniqueness of the remaining closed-loop state vectors (x_λ, x_μ) in an equilibrium. To present these conditions, we first recall several definitions and properties from convex optimization.

Let $\tilde{x}_p(w)$ denote a unique solution to (5.2) for some $w \in W$. Each $\tilde{x}_p(w)$ is characterized by a unique set of active constraints in (5.2c), where an inequality constraint $q_i(x_p) \leq r_i(w)$ is defined as active at $\tilde{x}_p(w)$ if $q_i(\tilde{x}_p(w)) = r_i(w)$. We define $\mathcal{I}_a(\tilde{x}_p(w))$ as the set of indices corresponding to the set of active inequality constraints at $\tilde{x}_p(w)$, i.e. $\mathcal{I}_a(\tilde{x}_p(w)) := \{i \mid 1 \leq i \leq k,\ q_i(\tilde{x}_p(w)) = r_i(w)\}$. Furthermore, we make the following standard definitions in connection with the optimization problem (5.2) and its solution $\tilde{x}_p(w)$.

Definition 5.6.1 (LICQ). If for $\tilde{x}_p(w)$ it holds that the set of vectors formed of all columns of $L^\top$ and $\nabla q_i(\tilde{x}_p(w))$, $i \in \mathcal{I}_a(\tilde{x}_p(w))$, is a linearly independent set, we say that the *linear independence constraint qualification* (LICQ) holds at $\tilde{x}_p(w)$. □

In other words, a linear independence constraint qualification states that at $\tilde{x}_p(w)$ the set of active constraints gradients is linearly independent. (Note that all the equality constrains are in the set of active constraints).

Definition 5.6.2 (*Primal degeneracy.*) If at $\tilde{x}_p(w)$ the LICQ is violated, optimization problem (5.2) is characterized by *primal degeneracy* at $\tilde{x}_p(w)$. □

Proposition 5.6.3 *Let $w \in W$ be given and let $\tilde{x}_p(w)$ denote the corresponding unique solution to* (5.2)*. If LICQ holds at $\tilde{x}_p(w)$, the set of Lagrange multipliers satisfying the corresponding KKT conditions* (5.3) *is a singleton.* □

Proof. The proposition trivially follows from (5.3). □

Finally, conditions for uniqueness of a closed-loop equilibrium are summarized in the following proposition.

Proposition 5.6.4 *For all constant input signals $w(t) = w \in W$ such that LICQ holds at $\tilde{x}_p(w)$, the closed-loop system has a unique equilibrium point.* □

Proof. The proposition trivially follows from Proposition 5.6.3. □

5.6.2 Stability analysis for a fixed $w \in W$

Global asymptotic stability analysis of the closed-loop system for a constant value of the exogenous signal $w(t) = w \in W$, for which $\tilde{x}_p(w)$ is not characterized by primal degeneracy, can be performed by searching for an appropriate Lyapunov function.

When the steady-state related optimization problem (5.2) is a quadratic program with linear constraints, the closed-loop system with a KKT controller is a linear complementarity system (Heemels et al., 2000a). Since both types of complementarity integrators can be presented in a piecewise affine framework, the closed-loop system can be equivalently presented as a piecewise affine system (Sontag, 1981; Johansson and Rantzer, 1998), for which efficient stability analysis methods have already been developed (Johansson and Rantzer, 1998; Hassibi and Boyd, 1998; Gonçalves et al., 2003).

When the steady-state related optimization problem (5.2) is given with a (higher order) polynomial objective function and (higher order) polynomial inequality constraints, the corresponding closed loop system is a "polynomial complementarity system". Since both types of complementarity integrators can be presented in a piecewise affine framework, the corresponding closed-loop system can always be represented as a piecewise polynomial system. For such systems, it is possible to perform stability analysis based on the sum-of-squares decompositions of non-negative polynomials (Parrilo, 2000; Lasserre, 2001). This analysis procedure is a direct higher order polynomial generalization of the PWA system analysis from (Johansson and Rantzer, 1998), and can be found in (Prajna and Papachristodoulou, 2003; Papachristodoulou and Prajna, 2005).

In general, for any nonlinear optimization problem (5.2), the closed-loop system can be approximated arbitrarily well with a piecewise affine (or a piecewise polynomial) system, and stability analysis can be performed in that framework.

Note that if a constant value of the exogenous signal $w(t) = w \in W$ is such that the optimum $\tilde{x}_p(w)$ is characterized by primal degeneracy, instead one equilibrium point there might exist a set of equilibria (not a singleton), which is then an invariant set for the closed-loop system. Each equilibrium in this set is characterized by different values of the state vectors (x_λ, x_μ), but unique values of the remaining states. Under additional generalized Slater constraint qualification, see (Pomerol, 1981) for details, the set of equilibria is guaranteed to be bounded. For stability analysis with respect to this set, one has to invoke LaSalle's invariance theorem, see (Khalil, 2002) for a general introduction and (Hespanha, 2004; Sanfelice et al., 2005; Çamlibel et al., 2006), including the references therein, for generalizations of the invariance theorem to hybrid systems.

Finally, note that if the value of the exogenous input signal w is a constant such that the steady-state optimization problem (5.2) is infeasible, the

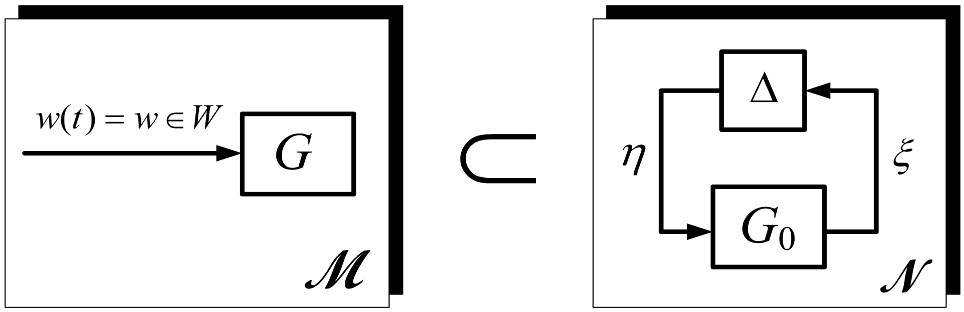

Figure 5.8: Analysis problem of the closed-loop system stability for all $w \in W$ as a robust stability analysis problem of an LTI system affected by structured uncertainties.

closed-loop system has no equilibrium and is therefore necessarily unstable [3].

