Numerical design of vehicles with optimal straight line stability on undulating road surfaces

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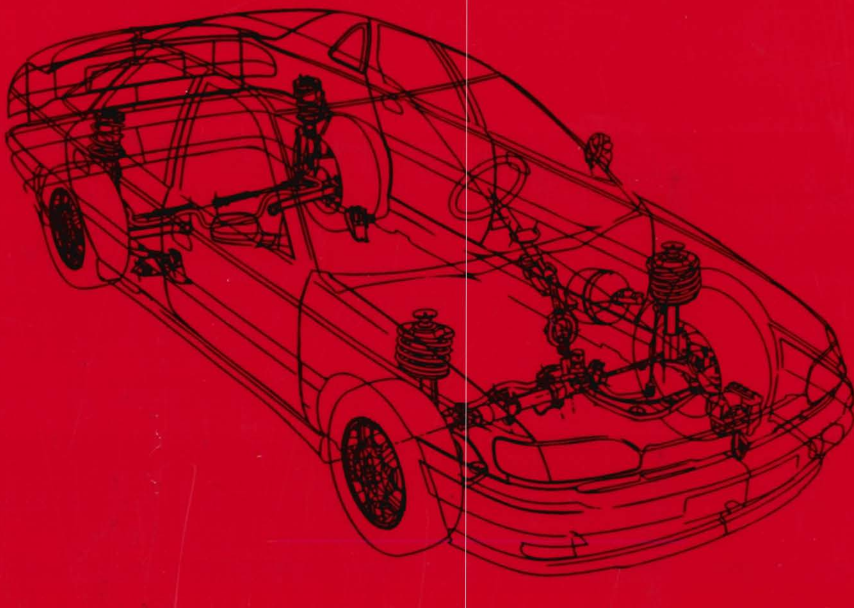

Gijsbert Roos

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PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de Rector Magnificus, prof.dr. J.H. van Lint, voor een commissie aangewezen door het College van Dekanen in het openbaar te verdedigen op donderdag 9 november 1995 om 16.00 uur

door

Gijsbert Roos

geboren te Leiden

Dit proefschrift is goedgekeurd door de promotoren:

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Contents

Summary

Straight line stability on undulating road surfaces is an important vehicle characteristic because it influences driver satisfaction and driver comfort. Nevertheless, in current industrial practice, straight line stability is optimized by tests with prototypes. It can hardly be taken into account in the early stages of the vehicle design process when many important parameters are fixed. Therefore, a numerical design method has been developed which can be used from the concept phase of vehicle development. This method is based on an estimation algorithm for the subjective judgment of the vehicle's straight line stability qualities.

For the development of the design method, a combined physical and statistical approach has been used. First, vehicle straight line stability on undulating road surfaces has been analyzed in order to find the elementary qualities on which this vehicle characteristic is based. Second, the models for the simulation of the elementary straight line stability qualities have been developed. Finally, the estimation algorithm has been derived from test and simulation results through regression.

The elementary straight line stability qualities have been derived by an analysis of the driver-vehicle closed loop system's behavior on undulating road surfaces. This has yielded one closed loop elementary straight line stability quality, viz. the sensitivity of the driver-vehicle system to road undulations, and three open loop qualities. The open loop qualities are the vehicle's sensitivity to road undulations, its controllability, and its steering feel.

The vehicle model that has been developed yields accurate simulation results of the responses to the two dominant inputs, the steering wheel angle input and the input from the road undulations. This has been validated by several experiments, including tests on undulating straight roads. Nevertheless, the steering system's specific behavior around zero cannot be taken into account in the numerical design for optimal straight line stability. Therefore, a completely numerical tuning of the steering system is not possible. However, all other important vehicle components have been fully integrated into the design method.

The estimation of the subjective judgment of a vehicle's straight line stability qualities is based on the results of two open loop simulations. One of the simulations is used for the assessment of the vehicle's road response. The other simulation yields information with respect to the steering response on even roads. Comparison with test results has shown that the numerical estimations are rather accurate.

Samenvatting

De rechtuitstabiliteit van een auto op golvende wegdekken heeft een grote invloed op het rijplezier en comfort van de bestuurder. Toch wordt de rechtuitstabiliteit in de huidige industriële praktijk met behulp van prototypes geoptimaliseerd. Zij kan nauwelijks worden meegewogen wanneer de belangrijkste ontwerpvariabelen van een nieuwe auto vroegtijdig in het ontwerpproces worden vastgelegd. Daarom is een numerieke ontwerpmethode ontwikkeld die vanaf de conceptuele fase van het voertuigontwerp kan worden gebruikt. Deze methode heeft als basis een schattingsalgoritme voor de subjectieve beoordeling van de rechtuitstabiliteit op golvende wegdekken.

