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DOI:
10.1111/j.1538-4632.1996.tb00925.x

Document status and date:
Published: 01/01/1996

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

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An Efficient Search Strategy for Site-Selection Decisions in an Expert System

This paper describes an algorithm for spatial search, which is used in an expert system for site selection. The algorithm, named ProfMat, is able to find the best site in the area of interest even when the number of possible sites is large and many decision criteria are involved. Compared to commonly used search procedures, ProfMat improves the efficiency of spatial search in two ways. First, the best site is identified through an iterative rather than a linear process of selection and evaluation of optional sites. Second, an area is searched by narrowing down the focus to increasingly smaller areas and, thus, sites are evaluated as much as possible groupwise. The ProfMat procedure is illustrated by analyzing the problem of retail site selection. A comparison with alternative search procedures shows that ProfMat considerably reduces the evaluation costs needed to find the best site. The implementation of the algorithm in an expert system shows how ProfMat can be used in combination with specialist's knowledge to solve site-selection problems. The efficiency of the procedure allows considering large sets of optional sites, so that it may improve the quality of the outcome.

1. INTRODUCTION

The selection of sites for locating an activity generally involves the evaluation of optional sites on several suitability criteria. Viewed this way, site selection can be approached as the problem of selecting one or more alternatives from a given set of discrete choice alternatives (possible sites) based on a common set of criteria, which is known as the discrete multi-criteria-decision-making (or in short, MCDM) problem. Discrete MCDM problems have received a lot of attention both in the fields of spatial planning (for example, Keeney 1980; Voogd 1983; Janssen 1991; Massam 1993) and business management (see Sawaragi, Inoue, and Nakayama 1986; Kirkwood 1992). In this research field, a large variety of methods has been developed for combining scores across criteria to arrive at an overall suitability score. The appropriate method depends on

This research was supported by the Technology Foundation (STW).

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Geographical Analysis, Vol. 28, No. 2 (April 1996) © 1996 Ohio State University Press

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characteristics of the decision problem, such as, for example, the goals of the decision maker and the measurement scale of data involved (for a review, see Ozernoy 1986). Discrete MCDM methods help decision makers to judge the suitability of specific sites, but they do not specify a search procedure for identifying the best sites from a given choice set.

As Keeney (1980, p. 15) has argued, in many cases there are too many possible sites in the study area to allow a thorough evaluation of all options. Often, part of the sites can easily be identified as being inappropriate by using one or more all-or-nothing (screening) criteria. However, even after eliminating sites that do not meet these criteria, the number of remaining sites may still be too large to permit an exhaustive search. A possible strategy would be to use additional screening criteria, to further reduce the choice set. However, this would introduce the risk of eliminating some or all of the best sites. Keeney concludes, therefore, that this problem requires the balancing of the costs and effort of evaluating a large number of options against the risk of eliminating suitable candidates. Given this balance, a more efficient site-selection procedure means a larger set of sites that can be considered and a smaller probability of removing suitable sites from the choice set. Due to this mechanism, improving the efficiency of the selection strategy may lead to better outcomes.

This paper introduces an efficient algorithm for identifying the best site from a given set of optional sites. The algorithm gives the same solution that would be generated by an exhaustive search procedure, but reduces search costs. The best site is identified through an iterative process of selection and evaluation of candidates and through focusing on increasingly smaller areas within the study area. The algorithm is implemented in an expert system. The system can be used to solve site-selection problems when it is complemented with knowledge specific for the activity to be located and the area to be searched. Therefore, the system is best viewed as an expert system for site-selection problems in general.

The paper is organized as follows. The first section briefly reviews search procedures used in existing systems for site selection, to situate our approach in a wider context. The next section introduces the algorithm we propose and illustrates the way it operates by analyzing a retail-site-selection problem. Furthermore, a comparison with alternative search procedures is made, to demonstrate the efficiency of the algorithm. Next, the generic expert system based on this algorithm is described. Finally, some conclusions and possible ways for future research are discussed.

2. REVIEW OF SEARCH STRATEGIES

A variety of search procedures are used in existing geographic information systems (GIS), decision support systems, and expert systems for site selection. In addition, several search procedures have been suggested in applied studies on location planning. In this section, we will briefly review these search procedures and place our approach against this background. We emphasize that in this study we focus on problems of selecting *single* sites, in contrast to problems of optimizing location *configurations* for networks of activities. Therefore, we will not consider procedures for locating networks, such as algorithms for solving location-allocation models (see Ghosh and Rushton 1987; Malczewski and Ogryczak 1990).

To deal with large choice sets, most studies suggest a two-staged procedure for finding the best sites. The aim of the first stage is to select candidate sites for further consideration. Then, in the second stage, the candidates are evaluated in depth to identify the best sites. Various strategies have been proposed for narrowing down the choice set in the first stage. First, in many cases it is appropriate to select feasible sites (or to eliminate alternatives) based on one or more screening criteria (Keeney 1980, p. 29; Tversky 1972; Mercurio 1984). This strategy has been used for site selection in a GIS environment (Carver 1991) and in several expert systems described in the literature (for example, Findikaki 1990; Suh, Kim, and Kim 1990; Han and Kim 1990; Rouhani and Kangari 1990).

Second, Reitsma (1990) suggests selecting the sites for which the attribute profile matches a set of requirements. The requirements explicitly depend on the characteristics and interests of the activity to be located. Reitsma stresses the importance of considering the attributes of a location as a set rather than individually, since locations with different attribute profiles may meet the same set of requirements. If the number of sites selected is larger than one, a second stage follows, wherein the best site is identified based on optimization criteria (for example, trading potential). This site-selection procedure has been implemented in the software REPLACE (Reitsma 1990) using decision support system, GIS, and expert-system technology.

Third, the spatial information system ISIS developed by Diamond and Wright (1988) supports a two-staged procedure, where the aim of the first stage is to select from a given choice set the subset of noninferior sites. Sites that are outperformed by at least one other site on all relevant criteria (objectives) are inferior, irrespective of the relative importance of objectives, and are eliminated from the choice set. In the second stage a final choice is made by (subjectively) weighting objectives.

Furthermore, several authors (Breheney 1988; Mercurio 1984; Ghosh and McLafferty 1987; Khosaka 1989) in retailing research have suggested hierarchical search procedures. Ghosh and McLafferty argue that for finding locations with high trading potential, three levels of scale should be distinguished. From high to low level these are market areas (for example, city areas), areas at intermediate level (say, market sectors), and sites. They advocate a top-down procedure, wherein market areas, market sectors, and sites are selected in a step-wise process. Mercurio describes a similar procedure in a (hypothetical) case of locating department stores, but he distinguishes shopping areas as an extra level in between the market-sector and site levels.

