Rolling schedule approaches for supply chain operations planning

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Judith Maria Spitter

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Spitter, Judith M.

Rolling schedule approaches for supply chain operations planning / by Judith Maria Spitter. – Eindhoven : Technische Universiteit Eindhoven, 2005. – Proefschrift.

ISBN 90-386-0021-6

NUR 804

Keywords: Production planning / Linear programming models / Rolling schedule / Supply chain management

Printed by Universiteitsdrukkerij Technische Universiteit Eindhoven

Cover designed by Paul Verspaget

Rolling Schedule Approaches for Supply Chain Operations Planning

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de
Technische Universiteit Eindhoven, op gezag van de
Rector Magnificus, prof.dr.ir. C.J. van Duijn voor een
commissie aangewezen door het College voor
Promoties in het openbaar te verdedigen
op maandag 25 april 2005 om 16.00 uur

door

Judith Maria Spitter

geboren te Vlaardingen

Dit proefschrift is goedgekeurd door de promotoren:

prof.dr. A.G. de Kok
en
prof.dr.ir. J.C. Fransoo

Copromotor:
dr.ir. N.P. Dellaert

Acknowledgements

This thesis would not be completed without the help of many others. It is my pleasure to express now my gratitude to all of them. First of all I would like to thank Ton de Kok. His excellent supervision and inexhaustible stream of suggestions and ideas contributed much to the contents of this thesis. I also thank him for giving me the opportunity to raise my son at home while finishing the last year of this research project. I would like to express my gratitude to my second supervisor Jan Fransoo for providing me with valuable remarks to increase the quality of, especially chapter 1, of this thesis. I also would like to thank my daily supervisor Nico Dellaert for his helpful comments. All participating members of my promotion committee are kindly acknowledged for their acceptance and interest in this defence.

I want to thank all my colleagues in the OPAC department for providing a comfortable working atmosphere. Special thanks go to Cristina Ivănescu for joining me at coffee, lunch, and tea breaks, to Gudrun Kiesmuller for answering all my questions (even the frequently asked questions), and to all the Ph.D. students in the OPAC department for cheering each other up during hard times occurring in this four year project.

Finally, I want to thank my friends and family for their continuing support and interest in my work. I especially want to thank Vincent for all his love and help. The last thank goes to Jasper, my little miracle, for reminding me with his always beautiful smile of the better things in life.

Judith Spitter,

April 2005

Contents

Chapter 1

Introduction

1.1 The SCOP problem

In the last decade of the twentieth century, Supply Chain Management (SCM) emerged as a new management concept. This management concept deals with the integration of planning, executing, and controlling all activities associated with the transportation, transformation and storage of goods from raw materials to end-users, as well as the associated information flows, in order to minimize total supply chain costs while satisfying customers demand. Intrigued by the problems and experiences that have emerged from business practice, Supply Chain Management has become a very visible and influential research topic in the field of operations research.

In this thesis we concentrate on only one part of SCM, namely the problem of **coordinating material and resource release decisions** in the **supply chain** such that predefined **customer service levels are met at minimal cost**. We refer to this coordination problem as the Supply Chain Operations Planning (SCOP) problem. For solving this problem, we make use of various mathematical programming methods and additionally build on results from multi-echelon inventory theory.

Let us discuss the three key features of the Supply Chain Operations Planning (SCOP) problem one-by-one, starting with the supply chain. First, we view the supply chain as a network, with suppliers delivering raw material and customers buying produced goods, see figure 1.1 for an example. To produce an end item, raw material must undergo several transformation activities, implying that raw materials transform into components, components into subassemblies, subassemblies into assemblies, and assemblies into finished products ready to be distributed. These transformation activities can be both physical, such as manufacturing or assembly activities and non-physical such as the transportation from one location to another.

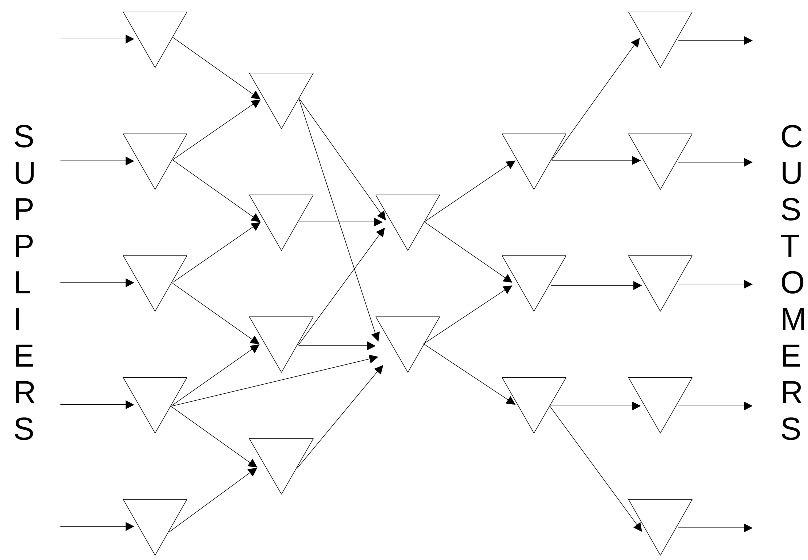

Figure 1.1: *An example of a Supply Chain.*

In this thesis we assume that supply chain structures have the following characteristics

- transformation activities may be preceded by multiple transformation activities,
- transformation activities may be succeeded by multiple transformation activities,
- resources have restricted capacity,
- holding costs of downstream items increase (i.e. added value is positive after each transformation activity), and
- lot sizes and setup cost are neglected.

Furthermore, we assume a make-to-stock environment, allowing us to produce without knowing customer demand exactly.

The second key feature of the SCOP problem is the coordination of material and resource release decisions. The SCOP function, depicted in figure 1.2, determines a production plan, i.e., the releases of both materials and resources, whereby we explicitly consider coordination of all release decisions simultaneously in a multi-item, multi-period setting. Special in our approach is way we apply the concept of planned lead times, time intervals in which the production of an order must take place: by which we decouple material release from resource consumption over time. Thus the capacity required to produce an order can be consumed at any point in time during the planned lead time. Former MP models with planned lead times can only be found

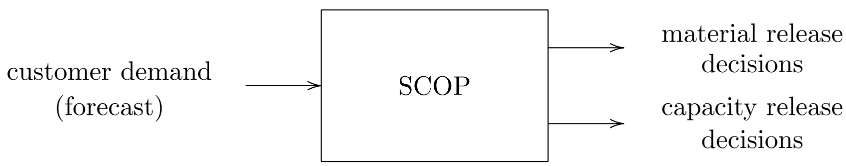

Figure 1.2: *Supply Chain Operations Planning.*

in Negenman (2000) and De Kok & Fransoo (2003). In example 1.1, we explain the release decisions of material and resources for one item using simple tables.

Example 1.1 *In this example we consider one item, namely item X. Item X is produced on a capacitated resource, that can only produce 15 units of item X in one period. Furthermore we assume that item X has a planned lead time of 2 periods. In table 1.1, the release decisions of orders are calculated and in table 1.2 we see how orders are allocated to a resource.*

	Period						
Item X	0	1	2	3	4	5	6
Gross requirements		20	5	23	5	25	10
Projected on-hand	28	8	3	-20			
Net requirements				20	5	25	10
Order receipts				20	5	25	10
Order release		20	5	25	10		

Table 1.1: *Material release.*

In table 1.1, we calculate the order release decisions based on the gross requirements for 6 successive periods. Projected on-hand are those items that are on stock at the current time. At the current time we have 28 units of item X on stock, which is enough to satisfy the gross requirements for the first two periods. To satisfy the gross requirements of the remaining periods, we need to produce item X. The number of units we have to produce is given by the net requirements. The order receipts indicate when the orders are produced and the order releases indicate the period in which we may start the production of an order. We assumed a planned lead time of 2 periods. The actual production time of item X is just one period, so items are either waiting one period before being produced or are produced immediately and wait for one period afterwards.

In table 1.2, we see that the order releases are copied for table 1.1. These are the number of items we have to produce. However we can only produce 15 units per period. We assume that there were no order releases in the past, that influence the capacity allocation. In period 1, there is an order release of 20 units, 15 units (maximal capacity of the resource) are produced immediately while 5 units remain waiting. These 5 units are produced in period 2 while the 15 units already produced are waiting the remainder of the planned lead time. In period 2, there is an order release of 5 units,

<table>
<tr><td>Item X</td><td colspan="2">1</td><td colspan="2">2</td><td colspan="2">3</td><td colspan="2">4</td><td colspan="2">5</td><td colspan="2">6</td></tr>
<tr><td>Order release</td><td colspan="2">20</td><td colspan="2">5</td><td colspan="2">25</td><td colspan="2">10</td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Capacity</td><td colspan="2">15</td><td colspan="2">15</td><td colspan="2">15</td><td colspan="2">15</td><td colspan="2">15</td><td colspan="2">15</td></tr>
<tr><td>Waiting before production</td><td colspan="2">5</td><td colspan="2">0</td><td colspan="2">10</td><td colspan="2">5</td><td colspan="2">0</td><td colspan="2"></td></tr>
<tr><td>Production</td><td>0</td><td>15</td><td>5</td><td>5</td><td>0</td><td>15</td><td>10</td><td>5</td><td>5</td><td></td><td></td><td></td></tr>
<tr><td>Waiting after production</td><td colspan="2">0</td><td colspan="2">15</td><td colspan="2">5</td><td colspan="2">15</td><td colspan="2">5</td><td colspan="2"></td></tr>
<tr><td>Order receipt</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">20</td><td colspan="2">5</td><td colspan="2">25</td><td colspan="2">10</td></tr>
</table>

Table 1.2: *Resource release.*

since we still have a capacity of 10 units left, these are produced immediately and wait in period 3 before they can be used. In period 3, we have again an order which is larger than the maximal capacity, so we produce 15 units in period 3 and 10 units in period 4. In period 4, we have an order of 10 units. Since we only have a capacity of 5 units left, 5 of the 10 units are produced in period 4 and 5 units are produced in period 5. So, now we also have the resource release decisions, these are given in the line "Production" in table 1.2.

The last key feature of the SCOP problem is the objective of meeting customer service levels at minimal cost. As service levels we a predefined non-stockout probability for all end items. With this objective we can measure and compare the performances, namely the total cost, for different SCOP functions. The performance of each SCOP problem is measured using a rolling schedule to incorporate the impact of stochastic end item demand (cf. figure 1.3). In accordance with the make-to-stock assumption, production plans are made assuming a demand forecast. For the successive periods $(t, t+1]$, $(t+1, t+2]$, ..., $(t+T-1, t+T]$ the SCOP is determined based on a forecast of the demand, whereby t is the current time and T is the planning horizon. Then, only the first period of the production plan is actually implemented by releasing both materials and resources due to be released in that first period. Directly after the first period, actual customer demand is known and a new production plan, based on the actual net inventory levels, a possibly updated forecast of future demand, and the outstanding orders generated by previous plans, is made for the coming planning horizon. The planning horizon for the SCOP function is in most industries several months with weekly time buckets. In some industries, e.g., bulk chemicals, this function may have a horizon as short as a couple of weeks with daily buckets, whereas in other industries, e.g., pharmaceuticals, the horizon may be as long as a couple of years with monthly time buckets.

Note that instead of shifting the planning horizon after each period, you can also shift only after, e.g., four periods. In this case the first four periods of the determined production plan are implemented, and only after the fourth period the a new production plan based on the actual demand of the last four periods and the demand forecast for the coming time horizon is calculated. Most probably this reduces the nervousness of the system, however the reaction time on deviations from customer demand forecast decreases. In this thesis we restrict ourself by shifting the planning horizon after each period.

The outline of this introductory chapter is as follows. In section 1.2 we describe the

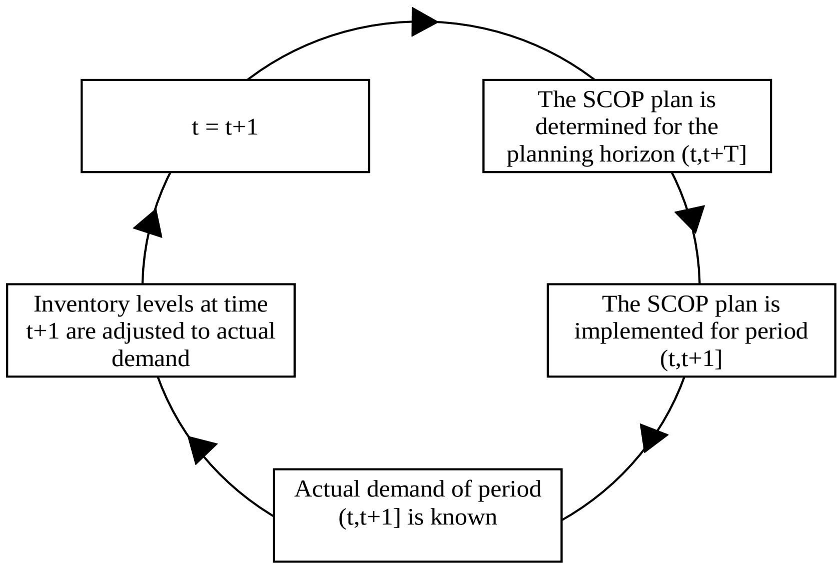

Figure 1.3: *Rolling schedule concept.*

motivation of the research and discuss the introduced planned lead time concept and literature on mathematical programming models to solve the SCOP problem. The problem statement and research questions are formulated in section 1.3. In section 1.4 we discuss the experimental framework we use to compare different SCOP functions. In section 1.5 the used methodology is discussed and we conclude with an outline of the thesis in section 1.6.

1.2 Motivation and background

1.2.1 Motivation

In this thesis we study the Supply Chain Operations Planning problem under demand uncertainty. Solving this problem is highly complex, and optimal control is even beyond mathematical tractability. Current research considers either tractable uncapacitated stochastic models of limited structural complexity (e.g. optimality of echelon order-up-to-levels for serial structures have been derived by Clark & Scarf (1960), for convergent structures by Rosling (1989), and for divergent structures by Diks & de Kok (1999)) or mathematical programming approaches to solve deterministic instances (e.g. Billington *et al.* (1983), Hopp & Spearman (2000), and Belvaux & Wolsey (2001)).

To incorporate demand uncertainty while dealing with complex supply chain structures, we use mathematical programming (MP) models (see section 1.2.3 for an overview of MP models) in a rolling schedule context. We refer this policy whereby MP models are used in a rolling schedule context as the MP based policy. Although the rolling schedule heuristic possibly results in non-optimal solutions, it is extremely relevant from a real-life perspective. In practice, expectations of future demands change, and current orders may be cancelled, delayed or expedited. Furthermore, production planning in a rolling schedule setting is a core element of the Material Requirements Planning (MRP) approach, see Bertrand *et al.* (1990). Moreover, a study of Blackburn & Millen (1980) in which the impact of a rolling schedule implementation on the performance of lot sizing methods for single-level assembly systems are examined, concludes with the following quote:

> It is hoped, ..., that future studies of new and old lot sizing methods will examine performance in a rolling schedule framework.

A recent rolling schedule study can be found in Stadtler (2003). In this paper a new time decomposition heuristic for solving multi-item, multi-level lot sizing problems for general product structures with single and multiple constrained resources as well as setup times is proposed. In this heuristic, lot sizing decisions are made sequentially within a internally rolling planning interval, while the capacities are considered over the entire planning horizon.

However not only customer demand, but also processing times are uncertain. In order to assume a 100% due date reliability, we introduce the planned lead time concept. The planned lead time is a time interval in which a released order must be produced, and it may consist of multiple periods whereby in each period a part of the order may be produced. In the next section, we discuss the importance of the planned lead time concept by positioning the SCOP problem in the planning hierarchy and by discussing the impact of demand uncertainty in the context of finite resource capacity.

1.2.2 Planned lead time concept

This section is defined into two parts. In the first part, we define planned lead times and explain the difference with the lead times in former MP models. In the second part we position the SCOP function in the Planning Hierarchy, and describe the role of planned lead times between planning levels.

Defining planned lead times

The phenomenon of lead times in manufacturing has been extensively studied. For an overview of this literature, which strongly relies on queuing network theory, we refer to Suri *et al.* (1993) and Buzacott & Shantikumar (1993). From this literature we learn that only a small part of the lead time is typically processing time, while its major part is waiting time. The waiting time is caused, as Karmarkar (1987) points out, by capacity restrictions at the resources, lot sizes, release times of batches and coordination of release times, sequencing at machines, production mix and the heterogeneity of items. Waiting time can occur both before and after the actual transformation activity.

By defining the planned lead time, we incorporate all variables determining the waiting time in a deterministic way. We fix the lead time for every item, and require that orders are finished within that time. Furthermore we assume that each item is transformed, (e.g. manufactured, assembled, or transported) within one period, while it waits the remainder of the planned lead time either before transformation or after transformation for the remainder of the planned lead time. Only after the planned lead time has elapsed a complete order becomes available for further use. Hence, the actual transformation of an item takes place in an arbitrary period during the planned lead time.

The introduction of planned lead times can also be seen as adding safety time to the problem. The planned lead time obviates the fluctuation in demand and increases the reliability of production units. One can argue that instead of safety time, safety stocks should be added to the problem. However, then the assumption that the production units have a 100% due date reliability cannot be held any longer. Furthermore, research of preference between safety time and safety stock in MRP systems is not conclusive. For example Whybark & Williams (1976) found in their simulation studies, that uncertainty in timing of demands or in timing of replenishments resulted in safety time being preferable to safety stock, while uncertainty in the quantity demanded or quantity produced resulted in safety stock being preferable to safety time. And by contrast, Grasso & Taylor (1984) found that safety time was never preferable to safety stock. However, since they only tried a couple of safety time alternatives, it is possible that some other choice of safety time might have outperformed systems with safety stock. Furthermore Buzacott & Shantikumar (1994) showed with a stochastic models that for single stage manufacturing system which is controlled by MRP, safety time is preferable to safety stock if there is a good forecast of future required shipments. If only the mean demand can be predicted, the models suggest that either safety stock of safety time can be used. Because earlier research is not conclusive and the advantage of decoupling material release from capacity allocation we advocate the use of planned lead times.

Until now, most of the MP models for solving production planning problems have zero-length or fixed lead times, see e.g. Billington *et al.* (1983), Chung & Krajewski (1984), Hopp & Spearman (2000), and Tempelmeier (2003). These fixed lead times, sometimes also called planned lead times, are modelling minimal lead times or delays, where capacity is allocated *at* this fixed time offset. In the model introduced in this thesis, capacity can be allocated at *any* period in time during the planned lead time interval. The introduction of planned lead times with multi-period capacity consumption thus provides an additional degree of freedom, since it decouples material release from resource consumption over time. Only in Negenman (2000) and De Kok & Fransoo (2003), this planned lead time concept is used before. However they did not exploit the extra flexibility created with planned lead times.

Note that we assume that each transformation activity is performed in one period, hence minimum transformation times longer than one period (e.g. when drying paint or transportation overseas) are not considered. However, these minimum transformation times can be taken into the model easily by defining latest starting times.

Positioning SCOP in the Planning Hierarchy

In figure 1.4, we depicted the position of Supply Chain Operations Planning in the Planning Hierarchy. We see that SCOP is positioned above the production unit control functions that are responsible for controlling lead time in a particular production unit of the supply chain (cf. Bertrand *et al.* (1990)). The information passed from the SCOP problem to the production unit control functions are the release decisions of the each items and its due date, i.e. its release date plus its planned lead time. The detailed production schedule is made on the level of the production unit control functions. On the SCOP level however, a capacity check for the released orders is already done.

Next to the SCOP function, an order acceptance function may be introduced in the control loop in order to control the total amount of work accepted by the supply chain. However we assume a make-to-stock environment, whereby all customer demand is satisfied either directly from stock or later by backorders.

Finally, a parameter setting function needs to coordinate the safety stocks, planned lead times, and workload parameters of the Supply Chain. In the remainder of this subsection we focus on the relation between the SCOP function and the production unit control functions.

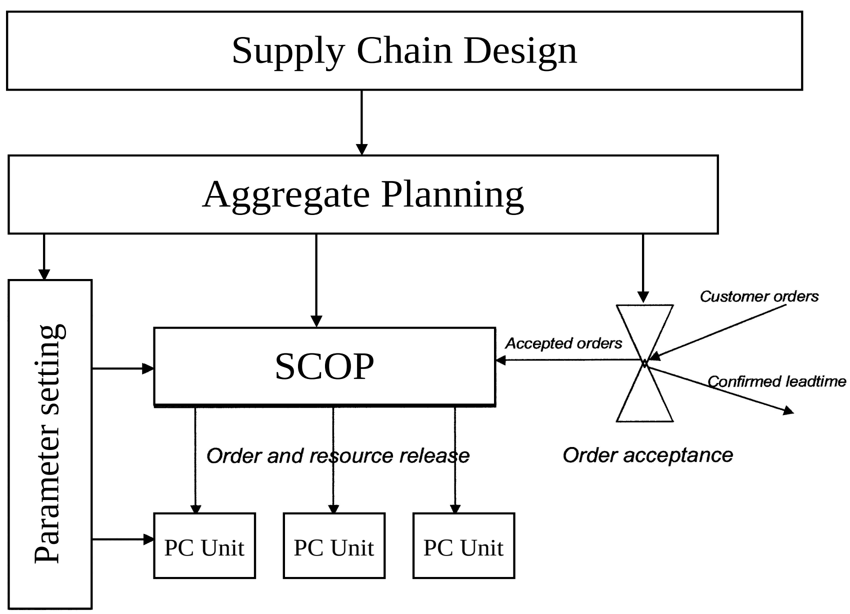

Figure 1.4: *Position of Supply Chain Operations Planning in the Planning Hierarchy (adapted from De Kok & Fransoo (2003), page 618).*

In this thesis we compare various SCOP functions based on the inventory cost, whereby we assume that lot sizing and setup time restrictions are irrelevant at the

SCOP level. More precisely, when we assume that in each period for all items a positive quantity is released, we can refrain from considering fixed set up or ordering cost. Implicitly this means that the planning period must be sufficient long. Note that especially for items upstream in the supply chain a long planning period dampens the fluctuations in the stochastic demand. However the stochastic demand still has a large impact on the production planning of downstream items, especially when they are produced on capacitated resources. In many practical applications lot sizing is not an issue at the SCOP level. Either because of sufficiently high manufacturing flexibility, or because of the time aggregation into weekly of monthly buckets. We believe that lot sizing restrictions either should take place on the aggregate planning level or on the level of production unit control functions. In the former, lot sizing restrictions are considered when defining the parameter settings, e.g. items can or cannot be produced in a certain period. In the latter, lot sizing restrictions are a part of the detailed production schedules.

Apart from the mathematical complexity of applying detailed scheduling on a supply chain wide scale, we also have to deal with the asymmetry in information. Information asymmetry basically entails the fact that when making a decision at a higher level, the amount and quality of information may be different from when the lower decision is made (later), and again different from when the actual execution of the decision is taking place. As a consequence of this information asymmetry, the actual schedule may be very different from the projected detailed schedule constructed to make the supply chain plan, because it is difficult to forecast producing times and schedules may depend heavily on actual order releases. In fact more detailed scheduling of materials will then lead to additional constraints on the operational level. Hence we consider the production unit control functions as black boxes which have to realize a set of targets set by the SCOP function. The targets are set by the released orders and their due dates. The due dates are defined by the planned lead times of an order.

At the SCOP level we have to deal with demand uncertainty. In Karmarkar (1993) the impact of demand uncertainty in the context of finite resource availability is extensively discussed. Released work orders queue either before a production department and/or within a production department at resources. The former is the case when applying workload control policies as proposed by Bertrand *et al.* (1990) and Hopp & Spearman (2000), or input/output control policies such as the MRP-I concept. The latter results from both demand and processing uncertainty. It is well-known that such queuing or waiting times account for 60%-80% of the total throughput time of work orders. We emphasize here that such waiting times cannot be explained by deterministic models. Since MP models do not take uncertainty into account, it is necessary to incorporate the impact of uncertainty through exogenous input variables, i.e. planned lead times and safety stocks. Only with reasonable due dates, and thus reasonable planned lead times, it is possible to find feasible schedules. Though planned lead times enable reliable due dates of *work* orders, the phenomenon of load-dependent *customer* order lead times cannot be resolved. The customer order process is exogenous and stochastic, so that available resources may be insufficient to guarantee a fixed or customer-specific lead time. In that case the only way to meet customer lead time requirements with high probability is to hold safety stocks. Thus our approach towards dealing with temporary increases in customer demand is in line with the workload control principles of Bertrand *et al.* (1990) and Hopp & Spearman (2000) who advocate to protect the production environment against temporary overloads and thereby shift excess throughput time from the production department to the customer order desk.

1.2.3 MP models for the SCOP problem

In section 1.2 we have stated that the aim of the SCOP problem is to ensure that the release of materials and the allocation of resources is done such that customer service levels are met at minimal costs. The first widely implemented concept to coordinate the release of materials is the Material Requirement Planning (MRP) concept, see e.g. Orlicky (1975), Vollmann *et al.* (1988), and Baker (1993). The major drawback of MRP, mentioned by many authors, e.g. Buxey (1989), Bertrand *et al.* (1990), and Baker (1993), is the lack of capacity constraints.

Since capacities are not considered, MRP has been extended to MRP-II (Wight (1981)). However, in MRP-II capacity and material violations can only be detected, but not resolved automatically (Hopp & Spearman (2000)). To cope with capacity constraints and material availability constraints mathematical programming (MP) models have been introduced. Voß & Woodruff (2003) illustrate the relationship between the MRP and MRP-II concepts and the mentioned deficiencies of MRP-II by formulating mathematical optimization models.

Although MP models for the planning of the release of materials and the allocation of resources were not very successful in practice at the start, see Shapiro (1993), some major disadvantages have been resolved today. Increasing computer power and more efficient algorithms make problems of realistic size and complexity solvable in a reasonable amount of time. Even more important is the fact that the anchoring of MRP systems in industry seems to be less and less solid, due to the growing awareness of the deficiencies of MRP, see Shapiro (1993) and Hopp & Spearman (2000). Consequently, knowing the performance of the SCOP function in a practical context (simulated with a rolling schedule) and understanding its behavior is extremely relevant.

The SCOP problem can be seen as a multi-item multi-level capacitated lot sizing problem (MLCLSP). Literature reviews of mathematical programming approaches for production planning and supply chain planning can be found in Shapiro (1993), Baker (1993), and Erenguc *et al.* (1999). The MLCLSP formulation is a more general formulation than the SCOP problem, and can be be used for a variety of planning problems in the Supply Chain Planning hierarchy. If MLCLSP is used for the SCOP problem, it is assumed that material release is not decoupled from resource consumption, as in the case of planned lead times. In the remainder of this section we review some papers concerning MLCLSP.

The first approach that formulates an LP problem, originates from Billington *et al.* (1983). Similar models can be found in Chung & Krajewski (1984) and Hopp & Spearman (2000). These models plan the orders in detail. The exact amount to be produced on prespecified discrete points in time is determined. For each time bucket, i.e. the time between two successive decision points in time, a known capacity measure serves as an upper bound on the production. Specific to this kind of modelling is that the actual production lead times of the orders, as they arise as a result of the finite capacity, are outputs of the model and depend on the size of the order and the available capacity only.

In the review by Erenguc *et al.* (1999), a number of MP models that include the notion of planned lead times is given. But none of these formulations distinguishes between the planned order and the capacity usage, which is important to model manufacturing flexibility by avoiding inefficient usage of available resources in accordance with the planned lead time concept.

Belvaux & Wolsey (2001) focus on MP models that lend themselves to efficient solutions with commercial mixed integer programming (MIP) software. They derive problem specific necessary inequalities that improve the performance of MIP solvers. Furthermore they make a distinction between problems where planning occurs infrequently, i.e. big bucket problems, and problems where planning occurs frequently, i.e. small bucket problems. In big bucket problems many setups for many different items occur during the single planning period under consideration, and in the small bucket problems we see typically a small number of setups for a limited number of items where multiple planning periods are considered. Hence models for the big bucket problems differ from the models for small bucket problems. The SCOP problem can both be seen as a small and a big bucket problem. A small bucket problem because planning occurs frequently, typically every week. A big bucket problem because we assume that in each period a positive quantity is release for all items, such that lot sizing restrictions and setup times do not play a role in the supply chain operations planning.

Other MP approaches for solving the MLCLSP are based on Lagrangean relaxation of either the capacity constraints or the inventory balance equations or both, see e.g. Tempelmeier & Derstroff (1996) and Özdamar & Barbarosoglu (2000). Relaxing the capacity constraints and introducing Lagrange multipliers associated with those constraints reduce the MLCLSP to a number of independent uncapacitated problems. If also the inventory balance equations are replaced, a number of single item lot sizing problems is obtained. The solutions to these independent problems are tied together by an iterative procedure for the Lagrange multipliers based on the calculation of subgradients.

This concludes the brief survey of MP models for the solving an instance of the SCOP problem. The major difference between our approach and earlier research is the interpretation of lead time. While the majority of MP models have fixed lead time with capacity allocation at this fixed time offset, the LP model we introduce has planned lead times creating additional flexibility by decoupling the order release from the capacity usage.

1.3 Problem statement and research questions

The aim of the research presented in this thesis is to make a contribution to the development of more effective rolling schedule approaches for solving the Supply Chain Operations Planning problem.

In order to meet this goal the following two questions are investigated.

1. Can we use the planned lead time concept to improve the performance of deterministic SCOP functions?

With the introduction of the planned lead time concept order release decisions and capacity allocation decisions are decoupled. An item may be produced at any time during its planned lead time. Thus, additional flexibility is created by the introduction of planned lead times. To make a proper use of the created flexibility, the following three sub-questions are investigated in a rolling horizon context.

1^a. What is the influence of production timing on the performance of the model?

1^b. Which factors influence the length of the optimal planned lead time?

1^c. Should items be available for further production steps directly after production, i.e., before the end of their planned lead time?

Besides mathematical programming methods applied within a rolling schedule context, there also exists stochastic models such as echelon stock (or base stock) control concepts as introduced by Magee (1958) and Clark & Scarf (1960) for solving the SCOP problem. These stochastic SCOP functions work very well for relative simple supply chain structures without capacity restrictions. If we can tighten the gap between stochastic and deterministic SCOP functions for uncapacitated situations, the deterministic SCOP function used in a rolling schedule can be a good alternative for large arbitrary supply chain structures with capacity restrictions. Hence we also investigated the following question.

2. How well do deterministic SCOP functions perform while using a rolling schedule to incorporate demand uncertainty compared to stochastic SCOP functions, and how can we tighten a possible gap?

1.4 The experimental framework

In this thesis various SCOP functions are compared. To make a proper comparison we define a costs structure and a performance criterion, similar as done in De Kok & Fransoo (2003). The cost structure is defined by the long-run average of the inventory plus work-in-process costs per period, and given by

$$\bar{C} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{n=1}^{N} \alpha_i \left(I_{it} + \hat{I}_{it} + \bar{I}_{it} \right) \tag{1.1}$$

where α_i are the holding costs, I_{it} are the physical inventory levels, $\hat{I}_{it}$ are the work-in-process levels before production, and $\bar{I}_{it}$ are the work-in-process levels after production of item i at time t, $\forall i, t$. For the precise definitions of these variables and parameters we refer to chapters 2 and 3.

The inventory and its associated cost are necessary to ensure sufficient customers service. Customer service levels have to be defined for all items with independent demand, i.e. for all end items. We define E as the set of all end items. As performance criteria we choose φ_i, $i \in E$ defined as

$$\varphi_i = \lim_{t\to\infty} P\{I_{it} > 0\}, \quad i \in E \qquad \text{non-stockout probability} \tag{1.2}$$

Likewise the case of $\bar{C}$ we assume existence of φ_i. Notice that φ_i is identical to the P_3-measure defined in Silver *et al.* (1998). For each SCOP functions $\mathcal{P}$ we want to solve the following problem P_φ:

$$\begin{array}{ll} \min & \bar{C}(\mathcal{P}) \\ \text{s.t.} & \varphi_i(\mathcal{P}) \geq \varphi_i^*, \quad i \in \mathcal{P} \end{array} \tag{1.3}$$

whereby φ_i indicates the desired minimal customer service level. It is implicitly assumed that the SCOP problem $\mathcal{P}$ satisfies its set of material and resource constraints derived in the coming chapters.

The SCOP problem is solved by deterministic model instances embedded in a rolling schedule setting. Hence the the concept of service level constraints does not make sense. In order to still ensure that within the deterministic setting of the problem priority is given to satisfaction of exogenous demand, we use beside linear holding costs also linear backorder cost (c.f. De Kok & Fransoo (2003)). Assuming that expensive items are more important than cheap items we assume that the cost per item i backlogged at the end of a period is β_i. This yield to the following objective function

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_i I_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i\in E} \beta_i B_{i,s-1} \tag{1.4}$$

with E all end items, β_i the backorder cost of item i, and B_{is} the backorders of item i at time s. In this thesis we assume $\beta_i \gg \alpha_i$.

This objective function does not completely solve our problem. We want to determine feasible plans that satisfy customer service level constraints at low cost, since it is easily seen that the above problem yields the same solution for any value of β_i larger than some value β_i^0. This implies that with this objective function we obtain customer service levels that may not satisfy our objective. To obtain the service level constraints we require additional decision variables. Following the inventory management literature (cf. Silver *et al.* (1998)) we introduce the concept of safety stocks in order to cope with demand uncertainty

S_i : safety stock parameter of item, i, $i = 1, 2, \ldots, N$

In order to control the customer service levels we modify the objective function as follows,

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_i (I_{i,s-1} - S_i)^+ + \sum_{s=t+1}^{t+T} \sum_{i \in E} \beta_i (S_i - I_{i,s-1})^+ \tag{1.5}$$

Although objective function (1.5) does not represent the real inventory holding costs and backorder costs, it still reflects the tradeoff between inventory holding and backorder costs. On top of that the safety stock parameters control the service levels. With the following lemma this becomes even more obvious.