5.6.3 Stability analysis for all $w \in W$

Ideally, one would desire a proof of the closed-loop system stability for all possible constant values of the exogenous signal $w(t)$, i.e. for $w(t) = w$ where w is any constant in W. Unfortunately, for a general steady-state optimization problem (5.2) and a corresponding KKT controller, proving this desired property is a very difficult task. Still, by accepting some level of conservatism and/or by appropriate modification of the KKT controller, such analysis is possible and, in some cases, can be very efficient.

In this section, we discuss a possibility of performing global stability analysis for all equilibrium points of a closed-loop system by using the results and tools from robust stability theory of linear time invariant (LTI) systems affected by structured uncertainties.

Suppose that $w \in W$ is given, and let G denote a model of the closed-loop system obtained by shifting the origin of the system to its equilibrium point. For each $w \in W$, G is a nonlinear autonomous system. Let $\mathcal{M}$ denote the (infinite) set of all systems G when w varies in W. The desired goal is to check asymptotic stability, with respect to the origin, of each element in $\mathcal{M}$.

One tractable approach to achieve this goal is to define a set of systems $\mathcal{N}$ such that $\mathcal{M} \subset \mathcal{N}$, and where each element of $\mathcal{N}$ has an *appropriate*

[3] By term unstable we mean lack of asymptotic stability with respect to an equilibrium point. Limit cycles or bounded chaotic behaviors are therefore here considered as unstable behaviors.

structure. Here, by appropriate structure we mean that $\mathcal{N}$ can be represented as an LTI system G_0 interconnected with a suitably characterized, structured uncertainty block Δ in the feedback loop, see Figure 5.8. When Δ is characterized by means of integral quadratic constraints (Megretski and Rantzer, 1997), proving stability of each element in $\mathcal{N}$ presents a classical robust stability analysis problem of an LTI system, see e.g. (Scherer and Weiland, 2000). Since $\mathcal{M} \subset \mathcal{N}$, proof of stability for each element in $\mathcal{N}$ implies stability of each element in $\mathcal{M}$.

Note that nonlinearities in a closed-loop system originate from the complementarity integrators in a KKT controller, and from the terms $q(x_p)$, $\nabla q(x_p)x_\mu$, $\nabla J(x_p)$, if indeed these terms are nonlinear in x_p and x_μ.

We have to emphasize that even if one succeeds in formulating the robust stability problem, i.e. in formulating the set $\mathcal{N}$ with an appropriate characterization of Δ, for a general steady-state optimization program (5.2), i.e. for a general KKT controller, this approach can be rather conservative.

Since a presentation of the above described procedure by considering a general steady-state optimization problem would not be insightful, in the following section we will restrict our attention to stability analysis of a closed-loop system when the steady-state related optimization problem (5.2) is quadratic program with linear constrains. In that case $q(x_p)$, $\nabla q(x_p)x_\mu$ and $\nabla J(x_p)$ are linear mappings of x_p and x_μ. Furthermore, for simplicity, we will consider a closed-loop system with a max-based KKT controller. These restrictions allow us to give an insightful presentation of certain problems, and their possible solutions, which are specific for a closed-loop system with a KKT controller.

Note that we do not aim to give an exposition of the relevant results from robust control theory. Still, for completeness of the presentation, we will shortly recall some of those results. For a detailed introduction and for many of the state-of-art results on this topic, we refer to the textbook (Scherer and Weiland, 2000).

Stability of a closed-loop system with max-based KKT controller

Let $w(t) = w \in W$ be a given constant-valued exogenous input to the closed-loop system with a max-based KKT controller and let $\tilde{x}_\mu(w)$ and $\tilde{x}_p(w)$ denote the values of the corresponding state vectors in an equilibrium point.

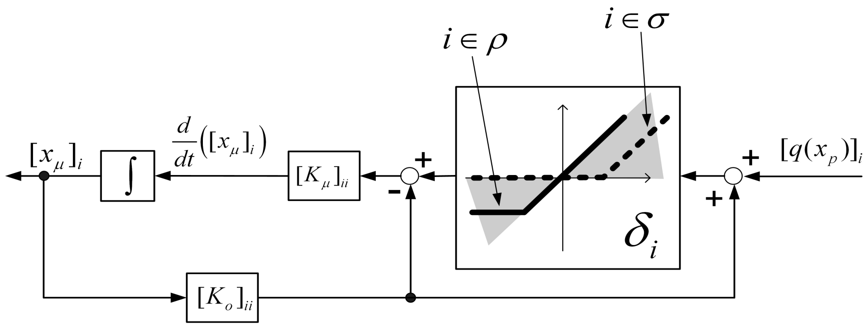

Figure 5.9: Max-based complementarity integrator after performing a state transformation to shift a closed-loop system equilibrium point to the origin. Piecewise affine characteristic in the block δ_i represents the max operator and always lies in the gray shaded sector.

Furthermore, let ρ, ϱ and σ be three index sets defined as follows:

$$\rho := \{i \mid [\tilde{x}_\mu(w)]_i > 0 = [q(\tilde{x}_p(w)) - r(w)]_i\}, \tag{5.32a}$$

$$\varrho := \{i \mid [\tilde{x}_\mu(w)]_i = 0 = [q(\tilde{x}_p(w)) - r(w)]_i\}, \tag{5.32b}$$

$$\sigma := \{i \mid [\tilde{x}_\mu(w)]_i = 0 > [q(\tilde{x}_p(w)) - r(w)]_i\}. \tag{5.32c}$$

After performing a state transformation to shift the equilibrium to the origin, the max-based complementarity integrator can be represented as an integrator interconnected with a static, continuous piecewise affine (PWA) characteristic, where for each set of indices ρ, ϱ and σ this characteristic is (qualitatively) different. However, the PWA characteristic always fulfils a sector bound condition, as illustrated in Figure 5.9. By treating the PWA characteristics as sector bounded *uncertainties*, stability analysis of a closed-loop system for *a set* of constant-valued exogenous signals w can be performed by analyzing (robust) stability of *one* (uncertain) *model*, which is depicted in Figure 5.10. Figure 5.10 presents a closed-loop system with a max-based KKT controller as interconnection of a *nominal system* G_0 with an uncertainty block Δ. The uncertainty Δ accounts for all PWA characteristics from complementarity integrators in all possible equilibria.