Finally, we can distinguish a group of methods that focuses on the problem of identifying promising or interesting alternatives as a first stage in site selection. To this end, various techniques are proposed. We mention genetic algorithms (Guimaraes Pereira, Peckham, and Antunus 1993), potential measures (Breheney 1988), and mathematical programming (see Brill et al. 1990). These methods do not guarantee finding the best site, but instead focus on the generation of alternatives that are likely to be good. Since, this problem is outside the focus of this paper, we will not further discuss these approaches here in any detail.

A common characteristic of the procedures reviewed above is the separation of the selection (reducing the choice set) and evaluation of candidate sites in distinct phases. However, such a linear procedure is inadequate in cases where the evaluation reveals shortcomings of the alternatives selected to such an extent that the decision maker (or in short, DM) wishes to reconsider candidates eliminated in the previous phase. Generally, the DM should be able to reconsider a selection made each time evaluation results are obtained. The procedure proposed in this paper supports such an iterative process of selection and evaluation (see Figure 1). An iterative procedure is not only more effective, but can

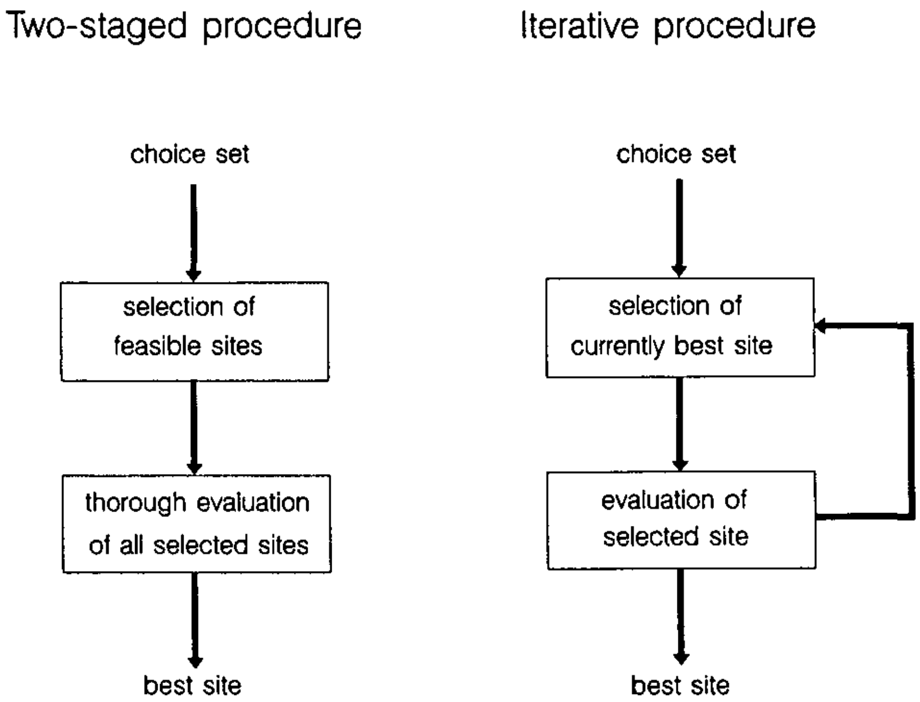

FIG. 1. Two-staged versus Iterative Procedures of Site Selection

also improve the efficiency. Evaluation costs can be reduced by selecting in each cycle the most promising option (given the currently available information) for evaluation.

Hierarchical procedures reduce evaluation costs in another way, namely, by utilizing the organization of sites in areas at different hierarchical levels. Evaluating areas means evaluating sites within that area as a group. Consequently, searching from high- to low-area level means maximally benefitting from a groupwise evaluation of sites. The procedure developed in this study combines an iterative and a hierarchical search strategy, to minimize evaluation costs. In contrast to existing hierarchical procedures, the iterative search process does not irreversibly proceed from top to bottom. The principle of iteration implies a *tentative* search process. If the evaluation of sublocations reveals shortcomings of an area, the process may return to a higher level to investigate an alternative area and so on. The next section gives a formal description of this iterative and hierarchical search procedure.

3. THE SEARCH ALGORITHM PROFMAT

The algorithm ProfMat (Profile Matcher) selects, from a given set of optional sites, the site that best matches a prespecified (ideal) attribute profile. Optionally, but not necessarily, the sites are ordered in groups (for example, areas) at different hierarchical levels (for example, area levels). This section discusses the assumptions and specification of ProfMat, respectively. Furthermore, an analysis of a retail-site-selection problem illustrates the application and operation of ProfMat. Finally, a comparison with alternative search procedures demonstrates the relative efficiency of the algorithm.

3.1 Assumptions

Locations are described in terms of the attributes that determine their suitability for a particular activity. These attributes form the profile of the site (see Findikaki 1990). On the other hand, an ideal attribute profile is specified, which reflects the objectives of the DM in locating the activity. The suitability of a specific location is expressed as the degree of similarity between the observed and ideal profile. The degree of similarity is considered a continuous variable called matching factor, *MF*. The *MF* is a function of the a priori *MF* and the discrepancy between observed and ideal attribute scores, as follows:

$$MF_i = MF_i^{ap} - \Sigma_j f_j(X_{ij}) \tag{1}$$

where:

MF_i is the *MF* score of the ith location;

MF_i^{ap} is the a priori *MF* of the ith location;

X_{ij} is the jth attribute score of the ith location;

f_j is the jth function defining the discrepancy (in *MF* units) between a score and the jth ideal score;

$f_j(X) = 0$, if X is unknown.

The second term on the RHS expresses the decrease in *MF* caused by observed discrepancies with the ideal profile. The function f_j incorporates both the ideal score (or class of scores) and a way of measuring the degree of mismatch (discrepancy) with observed scores. The specification of f_j depends largely on the scale used to measure the attribute. If j is a quantitative (rational or interval) attribute, determining the degree of mismatch involves the steps of calculating the difference between observed and ideal score and rescaling this difference to units *MF*. The latter step can be decomposed in a standardization and a weighting operation, to account for differences in measurement unit and relative importance across attributes, respectively. Standardization can be achieved by rescaling observed discrepancies on all attributes to a common zero-one scale. If the attribute concerned is not linearly related to *MF*, standardization involves the extra step of transforming the rescaled discrepancy to a linear scale.

Rescaling an observed discrepancy to a zero-one scale can be realized by

$$D_{ij} = \frac{|X_j^{ideal} - X_{ij}^*|}{X_j^{\max} - X_j^{\min}} \tag{2}$$

where

D_{ij} is the discrepancy on a zero-one scale of the ith location regarding the jth attribute;

$$X_{ij}^* \begin{cases} = X_{ij} & X_j^{\min} \le X_{ij} \le X_j^{\max} \\ = X_j^{\max} & X_{ij} > X_j^{\max} \\ = X_j^{\min} & X_{ij} < X_j^{\min} \end{cases}$$

X_j^{ideal} is the ideal score of the jth attribute;

$X_j^{\max} > X_j^{\min}$.