Sample path lemma
Suppose a sample path of the demand process and a sample path of the forecasting process are given. Furthermore assume that for all end items

$$I_{i,t} = S_i, i \in E$$

Then the solution to the problem expressed in terms of the material order releases R_{is} and the capacity claims V_{ius} (the definitions of these decision variables are given in chapter 2) with objective function (1.5) subject to the LP constraints (also given in chapter 2 is the same for each value of S_i, $i \in E$, for all $s \geq t$, with t is the current time.

The proof of the sample path lemma is based on induction. Given the initial inventory levels it is clear that the objective function (1.5) implies an optimal solution (R_{is},V_{ius}) at $t = 1$ based on demand forecast that is the same for any value of S_i. This implies that the solution ($R_{i,1}$,$V_{iu,1}$) at $t = 1$ based on the actual demand is the same for any value of S_i. But then $I_{i,1} - B_{i,1} - S_i$ is the same for any value of S_i. This argument can be repeated for any value of t. For a formal proof we refer to Kohler-Gudum & de Kok (2001).

Corollary to the sample path lemma
The problem P_φ for the SCOP function defined by the LP constraints and the objective function (1.5) have a unique solution $S_{i i \in E}$, where each S_i, $i \in E$, can be determined independent of all other S_j, $j \in E$,$j \neq i$.

Noticing that the objective function (1.5) is identical to (1.4) with $S_i = 0$, $i \in E$, the corollary to the sample path lemma justifies the following procedure.

1. Run a discrete event simulation of the system with $S_i = 0$, where at the start of each period $t = 1, 2, \ldots,$ we solve the LP based on the forecast of the demand.

2. From the discrete event simulation compute the empirical distribution function of $I_{it} - B_{it} - S_i$.

3. Given this empirical distribution function compute S_i^*, such that the required end-item service level is achieved.

4. Run another simulation with S_i^* in order to compute $\bar{C}(\mathcal{P})$.

1.5 Research design

The aim of this thesis is to make a contribution to the development of effective linear programming models for solving the Supply Chain Operations Planning problem. In this section we describe the steps that we have taken to reach this goal.

First of all we have to mention that already a lot of work has been done in this research area. The number of mathematical programming approaches for solving the SCOP or related problems is high (reviews on MP models can be found in Shapiro (1993), Baker (1993), and Erenguc *et al.* (1999)). However, these MP approaches are only solved for instances based on a forecast of customer demand. Since we consider demand uncertainty, we study an MP approach in a rolling schedule setting.

The LP model we use in this thesis originates from Negenman (2000). In chapter 3 of his thesis, he introduces a based LP policy for solving the SCOP problem, whereby the capacity restrictions are taken into consideration in an iterative way. We adapted this model such that the capacity constraints are a part of the LP model. We used this model instead of any other, because we liked the idea of planned lead times which decouple the order release from the capacity usage. This decoupling of order release from capacity usage not only gives a better reflection of the real-life situation, but we expected that by using the extra flexibility of planned lead times the performance of the LP based policy also would increase.

Throughout the thesis we explored various ways to improve the performance of the LP stategy. After each adjustment we studied the adjusted LP based policy under demand uncertainty. Since an analytic study on the performance of the model is impossible, we use a discrete event simulation to gain insights in the behavior of the decision variables.

In order to obtain good estimates for the different performance statistics we successively computed a number of m periods. To obtain m we computed the total cost per period. The value of m was determined such that the average costs have a relative error of at most 0.025 and a confidence level of 95% using a standard procedure, see Law & Kelton (2000). The difference in performance between the various parameter settings, especially between the "low" and the "high" value of a certain input parameter, is sufficient to be relevant for obtaining insights. Hence, all conclusions are based on conservative estimates of the significant effects.

For the simulation studies we used small, well-chosen supply chain structures. The reason we use small structures is twofold. First, it is easier to find relations between input parameters, decision variables, and the performance when the product structure is not too complicated. Second, the number of variables and constraints in the LP representation of the supply chain depends on the size of the supply chain. Larger supply chains means larger LP models, and larger LP models gives long computational times solving the LP problem. Since we use several simulation studies to gain insights in the performance of a certain LP model used in a rolling schedule concept, the computation time of an instance should be short to avoid extremely time consuming experiments.

Besides being small the supply chain structures must be representative. Hence, in chapters 3 and 6, we used the so-called W-structure. This 5-item structure consist of two end items, and each end item consist of a specific and a common child item. Thus the chosen supply chain consist of both divergent and convergent parts as most real-life supply chains have, therefor this supply chain is a good choice to measure the performance of the adjusted models. In chapters 4 and 5, this W-structure was not sufficient to study the mutual relations between items produced on capacitated resources Hence in these chapters we use an other set of supply chain structures. In chapter 4 the representativeness of these supply chain structures is discussed. In all experiments we vary the squared coefficient of variation of the demand distribution, the utilization rate of the resources and the cost structure. Furthermore at the end of chapters 3, 4, 5, and 6, we show for the same 18-item supply chain setting with both divergent and convergent parts and a limited set of different input parameters, the savings made with the proposed adjustments in that chapter.

1.6 Outline of the thesis

The remainder of this thesis is organized as follows. In chapter 2, two equivalent LP models with planned lead times for solving the SCOP problem are introduced. The difference between the two models is the translation of the capacity restrictions into constraints for the LP model. The computational times of the two models are compared for different supply chain structures using different LP solvers. This chapter is based on Spitter *et al.* (2005a).

In chapter 3, which is based on Spitter *et al.* (2005b), we take a closer look at the planned lead time concept. We compare early and late production in a rolling schedule setting. With early production we mean that the production starts as soon as possible after the order is released. This limits the amount of unused capacity. By late production the production starts as late as possible to obtain lower work-in-process costs. We show that for items produced on resources with high utilization rates early production is beneficial.

In chapter 4, which is based on Spitter *et al.* (2003), we look at the factors influencing the optimal length of planned lead times. Similar to queuing theory we see that the variability of the demand and the capacity of the resources influence the optimal length of the lead time. Unfortunately a third factor, namely the difference in costs between end items and items produced on capacitated resources, plays a key role in determining optimal planned lead times. Therefore, predictions on the optimal length of planned lead times are hard to make beforehand.

In accordance with the planned lead time concept, orders are only available for succeeding production steps at the end of the planned lead time. Thus items may wait several periods before they can be used in succeeding production steps. Hence, in chapter 5, we developed a model in which items are available directly after production. Results of a test cases using supply chain structures with optimal planned lead times, show us that immediate availability does not seems to improve the performance of the model. Therefore we adjust the model, such that items are only available before the end of their planned lead time if they can avoid or reduce backorders. The same test cases now show an improvement in the performance of the LP based policy.

In chapter 6 we concentrate on the allocation of materials. In divergent parts of the supply chain child items have to be allocated among different parent items. In this chapter we put allocation rules into the LP model forcing the model to balance the allocation of child items. Another method for balanced allocation is replacing the linear objective function by a quadratic one. For uncapacitated supply chains, the allocation strategies are compared with the base stock policies, described in De Kok & Fransoo (2003), for supply chains with infinite resource capacity. We see that by using allocation strategies the existing gap in performance between stochastic SCOP functions and deterministic SCOP functions used in a rolling schedule context is tightened. For capacitated supply chains the performance of the allocation strategies is measured by comparing the inventory costs between the models with and without allocation strategies.

Finally in chapter 7 we complete this thesis by summarizing the results, formulating conclusions, and giving some recommendations for future research.

Chapter 2

Two LP models for solving the SCOP problem

2.1 Introduction

The basic problem of Supply Chain Operations Planning (SCOP) is to ensure that, given the constraints of the system, i.e. resource and material availability constraints, the best possible quantity of every item is released at the best possible time, such that customer levels are met at the lowest cost. Often Material Requirements Planning (MRP) systems are used for solving the SCOP problem, see, e.g., Orlicky (1975), Vollmann *et al.* (1988), Bertrand *et al.* (1990), and Hopp & Spearman (2000). The MRP logic has a major drawback. As mentioned by many authors, e.g. Buxey (1989) and Krajewski & Ritzman (1993), MRP logic does not take capacity constraints into account. As MRP systems do not provide a solution to this fundamental issue, it must be handled by human interventions, such are usage of safety stocks and replanning.

In this chapter, we propose two alternative formulations of a Linear Programming (LP) model to solve the SCOP problem. Compared to earlier developed mathematical programming models, e.g. see Billington *et al.* (1983), Chung & Krajewski (1984), Hopp & Spearman (2000), Voß & Woodruff (2003), and Tempelmeier (2003) for a comprehensive overview, the proposed LP model differs through the introduction of so-called planned lead times with multi-period capacity consumption. In Billington *et al.* (1983), Chung & Krajewski (1984), and Hopp & Spearman (2000), it is assumed that orders released at the start of a period are also processed in that period. In our model we assume that orders released at the start of a period must be processed before the given planned lead time expires, i.e. before its due date. The capacity required to produce the order is consumed during the planned lead time interval, i.e. capacity consumption in multiple periods is allowed. Billington *et al.* (1983) also introduce so-called planned lead times, yet these planned lead times are modelling minimal lead times or delays, where capacity is allocated *at* this fixed time offset. In the model presented in this chapter capacity can be allocated at *any* point in time within the planned lead time interval. The introduction of planned lead times with multi-period capacity consumption thus provides an additional degree of freedom, since it decouples material release from resource consumption over time. In general, introducing additional degrees of freedom allow for lower-cost solutions. When setting cost-optimal planned lead times we also have to take into account, besides the inventory at the stockpoints, the work-in-process inventory. In the models of Billington *et al.* (1983), Chung & Krajewski (1984), Hopp & Spearman (2000), and Tempelmeier (2003), only the inventory at the stockpoints are of interest when finding cost-optimal settings. In this chapter we restrict ourselves to the formulation of the LP model as such. The question of setting cost-optimal planned lead times is “extremely” complex and requires an extensive experimental study, which is left for further research. In chapter 4, initial results of this issue can be found. Note that since the end item demand is stochastic and the proposed model is deterministic, the SCOP problem is solved using a forecast of the end item demand during the planning horizon. If the planning horizon is sufficiently long seasonal effects can also be taken into account.

We assume that released orders are available at their due date (i.e. at the end of the planned lead time) only. Thus the shop executing the orders is modelled at the SCOP level by anticipating the throughput times of items and the resource availability of bottlenecks. We assume a 100% due date reliability from the shop because material and resource constraints are incorporated in the model. This particular anticipation assumption is in line with the literature on controlled Work in Progress (cf. Bertrand *et al.* (1990) and Hopp & Spearman (2000)). This literature recognizes the fact that in the context of periodic planning, throughput times of released orders may be multiple periods, due to complex routing along multiple resources and due to queueing at those resources. For an extensive discussion of the concept of anticipation in the context of hierarchical planning, we refer to Schneeweiss (2003) and De Kok & Fransoo (2003). We notice that Negenman (2000) was the first to propose a model with planned lead times with multi-period capacity consumption and applied this model to a two-level hierarchy, where the top level is modelled as described in this chapter and the base level is modelled as a flow shop.

Our LP model does not allow for inclusion of lot sizing constraints and set-up cost. However, in principle our modelling of the planned lead times with multi-period capacity consumption can be incorporated into multi-level lot sizing problems, yet the algorithmic consequences are clearly a subject for further research.

The remainder of this chapter is organized as follows. In Section 2.2 we consider the production dynamics. The notation which is used throughout this thesis is introduced in section 2.3. In section 2.4, the uncapacitated linear programming model with planned lead times is given, in sections 2.4.2 and 2.4.3 two sets of capacity constraints are given. The two models are compared on their computational times in section 2.5. In section 2.6, conclusions can be found.

2.2 Production dynamics

Consider the assembly department of item i. At time t a work order of quantity R_{it} is released. For each work order there exists a planned lead time τ_i restricting the department in the following way: a work order released at time t will *only* be available for the next stage at time $t + \tau_i$, i.e. the finished units can be used to fulfill independent or dependent demand at time $t + \tau_i$, but not before. Execution of the work order consists of several consecutive steps during the time interval $(t, t + \tau_i]$.

First, after releasing the work order, all its respective amounts of components are gathered. If item j is a component of i and we need h_{ji} units of j for the production of a single unit of item i, hence for the whole work order $h_{ji}R_{it}$ units of component j are needed at time t.

During the time period $(t, t + \tau_i]$ the work order is executed. For all resources, and in the successive periods $(t, t + 1], (t + 1, t + 2], \ldots, (t + \tau_i - 1, t + \tau_i]$ the capacity constraints should be met. Notice that the released work order R_{it} can be split up and that each part may be produced on several resources that can operate in parallel.

The capacity needed for producing one unit of item i equals p_i, so the total capacity for the released work order R_{it} equals p_iR_{it}. The capacity constraints imply that for each available resource during each appropriate time period the amount of work of the released work order R_{it} is not higher than the available capacity.

Finally, at time $t + \tau_i$, R_{it} units of items i are released and available. The *actual* completion of the complete work order can be before time $t + \tau_i$, depending on the outcome of the actual detailed planning and execution, but we assume that the work order is not available before time $t + \tau_i$. At time $t + \tau_i$ the R_{it} units are either stored at the inventory of i, or directly passed to fulfill dependent or independent demand.

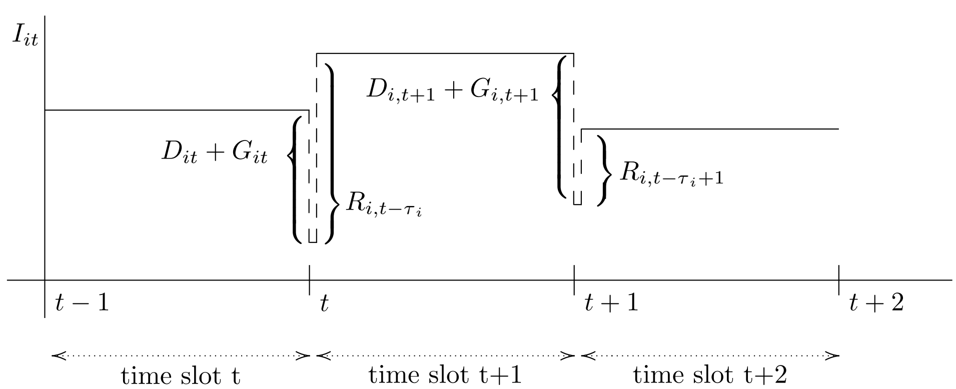

Figure 2.1: *Inventory position of item i for a number of periods.*

In figure 2.1, the inventory position of item i during a few periods is given. Assume a certain amount of inventory at time $t-1$, and observe the changes to the inventory in time slot t, i.e. in period $(t-1, t]$. We simplify the model by assuming that all changes occur just before time t. First, if item i is an end item then there can be exogenous demand D_{it} from customers. Second there might be endogenous demand from parent items of i. Third, orders $R_{i,t-\tau_i}$ released at time $t-\tau_i$ are received. Hence the inventory position at time t is equal to $I_{i,t-1} - D_{it} - G_{it} + R_{i,t-\tau_i}$.

2.3 Notation

The notation introduced in this section is needed for the formulation of the LP model for solving the SCOP problem. The notation is used throughout the whole thesis. Additional notation needed in succeeding chapters for the adjustment of the introduced LP models can be found in the concerned chapter.

Input parameters

t : $t \in \mathbb{N}$, the current time.

T : $T \in \mathbb{N}$, the planning horizon. The time period $[t-\tau_i, t+T]$ is divided in a starting time $t-\tau_i$ and $T+\tau_i$ time slots s, defined as $(s-1, s]$, $s = t-\tau_i+1, \dots, T$.

N : $N \in \mathbb{N}$, the number of different items. The items are labeled $1, \dots, N$.

p_i : $p_i > 0$, the capacity required for producing one unit of item i, $i = 1, \dots, N$. We assume this is the same for each of the facilities that could assemble i.

H : $H = (h_{ij})_{1 \le i,j \le N}, h_{ij} \in \mathbb{N}$, is the matrix representing the bills of material, i.e., h_{ij} is the number of units of item i needed for the production of a single unit of item j. We assume that the matrix H is lower triangular and that the diagonal of H contains only zero elements. This implies that no item can be (directly or indirectly) a component of itself.

τ_i : $\tau_i \in \mathbb{N}, \tau_i \ge 1$, $i = 1, \dots, N$, the planned lead time of item i.

α_i : $\alpha_i > 0, i = 1, \dots, N$, the inventory costs of holding one unit of item i.

β_i : $\beta_i \ge 0, i = 1, \dots, N$, the costs due to backordering, i.e., the unit penalty costs induced by backorders of item i.

D_{is} : $D_{is} \in \mathbb{N}, i = 1, \dots, N, s = t, \dots, t+T-1$, the exogenously determined demand for item i during time slot s.

L : $L \in \mathbb{N}$, the number of different resources. The resources are labeled $1, \dots, L$.

c_{ms} : $c_{ms} \in \mathbb{N}, m = 1, \dots, L, s = t+1, \dots, t+T$, the maximum assembly time available on resource m in time slot s.

$\mathcal{R}_i$: the set of resources that can be used by item i, $i = 1, \dots, N$.

$I_{i,t-1}$: the inventory of item i at time $t-1$, $i = 1, \ldots, N$.

$B_{i,t-1}$: the amount of backorders of item i at time $t-1$, $i = 1, \ldots, N$.

R_{is} : the planned orders of item i already determined at time $t-\tau_i, t-\tau_i+1, \ldots, t-1$, $i = 1, \ldots, N$, $s = t-\tau_i, \ldots, t-1$. These orders become available at time $t, t+1, \ldots, t+\tau_i-1$, respectively.

V_{ims} : the capacity claimed at resource m during time slot s for producing (part of) the orders $R_{i,t-\tau_i}, R_{i,t-\tau_i+1}, \ldots, R_{i,t-1}$, $i = 1, \ldots, N$, $m = 1, \ldots, L$, $s = t-\tau_i+2, \ldots, t$.

Z_{isu} : the part of planned order R_{is} of item i released at time s and already executed in time slot u, $i = 1, \ldots, n$, $s = t-\tau_i+1, \ldots, t-1$, $u = s+1, \ldots, t$.

Decision variables

I_{is} : the inventory of item i at time s, $i = 1, \ldots, N$, $s = t, \ldots, t+T-1$.

G_{is} : the requirements of item i at time s to produce its parent items, $i = 1, \ldots, N$, $s = t, \ldots, t+T-1$.

R_{is} : the planned orders of item i at time s, $i = 1, \ldots, N$, $s = t-\tau_i, \ldots, t+T-1$.

B_{is} : the backorders of item i at time s, $i = 1, \ldots, N$, $s = t, \ldots, t+T-1$.

V_{ims} : the capacity allocated to item i at resource m in time slot s, $i = 1, \ldots, N$, $m = 1, \ldots, L$, $s = t+1, \ldots, t+T$.

Z_{isu} : the part of the planned order R_{is} of item i released at time s and executed in time slot u, $i = 1, \ldots, n$, $s = t, \ldots, t+T-1$, $u = s+1, \ldots, s+\tau_i, u \leq T$.

Note that the characterization of the variables I_{is}, G_{is}, R_{is}, B_{is}, V_{ims} and Z_{ims} depends on s. If $s < t$ these variables are inputs to our model, if $s \geq t$ they are outputs of our model.

2.4 Linear Programming model

For solving the SCOP problem, we introduce a linear programming model with planned lead times. First we give the LP model without the capacity constraints. and in sections 2.4.2 and 2.4.3 capacity constraints are added in two separate ways. In section 2.4.2, the capacity restrictions are modelled with cumulative constraints and in section 2.4.3 balance equations are used. The equivalence between these two sets of capacity constraints can be found in appendix A.

2.4.1 Uncapacitated LP model

The aim of the SCOP problem is to minimize the inventory costs, hence the inventory costs belong to the objective function of the LP model. To ensure also that the exogenous customer demand is satisfied, as explained in section 1.4, we add backorder costs to the objective. This yields to the following objective function

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_i I_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \beta_i B_{i,s-1} \tag{2.1}$$

Since we minimize the inventory and the backorder costs, we have to know the inventory status. The net inventory $I_{is} - B_{is}$ of item i at time s is equal to the net inventory at time $s-1$ plus the received order $R_{i,s-\tau_i}$ which is released at $s-\tau_i$, minus the exogenous and endogenous demand of item i during time slot s, thus

$$I_{is} = I_{i,s-1} + R_{i,s-\tau_i} - D_{is} - G_{is} + B_{is} - B_{i,s-1}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{2.2}$$

The endogenous demand is equal to the summation over all order requests of parent items, and given by

$$G_{is} = \sum_{j=1}^{N} h_{ij} R_{js}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{2.3}$$

The following constraint ensures that the number of backorders cannot increase more than the exogenous demand. Since there is not an exogenous demand for intermediate items, this constraint also ensures that backorders can only occur at end items.

$$B_{is} - B_{i,s-1} \leq D_{is}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{2.4}$$

The decision variables must be all equal to or larger than 0, so the following boundary constraints are also added to the model.

$$R_{is}, B_{is}, I_{is} \geq 0, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{2.5}$$

Note that although the planning horizon equals $t+T$, solutions are only obtained up to time $t+T-1$. This is due to the fact that values of $R_{i,t+T}$ only depend on future inputs determined after time $t+T$.

2.4.2 Cumulative capacity constraints

As already mentioned, we give two sets of capacity constraints, namely cumulative capacity constraints and balanced capacity constraints. Both sets of capacity constraints give equal minimal costs, only the allocation of orders to the resources is different. We begin with the cumulative constraints. In the cumulative capacity constraints, the required capacity for order R_{is} of item i at time s is divided over the so-called capacity claims V_{ims}. Capacity can only be claimed from resources that are able to produce item i. Inequality (2.6) ensures that capacity is not claimed before the request is made and inequality (2.7) sees to it that the requests are produced within their planned lead time. We assume here that the production of an order does not start before all earlier requests have been fulfilled. We explain these inequalities in example 2.1.

$$\sum_{u:-\tau_i < u \le s} p_i R_{iu} \ge \sum_{m \in \mathcal{R}_i} \sum_{u:-\tau_i+1 < u \le s+1} V_{imu} \quad i = 1,\ldots,N,\ s = t-\tau_i+1,\ldots,t+T-1 \tag{2.6}$$

$$\sum_{u:-\tau_i < u \le s} p_i R_{iu} \le \sum_{m \in \mathcal{R}_i} \sum_{\substack{u:-\tau_i+1 < u \le s+\tau_i,\\ u < t+T}} V_{imu} \quad i = 1,\ldots,N,\ s = t-\tau_i+1,\ldots,t+T-1 \tag{2.7}$$

Example 2.1 *We explain inequalities (2.6) and (2.7) with an example. Assume that for a certain item the planned lead time is equal to* 3 *and that this item is produced on resource* u*. Assume furthermore that there are no orders or capacity claims from the past, and* $p_i = 1$*. Then we have*

$$V_1 \le R_0 \le V_1 + V_2 + V_3 \tag{2.8}$$

$$V_1 + V_2 \le R_0 + R_1 \le V_1 + V_2 + V_3 + V_4 \tag{2.9}$$

$$V_1 + V_2 + V_3 \le R_0 + R_1 + R_2 \le V_1 + V_2 + V_3 + V_4 + V_5 \tag{2.10}$$

$$\cdots \tag{2.11}$$

For simplicity, only the time indices are used. In (2.8) we see that capacity for order R_0 *can be claimed in time slots* 1, 2, *and* 3*. In (2.9) it is immediately clear that capacity for order* R_1 *can be claimed up to time slot* 4*, it is not so clear that capacity for this order cannot be claimed before time slot* 2*. Assume that capacity is claimed for* R_1 *in time slot* 1*. Since production of an order does not start before all earlier orders are produced, all the capacity needed for order* R_0 *is also claimed in time slot* 1*. Thus the capacity claim* V_1 *consist of the whole order* R_0 *plus (a part of) order* R_1*. This is in contradiction with the left-hand-side of inequality (2.8). Hence capacity for order* R_1 *cannot be claimed before time slot* 2*.*

The capacity claim V_{iut} is the capacity allocated to some items i at resources u during time slot t. The total capacity claimed on a certain resource during a time slot cannot exceed the capacity of that resource, which is ensured by the following inequality

$$\sum_{i:m \in \mathcal{R}_i} V_{ims} \le c_{ms} \quad m = 1,\ldots,L,\ s = t+1,\ldots,t+T \tag{2.12}$$

Capacity claims are always equal or larger than 0, so the following boundary constraint is also added to the model.

$$V_{ims} \ge 0, \quad i = 1,\ldots,N,\ m = 1,\ldots,L,\ s = t+1,\ldots,t+T \tag{2.13}$$

2.4.3 Balanced capacity constraints

In this section the capacity constraints are met by using balanced capacity constraints instead of the cumulative capacity constraints. An order R_{it} can be produced during whole its planned lead time. Therefore, the order can be split in several parts dependent on the length of the planned lead time. And each part of the order is produced in a separate time slot. Mathematically, this is given by

$$p_i R_{is} = \sum_{\substack{u:s+1 \leq u \leq s+\tau_i, \\ u \leq s+T}} Z_{isu}, \quad i = 1, \ldots, n, \; s = t - \tau_i + 1, \ldots, t + T - 1 \tag{2.14}$$

Then, the items $\sum_{u:s-\tau_i \leq u < s} Z_{ius}$, released in periods $s - \tau_i, s - \tau_i + 1, \ldots, s - 1$ and produced in time slot s, are allocated to a feasible resource m.

$$\sum_{u:s-\tau_i \leq u < s} Z_{ius} = \sum_{m \in \mathcal{R}_i} V_{ims}, \quad i = 1, \ldots, n, \; s = t + 1, \ldots, t + T \tag{2.15}$$

Of course, the total capacity $\sum_{i:m \in \mathcal{R}_i} V_{imt}$ claimed on resource m during time slot s cannot exceed the capacity of that resource, thus

$$\sum_{i:m \in \mathcal{R}_i} V_{ims} \leq c_{ms} \quad m = 1, \ldots, L, \; s = t + 1, \ldots, t + T \tag{2.16}$$

Again we have a set of boundary equations such that negative values do not occur

$$V_{ims} \geq 0, \quad i = 1, \ldots, n, \; m = 1, \ldots, L, \; s = t + 1, \ldots, t + T \tag{2.17}$$

$$Z_{isu} \geq 0, \quad i = 1, \ldots, n, \; s = t, \ldots, t + T - 1, \; u = s + 1, \ldots, s + \tau_i \tag{2.18}$$

The only difference between the two models is the way in which orders are allocated to the available capacity. Hence the models only differ in work-in-process cost, inventory costs are equal for both models. Therefore, the models are only compared on their computational times. The results can be found in the next section.

2.5 Comparing the models

The computational time of LP models depends on many factors. Some of these factors are related to the LP matrix, such as the number of variables, the number of constraints, the number of non-zero elements in the LP matrix, and the "shape" of the number of non-zero elements in the LP matrix. The computational time is also dependent on the method used for solving the LP model. In section 2.5.1 the supply chain under consideration is given and in section 2.5.2 the results are given and discussed.

2.5.1 Experimental design

For the comparison of the two LP models, the supply chain must be large enough such that the computational time is at least a few seconds. This lightens the comparison and the sight effects of system activities (of the computer) can be neglected. Of course we also wish to have the new features, namely multi-period capacity consumption created by planned lead times longer than one period and capacity allocation whereby several different items can be produced on the same resource and a resource can produce several different items, in the supply chain. Hence, for the comparison of the LP models we used a supply chain with 263 items, including 20 end items (see figure 2.2). The structure of the supply chain is convergent and consists of 5 levels, 20 end items on the first level, 30 items on the second level, 45 items on the third level, 67 items on the fourth level, and 101 items on the fifth level. Each item in the supply chain is used in at most one other item and consists of at least one other item. There are a finite number of resources on the first, third and fifth level. The number of resources on each level is equal to the number of items on that level divided by two. So on average one resource can be used by two different items. We assume a time horizon of 15 time slots, thus $T = 15$.

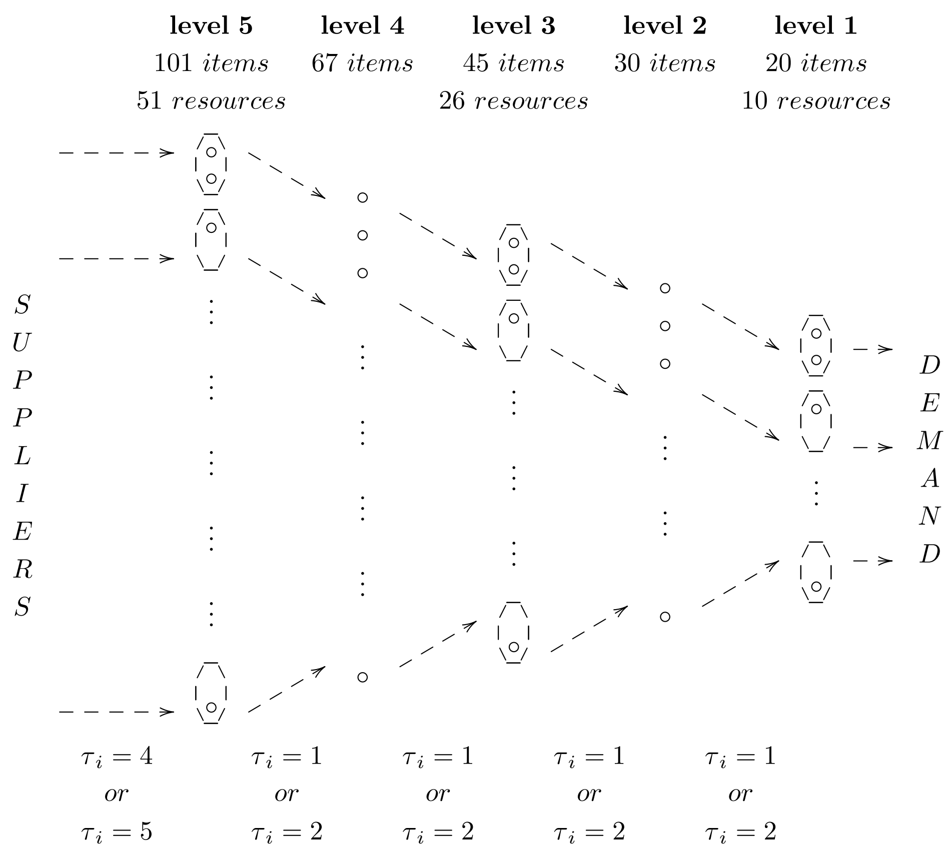

Figure 2.2: *Scheme of the supply chain structure.*

For the distribution of the end item demand we assume a gamma distribution with a mean of 50 units and a squared coefficient of variation of 0.25. There is no exogenous demand for components.

The planned lead times of the items on the first four levels are arbitrarily set to 1 or 2. The lead times of suppliers are a bit longer, we have set the planned lead times of the items on the lowest level arbitrarily to 4 or 5.

The inventory costs α_i of holding one unit of item i during a time slot t are equal for all time slots and consist of the summation of inventory costs of the preceding items and some additional value that depends on the level of the item. The unit penalty costs β_i induced by backorders of item i during time slot t are set to 10 α_i.

As starting values we have assumed that there are neither inventories nor backorders for any item. Furthermore we have assumed that the pipelines are filled with the average demand per period.

In the experiments we consider four different settings, namely the setting as given above, a setting with higher variation, a setting with a higher commonality factor, and a setting in which the number of resources are doubled.

Initial setting

The number of constraints, the number of variables, and the number of non-zero elements of the initial setting are given in table 2.1. We programmed the LP models such that only the variables influencing the optimal production plan are considered; if an item is produced on a resource with infinite capacity, defining variable V_{ims} is unnecessary since there will always be enough capacity available. Reducing the number of constraints, we have substituted equations (2.3) in equations (2.2) during the implementation.

	Number of constraints	*Number of variables*	*Number of non-zero elements*
LP-CUM	14175	14325	119640
LP-BAL	14175	22605	55710

Table 2.1: *Number of constraints, variables, and non-zero elements in the LP models.*

Higher variation in demand

In the initial setting the end item demand follows a gamma distribution with a mean of 50 units and a squared coefficient of variation of 0.25. In the experiments with higher variation, we increased the squared coefficient of variation from 0.25 to 2.

Higher commonality factor

In the initial setting, each item in the supply chain is used in at most one other item and consists of at least one other item, i.e. an item has at most one parent item. We say then that the commonality factor is equal to 1. In this setting we increase the commonality factor to 1.5. This means that on average each item has 1.5 parent items. The number of parent items of a child item is not restricted, and parent and child items do not have to be on successive levels.

If we study equation (2.3), we observe that if a child item has more than one parent item, the endogenously determined demand, G_{it}, consists of more than one order request. Hence the number of non-zero elements in the LP matrix increases, see table 2.2.

	Number of constraints	*Number of variables*	*Number of non-zero elements*
LP-CUM	14175	14325	121613
LP-BAL	14175	22605	57683

Table 2.2: *Number of constraints, variables, and non-zero elements in the LP models when the commonality factor is* 1.5.