De ontwikkeling van de ontwerpmethode is in drie fasen uitgevoerd. Eerst zijn de basiseigenschappen waarop rechtuitstabiliteit op golvende wegdekken is gebaseerd analytisch afgeleid. Daarna zijn modellen ontwikkeld voor de simulatie van deze basiseigenschappen. Tenslotte is het schattingsalgoritme met behulp van regressie afgeleid uit test- en simulatieresultaten.

Voor het verkrijgen van inzicht in het fenomeen rechtuitstabiliteit moet het gesloten systeem van auto en bestuurder worden geanalyseerd. Analyse van dit systeem heeft een aantal basiseigenschappen opgeleverd. De gecombineerde basiseigenschap van auto en bestuurder is de gevoeligheid van het gesloten systeem voor wegdekgolvingen. Andere basiseigenschappen zijn de controleerbaarheid, het stuurgevoel en de gevoeligheid voor wegdekgolvingen van de auto.

Het ontwikkelde voertuigmodel geeft nauwkeurige simulatieresultaten van de responsies op de twee belangrijkste inputs, de stuurwielhoek-input en de input vanuit het wegdek. Dit is met verschillende experimenten, waaronder tests op golvende wegdekken, gevalideerd. Echter, met een van de waargenomen effecten, namelijk het specifieke gedrag van het stuursysteem rond de rechtuitstand, kan tijdens het ontwerpen op optimale rechtuitstabiliteit geen rekening worden gehouden. Daarom kan de afstemming van het stuursysteem niet geheel numeriek worden uitgevoerd. Alle andere belangrijke voertuigcomponenten zijn wel volledig in de ontwerpmethode geïntegreerd.

De schatting van de subjectieve beoordeling van de rechtuitstabiliteit is gebaseerd op de resultaten van twee simulaties. Een van de simulaties geeft een beeld van het voertuiggedrag op golvende wegdekken. De andere simulatie betreft de stuurrespons op een vlak wegdek. Vergelijking met testresultaten heeft aangetoond dat de nauwkeurigheid van de schattingen vrij goed is.

Résumé

La tenue de cap sur chaussée ondulante est un élément important de la tenue de route parce qu'elle influence la satisfaction et le confort du conducteur. Dans la pratique industrielle actuelle, la tenue de cap est optimisée à l'aide de prototypes. Elle ne peut guère être prise en compte dans la conception de base du véhicule. Pour ces raisons, une méthode de conception a été mise au point qui permet d'estimer le jugement subjectif de la tenue de cap et qui peut être utilisée dès les premiers stades de l'étude d'un nouveau véhicule.

La méthode de conception a été mise au point en trois étapes. D'abord, les caractéristiques de base sur lesquelles la tenue de cap sur chaussée ondulante est fondée ont été cherchées. Ensuite, des modèles ont été développés pour la simulation des caractéristiques de bases. Finalement, l'algorithme d'estimation a été déduit de résultats d'essais et de simulations par régression.

Le système à analyser pour mieux comprendre la tenue de cap sur chaussée ondulante est la boucle fermée véhicule-conducteur. L'analyse de ce système a donné une caractéristique de base de la boucle fermée et plusieurs caractéristiques de base du véhicule en boucle ouverte. La caractéristique boucle fermée est la sensibilité du système véhicule-conducteur aux ondulations de la chaussée. Les autres caractéristiques de base sont la sensibilité aux ondulations de la chaussée, la facilité de correction et l'agrément de direction du véhicule.

Les entrées les plus importantes du véhicule sont l'angle volant et la sollicitation de la chaussée. Des validations, entre autres sur route ondulante, ont montré que le modèle de véhicule qui a été développé est capable de simuler les réponses à ces entrées avec une assez bonne précision. Cependant, un des phénomènes observés, le comportement spécifique du système de direction autour de zéro, ne peut pas être pris en compte dans les simulations de la tenue de cap. Par conséquent, une mise au point entièrement numérique du système de direction n'est pas possible.

L'estimation du jugement subjectif de la tenue de cap est fondée sur les résultats de deux simulations. La première simulation est une simulation de la réponse du véhicule aux ondulations de la chaussée. L'autre simulation permet l'évaluation de la réponse au volant du véhicule. La corrélation entre les estimations du jugement subjectif et les résultats d'essais est assez bonne.