The ideal score, X_j^{ideal}, equals the maximum score, X_j^{max}, if j is positively related to *MF*, and is set to the minimum score, $X_j^{\min}$, if j is negatively related to *MF*. Otherwise, if the attribute has first positive and then negative returns, or vice versa, the ideal score is set to the optimum in between the range of j. In any case, the minimum and maximum scores on the jth attribute must be known. We emphasize that it is not necessary to know the actual range of scores on an attribute for determining the extremes. The extremes are set to levels within which the DM wishes to differentiate alternatives on *MF*. The range specified does not necessarily correspond with the actual range of the attribute, but defines the minimum and maximum decrease in *MF* possible on the attribute.

If the jth attribute is linearly related to *MF*, the standardized discrepancy simply equals the rescaled discrepancy:

$$D_{ij}^* = D_{ij} \tag{3}$$

where D_{ij}^* is the standardized discrepancy of the ith location on the jth attribute.

Alternatively, if j is an exponential function of *MF*, the following standardization function can be used:

$$D_{ij}^* = (D_{ij})^a \qquad a > 1. \tag{4}$$

This exponential function attenuates small discrepancies (near zero) and magnifies large discrepancies (near one). In fact, equation (4) summarizes a *set* of standardization functions, one for each value of exponent $a > 1$. The larger a the stronger the attenuation and magnification effects are, that is, the stronger the all-or-nothing character of discrepancies. Finally, an attribute may have an S-shaped relationship with *MF*. Then, the following standardization function is useful:

$$D_{ij}^* = \sin(\tfrac{1}{2}\,\pi D_{ij}). \tag{5}$$

This function attenuates both small and large discrepancies, whereby the attenuation effect increase as discrepancies approach the extremes (zero or one).

After standardization, the discrepancy, D_{ij}^*, of each attribute is expressed on a zero-one scale, which is linearly related to *MF*. Therefore, the decrease in *MF* is simply found by weighting the discrepancy according to the relative importance of the jth attribute:

$$f_j(X_{ij}) = w_j D_{ij}^* \tag{6}$$

where $f_j(X_{ij})$ is the decrease in *MF* of the ith location on the jth attribute; w_j is the weight of the jth attribute.

The discrepancy functions described above can be used only for attributes that are a regular (linear, exponential, or S-shaped) function of *MF*. In case of quantitative attributes for which no such function exists, the DM should define classes of scores with the same impact on *MF* and specify a decrease in *MF* for each class. Also, if j is a qualitative (nominal or ordinal) attribute, it is not possible to use a standard discrepancy function. Then, similarly, the DM should specify a decrease in *MF* for each possible score of the jth attribute. An irregularly shaped discrepancy function can be defined by a set of if-then rules (a logic-based model), which relates decreases in *MF* to specific scores or classes of scores of an attribute. In sum, many different specifications of the discrepancy function are possible, dependent on the nature of the attribute concerned. In site-selection problems, the set of relevant attributes is typically heterogeneous, including both quantitative and qualitative attributes. Therefore, the discrepancy functions should be defined for specific attributes or groups of attributes.

In the suitability function [equation (1)] a best-scenario strategy is adopted, in the sense that an attribute is assumed to match the ideal unless there is evidence of the contrary. Formally, this implies that the discrepancy function, f, returns a zero value if an attribute score is unknown. Consequently, in any stage of the search process, the *MF* score reflects the maximum score of a location: evaluating an unknown attribute score may lead to a decrease in *MF*, but cannot result in an increase in *MF*. Thanks to this best-scenario assumption, not necessarily all locations need to be evaluated thoroughly to identify the best site. Search can be terminated the moment the currently best site has been fully evaluated.

The a priori *MF* (the first term on the RHS) is an exogenous constant, which expresses the a priori preference for a site (the suitability judgment if all attribute scores are unknown). A priori preferences may exist if not all relevant attributes are included in the attribute profile. Then, the a priori *MF* is differentiated across locations according to their performance on missing attributes. However, normally there are no a priori preferences. Then, the a priori *MF* of locations is set to the same arbitrary level (for example, zero).

As expressed by equation (1), the *MF* of locations is determined by summing up discrepancies across attributes. The additive form of the suitability function assumes that a mismatch on one attribute can be evaluated independently of other attribute scores. In other words, interactions between attributes are assumed to be absent. We emphasize that this assumption of independent attributes does not limit ProfMat to cases where interactions between suitability factors are absent. Instead, it requires that attributes are defined in such a way that they are independent. A possible strategy to arrive at a set of independent attributes is to list all relevant factors and, next, to define combinations of interacting factors as single attributes. (In the extreme case where all factors interact, only one attribute would result, which can be interpreted as site suitability.) Therefore, the independence of attributes is not an assumption on the state of affairs in the real world, but rather a matter of definition. As such, it does not limit the applicability of the method.

ProfMat assumes the general case, where alternatives are organized in groups at one or more hierarchical levels. A pure hierarchy is assumed, that is, an alternative cannot belong to more than one group at higher level. Attribute j is considered to be related to level n if alternatives within groups at level n have the same score on j, whereas alternatives between groups differ on j. In a spatial context, groups can often be defined by delineating (contiguous) areas. Then, groups of different order correspond to areas at different levels of scale. In the following, we will assume the special case of such a geographically defined hierarchy, although ProfMat is general for all kinds of hierarchies. The definition of *MF* [equation (1)] can be generalized for a hierarchy of options, as follows:

$$\begin{aligned} MF_i &= \max_k(MF_k) && k \in S_i \quad S_i \neq \emptyset \\ &= MF_i^{ap} - \Sigma_j f_j(X_{ij}) && S_i = \emptyset \end{aligned} \tag{7}$$

```
(0)  initialize the a-priori MF of optional sites
(1)  set i to the study area
     set Best Alternative to the second best sublocation of i
(2)  if  i has sublocations
     then
     - if  the Best Alternative is better than the best sublocation of i
       then
       - go to (1)
       else
       - if  not all attributes of the best sublocation of i are known
         then
         - evaluate the first unknown attribute of the best sublocation of i
         - go to (2)
         else
         - if  i = study area
           then
           - set Best Alternative to the second best sublocation of i
         - set i to the best sublocation of i
         - go to (2)
     else
     - select i
```

FIG. 2. The ProfMat Algorithm

where S_i is the set of sublocations of the ith location, and j is the jth element of the set of all (including the higher-level) attributes, and other elements are defined as above. In words, if location i has sublocations, then the *MF* of i equals the *MF* of the best sublocation, otherwise (i is a location at the lowest level), the *MF* of i equals the a priori score minus the total of discrepancies across all (including higher-level) attributes. This rule expresses the idea that an area is as suitable as the most suitable subarea and that the suitability of lowest-level locations is a function of all (including higher-level) attributes. So, the *MF* of higher-order locations is recursively defined in terms of the *MF* of their sublocations. Therefore, to evaluate the *MF* of a higher-order location, the *MF* of all sublocations must be determined and so on, until the lowest level is reached. When the *MF* of the lowest-level locations are known, the locations one level higher are set to the maximum *MF* of their sublocations and so on, until the highest level is reached. Finally, we note that the a priori *MF* of higher-order locations is derived from the a priori *MF* of their sublocations. Therefore, only the a priori *MF* of lowest locations need to be specified.