Double number of resources

In the last setting the number of resources is doubled. Every item on the levels 1, 3, and 5 is now produced on its own resource. For the number of constraints, the number of variables, and the number of non-zero elements in the LP matrix the differences are minimal, see table 2.3. Note that, because we doubled the number of resources and simultaneously restricted the number of resources an items can be produced to one, the number of variables is not changed. We explain this with an example. Assume there are two products and one resource, then we have the variables $V_{1,1,t}$ and $V_{2,1,t}$. By doubling the resources each item is produced on its own resource and we have the variables $V_{1,1,t}$ and $V_{2,2,t}$, so the number of constraints is not changed.

	Number of constraints	*Number of variables*	*Number of non-zero elements*
LP-CUM	15360	14325	119520
LP-BAL	15360	22605	55590

Table 2.3: *Number of constraints, variables, and non-zero elements in the LP models when doubling the resources.*

2.5.2 Results

The experiments have been executed on a Toshiba Satellite Pro 4600 with a 750 Mhz Intel Celeron Processor. The LP models are solved using the CPLEX 6.6 solver. For a large number of supply chain settings, we have recorded the computational time for solving the models. The models are solved by four different solver options, namely the Primal Simplex, the Dual Simplex, the Primal-Dual Barrier, and the Network optimizer. The results can be found in table 2.4. Besides the initial setting given in the previous section, we have also recorded the computational time when the variation in the demand is higher, when the commonality factor is higher, and when the number of resources are doubled.

In table 2.4, we observe that the Dual Simplex option is fastest for all supply chain settings. This is also the solver option with the least difference in computational time between the different supply chain settings. Furthermore we see that for the Dual Simplex option LP-BAL is solved approximately twice as fast as LP-CUM.

For most experiments LP-BAL is solved faster then LP-CUM. Only if the Network Simplex option is used, LP-CUM is solved faster for some of the settings.

Another result is that the computational time of the settings with a higher commonality factor is longer then the computational time in the initial setting. Only for the Dual Simplex option, the difference is small. The Barrier option solved the setting with a double number of resources significantly faster then the initial setting. This is not the case for the other solver options.

When we look at the utilization rates of the resources we see a slight tendency to longer computational times if the utilization rates are higher. This is the case for all used solver options.

From the results we can conclude that the Dual Simplex solver option is most suitable for this kind of problems. When solving the SCOP problem, the model using equality constraints for capacity restrictions (LP-BAL) is preferable.

2.6 Conclusion

In this chapter we propose to incorporate the concept of planned lead times with multi-period capacity consumption in the formulation of quantitative models for Supply Chain Operations Planning. This concept implies that work orders released at the start of a period must be completed before the due date, i.e. before the end of the planned lead times, but can be processed in any period between date of release and due date. So by adding planned lead time, which allow multi-period capacity consumption, to the model additional flexibility is created compared to the original models with fixed lead times.

We have shown that the concept of planned lead times with multi-period capacity consumption can be elegantly incorporated into an LP model of the general capacitated assembly problem. We proposed two alternative LP models. In the first formulation, LP-CUM, the capacity restrictions are incorporated using cumulative inequalities. In the second formulation the cumulative inequalities are replaced by balance equations. From a set of experiments with various supply chain settings and 4 different solver options, we conclude that the Dual Simplex solver option finds the optimal solution for the SCOP problem fastest. Using the Dual Simplex option LP-BAL solves the problem faster than LP-CUM.

Utilization rate of the resources, on level			*Initial setting*		*Higher variation*		*Higher commonality factor*		*Double number of resources*	
1	3	5	C	B	C	B	C	B	C	B
Primal Simplex										
50%	50%	50%	7.5	3.1	6.7	3.1	29.5	4.3	7.6	2.9
90%	50%	50%	7.3	3.2	6.6	3.1	23.0	5.7	7.4	3.1
50%	90%	50%	8.6	3.8	7.6	3.4	26.4	6.0	7.8	3.1
50%	50%	90%	8.6	3.6	7.3	3.3	28.2	6.1	8.5	3.1
90%	90%	50%	8.3	4.0	7.7	3.5	37.1	8.2	7.3	3.3
90%	50%	90%	8.2	3.6	7.0	3.3	39.6	7.4	7.7	3.3
50%	90%	90%	10.0	3.9	8.1	3.6	30.9	8.0	9.1	3.3
90%	90%	90%	9.2	4.1	7.7	3.6	45.2	10.9	8.1	3.3
Dual Simplex										
50%	50%	50%	4.0	2.2	4.4	2.6	4.6	2.7	4.0	1.9
90%	50%	50%	4.0	2.2	4.4	2.6	4.7	2.8	4.1	1.9
50%	90%	50%	4.5	2.6	4.7	2.7	5.5	3.0	4.2	2.1
50%	50%	90%	4.2	2.5	4.7	2.7	4.9	2.9	4.2	2.4
90%	90%	50%	4.4	2.5	4.7	2.8	5.6	3.0	4.1	2.0
90%	50%	90%	4.1	2.5	4.6	2.7	5.0	3.1	4.3	2.0
50%	90%	90%	4.6	2.7	5.0	2.8	5.8	3.2	4.3	2.5
90%	90%	90%	4.5	2.6	5.0	2.8	5.8	3.3	4.2	2.0
Barrier										
50%	50%	50%	12.5	9.4	13.2	8.8	31.7	23.4	6.0	3.8
90%	50%	50%	13.3	9.9	14.5	9.2	37.5	28.2	6.9	4.4
50%	90%	50%	14.9	11.4	14.5	10.3	38.0	32.1	7.0	4.5
50%	50%	90%	13.5	10.0	12.6	8.6	32.5	26.2	6.1	3.9
90%	90%	50%	14.9	12.1	14.6	10.0	42.9	31.0	6.9	4.4
90%	50%	90%	12.7	9.7	13.1	9.6	37.6	28.0	6.9	4.4
50%	90%	90%	14.6	11.1	14.2	13.2	42.6	34.7	7.0	4.5
90%	90%	90%	14.8	11.2	14.6	9.2	51.0	30.3	7.1	4.4
Network Simplex										
50%	50%	50%	4.8	6.5	5.3	6.1	11.1	9.2	4.0	3.4
90%	50%	50%	4.5	6.5	5.3	6.6	12.0	7.6	4.1	3.3
50%	90%	50%	5.0	6.8	6.0	7.9	9.4	8.7	4.3	3.4
50%	50%	90%	4.7	5.9	5.2	6.9	9.1	9.2	4.3	3.7
90%	90%	50%	5.1	7.7	6.3	7.4	10.7	9.2	4.1	3.3
90%	50%	90%	4.8	5.7	5.6	6.7	7.9	8.4	4.2	3.5
50%	90%	90%	5.2	6.6	6.3	6.8	8.7	7.3	4.5	3.5
90%	90%	90%	5.0	6.4	5.8	6.8	8.2	8.6	4.2	3.3

Table 2.4: *Computational times in seconds for several Supply Chain settings and several solution methods for both LP-CUM (C) and LP-BAL (B).*

Chapter 3

Early vs late production during the planned lead time

3.1 Introduction

The Linear Programming models for solving the Supply Chain Operations Planning problem introduced in the previous chapter are typically used in a rolling schedule setting, where periodically plans are derived based on forecasts of future demand. When the forecast and the actual demand do not correspond, even a seemingly optimal production plan can turn out to be a poor production strategy. In this chapter we increase the performance of the LP model in a rolling schedule setting by exploiting the features of planned lead times. Planned lead times are defined such that an order is produced within the planned lead time, and only at the end of the planned lead time the order becomes available for further use. Although a part of the order may be finished earlier, this part is assumed to be work-in-process (WIP) at the factory floor, and is not available before the end of its planned lead time.

In the models of, e.g., Billington *et al.* (1983), Chung & Krajewski (1984), and Belvaux & Wolsey (2001), it is assumed that orders requested at the beginning of a period are processed in that period, or are processed in a multitude of periods whereby the production actually takes place during the whole time interval. So, when solving their mathematical programming models, the production of an order always begins as late as possible to avoid large inventory. Since we use the planned lead time concept, we have an additional decision parameter, namely the timing of production during the planned lead time. When producing as late as possible the WIP costs are low, since holding child items is in general cheaper than holding parent items. But when producing early, a part of the resource capacity is still available for the remainder of the planned lead time, and since we work in a rolling schedule setting with demand uncertainty, this leftover capacity can be used to satisfy future demand. So, early production can decrease the safety stocks required to satisfy given service levels. The net effect of these two opposites, late production to avoid WIP costs and early production to prevent capacity loss, is not clear. Therefore we set up experiments to develop a deeper understanding. In this chapter, we compare early production with late production for different input parameters.

The remainder of this chapter is organized as follows. In the next section, we show with an example the influence of the production timing on the available capacity. In chapter 2 we did not include the work-in-process cost when solving the SCOP problem, hence in section 3.3 we show how we can calculate them after each iteration step (of the rolling schedule). In section 3.4 early and late production are compared in a numerical experiment, and we end this chapter with conclusions in the last section.

3.2 The use of capacity; an example

We investigate how the timing of production influences the costs of inventory and work-in-process. As mentioned earlier, by producing at the beginning of the planned lead time we may have some capacity left at the end. Since we work with a rolling schedule and assume demand uncertainty, this capacity might be useful in the coming period. In this section we give an example to illustrate two different approaches of using the available capacity.

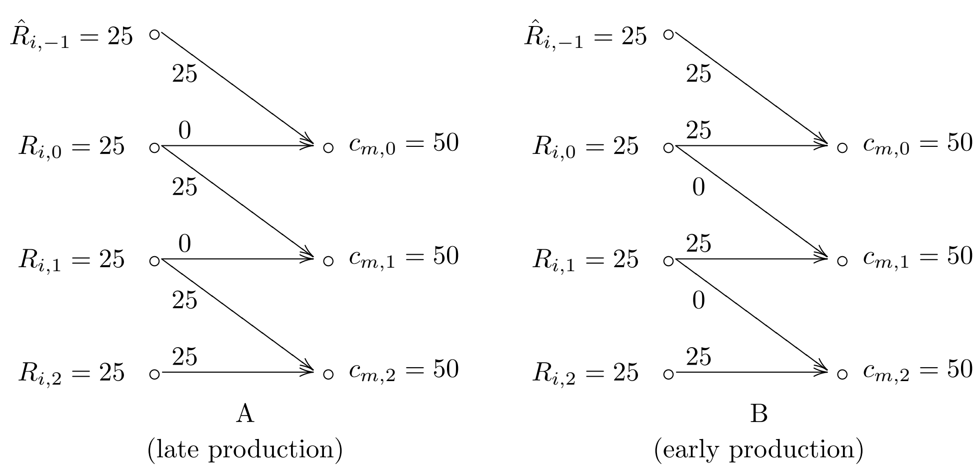

Figure 3.1: *Network flow model at $t = 0$. Producing late is shown in figure A and producing early in right figure B.*

We visualize the capacity use using a network flow model. Consider the production of an item i with planned lead time $\tau_i = 2$ on a single resource m. The capacity c_{mt} of that resource is equal to 50 during each time slot. The model is optimized for a time horizon of 3 periods and the requests R_{it} are known for the coming 3 periods. We assume that the capacity required for producing one unit of item i is 1, thus $p_i = 1$. In figure 3.1, we show the flows in the network. $\hat{R}_{it}$ is the part, of an earlier request R_{it}, that is not produced yet. We have $\hat{R}_{i,-1} = 25$; this part of the earlier request $R_{i,-1}$ has to be produced at $t = 0$. At $t = 0$, we also have a new request $R_{i,0}$ of 25 items. We still have a capacity of 25 items left, so we can choose to produce the complete order now or in the next period, which correspond with producing early and producing late respectively. In figure 3.1, the figure A shows the capacity use if producing late and the figure B if producing early.

If we look at the used capacity, we see that the available capacity at $t = 0$ is used completely if we decide to produce early. By producing late we still have half of the capacity left, which never can be used again.

We study production planning models with a rolling schedule setting. Hence we calculate an optimal production plan for a certain horizon based on demand forecast. After a period, we know the actual demand of that period, and the parameters of the model are adjusted according to the actual demand. For the coming time horizon a new production plan is calculated.

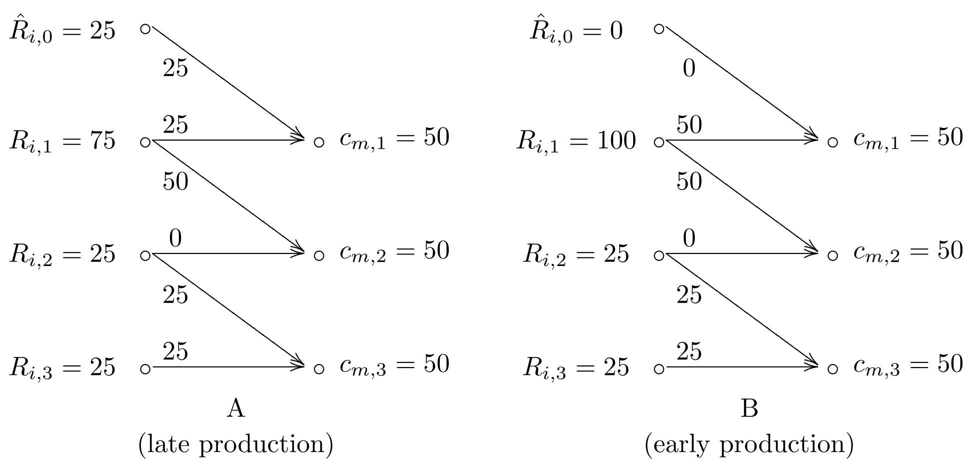

Figure 3.2: *Network flow model at $t = 1$. Producing late is shown in figure A' and producing early in figure B'.*

In figure 3.2, we shifted the time horizon with one period. Since we already met this request at $t = 0$, we see in figure B' that $\hat{R}_{i,0} = 0$ for the case of early production. For the case of late production, we still have to produce 25 items. Now we consider $R_{i,1}$, and we want to know the largest request possible. In the case of early production, figure B', we still have all capacity available at $t = 1$ and at $t = 2$. Since we have a planned lead time of 2 periods, we have two periods to meet the request. So the largest request possible is 100 items. In the case of late, figure A', production we still have to finish an earlier request of 25 items, so the largest possible request is 75 items.

We have seen that early production increases the number of possible requests, this is beneficial in models with demand uncertainty. By producing early the safety stock might be become smaller. A disadvantage of early production is the additional work-in-process inventory costs. In the LP model, given in chapter 2, the work-in-process is not included. In the next section we give formulas to calculate the work-in-process inventory. However we do not include them into the LP model. It is not desirable to include them, since we study the specific scenarios whereby production starts as late or as early as possible.

3.3 Work-in-process inventory

The production of an item takes place during its planned lead time. For an item i this means that it is in stock until there is a request for it. At that time this item is taken out of the stock and waits to be used. At a certain moment during its planned lead time the item is transformed into its parent item j together with the other child items of j. This parent item has to wait during the remaining lead time before it is put into the stock of items j. Waiting as child item before production is generally less expensive than waiting as parent item after production, due to the fact that production adds value in the form of labor, resource usage etc. Thus producing early might reduce the safety stock levels, while it increases the work-in-process inventory costs. In this section we give expressions for the amount of work-in-process inventory. Towards this end we define:

$\hat{I}_{is}$: the work-in-process inventory of item i at time s waiting after being produced, $i = 0, \ldots, N$, $s = t, t+1 \ldots, t+T-1$.

$\bar{I}_{is}$: the work-in-process inventory of item i at time s waiting to be used in production, $i = 0, \ldots, N$, $s = t, t+1 \ldots, t+T-1$.

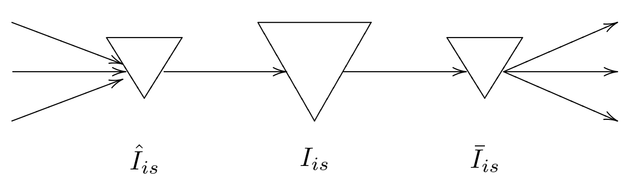

Figure 3.3: *Inventory and work-in-process inventory of item i.*

In figure 3.3 the three inventory points of an item i are shown. The work-in-process inventory $\hat{I}_{is}$ waiting after production of item i at time s, is equal to the inventory $\hat{I}_{i,s-1}$ plus the items that have been produced during time slot s minus the items at the end of their planned lead times. These are the items $R_{i,s-\tau_i}$ which are added to the inventory I_{is} of item i at time s. In equations (3.1) the work-in-process inventory balances of $\hat{I}_{is}$ are given.

$$\hat{I}_{is} = \hat{I}_{i,s-1} + \sum_{m \in \mathcal{R}_i} V_{ims} - R_{i,s-\tau_i}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \qquad (3.1)$$

The work-in-process inventory $\bar{I}_{is}$ waiting before production is equal to the inventory position of the previous period plus the total requests of parent items for child item i at time s minus that part of the request that had gone into production at time s. In equations (3.2) the work-in-process inventory balance of $\bar{I}_{is}$ are given. Note that these equations are non-existent for the end items. Since, they do not have parent items, they cannot get a request from them.

$$\bar{I}_{is} = \bar{I}_{i,s-1} + G_{is} - \sum_{j=1}^{N} \sum_{m \in \mathcal{R}_j} h_{ij} V_{j,m,s+1}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \quad (3.2)$$

Note that these two sets of work-in-process inventory balances are not added to the LP model with an accompanying cost function. By adding the work-in-process costs to the objective function, the LP strategu will find, for each instance, the optimal production plan with minimal cost. However, in this chapter we look at the extreme cases of early and late production in a rolling schedule setting.

3.3.1 Objective functions for early and late production

In the previous section we have seen that the work-in-process costs are not considered in the LP model. Hence, the actual production of a released order can take place during an arbitrary time within the planned lead time. To restrict the model to early or late production, an additional term is added to the linear objective function (2.1). The capacity claims V_{ims} reflect the usage of capacity, early production give large claims for small s and late production give large claims for large s. Hence the capacity claims should be added to the objective function, whereby the accompanying costs are relatively expensive for small s if we have late production and relative large costs for large s if we have early production. However, the additional costs for capacity claims must be small, such that optimal release strategy is not influenced by the timing of production. The objective function for late production becomes

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_i I_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \beta_i B_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \sum_{m=1}^{L} (T-s) \varepsilon V_{ims} \qquad (3.3)$$

whereby ε reflects a very small cost. We see that claiming capacity at the beginning of the time horizon is a bit more expensive than at the end of the time horizon. Since the objective function is minimized, production starts late. For producing as early as possible we change the objective in a similar way, see equation (3.4). The capacity claims at the end of the time horizon are more costly, so production starts early.

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_i I_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \beta_i B_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \sum_{m=1}^{L} s\varepsilon V_{ims} \tag{3.4}$$

3.4 Numerical study using a rolling schedule

In this section we study the performance of the standard LP strategy, i.e. the LP model given in chapter 2 in a rolling schedule context, compared with the two extreme cases of early and late production in a rolling schedule setting. For a fixed time period early production is always more costly, because of the higher work-in-process cost. However, in a rolling schedule with early production the leftover capacity can be used to satisfy future demand. And this can reduce the inventory costs. In section 3.4.1 we present the experimental setting in which we compare the performance of these three models, and the results of the comparison are given in section 3.4.2.

3.4.1 Experimental setting

For the comparison of the three models we use a simple arbitrary supply chain structure which makes a distinction between specific and common child items. The so-called W-structure, see figure 3.4, consists of both divergent and convergent parts. The two end items consist both of one specific child item and one common child item.

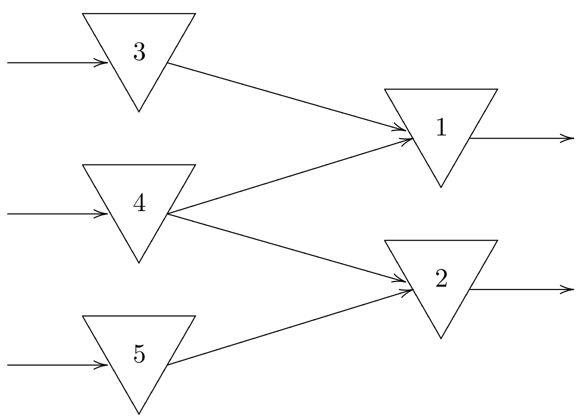

Figure 3.4: *Studied supply chain.*

For each parent item, we assume that one unit i of each child item is needed for the production of a single unit of the parent item j, thus $h_{ij} = 1$ for all $i = 1, \ldots, N$ and for all $j = 1, \ldots, N$ whereby j is a parent item of item i. The capacity requirement p_i for producing one item i is equal to one for all items. The number of resources is equal to the number of items, i.e. items are produced on dedicated resources. The raw materials (items 3, 4, and 5) are delivered by suppliers at the end of the planned lead time.

Initially we start with 24 test cases, whereby the planned lead time τ_i is equal to 2 periods for all i. The 24 test cases are divided into 4 scenarios. The 4 scenarios are given in table 3.1. In each scenario the items are assembled on resources with limited capacity. We study utilization rates of $0.7, 0.75, 0.8, 0.85, 0.9$, and 0.95. Furthermore we assume that $\beta_i = 10\alpha_i$ for all $i = 1, \ldots, N$.

	$cv_1^2 = cv_2^2$	$\alpha_1 = \alpha_2$
scenario 1	0.25	10
scenario 2	0.25	15
scenario 3	2	10
scenario 4	2	15

Table 3.1: *Parameter settings for four different scenarios*

In a fifth scenario, instead of looking at increasing utilization rates, we look at increasing planned lead times to get insight in the influence of the planned lead time on the production strategy. In scenario 5, we have $cv_1^2 = cv_2^2 = 0.25$, $\alpha_1 = \alpha_2 = 15$, and the utilization of the resources are set equal to 0.9. The planned lead time increases from 1 until 6 periods.

As already mentioned we study this supply chain structure in a rolling schedule. For the planning horizon T=10, the optimal production plan is calculated based on the mean demand. After a period is ended, the actual demand is known and the parameters are adjusted to this demand (cf chapter 1). The actual demand follows a gamma distribution with mean 100. Safety stocks are selected such that in 95% of the time no backorders occur. In chapter 1 the selection procedure of the safety stocks is discussed.

3.4.2 Results

To measure the performance of the models with late and early production, we take the inventory and the work-in-process costs. In figures 3.5 and 3.6, the relative and the actual difference in cost between the standard model and the adjusted models is given. In table 3.2 the required safety stock to meet the service level of 95% non-stockout probability, for scenarios 2 and 4, are given. We see that the required safety stocks are lower for early production compared with late production.

The results show us that in general producing as soon as there is available capacity during the planned lead time is cheaper then producing as late as possible. Especially for $cv_i^2 = 0.25$ and an utilization rate of 90% the savings obtained by early production are high. Only in scenario 2, for the relative low utilization rates it is more expensive

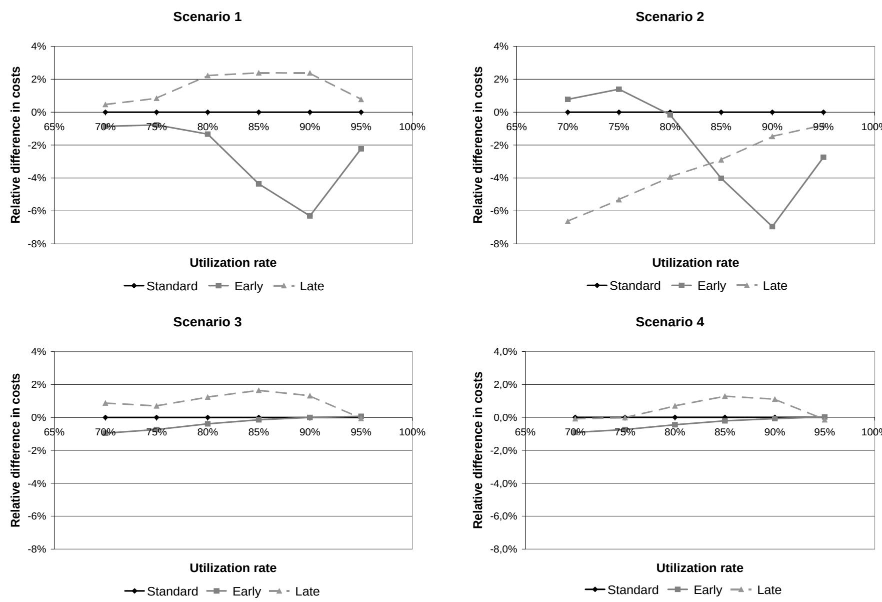

Figure 3.5: *Relative difference of inventory and work-in-process costs of producing late and producing early compared with the standard LP model of chapter 2.*

to produce early then late. This is due to the combination of small utilization rates and high holding cost of the end items. If the utilization rate is low, it is less necessary to save up capacity for worse times, because it is less likely to run out of capacity. On the other hand, the work-in-process cost for producing an item just-in-time is equal to 10 (with late production two raw materials have to wait the first period of the planned lead time), while the work-in-process cost for producing an item in the first period the work-in-process costs are 15 (one end items have to wait the last period of the planned lead time). So we although the safety stocks for early production are less (see table 3.2) then for late production, the savings are less than the additional work-in-process cost and producing late is cheaper.

Furthermore we see that for utilization rates of 95% the differences in inventory costs are small. This is caused by a lack of flexibility; the utilization rate is so high that there is less flexibility in the timing of production. Requests are forced to be produced late, because earlier request are not finished yet. Especially, if the variation in the demand is high, flexibility is desired, and the lack of it forces the models to produce late and the differences in costs become small.

If we look at the standard LP strategy, we see that this model is never the cheapest in the four scenarios. In the standard LP model there is not an explicit strategy for the timing of production, so the safety stock is not explicitly low like when producing early, while the work-in-process inventory costs are not as low as when producing late. However remark that there might be cases in which the LP strategy performs better, since early and late production are just two extreme cases. A combination of these

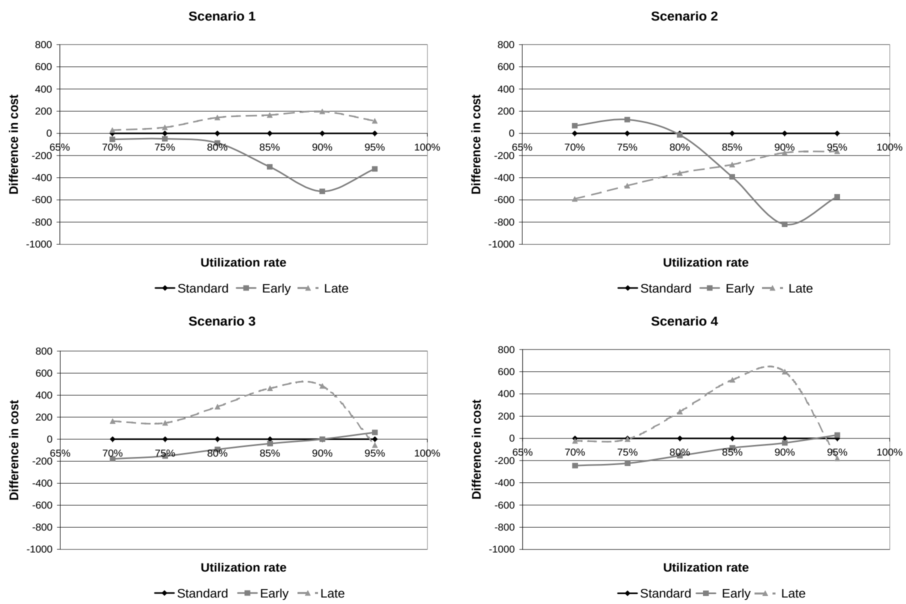

Figure 3.6: *Difference of inventory and work-in-process costs of producing late and producing early compared with the standard LP model of chapter 2.*

production strategies might be optimal.

Note that the safety stocks, see table 3.2, of the identically distributed end-items strongly differ. Apparently the used CPLEX solver, although the holding and back-order costs of both end items are equal, always favors item 1 when allocating the available child items. This allocation problem is also noticed in Negenman (2000) and in De Kok & Fransoo (2003). In chapter 6 of this thesis, we discuss the allocation problem thoroughly.

In scenario 5 we have a setting with a squared coefficient of variation equal to 0.25 and holding costs of end items equal to 15. We have set the utilization factor equal to 90% for all the resources and we increase the planned lead times from one period to six periods. Because of the long planned lead times the time horizon is increased to 20 periods. The results are plotted in figure 3.7.

First of all the total inventory and work-in-process costs are equal for all three models if the planned lead time is equal to one period. This result is not remarkable, since shifting with production timing is not possible for this value of the planned lead time.

Furthermore we see that the relative difference between the standard model and the model whereby production starts as late as possible increases rapidly for longer planned lead times. The explanation for this phenomenon is simple. Due to the long planned lead times, there is a lot of work-in-process. To minimize the accompanying cost, production should start as late as possible.

At last we see that for long planned lead times, any strategy is better then producing

Scenario 2	*Standard*		*Early*		*Late*	
Utilization rate	Item 1	Item 2	Item 1	Item 2	Item 1	Item 2
70%	205	225	203	216	209	228
75%	208	228	203	223	213	240
80%	215	246	208	233	225	266
85%	229	302	216	266	252	320
90%	314	431	279	374	346	448
95%	688	913	646	866	717	940
Scenario 4	*Standard*		*Early*		*Late*	
Utilization rate	Item 1	Item 2	Item 1	Item 2	Item 1	Item 2
70%	874	1061	866	1033	900	1068
75%	1004	1271	990	1253	1022	1293
80%	1166	1577	1155	1559	1193	1609
85%	1422	1982	1418	1966	1467	2027
90%	2135	2748	2129	2739	2183	2808
95%	5578	6243	5568	6243	5602	6261

Table 3.2: *Required safety stocks of the end items, for scenarios 2 and 4, to guarantee a service level of* 95% *non-stockout probability.*

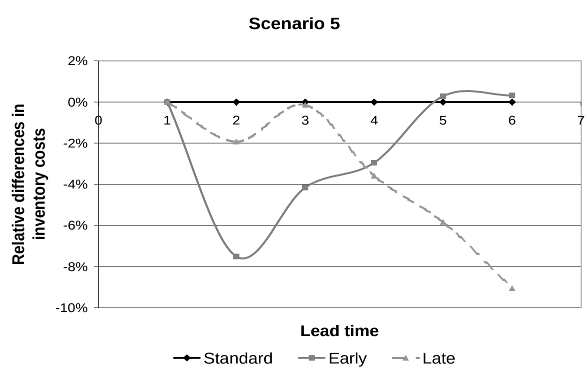

Figure 3.7: *Relative difference of inventory and work-in-process costs of producing late and producing early compared with the standard LP model of chapter 2.*

as soon as their is available capacity. Because of the long planned lead times, early produced items have to wait several periods before they can be used to satisfy end item demand. This results in high work-in-process cost. The achieved savings in safety stocks when using early production are cancelled out by the large work-in-process costs.

Summarizing the results we can conclude that production as soon as their is available capacity is beneficial if resources have high utilization rates and planned lead times are not too long. In these cases the additional work-in-process costs when using an early production strategy are small compared with the savings in safety stocks.

3.5 The 18-item model

To gain a better insight in the performance of the proposed adjustments to the standard LP strategy, we perform simulation studies with the same supply chain structure and parameter setting at the end of every chapter. For these simulation studies we use a supply chain structure with 18 items, namely 4 end items, 7 intermediate items, and 7 begin items. Each intermediate item consist of one begin item, and each end item consist of three intermediate items of which one specific item, one semi-common item, and one common item. This supply chain structure is given in figure 3.8.

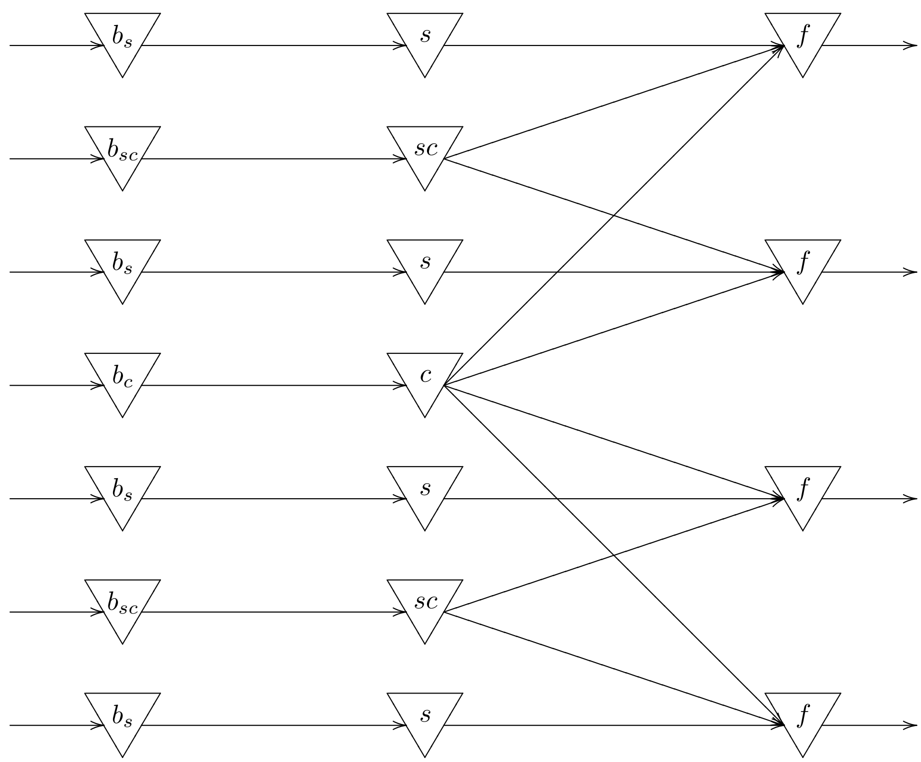

Figure 3.8: *Schematic representation of 18-item model.*

We assume that the demand for the end-items is stationary. More precisely, demand for end-item i in consecutive periods is i.i.d. We also assume that the demand processes for different end-items are uncorrelated. The mean demand is 100 for all end items. To get insight into the impact of demand variability on the performance we vary the squared coefficient of variation cv_i^2 for each end-item i as 0.25 and 2.