The following well-known theorem presents a sufficient condition for exponential stability of a closed-loop system depicted on the right hand side in Figure 5.8. The theorem, as well as an insightful proof, are given for the completeness of the presentation.

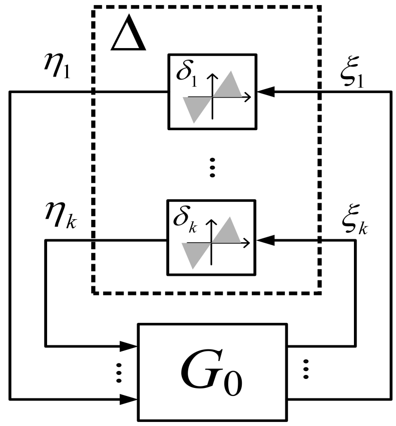

Figure 5.10: Closed-loop system with PWA characteristics from complementarity integrators represented as uncertainties.

Theorem 5.6.5 *Let the system G_0 from Figure 5.8 be an LTI given by its state-space realization*

$$\dot{x} = Ax + B\eta, \tag{5.33a}$$
$$\xi = Cx + D\eta, \tag{5.33b}$$

and let

$$\eta = \Delta(\xi) \tag{5.34}$$

denote the relation (in time-domain) between interconnecting signals ξ and η. The system given by a (5.33), (5.34) is exponentially stable if there exists a symmetric matrix $P \succ 0$ and a Hermitian multiplier Π with

$$\begin{pmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{pmatrix} + \begin{pmatrix} 0 & I \\ C & D \end{pmatrix}^\top \Pi \begin{pmatrix} 0 & I \\ C & D \end{pmatrix} \prec 0, \tag{5.35}$$

such that

$$\begin{pmatrix} \Delta(\xi) \\ \xi \end{pmatrix}^\top \Pi \begin{pmatrix} \Delta(\xi) \\ \xi \end{pmatrix} \geq 0 \qquad \text{for all} \quad \xi. \tag{5.36}$$

□

Proof. Let $P \succ 0$ and Π be such that the matrix inequalities (5.35) and (5.36) hold. Then there exists $\varepsilon \in \mathbb{R}$, $\varepsilon > 0$ such that

$$\begin{pmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{pmatrix} + \begin{pmatrix} 0 & I \\ C & D \end{pmatrix}^\top \Pi \begin{pmatrix} 0 & I \\ C & D \end{pmatrix} + \begin{pmatrix} \varepsilon P & 0 \\ 0 & 0 \end{pmatrix} \prec 0. \tag{5.37}$$

Let $x(t)$, $\eta(t)$ and $\xi(t)$ be arbitrary system trajectories. Left multiplication of (5.37) with $\mathrm{col}(x(t),\eta(t))^\top$ and right multiplication with $\mathrm{col}(x(t),\eta(t))$ implies

$$\frac{d}{dt}x(t)^\top Px(t) + \begin{pmatrix}\eta(t)\\ \xi(t)\end{pmatrix}^\top \Pi \begin{pmatrix}\eta(t)\\ \xi(t)\end{pmatrix} + \varepsilon x(t)^\top Px(t) \leq 0, \tag{5.38}$$

what with (5.36) and (5.34) further implies

$$\frac{d}{dt}x(t)^\top Px(t) + \varepsilon x(t)^\top Px(t) \leq 0. \tag{5.39}$$

Hence, the function $V(x) = x^\top Px$ is a Lyapunov function which proves the stability of the system. □

In a case when Δ represents sector bounded PWA characteristics of the complementarity integrations, see Figure 5.10, it is easy to derive a class of multipliers Π which satisfy the condition (5.36). For example, one can use the standard multipliers for sector bounded static nonlinearities, see e.g. (Megretski and Rantzer, 1997) and the example in the following section. Furthermore, each uncertainty block δ_i from Figure 5.10 can be interpreted as multiplication (in time domain) with a time-varying uncertainty $\delta_i(t)$, where $0 \leq \delta_i(t) \leq 1$ for all $i \in \{1,\ldots,k\}$ and for all $t \in \mathbb{R}$, and one can reside on standard repeated-blok multipliers from classical structured singular value theory, or on less conservative full-block multipliers, see (Scherer and Weiland, 2000) for details.

Since the condition (5.35) is a linear matrix inequality (LMI) in P and Π, its feasibility can be efficiently tested by a dedicated software, e.g. (Sturm, 2001). However, Theorem 5.6.5 will in practice often fail to give any conclusive answer in stability analysis of the closed-loop system with a max-based KKT controller. This is explained with the following observations.

Note that by treating the PWA characteristics from complementarity integrators as sector bounded uncertainties, we do not capture any information about the set W, i.e. about the set of admissible values of the exogenous input w. Furthermore, we do not capture any additional information about the terms $r_i(w)$, $i = 1,\ldots,k$ and $h(w)$ from (5.2).