3.2 Specification

The ProfMat algorithm is represented verbally in Figure 2 and schematically in Figure 3. The process starts at the highest level by setting the current location i to the study area. Area i is searched through an iterative process of selecting the sublocation with the highest *MF* and evaluating the first unknown attribute of that sublocation. The *MF* of each sublocation is calculated using equation (7). As implied by this equation, the *MF* of a location equals the *MF* of the best sublocation. Therefore, initially, when all attribute scores are unknown, the *MF*

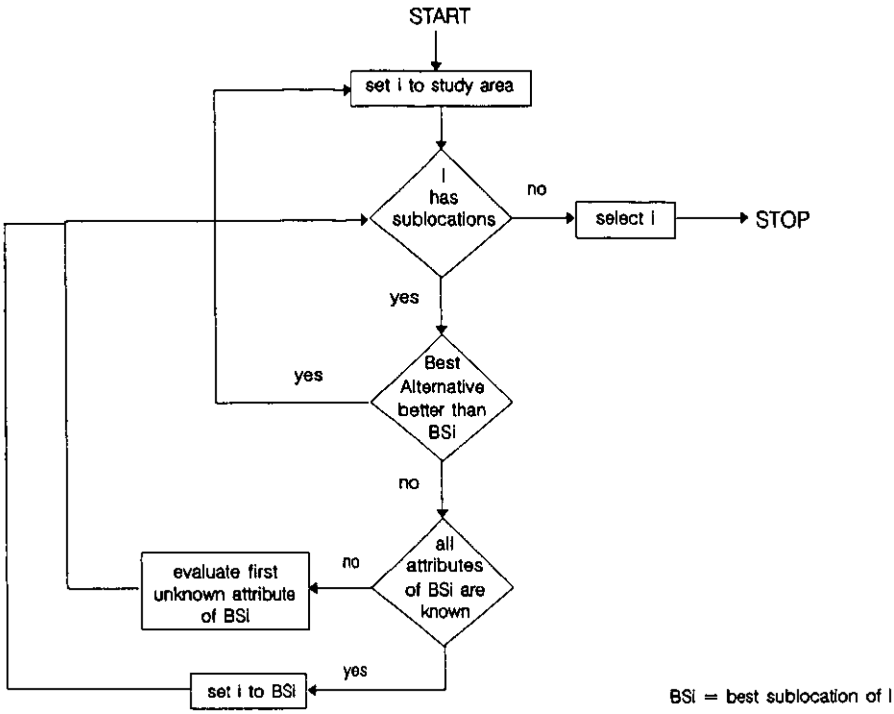

FIG. 3. Schematic Representation of the ProfMat Algorithm

of areas equals the a priori *MF* of their best site. Unless a priori preferences exist, the initial *MF* scores of the best sublocations of *i* are the same and the first one is selected as the best alternative and will be evaluated on the first unknown attribute. A discrepancy on an area attribute causes a decrease in the *MF* of all sites within that area [the second part of equation (7)]. Consequently, the *MF* of the best site of the area and therefore the *MF* of the area itself will also decrease to the same extent. Therefore, if evaluation of the first attribute of the best sublocation of *i* reveals a discrepancy, the *MF* of the currently best sublocation and also the *MF* of all its descendants (locations at lower levels) decrease accordingly. Then, the best sublocation may lose its priority, with the consequence that another sublocation will be selected as the currently best sublocation in the next cycle and so on. This process continues until all attributes of the currently best sublocation are known (have been evaluated). If the best sublocation has sublocations, then the current location *i* is set to the best sublocation. Setting *i* to its best sublocation means narrowing down the focus to a smaller area. This smaller area, *i*, is then searched in the same way by repeatedly selecting the best sublocation of *i* and evaluating the first unknown attribute. Again, if the currently best sublocation has been fully evaluated and has itself sublocations, then *i* is set to this sublocation and the whole foregoing process is repeated from this lower-level location.

In going downward the *MF* of the best sublocations decrease monotonically as evaluation reveals more and more discrepancies. Since the suitability of areas equals the suitability of their best sublocation, the *MF* of the higher-level areas currently being searched tend to decrease as a consequence. If an area at a higher level loses priority, then the process tracks back to the higher level, where the alternative area that has gained priority is selected for further evaluation and so on. So, the process keeps track of the "best alternative," that is, the second best subarea of the study area. The best alternative is considered as an additional option each time a selection has to be made. Selecting this alternative means tracking back to the top level to select an alternative path.

The process stops when a sublocation is selected of which all attributes are known and that does not have sublocations itself. A sublocation that meets this condition is indeed the best option, since it is better than the best alternative and has been completely evaluated. In sum, ProfMat evaluates attributes one at a time from high- to low-location level. In focusing on increasingly smaller areas, the branch being searched may lose priority. Then, the process tracks back to investigate an alternative path that has become more promising. The process stops when the currently best site has been completely evaluated, which is then identified as the definite best site.

3.3 *Illustration*

The first step in the ProfMat procedure is the specification of the site selection problem in terms of relevant attributes and attached discrepancy functions, dependent on the objectives of the DM. Next, the optional locations in the area of interest are identified at different location levels and ProfMat is run to find the best site. In this section, we illustrate both the problem specification and the operation of the ProfMat procedure using a case of retail site selection. The problem specification is derived from Ghosh and McLafferty (1987) and Mercurio (1984), but it should be noted that we have adjusted and simplified the problem specification somewhat for illustration purposes. A hypothetical location hierarchy is used to demonstrate the steps of the ProfMat procedure.

The Problem. In this example, we consider the problem of finding one or more sites with high trading potential in a given market area. For reasons of convenience, we assume that the choice of the market area is fixed, although in real problems of this kind several optional market areas will be considered on attributes such as *retail saturation* and *market expansion potential* (Ghosh and McLafferty 1987). The location attributes that are generally relevant for determining trading potential are listed in Table 1. More attributes may be

TABLE 1
The Set of Location Attributes

Attribute	Level	Model/Factors	Scale
Sales potential	market sector	—population size —growth rate —income —profile	three-point scale
Competition level	market sector	—number of shops —total selling space —competitive strength	three-point scale
Regional accessibility	shopping area	—access to public transport —access to main road —level of street congestion —quality of access to streets	five-point scale
Retail environment	shopping area	—number of branches —total selling space —quality of presentation	five-point scale
Site accessibility	site	—availability of parking space —quality of ingress/egress	five-point scale
Trading potential	site	—spatial interaction model	continuous

relevant in specific problems of this type, particularly, at the level of sites, but this list is commonly appropriate and suffices to demonstrate our approach.