Furthermore we assume that there are three capacitated resources in the supply chain.

All intermediate items are produced on these resources, whereby the specific items, the semi-common items, and the common items are produced on separate resources. To get insight into the impact of resource utilization, we vary the utilization rates $(\rho_s, \rho_{sc}, \rho_c)$ as follows

$(0.85, 0.9, 0.95)$ resource producing common intermediate items high utilization rate, resource producing specific intermediate items low utilization rate

$(0.95, 0.9, 0.85)$ resource producing common intermediate items low utilization rate, resource producing specific intermediate items high utilization rate

To gain insight into the impact of the cost structure, we also vary the holding costs $(\alpha_f, \alpha_s, \alpha_{sc}, \alpha_c, \alpha_{b_s}, \alpha_{b_{sc}}, \alpha_{b_c})$ as follows

$(240, 20, 40, 60, 10, 20, 30)$ large added value at the end items

$(240, 30, 60, 90, 10, 20, 30)$ large added value at the intermediate items

For the planned lead times we assume the items produced on the bottleneck resource, i.e. the resource with the highest capacity, have a planned lead time of 2 periods. All other items have a planned lead time of 1 period. Thus, only for the items produced on the bottleneck resource there is a flexibility in the timing of production.

The simulation study is performed as described in section 1.4. The safety stocks are chosen such that we obtain a non-stockout probability of 95%. The results of this study can be found in table 3.3, whereby the relative differences are calculated by

$$\Delta_{\text{early vs standard}} = \frac{\bar{C}_{standard} - \bar{C}_{early}}{\bar{C}_{standard}}, \tag{3.5}$$

$$\Delta_{\text{late vs standard}} = \frac{\bar{C}_{standard} - \bar{C}_{late}}{\bar{C}_{standard}}, \tag{3.6}$$

and

$$\Delta_{\text{early vs late}} = \frac{\bar{C}_{late} - \bar{C}_{early}}{\bar{C}_{late}} \tag{3.7}$$

If we look at the results the first thing we notice is that for all but two cases the model with production as soon as possible performs best. Production as late as possible is in two cases better then the standard LP strategy, but always worse then producing as soon as possible. While in previous experiments in this chapter $LP_{standard}$ was never the best, in this experiment it gives the lowest inventory cost in two experiments. The explanation of these results are twofold. First early and late production are extreme cases to get insight in the problem, thus other production strategies might be better is some cases. Second, since both the specific intermediate items and the semi-common intermediate items are produced on one resource, the production timing of the different items is also an issue. Additional studies can be carried out to measure the influence of the production sequence of various items on the performance of the SCOP function.

			Early vs standard	Late vs standard	Early vs late
cv_i^2	$(\rho_s, \rho_{sc}, \rho_c)$	$(\alpha_s, \alpha_{sc}, \alpha_c)$			
0.25	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	11.76%	3.58%	8.49%
0.25	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	8.16%	-2.36%	10.25%
2	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	-0.30%	-3.15%	2.65%
2	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	1.33%	-4.37%	5.40%
0.25	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	8.33%	4.14%	4.43%
0.25	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	7.38%	-2.03%	9.18%
2	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	0.89%	-5.12%	5.72%
2	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	-1.39%	-5.18%	3.63%

Table 3.3: *Relative difference between the three strategies: the standard LP strategy, the LP strategy with early production, and the LP strategy with late production*

Furthermore we see that comparatively the largest saving can be achieved if the squared coefficient of variation is small. When the squared coefficient of variation is high customer demand is very whimsical, i.e. the amount of demand is either high of low. If the demand is low for several periods, it is better to produce late. However is the demand is high, all capacity must be used and production starts as soon as possible. Hence by choosing one of the extreme production strategies, the performance of the SCOP function does not increase considerately.

Comparing the results on cost structure, we again see that for the cost structure with a high added value at the end items the relative savings in performance are higher for early production versus standard LP strategy. In these cases the savings in safety stock cost are relatively high compared to the additional work-in-process cost when producing as soon as possible.

Last, we see small differences between having the bottleneck machine, i.e. the machine with highest utilization rate, producing specific of common intermediate items. In general the savings are less, when the specific items are produced on the bottleneck machine. In these cases the sequence of producing the four specific items also plays role.

3.6 Conclusion and further research

In this chapter we studied the timing of production within the planned lead time of an item in a rolling schedule setting. For this study, we used the LP model introduced in chapter 2. We adjusted the objective function of this model to influence the timing of production within the planned lead time. We considered two extreme production strategies, namely production as soon as there is available capacity and production as late as possible. Producing early has the benefit that safety stocks are lower. Less unused capacity is thrown away at the end of a period so there is more capacity left over for the coming periods. This means that fluctuation in the actual demand can be handled more easily and less safety stocks are necessary. A disadvantage is that the work-in-process inventory costs are higher than when producing late, because stocking a whole item is more expensive than stocking the materials it consists of. Producing late can be compared with the models of Billington *et al.* (1983), of Chung & Krajewski (1984), and of Belvaux & Wolsey (2001). In these models the lead time is variable and calculated according to the demands on available capacity. To avoid high inventory, orders are produced just in time.

In this chapter we studied a well-structured two echelon supply chain , whereby all items had a planned lead time of two periods, and we studied the 18-item model whereby only the items produced on the bottleneck resource had a planned lead time of two periods, all other items had a planned lead time of one period. For these two structures we can conclude that in general producing early is cheapest, only if the added value of the production step is too high and/or the planned lead times are too long, the work-in-process inventory costs for producing early become so high that producing late is more beneficial. For larger, multi-echelon supply chains more research has to be done, but the general idea still holds. If the savings in safety stocks are larger than the additional work-in-process costs, early production is advisable. Since work-in-process inventory can be found at every level of the supply chain, and safety stocks only at the highest level, early production should be restricted to the items produced on resources with high utilization rates.

We have seen that for utilization factors of 95%, the flexibility in the timing of production is very low. By increasing the planned lead times, the flexibility in the timing of production also increases. However, long planned lead time give high work-in-process costs. Hence, an interesting research topic is the determination of optimal planned lead times for arbitrary supply chains with uncertain demand. In the next chapter we identify factors which influence the optimal planned lead time.

Chapter 4

Factors which influence the optimal planned lead time

4.1 Introduction

In chapter 2, we introduced an LP model with planned lead times for solving the Supply Chain Operations Planning problem. The reason we use planned lead times is two-fold. First, by using planned lead times we can incorporate all kind of queueing effects. As Karmarkar [1987] points out, on the scheduling level several other variables besides the capacity, affect the queueing behavior in complex production environments. These variables are, amongst others, lot sizes, release times of batches and the coordination of these release times, sequencing at machines, production mix, and the heterogeneity of items. Furthermore, linear programming models are applied in a rolling schedule setting, where periodically plans are derived based on forecasts of future demand and other exogenous information. The difference between the actual demand and its forecast induces the typical queueing behavior that arises when finite resource availability is confronted with stochastic resource requirements. This queueing behavior cannot be made endogenous to a deterministic model.

Secondly, by using planned lead times additional flexibility in the timing of production is created. While in models without planned lead times production starts always starts as late as possible in order to meet customer service levels at minimal costs, in the model with planned lead times the production can, e.g., already start immediate after an order is released if there is capacity available. Note that orders in this model are released earlier than in the former models, e.g. Billington *et al.* (1983), Chung & Krajewski (1984), and Hopp & Spearman (2000)), because of the (longer) planned lead times. In the previous chapter, we have shown that producing early, especially when utilization rates are high, is beneficial. By producing early, not only the amount of unused capacity is reduced, but also the safety stocks. Since early production increases the work-in-process costs, the correct setting of planned lead times is important.

In this paper we investigate the setting of planned lead times when using LP strategies. Our research question in this chapter is: Which factors influence the planned lead times, and how? Our approach to this research question is both analytical and experimental. We first identify factors influencing the planned lead times by using queueing theory and previous research. Then we formulate testable hypotheses about the interesting experimental settings we created. We use the LP model with discrete event simulations to test these hypotheses.

4.2 Factors influencing the planned lead time

In this section we investigate which factors influence the optimal planned lead time. Since waiting time plays an important role in the lead time, we first discuss the expected waiting time for a single server queue when using queueing theory.

We consider a GI/G/1 queue, where the service time and the inter-arrival time between the orders have a general distribution. When an order arrives and the server is busy, the order waits in a queue. When all orders that arrived earlier have been finished, the waiting order is produced. So, we have a first in, first out (FIFO) system. Assume that the distribution of the service times is also general. Let the inter-arrival distribution A have mean $1/\lambda$ and variance σ_a^2, and let the service distribution G have mean $1/\mu$ and variance σ_g^2. For a stable system we must have $0 < \rho = \lambda/\mu < 1$. Let $cv_a^2 = \lambda^2\sigma_a^2$ and $cv_g^2 = \mu^2\sigma_g^2$, cv^2 is the squared coefficient of variation. We are interested in the average waiting time of this system. The approximation for the waiting time, which was first investigated by Kingman (1962), and is given by:

$$W_q = \frac{\lambda(\sigma_a^2 + \sigma_g^2)}{2(1-\rho)} \tag{4.1}$$

In our SCOP function it is assumed that demand occurs once per period, the demand distribution is general and we only know the expected demand $E[D]$ and the squared coefficient of variance cv_d^2. The orders derived from the demand are produced in order of arrival, and the utilization rate ρ of the server is known. There is a demand for products at the beginning of each period, so the inter-arrival time of orders is deterministic and equal to one period. Thus we have a variance of $\sigma_a^2 = 0$ and, since $E[A] = 1/\lambda$, $\lambda = 1$. The service time per unit is fixed, so the service time of an order only depends on the size of the order. Hence, the coefficients of variation of the distribution functions of both the demand and the server are equal, thus $cv_g^2 = c_d^2$. Further we know that $\sigma_g^2 = cv_g^2/\mu^2 = cv_d^2/\mu^2$ and $\rho = \lambda/\mu = 1/\mu$, thus $\sigma_g^2 = \rho^2 cv_d^2$. The expected waiting time is now given by:

$$W_q = \frac{\rho^2 cv_d^2}{2(1-\rho)} \tag{4.2}$$

In (4.2), we see that the waiting time is dependent on the utilization rate of the machine and on the coefficient of variation. Both higher utilization rates and higher variance in the demand increase the waiting time. Hence, the utilization rate and the coefficient of variation are two factors influencing the planned lead time.

So far we have identified two factors influencing the planned lead time, a third factor can be found in chapter 3. There we studied the timing of production. We considered early and late production within the planned lead time. The timing is amongst other factors dependent on the holding costs of the items. Producing early is only beneficial if the safety stocks savings are larger than the increased work-in-process costs. For planned lead times a similar reasoning is applicable. Longer planned lead times can reduce the safety stocks and thus the involved costs, however on the other hand the work-in-process inventory and thus the involved costs increase with longer planned lead times. So the difference between the holding costs of the end items and the holding costs of the items produced on capacitated resources is also a factor influencing the optimal length of the planned lead times.

4.3 Numerical experiments

In this section we look at the three factors, identified in the previous section, which have influence on the optimal planned lead time in an experimental setting. The chosen supply chain structures for the experiments are discussed in section 4.3.1. In section 4.3.2, we pose hypotheses about the setting of the planned lead time. The parameter setting in the experiments are discussed in section 4.3.3 and the results can be found in section 4.3.4.

4.3.1 Experimental setting

As we have seen in equation (4.2), long waiting times only occur if the utilization rate of a resource is sufficiently high. Hence, to make the experiments interesting, we use capacitated resources with high utilization rates. In the experiments we investigate the optimal planned lead times for various parameter settings. For each parameter setting we first have to find the optimal planned lead time, which means that for each parameter setting we have to try several different planned lead times to identify the optimal one. Since finding optimal planned lead times is time consuming, we restrict ourself to small supply chain structures with at most two capacitated resources. In the experiments we use three different supply chain structures, see figure 4.1. The items produced on capacitated resources are indicated with a circle, we are interested in the optimal planned lead time of these items. The planned lead times of the other items are set equal to one period, whereby we assume that this is optimal because waiting times at uncapacitated resources are very small.

The three chosen supply chain structures are representative for all kind of supply chain structures. Structure 1 is a very simple serial supply chain structure with only one capacitated resource. Since there is only one capacitated resource the optimal planned lead time is only influenced by the identified factors, and not by other capacitated resources. Of course, the effects of more than one capacitated resource in one supply chain is also an interesting topic. Hence we distinguish two other situations. In the first situation, structure 2, we have a convergent structure where two child items are processed on two different capacitated resources and used in the same parent item. The optimal planned lead times are influenced by each other because production of the parent item is only possible if all child items are available. In the second situation, structure 3, we have a serial structure with two capacitated resources. In this situation, the endogenous demand of the child item is influenced by the production rate of the parent item, and the production rate of the parent item is, among other factors, influenced by the length of its planned lead time. So the optimal planned lead time of the upstream items is influenced by the optimal planned lead time of the downstream items.

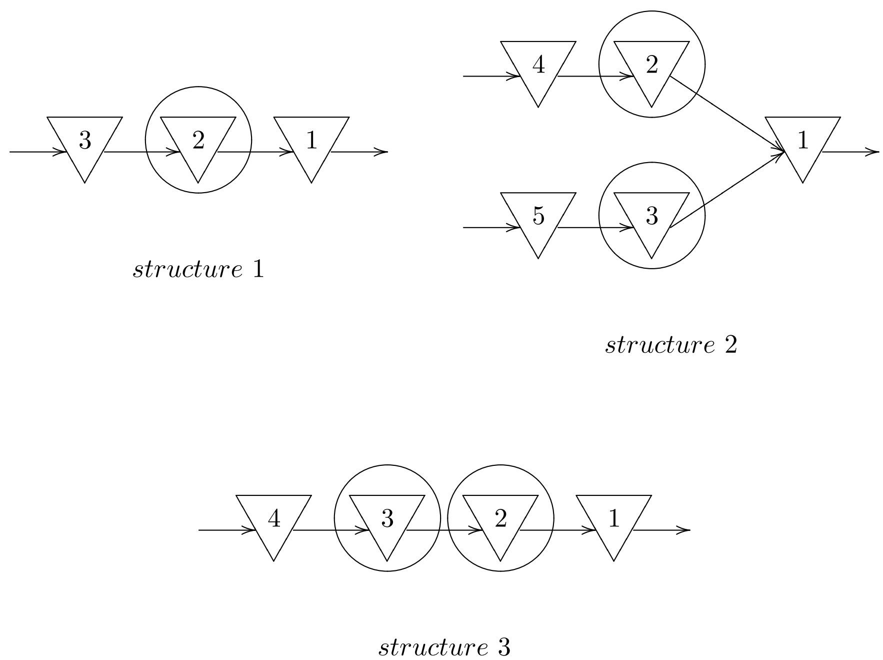

Figure 4.1: *Studied supply chain structures.*

We assume a one-to-one production ratio, where one unit of each child item is needed for the production of a single unit of a parent item. The capacity requirement p_i for producing one item i is equal to one for all items. The raw material is delivered by suppliers. We assume that these suppliers deliver exactly at the end of the lead time, so we do not have work-in-process inventory costs for these items.

We study these supply chain structures in a rolling schedule setting, as discussed in chapter 1. Safety stocks are selected, with the method discussed in chapter 1, such that in 95% of the time no backorders occur.

4.3.2 Hypotheses

In section 4.2, we distinguished three factors, namely the variation in demand, the utilization rate of the resources, and the holding cost structure in the supply chain, which influence the optimal planned lead time. In this section we pose hypotheses, to be tested in a rolling schedule setting, based on these three factors.

Hypothesis 1 *The optimal planned lead time of an item is longer if the utilization rate of the resource it is produced on is higher.*

Hypothesis 2 *The optimal planned lead time of an item is longer if the variation in the demand is higher.*

In the approximation of the waiting time, equation (4.2), we see that higher utilization rates and more variation result in longer waiting times. Thus, these two hypotheses are consistent with the findings in section 4.2. Hence, we expect that the optimal planned lead time increases if the utilization rate and/or the demand variation increase.

Hypothesis 3 *The optimal planned lead time of an item is longer if the difference in holding costs between this item and the end item is larger.*

Since longer planned lead times may reduce the safety stocks and thus the involved costs, longer planned lead times also create more work-in-process costs. Hence, the relation between the savings of the safety stocks reduction and the additional costs of increased work-in-process inventory influences the optimal planned lead time. So, we expect that the optimal lead time is longer if the difference between safety stock cost and the work-in-process costs is large.

4.3.3 Experimental parameters

To verify the posed hypotheses, we use simulation studies. The factors we investigate are the squared coefficient of variation, the utilization rate of the resources, and the holding cost structure. Finding optimal planned lead times is time consuming; for a certain parameter setting the simulation must be executed sufficient times with different planned lead times, and the planned lead time creating lowest cost is optimal. Hence, the number of experiments we can carry out within a limited time is restricted, so we have to choose the parameter settings carefully.

Squared coefficient of variation

In hypothesis 1, it is claimed that more variation in demand results in longer optimal planned lead times. Hence, in the experiments we vary between a small squared coefficient of variation, and a large squared coefficient of variation. As specific values we use $cv_1^2 = 0.25$ and $cv_1^2 = 2$.

Utilization rate of resource

Hypothesis 2 states that high utilization rates result in long optimal planned lead times. Hence, in the experiments we investigate different utilization rates, see table 4.1. For supply chain structure 1, we look at 4 increasing utilization rates and expect to find increasing optimal planned lead times. In structures 2 and 3, we consider equal and unequal utilization rates for the two capacitated resources in order to study the influence of the capacitated resources on the optimal planned lead times of the items produced on these capacitated resources.

Structure 1	Structure 2		Structure 3	
Item 2	Item 2	Item 3	Item 2	Item 3
$\rho_2 = 0.85$	$\rho_2 = 0.85$	$\rho_3 = 0.85$	$\rho_2 = 0.85$	$\rho_3 = 0.85$
$\rho_2 = 0.88$	$\rho_2 = 0.88$	$\rho_3 = 0.85$	$\rho_2 = 0.88$	$\rho_3 = 0.85$
$\rho_2 = 0.91$	$\rho_2 = 0.88$	$\rho_3 = 0.88$	$\rho_2 = 0.85$	$\rho_3 = 0.88$
$\rho_2 = 0.94$			$\rho_2 = 0.88$	$\rho_3 = 0.88$

Table 4.1: *The utilization rates used in the experiments.*

Holding cost structure

We characterize the holding cost structure by the added value per production level. We consider 3 holding cost structures for supply chain structures 1 and 2, and 4 holding cost structures for supply chain structure 3, see table 4.2. In each cost structure a large added value is added to the holing cost on a different level in the supply chain. For example, in holding cost structure A of supply chain structure 1 the large added value is added when the end item is produced, i.e. the holding costs are low for items 2, and 3 and high for item 1, while in structure C stocking item 3 is already expensive.

		Item 1	Item 2	Item 3	Item 4	Item 5
Structure 1	A	6	2	1		
	B	6	5	1		
	C	6	5	4		
Structure 2	A	8	2	2	1	1
	B	11	5	5	1	1
	C	11	5	5	4	4
Structure 3	A	7	3	2	1	
	B	7	6	2	1	
	C	7	6	5	1	
	D	7	6	5	4	

Table 4.2: *The holding costs for the different items used in the experiments.*

We perform the experiments with all combination of the parameter settings per supply chain structure, so in total, we have $2 * 4 * 3 = 24$ different parameter settings for supply chain structure 1, $2 * 3 * 3 = 18$ different settings for supply chain structure 2, and $2 * 4 * 4 = 32$ different settings for supply chain structure 3.

4.3.4 Results

The results of the experiments, namely the optimal planned lead times, for the three supply chain structures with various parameter settings are given in table 4.3.

			$cv_1^2 = 0.25$				$cv_1^2 = 2$			
str.	item	ρ	A	B	C	D	A	B	C	D
1	2	0.85	1	1	1		8	1	1	
1	2	0.88	2	1	1		10	1	1	
1	2	0.91	3	1	1		17	1	1	
1	2	0.94	4	1	1		31	1	1	
2	2 - 3	0.85 - 0.85	1 - 1	1 - 1	1 - 1		5 - 5	1 - 1	1 - 1	
2	2 - 3	0.88 - 0.85	2 - 1	1 - 1	1 - 1		8 - 4	4 - 1	4 - 1	
2	2 - 3	0.88 - 0.88	1 - 1	1 - 1	1 - 1		9 - 9	1 - 1	1 - 1	
3	2 - 3	0.85 - 0.85	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1
3	2 - 3	0.88 - 0.85	1 - 1	1 - 1	1 - 1	1 - 1	3 - 1	1 - 1	1 - 1	1 - 1
3	2 - 3	0.85 - 0.88	1 - 1	1 - 1	1 - 1	1 - 1	1 - 5	1 - 5	1 - 2	1 - 1
3	2 - 3	0.88 - 0.88	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1	1 - 1

Table 4.3: *Optimal planned lead times.*

Let us first look at supply chain structure 1, we see clearly that hypotheses 1 and 2 hold for the used parameter setting with holding cost structure A. Both for higher demand variation and higher utilization rates the optimal planned lead times increase. For cost structures B and C, the optimal planned lead times are all equal to one period. Apparently the difference in holding cost between the end item and the item produced on the capacitated resource is too small and the savings in the safety stocks are cancelled out by the additional work-in-process cost.

Supply chain structure 2 with cost structure A and high demand variation gives optimal planned lead times longer than one period. So higher demand variation increases the length of optimal planned lead times if the added value is high at the highest supply chain level. Looking at the utilization rates it is not so obvious that hypotheses 2 is true. If the utilization are of both resources are equal and both utilization rates increase with equal amount, the optimal planned lead times indeed also become longer. However, if only the utilization rate of one of the resources increases, the optimal planned lead time of the items on the other resource may even decrease (cf. $(\rho_2, \rho_3) = (0.85, 0.85)$ with $(\rho_2, \rho_3) = (0.88, 0.85)$ for holding cost structure A and $cv_1^2 = 2$). Thus next to the three identified factors which influence the optimal planned lead times, also other capacitated resources have influence on the optimal planned lead time of an item produced on a capacitated resource. In figure 4.2, we give the inventory costs of various planned lead times for the parameter setting with $cv_1^2 = 2$, $(\rho_2, \rho_3) = (0.85, 0.85)$, and holding cost structure A. We see that for symmetric supply chain structures the optimal planned lead times should be

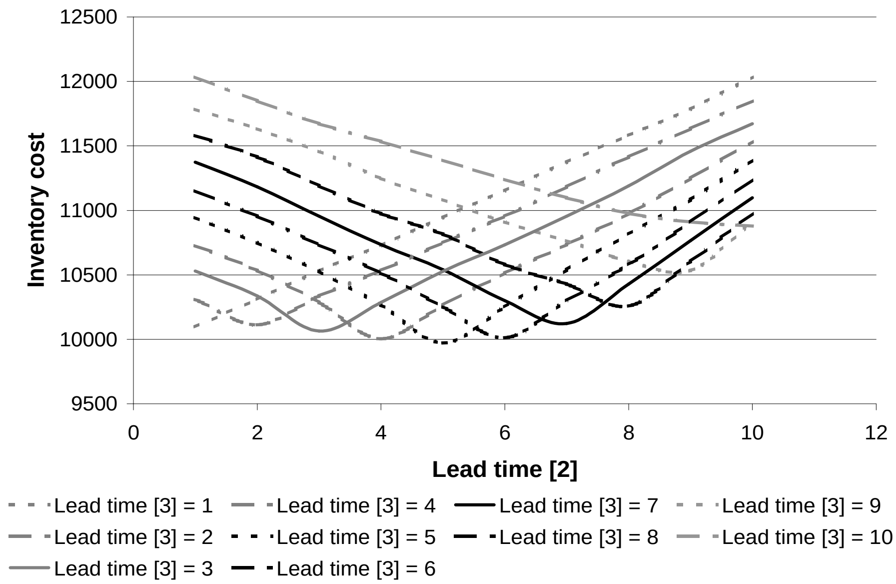

Figure 4.2: *Cost for different planned lead times.*

equal, and that the minimal cost are reached if $(\tau_2, \tau_3) = (5, 5)$.

For most parameter settings in supply chain structure 3, the optimal planned lead times are equal to one period. Thus possible costs savings by reduction in safety stocks are cancelled out by the additional work-in-process costs. Furthermore we see that if the optimal planned lead time is longer than one period, then only the item produced on the resource with the highest utilization rate has a 'long' optimal planned lead time. We see also that the optimal planned lead times are longer if the most upstream resource has the highest utilization rate, compare for example $(\rho_2, \rho_3) = (0.85, 0.88)$ with $(\rho_2, \rho_3) = (0.88, 0.85)$. These results are not surprising, since the holding cost of upstream items is lower, the additional work-in-process costs caused by longer planned lead times are also lower.

Having planned lead times with multi-period capacity consumption is a feature of the models introduced in this thesis. However, if we set the planned lead times equal to one period, the model is comparable with former models such as defined in Billington *et al.* (1983), in Hopp & Spearman (2000) and in Tempelmeier (2003). In table 4.4, we give the relative difference in inventory plus work-in-process cost between using the optimal planned lead times and planned lead times equal to 1 period, the difference is given by

$$\frac{\text{cost for } \tau \text{ equal to 1 - cost for optimal } \tau}{\text{cost for optimal } \tau} \times 100\% \tag{4.3}$$

First of all, we see that savings up to 33.7% can be reached. Furthermore we see that

			$cv_1^2 = 0.25$				$cv_1^2 = 2$			
str.	item	ρ	A	B	C	D	A	B	C	D
1	2	0.85	-	-	-		7.6%	-	-	
1	2	0.88	3.56%	-	-		14.6%	-	-	
1	2	0.91	7.82%	-	-		21.6%	-	-	
1	2	0.94	19.13%	-	-		33.7%	-	-	
2	2 - 3	0.85 - 0.85	-	-	-		1.3%	-	-	
2	2 - 3	0.88 - 0.85	0.2%	-	-		8.5%	3.7%	1.3%	
2	2 - 3	0.88 - 0.88	-	-	-		4.7%	-	-	
3	2 - 3	0.85 - 0.85	-	-	-	-	-	-	-	-
3	2 - 3	0.88 - 0.85	-	-	-	-	2.0%	-	-	-
3	2 - 3	0.85 - 0.88	-	-	-	-	5.5%	5.4%	0.5%	-
3	2 - 3	0.88 - 0.88	-	-	-	-	-	-	-	-

Table 4.4: *Relative difference in inventory plus work-in-process costs between the optimal planned lead times and planned lead times equal to 1 period.*

the holding costs again play an important role in determining the optimal planned lead times. While in supply chain structure 1, with $cv_1^2 = 2$ and $\rho_2 = 0.94$, the optimal planned lead time of 31 periods gives savings of 33.7% for cost scenario A, in scenarios B and C the optimal planned lead time is equal to 1 period. So no savings are reached by longer planned lead times in these two scenarios. Although the safety stocks decrease by the longer planned lead times in these scenarios, the cost savings of the decreasing safety stocks are smaller than the increase in work-in-process cost caused by the longer planned lead times. So longer planned lead times are only beneficial if the added value between the item produced on the capacitated resource and the end item is large enough.

	$\rho = 0.85$	$\rho = 0.88$	$\rho = 0.91$	$\rho = 0.94$
$cv_d^2 = 0.25$	0.60	0.81	1.15	1.84
$cv_d^2 = 2$	4.82	6.45	9.20	14.73

Table 4.5: *Approximated waiting times using equation (4.2).*

In section 2, we determined an approximation for the waiting times in a single item, single server queue. With this approximation we determined factors influencing the optimal planned lead times. Although the customer demand distribution differ from the endogenous demand distribution upstream in the supply chain, we still investigate if the approximated waiting times, obtain with equation (4.2), are usable approximations of the optimal planned lead times. In table 4.5, the approximated waiting times for different demand distributions and utilization rates are given.

In table 4.6, the relative difference in inventory plus work-in-process costs between the optimal planned lead times and the planned lead times derived by rounding up the approximated waiting times is given. The relative difference is calculated by

			$cv_1^2 = 0.25$				$cv_1^2 = 2$			
str.	item	ρ	A	B	C	D	A	B	C	D
1	2	0.85	-	-	-		1.2%	3.2%	12.9%	
1	2	0.88	3.56%	-	-		3.8%	1.3%	12.5%	
1	2	0.91	0.90%	-	-		9.2%	0.6%	12.4%	
1	2	0.94	14.64%	-	-		20.8%	0.9%	13.5%	
2	2 - 3	0.85 - 0.85	-	-	-		-	4.8%	15.5%	
2	2 - 3	0.88 - 0.85	0.2%	-	-		1.4%	4.3%	10.8%	
2	2 - 3	0.88 - 0.88	-	-	-		0.6%	3.2%	15.4%	
3	2 - 3	0.85 - 0.85	-	-	-	-	4.1%	17.2%	31.8%	40.0%
3	2 - 3	0.88 - 0.85	-	-	-	-	6.4%	14.1%	28.9%	34.5%
3	2 - 3	0.85 - 0.88	-	-	-	-	7.7%	15.3%	24.1%	32.4%
3	2 - 3	0.88 - 0.88	-	-	-	-	7.3%	16.8%	16.8%	43.4%

Table 4.6: *Relative difference in inventory plus work-in-process costs between the optimal planned lead times and approximated planned lead times.*

$$\frac{\text{cost for approximated } \tau \text{ - cost for optimal } \tau}{\text{cost for optimal } \tau} \times 100\% \tag{4.4}$$

We see that, since differences up to 40% are reached, the approximated waiting times are not suitable for determining good planned lead times. So we have to conclude that extended research has to be carried out to find heuristics for determining good planned lead times. In these studies not only the variation in demand and the utilization rate of the resource, but especially the holding cost structures must be taken into account. We have seen that holding cost structures play a key role in determining the optimal planned lead times. Longer planned lead times can decrease the safety stock costs, but this is only advantageous if the work-in-process costs do not increase faster.

4.4 The 18-item model

In the previous chapter we introduced the 18-item model, we saw that for most examined cases producing as soon as there is capacity available for the released item is cheapest. In this chapter we use simulation studies to determine the optimal planned lead times for the model with early production, whereby we assumed that items produced on the same machine have equal planned lead times. This is justified since the demand for all end-items in consecutive periods is i.i.d. and the mean and squared coefficient of variation are equal for all end items. The results are given in table 4.7

Also for the supply chain chapter we see that the optimal planned lead times are longer than one period. Furthermore we see that high variation and/or high added value at the end item level results in long optimal planned lead times. In the last column of table 4.7, we see the relative difference between the model with planned lead times equal to one period and the model with optimal planned lead times. This difference is given by

cv_i^2	$(\rho_s, \rho_{sc}, \rho_c)$	$(\alpha_s, \alpha_{sc}, \alpha_c)$	$(\tau_s, \tau_{sc}, \tau_c)$	Δ
0.25	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	$(1, 1, 2)$	7.72%
0.25	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	$(2, 1, 1)$	11.88%
2	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	$(1, 2, 8)$	17.67%
2	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	$(9, 2, 1)$	14.21%
0.25	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	$(1, 1, 2)$	3.18%
0.25	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	$(2, 1, 1)$	10.05%
2	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	$(1, 1, 3)$	6.46%
2	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	$(8, 1, 1)$	11.76%

Table 4.7: *Optimal planned lead times, and the relative difference between the model with planned lead time equal to one period and the model with the optimal planned lead time.*

$$\Delta = \frac{\bar{C}_{\tau_i=1} - \bar{C}_{\tau_i=\tau_i^*}}{\bar{C}_{\tau_i=1}}. \tag{4.5}$$

We see that, especially in the cases whereby the added value at the end item level is high, the savings are considerable. So we can conclude that spending time in finding optimal of at least good planned lead times is worthwhile.

4.5 Conclusions

In this chapter we focussed on the factors which influence the optimal planned times when using the SCOP function in a rolling schedule setting with demand uncertainty. Notice that on one hand the work-in-process cost increases when using longer planned lead times, but on the other hand in most case the longer planned lead times give a reduction in safety stock, and thus in the related cost. Hence finding the optimal planned lead times is equivalent to finding the optimal balance between work-in-process and safety stocks cost.

From queueing theory we know that the waiting time depends on the variance of the inter arrival times and on the utilization rate of the server. Since lead times consist for a large part of waiting time, we investigated if these two factors also have influence on the optimal planned lead time. A third factor we considered is the holding cost of the items in the supply chain. If the safety stocks are very expensive compared with the work-in-process cost, the optimal planned lead time is probably longer than if the difference in cost is small.

We studied these three factors in a numerical study with discrete event simulation for three specific supply chain structures. For all three structures we can conclude that both higher demand variation and higher utilization rates of the resources increases the optimal planned lead time. However the holding cost structure plays a key role in determining the optimal safety stocks. Safety stocks can decrease by longer planned lead times if the variation in demand and/or the utilization rate of the resources is high, but this is only advantageous if the additional work-in-process cost are smaller that the reduction in safety stock cost.

We also looked at supply chain structures with two capacitated resources, and we saw that the resource with the highest utilization rate has the longest optimal planned lead time. If the resources are serial, the optimal planned lead time of the resource with the lowest utilization rate has an optimal planned lead time of one period. For symmetric supply chains, the optimal planned lead time of two items is equal if they are both produced on resources with equal utilization rates.