Chapter 1

General Introduction

1.1 Vehicle straight line stability on undulating road surfaces

Roadholding of a vehicle is not only important on winding roads or for the performance of swerves in emergency situations; it is also relevant on undulating straight roads, especially if driving speed is relatively high (e.g. above 100 km/h). Under these circumstances, the roadholding is often referred to by the term "straight line stability". This terminology is adopted for this thesis. Despite the fact that the actual stability of modern cars is hardly ever endangered on straight roads without sudden traffic interactions, the straight line stability qualities of vehicles vary widely between car models and car makes.

The main contrast between a vehicle having a good and a vehicle having a bad straight line stability on undulating road surfaces is probably a difference in the psychological pressure or in the physical stress imposed upon the driver. The vehicle having a bad stability requires more driver attention if it is driven on an undulating road. Through the nature of its input, vehicle straight line stability on undulating road surfaces is a dynamic vehicle characteristic. Furthermore, the importance of driver-vehicle interaction is evident. Hence, the driver-vehicle closed loop system must be included in the analysis of vehicle straight line stability on undulating road surfaces.

For car manufacturers, vehicle straight line stability on undulating road surfaces is an important vehicle characteristic because it has an influence on driver satisfaction, driver comfort, and even on road safety. Moreover, every road can be considered to undulate to a certain extent. Vehicle straight line stability on undulating road surfaces can be directly related to road safety in case the driver temporarily ignores his control tasks when the vehicle comes across a major obstacle or a critical series of obstacles. It has an indirect relationship to road safety through driver fatigue.

Until now, vehicle straight line stability on undulating road surfaces could only be tested and optimized with the aid of prototypes on actual undulating roads. Vehicle straight line stability could hardly be taken into account in the early stages of vehicle development when the basic design parameters are fixed. It is clear that this is not an ideal situation and that simulation tools and knowledge of the relevant phenomena are desired.

Studies of vehicle straight line stability on undulating road surfaces have only rarely been reported in literature. Moreover, most of the reported research studies were purely experimental. For instance, Tanaka [46] proposed the "negative steering workload", a standard based on the couple and angular velocity at the steering wheel, as test criterion for vehicle straight line stability at high speeds. Another example of a search for test standards is the research reported by Ehlich in [9]. Deppermann [8], on the other hand, has reported a rather extensive experimental and theoretical study. First, he deduced measurement criteria for straight line stability from subjective tests and measurements on highways. Later, he used numerical simulations to determine the influence of vehicle parameter variations on these measurement criteria.

A driving quality which is related to straight line stability is the so-called "on center feeling", the vehicle response to small steering wheel angles added to a straight line motion. On center feeling probably plays an important part in the driver-vehicle interaction. Driving qualities such as stability against crosswind perturbations and pulling, which have been discussed rather extensively in literature, differ from straight line stability on undulating road surfaces. Stability against crosswind perturbations has a different input as straight line stability, whereas pulling, which may have the same input as straight line stability, is a stationary instead of a dynamic vehicle characteristic. In order to facilitate the reading, the specification "on undulating road surfaces" will often be omitted in the remainder of this thesis.

1.2 Modelling of the driver-vehicle system

The study and simulation of vehicle straight line stability on undulating road surfaces requires, in principle, an accurate model of the driver-vehicle closed loop system. This is schematically shown in Figure 1.1. The quality of the closed loop model depends on the accuracy, under the relevant circumstances, of both the driver model and the vehicle model. The vehicle model should yield accurate responses to the road and steering wheel inputs, whereas the driver model should represent a realistic driver reaction to the directional behavior of the vehicle.

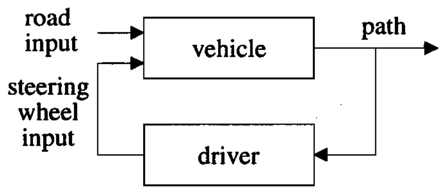


Figure 1.1 Driver-vehicle closed loop system on an undulating road

1.2.1 Vehicle modelling

Two vehicle elements are particularly important for straight line stability on undulating road surfaces, viz. the steering system and the tire. The steering system, which contains everything between the steering wheel and the front wheels, is important because it plays an important part in the driver-vehicle interaction. For instance, it transfers the input from the driver to the front wheels. The other important element is the tire which is always essential if roadholding is concerned.