The attribute list relates to three location levels. From high to low, the location levels involved are market sectors, shopping areas, and sites. Market sectors are the subareas of a market area (for example, city area) that define potential trade areas of new stores. They are delineated based on natural barriers (for example, highways) and spatial interaction patterns of the population. The attributes *sales potential* and *competition level* determine the attractiveness of market sectors. Shopping areas, one level lower, are concentrations of shops and are defined by drawing proximity bands around existing (and planned) retail facilities. *Regional accessibility* and *retail environment* are considered the relevant attributes of shopping areas. Finally, sites are locations at the lowest level. Optional sites are evaluated on *site accessibility* and *trading potential*.

The attributes have different measurement scales. The *trading potential* of sites is a quantitative attribute, which is measured as predicted turnover using a spatial interaction model. The other attributes are measured on a five-point or three-point ordinal scale. These qualitative attributes may be evaluated based on logic-based models or the judgment of the retailer concerned.

Attached to each attribute is a discrepancy function [f_j in equation (7)], which returns a decrease in *MF* given an attribute score. In this example, all attributes (continuous or discrete) are assumed to be linearly related to *MF*. Therefore, the linear discrepancy function [composed of equations (2), (3), and (6)] is used for all attributes. The minimum and maximum scores of discrete attributes are fixed (that is, one and five for the five-point scale and one and three for the three-point scale). The extremes of *trading potential*, the continuous attribute, on the other hand, is set by the retailer (or analyst) to the range within which he or she wishes to differentiate between alternatives. The ideal score of *competition level* is set to a minimum level and the ideal scores of the other attributes, which are positively related to *MF*, are set to maximum scores. Finally, the attribute weights are set according to the relative importance of attributes for determining trading potential.

The Procedure. Having specified the set of attributes and attached discrepancy functions, the next step is to identify the optional locations at the different location levels involved. First, the optional market sectors are defined by subdividing the market area in smaller areas (based on natural barriers and spatial interactions). Then, optional shopping areas within market sectors are defined by delineating concentrations of existing shops. Finally, the candidate sites are identified by an in-field inspection of the shopping areas.

To illustrate the operation of the algorithm, we consider the example location hierarchy which is schematically shown in Figure 4. Market M encompasses eight optional sites (S_1 to S_8) which are grouped in four shopping areas (SA_1 to SA_4) and two market sectors (MS_1 and MS_2). The scores associated with each location denote the initial *MF* and decreases in *MF*, respectively. The initial *MF* of areas equals the initial *MF* of best sublocations and the initial *MF* of sites equals the a priori *MF*. In this example, we assume that the retailer has formulated a priori preferences based on attributes (for example, the interception rate and visibility of sites) that are not included in the suitability function. The a priori *MF* of sites is set by the retailer in correspondence with the existing a priori preferences. Decreases in *MF* follow from nonoptimal attribute scores and are the outcome of evaluation.

The search process starts at the highest level by setting the current location i to the market area M. The best sublocation, MS_1 ($MF = 10$), is selected for