At last we determined the optimal planned lead times that belong to the 18-item model. Also here we see that higher demand variation gives longer optimal planned lead times. Furthermore we saw that the cost savings are higher when their is a high added value downstream in the supply chain.

LP models with planned lead times with multi-period capacity consumption are not common. Most models, see e.g. Billington *et al.* (1983), Hopp & Spearman (2000) and Tempelmeier (2003), have a fixed lead time where capacity is allocated at this fixed time offset. If we set the planned lead time in our model equal to one period, it is comparable with those common models. We can conclude that using planned lead times give better results as long as they are well-chosen.

Finding optimal planned lead times is very time consuming, especially for large supply chain structures. Hence in future research there may be looked for an heuristic to find good planned lead times. For large supply chain structures you can probably restrict yourself to find good or even optimal planned lead time of the items produced on capacitated or bottleneck resources only. All other planned lead times can be set equal to one period.

Chapter 5

Early availability of produced items

5.1 Introduction

In the previous chapters, items are only available for further usage at the end of their lead times. When produced at the beginning of its planned lead time an item may wait a large part of its planned lead time while possibly needed downstream. To avoid this unnecessary, or even undesirable waiting time, we adapt the LP model introduced in chapter 2, whereby we assume that production starts as soon as possible (see chapter 3). The first remedy proposed is to adjust the LP model, such that items are available for further usage as soon as they are produced. Thus the flexibility of multi-period capacity allocation remains in the model, but produced items do not have to wait the remainder of the planned lead time before they can be used in succeeding production steps. We test this model using the supply chain structures introduced in chapter 4 which have optimal planned lead times longer than one period.

As an alternative remedy, we adjust the LP model, and make items available before the end of their planned lead time if they are actually needed in succeeding production steps. This means that produced items are only available early if backorder are avoided or reduced. Again the model is tested.

The remainder of this chapter is organized as follows. We start with a brief overview of the model presented in chapter 2, with the adjustments made in chapter 3 in order to produce as soon as their is available capacity. In section 5.3, the model with immediate availability of items after being produced and the performance of this model is discussed. And in section 5.4, we discuss the model and its performance whereby produced items can be used in proceeding production steps before the end of their lead time if necessary. We end this chapter with conclusions in section 5.6.

5.2 LP model with early production

In this section we briefly discuss the LP model, introduced in chapter 3, whereby production of items always starts as early as possible. We give the mathematical model and a brief explanation whereby we concentrate on the objective function and those constraints that play a role when we want to release items early.

The LP model with early production is given by

$$\min \sum_{s=t+1}^{t+T}\sum_{i=1}^{N}\alpha_{is}I_{i,s-1} + \sum_{s=t+1}^{t+T}\sum_{i=1}^{N}\beta_{is}B_{i,s-1} + \varepsilon\sum_{s=t+1}^{t+T}\sum_{i=1}^{N}\sum_{m\in\mathcal{R}_i} sV_{ims} \tag{5.1}$$

subject to

$$I_{is} = I_{i,s-1}+R_{i,s-\tau_i}-D_{is}-\sum_{j=1}^{N}h_{ij}R_{js}+B_{is}-B_{i,s-1}, \quad i=1,\ldots,N,\ s=t,\ldots,t+T-1 \tag{5.2}$$

$$\sum_{u:-\tau_i<u\leq s} p_iR_{iu} \geq \sum_{m\in\mathcal{R}_i}\ \sum_{u:-\tau_i+1<u\leq s+1} V_{imu} \quad i=1,\ldots,N,\ s=t-\tau_i+1,\ldots,t+T-1 \tag{5.3}$$

$$\sum_{u:-\tau_i<u\leq s} p_iR_{iu} \leq \sum_{m\in\mathcal{R}_i}\ \sum_{\substack{u:-\tau_i+1<u\leq s+\tau_i,\\ u<t+T}} V_{imu} \quad i=1,\ldots,N,\ s=t-\tau_i+1,\ldots,t+T-1 \tag{5.4}$$

$$\sum_{i:m\in\mathcal{R}_i} V_{ims} \leq c_{ms} \quad m=1,\ldots,L,\ s=t+1,\ldots,t+T \tag{5.5}$$

$$B_{is}-B_{i,s-1} \leq D_{is}, \quad i=1,\ldots,N,\ s=t,\ldots,t+T-1 \tag{5.6}$$

$$R_{is}, B_{is}, I_{is} \geq 0, \quad i=1,\ldots,N,\ s=t,\ldots,t+T-1 \tag{5.7}$$

$$V_{ims} \geq 0, \quad i=1,\ldots,N,\ m=1,\ldots,L,\ s=t+1,\ldots,t+T \tag{5.8}$$

Constraints (5.2) are the inventory balances for all $i = 1,2,\ldots,N$ and $s = t, t+1,\ldots,t+T-1$ and constraints (5.3), (5.4), and (5.5) deal with multiple capacity allocation. Constraints (5.6) ensure that the number of backorders can only increase by the exogenous demand for the concerned item. The last two constraints are boundary constraints. By adding the term $\varepsilon\sum_{s=t+1}^{t+T}\sum_{i=1}^{N}\sum_{m\in\mathcal{R}_i} sV_{ims}$ to the objective function, the capacity allocation is done such that capacity is allocated to released items as soon as their is capacity available. Focussing on the inventory balance (5.2), we see that the net inventory $I_{is} - B_{is}$ at time s is equal to the net inventory $I_{i,s-1} - B_{i,s-1}$ at time $s-1$, minus endogenous and exogenous demand plus the incoming orders $R_{i,s-\tau_i}$. These orders are released a planned lead time ago, and produced during its planned lead time. At time s the order is available for further production steps or the satisfy customer demand. In the coming two subsections we adjust the LP models such that order become available before the end of their planned lead time. So, it is clear that the decision variable $R_{i,s-\tau_i}$ have to be replaced by other, yet to define, variables.

In chapter 3 we introduced inventory balance equations for work-in-process. We made a distinction between released items waiting until they are produced, see equation (5.9), and items that are already produced but have to wait until the end of the planned lead time before they are available for succeeding production steps, see equation (5.10). In this equation, the number of work-in-process items $\hat{I}_{is}$ waiting until the end of their planned lead time at time s is equal to the number of work-in-process items $\hat{I}_{I,s-1}$ at time $s-1$ plus the items produced during time slot s minus the orders $R_{i,s-\tau_i}$ released a planned lead time ago. When we make items available before the end of their planned lead time, it is clear that the again $R_{i,s-\tau_i}$ will be replaced by other, yet to define, variables.

$$\bar{I}_{is} = \bar{I}_{i,s-1} + \sum_{j=1}^{N} h_{ij} R_{js} - \sum_{j=1}^{N} \sum_{m \in \mathcal{R}_j} h_{ij} V_{j,m,s+1}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{5.9}$$

$$\hat{I}_{is} = \hat{I}_{i,s-1} + \sum_{u \in \mathcal{R}_i} V_{ius} - R_{i,s-\tau_i}, \quad i = 1, \ldots, n, \; s = t, \ldots, t+T-1 \tag{5.10}$$

with

$\bar{I}_{is}$: the work-in-process inventory of item i at time s waiting to be used in production, $i = 0, \ldots, N$, $s = t, t+1 \ldots, t+T-1$.

$\hat{I}_{is}$: the work-in-process inventory of item i at time s waiting after being produced, $i = 0, \ldots, N$, $s = t, t+1 \ldots, t+T-1$.

5.3 Immediate availability of produced items

In this section we consider a LP strategy whereby items are available as soon as they are produced. In section 5.3.1 the changes in the mathematical representation of the LP model are given. We are especially interested in the performance of this strategy compared with the standard LP strategy. Hence in section 5.3.2 and 5.3.3 an experimental setting with its results is discussed.

5.3.1 Mathematical representation of the model

In the previous section, we have mentioned that the inventory balance, and the work-in-process inventory balance after production changes if we adjust the LP model such that items are available for further usage immediately. In this section we discuss these changes, and we also adjust the objective function.

Inventory balance

The original inventory balance,

$$I_{is} = I_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j=1}^{N} h_{ij} R_{js} + B_{is} - B_{i,s-1}, \quad i = 1,\dots,N,\ s = t,\dots,t+T-1, \tag{5.11}$$

states that the net inventory, $I_{is} - B_{is}$, at time s is equal to the net inventory, $I_{i,s-1} - B_{i,s-1}$, at time s plus released orders, $R_{i,s-\tau_i}$, minus endogenous and exogenous demand, $\sum_{j=1}^{N} h_{ij} R_{js} + D_{is}$. The orders, $R_{i,s-\tau_i}$, are released a planned lead time ago and are produced during this planned lead time. If the order is produced at the beginning of the planned lead time, it has to wait several periods until the end of its planned lead time. In this section, instead of waiting until the end of the planned lead time, we release items immediately after production. We know that during time slot s, capacity $\sum_{m\in\mathcal{R}_i} V_{ims}$ is allocated at the available resources for the production of item i. Hence at the end of time slot s these items are produced and ready for future production steps. Since we want these items to be available for further usage immediately after production we have to replace the incoming orders, $R_{i,s-\tau_i}$, in the inventory balance by the produced orders, $\sum_{m\in\mathcal{R}_i} V_{ims}$. So, the new inventory balance becomes

$$I_{is} = I_{i,s-1} + \sum_{m\in\mathcal{R}_i} V_{ims} - D_{is} - \sum_{j=1}^{N} h_{ij} R_{js} + B_{is} - B_{i,s-1}, \quad i = 1,\dots,N,\ s = t,\dots,t+T-1 \tag{5.12}$$

Work-in-process inventory balance

Since items are available for further usage immediately after production, we no longer have work-in-process waiting the remainder of the planned lead time. Hence the work-in-process inventory balance after production is no longer needed. Note that, we still have work-in-process waiting to be produced, so the work-in-process inventory balance before production remains in the model.

Objective function

In chapter 3, we show that in a lot of situations, especially when capacity utilization is high, early production is beneficial for the inventory costs. Early production means that requested orders are produced as soon as there is available capacity. To ensure early production we adjusted the objective function, in chapter 3 by

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_{is} I_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \beta_{is} B_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \sum_{m\in\mathcal{R}_i} \varepsilon s V_{ims} \tag{5.13}$$

Unfortunately, this objective function is not suitable when items are available immediately after production. When items are made available for further production or to satisfy customer demand, they turn up as inventory, and as we can see in the objective function, inventory is costly. So when solving the production planning problem, the LP model makes a plan in which the production, and thus also the availability, is as late as possible (avoiding large inventory). To avoid this scenario, we have to add additional cost to the production of items. Items can be released at most a planned lead time minus a period ($\tau_i - 1$) before the end of the planned lead time (actual production takes place within one period), and if not used for further production, these items spend at most ($\tau_i - 1$) periods longer as inventory. We have to compensate for the time additionally spend as inventory, so the objective function becomes

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_{is} I_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \beta_{is} B_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \sum_{m \in \mathcal{R}_i} ((\tau_i - 1)\alpha_{is} + \varepsilon s) V_{ims} \tag{5.14}$$

whereby α_{is} and β_{is} are chosen such that

$$(\tau_i - 1)\alpha_{is} < 10\beta_{is}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{5.15}$$

5.3.2 Experimental setting

In this section we five the supply chain structures with appropriate parameter settings to measure the performance of the strategy in which items are available for further usage immediate after production. We measure the performance by comparing the inventory and work-in-process cost of the LP strategy including early production with the LP strategy including early production and early availability.

For a fair comparison of the performance between the LP strategies, the choice of the planned lead time is important. If the planned lead times are extremely long, early availability of produced items most probably improves the performance of the model. And if the chosen planned lead times are already too small, early availability is not an issue. Hence, we use supply chain structures of which we know the optimal planned lead times, see chapter 4. If the optimal planned lead time is equal to one period, items are always available immediate after production. Hence, in this research we only use the experimental cases whereby the optimal planned lead time is longer than one period. In figure 5.1 the used supply chain structures are given. The circles indicate those items which are produced on capacitated resources.

We assume a one-to-one production ratio, where one unit of each child item is needed for the production of a single parent item. The capacity requirement p_i for producing one item i is equal to one for all items. The planned lead times of the items produced on uncapacitated resources are all set equal to one period. The inventory cost of the various items are given in table 5.1.

The performance of the early release strategy is studied using a rolling schedule setting, see chapter 1. The time horizon is chosen such that it is longer than the cumulative lead time, and the actual demand on end items follows a gamma distribution with mean 100. For each experiment we vary the squared coefficient of variation of

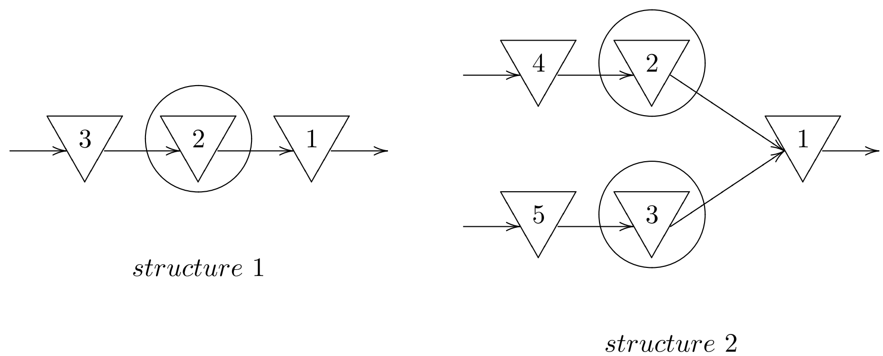

Figure 5.1: *Studied supply chain structures.*

	item 1	item 2	item 3	item 4	item 5
structure 1	6	2	1		
structure 2	8	2	2	1	1

Table 5.1: *The holding costs used in the experiments.*

actual demand and the utilization rate of the capacitated resources. Per parameter setting we use three values for the planned lead time of the items produced on the capacitated resources, namely the optimal planned lead times, the optimal planned lead times minus one period, and the optimal planned lead time plus one period. The optimal planned lead times for each parameter setting are given in table 5.2.

Structure 1			Structure 2		
cv_1^2	ρ_2	τ_2	cv_1^2	(ρ_2, ρ_3)	(τ_2, τ_3)
0.25	0.88	2	0.25	$(0.88, 0.85)$	$(2, 1)$
0.25	0.91	3	0.25	$(0.91, 0.85)$	$(3, 1)$
0.25	0.94	4			
2	0.85	8	2	$(0.85, 0.85)$	$(5, 5)$
2	0.88	10	2	$(0.88, 0.85)$	$(8, 4)$
2	0.91	17	2	$(0.88, 0.88)$	$(9, 9)$
2	0.94	31	2	$(0.91, 0.85)$	$(19, 1)$

Table 5.2: *Parameter settings used in the experiments.*

5.3.3 Results

The results of the experiments can be found in table 5.3. In this table the relative differences, see equation (5.16), between the model with and without the immediate availability of produced items are given.

$$\Delta = \frac{\bar{C}_{standard} - \bar{C}_{immediate}}{\bar{C}_{standard}} \tag{5.16}$$

Structure 1					Structure 2				
cv_1^2	ρ_2	$\Delta_{\tau_i^*-1}$	$\Delta_{\tau_i^*}$	$\Delta_{\tau_i^*+1}$	cv_1^2	(ρ_2, ρ_3)	$\Delta_{\tau_i^*-1}$	$\Delta_{\tau_i^*}$	$\Delta_{\tau_i^*+1}$
0.25	0.88	0%	-6%	-4%	0.25	$(0.88, 0.85)$		-5%	-1%
0.25	0.91	-10%	-16%	-13%	0.25	$(0.91, 0.85)$		-14%	-12%
0.25	0.94	-19%	-26%	-26%					
2	0.85	-12%	-12%	-10%	2	$(0.85, 0.85)$	-5%	-5%	-7%
2	0.88	-20%	-21%	-19%	2	$(0.88, 0.85)$	-13%	-15%	-16%
2	0.91	-31%	-31%	-31%	2	$(0.88, 0.88)$	-16%	-15%	-15%
2	0.94	-40%	-39%	-38%	2	$(0.91, 0.85)$		-14%	-12%

Table 5.3: *Relative difference, for all parameter settings, between the model with and without immediate availability after production.*

We see only negative results. This means that the inventory cost of the LP strategy with early production are lower than of the strategy with immediate availability of produced items. So, availability of produced items immediately after production does not give the desired results. Furthermore we see that the results are worse if the optimal planned lead times become longer (compare table 5.2 with table 5.3. Apparently, the savings achieved by the introduction of the optimal planned lead time are cancelled out when items are available for further usage after production. A possible explanation is that when items are available immediately after production the maximal number of available items per period is equal tot the capacity of the resource per period. Hence the release of orders larger than the maximum capacity per period, which is commonly with longer planned lead times, makes not longer sense.

5.4 Early availability of produced items when needed

Another option to avoid undesired waiting of produced items is to make these items available before the end of their planned lead time, but only when needed to avoid or reduce backorders. This implies that parts of an orders are available earlier, while the rest of the order waits the remainder of the planned lead time. The mathematical representation of this model is given in section 5.4.1. The experimental setting to measure the performance of the model is given in section 5.4.2, and the results can be found in 5.4.3.

5.4.1 Mathematical representation of the model

Again we have to adjust the inventory balance, the work-in-process inventory balance after production and the objective function. To keep track of the number of items already used in succeeding production steps and the number of items waiting the remainder of the planned lead time, we first defined additional notation. Thereafter the changes in the model are discussed one by one.

Additional notation

Input parameters

γ_{is} : $\gamma_{is} < \beta_{is}, i = 1, \ldots, N, s = t+1, \ldots, t+T$, the fictive costs for releasing item i during time slot s.

Decision variables

X_{is} : the items i at time s of order $R_{i,s-\tau_i}$ which remain as work-in-process until the end of the planned lead time, $i = 1, \ldots, N$, $s = t, \ldots, t+T-1$.

Y_{isu} : items i used before the end of their planned lead time at time s of order R_{iu}, $i = 1, \ldots, N$, $s = t, \ldots, t+T-1$, $u = s+1, \ldots, s+\tau_i-1$.

Inventory balance

The released order R_{is} can split up by the items used before the end of the planned and the items that remained as work-in-process until the end of the planned lead time, by

$$X_{is} = R_{i,s-\tau_i} - \sum_{u=s-\tau_i+1}^{s-1} Y_{i,u,s-\tau_i} \quad i = 1, \ldots, N, \ s = t, \ldots, t+T-1 \tag{5.17}$$

X_{is} are the items of the released order $R_{i,s-\tau_i}$ which remain waiting until the end of the planned lead time and $\sum_{u=s-\tau_i+1}^{s-1} Y_{i,u,s-\tau_i}$ are the items of order $R_{i,s-\tau_i}$ that are used in succeeding production steps before the end of the planned lead time.

Hence, in the inventory balance

$$I_{is} = I_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j=1}^{N} h_{ij} R_{js} + B_{is} - B_{i,s-1}, \quad i = 1, \ldots, N, \ s = t, \ldots, t+T-1, \tag{5.18}$$

the order R_{is} must be replaced by the items $\sum_{u=s-\tau_i+1}^{s-1} Y_{isu}$ which are needed in succeeding productions steps and the items X_{is} which waited until the end of their planned lead time. The inventory balance is now given by

$$I_{is} = I_{i,s-1} + X_{is} + \sum_{u=s-\tau_i+1}^{s-1} Y_{isu} \quad - \quad D_{is} - \sum_{j=1}^{N} h_{ij} R_{js} + B_{is} - B_{i,s-1}, \qquad i = 1, \ldots, N, \ s = t, \ldots, t+T-1 \tag{5.19}$$

Work-in-process inventory balance

Since parts of an order may be released earlier, the work-in-process inventory balance after production

$$\bar{I}_{is} = \bar{I}_{i,s-1} + \sum_{j=1}^{N} h_{ij} R_{js} - \sum_{j=1}^{N} \sum_{m \in \mathcal{R}_j} h_{ij} V_{j,m,s+1}, \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{5.20}$$

also changes. In the above balance equation, the work-in-process inventory position after production $\hat{I}_{is}$ at time s is determined by adding produced items $\sum_{m \in \mathcal{R}_i} V_{i,m,s}$ and subtracting order $R_{i,s-\tau_i}$ from the work-in-process after production inventory position at time $s-1$. Since the order $R_{i,s-\tau_i}$ is split up, the work-in-process inventory balance after production becomes

$$\hat{I}_{is} = \hat{I}_{i,s-1} + \sum_{m \in \mathcal{R}_i} V_{i,m,s} - \sum_{u=s-\tau_i+1}^{s-1} Y_{isu} - X_{i,s} \quad i = 1, \ldots, N, \; s = t, \ldots, t+T-1 \tag{5.21}$$

The work-in-process inventory after production give the number of produced items waiting the remainder of their planned lead time. Hence this quantity gives the maximum number of items which can be used for succeeding production steps if necessary. So, this quantity must be known during the optimization process. Therefore, we have to add this work-in-process balance to the list of constraints of the LP model instead of calculating the work-in-process inventory afterwards.

Objective function

The aim of this model is to make produced items available for succeeding production steps before the end of their planned lead times, but only if they are needed for further production or customer sales. This means that produced items are only available before the end of their planned lead time if they can avoid or reduce backorders. We can reach this goal by adding an additional term to the objective function. This additional term should penalizes early availability of produced items if they are not needed to avoid backorders. Hence the cost factor for early availability must be smaller than the backorder costs, otherwise it is cheaper to have backorders. So the original objective function

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_{is} I_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \beta_{is} B_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \sum_{m \in \mathcal{R}_i} \varepsilon s V_{ims} \tag{5.22}$$

is replaced by

$$\begin{aligned} \min \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \alpha_{is} I_{i,s-1} \quad &+ \quad \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \beta_{is} B_{i,s-1} + \sum_{s=t+1}^{t+T} \sum_{i=1}^{N} \sum_{m \in \mathcal{R}_i} \varepsilon s V_{ims} \\ &+ \quad \sum_{s=t+1}^{t+T} \sum_{u=s-\tau_i+1}^{s-1} \sum_{i=1}^{N} \gamma_{is} Y_{i,s-1,u} \end{aligned} \tag{5.23}$$

with

$$\gamma_{is} < \beta_{is}, \quad i = 1, \dots, N, \; s = t, \dots, t + T - 1 \tag{5.24}$$

5.4.2 Experimental setting

To measure the performance of this new model, whereby produced items are only available earlier if they are needed to avoid or reduced backorders, we use the same parameter setting as given in section 5.3.2, so the model is tested for various values of the squared coefficient of variation, for various utilization rates, and various planned lead times.

5.4.3 Results

In this section we discuss the results of the test cases, the results can be found in table 5.4. In this table the relative differences, see equation (5.25), between the model with and without early availability of produced items.

$$\Delta = \frac{\bar{C}_{standard} - \bar{C}_{necessary}}{\bar{C}_{standard}} \tag{5.25}$$

Structure 1					Structure 2				
cv_1^2	ρ_2	$\Delta_{\tau_i^*-1}$	$\Delta_{\tau_i^*}$	$\Delta_{\tau_i^*+1}$	cv_1^2	(ρ_2, ρ_3)	$\Delta_{\tau_i^*-1}$	$\Delta_{\tau_i^*}$	$\Delta_{\tau_i^*+1}$
0.25	0.88	0%	5%	17%	0.25	$(0.88, 0.85)$		1%	13%
0.25	0.91	-1%	7%	16%	0.25	$(0.91, 0.85)$		1%	11%
0.25	0.94	-1%	6%	13%					
2	0.85	6%	9%	14%	2	$(0.85, 0.85)$	1%	2%	4%
2	0.88	4%	7%	10%	2	$(0.88, 0.85)$	0%	1%	3%
2	0.91	9%	12%	14%	2	$(0.88, 0.88)$	3%	5%	7%
2	0.94	21%	22%	24%	2	$(0.91, 0.85)$		2%	3%

Table 5.4: *Relative difference, for all parameter settings, between the model with and without early availability after production when necessary.*

We see all positive results for the cases with optimal planned lead times. This implies that the model whereby produced items can be used before the end of their planned lead time to avoid or reduce backorders, gives lower costs than the LP strategy with early production whereby each produced item has to wait the remainder of its planned lead time before it can be used. Note, that there is not a clear correlation between the savings and the parameter settings. Only for the cases whereby the items produced on capacitated resources have a planned lead time equal to the optimal planned lead time plus one period, the savings are relative high. This high reduction in cost is an indication that the optimal planned lead times may be longer when the early release strategy is used. To confirm this thought, we plotted for increasing planned lead time the inventory cost of the case with $cv_1^2 = 0.25$ and $\rho_2 = 0.94$. The result can be found in figure 5.2.

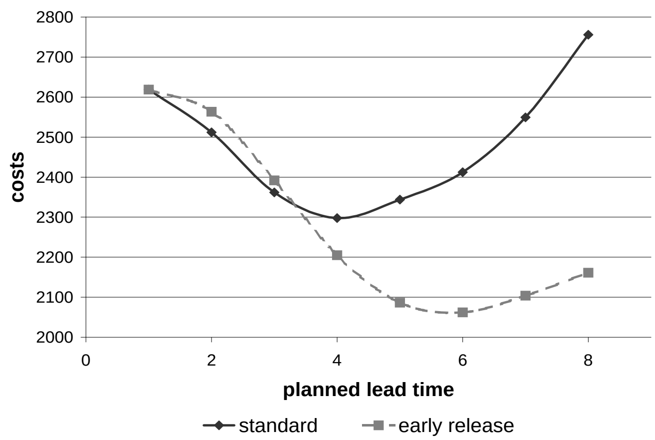

Figure 5.2: *Cost for different planned lead times.*

When we look at figure 5.2, we see that the LP strategy with early release strategy has an optimal planned lead time of 4 periods, while for the LP strategy with the early release and the early availability strategy the optimal planned lead times is equal to 6 periods. Apparently, additional flexibility created by longer planned lead times can reduce the inventory costs if produced items can be used for further production if needed. Furthermore it is clear that if the planned lead times are too long, early availability of produced items is always better. Also the opposite is true: if the planned lead time is smaller than the optimum planned lead time the model with the early availability strategy performs worse.

5.5 The 18-item model

Also in this chapter we look at the performance of the 18-item model taking into account the adjustments we made to the model. We only look at the model in which items are available before the end of their planned lead time if the are needed to avoid or reduce backorders. We use the parameter setting given in chapter 2. However, for the planned lead times we use the optimal planned lead times determined in chapter 4. The results can be found in table 5.5, whereby the one but last column gives the relative difference between the LP strategy with early production and the LP strategy with early production and availability for optimal planned lead times, given by

$$\Delta_{\tau_i=\tau_i^*} = \frac{\bar{C}_{\tau_i=\tau_i^*} - \bar{C}_{necessary}}{\bar{C}_{\tau_i=\tau_i^*}}, \tag{5.26}$$

between the LP strategy with early production using optimal planned lead times and the adjusted strategy with early production and early availability of released orders when necessary. In the last the relative difference between the standard LP strategy with planned lead times equal to one period and the early availability strategy using optimal planned lead times, given by

$$\Delta_{\tau_i=1} = \frac{\bar{C}_{\tau_i=1} - \bar{C}_{necessary}}{\bar{C}_{\tau_i=1}}, \tag{5.27}$$

between the standard LP strategy and the adjusted strategy with early availability of released orders when necessary.

cv_i^2	$(\rho_s, \rho_{sc}, \rho_c)$	$(\alpha_s, \alpha_{sc}, \alpha_c)$	$\Delta_{\tau_i=\tau_i^*}$	$\Delta_{\tau_i=1}$
0.25	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	13.58%	20.25%
0.25	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	10.16%	20.83%
2	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	14.09%	29.27%
2	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	21.30%	32.49%
0.25	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	9.59%	12.46%
0.25	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	9.52%	18.62%
2	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	12.81%	18.44%
2	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	16.75%	26.54%

Table 5.5: *Relative difference between the strategies with early production and early availability and the strategy with early production and optimal planned lead times, and between the strategy with early production and early availability and the standard LP strategy (with planned lead times equal to one period).*

Again we see large savings, especially when we compare the adjusted model with the standard LP strategy. This standard LP strategy is comparable with former MP model with fixed lead times (e.g. Billington *et al.* (1983), Hopp & Spearman (2000), and Belvaux & Wolsey (2001)). Hence when using LP models in a rolling schedule setting, planned lead times with early availability of released orders when necessary to avoid or reduce backorders are a good way to incorporate demand uncertainty.

5.6 Conclusions

In chapter 1 we defined the planned lead time such that items can be produced during their planned lead time, and if they are finished before the end of there planned lead time, they have to wait until the end of their planned lead time before they can be used in the next production step. In this chapter we have investigated two strategies whereby the early produced items are released before the end of their planned lead time. In the first strategy items are made available for further production steps or customer demand as soon as they are produced. And in the second strategy items are only made available when they are necessary to avoid or reduce backorders.

When measuring the performance of the two models it is important to use optimal and nearly optimal planned lead times. With relative long planned lead times it is always better to make items available before the end of their planned lead time, and with relative short planned lead times it makes no sense to even shorten your planned lead times by making produced items available early. We consider three values for the planned lead time in the experiments, namely the optimal planned lead time (found in chapter 4), the optimal planned lead time plus 1, and the optimal planned lead time minus 1. The results show that for all cases the model whereby items are available immediately after production is worse, and the model whereby items are available after production when necessary to avoid or reduce backorders is better than the standard LP strategy. Furthermore we have seen that if items are available early when needed to avoid or reduce backorders, their optimal planned lead times becomes longer than in the LP strategy with early production but without early availability.

Chapter 6

Balance allocation of shortages

6.1 Introduction

In the previous three chapters we improved the performance of the original LP model formulated in chapter 1 by introducing planned lead times with multi-period capacity consumption. In this chapter the focus shifts to material allocation from child items to parent items. In chapter 3, we noticed an imbalance in the allocation of materials. This imbalance is also detected in De Kok & Fransoo (2003). They also showed that the LP-based SCOP function is outperformed by a so-called synchronized base stock (SBS) policy developed by De Kok & Visschers (1999). The SBS policy uses allocation mechanisms, derived from the analysis of divergent systems (c.f. Van der Heijden *et al.* (1997), Diks & de Kok (1998), Diks & de Kok (1999), and De Kok & Visschers (1999)) in general supply chain structures. However, the results of De Kok & Fransoo (2003) are restricted to uncapacitated supply chains, because SBS policies do not take into account capacity restrictions. The focus of this chapter is to develop LP strategies for capacitated SCOP problems with allocation policies. We restrict ourselves to material allocation and assume a one-to-one relation between item and resource. The introduction of allocation policies into the LP model implies that we add additional constraints. Therefore the optimal solution to the new problem may have a higher cost than in the original formulation. However, the LP model is part of a rolling schedule implementation. We will show that, despite the increase in costs on an instance, the actual costs incurred under uncertainty can be substantially lower. In example 6.1 we show how material allocation can decrease your actual costs when the actual demand differs a little from the actual demand.

Example 6.1 *Consider a small supply chain with three items, of which two end items (A & B). Both end items are supplied by the third item (C). For both end items, the inventory costs are equal to 1 per unit on stock and the backorder costs are equal to 10 per unit short. Assume that item C has 150 units available for the production of* *the two end items, and the expected demand of the two end items for item C is 100 units each. So there is a shortage of 50 units. In table 6.1 the available items are allocated in two different manners. In case I, item A receives 100 items and item B only 50, and in case II both end items get 75 items. In the lower table we see that, although the costs for both given allocation policies are equal when considering the forecast, the allocation policies do play a role when the actual demand turns out to be a little different from the forecast.*

	Item	Forecast demand	Allocation	Inventory	Backorders	Costs
Case I	A	100	100	0	0	0
	B	100	50	0	50	500
Case II	A	100	75	0	25	250
	B	100	75	0	25	250

	Item	Actual demand	Allocation	Inventory	Backorders	Costs
Case I	A	98	100	2	0	2
	B	102	50	0	52	520
Case II	A	98	75	0	23	230
	B	102	75	0	27	270

Table 6.1: *Amounts of inventory, backorders and costs for different allocation policies of example 6.1.*

Allocation problems only occur in the divergent parts of arbitrary supply chains. For pure divergent systems allocation rules have received considerable attention. An overview on this line of research, where allocation policies are determined to minimize holding and shortage costs, is given by Axsäter (2003). Eppen & Schrage (1981) introduced a fair share allocation rule, based on equal stock-out probability at end-stockpoints, for a two-echelon system without intermediate stocks. In De Kok (1990) the Consistent Appropriate Share (CAS) rationing policy, with the fill rate (the fraction of demand delivered immediately from stock on hand) as service criterion, for 2-echelon systems is introduced. With the CAS rationing policy inventory is allocated to the local stockpoints based on safety stock ratios. Verrijdt & de Kok (1996) have modified the CAS rationing policy to cope with significantly differing fill rate targets. In Verrijdt & de Kok (1995) the CAS rationing policy is generalized for arbitrary divergent N-echelon systems where only end-stockpoints are allowed to hold stocks. A generalization of the CAS rationing policy for 2-echelon divergent systems is the Balanced Stock (BS) rationing introduced by Van der Heijden (1997). In Van der Heijden *et al.* (1997) this allocation rule is extended for general N-echelon distribution systems where all upstream, downstream and intermediate stockpoints are allowed to hold stock. In De Kok & Fransoo (2003), a correction of the formula for allocation fractions is given for non-identical successors.