In literature, several studies of steering systems and of their characteristics have been reported but most of these concern steering wheel angles that are larger than the angles which are normally employed while keeping the straight line. Nevertheless, they can be used as starting point for the development of a steering system model for the simulation of straight line stability. Shimomura [43] studied the influence of several vehicle parameters on the relationship between the steering wheel angle and the steering wheel couple. The model he used has eight degrees of freedom and contains a detailed steering system model with Coulomb frictions, springs, and inertia. It yields simulation results that, under the employed harmonic excitation with a steering wheel angle amplitude of 14 degrees, correspond rather well with measurement results. Ashley [3] measured a dynamic amplification and a phase lead of the front wheel steering angles with respect to the steering wheel angle. These phenomena were reproduced by Segel [42] with the aid of a linear vehicle model containing a dynamic steering system model.

An accurate description of tire behavior is very important for roadholding analyses. In addition, tire modelling is a rather difficult subject because the tire is a complicated vehicle component. As a result, several tire models have been reported in literature (see for instance the review on tire modelling that was presented by Pacejka in [31]). A combination of elements from these models might yield an accurate model for the study of vehicle straight line stability. Tire models for roadholding simulations can be divided into two classes: physical and empirical models. Models from both classes have been used for the three applications that are relevant for the simulation of vehicle straight line stability, viz. for the simulation of vehicle maneuvering on even roads, for the examination of the influence of road unevenness on vehicle cornering, and for the study of front wheel vibrations.

The models of Sakai [39] and Gim [12, 13] are examples of physical tire models for the simulation of vehicle maneuvering on even roads. An example of an empirical model for the same purpose is the "Magic Formula" model which has been presented by Bakker [4, 5]. In this model and in many other tire models for the simulation of vehicle maneuvering, the tire's dynamic behavior is described by a first order filter with a relaxation length as time constant. However, second order filters are also used. Nast [26] found a rather good correlation between measurements of a tire's dynamic response to steering angle inputs and a second order filter model. Heydinger [14] also used a second order filter in his tire model for vehicle handling simulation.

Studies of the influence of road unevenness, through wheel load variations, on tire characteristics such as the effective cornering stiffness also contain interesting elements for the analysis of vehicle straight line stability on undulating road surfaces. Laermann [15] and Mühlmeier [24] examined this phenomenon with the aid of rather detailed physical tire models that include belt and contact patch modelling. Takahashi [45] and Pacejka [32] obtained a good correspondence with measurement results with the aid of a much simpler semi-empirical model.

Finally, tire models for the investigation of front wheel vibrations are relevant for the simulation of vehicle straight line stability. These models are different from the models for the simulation of vehicle cornering because the front wheel vibrations occur at higher frequencies. Pacejka [28, 29, 30] and Strackerjan [44] have developed tire models that accurately reproduce tire responses to small steering wheel angle inputs up to frequencies of about 10 to 15 Hz. Their models contain a rigid tire belt with inertia while the tire width is also taken into account.

The simulation of vehicle straight line stability does not require a specific description of the vehicle's suspension. This description can be carried out by conventional methods. Two different approaches may be used. In the first approach, the vehicle and its suspension are modelled by a detailed multi-body model which includes descriptions of the links and joints of the suspension. Examples of applications of multi-body models for roadholding simulations were reported by Ross-Martin [38] and Venhovens [48]. In the second approach, a lumped parameter model of the vehicle is used. This kind of model includes descriptions of only those vehicle characteristics that are important under the relevant circumstances. Examples of this approach are given by Landreau [16] and Shimomura [43].

1.2.2 Driver modelling

Studies of driver behavior and of driver-vehicle interaction have rather often been reported in literature because these subjects are important for the optimization of vehicle dynamics. They are not only important for the optimization of vehicle straight line stability but also for the optimization of roadholding in general, for instance when active systems like four-wheel steering are employed. In addition, the modelling of driver behavior is a difficult subject. Each driver has his own particular behavior while a single driver behaves differently in different cars. Therefore, driver behavior has only partly been investigated to the present day.

In driver modelling, two different approaches can be distinguished. Some driver models control the vehicle through the steering wheel couple while other models control through the steering wheel angle. An example of a model from the first class was described by Nagai [25]. The so-called "cross-over model" of driver straight line keeping which has been developed by McRuer [20, 21, 22] and Allen [2] belongs to the second class. It is a linear model with parameters that depend on the behavior of the vehicle. Simulation results obtained with the model of McRuer and Allen were also reported by Garrot [11]. A simplified version of the model has been used by, for instance, Legouis [17, 18] and Tousi [47]. This model does not include driver adaptation.