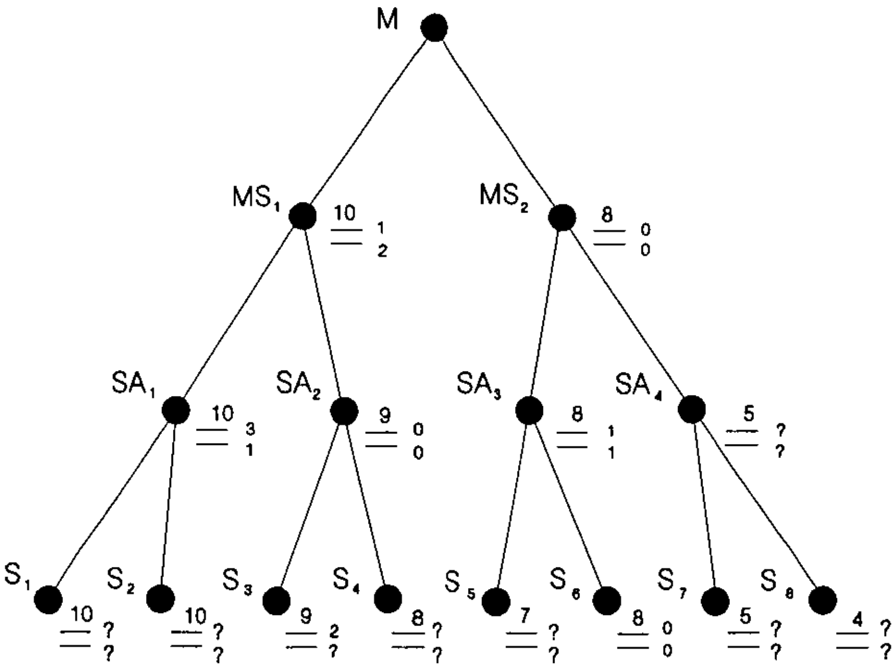

FIG. 4. Schematic Example of a Hierarchy of Options

evaluation and the best alternative is set to the second best market sector, which is MS_2 ($MF = 8$). Evaluation of the first attribute (*sales potential*) of MS_1 reveals a discrepancy, which causes a decrease of one *MF* unit for all sites within MS_1 and, consequently, also for all shopping areas within MS_1 and for MS_1 itself. After this evaluation step, the set of sublocations of i ($= M$) is reconsidered to identify the best option. MS_1 is still the best option and is reselected for evaluating the next unknown attribute (*competition level*). This time the *MF* of MS_1 and also all descendants of MS_1 decrease with two units. In the next cycle, MS_2 ($MF = 8$) is selected for evaluation and the best alternative is set to MS_1 ($MF = 7$). Evaluation of the first attribute of MS_2 does not lead to a decrease in *MF*, so that this option keeps priority and is reselected in the next cycle. Also the evaluation of the second attribute of MS_2 in the next cycle does not lead to a decrease in *MF*. Since the best sublocation ($= MS_2$) of i has been completely evaluated, the focus narrows down by setting i to MS_2 and the same process is repeated from this lower-level location. The set of sublocations (SA_3, $MF = 8$, and SA_4, $MF = 5$) and the best alternative (MS_1, $MF = 7$) are considered to identify the best option. In the cycles that follow, SA_3 is evaluated first on the first attribute (*regional accessibility*, decrease in $MF = 1$, new $MF = 7$, keeps priority) and next on the second attribute (*retail environment*, decrease in $MF = 1$, new $MF = 6$, loses priority). At this stage, the best sublocation (SA_3, $MF = 6$) is outperformed by the best alternative (MS_1, $MF = 7$) and, consequently, the process tracks back to the top level by setting i to the superlocation ($= M$) of the best alternative. Again, the best sublocation (MS_1, $MF = 7$) and the best alternative (MS_2, $MF = 6$) are determined. Since all attributes of the best sublocation ($= MS_1$) are known, the focus narrows down by setting i to this sublocation. The same cycles of selecting and evaluating the best sublocation follow from this lower-level location. After having evaluated the first attribute (*regional accessibility*) of SA_1 (decrease in $MF = 3$, new $MF = 4$, loses priority), the first attribute of SA_2 (decrease in $MF = 0$), and the second attribute (*retail environment*) of SA_2 (decrease in $MF = 0$), all attributes of the best sublocation (SA_2, $MF = 6$) are known and the focus further narrows down by setting i to this best sublocation, SA_2. After having evaluated the first attribute (*site accessibility*) of S_3 (decrease in $MF = 2$, new $MF = 4$, loses priority), the best sublocation (S_4, $MF = 5$) is worse than the best alternative (MS_2, $MF = 6$), so that the process tracks back to the top level by setting i to M. From M, the focus is narrowed down first to MS_2 and next to SA_3, which are the currently best and completely evaluated sublocations. After having evaluated both attributes of S_6 (no decreases in MF, $MF = 6$), it is still the best sublocation and better than the best alternative (MS_1, $MF = 5$). Since S_6 has no sublocations and has been fully evaluated, it is finally selected as the best site.

Discussion. The example illustrates several general properties of the ProfMat procedure. First, options with a higher a priori *MF* have a higher priority for evaluation. Therefore, a priori preferences influence the initial direction of the search process. Second, the attributes of a location are evaluated from high to low level. Higher-level attributes are not necessarily more important, but they are more informative since they relate to larger groups of sites. So, considering attributes in this order means evaluating sites as much as possible groupwise. From the point of view of efficiency, this way of ordering attributes is better than an ordering based on relative importance, as used in other hierarchical methods (for example, Tversky 1972). Third, the selection of the best sublocation is reconsidered each time an attribute has been evaluated. This property implies that evaluation efforts are always directed to the most promising option. Fourth, not necessarily all locations need to be fully evaluated for finding the best site (for example, S_1, S_2, S_4, S_5, S_7 and S_8 were skipped). If the currently best site is fully evaluated (all attribute scores are known), it is known to be the best, even if alternative locations are only partly evaluated. The best-scenario assumption implies that the *MF* of locations reflect the maximum scores possible. Evaluating additional attributes of alternatives cannot lead to increases in *MF* and, therefore, cannot change the outcome. ProfMat reduces the expected number of attribute evaluations by directing evaluation efforts exclusively to the most promising (the currently best) location and most informative (from high- to low-level) attributes in each stage of the search process.

The Efficiency of the Procedure. The major cost factor in site-selection procedures is the collection of data needed for identifying candidate locations and evaluating location attributes. Compared to an exhaustive and flat search (evaluating all locations on all attributes), ProfMat reduces the number of attribute evaluations (and therefore the data needs) by using an iterative and hierarchical search strategy. To obtain an indication of the relative efficiency of ProfMat, we have compared ProfMat to alternative search procedures on the total number of attribute evaluations performed in a representative sample of site-selection problems (cases).

Four different search procedures are compared: (i) an iterative and hierarchical search (ProfMat); (ii) an exhaustive and hierarchical search; (iii) an iterative and flat search; and (iv) an exhaustive and flat search. In exhaustive procedures (ii and iv) all locations are evaluated on all attributes. In contrast, in iterative procedures (i and iii) the currently best location is selected and evaluated on an unknown attribute (one at a time) in a cyclic process until all attributes of the currently best location are known. Furthermore, a distinction can be made between hierarchical and flat search procedures. As opposed to flat search, in hierarchical procedures (in combination with either an exhaustive or an itera-

TABLE 2
The Number of Attribute Evaluations in Twenty Cases of a 10^3 Tree and a 5^4 Tree

		iterative, hierarchical search	exhaustive, hierarchical search	iterative, flat search	exhaustive, flat search
10^3 tree	mean (% of max)	102.6 (1.71)	2220 (37.00)	2231.6 (37.19)	6000 (100)
	stand. dev.	51.0	0	481.9	0
	range	149	0	1724	0
5^4 tree	mean (% of max)	125.2 (2.50)	1510 (30.20)	2391.3 (47.83)	5000 (100)
	stand. dev.	68.4	0	498.0	0
	range	298	0	1610	0

tive procedure), locations are evaluated as groups rather than individually on higher-level attributes.

To obtain a representative sample of cases, we considered two types of location hierarchies. The first hierarchy has three levels and ten locations per area (at each level) and is referred to as a 10^3 tree. The second hierarchy consists of four levels and five locations per area (at each level) and is referred to as a 5^4 tree. So, the 10^3 tree has a total of 1,000 sites ordered in groups of ten sites at three levels, whereas the 5^4 tree has a total of 625 sites ordered in groups of five sites at four levels. In both hierarchies, two attributes are associated with each level. For each tree, a sample of twenty cases was obtained by twenty times assigning random discrepancy values (between 0 and 9 *MF* units) to location attributes. The $2 * 20$ cases were analyzed using each of the four procedures, whereby the total number of attribute evaluations was counted. Of course, all procedures gave the same selection outcome; therefore, their efficiency can be compared. Table 2 shows the results.

The number of evaluations of the exhaustive procedures does not depend on the actual attribute profiles of locations. In the flat search variant (iv) the number of evaluations required are $1{,}000 * 3 * 2$ (= 6,000) and $625 * 4 * 2$ (= 5,000) for the 10^3 tree and 5^4 tree. On the other hand, in the hierarchical search variant (ii) the number of evaluations in the 10^3 is $(10 + 10^2 + 10^3) * 2 = 2,220$ and in the 5^4 tree $(5 + 5^2 + 5^3 + 5^4) * 2 = 1,510$.

In iterative procedures (i and iii), the number of evaluations needed depends on the attribute profiles of locations. To find the best site in each of the twenty cases of a 10^3 and a 5^4 tree, a flat search strategy (procedure iii) requires, on average, 2231.6 and 2391.3 evaluations in the 10^3 tree and the 5^4 tree. For the hierarchical search procedure (i), these figures are 102.6 and 125.2.