In this chapter we extend the LP model introduced in chapter 2 with linear allocation rules. These linear allocation rules divide shortages among the parent items by predefined allocation fractions. In a second model the allocation rules are relaxed by introducing lower bounds for the amount of shortages that have to be allocated to parent items. Furthermore we introduce a model, whereby the linear objective function is replaced by a quadratic objective function. The idea behind this is, that similar to the linear allocation rules, the quadratic objective function ensures a balanced rationing of the shortages among the paren items. Under a linear objective function shortages are typically rationed among “cheap” parent items, while “expensive” items do not take part in the shortages at all.

In De Kok & Fransoo (2003), it is shown that the standard LP strategy is outperformed by the so-called SBS policy. In this chapter, we compare the SBS policy with the introduced LP strategies with allocation policies. We see that the SBS policy still performs a little better, but the differences are small. Since, the SBS policy is only suitable for uncapacitated supply chains, the introduced LP strategies with allocation policies provide the required generality needed to handle arbitrary capacitated supply chains. Hence, at the end of this chapter we also measure the performance of the standard LP strategy with the LP strategies with allocation policies for capacitated supply chains and various allocation fractions.

The remainder of this chapter is organized as follows. In section 6.2 the three different allocation policies are introduced and formulated as mathematical models. In section 6.4, the different allocation policies for uncapacitated SCOP problems are compared with the SBS policy. In section 6.5, the performances of the different allocation policies, in capacitated supply chains, are examined by comparing the strategies with allocation policies with a standard LP strategy. The conclusions are presented in section 6.7.

6.2 Illustration of the allocation policies

In this section we introduce the three models with allocation policies with an example. In the first model, allocation of material takes place by adding linear allocation rules, as constraints, to a standard LP model. These linear allocation rules allocate shortages of a child item to its parent items by a predefined ratio. The second model also uses linear allocation rules, but instead of allocating all shortages by a specific ratio, lower bounds are given for the amount of shortages that have to be allocated to a parent item. In the third model, balanced allocation of child items is enabled by a quadratic objective function.

The allocation policies are illustrated with simple MP models. The models are given in table 6.2 and are depicted in figure 6.1.

Standard LP model

The standard LP model is not depicted in figure 6.1. However, it is equal to the pictures of the model with allocation rules, but without lines a and b. Line z is the line on which the optimum value of the LP model lies. So, we see that in the model $min\, x + y$, the whole line segment between A and B is optimal. And for the model $min\, 3x + 4y$, the optimum value lies in point B.

Standard	Allocation	Lowerbound	Quadratic
$min\; x+y$	$min\; x+y$	$min\; x+y$	$min\; x^2+y^2$
$x+y \geq 5$	$x+y \geq 5$	$x+y \geq 5$	$x+y \geq 5$
$x \geq 1$	$x \geq 1$	$x \geq 1$	$x \geq 1$
$y \geq 1$	$y \geq 1$	$y \geq 1$	$y \geq 1$
	$5x+y \geq 15$	$5x+y \geq 12.5$	
	$x+5y \geq 15$	$x+5y \geq 12.5$	
$min\; 3x+4y$	$min\; 3x+4y$	$min\; 3x+4y$	$min\; 3x^2+4y^2$
$x+y \geq 5$	$x+y \geq 5$	$x+y \geq 5$	$x+y \geq 5$
$x \geq 1$	$x \geq 1$	$x \geq 1$	$x \geq 1$
$y \geq 1$	$y \geq 1$	$y \geq 1$	$y \geq 1$
	$5x+y \geq 15$	$5x+y \geq 12.5$	
	$x+5y \geq 15$	$x+5y \geq 12.5$	

Table 6.2: *Models corresponding figure 6.1.*

Allocation rules

In the model where we minimize $x + y$, all points on the line between A and B give optimum values. To restrict the LP solver to one point we add allocation rules, represented by lines a and b. Now, the optimum is in point C. For the model minimizing $3x+4y$ with allocation, the optimum is also found in point C. The objective function value of this point is higher than the value of point B, which is the optimum for the standard LP model. So in cases with unequal cost, allocation rules can increase the costs. However, in a rolling horizon setting, the inventory costs can still decrease, see example 6.1 and the results in section 6.5.2.

Lower bounds

In the model $min\, x+y$ with lower bounds the allocation rules are relaxed. In stead of a fixed optimum in point C, the LP solver can choose any point on the line between points C and D. For the model $min\; 3x + 4y$, the lower bounds put the optimum in point D, which value is greater than point B, but smaller than the value of point C in the model with allocation rules.

Quadratic

A completely different method to dictate the solver to one point on the line AB, is by replacing the linear objective function $x + y$ by the quadratic function $x^2 + y^2$. The optimum, point C, is found on the point where line $x + y = 5$ is touching circle z. This point corresponds with point C in the model with allocation rules. The model $min\; 3x + 4y$ is replaced by $min\; 3x^2 + 4y^2$, the optimum E can be found on the oval z and lies a little lower on line AB then point C in the upper picture.

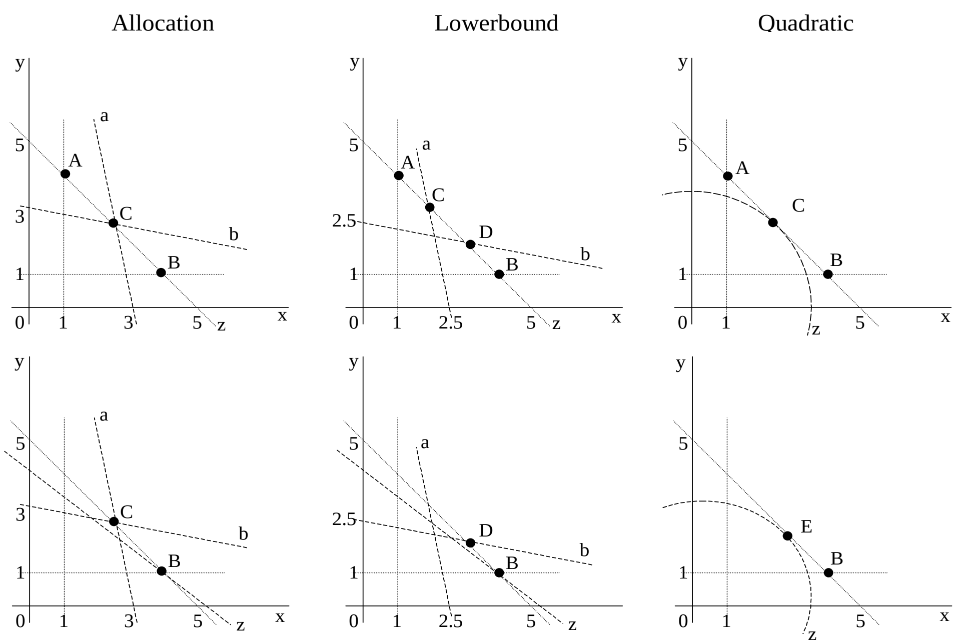

Figure 6.1: *An example of a linear and a quadratic programming model.*

6.3 Mathematical representation of the allocation policies

6.3.1 Notation

For the mathematical representation of the allocation policies we need additional notation. The notation concerning the product structure is illustrated in figure 6.2.

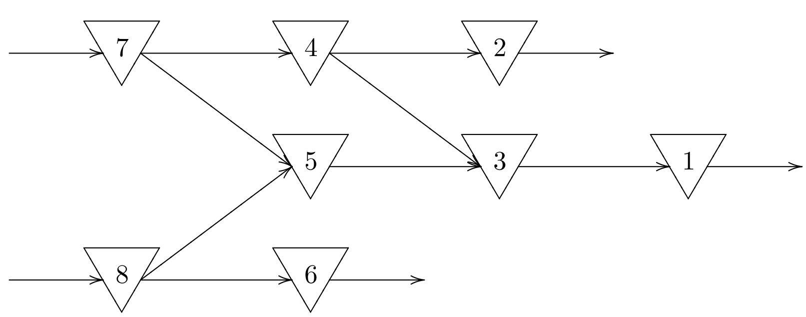

Figure 6.2: *Schematic representation of product structure.*

System layout

$ech(i)$: all items that constitutes the echelons of item i, $i = 1, \ldots, n$ (e.g., $ech(4) = \{1, 2, 3, 4\}$).

$prec(i)$: set of preceding items of item i, $i = 1, \ldots, n$ (e.g., $prec(3) = \{4, 5\}$).

$succ(i)$: set of succeeding items of item i, $i = 1, \ldots, n$ (e.g., $succ(4) = \{2, 3\}$).

E : set of all end items (e.g., $E = \{1, 2, 6\}$).

E_i : all end items in $ech(i)$, $i = 1, \ldots, n$ (e.g., $E_8 = \{1, 6\}$).

E_i^c : all item which are not end items in $ech(i)$, $i = 1, \ldots, n$ (e.g., $E_8^c = \{3, 5, 8\}$).

Input parameters

q_{ij} : allocation fraction from item i to item j, whereby $i \in prec(j)$, $i = 1, \ldots, n,\ j = 1, \ldots, m$.

$\tilde{h}_{ij}$: the number of units i incorporated in a single unit of item j, $i = 1, \ldots, n,\ j = 1, \ldots, n$.

k_{ij} : $k_{ij} \in \mathbb{N}$, the number of different routes between i and j, $i = 1, \ldots, n,\ i \notin E,\ j = 1, \ldots, n$. The routes are labelled $1, \ldots, k_{ij}$.

$\hat{\tau}_{ijl}$: cumulative lead time from product i to product j via route l, $i = 1, \ldots, n,\ i \notin E,\ j = 1, \ldots, n,\ l = 0, \ldots, k_{ij}$.

$\hat{h}_{ijl}$: number of products i incorporated in a single unit of item j via route l, $i = 1, \ldots, n,\ i \notin E,\ j = 1, \ldots, n,\ l = 0, \ldots, k_{ij}$.

$\hat{D}_{is}$: cumulative demand of item i at time s, $i = 1, \ldots, n,\ i \notin E,\ s = t, \ldots, t + T - 1$.

Decision variables

$\tilde{I}_{is}$: the echelon inventory position of item i at time s, $i = 1, \ldots, n,\ s = t, \ldots, t + T - 1$.

S_{is}^- : shortage of item i at time s to satisfy the needs of the parent items of i, $i = 1, \ldots, n,\ i \notin E,\ s = t, \ldots, t + T - 1$.

S_{is}^+ : slack of item i at time s to satisfy the needs of the parent items of i, $i = 1, \ldots, n,\ i \notin E,\ s = t, \ldots, t + T - 1$.

6.3.2 Allocation rules

To apply the allocation rules, we incorporate echelon stocks into our mathematical model. The concept of echelon stock has been introduced by Magee (1958) and Clark & Scarf (1960). With these echelon stocks we can determine shortages of child items and allocate them to the parent items.

Echelon stock

Echelon stocks are added to the model by an echelon inventory balance equation, derived in this section. We start by giving the definition of the echelon inventory position of Diks (1997):

Definition The *echelon stock* of an item equals the sum of its physical stock plus the amount in transit to or on hand at its downstream items minus possible backorders at its end items. The *echelon inventory position* of an item equals its echelon stock plus all material in transit to that item.

Mathematically the echelon inventory position is given by

$$\tilde{I}_{is} = I_{i,s-1} + \sum_{j=1}^{n} \sum_{u=s-\tau_j+1}^{s} h_{ij} R_{ju} + \sum_{j=1}^{n} h_{ij} \tilde{I}_{js} - B_{is} \quad i = 1,\ldots,n,\ s = t,\ldots,t+T-1 \tag{6.1}$$

The parameter $\tilde{h}_{ij}$, the number of units i incorporated in a single unit of item j, is introduced to calculate the summation of the demand of all end items of $ech(i)$ at time t. So, we have

$$\sum_{j \in E_i} \tilde{h}_{ij} D_{is} = \sum_{j \in E_i} h_{ij} D_{js} + \sum_{j \in E_i^c} h_{ij} \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} \quad i = 1,\ldots,n,\ s = t,\ldots,t+T-1 \tag{6.2}$$

The echelon inventory position of item i at time s is determined by the echelon inventory position of that item at time $s-1$ plus the orders requested minus the total demand of the end items of $ech(i)$. So, the echelon inventory balance is given by

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + R_{i,s-\tau_i} - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} - D_{is} \quad i = 1,\ldots,n, \quad s = t,\ldots,t+T-1 \tag{6.3}$$

Since the echelon inventory position and the physical inventory position are related to each other in a 1-1 fashion, the echelon inventory balance (6.3) must be equivalent to the physical inventory balance (2.2). In appendix B it is shown to be true.

Determination of shortages

With the echelon inventory position we are able to determine the shortages. We have to realize that, since we deal with arbitrary supply chain structures, two or more items can be used in the same product but in a different way. For example a screw can be used in a intermediate item, but can also be used to fasten the intermediate item to the end item. So there are two screws incorporated in the end product which follow different routes. Since the cumulative lead time of between screw and end item might be different in different routes, the number of routes k_{ij} from item i to item j and the cumulative lead time $\hat{\tau}_{ijl}$ from item i to item j via route l are calculated.

The total future demand for $i \notin E$ is then calculated by

$$\hat{D}_{is} = \sum_{j \in E_i} \sum_{l=0}^{k_{ij}} \sum_{u=s+1}^{s-\hat{\tau}_{ijl}} \hat{h}_{ijl} D_{ju}, \quad i = 1, \ldots, n, \ i \notin E, \ s = t, \ldots, t+T-1 \tag{6.4}$$

The shortage (slack) of item $i \notin E$ is defined by the negative (positive) difference between the available amount of item i and the desired amount of item i.

$$\underbrace{S_{is}^{-}}_{\text{shortage}} - \underbrace{S_{is}^{+}}_{\text{slack}} = \underbrace{\sum_{j=1}^{n} h_{ij} \hat{D}_{j,s+\tau_j} - \sum_{j=1}^{n} h_{ij} \tilde{I}_{j,s+\tau_j}}_{\text{desired amount of item } i} - \underbrace{I_{i,s-1} - R_{i,s-\tau_i}}_{\text{available amount of item } i}, \quad i = 1, \ldots, n, \ i \notin E, \ s = t, \ldots, t+T-1 \tag{6.5}$$

Allocation of shortages

When there are not enough units of a child item available to satisfy the demand of its parent items, the shortage is divided among all the parent items. So, a parent item can only get its total future demand minus its echelon stock (i.e., this is its demand for child items) minus a fraction of the shortage, see inequality (6.6).

$$h_{ij} R_{j,s-\tau_j} \geq h_{ij} \hat{D}_{js} - h_{ij} \tilde{I}_{js} - q_{ij} S_{i,s-\tau_j}^{-} \quad \forall i \in prec(j), \quad j = 1, \ldots, n, \ s = t, \ldots, t+T-1. \tag{6.6}$$

whereby

$$q_{ij} \geq 0, \quad i = 1, \ldots, n, \ j = 1, \ldots, n \tag{6.7}$$

$$\sum_{j=1}^{n} q_{ij} = 1, \quad i = 1, \ldots, n \tag{6.8}$$

Notice that in inequality (6.6), the parent can at least order its demand minus a fraction of the shortage. In the LP model, this ensures that the parent item can order more then necessary (e.g. to avoid backorders in the future) when there is slack. In appendix C, it is shown that for pure divergent supply chains equality holds if there are shortages. For arbitrary supply chains this is not true. The order R_{jt} can even be smaller than inequality (6.6) indicates, namely when the ordered amount is restricted by the available number of other child items of parent item j. The model is able to ignore constraint (6.6) by adding slack and shortages to the problem, such that the difference between slack and shortages remain equal.

Objective function

By adding constraints (6.3), (6.5), and (6.6) to the linear programming model of chapter 2 we are not finished. When optimizing the problem, the LP solver compensates any shortage with imaginary slack, such that the order request are not restricted. So, we have to penalize slack by adding a cost component for slack in the objective function. The penalty for imaginary slack must be high enough so that it is not beneficial to create it, but must not be so high that having slack is more costly than backorders. Thus, the related costs should be smaller than the minimum backorder costs, but cannot be too small. Hence, we subtract small ε costs of the backorder costs, and use that as penalty costs for creating imaginary slack. So, we consider the following objective function

$$\min \sum_{t=1}^{T}\sum_{i=1}^{n} \alpha_i I_{i,t-1} + \sum_{t=1}^{T}\sum_{i=1}^{n} \beta_i B_{i,t-1} + \sum_{t=1}^{T}\sum_{i=1}^{n} \left(\min_{j \in E_i} \beta_{jt} - \varepsilon \right) \left(S_{it}^{+} + S_{it}^{-} \right) \tag{6.9}$$

whereby ε is a very small number.

Allocation fractions

The determination of allocation fractions, q_{ij}, has received considerable attention in literature. We consider four different allocation fractions. The first allocation fraction is based on the number of parent items, shortages are allocated proportional to the number of parent items by

$$q_{ij}^{|N|} = \frac{1}{|succ(i)|} \quad i = 1, \ldots, n, \ j = 1, \ldots, n \tag{6.10}$$

In the second allocation fractions we also take the mean demand of the parent items into account. This allocation fraction with

$$\mu_j = \sum_{k \in E_j} E[D_k] \quad j = 1, \ldots, n$$

is given by

$$q_{ij}^{\mu} = \frac{\mu_j}{\sum_{k \in succ(i)} \mu_k} \quad i = 1, \ldots, n, \ j = 1, \ldots, n \tag{6.11}$$

The third allocation fraction arises from Eppen & Schrage (1981). They introduce a fair share allocation rule for a two-echelon system without intermediate stocks. The allocation rule ensures that the end-stockpoint probabilities are equalized. Defining

$$\sigma_j = \sigma\left(\sum_{k\in E_j} D_k\right) \quad j = 1,\dots,n$$

the allocation fraction of Eppen & Schrage (1981) is represented by

$$q_{ij}^{ES} = \frac{\sigma_j}{\sum_{k\in succ(i)} \sigma_k} \quad i = 1,\dots,n,\ j = 1,\dots,n \tag{6.12}$$

Van der Heijden (1997) determined linear allocation policies minimizing the probability of imbalance. Imbalance is the allocation of a negative quantity from an intermediate stockpoint to at least one of its successors. De Kok & Fransoo (2003) corrected an error in the analysis of Van der Heijden (1997) and they obtained the following expression for the allocation fraction

$$q_{ij}^{HKF} = \frac{\sigma_j^2}{2\sum_{k\in succ(i)} \sigma_k^2} + \frac{\mu_j^2}{2\sum_{k\in succ(i)} \mu_k^2} \quad i = 1,\dots,n,\ j = 1 \tag{6.13}$$

6.3.3 Lower bounds for allocation rules

In the second allocation policy, the strict allocation rules are relaxed by introducing lower bounds. So, instead of fixing the number of items allocated to parent items, as done in constraint (6.6), we adjust the constraint such that at least a percentage of order determined by (6.6) is allocated to the specific parent item. The lower bound is realized by adding a percentage to constraint (6.6) as follows

$$\begin{aligned} h_{ij}R_{j,s-\tau_j} \geq (1-\gamma)(h_{ij}\hat{D}_{js} - h_{ij}\tilde{I}_{js} - q_{ij}S^-_{i,s-\tau_j}) \quad \forall i \in prec(j) \\ j = 1,\dots,n,\ s = t,\dots,t+T-1. \end{aligned} \tag{6.14}$$

whereby

$$q_{ij} \geq 0, \quad i = 1,\dots,n,\ j = 1,\dots,n \tag{6.15}$$

$$\sum_{j=1}^{n} q_{ij} = 1, \quad i = 1,\dots,n \tag{6.16}$$

$$0 \leq \gamma \leq 1 \tag{6.17}$$

Notice that $\gamma = 0$ gives the LP model with strict allocation policies, and for $\gamma = 1$ the model correspond to the standard LP model.

6.3.4 Quadratic objective function

In the previous sections we balanced the allocation of shortages by adding allocation rules to the LP model. In this section we balance the allocation by replacing the linear objective function by a quadratic objective function.

The objective function in SCOP function becomes

$$\min \sum_{s=t+1}^{t+T} \sum_{i=1}^{n} \tilde{\alpha}_i I_{i,s-1}^2 + \sum_{s=t+1}^{t+T} \sum_{i=1}^{n} \tilde{\beta}_i B_{i,s-1}^2 \tag{6.18}$$

where $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ for $i = 1, \ldots, n$ are constants to indicate the relations between the inventory and backorder costs of the items.

Note that we can approximate the quadratic objective function by a piecewise linear one. Using a piecewise linear objective function brings some additional complications. First of all, the number of steps and the associated cost per step have to be decided on. This implies some additional decision variables. And second, the number of constraints in the model increases rapidly by an increasing number of steps. In this chapter we leave the piecewise linear objective function out of consideration.

Weight factors in quadratic objective function

In this section we consider three different kind of weight factors. The first set of weight factors is equal to the used cost structures. Thus

$$\tilde{\alpha}_i = \alpha_i \quad i = 1, \ldots, n \tag{6.19}$$
$$\tilde{\beta}_i = \beta_i \quad i = 1, \ldots, n \tag{6.20}$$

The two other sets of weight factors are inspired by the allocation fractions of Eppen & Schrage (1981) and of Van der Heijden (1997) and De Kok & Fransoo (2003), so instead of only cost we also take the demand distribution into account. The weight factors are chosen such that the inventory and the backorder costs of one item per period are higher for items with smaller allocation fractions. The weight factors are given by

$$\tilde{\alpha}_i^{ES} = \left(\min_{j \in prec(i)} \frac{\sum_{k \in E_j, k \neq j} \sigma_k}{\sum_{k \in E_j} \sigma_k} \right) \alpha_i^2 \quad i = 1, \ldots, n \tag{6.21}$$
$$\tilde{\beta}_i^{ES} = 10 \tilde{\alpha}_i^{ES} \quad i = 1, \ldots, n \tag{6.22}$$

and

$$
\begin{aligned}
\tilde{\alpha}_i^{HKF} &= \left(\min_{j \in prec(i)} \frac{\sum_{k \in E_j, k \neq j} \sigma_k^2}{\sum_{k \in E_j} \sigma_k^2} + \frac{\sum_{k \in E_j, k \neq j} \mu_k^2}{\sum_{k \in E_j} \mu_k^2} \right) \alpha_i^2 \\
& \qquad\qquad\qquad\qquad i = 1, \ldots, n,\ t = 1, \ldots, T \qquad (6.23) \\
\tilde{\beta}_i^{HKF} &= 10 \tilde{\alpha}_i^{HKF} \qquad\qquad i = 1, \ldots, n \qquad (6.24)
\end{aligned}
$$

6.4 MP models with balanced allocation vs SBS concept

De Kok & Fransoo (2003) show that synchronized base stock (SBS) concepts performs better than existing LP strategies, for uncapacitated supply chains with stochastic demand. In this section we compare this SBS concept with the introduced mathematical programming models with allocation policies. In the first section the SBS concept is discussed, and in section 6.4.2 the experimental setting, also used in De Kok & Fransoo (2003), for comparing the model is given. We conclude with the results of the comparison.

6.4.1 Synchronized base stock control concept

The synchronized base stock (SBS) control concept, introduced by De Kok & Visschers (1999), is a class of policies that can be applied to general supply chain structure. The allocation mechanisms used in the SBS concept are derived from the analysis of divergent systems (c.f. Van der Heijden *et al.* (1997)). With the SBS concept it is possible to characterize the optimal policy for non capacitated situations under i.i.d. exogenous demand, and near optimal policies can be found numerically.

From De Kok & Fransoo (2003) we learn that the SBS concept considerable outperforms the LP-based concept. Explanation for the superiority of the SBS concept was found in the way LP tends to prioritize items in case of shortage of upstream availability instead of rationing among the items that need this upstream availability. The models proposed in this chapter have allocation policies for rationing these shortages. Hence, we compare the stochastic SBS concept with the deterministic MP models with allocation policies using a rolling horizon.

In the numerical comparison we restrict to the situations with infinite resource availability, since the SBS policy cannot deal with finite resources. Results for capacitated systems are only available for single item, single stage systems (see e.g. De Kok (1989)), serial systems (Tayur (1993)), or for divergent systems where only the most upstream stage is capacitated (see De Kok (2000)).

6.4.2 Numerical experiment

For the comparison of the SBS concept and the MP strategies with allocation policies, we use the same experimental setting as De Kok & Fransoo (2003). We consider the product structure consisting of the 11 items given in figure 6.3. The 11 item product structure consists of 4 end-items. All end-items contain common component 11. End-items 1 and 2 share component 9, while end-items 3 and 4 share component 10. And each end-item contains a specific component.

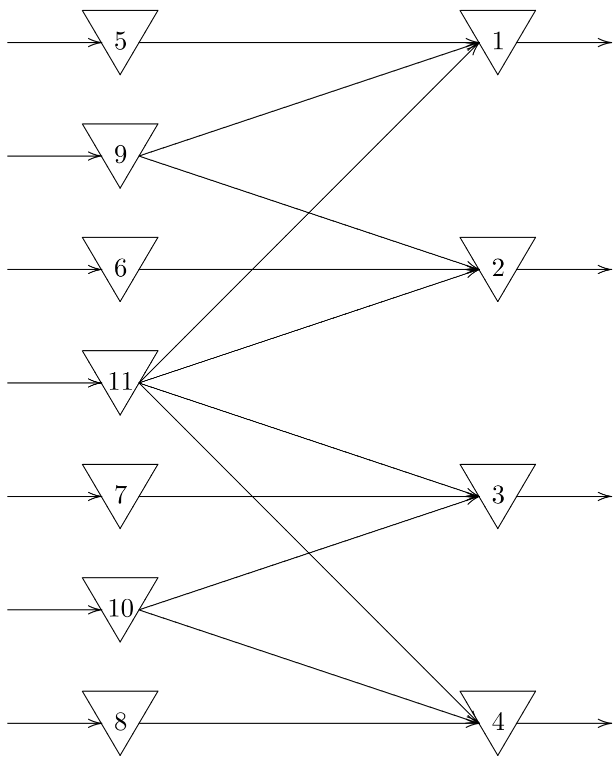

Figure 6.3: *Schematic representation of 11 item model.*

We assume that the demand for the end-items is stationary. More precisely, demand for end-item i in consecutive periods is i.i.d. We also assume that the demand processes for different end-items are uncorrelated. The mean demand is 100 for all end items. To get insight into the impact of demand variability on the choice of a SCOP function we vary the squared coefficient of variation cv_i^2 for each end-item i as 0.25, 0.5, 1 and 2. The costs structure is as follows

$\alpha_i = \alpha_f = 100$ inventory costs end-items, i=1,2,3,4

$\alpha_i = \alpha_s = 10$ inventory costs specific components, i=5,6,7,8

$\alpha_i = \alpha_{sc} = 30$ inventory costs semi-common components, i=9,10

$\alpha_i = \alpha_c = 50$ inventory costs common components, i=11

For the planned lead times we have analogously to the costs structure the following variables

$\tau_i = \tau_f$ planned lead time end-items, i=1,2,3,4

$\tau_i = \tau_s$ planned lead time specific components, i=5,6,7,8

$\tau_i = \tau_{sc}$ planned lead time semi-common components, i=9,10

$\tau_i = \tau_c$ planned lead time common components, i=11

We vary the planned lead times (τ_s,τ_{sc},τ_c) as follows (1,2,4), (4,2,1) and (1,4,2). The safety stocks are chosen such that we obtain a non-stockout probability of 95%.

6.4.3 Results

The results of the comparison between the SBS concept and the LP strategies with allocation policies are given in table 6.3. First of all we remark that the inventory costs for the standard LP strategy found by De Kok & Fransoo (2003) are larger than the results we found. The differences are caused by a mistake in the implementation of the LP strategy by De Kok & Fransoo (2003). Although the results of the standard LP strategy are better than in De Kok & Fransoo (2003), the difference between the SBS concept and the standard LP strategy is still significant, and using allocation policies might give better results.

		Supply chain inventory cost						
cv_i^2	$(\tau_s, \tau_{sc}, \tau_c)$	SBS	LP_{st}	Δ_{LP}	LP_{alloc}	Δ_{alloc}	QP	Δ_{QP}
0.25	(1,2,4)	71682	79477	10.9%	73458	2.5%	73332	2.3%
0.25	(4,2,1)	76476	78133	2.2%	78853	3.1%	76765	0.4%
0.25	(1,4,2)	73550	80620	9.6%	74197	0.9%	74172	0.8%
0.5	(1,2,4)	104448	115227	10.3%	106923	2.4%	106679	2.1%
0.5	(4,2,1)	112316	114659	2.1%	115696	3.0%	113381	0.9%
0.5	(1,4,2)	107616	115386	7.2%	108376	0.7%	108362	0.7%
1	(1,2,4)	152203	168533	10.7%	158101	3.9%	157752	3.6%
1	(4,2,1)	165328	169122	2.3%	170590	3.2%	167180	1.1%
1	(1,4,2)	157034	169030	7.6%	159588	1.6%	159722	1.7%
2	(1,2,4)	218551	247578	13.3%	233044	6.6%	232392	6.3%
2	(4,2,1)	245998	249849	1.6%	250075	1.7%	247705	0.7%
2	(1,4,2)	228789	247571	8.2%	235761	3.0%	235694	3.0%

Table 6.3: *Inventory costs of different SCOP functions.*

Looking at the results we see that with uncapacitated resources, the allocation policy with quadratic objective function outperforms the strategy with allocation rules. Furthermore we see for the cases where specific items have a planned lead time of four periods, the performance of the quadratic model is only a little better, and the performance of LP_{alloc} even worse, than the performance of the standard LP strategy. If the specific items have the longest planned lead time, they are the bottlenecks of the system, this means that the (semi-)common items do not have an allocation problem. So, for supply chain structures where shortages can be allocated to several parent items, linear allocation rules improve the performance. But in those supply chain structures without an actual allocation problem, interference of allocation rules diminish the performance of the LP strategy. On the other hand for the problems where the common item has the longest planned lead time, the common item is the bottleneck, and allocation rules improve the performance of the LP strategy.

In the last column of table 6.3, the relative difference between SBS_{sim} and the best allocation policy, namely QP, are given. We see that SBS policies still outperforms the mathematical programming models, but the differences are small. The largest differences are found when the common planned lead time is equal to four periods, so apparently the SBS concept better exploits the commonality. From the discussion in De Kok & Fransoo (2003) it follows that this may be caused by better parameter settings. The echelon stock control policy allows for implementing "push" policies where upstream stocks are zero, while the LP strategy tends to build up stock upstream. A possible explanation for this is that the LP strategy aims at satisfying the end item demand forecast. After satisfying this forecast remaining stocks of components are not used for assembly of end items, because stocks of components are cheaper than stocks of assembled end items.

6.5 Numerical experiment with capacitated supply chains

In the previous section it turned out that the SBS policy still outperforms MP based concepts. However the differences are small, and the SBS policy is restricted to uncapacitated problems. Hence, MP based concepts are good alternatives for capacitated problems. In this section the performance of the allocation policies is measured by comparing them with the standard LP strategy.

6.5.1 Experimental design

Allocation problems only occur in divergent parts of the supply chain. Hence, for the experiments we need a supply chain with a divergent part. We have chosen a so-called "W-structure", whereby we have two end items, and each of these end items consist of a common and a specific item. The "W-structure" is given in figure 6.4. We use the "W-structure" instead of a pure divergent structure to show that the allocation policies also work if the allocation of common child items is restricted by the number of specific child items.

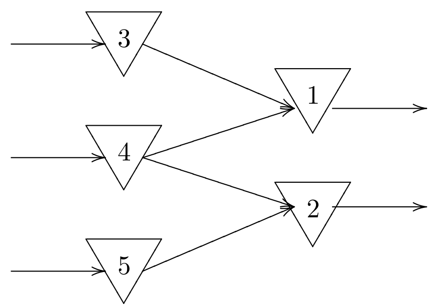

Figure 6.4: *Schematic representation of 2-echelon system.*

We consider a one-to-one relation between items and resources, i.e. each item is produced on its own resource. For simplicity we number the resources according to the number of the item it produces. The end items are produced on uncapacitated resources, while the specific and common child items are not. In the results, see section 6.4.3, of the comparison between the SBS and the MP models with allocation policies we have seen that if the specific items have long planned lead times the allocation policies did not have any effect on the inventory cost. Long lead times are comparable with high utilization rates of resources. Hence we restrict the experiments to test cases whereby the common item is production the resource with the highest utilization rate. In section 6.2, we have seen that the cost structure of the end items plays an important role in finding an optimal solution. For equal cost the optimal solution could be found on a whole line segment, while in the case of unequal cost we found only one optimal solution. Hence we consider two cost-structures, and expect large reductions in inventory cost especially when the costs of end items are equal. To identify more situations in which allocation policies reduce the inventory costs, we also look at the following settings

- equal vs. unequal mean demand,
- equal vs. unequal squared coefficient in variation of the demand, and
- several utilization rates for the specific and common child items.

The used input parameters are given in 6.4.

All possible parameters yield $3 * 2^5 = 96$ cases, but due to the symmetry of the supply chain system the number of cases can be reduced to 78. The performance of the models is tested with a discrete event simulation of 26,000 time periods. The models are solved using a CPLEX solver.