For each PWA characteristic we only utilize the information that it belongs to a certain sector and therefore the model from Figure 5.10 accounts not only for the set of systems whose stability we desire to prove, but it also accounts for an infinite set of additional systems. For practical reasons, let us denote the latter set with $\mathcal{G}$. The set $\mathcal{G}$ necessarily includes a system

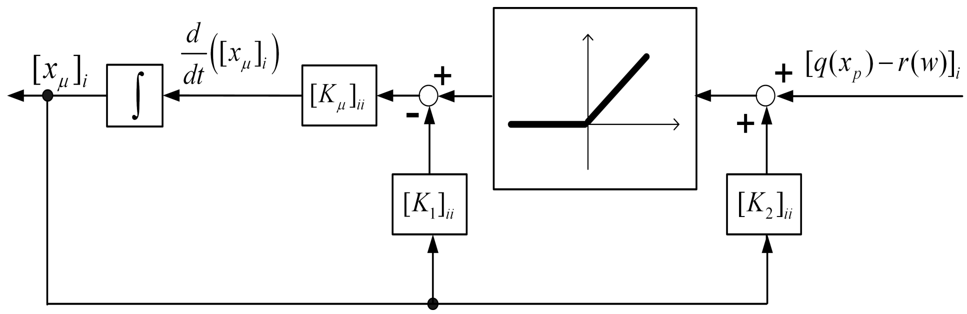

Figure 5.11: First order approximation of max-based complementarity integrator.

which is obtained when each complementarity integrator in the closed-loop system is replaced by pure integrator. This is easy to observe, since the uncertainty block δ_i from Figure 5.9 also accounts for a constant positive gain less or equal then 1, in which case the whole scheme from Figure 5.9 reduces to a pure integrator with a constant gain. If the total number of constraints in the optimization problem (5.2) is bigger that the dimension of the partial state vector x_p, i.e. if $l + k > m$ (see (5.2)), the above described robust stability analysis approach will necessarily fail to give any conclusive answer. Reason for this is the following: when $l + k > m$, state matrix (matrix "A" in a state-space realization) of the closed-loop system where each complementarity integrator is replaced with a pure integrator is necessarily singular. Since this particular system is an element of $\mathcal{G}$, we conclude that, when $l + k > m$, there always exists at least one system in $\mathcal{G}$ for which there does not exist a Lyapunov function with strictly negative definite time derivative along the system trajectories. Therefore, when $l + k > m$, linear matrix inequalities from a robust stability analysis test cannot be strictly feasible.

There is a simple remedy for the above problem. In a case when $l+k > m$, one can easily modify a max-based KKT controller to obtain a closed-loop system for which it is possible to perform stability analysis for all values of $w \in W$. This modification includes replacement of (some of) the complementarity integrators with their first order approximations. Here, by the term *first order approximation of a max-based complementarity integrator* we denote a system represented with a block scheme in Figure 5.11 where $[K_1]_{ii} \neq [K_2]_{ii}$ (for comparison, see the scheme of a max-based complementarity integrator from Figure 5.1). Replacement of a max-based complementarity integrator with its first order approximation is seen as equivalent to replacement of a pure integrator with a first order LTI system.

Note that a max-based complementarity integrator consists of two dynamical modes: one corresponds to a first order LTI system and the other to a pure integrator. In a first order approximation of a max-based complementarity integrator, both dynamical modes correspond to a first order LTI system. Usage of the first order approximations does not guarantee that the corresponding inequality constraints (5.2c) will be satisfied in a steady-state. However, for a fixed $w \in W$, one can always guarantee that the violations of these constraints will be arbitrarily small, if the gains $[K_1]_{ii}$ and $[K_2]_{ii}$ (see Figure 5.11) are chosen in such a way that the value $|[K_1]_{ii} - [K_2]_{ii}|$ is made sufficiently small. The benefit of using the first order approximations is that it allows for the usage of robust stability analysis tools in stability analysis of a closed-loop system for all $w \in W$.

If the total number of constraints in the optimization problem (5.2) is bigger that the dimension m of the partial state vector x_p, i.e. if $n_i := l+k > m$, and if $n_i - m$ complementarity integrators in the KKT controller are replaced with its first order approximations, the stability analysis is possible, i.e. the LMIs in the analysis procedure are no longer deemed to be not strictly feasible.

Stability analysis procedure of a closed-loop system with a KKT controller for all $w \in W$ is illustrated in the following example.

5.6.4 Example

Consider a third-order system of the form (5.1) with (5.1a) given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \hline \dot{x}_3 \end{pmatrix} = \left(\begin{array}{cc|c} -2.5 & 0 & -5 \\ 0 & -5 & -15 \\ \hline 0.1 & 0.1 & -0.2 \end{array}\right) \begin{pmatrix} x_1 \\ x_2 \\ \hline x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \hline -0.1 \end{pmatrix} w + \begin{pmatrix} 2.5 & 0 \\ 0 & 5 \\ \hline 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad (5.40)$$