Using the exhaustive and flat search (the fourth column) as a reference, the introduction of an iterative search strategy reduces the average number of attribute evaluations to 37.19 percent (in the 10^3 tree) and 47.83 percent (in the 5^4 tree). For the introduction of an hierarchical search strategy, these figures are 37.00 percent and 30.20 percent. The combination of the iterative and hierarchical strategy (ProfMat) results in a considerable efficiency improvement. On average, only 1.71 percent and 2.50 percent of the total of possible attribute evaluations are needed in the 10^3 tree and 5^4 tree.

4. A GENERIC EXPERT SYSTEM

As several authors (Ortolana and Perman 1990; Wright 1990; Curry and Moutinho 1992) have argued, expert system techniques are useful for developing computer-based systems for site selection. Site selection generally requires the combination of several factors (for example, population characteristics) for determining the suitability (for example, sales potential) of candidate locations (for example, market sectors). Often, quantitative models (for example, spatial interaction model) or generally applicable methods (for example, a weighted mean) can be used for factor combination. However, in many cases the combination of factors requires qualitative reasoning based on knowledge specific for the problem at hand. Expert systems (or knowledge-based systems) provide the means for modeling qualitative knowledge. Also, quantitative models can be used in these systems, possibly, by including calls to external procedures (possibly, written in a procedural language). Therefore, expert systems are potentially useful for modeling site-selection problems and have been proven to be successful in several studies (for example, Findikaki 1990; Suh, Kim, and Kim 1990; Han and Kim 1990).

In this study we have implemented ProfMat in an expert system using Prolog. When complemented with knowledge specific for a certain site-selection problem, the system finds the best sites (possibly in interaction with the user) using the ProfMat procedure. Therefore, the system is best viewed as a *generic* expert system for solving large site-selection problems. The problem-specific knowledge is provided by a prespecified knowledge base and, complementarily, by the user during a consultation session. In this section we discuss the functionality of the system by describing the structure of the input knowledge base and the user-system interaction during consultations.

4.1 Structure of the Domain-Knowledge Base

Table 3 summarizes the ProLog predicates available for building the problem-specific knowledge base. A distinction can be made between predicates related to the attributes involved (the ideal specification) and predicates that describe the optional locations. These two components of the knowledge base will be described in turn.

The following predicates are available for defining the attribute-related knowledge:

> *attribute_subset(Level, Attr_list);*
> *attribute_score(Attribute, Location, Score);*
> *attribute_discrepancy(Attribute, Score, Discrepancy);*
> *relevant_prop(Attribute, Prop_list).*

TABLE 3
Summary of the Predicates of the Input Knowledge Base

Predicates
attribute subset (*Level, Attr list*)
defines for a location level the relevant location attributes
attribute score (*Attribute, Location, Score*) (optionally)
defines for an attribute the evaluation of the attribute score, given the location
attribute discrepancy (*Attribute, Score, Discrepancy*) (optionally)
defines for an attribute the decrease in *MF*, given the attribute score
relevant prop (*Attribute, Prop list*) (optionally)
defines for an attribute the characteristics that are relevant for user's evaluation of the attribute score
location option (*Location, Level, Subloc list*)
defines for a location the location level and the set of sublocations
a priori MF (*Site, Score*)
defines for a lowest-level location the a priori *MF*
location data (*Location, Property, Score*) (optionally)
defines for a location characteristic the score

The predicate *attribute_subset* lists the attributes per location level. For example, the fact,

attribute_subset(market_sector, [sales_potential, competition_level]),

defines the relevant attributes at the market-sector level in the retail-site-selection problem described in the former section. The position of attributes in the list defines the priority for evaluation. So, in the search process attributes are evaluated in the order they are listed in *Attr_list*.

The predicate *attribute_score(Attribute, Location, Score)* can be used to define functions for evaluating attribute scores; it binds the variable *Score* with a specific score, given bindings of *Attribute* and *Location*. The definition of this predicate typically consists of a set of rules that relates scores to location characteristics. For example, the rule,

```
attribute_score(retail_environment, Location, moderate):-
    location_data(Location, number_of_branches, X),
    location_data(Location, total_selling_space, Y),
    location_data(Location, quality_of_presentation, Z),

    X > 5, X < 8,
    Y > 4500, Y < 6000,
    Z = excellent,
```

relates a moderate retail environment of *Location* to a specific combination of number of branches, total selling space, and quality of presentation. Or, in procedural terms, the retail environment of *Location* is set to moderate if *Location* meets the specified conditions. In case of quantitative attributes, functions may take the form of a mathematical model, such as a spatial interaction model for estimating sales potential. If large calculations are involved, mathematical functions are typically built in by a call to a routine written in a procedural language.

The predicate *attribute_discrepancy(Attribute, Score, Discrepancy)* is available for defining discrepancy functions [f_j in equation (7)]; it binds the variable *Discrepancy* with a specific discrepancy, given bindings of *Attribute* and *Score*. For example, the rule,

attribute_discrepancy(regional_access, medium, 2),

relates a decrease in *MF* of two units to medium regional accessibility. Or, in procedural terms, *Discrepancy* is set to two units if the regional accessibility is medium. Standard methods for calculating discrepancy scores are realized by general, rather than attribute-specific predicates, such as,

```
linear_discrepancy(Weight, Score, MaxScore, MinScore, IdealScore,
    Discr):-
    standard_discr(Score, MaxScore, MinScore, IdealScore, D),
    Discr = Weight * D.
```

Linear_discrepancy can be used for attributes, which are linearly related to *MF*. The predicate *standard_discr* binds *D* with the discrepancy on a zero-one scale.

Alternatively, attribute scores or discrepancies may be defined in interaction with the user, rather than by built-in functions. ProfMat assumes that this is the case if built-in functions are missing. So, the interactive way of evaluating an attribute is realized simply by not including an evaluation function for that attribute. When an attribute is evaluated interactively, ProfMat will try to find data (stored in *location_data*) in the knowledge base relevant for evaluating that attribute. These data are then displayed on the screen simultaneously with a prompt to enter a value, to support the user's decision.

The predicate *relevant_prop(Attribute, Prop_list)* can be used to define the relevance of location factors to an attribute. The elements of *Prop_list* refer to the variable *Property* in *location_data*. Every time the user is asked to enter an attribute score, ProfMat presents the data stored in *location_data* referred to by *relevant_prop*. So, for example, if number of shops, total selling space, and competitive strength are relevant for evaluating the competition level of market sectors, the fact

relevant_prop(competition_level, [number_of_branches,
total_selling_space, competitive_strength])

is included in the fact base. Then, ProfMat displays the properties listed to support the evaluation of the competition level of a market sector.

The second component of the knowledge base describes the optional locations in the area to be searched, using the following predicates:

location_option(Location, Level, Subloc_list);
a_priori_MF(Site, Score);
location_data(Location, Property, Score).