Parameter	Description	Values
μ_1	The mean demand per period at item 1	50
μ_2	The mean demand per period at item 2	50, 100
c_1^2	The squared coefficient of variation at item 1	0.25, 2
c_2^2	The squared coefficient of variation at item 2	0.25, 2
α_1	Holding costs of item 1	25, 50
α_2	Holding costs of item 2	25, 50
τ_1	Planned lead time of item 1	1
τ_2	Planned lead time of item 2	1
φ_1	Service level of item 1	0.95
φ_2	Service level of item 2	0.95
ρ_1	Utilization rate of resource 1	∞
ρ_2	Utilization rate of resource 2	∞

Parameters	Description	Values
$(\alpha_3, \alpha_4, \alpha_5)$	Holding costs of items 3, 4,and 5	(10,10,10)
(τ_3, τ_4, τ_5)	Planned lead times of items 3, 4,and 5	(1,1,1)
(ρ_3, ρ_4, ρ_5)	Utilization rates of resources 3, 4,and 5	(0.8, 0.85, 0.8)
		(0.9, 0.95, 0.9)
		(0.9, 0.99, 0.9)

Table 6.4: *Parameter values.*

6.5.2 Results

The performance of each strategy with allocation policies is determined by the relative difference in inventory costs between the model with and the model without an allocation policy. In table 6.5, the average, the maximum, and the minimum relative differences are given. Note that "QUANT" correspond with the allocation fraction $q_{ij}^{|N|}$ and "MEAN" with the allocation fraction q_{ij}^{μ}.

If we look at the results we see average reductions in inventory costs of more than 20%. The best performance is reached by using a quadratic objective function; the average reduction exceeds 25%. Looking at the maximum reductions, savings of 65% can be made. Unfortunately, there are also cases in which the inventory costs become larger when using allocation policies. However, the number of cases with negative results is small. We noticed that negative results only in cases with totally unequal parameter settings of the end items and high utilization rates of the resources, whereby the highest cost correspond with the item having the biggest demand and highest variation. Apparently too much shortages are allocated to these items when using allocation policies. Using a quadratic objective function with weight factors based on the allocation fractions discussed in Eppen & Schrage (1981) and in De Kok & Fransoo (2003), no negative results occur at all.

Furthermore we see that allocation fractions which also takes the variance in the demand into account perform better than both $q_{ij}^{|N|}$ and q_{ij}^{μ}. Especially in the worst case scenario with unequal demand and coefficient of variance the results are depressing,

	Average	Maximum	Minimum
Allocation QUANT	18.83%	65.43%	-48.65%
Allocation MEAN	20.27%	65.51%	-26.43%
Allocation ES	21.66%	65.45%	-4.73%
Allocation HKF	21.79%	65.51%	-14.85%
Lower bound 25% QUANT	18.83%	65.43%	-48.65%
Lower bound 25 % MEAN	20.27%	65.51%	-26.43%
Lower bound 25% ES	18.89%	65.36%	-48.41%
Lower bound 25% HKF	20.28%	65.38%	-26.15%
Quadratic	24.28%	66.94%	-13.49%
Quadratic ES	26.73%	67.60%	0.71%
Quadratic HKF	26.98%	67.42%	0.68%

Table 6.5: *Average, maximum, and minimum relative difference in inventory costs, compared to the standard LP strategy.*

so it is worthwhile to use more sophisticated allocation fractions. The differences between the allocation fractions of Eppen & Schrage (1981) and De Kok & Fransoo (2003) is very small, but in general q_{ij}^{HKF} gives better results. The performance of the allocation policies with lower bounds is equivalent to the performance of the model with linear allocation rules. Hence in the remainder of the chapter we only focus on the allocation policy with linear allocation rules.

In the models with quadratic objective functions we distinguished three different sets of weight factors. In the first set the weight factors are similar to the weight factors in the linear objective function and the other two sets of weight factors are inspired by the allocation fractions of Eppen & Schrage (1981) and De Kok & Fransoo (2003). We see that the weight factors in which we also take the mean and variance of the actual demand into consideration performs better, but the differences with the first set of weight factors is small (around 2%). Hence using a quadratic objective function is a good and robust alternative for using linear objective functions.

Since we noticed, in section 6.1 that allocation problems typically occur in situations with equal inventory costs for the end items, we separately give, in figure 6.5, the performances of the models with allocation policies for both equal and unequal inventory costs. Indeed, we see large reductions when the holding costs of the end items are equal, but for unequal holding cost the reductions are still considerable. Notice that the difference between the model with linear allocation rules and the model with quadratic objective function is larger when the holding costs are unequal. So, when using the linear allocation rules too much shortages are allocated to the expensive parent item.

In figure 6.6, we give the performance of the allocation policies for different utilization rates. We see that cost savings are primarily made if the utilization rates of the resources are high. Since shortages occur more often when the utilization rates are high, the savings are higher when allocation policies are used. Hence, for $(\rho_3, \rho_4, \rho_5) = (0.8, 0.85, 0.8)$ the savings are only 5% while for $(\rho_3, \rho_4, \rho_5) = (0.9, 0.99, 0.9)$ the savings are more than 35%.

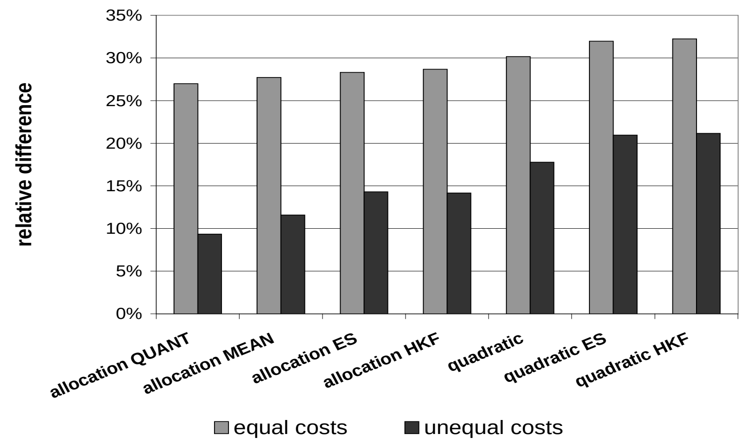

Figure 6.5: *Relative difference between allocation policies and the standard LP strategy for equal and unequal costs.*

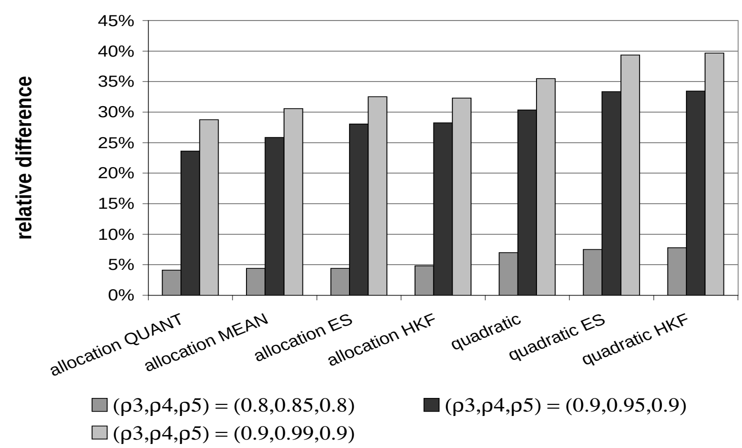

Figure 6.6: *Relative difference between allocation policies and the standard LP strategy for different capacities.*

In figures 6.7 and 6.8, we separated the results by equal and unequal demand and equal and unequal squared coefficient of variation respectively. We see that the allocation rules preform slightly better if the demand of the end items is unequal. The difference in performance between equal and unequal squared coefficient of variance are small. Apparently, in the LP strategy without allocation rules not enough shortages are allocated to the item with highest demand. If we compare the models with linear allocation and different allocation fractions, we see that $q_{ij}^{|N|}$ also does not allocate enough shortages to the item with the highest demand. Furthermore we see clearly that allocation fractions that consider both the mean demand and the demand

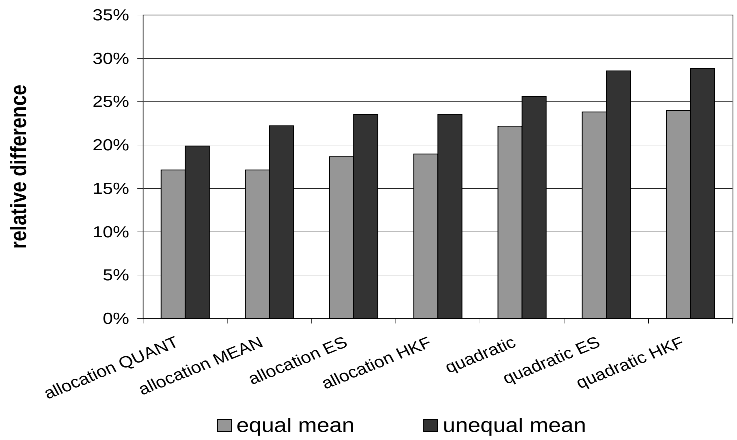

Figure 6.7: *Relative difference between allocation policies and the standard LP strategy for equal and unequal mean.*

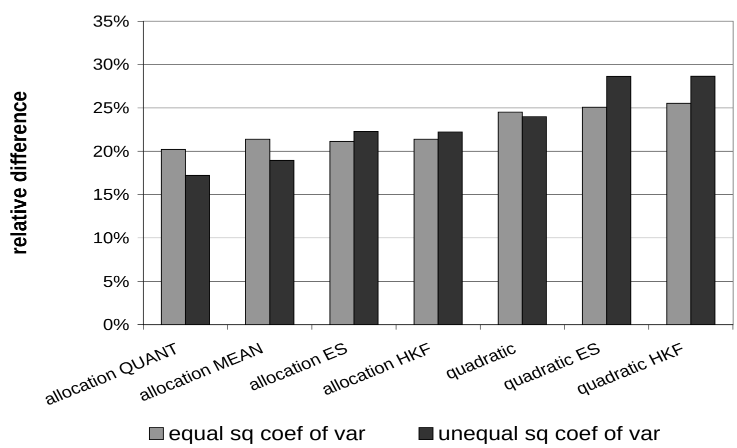

Figure 6.8: *Relative difference between allocation policies and the standard LP strategy for equal and unequal squared coefficient of variation.*

variation perform better.

We also look at the time needed per test case. The experiments are done on a computer with a 2.00 GHz Intel Pentium 4 processor. In table 6.6, the used time is shown. Of course the standard LP strategy is fastest, since this is the smallest model. However, on the second place we see the QP strategy. This is most convenient, being the strategy using allocation rules with the best performance.

Model	Time
Standard	514 sec.
Allocation rules	764 sec.
Lower bound	773 sec.
Quadratic	672 sec.

Table 6.6: *Time needed per test case.*

6.6 The 18-item model

For the last time, we look at the 18-item model. In this section we compare the LP strategy with early production and optimal planned lead times with the introduced strategies with allocation rules. The results are given in table 6.7, whereby $\Delta_{alloc_{(\tau_i=\tau_i^*)}}$, and $\Delta_{QP_{(\tau_i=\tau_i^*)}}$ are given by

$$\Delta_{alloc_{(\tau_i=\tau_i^*)}} = \frac{\bar{C}_{early_{(\tau_i=\tau_i^*)}} - \bar{C}_{alloc_{(\tau_i=\tau_i^*)}}}{\bar{C}_{early_{(\tau_i=\tau_i^*)}}}, \tag{6.25}$$

and

$$\Delta_{QP_{(\tau_i=\tau_i^*)}} = \frac{\bar{C}_{early_{(\tau_i=\tau_i^*)}} - \bar{C}_{QP_{(\tau_i=\tau_i^*)}}}{\bar{C}_{early_{(\tau_i=\tau_i^*)}}}. \tag{6.26}$$

cv_i^2	$(\rho_s, \rho_{sc}, \rho_c)$	$(\alpha_s, \alpha_{sc}, \alpha_c)$	$\Delta_{alloc_{(\tau_i=\tau_i^*)}}$	$\Delta_{QP_{(\tau_i=\tau_i^*)}}$
0.25	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	8.49%	11.43%
0.25	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	7.20%	9.25%
2	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	9.08%	9.10%
2	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	7.06%	8.57%
0.25	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	8.15%	10.31%
0.25	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	7.40%	9.85%
2	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	15.14%	13.78%
2	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	4.25%	4.83%

Table 6.7: *The relative difference between the LP strategy with early production and optimal planned lead time and the strategies with allocation rules and optimal planned lead times*

Again we see savings, when we use allocation policies. The savings are highest when the utilization rate of the resource producing the common item is equal to 95%. But also if the resource on which the specific items are produced have an utilization rate is 95% the savings are remarkable. Notice that in the 11-item model, section 6.4, there were not much savings when the specific items had the longest planned lead time. This difference may be caused by the difference in the number of items per resource. In the 11-item model the specific items were all produced on their own resource, however in the 18-item model all four specific intermediate items are produced on one resource. The allocation policy apparently gives an indication of the amounts in which the specific items must be produced in case of capacity shortages.

Remark that the experimental setting with $cv_i^2 = 2$, $(\rho_s, \rho_{sc}, \rho_c) = (0.85, 0.90, 0.95)$, and $(\alpha_s, \alpha_{sc}, \alpha_c) = (30, 60, 90)$ is the only case that LP strategy with linear allocation rules performs better than the strategy with quadratic objective function. This deviant result might suggest that when using allocation rules, the optimal planned lead times change. To prove this idea, we determined the optimal planned lead time for the allocation policies and compare the results with the LP strategy with early production and optimal planned lead times. The results can be found in table 6.8.

				LP_{alloc}	
cv_i^2	$(\rho_s, \rho_{sc}, \rho_c)$	$(\alpha_s, \alpha_{sc}, \alpha_c)$	$(\tau_s, \tau_{sc}, \tau_c)$	Δ_{alloc}	$(\tau_s, \tau_{sc}, \tau_c)$
0.25	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	$(1, 1, 2)$	10.92%	$(1, 1, 1)$
0.25	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	$(2, 1, 1)$	7.37%	$(1, 1, 1)$
2	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	$(1, 2, 8)$	10.87%	$(1, 1, 3)$
2	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	$(9, 2, 1)$	11.16%	$(3, 1, 1)$
0.25	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	$(1, 1, 2)$	14.32%	$(1, 1, 1)$
0.25	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	$(2, 1, 1)$	9.02%	$(1, 1, 1)$
2	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	$(1, 1, 3)$	15.14%	$(1, 1, 3)$
2	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	$(8, 1, 1)$	7.45%	$(5, 1, 1)$

				QP	
cv_i^2	$(\rho_s, \rho_{sc}, \rho_c)$	$(\alpha_s, \alpha_{sc}, \alpha_c)$	$(\tau_s, \tau_{sc}, \tau_c)$	Δ_{QP}	$(\tau_s, \tau_{sc}, \tau_c)$
0.25	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	$(1, 1, 2)$	14.50%	$(1, 1, 1)$
0.25	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	$(2, 1, 1)$	10.97%	$(1, 1, 1)$
2	$(0.85, 0.9, 0.95)$	$(20, 40, 60)$	$(1, 2, 8)$	14.26%	$(1, 1, 3)$
2	$(0.95, 0.9, 0.85)$	$(20, 40, 60)$	$(9, 2, 1)$	18.83%	$(3, 1, 1)$
0.25	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	$(1, 1, 2)$	16.77%	$(1, 1, 1)$
0.25	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	$(2, 1, 1)$	11.76%	$(1, 1, 1)$
2	$(0.85, 0.9, 0.95)$	$(30, 60, 90)$	$(1, 1, 3)$	18.85%	$(1, 1, 1)$
2	$(0.95, 0.9, 0.85)$	$(30, 60, 90)$	$(8, 1, 1)$	13.33%	$(1, 1, 1)$

Table 6.8: *The relative difference between the LP strategy with early production and optimal planned lead time and the strategies with allocation rules and their optimal planned lead times*

We see that indeed the optimal planned lead time when using allocation rules differ from the optimal planned lead times of the LP strategy with early production. Remarkable results are found for the cases with $cv_i^2 = 2$ and $(\alpha_s, \alpha_{sc}, \alpha_c) = (20, 40, 60)$. In these two cases the savings made by using allocation rules are larger if the specific item is produced on the bottleneck resources, while in all other cases the savings are highest if the common item is produced on the bottleneck resource. These large savings in case with $cv_i^2 = 2$ and $(\alpha_s, \alpha_{sc}, \alpha_c) = (20, 40, 60)$ are not only caused by the allocation policies, but also by the larger reduction in planned lead times. Hence we conclude that when using allocation policies the optimal planned lead times become shorter.

6.7 Conclusion

Supply Chain Operations Planning problems of multi-item, multi-echelon systems with demand uncertainty can be solved using Linear Programming models in a rolling horizon setting. Negenman (2000) and De Kok & Fransoo (2003) noticed that LP strategies do not seem to find the right balance in the allocation of stock among the different items. In this chapter we focussed on material allocation in LP models, using the LP model introduced in chapter 2. Notice that although capacity allocation is also a part of this model, in this chapter we restricted ourselves to material coordination.

In this chapter we include material allocation policies in the LP model by adding linear allocation rules to the LP model. The linear allocation rules allocate shortages of a child item to all its parent items. To identify shortages, the echelon inventory position is added to the model. In the second model the linear allocation rules are relaxed by introducing lower bounds for the amount of shortages allocated to the parent items. A third strategy for solving the allocation imbalance is by replacing the linear objective function by a quadratic one. Note that, by the introduction of allocation policies, additional constraints are added to the LP model. Therefore the optimal solution to the problem may have a higher cost then the original solution. However, the LP model is part of a rolling schedule implementation. We have shown that, despite the increase in cost on an instance, the actual costs incurred under uncertainty can be substantially lower.

In De Kok & Fransoo (2003) it is shown that for supply chains with infinite capacity and identical end items, the so-called synchronized base stock (SBS) policies outperform MP models. De Kok & Fransoo (2003) state that LP-based concepts do not seem to find the right balance in allocating stock among the different items. Hence, in this chapter we compared the SBS policy with the introduced MP models with allocation policies. The results are quite promising. Especially the model with the quadratic objective function closes the gap between SBS and LP. Since the strategy with quadratic objective function (QP strategy) is, in contrast to the SBS policy, applicable to capacitated problems the QP model is a good alternative for problems that cannot be solved by the SBS policy.

Furthermore we measured the performance, compared to the standard LP strategy, of the allocation policies for capacitated supply chains. For the experiments we used a so-called "W-structure", with two end items whereby each item is consist of a specific and a common child item. The results show us that models with allocation policies perform better than the standard LP strategy, whereby the QP strategy performs best. The QP strategy gives not only lower inventory cost, it also is faster in solving the SCOP problem.

As already mentioned capacity allocation is not taken as part of the model. Although it may turn out that capacity allocation is no longer an issue when material allocation rules are used, it is still an interesting topic for future research.

Chapter 7

Conclusions and future research

7.1 Main research findings

In this thesis we studied rolling schedule approaches to solve the Supply Chain Operations Planning (SCOP) problem. The SCOP problem is the problem of coordinating the material and resource release decisions while obeying capacity restrictions in a supply chain such that predefined customer service levels are met at minimal costs. The SCOP problem assumes that customer demand and processing times are stochastic. In this thesis we restricted ourself to stochastic demand only. The aim of this research is to make a contribution to the development of more effective approaches for solving the SCOP problem. Towards this end we introduced, in chapter 2, a linear programming (LP) model with planned lead times. The reason we use an LP model, or MP models in general, is twofold. First, MP models can easily be incorporated in existing Advanced Planning Systems (APS), which provide an appropriate framework for putting OR developments in practice (see Fleischmann & Meyr (2003)). Second, a big advantage over multi-echelon concepts is the possibility to add constraint, such as lot-sizing and capacity constraints to the problem. While the multi-echelon concept can only be used for uncapacitated supply chains and some strictly defined capacitated supply chains, MP models can be used for all kind of problem. To incorporate stochastic customer demand in the deterministic LP model, we studied the LP model in a rolling schedule context. While using this model we pursued the goal of the research by elaborating on the following two research questions.

1. Can we use the planned lead time concept to improve the performance of deterministic SCOP functions?

By introducing planned lead times, additional flexibility is added to the model, namely by decoupling the capacity allocation decision from the order release decision. Hence we were interested if we could use this additional flexibility to reduce the inventory cost of the model.

Besides mathematical programming models applied in a rolling schedule context, there also exist stochastic models to solve the SCOP problem, such as echelon stock (or base stock) control concepts introduced by Magee (1958) and Clark & Scarf (1960). These stochastic models can only solve uncapacitated supply chain structures and a few specially designed capacitated supply chain structures. In De Kok & Fransoo (2003), it is shown that the stochastic models obtain lower cost than the deterministic SCOP functions. To tighten the gap between these two approaches, we ask ourself the following question:

2. How well do deterministic SCOP functions perform while using a rolling schedule to incorporate demand uncertainty compared to stochastic SCOP functions, and how can we tighten a possible gap?

In the following subsections we summarize our conclusions regarding these questions.

7.1.1 Optimal usage of planned lead time concept

In chapter 2 we formulated a linear programming model with planned lead times. These planned lead times consist of one or more periods in which a released order must be produced. We assumed that the production of one item always can be done within one period, and that an order can be split up in several parts. Each part is produced during one period of the planned lead time. This approach of planned lead times decouples the order release from the capacity usage, and creates additional flexibility in the model. The decoupling of the order release decision from the capacity usage by using planned lead times is new. In former MP models, see e.g. Billington *et al.* (1983), Hopp & Spearman (2000), and Tempelmeier (2003), the lead time is assumed to be equal to zero of fixed. These fixed lead times are modelling minimal lead times or delays whereby capacity is allocated at this fixed time offset.

In chapters 3, 4, and 5, we studied the planned lead time concept and its possibilities thoroughly. In chapter 3 we looked at the timing of production during the planned lead time. We considered two situations, namely the situation in which production starts as soon as there is available capacity and the situation in which the production of items starts as late as possible. Starting the production as late as possible seems cheapest, since stocking child items is less expensive than stocking parent items. However, since customer demand is uncertain and the LP model is used in a rolling schedule concept saving capacity for later might also be a good strategy. Experiments show that especially for items produced on resources with high utilization rates, starting the production as soon as there is capacity available, is most effective

Planned lead times have to be determined beforehand, however strategies to find values of the planned lead times which minimize the cost are not available. In chapter 4 we have discovered three exogenous factors influencing the optimal planned lead time. From queuing theory we already knew that the utilization rate of the server and the variability in demand influences the waiting time. Since waiting time is a large part of a lead time, it is not surprising that both mentioned parameters also influence the optimal planned lead time. Higher utilization rates and/or larger demand variability increases the the optimal planned lead time. However, the holding costs structure of the supply chain plays the leading role in determining optimal planned lead times. Increasing the planned lead time has two effects. First, the safety stocks may decrease. Second, the amount of work-in-process increases. Thus longer planned lead times are only advantageous if the additional work-in-process costs are outweighed by the savings in safety stocks.

Planned lead times are defined such that released items are available for succeeding production steps at the end of their planned lead time only. Items produced before the end of the planned lead time, wait the remainder of the planned lead time. In chapter 5 two strategies are given which make it possible to use produced items before the end of their planned lead time. Under the first strategy items are made available for further usage directly after production. Experiments showed that this strategy does not improve the performance of the LP strategy. When items are available immediately after production, the maximal number of available items per period is equal to the capacity of the resources. This decreases the variability in the number of available items per period. Hence, the number of available items do not follow the fluctuation in the demand as closely as when items are available at the end of their planned lead time. This increases the number of backorders, which results in higher inventory costs. In the second strategy the produced items are only available before the end of their planned lead time if needed to avoid or reduce backorders. It turned out that when using planned lead times that are optimal or longer, reductions in costs are made. If the planned lead times are smaller then the optimal planned lead times, using this strategy increases the costs. With smaller planned lead times there is less flexibility in the capacity usage and backorders occur more often. Hence, the number of times items have to be released before the end of their planned lead times increases, and if it happens too often the number of items available per period becomes equal to the maximum capacity per period. In the first strategy we already showed that this increases the cost. If the planned lead times are longer than the optimal planned lead times, the strategy shortens the actual lead time and this gives a reduction in cost.

7.1.2 Performance of LP versus stochastic strategies in rolling schedule setting

In De Kok & Fransoo (2003) it is shown that LP strategies are outperformed by stochastic base stock policies for uncapacitated supply chains. They noticed, and so did we in chapter 3, that the safety stocks for identically distributed end-items strongly differ. Apparently the LP solver always favors one item over another, when ties must be broken. To solve this problem, we introduce two types of allocation strategies in chapter 6. In the first strategy, linear allocation rules are added to the model. These allocation rules divide shortages of a child item among the parent items using predefined allocation fractions. In the second strategy the linear objective function is replaced by a quadratic one. The LP strategies with an allocation policy are again compared with stochastic base stock policies. The results show that the LP policies are still outperformed by the stochastic base stock policies, but the differences are small. This suggest that these adjusted LP strategies are good alternatives for capacitated supply chain structures for which the SCOP problem cannot be solved by stochastic base stock policies.

We also compared the performance of the strategies with an allocation policy with the standard LP strategy for capacitated supply chains. The results show that the models with an allocation policy perform better. A last remarkable result is that for both uncapacitated as capacitated supply chains replacing the linear objective function by a quadratic objective (QP strategy) gives better results than adding linear allocation rules to the LP model. The QP strategy does not only give lower inventory costs, it is easy to implement and it also can be solved efficiently by existing software.

7.1.3 General conclusion

A general conclusion of this thesis is that existing LP models can be used to solve Supply Chain Operations Planning problems with demand uncertainty by using them in a rolling schedule context. Although LP models give optimal production plans for a fixed planning horizon, in a rolling schedule context whereby the customer demand is uncertain, these plans are most probably not optimal. In this thesis we showed that the performance of the model can be improved by the introduction of planned lead times. The proper coordination of the timing of production within the planned lead time can reduce the cost even more. The performance can also be improved by using allocation strategies for allocating shortages among parent items. It turned out that the models with a quadratic objective function and linear constraints are most suitable for solving the SCOP problem; they give the lowest inventory costs of the studied approaches, and can be solved in a reasonable time.

7.2 Future research

In chapter 4 of this thesis, we studied factors which influence the optimal planned lead times. We learned that the variation in demand and the utilization rates of the resources influence the value of the optimal planned lead time. However, the cost structure plays the key role in determining optimal planned lead times. We found optimal planned lead times by comparing the inventory costs of simulation studies for various planned lead times. This procedure is very time consuming. Hence, an interesting research topic is to find a heuristic, based on these three factors, to determine optimal or nearly optimal planned lead times. Note that longer planned lead times gives more flexibility in capacity allocation. Hence, if it is possible to estimate the safety stocks based on the utilization rate and demand variation, and if it is possible to estimate the effects on the safety stock obtained by the additional flexibility in capacity allocation created by longer planned lead times, it might be possible to find good planned lead times by balancing the safety stock savings and the increased work-in-process inventory cost. However if more than one item in the supply chain is produced on a capacitated resource, there is also a dependency between the optimal planned lead times, i.e., the planned lead time of one item influence the optimal planned lead time of another item.

With the introduction of allocation strategies we tighten the gap between the better stochastic base stock policies and the policy based on linear programming models for uncapacitated supply chain structures. Although the gap between the two models is not too big, it is still interesting to tighten the gap even more. From the discussion in De Kok & Fransoo (2003) it follows that this existing gap may be caused by better parameter settings in the echelon stock control policy. The echelon stock control policy allows for implementing "push" policies where upstream stocks are zero, while the LP strategy tends to build up stock upstream. A possible explanation for this is that the LP strategy aims at satisfying the end item demand forecast. After satisfying this forecast remaining stocks of components are not used for assembly of end items, because stocks of components are cheaper than stocks of assembled end items. By adjusting the LP strategy such that instead of stocking at upstream levels more items are stocked at downstream levels the gap might be even more tightened.

In this thesis we studied only relatively small supply chain structures. To see the full effect of all modifications to the standard LP strategy one should compare the inventory cost of the standard LP strategy with the inventory cost of the adjusted LP strategy in a real-life situation. For large supply chain structures, having planned lead times longer than one period is only profitable for those items that are produced on tightly capacitated resources. Early availability of these items decreases the inventory costs, but early production is only beneficial if the difference savings in safety stocks cost are larger then the additional work-in-process inventory cost.

To create additional flexibility in the timing of production we used the planned lead times concept. These planned lead times can be seen as adding safety time to the model. An alternative for safety time is safety stock. Hence an interesting research topic is the introduction of safety stocks at upstream items. It will be a challenge to find optimal safety stocks for every items on the supply chain. Backorders are only allowed at the end item level, so a method is needed to to measure the shortages of child items that are now resolved by backorders on the end item level.

If one of the LP strategies or the QP strategy is used in a real-life setting the needed safety stocks must be determined beforehand. An interesting study is to determine good safety stocks, while taken into account future demand. Throughout this thesis we have seen that the needed safety stocks are reduced by every adjustment we made in the LP model, so the needed amount of safety stock is dependent of many factors. Hence simple rules to determine safety stocks may not be sufficient.

A last mentioned interesting research topic is the relation between SCOP and the unit control function. In this thesis we assumed an 100% due date reliability from the unit control functions, but what happens if the unit control functions are not for 100% reliable? Which information should be exchanged between the two levels to reach customer service levels at minimal cost?

7.3 Retrospect

The original idea of this research wa to develop effective concepts to solve Supply Chain Operations Planning problems of capacitated supply chain structures under demand uncertainty.

Capacity constraints and all kind of other restrictions of the supply chain can easily be added to an LP model. However, an LP model can only solve deterministic problems. Hence, in order to use the easiness of LP but simultaneously incorporate demand uncertainty, we investigated the performance of an LP model in a rolling schedule concept.

In the proposed LP problem, we introduced planned lead times with multi-period capacity allocation and studied the possibilities of the created additional flexibility. It turned out that the inventory costs reduce by a proper use of these planned lead times.

We also studied the use of allocation policies within the SCOP concept. These allocation policies allocate shortages of a child item fractionally, taken into account the mean and variance of the demand distribution, among its parent items. We have seen that allocation policies, especially the QP strategy, improve the performance of the SCOP concept considerably.

In retrospect we may conclude this work by stating that capacitated SCOP problems with demand uncertainty can easily solved by MP models, especially the model with quadratic objective function, in a rolling schedule setting.

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Appendix A

The equivalence of the two sets of capacity constraints

In this appendix we prove the equivalence between the two sets of capacity constraints from sections 2.4.2 and 2.4.3. Without loss of generality we assume all variables determined in the past are equal to zero and we consider the time horizon $t = 0, \ldots, T$.

Theorem A.1 *If we assume that a work order is not produced before all earlier work orders of that item are finished then constraints*

$$\sum_{s:-1< s\leq t} p_i R_{is} \geq \sum_{m\in\mathcal{R}_i} \sum_{s:0< s\leq t+1} V_{ius} \quad i = 1, \ldots, n,\ t = 0, \ldots, T-1$$

and

$$\sum_{s:-1< s\leq t} p_i R_{is} \leq \sum_{m\in\mathcal{R}_i} \sum_{\substack{s:0< s\leq t+\tau_i,\\ s\leq T}} V_{ims} \quad i = 1, \ldots, n,\ t = 0, \ldots, T-1$$

are equivalent to constraints

$$p_i R_{it} = \sum_{\substack{s:t+1\leq s\leq t+\tau_i,\\ s\leq T}} Z_{its}, \quad i = 1, \ldots, n,\ t = 0, \ldots, T-1 \tag{A.3}$$

and

$$\sum_{\substack{s:t-\tau_i\leq s<t,\\ s\geq 0}} Z_{ist} = \sum_{m\in\mathcal{R}_i} V_{imt}, \quad i = 1, \ldots, n,\ t = 1, \ldots, T \tag{A.4}$$

Proof Constraints (A.1) and (A.2) $\Rightarrow$ constraints (A.3)and (A.4): With constraints (A.1) and (A.2) the part of the released work order R_{it} executed in time slot s can be the determined. If we define Z_{its} as

Z_{its} : the part of the planned work order release of item i requested on at time t and executed in time slot s, $i = 1, \ldots, n$, $t = 0, \ldots, T-1$, $s = t+1, \ldots, t+\tau_i, s \leq T$.

then

$$Z_{its} = \min\{\underbrace{(\sum_{m \in \mathcal{R}_i} V_{ims} - \sum_{\substack{r: s-\tau_i \leq r < t,\\ r \geq 0}} Z_{irs})}_{\text{available capacity}}, \underbrace{(p_i R_{it} - \sum_{r: t<r<s} Z_{itr})}_{\text{needed capacity}}\} \tag{A.5}$$

The "available capacity" consists of the total capacity claim in time slot s minus the capacity already allocated to earlier released work orders. The "needed capacity" consists of the needed capacity of released work order R_{it} minus the capacity already allocated to this work order. Since (A.1) and (A.2) hold there is always enough capacity claimed for the released work orders and

$$\sum_{\substack{s: t+1 \leq s \leq t+\tau_i,\\ s \leq T}} Z_{its} = p_i R_{it}, \quad i = 1, \ldots, n,\ t = 0, \ldots, T-1 \tag{A.6}$$

Remains to show that constraint (A.4) also holds. Looking at the definition of V_{imt}

V_{imt} : the capacity allocated to an item i at resource u in time slot t, $i = 1, \ldots, n$, $m = 1, \ldots, L$, $t = -\tau_i + 2, \ldots, T$.

and the definition of Z_{its}, we see that constraint (A.4) always hold by definition.

Constraints (A.3)and (A.4) $\Rightarrow$ constraints (A.1) and (A.2): We first concentrate on showing that constraint (A.1) holds.

Since $Z_{its} \geq 0$ for $i = 1, \ldots, n$, $t = 0, \ldots, T-1$, $s = t+1, \ldots, t+\tau_i, s \leq T$, we have shown that (A.1) holds, if we show that

$$\sum_{0 \leq s \leq t} p_i R_{is} = \sum_{s: 1 \leq s \leq t+1} \sum_{m \in \mathcal{R}_i} V_{ims} + \sum_{r: t-\tau_i+2 \leq r \leq t} \sum_{\substack{q: t+2 \leq q \leq r+\tau_i,\\ q \leq T}} Z_{irq} \tag{A.7}$$

holds.