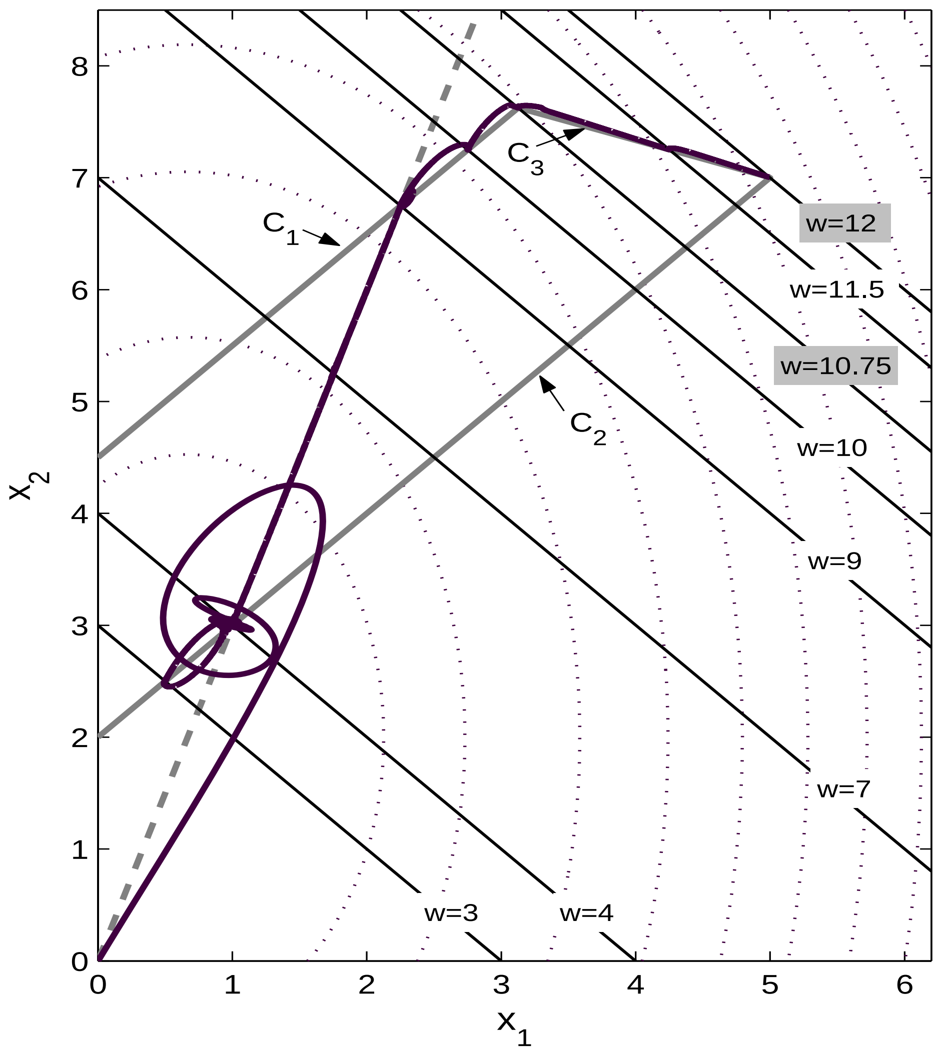

Figure 5.12: Simulated trajectory of the state vector x_p.

where $x_p = \mathrm{col}(x_1, x_2)$, $x_q = x_3$ and $u = \mathrm{col}(u_1, u_2)$. The associated steady-state related optimization problem is defined as follows:

$$\min_{x_p} \quad \frac{1}{2} x_p^\top H x_p + a^\top x_p \tag{5.41a}$$

subject to

$$x_1 + x_2 = w, \tag{5.41b}$$

$$-x_1 + x_2 \leq 4.5, \tag{5.41c}$$

$$x_1 - x_2 \leq -2,, \tag{5.41d}$$

$$x_1 + 3x_2 \leq 26, \tag{5.41e}$$

where $H = \text{diag}(6, 2)$, $a = \text{col}(-4, -4)$, and the value of the exogenous signal w is limited to be in the interval $W = [3, 12]$.

It can easily be verified that Assumption 5.2.1 and Assumption 5.2.3 hold for (5.40), (5.41) and the given set W. We assume that the complete state vector is available for control, i.e. that the output equation (5.1b) is given by $y = x$. From the dynamics of the state x_3, it follows that in steady-state the equality $x_1 + x_2 - 2x_3 = w$ holds. Therefore, in steady-state, $x_3 = 0$ implies fulfilment of the constraint (5.41b). This implies that we can use the value of the state x_3 as a measure for violation of this constraint (see Remark 5.3.2).

Controller and simulation results. For the simulation of the closed-loop system response, we have used the max-based KKT controller with the gains $K_\lambda = 0.5$, $K_\mu = 0.4I_3$, $K_c = -8I_2$ and $K_o = I_3$. The results of the simulation are presented in Figure 5.12. The black, thick, solid line in the figure represents the trajectory of the partial state vector $x_p = \text{col}(x_1, x_2)$ from the closed-loop system response to the step-wise changes in the exogenous input w. The straight, black solid lines labeled with w represent the equality constraint (5.41b) and for each line the corresponding value of w is indicated in the figure. At the beginning of the simulation, the state of the closed-loop system in at the origin and $w = 3$. At the time instant $t = 100$, the step change of the exogenous signal w is applied, in which w is set to 4. This step-wise strictly increasing changes of w are continued in the time intervals of $\Delta t = 50$ and the value of w after each change is indicated in the figure. The last applied change set the value of w to 12. Gray, solid straight lines labeled C_1, C_2 and C_3 represent the inequality constraints (5.41c), (5.41d) and (5.41e), respectively, and the steady-state feasible region is in the interior and on the boundary of the convex polyhedron presented in the figure. Thin dotted lines represent the contour lines of the objective function (5.41a), while the straight dashed line represents the locus of the optimal point $\tilde{x}_p(w)$ for the whole range of values w in the case when all the inequality constraints in the optimization problem (5.41) would be left out.

The obtained simulation results clearly illustrate that the desired closed-loop system behavior is achieved: for any admissible constant-valued w the close-loop system has settled in the optimal steady-state point, where optimality is defined with (5.41).

Primal degeneracy. Consider the solution $\tilde{x}_p(w)$ of the optimization problem (5.41) for $w = 10.75$ and for $w = 12$. In both cases, $\tilde{x}_p(w)$ is characterized by three active constraints (two inequality and one equality constraint, see Figure 5.12). Since the dimension of x_p is two, the linear independence constraint qualification cannot hold for these points, i.e. for $w = 10.75$ and for $w = 12$, $\tilde{x}_p(w)$ is necessarily characterized by primal degeneracy. It can be verified that for $w = 10.75$ and for $w = 12$, the closed-loop system is characterized by a set of equilibria, which is not a singleton. For example, when $w = 10.75$ and the closed-loop system is in steady-state, the state vectors (x_λ, x_μ) of the KKT controller are given by $x_\lambda = (-13 - 2\gamma)$, $x_\mu = \text{col}(1.75 - \gamma, 0, \gamma)$ where γ is a number between 0 and 1.75.

Due to the existence multiple equilibria, any search for a Lyapunov function to prove asymptotic stability of an equilibrium point corresponding to $w = 10.75$ or $w = 12$ will necessarily fail to provide a conclusive answer. A possible remedy for this problem is to replace some of the complementarity integrators with their first order approximations, as we will see later in this example.

Note also that for any constant-valued input w, such that $w > 12$, the closed-loop system is necessarily unstable since in this case the constraints from (5.41) are infeasible.