The predicate *location_option* stores for each location the identifier, level, and the list of (identifiers of) sublocations. The study area itself is considered the highest-level location. For example, the following fact defines the study area as a list of optional market sectors:

location_option(market_A, study_area, [market_sector_A1,
market_sector_A2]).

where *study_area* is the reserved keyword that indicates the starting point of the search. Lowest-level options are characterized by an empty sublocation list. Finally, the predicate *location_data* stores values on location factors (properties) needed for evaluating location attributes, such as

location_data(shopping_area_A, number_of_branches, 7),

The predicates described above make up the problem-specific knowledge, which is input to the ProfMat system. During the search process, ProfMat adds to this knowledge-based evaluation results using the following predicate:

location_profile(Location, MF, Score_list, Discr_list).

The variable *Score_list* contains the scores and *Discr_list* the discrepancies of the attributes listed in *Attr_list* of *attribute_subset*. *MF* represents the *MF* of *Location*. Before starting the search process, a set of *L* facts of this type is created to store the profile of *L* location options, whereby *Score_list* and *Discr_list* are initialized with unknown values and zero values, respectively. During the search process *location_profile* is updated every time evaluation results are obtained.

4.2 User Interaction

The first step in a consultation session is the specification of the input-knowledge base that defines the site-selection problem. Then, the system evaluates optional locations according to the ProfMat procedure. When a location attribute is needed while it is unknown, the system tries to find the needed value by searching the knowledge base for appropriate facts (*location_data*) or evaluation functions (*attribute_score* or *attribute_discrepancy*). If this search fails, the system starts a procedure for an appropriate interaction with the user. In case of an attribute score, the system first searches the *relevant_prop* facts and upon success the *location_data* facts, to collect supportive data. Collected data, if any, are displayed in an information window. In case of a discrepancy score, the presented information consists of the score on the concerning attribute. Simultaneously, a dialogue window is opened to prompt the user for entering the needed value. Each time the system asks for a value, information can be obtained on the reason of the question. This information consists of the profile of the option under evaluation in terms of the score, ideal score, and discrepancy of all attributes at that level and the higher levels.

If the stop condition is met, that is, if the site with the highest *MF* is fully evaluated, the system presents the name, *MF*, and attribute-profile of the selected site. Optionally, the next-best solution is generated by repeating the procedure, whereby the a priori *MF* of the selected (best) site is set to some minimum value, so that it is ignored in the subsequent search process. This can be repeated to generate the third-best solution and so on.

The degree of user interaction in evaluating options depends on the specification of the input knowledge base. The system works in a noninteractive mode, if functions are built in for evaluating attribute scores and discrepancies for all attributes. In the other extreme, if evaluation functions and (references to) relevant data are lacking, the scores and discrepancies on all attributes are determined by the user without supportive information. Many variants are possible in between these extremes. In one variant, which may be useful in some situations, the system evaluates (and presents) attribute scores and the user determines the corresponding decreases in *MF*. In another typical variant the user decides both on the scores and discrepancies of attributes and the system provides data for supporting the decisions. Through the specification of the input knowledge base, the user is able to prescribe the desired type of evaluation per attribute.

5. CONCLUSIONS AND DISCUSSION

The ProfMat algorithm developed in this study selects from a given set of optional sites the site that best matches a specified ideal profile. The procedure differs from most commonly used procedures for site selection in two respects. First, an iterative rather than linear process of selecting and evaluating candidate sites is used. Second, rather than using a flat search, the study area is searched at different levels of scale, from high to low level. The iterative and hierarchical procedure typically results in a tentative search process of narrowing down the focus to increasingly smaller areas and returning to higher levels to investigate alternative paths. This recursive process stops if the currently best site is fully evaluated.

This procedure improves the efficiency of the site-selection process, without affecting the quality of the outcome. The best option may be identified without having evaluated all options completely. Unknown attributes are assumed to be ideal, so that partly evaluated options can be excluded if they are outperformed by any other completely evaluated alternative. The procedure makes sure that evaluation efforts are always directed to the most promising option and the most informative attribute. Consequently, the expected number of attribute evaluations is reduced to a minimum.

Given the size of the choice set, the efficiency improvement of the search process leads to a reduction of costs associated with data collection and computation. On the other hand, given a certain amount of resources available for search, the efficiency improvement enables the DM to consider a larger choice set. The latter is significant in many site-selection problems, where the size of the choice set is limited by available resources. Then, by allowing larger choice sets, the ProfMat procedure may improve the quality of the solution. Furthermore, the ProfMat procedure probably corresponds more closely to the generally iterative nature of the human decision-making process and the top-down orientation of human spatial search. Therefore, the incorporation of ProfMat in a decision support system (DSS) or expert system for site selection may improve the accessibility and face validity of the systems.

ProfMat addresses the multicriteria, single-site problem. It cannot be used to find the optimal location of a network of activities (Lawrence and Ostresh 1978). However, in real-world situations, networks are rarely developed from scratch. Instead, the problem is often to find an additional facility to expand an existing network. ProfMat is useful for these problems, especially if the number of potential locations is large.

ProfMat has been implemented in an expert system for site selection. The user of the system specifies the problem-specific knowledge using a prespecified set of Prolog predicates. The system controls the input knowledge to solve specific problems using ProfMat and supports various modes of evaluating options varying from highly interactive to noninteractive. Currently, the system does not provide facilities for supporting the development and maintenance of the knowledge base. Both the user friendliness and accessibility of the system would be improved by adding an interface between user and knowledge base that insulates the user from implementation details.

Furthermore, the usefulness of the system could be improved by adding facilities for supporting the interactive evaluation of attributes. First, the user should be able to query any supportive information from an area database. Second, the user-system interaction should take place through a map of the study area. Then, data queries and data presentations by the user or the system would be linked to the concerning locations on the map.

In case of selection problems that are considered too ill-structured to formalize the search procedure, a DSS provides a more appropriate problem-solving environment than an expert system. In a DSS approach, the user would control the ProfMat type of search and the system would provide supportive information. The expert system and DSS approaches can be integrated as two optional modes of the same (hybrid) system. The user of such a system is able to choose between a system or user-controlled search process, dependent on the complexity of the problem.

Finally, the ProfMat procedure, whether system or user-controlled, would benefit from facilities available in most existing GIS. First, GIS provides database management functions for storing, managing, and querying location, attribute, and topologic data, which describe the study area. Second, functions for spatial analysis typically available in GIS can be used for identifying candidate locations (for example, overlay-analysis, selection functions), for defining subareas (for example, regionalization or geographic-based functions) and for generating attribute data (for example, network analysis, spatial interaction models, aggregation functions). Finally, GIS visualization tools can be used for producing maps for user-interfacing, display, and report purposes. The integration of ProfMat in a GIS would not only be beneficial for ProfMat, but would also enhance the usefulness of GIS for spatial search.

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