By using iteration we show that (A.7) holds. First we show that it is true for $t = 0$. For $t = 0$ equation (A.7) becomes:

$$\begin{aligned} p_i R_{i0} &= \sum_{m \in \mathcal{R}_i} V_{im1} + \sum_{r: -\tau_i+2 \leq r \leq 0} \sum_{\substack{q: 2 \leq q \leq r+\tau_i,\\ q \leq T}} Z_{irq} \\ &= \sum_{m \in \mathcal{R}_i} V_{im1} + \sum_{\substack{q: 2 \leq q \leq \tau_i,\\ q \leq T}} Z_{i0q} \end{aligned} \tag{A.8}$$

From constraint (A.3) we derive that

$$p_i R_{i0} = \sum_{\substack{s: 1 \leq s \leq \tau_i,\\ s \leq T}} Z_{i0s} \tag{A.9}$$

and constraint (A.4) gives us

$$Z_{i01} = \sum_{m \in \mathcal{R}_i} V_{im1} \tag{A.10}$$

so we have

$$p_i R_{i0} = \sum_{m \in \mathcal{R}_i} V_{im1} + \sum_{\substack{s:2 \leq s \leq \tau_i, \\ s \leq T}} Z_{i0s} \tag{A.11}$$

which is equal to equation (A.8).

Now we assume that equation (A.7) is true for t and prove that it is also true for $t+1$.

$$\begin{aligned}
\sum_{0 \leq s \leq t} p_i R_{is} &= \sum_{s:1 \leq s \leq t+1} \sum_{m \in \mathcal{R}_i} V_{ims} + \sum_{r:t-\tau_i+2 \leq r \leq t} \sum_{\substack{q:t+2 \leq q \leq r+\tau_i, \\ q \leq T}} Z_{irq} \\
&= \sum_{s:1 \leq s \leq t+1} \sum_{m \in \mathcal{R}_i} V_{ims} + \sum_{r:t-\tau_i+2 \leq r \leq t} Z_{i,r,t+2} \\
&+ \sum_{r:t-\tau_i+3 \leq r \leq t} \sum_{\substack{q:t+3 \leq q \leq r+\tau_i, \\ q \leq T}} Z_{irq} \\
&= \sum_{s:1 \leq s \leq t+1} \sum_{m \in \mathcal{R}_i} V_{ims} + \sum_{m \in \mathcal{R}_i} V_{i,m,t+2} - Z_{i,t+1,t+2} \\
&+ \sum_{r:t-\tau_i+3 \leq r \leq t} \sum_{\substack{q:t+3 \leq q \leq r+\tau_i, \\ q \leq T}} Z_{irq} \\
&= \sum_{s:1 \leq s \leq t+2} \sum_{m \in \mathcal{R}_i} V_{ims} - p_i R_{i,t+1} + \sum_{\substack{s:t+3 \leq s \leq t+\tau_i+1, \\ s \leq T}} Z_{i,t+1,s} \\
&+ \sum_{r:t-\tau_i+3 \leq r \leq t} \sum_{\substack{q:t+3 \leq q \leq r+\tau_i, \\ q \leq T}} Z_{irq} \\
&= \sum_{s:1 \leq s \leq t+2} \sum_{m \in \mathcal{R}_i} V_{ims} - p_i R_{i,t+1} \\
&+ \sum_{r:t-\tau_i+3 \leq r \leq t+1} \sum_{\substack{q:t+3 \leq q \leq r+\tau_i, \\ q \leq T}} Z_{irq}
\end{aligned} \tag{A.12}$$

Thus, we see that equation (A.7) also holds for $t+1$

$$\sum_{0 \leq s \leq t+1} p_i R_{is} = \sum_{s:1 \leq s \leq t+2} \sum_{m \in \mathcal{R}_i} V_{ims} + \sum_{r:t-\tau_i+3 \leq r \leq t} \sum_{\substack{q:t+3 \leq q \leq r+\tau_i, \\ q \leq T}} Z_{irq} \tag{A.13}$$

The only thing left to show is that inequality (A.2) also holds. By substituting constraint (A.4) in (A.3), we get:

$$\begin{aligned}
p_i R_{it} &= \sum_{\substack{s:t+1 \leq s \leq t+\tau_i, \\ s \leq T}} Z_{its} = \sum_{\substack{s:t+1 \leq s \leq t+\tau_i, \\ s \leq T}} \Big(\sum_{m \in \mathcal{R}_i} V_{imt} - \sum_{\substack{r:s-\tau_i \leq r < t, \\ s \leq T}} Z_{irs} - \sum_{\substack{r:t < r < s, \\ s \leq T}} Z_{irs} \Big) \\
&\leq \sum_{\substack{s:t+1 \leq s \leq t+\tau_i, \\ s \leq T}} \sum_{m \in \mathcal{R}_i} V_{imt}
\end{aligned} \tag{A.14}$$

From inequality (A.14) it is easy to see that also (A.2)holds.

□

Appendix B

The equivalence between physical and echelon inventory balance

In this section we show that the echelon inventory balance introduced in section 6.3.2 is equal to the physical inventory balance given in the model in section 2.4.

Theorem B.1 *The physical inventory balance*

$$I_{is} = I_{i,s-1} + R_{i,s-\tau_i} - D_{is} - G_{is} + B_{is} - B_{i,s-1}, \quad i = 1, \ldots, n, \; s = t, \ldots, t+T-1 \tag{B.1}$$

is equal to the echelon inventory balance

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + R_{i,s-\tau_i} - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} - D_{ist}, \quad i = 1, \ldots, n, \; s = t, \ldots, t+T-1 \tag{B.2}$$

Proof The physical inventory balance (B.1) $\Rightarrow$ the echelon inventory balance (B.2). We start with substituting the expression for the echelon inventory position

$$\tilde{I}_{is} = I_{is} + \sum_{j=1}^{n} \sum_{u=s-\tau_j+1}^{s} h_{ij} R_{ju} + \sum_{j=1}^{n} h_{ij} \tilde{I}_{js} - B_{is} \quad i = 1, \ldots, n, \; t = 0, \ldots, T-1 \tag{B.3}$$

into the physical inventory balance, thus

$$\begin{aligned} \tilde{I}_{is} &- \sum_{j=1}^{n} \sum_{u=s-\tau_j+1}^{s} h_{ij} R_{ju} - \sum_{j=1}^{n} h_{ij} \tilde{I}_{js} + B_{is} = \tilde{I}_{i,s-1} - \sum_{j=1}^{n} \sum_{u=s-\tau_j}^{s-1} h_{ij} R_{ju} \\ &- \sum_{j=1}^{n} h_{ij} \tilde{I}_{j,s-1} + B_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j=1}^{n} h_{ij} R_{js} + B_{is} - B_{i,s-1} \end{aligned} \tag{B.4}$$

By rearranging the terms, we get

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + \sum_{j=1}^{n} h_{ij}\Big(\sum_{u=s-\tau_j+1}^{s} R_{ju} - \sum_{u=s-\tau_j}^{s-1} R_{ju} - R_{js} + \tilde{I}_{js} - \tilde{I}_{j,s-1}\Big) - D_{is} + R_{i,s-\tau_i} \tag{B.5}$$

which is equal to

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + \sum_{j=1}^{n} h_{ij}(\tilde{I}_{js} - \tilde{I}_{j,s-1} - R_{i,s-\tau_i}) + R_{i,s-\tau_i} - D_{is} \tag{B.6}$$

If i is an end item, then $h_{ij} = 0$ for all $j = 1, \ldots, n$, and equality (B.6) becomes

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + R_{i,s-\tau_i} - D_{is} \tag{B.7}$$

this is the echelon inventory balance, since $\sum_{j \in E_i} \tilde{h}_{ij} D_{js} = 0$ for end items i.

So for end items we have proven that the physical inventory balance (B.1) $\Rightarrow$ the echelon inventory balance (B.2).

Now assume that for all parent items (B.1) $\Rightarrow$ (B.2) holds, thus

$$\forall j \in V_i : \qquad \tilde{I}_{js} = \tilde{I}_{j,s-1} + R_{j,s-\tau_i} - \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} - D_{js} \tag{B.8}$$

then equation (B.6) becomes

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j=1}^{n} h_{ij}\Big(\sum_{k \in E_j} \tilde{h}_{jk} D_{ks} + D_{js}\Big) \tag{B.9}$$

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j \in E_i} h_{ij}\Big(\sum_{k \in E_j} \tilde{h}_{jk} D_{ks} + D_{js}\Big) - \sum_{j \in E_i^c} h_{ij}\Big(\sum_{k \in E_j} \tilde{h}_{jk} D_{ks} + D_{js}\Big) \tag{B.10}$$

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j \in E_i} h_{ij} D_{js} - \sum_{j \in E_i^c} h_{ij} \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} \tag{B.11}$$

and by substituting the expression for the total demand of item i incorporated in the end items of $ech(i)$ at time t.

$$\sum_{j \in E_i} \tilde{h}_{ij} D_{is} = \sum_{j \in E_i} h_{ij} D_{js} + \sum_{j \in E_i^c} h_{ij} \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} \tag{B.12}$$

we finally get

$$\tilde{I}_{is} = \tilde{I}_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \tag{B.13}$$

So, if the assumption "(B.1) $\Rightarrow$ (B.2) for all parent items" holds, then we have proved that the physical inventory balance $\Rightarrow$ the echelon inventory balance.

We still have to prove that indeed (B.1) $\Rightarrow$ (B.2) holds for all parent items. Parent items are

1. end items, or

2. child items of other parent items

We know that (B.1) $\Rightarrow$ (B.2) is true for end items. For child items it is true if it is true for parent items. Since, in the end, parent items are end items. The theorem is also true for child items.

To proof the theorem, we also have to proof that echelon inventory balance (B.2) $\Rightarrow$ the physical inventory balance (B.1). We start by substituting the expression for the echelon inventory position (B.3) into the echelon inventory balance, then we get

$$\begin{aligned} I_{is} \quad &+ \quad \sum_{j=1}^{n} \sum_{u=s-\tau_j+1}^{s} h_{ij} R_{ju} + \sum_{j=1}^{n} h_{ij} \tilde{I}_{js} - B_{is} = I_{i,s-1} + \sum_{j=1}^{n} \sum_{u=s-\tau_j}^{s-1} h_{ij} R_{ju} \\ &+ \quad \sum_{j=1}^{n} h_{ij} \tilde{I}_{j,s-1} - B_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \end{aligned} \tag{B.14}$$

rearranging the terms gives

$$\begin{aligned} I_{is} \quad &= \quad I_{i,s-1} + R_{i,s-\tau_i} - D_{is} + B_{is} - B_{i,s-1} - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \\ &+ \quad \sum_{j=0}^{n} h_{ij} \big(\sum_{u=s-\tau_j}^{s-1} R_{ju} - \sum_{u=s-\tau_j+1}^{s} R_{ju} - \tilde{I}_{js} + \tilde{I}_{j,s-1} \big) \end{aligned} \tag{B.15}$$

$$\begin{aligned} I_{is} &= I_{i,s-1} + R_{i,s-\tau_i} - D_{is} + B_{is} - B_{i,s-1} - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \\ &+ \sum_{j=0}^{n} h_{ij}(R_{j,s-\tau_j} - R_{j,s} - \tilde{I}_{js} + \tilde{I}_{j,s-1}) \end{aligned} \tag{B.16}$$

$$\begin{aligned} I_{is} &= I_{i,s-1} + R_{i,s-\tau_i} - D_{is} + B_{is} - B_{i,s-1} - \sum_{j=1}^{n} h_{ij} R_{js} \\ &+ \sum_{j=0}^{n} h_{ij}(R_{j,s-\tau_j} - \tilde{I}_{js} + \tilde{I}_{j,s-1}) - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \end{aligned} \tag{B.17}$$

By substituting the echelon inventory balance into this expression, we get

$$\begin{aligned} I_{is} &= I_{i,s-1} + R_{i,s-\tau_i} - D_{is} + B_{is} - B_{i,s-1} - \sum_{j=1}^{n} h_{ij} R_{js} \\ &+ \sum_{j=0}^{n} h_{ij}(D_{is} + \sum_{k \in E_j} \tilde{h}_{jk} D_{ks}) - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \end{aligned} \tag{B.18}$$

$$\begin{aligned} I_{is} &= I_{i,s-1} + R_{i,s-\tau_i} - D_{is} + B_{is} - B_{i,s-1} - \sum_{j=1}^{n} h_{ij} R_{js} \\ &+ \sum_{j \in E_i} h_{ij} D_{is} + \sum_{j \in E_i^c} h_{ij} D_{is} \\ &+ \sum_{j \in E_i} h_{ij} \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} + \sum_{j \in E_i^c} h_{ij} \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} \\ &- \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \end{aligned} \tag{B.19}$$

Since $\sum_{j \in E_i^c} h_{ij} D_{is} = 0$ and $\sum_{j \in E_i} h_{ij} \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} = 0$, we get

$$\begin{aligned} I_{is} &= I_{i,s-1} + R_{i,s-\tau_i} - D_{is} + B_{is} - B_{i,s-1} - \sum_{j=1}^{n} h_{ij} R_{js} \\ &+ \sum_{j \in E_i} h_{ij} D_{is} + \sum_{j \in E_i^c} h_{ij} \sum_{k \in E_j} \tilde{h}_{jk} D_{ks} - \sum_{j \in E_i} \tilde{h}_{ij} D_{js} \end{aligned} \tag{B.20}$$

Finally by substituting (B.12), we get

$$I_{is} = I_{i,s-1} + R_{i,s-\tau_i} - D_{is} - \sum_{j=1}^{n} h_{ij} R_{js} + B_{is} - B_{i,s-1} \tag{B.21}$$

□

Appendix C

Strict allocation

Theorem C.1 *In a pure divergent supply chain holds*

$$h_{ij}R_{j,s-\tau_j} = h_{ij}\hat{D}_{js} - h_{ij}\tilde{I}_{js} - q_{ij}S^-_{i,s-\tau_j} \quad \forall i \in prev(j)$$
$$j = 1,\ldots,n,\ s = t,\ldots,t+T-1. \tag{C.1}$$

if

$$I_{is} = I_{i,s-1} + R_{i,s-\tau_i} - D_{is} - G_{is} + B_{is} - B_{i,s-1}, \quad i = 1,\ldots,n,\ s = t,\ldots,t+T-1 \tag{C.2}$$

$$G_{is} = \sum_{j=1}^{n} h_{ij}R_{js}, \quad i = 1,\ldots,n,\ s = t,\ldots,s+T-1 \tag{C.3}$$

$$B_{is} - B_{i,s-1} \le D_{is}, \quad i = 1,\ldots,n,\ s = t,\ldots,t+T-1 \tag{C.4}$$

$$S^-_{is} - S^+_{is} = \sum_{j=1}^{n} h_{ij}\hat{D}_{j,s+\tau_j} - \sum_{j=1}^{n} h_{ij}\tilde{I}_{j,s+\tau_j} - I_{i,s-1} - R_{i,s-\tau_i},$$
$$i = 1,\ldots,n,\ i \notin E,\ s = t,\ldots,t+T-1 \tag{C.5}$$

$$h_{ij}R_{j,s-\tau_j} \ge h_{ij}\hat{D}_{js} - h_{ij}\tilde{I}_{js} - q_{ij}S^-_{i,s-\tau_j} \quad \forall i \in prev(j)$$
$$j = 1,\ldots,n,\ s = t,\ldots,t+T-1. \tag{C.6}$$

$$\sum_{j=1}^{n} q_{ij} = 1, \quad i = 1,\ldots,n \tag{C.7}$$

$$R_{is}, B_{is}, I_{is} \ge 0, \quad i = 1,\ldots,n,\ s = t,\ldots,t+T-1 \tag{C.8}$$

$$S^+_{is} = 0, \quad i = 1,\ldots,n,\ i \notin E,\ s = t,\ldots,t+T-1 \tag{C.9}$$

Proof We proof this theorem by showing that

$$h_{ij}R_{j,s-\tau_j} > h_{ij}\hat{D}_{js} - h_{ij}\tilde{I}_{js} - q_{ij}S^-_{i,s-\tau_j} \quad \forall i \in prev(j) \tag{C.10}$$

leads to a contradiction. Since $S^+_{is} = 0$, we have

$$S^-_{is} = \sum_{j=1}^{n} h_{ij}\hat{D}_{j,s+\tau_j} - \sum_{j=1}^{n} h_{ij}\tilde{I}_{j,s+\tau_j} - I_{i,s-1} - R_{i,s-\tau_i} \tag{C.11}$$

Furthermore,

$$h_{ij}R_{j,s-\tau_j} > h_{ij}\hat{D}_{js} - h_{ij}\tilde{I}_{js} - q_{ij}S^-_{i,s-\tau_j} \quad \forall i \in prev(j) \tag{C.12}$$

by summing over all parent items, we get

$$\sum_{j=1}^{n} h_{ij}R_{j,s-\tau_j} > \sum_{j=1}^{n} h_{ij}\hat{D}_{js} - \sum_{j=1}^{n} h_{ij}\tilde{I}_{js} - \sum_{j=1}^{n} q_{ij}S^-_{i,s-\tau_j} \quad \forall i \in prev(j) \tag{C.13}$$

Since $\sum_{j=1}^{n} q_{ij} = 1$, the inequality becomes

$$\sum_{j=1}^{n} h_{ij}R_{j,s-\tau_j} > \sum_{j=1}^{n} h_{ij}\hat{D}_{js} - \sum_{j=1}^{n} h_{ij}\tilde{I}_{js} - S^-_{i,s-\tau_j} \quad \forall i \in prev(j) \tag{C.14}$$

and thus

$$\sum_{j=1}^{n} h_{ij}R_{js} > \sum_{j=1}^{n} h_{ij}\hat{D}_{j,s+\tau_j} - \sum_{j=1}^{n} h_{ij}\tilde{I}_{j,s+\tau_j} - S^-_{is} \quad \forall i \in prev(j) \tag{C.15}$$

and by substituting equality (C.11) in the above inequality, we have that

$$\sum_{j=1}^{n} h_{ij}R_{js} > I_{i,s-1} + R_{i,s-\tau_i} \quad \forall i \in prev(j) \tag{C.16}$$

Notice that $\sum_{j=1}^{n} h_{ij}R_{jt} = G_{it}$. Now we can substitute (C.16) in the inventory balance (C.17), and we get

$$\begin{aligned} I_{is} &< I_{i,s-1} + R_{i,s-\tau_i} - D_{is} - I_{i,s-1} - R_{i,s-\tau_i} + B_{is} - B_{i,s-1} \\ &= -D_{is} + B_{is} - B_{i,s-1} \\ &\qquad i = 1,\ldots,n,\ i \notin E,\ s = t,\ldots,t+T-1 \end{aligned} \tag{C.17}$$

Since constraint (C.4) holds, we get $I_{is} < 0$ which is in contradiction with the under bound (C.8) for the inventory I_{is}.

□

Summary

Supply Chain Operations Planning (SCOP) involves the determination of an extensive production plan for a network of manufacturing and distribution entities within and across organizations. The production plan consist of order release decisions that allocate materials and resources in order to transform these materials into (intermediate) products. We use the word item for both materials, intermediate products, and end-products. Furthermore, we consider arbitrary supply chains, i.e. the products produced by the supply chain as a whole and sold to customers consist of multiple items, where each item may in turn consists of multiple items and where each item may be used in multiple items as well. The aim of SCOP is not only to obtain a feasible production plan, but the plan must be determined such that pre-specified customer service levels are met while minimizing cost.

To obtain optimal production plans we use a linear programming (LP) model. The reason we use an LP model is twofold. First, LP models can easily be incorporated in existing Advanced Planning Systems (APS). Second, while the multi-echelon inventory concept can only be used for uncapacitated supply chains and some special cases of capacitated supply chains, capacity constraints but also other restrictions can easily added to LP models. In former mathematical programming (MP) models, the needed capacity was allocated at a fixed time offset. This time offset was indicated by fixed or minimum lead times. By the introduction of planned lead times with multi-period capacity allocation, an additional degree of freedom is created, namely the timing of capacity allocation during the planned lead time. When using the LP model in a rolling schedule context, timing the capacity allocation properly can reduce the inventory cost.

Although the number of studies on MP models for solving the SCOP or related problems are carried out by various researchers is enormous, only a few of these studies use a rolling schedule. Production plans are only calculated for a fixed time horizon based on the forecast of customers demand. However, since customer demand is uncertain, we emphasize the use of a rolling schedule. This implies that a production plan, based on sales forecasts, is calculated for a time interval $(0, T]$, but only executed for the first period. At time 1, the actual demand of the first period is known, and the inventory status of the consumer products are adjusted according the actual demand. For time interval $(1, T+1]$, a new production plan is calculated.

In this thesis, we studied the proposed LP strategy with planned lead times in a

rolling schedule setting whereby we focused on the following topics:

- timing of production within the planned lead time,
- factors influencing the optimal planned lead time,
- early availability of produced items, i.e. availability of items before the end of their planned lead time, and
- balanced material allocation.

In the first three studies we explore the possibilities of using planned lead times. In the first study, timing of production, we compare the situation whereby released items are produced as soon as there is available capacity with the situation whereby released items are produced as late as possible within the planned lead time. If items are produced as soon as possible, there is more capacity left for future production. Since we work with uncertain customer demand whereby demand may be larger than expected, this capacity might be very useful. A drawback of production as soon as possible are the additional work-in-process cost. The results of simulation studies show that if the utilization rates of resources and/or the variation in demand are high, producing early is better. However this is only the case if the added value between the concerned item and the end item is high.

The second study deals with factors influencing the optimal planned lead time. From queuing theory it is already known that the variance in demand and the utilization rate of the resources determine the waiting time. More variation and/or higher utilization rates give longer waiting times. Since lead times consist for a large part of waiting time, these two factors most probably also influence the length of the optimal planned lead time. For a set of representative supply chain structures we showed that this was indeed the case. With longer planned lead times, the flexibility in capacity allocation is higher. Additional flexibility gives lower safety stocks, but longer planned lead times also means more work-in-process. Hence, an important third factor which influence the optimal planned lead time is the holding costs structure.

When using planned lead times, early produced items have to wait the remainder of their planned lead time. This seems contradictory, especially if these items are necessary to avoid or reduce backorders. Therefore we adapt the standard LP model in two ways. In the first model, items are made available for succeeding production steps directly after they are produced. And in the second model, produced items are only made available for succeeding production steps if they are needed to avoid or reduce backorders. Experiments showed that the first model does not improve the performance of the standard LP strategy. The advantages of planned lead times longer than one period are nullified by early availability of produced items. The second model indeed improves the performance of the standard LP strategy, but only when the planned lead times are optimal or longer.

Comparing the introduced LP strategy with a so-called synchronized base stock policy under the assumption of infinite capacity, it turned out that the LP strategy is outperformed by the base stock policy. In order to obtain a better performance, we added linear allocation rules to the LP model. With these allocation rules shortages of child items are divided among the parent items using a predefined allocation fraction. A second way of balanced allocation of child items is obtained by replacing the linear objective function by a quadratic one. The results of a well-chosen set of experiments showed that although the synchronized base stock policy also outperforms the adjusted LP strategies, the difference in performance is small. Hence, the adjusted LP strategies are good alternatives for large, capacitated supply chain structures which cannot be solved by synchronized base stock policies. Comparing the model with linear allocation rules with the model with quadratic objective function, the preference is given to the latter model. This model does not only give the lowest inventory costs, it also has the shortest computation time. Furthermore, this model can easily be implemented and solved by existing software.

Summarizing the main results of this thesis, we conclude that deterministic LP models can be used to solve the SCOP problem with stochastic demand by using the LP model in a rolling schedule concept. By using optimal planned lead times with multi-period capacity allocation, early production during the planned lead times, and early availability of needed produced items before the end of the planned lead time, we can decrease the inventory costs. The costs can also be reduced by using allocation strategies to allocate shortages among parent items proportionally. Especially the results for the model with quadratic objective function are promising.

Samenvatting

In Supply Chain Operations Planning (SCOP) gaat het om het bepalen van een uitgebreide productieplanning voor een netwerk van productie- en distributiebedrijven, zowel binnen als tussen organisaties. Een goede productieplanning zorgt ervoor dat producten en componenten aanwezig zijn op het juiste moment, op de juiste plaats, en op efficiënte wijze. We gebruiken het woord component voor zowel grondstoffen, halffabrikaten, als eindproducten. We beschouwen supply chains waarbij de producten die verkocht worden aan de consument uit meerdere componenten bestaan. Elk component kan op zijn beurt ook uit meerdere componenten bestaan, en elk component kan worden verwerkt in meerdere andere componenten. SCOP heeft niet alleen als doel een uitvoerbare productieplanning te genereren, maar de planning moet zodanig zijn dat de klantenservice een van tevoren vastgesteld niveau bereikt terwijl de kosten zo laag mogelijk gehouden worden.

Voor het verkrijgen van optimale productieplanningen gebruiken we een lineair programmering (LP) model. De reden dat we voor een LP model kiezen is tweeslachtig. Ten eerste kunnen LP modellen eenvoudig opgenomen worden in de bestaande 'Advanced Planning Systems' (APS). En ten tweede kan in LP modellen op eenvoudige wijze capaciteit en andere beperkingen worden toegevoegd. Dit is niet het geval in multi-echelon voorraad modellen die dan ook alleen voor supply chains met oneindige capaciteit en een aantal specifieke gecapaciteerde supply chains, gebruikt kunnen worden. In eerdere mathematische programmerings (MP) modellen werd de benodigde capaciteit toegekend gedurende een vast tijdsinterval, en dit tijdsinterval wordt bepaald door de vaste of minimale doorlooptijd. Met de introductie van geplande doorlooptijden, waarbij capaciteitgebruik gedurende meerdere periodes mogelijk is, wordt een extra vrijheidsgraad gecreëerd, namelijk de beslissing wanneer tijdens de geplande doorlooptijd de benodigde capaciteit wordt toegekend aan de vrijgegeven orders. Wanneer LP wordt gebruikt in een rollende horizon context, kunnen de voorraad kosten gereduceerd worden door op het juiste moment capaciteit toe te kennen aan een vrijgegeven order.

Hoewel het aantal onderzoeken naar MP modellen voor het oplossen van het SCOP en aanverwante problemen groot is, is het aantal onderzoeken waarbij een rollende horizon gebruikt wordt klein. In de meeste onderzoeken wordt de productieplanning, gebaseerd op een voorspelling van vraag naar artikelen, alleen berekend voor een vaste tijdshorizon. Echter, omdat de vraag van klanten onzeker is, leggen wij de nadruk op het gebruik van een rollende horizon. Dit houdt in dat er een productieplanning, gebaseerd op vraagvoorspelling, wordt berekend voor de tijdsinterval $(0, T]$, maar alleen de beslissingen voor de eerste periode worden daadwerkelijk uitgevoerd. Na de eerste periode is de werkelijke vraag bekend en worden de voorraadniveaus aangepast aan deze vraag. Daarna wordt er voor de tijdsinterval $(1, T+1]$ een nieuwe productieplanning berekend.

In dit proefschrift onderzochten we het voorgestelde LP model voorzien van geplande doorlooptijden, waarbij we ons concentreerden op de volgende onderwerpen:

- moment van produceren tijdens de geplande doorlooptijden,
- elementen die de optimale geplande doorlooptijden beïnvloeden,
- vroegtijdig beschikbaar maken van geproduceerde producten, dat wil zeggen het beschikbaar maken van geproduceerde producten voor het eind van de geplande doorlooptijden, en
- evenwichtige materiaal toekenning.

In het eerste onderzoek, het moment van produceren, vergelijken we de situatie waarbij vrijgegeven orders worden geproduceerd zodra er capaciteit is met de situatie waarbij vrijgegeven orders zo laat mogelijk binnen de geplande doorlooptijden geproduceerd worden. Als items direct worden geproduceerd, blijft er meer capaciteit over voor toekomstige productie. Aangezien we met onzekere vraagprocessen werken, waarbij de werkelijke vraag groter kan zijn dan de verwachte vraag, kan deze bewaarde capaciteit erg nuttig zijn. Een nadeel van zo vroeg mogelijk produceren is de grotere kosten van onderhanden werk. De resultaten van de simulatie studie wijzen uit dat als de bezettingsgraad van machines en/of de variatie in de vraag hoog zijn, zo vroeg mogelijk produceren goedkoper is. Echter, dit is alleen het geval wanneer de toegevoegde waarde tussen het betreffende component en het eindproduct groot is.

In het tweede onderzoek bekijken we de elementen die de optimale geplande doorlooptijden beïnvloeden. Van wachttijdtheorie weten we dat de variatie van de vraag en de bezettingsgraad van de machines de wachttijd bepalen. Meer variatie en/of hogere bezettingsgraden resulteren in langere wachttijden. Aangezien doorlooptijden voor een groot gedeelte bestaan uit wachttijd, beïnvloeden deze twee elementen waarschijnlijk ook de optimale lengte van de geplande doorlooptijden. Voor een aantal representatieve supply chain structuren laten we zien dat dit inderdaad het geval is. Langere geplande doorlooptijden zorgen voor meer flexibiliteit in het toekennen van capaciteit aan vrijgegeven orders. Deze extra flexibiliteit zorgt voor lagere veiligheidsvoorraden, dus ook de kosten voor het aanhouden van veiligheidsvoorraden gaan omlaag. Echter, langere geplande doorlooptijden zorgen voor meer onderhanden werk met bijbehorende kosten. Vandaar dat de kostenstructuur van de supply chain een derde belangrijk element is die de optimale lengte van de geplande doorlooptijden beïnvloedt.

Wanneer we geplande doorlooptijden gebruiken, moeten geproduceerde producten wachten tot het eind van hun geplande doorlooptijd. Dit lijkt tegenstrijdig, vooral als deze producten nodig zijn om tekorten te vermijden of te verminderen. Daarom passen we de standaard LP strategie op twee manieren aan. In het eerste model worden producten beschikbaar gemaakt voor verdere productie zodra ze geproduceerd zijn. En in het tweede model worden geproduceerde items alleen maar beschikbaar gemaakt voor het eind van de geplande doorlooptijd als ze nodig zijn om nabestellingen te voorkomen of te verminderen. Experimenten tonen aan dat het eerste model niet het gewenste resultaat, namelijk lagere kosten, geeft. De voordelen van geplande doorlooptijden worden tenietgedaan door het direct beschikbaar zijn van geproduceerde producten. Het tweede model verbetert inderdaad de prestatie van de standaard LP strategie, maar alleen als de geplande doorlooptijd de optimale lengte of langer heeft.

Vergelijken we de geïntroduceerde LP strategie met het zogenoemde 'synchronized base stock' concept voor kleine supply chain structuren zonder capaciteit restricties, dan blijkt dat de LP strategie slechter presteert dan het 'synchronized base stock' concept. Om de prestatie van de LP strategie te verbeteren, voegen we lineaire allocatie regels toe. Met deze allocatie regels wordt het tekort aan deelcomponenten verdeeld over alle opvolgende componenten waarbij gebruik gemaakt wordt van vooraf gedefinieerde allocatie fracties. Een tweede manier om deelcomponenten evenwichtig toe te kennen aan alle opvolgende componenten, wordt bereikt door de lineaire doelfunctie te vervangen door een kwadratische doelfunctie. Experimenten tonen aan dat het 'synchronized base stock' concept nog steeds beter presteert, hoewel de verschillen klein zijn geworden. We kunnen dan ook concluderen dat de verbeterde LP strategieën goede alternatieven zijn voor grote, supply chain structuren met capaciteit restricties die niet opgelost kunnen worden door het 'synchronized base stock' concept. Vergelijken we het model met lineaire allocatie regels met het model met kwadratische doelfunctie, dan moeten we concluderen dat laatstgenoemde model de voorkeur heeft. Niet alleen geeft het de laagste kosten, het heeft ook de kortste rekentijd en is op eenvoudige wijze te implementeren in en op te lossen door bestaande programmatuur.

Vatten we nog even de belangrijkste bevindingen uit dit onderzoek samen, dan concluderen we dat deterministische LP modellen gebruikt kunnen worden voor het oplossen van het SCOP probleem met stochastische vraag door de modellen te gebruiken in een rollende horizon. Door gebruik te maken van optimale doorlooptijden waarbij capaciteitgebruik gedurende meerdere periode mogelijk is, kunnen we met vroeg produceren gedurende de geplande doorlooptijd, en met het beschikbaar stellen van geproduceerde componenten die nodig zijn voor het eind van de geplande doorlooptijd, de voorraadkosten reduceren. Een andere manier om de kosten te verkleinen is het gebruik van allocatie strategieën waarbij tekorten verdeeld worden over verschillende producten. Vooral het model met een kwadratische in plaats van een lineaire doelfunctie is veelbelovend.

Curriculumn Vitae

Judith Maria Spitter was born on November 17th 1975, in Vlaardingen, the Netherlands. In 1993 she received her HAVO diploma from the Angelus Merula school in Spijkenisse, and in 1995 she received her VWO diploma from the Penta College also in Spijkenisse. In September 1995 she started her study in Technical Mathematics at Delft University of Technology. She received her M.Sc. degree in 2000 after a research on a model for determining optimum brewery layouts at Heineken Technical Services. In January 2001 she started as a Ph.D. student at the Technische Universiteit Eindhoven under supervision of prof. dr. A.G. de Kok and dr. ir. N.P. Dellaert. The research consisted of the development of more effective linear programming models for solving the Supply Chain Operations planning problem under demand uncertainty. This thesis concludes the research. Apart from the thesis, the Ph.D. research has resulted in a number of published papers that have been presented at various international conferences.