Controller modification. Since the total number of constraints in (5.41) is 4, and since the dimension of the partial state vector x_p is 2, to be able to perform asymptotic stability analysis for equilibria corresponding to all constant values of w, we have replaced two complementarity integrators in the KKT controller with their first order approximations. The modified integrators are those corresponding to the constraints (5.41d) (C_2 in Figure 5.12) and (5.41e) (C_3 in Figure 5.12), and in both modifications we have used the gains $[K_1]_{ii} = 1$ and $[K_2]_{ii} = 0.999$ (see Figure 5.11), while all the other gains in the controller are left unchanged.

One of the consequences of this modification is that the closed-loop system is no longer guaranteed to satisfy the corresponding constraints for all admissible values of w. However, the violations of those constrains, which are shown in Figure 5.13, are very small and for some practical application could be neglected.

The three labeled curves in Figure 5.13 represent the steady-state violations of the three inequality constrains. For each w, the value for which the corresponding point in the curve C_2 exceeds zero, is the value for which the constraint (5.41d) will be violated in the equilibrium of the closed-loop system corresponding to that w. Line C_1 in Figure 5.13 corresponds to the violation of the constraint (5.41c), while the line C_3 corresponds to the violation of (5.41e). Note that, since the complementarity integrator corresponding to the constraint (5.41c) (C_2) was not replaced by a first order approximation, this constraint will be satisfied for all w.

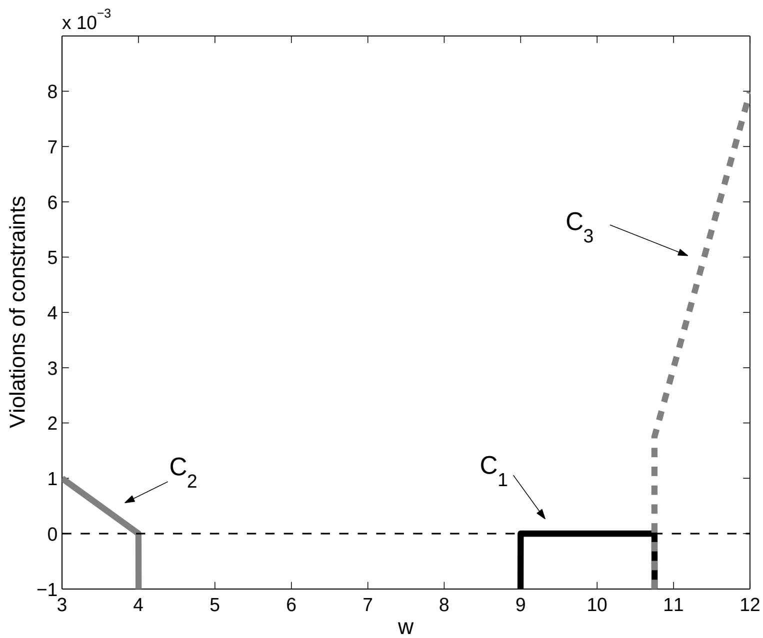

Figure 5.13: Violations of steady-state constraints for different values of the exogenous input w.

As a result of the modification of the KKT controller, for any w there now exists a *unique* equilibrium point (including all $w > 12$). Furthermore, we can now prove asymptotic stability of the equilibrium for each $w \in \mathbb{R}$, what is done in the following paragraph.

Multipliers and stability analysis. Let Figure 5.10 represent the closed-loop system with the *modified* KKT controller, and with δ_i, $i = 1, \ldots, 3$, corresponding to the three PWA characteristics from the (modified) complementarity integrators. For practical reasons, we denote the relations between the signals ξ_i and η_i, see Figure 5.10, with $\eta_i = \delta_i(\xi_i)$, $i = 1, \ldots, 3$, and define the following abbreviations $\xi = \mathrm{col}(\xi_1, \xi_2, \xi_3)$, $\eta = \mathrm{col}(\eta_1, \eta_2, \eta_3)$, $\Delta(\xi) = \mathrm{col}(\delta_1(\xi_1), \delta_1(\xi_2), \delta_1(\xi_3))$, i.e. $\eta = \Delta(\xi)$. The fact that each $\delta_i(\cdot)$ belongs to the sector as presented in Figure 5.9, implies the following inequality:

$$(\xi_i - \delta_i(\xi_i))\delta_i(\xi_i) \geq 0, \quad \text{for all} \quad \xi_i \in \mathbb{R} \quad \text{and} \quad i = 1, 2, 3, \qquad (5.42)$$

or equivalently

$$\begin{pmatrix} \delta_i(\xi_i) \\ \xi_i \end{pmatrix}^\top \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \delta_i(\xi_i) \\ \xi_i \end{pmatrix} \geq 0, \quad \text{for all} \quad \xi_i \in \mathbb{R} \quad \text{and} \quad i = 1, 2, 3. \tag{5.43}$$

From (5.43) it is easy to verify that the inequality

$$\begin{pmatrix} \Delta(\xi) \\ \xi \end{pmatrix}^\top \begin{pmatrix} Q(y) & S(y) \\ S^\top(y) & 0 \end{pmatrix} \begin{pmatrix} \Delta(\xi) \\ \xi \end{pmatrix} \geq 0 \tag{5.44}$$

holds for all $y = \text{col}(y_1, y_2, y_3) \in \mathbb{R}^3$, $y > 0$, and all $\xi \in \mathbb{R}^3$, where $Q(y) = \text{diag}(-2y_1, -2y_2, -2y_3)$ and $S(y) = \text{diag}(y_1, y_2, y_3)$. In other words (5.44) defines a set of multipliers Π (these are standard multipliers for sector bounded nonlinearities) with the vector y as variables.

With the above derived multipliers, an LMI solver successfully finds a feasible solution for the corresponding LMIs in Theorem 5.6.5, what proves asymptotic stability of the equilibrium of the closed-loop system for each $w \in \mathbb{R}$.

5.7 Conclusions

We have presented a control design procedure as a solution to the problem of regulating a general linear time-invariant dynamical system to a time-varying economically optimal operating point. The system was characterized with a set of exogenous inputs as an abstraction of time-varying loads and disturbances acting on the system. Economic optimality was defined through a constrained convex optimization problem with a set of system states as decision variables, and with the values of exogenous inputs as parameters.

A distinguishing, advantageous feature of the presented approach is that it offers an explicitly defined controller structure as a solution, i.e. the resulting controller is not based on solving on-line the corresponding optimization problem. We have presented well-posedness conditions for the developed control structure.

The presented control design procedure is a formalization, generalization and extension of the methods we have used to design a price-based power balance and congestion management controller in Chapter 2.

6

Conclusions and recommendations

In this thesis we have investigated the use of price signals in real-time power balance control, congestion control and ancillary service scheduling in electrical power systems. In this chapter we present an overview of the main results and contributions and discuss open problems, possible directions and starting points for future research.

6.1 Contributions

The main contributions of the thesis are:

- A novel, distributed, price-based control structure for real-time optimal power balance control and congestion management.
- A novel, robust, hybrid MPC control scheme for power balance control with hard constraints on line power flows and network frequency deviations.
- The concept of Autonomous Power Networks as a concise formulation to deal with economic, technical and reliability issues in power systems with a large penetration of dispersed generation units.
- A novel operational structure of ancillary service markets.
- A constrained steady-state optimal control based on dynamical complementarity controllers derived from the Karush-Kuhn-Tucker optimality conditions.

6.1.1 Real-time price-based power balance and congestion control

Due to large-scale integration of distributed generation with large amounts of renewable energy based units, future power systems will be characterized by significantly larger uncertainties. Success of power balance and congestion management control schemes in present power systems heavily relies on relatively accurate predictions of future systems state, as the vast majority of power production in those systems is scheduled in an open-loop manner. Feasibility, reliability and economical efficiency and effectiveness of future power systems will increasingly relay on real-time, feedback control solutions.

In Chapter 2, we have considered the problem of real-time, price-based, economically optimal power balance control and congestion management. We have designed a dynamic feedback controller for optimal real-time update of electricity prices. The developed controller reacts on a measure of power imbalance in the system and on measured violations of line flow limits in a transmission network. The output of the controller is a vector of nodal prices. Each nodal price is communicated to the production/consumption units at the corresponding network node, which then autonomously react on this price by adjusting their production/consumption levels to maximize their own benefits from producing/consuming electrical energy.

Under the hypothesis of global asymptotic stability of the closed-loop system, we have proven that the developed controller will continuously drive the system towards the equilibrium where all the network constraints are satisfied, and where the total economical benefit of the system is maximized. In other words, we have proven that the controller will, based on the measurements from the current state of the system, always provide the correct nodal prices. Furthermore, the proposed control structure is characterized by certain properties which makes it especially suitable for practical applications. Summarized, those properties are:

- The *only* system parameters that are explicitly included in the control law are the transmission network parameters, i.e. network topology and line impedances. To provide the correct nodal prices, the controller requires no knowledge of marginal cost/benfit functions of producers/consumers in the system (neither is it based on the estimates of those functions).
- The controller is given in an explicit form, i.e. it is not based on solving

an optimization problem on-line. The nodal prices updates are based on simple, explicitly defined and easily comprehensible rules.

- The transmission network structure is preserved in the controller, allowing for its distributed implementation. More precisely, the control law can be implemented through a set of *nodal controllers*, where one nodal controller (NC) is assigned to each node in the network. Each NC acts only on locally available information, i.e. on measurements from the corresponding node and on the information obtained from NC's of the adjacent nodes. The communication network graph among NC's is therefore the same as the graph of the underlying physical network. Any change is the network topology requires only simple adjustments in NC's that are local to the location of the change. The distributed control structure is specially advantageous taking into account the large-scale of electrical power systems.

The effectiveness of the proposed distributed control scheme has been illustrated on the IEEE 39-bus New England test system with 10 price elastic generating units.

The main assumption of the developed control scheme is that the DC approximation of the power flows in the network is allowed.

6.1.2 Hybrid MPC control scheme for power balance control

On the time scale of one to several seconds, increased uncertainties in future power systems will introduce relatively large, suddenly occurring power fluctuations in line power flows, threatening the security of the system by approaching its stability limits. Due to the decrease in inertia of the system, fast acting control loops for frequency and power balance control will become crucial.

With the novel model predictive control (MPC) scheme presented in Chapter 3, we have shown the potential of using price signals in contributing to control of fast sudden imbalances. The developed control scheme is shown to be efficient and effective in limiting large frequency excursions and extensive line overloads.

Line flow constraints in a transmission system are specified for steady-state operation of the system, and we have shown that the price-based optimal controller developed in Chapter 2 guarantees fulfilment of those constraints in steady-state. In other words, in Chapter 2 those constraints were treated as *soft constraints*, i.e. their temporary violations were allowed during transient periods.

By combining the "basic" nodal-price controller developed in Chapter 2 with a suitably defined MPC controller, we could impose *hard constraints* on the maximal transient violations of the steady-state related line flow limits. Furthermore, we could impose *hard constraints* on the maximal network frequency deviations.

The MPC controller served only to add corrective signals to the output of the basic nodal-price controller, i.e. to the nodal prices, and acted only when the predictions indicated that the imposed hard constraint will be violated. In any other case, output of the MPC controller was zero and only the basic nodal-price controller was active. Due to this exclusively *corrective function* of the MPC control action, the closed-loop system performance is less sensitive to errors in the model used for predictions.

Under certain hypotheses, we have proven asymptotic stability of the complete closed-loop system. Simulations illustrated the effectiveness of the proposed control scheme.

6.1.3 Autonomous power networks

Due to the increased uncertainties, reliable and economically optimal provision of ancillary services will become increasingly important in future power systems. Price inelasticity of customers is one of the biggest flaws of the present electricity markets. Active involvement of consumers in both energy and ancillary service markets, is an issue with the largest, yet unexploited potential for increasing efficiency and effectiveness of electrical power systems.

In Chapter 4 we have presented the concept of Autonomous Power Networks as a concise formulation to deal with economic, technical and reliability issues in future power systems characterized by large amount of distributed generation units. The autonomous power network (AN) was defined as an aggregated set of producers and consumers, which is capable and responsible for complying to the set of integration protocols required for efficient and reliable operation of the overall power system. In other words, an AN was presented as a major building block of a power system in all of its layers, i.e. physical, economic and reliability.

Specifically, we have introduced an AN as a new market entity that enables creation of competitive markets for ancillary services which are characterized by large amount of players on both supply and consumption side. Each

AN is presented to the rest of the system as both potential *producer and consumer* of electrical power and ancillary services. This is in contrast with the present power systems, where an independent system operator (ISO) coordinates the ancillary service markets and at the same time provides the only demand for them. In an AN based power system, the ISO remains as a coordinator of the markets.

Furthermore, an AN enables active involvement of virtually all consumers in energy and ancillary service markets. Dealing with a limited set of consumers, an AN presents both the technical infrastructure and the market support for a small consumers, e.g. residential loads, for its integration into the markets.

6.1.4 Constrained steady-state optimal control

In Chapter 5 we have considered the problem of constrained steady-state optimal control. We have presented a control design procedure as a solution to the problem of controlling a general linear time-invariant dynamical system to an economically optimal operating point. The system was characterized with a set of exogenous inputs as an abstraction of time-varying loads and disturbances acting on the system. Economic optimality was defined through a constrained convex optimization problem with a set of system states as decision variables, and with the values of exogenous inputs as parameters.

A distinguishing, advantageous feature of the presented approach is that it offers an explicitly defined controller structure as a solution, i.e. the resulting controller is not based on solving on-line the corresponding optimization problem.

The presented control design procedure is a formalization, generalization and extension of the methods we have used to design a price-based power balance and congestion management controller in Chapter 2.

6.2 Open problems and ideas for future research

There are several interesting research directions and open problems in connection to the research presented in this thesis. In the following several subsections we address and discuss those issues.

Distributed MPC

Recently, the topic of distributed model predictive control (MPC) has gained a significant interest in control system community. Many theoretical results and application case studies have already been reported, however, almost exclusively dealing with linear systems.

In Chapter 3, we have complemented the *distributed* price-based controller with a *centralized* hybrid MPC. Development of efficient distributed hybrid MPC is therefore of great interest for this particular application.

Stability analysis by exploiting sparsity of power system models

Power system dynamic behavior in the low frequency range is described through a set of sparse differential algebraic equations (DAE), see e.g. (Pal and Chaudhuri, 2005). The sparsity in the DAE comes through algebraic power flow equations. In virtually all current stability analysis and control synthesis approaches for power systems, sparse algebraic equations are eliminated at the loss of this structure (Pal and Chaudhuri, 2005).

In Chapter 2, in designing the price-based real-time power balance controller, we have successfully exploited the sparsity of power flow equations, with the result of obtaining a *distributed* control structure. One interesting and important research question is the following:
Is it possible to develop a stability analysis and control synthesis technique that exploits the sparsity inherent to power system models, and which would not be too conservative for practical applications?

Receding horizon based operation of power and ancillary services markets

In Chapter 4 we have presented an autonomous networks based power system, where each autonomous network (AN) schedules its production and ancillary services by its active involvement in competitive energy and ancillary servies markets. If each AN performs this scheduling (including unit commitment) in a receding horizon, the markets also operate on a receding horizon principle and coordinate the actions among ANs. Due to the coupling between subsequent time intervals, e.g. ramp constraints, minimum up and down times in unit commitment, etc., and due to the couplings between energy and ancillary services markets, the whole system becomes a complex dynamical system. A challenging task is to analyse the behavior of such a system and to put requirements on each player so that feasibility, stability and optimality will be guaranteed at all times.

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Acknowledgements

During the past four years, my work has been supported, encouraged and inspired by many people. It is my great pleasure that I have reached the point when I can look back in time and express my gratitude to them all.

First of all, I would like to thank my first promotor, Prof. Paul van den Bosch, for his guidance and continuous support during the last four years. I will always be grateful for the patience, time and effort he has invested in my work. I would especially like to thank him for his trust in me during the early stages, and his strong support during the final stages of my PhD journey.

I would like to thank my second promotor, Prof. Wil Kling, for his support as the project leader and for many valuable suggestions and comments.

My gratitude also goes to my co-promotor, Dr. Mircea Lazar, for a fruitful and pleasant collaboration during the last year. I truly appreciate his contribution and his efforts in improving readability of my thesis and deeply admire his dedication to research and his hard-working attitude. I also thank him for all the fun we had during the past years.

I would like to thank Dr. Ad Damen for our pleasant discussions during the first years of my PhD studies. I very much enjoyed being a coach for his course on robust control, what was also a perfect learning opportunity for me.

I am grateful to Prof. Okko Bosgra, Prof. Ian Hiskens and Prof. Hans Schumacher for being the members of the core committee for this thesis and for their valuable suggestions and comments. A special word of gratitude goes to Prof. Ian Hiskens for making an effort to travel to Eindhoven.

Furthermore, I would like to thank Dr. Siep Weiland for several pleasant and intriguing discussions, and Dr. Maurice Heemels for his advices and help on the topic of well-posedness of complementarity systems.

Thanks to my fellow PhD students (some already Dr.'s) Mark, Maarten, Nelis, John, Aleksandar, Michiel, Heico, Patricia, Bart, Leo, Satya, Femke, Patrick, Michal and Jasper for creating a relaxed and friendly atmosphere. Special thanks to Nelis for being a perfect officemate; to Mark and Maarten for the company and all the fun we had during the late working hours; to Aleksandar for his help on many different occasions.

I also wish to thank my colleagues in the IOP-EMVT "Intelligent power systems" project, Johanna, Lou, Pieter, Anton, Frans, Johan, Georgios, Muhamad, Roald, Jody, Cai and Sjef, for a nice time during our numerous meetings. Special thanks to Anton for his help in many various situations.

Thanks to my former master students Kemal, Erik and Giel. I honestly enjoyed working with them all.

Also, thanks to Barbara and Udo. They are the reason I never had to worry about any administrative and computer problem that came along.

Finally, I would like to thank my family for all their love and support during these four years.

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Eindhoven, July, 2007

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