1.2.3 Validation

Validation with the aid of experiments is an important stage in the development of new simulation models. In the case of a model for the study of vehicle straight line stability on undulating road surfaces, the most direct validation is a validation of the response of the driver-vehicle closed loop system to the relevant road input. However, this validation is inappropriate because one element of the closed loop system, the driver, is particularly difficult to model. In addition, the validation of the closed loop system yields little information on the location of possible modelling inaccuracies. Hence, separate validations of the vehicle and driver models must also be considered.

The control of a vehicle on a straight road requires only small steering wheel angles. Therefore, a steering response validation with a small steering wheel angle input is more appropriate for the study of vehicle straight line stability than a validation with medium or large steering wheel angle inputs. However, measurements of vehicle responses to small steering wheel angle inputs are rather difficult to carry out and have, to the knowledge of the author, not been reported in literature. Test results of vehicle responses to medium steering wheel angle inputs are easier to obtain. Such results were reported by, for instance, Ashley [3], Chrstos [7], Ehlich [10], and Shimomura [43].

The directional response of a vehicle to road undulations inputs is also difficult to measure. In relation to this response, only test results obtained with an artificial excitation on an even road surface have been reported (cf. Roos [36]). The applied artificial excitation was produced by a mass which was translated laterally within the car, and which, like road undulations, yielded a roll moment and a lateral force on the vehicle. Test results with respect to the modelling of driver straight line keeping have been reported by Allen [2] and McRuer [21].

1.3 Subjective judgment

Driver satisfaction is a legitimate objective for the optimization of vehicle straight line stability on undulating road surfaces. As a consequence, knowledge of the subjective judgment of vehicle straight line stability on undulating road surfaces is important for the development of a design method for this purpose.

Deppermann [8] studied the judgment of vehicle straight line stability rather extensively. He conducted experiments with several drivers and several vehicles and concluded that, on highways, a driver's judgment is influenced by the steering effort, by the on-center-feeling, and by the open loop sensitivity of the vehicle to road undulations.

1.4 Thesis

This thesis describes the development of a method for the numerical design of vehicles with optimal straight line stability on undulating road surfaces. It shows that the standard for vehicle straight line stability on undulating road surfaces, being the subjective judgment by a professional test driver, can be forecasted rather accurately from early stages of vehicle development. The possibility to take vehicle straight line stability on undulating road surfaces into account from early stages of vehicle development means a large improvement with respect to the current situation where vehicle straight line stability can only be tested and optimized with the aid of prototypes.

1.5 Method

The numerical design method which has been developed is based on an estimation algorithm for the subjective judgment of the vehicle's straight line stability qualities. With the aid of the estimation algorithm, design alternatives can be compared with each other and with existing car models. This basis has been selected because it can be used as an early safeguard against bad straight line stability qualities as well as for optimization purposes.

For the development of the design method, three different approaches could be chosen, viz. a statistical, physical, or combined physical and statistical approach. However, an entirely statistical development is very difficult because vehicle straight line stability on undulating road surfaces contains many interactions between the vehicle parameters. In addition, a statistical study yields only little knowledge of the occurring phenomena. A purely physical approach, on the other hand, also has its disadvantages. From empirical experience it is known that a vehicle's straight line stability qualities depend on several basic vehicle characteristics and the importance of these basic characteristics cannot be estimated physically.

Therefore, the combined physical and statistical approach has been chosen. First, vehicle straight line stability on undulating road surfaces has been analyzed in order to find the elementary qualities on which this driving characteristic is based. Second, the components of the closed loop system, the vehicle and the driver, have been modelled for the simulation of the elementary straight line stability qualities, and, finally, the estimation algorithm has been composed from the elementary qualities through regression with the aid of test results.

In the modelling of the components of the closed loop system, priority has been given to vehicle modelling. Vehicle behavior on undulating straight roads has hardly been explored before and the vehicle is the element to be optimized. The objective of the vehicle modelling was an accurate simulation of the elementary straight line stability qualities under the relevant circumstances, i.e. with only small steering angles, at relatively high driving speed, etc.

The experiments and simulations that are reported in this thesis have been carried out with three different vehicles. These vehicles are indicated by the letters A, B, and C (see Table 1.1). Since several configurations of the vehicles have been used, a number is added for the specific configuration. For instance, Vehicle A1 refers to a particular configuration of Vehicle A. The configurations differ as a result of modified suspensions, different tire pressures, etc. Moreover, next to experiments on flat road surfaces, measurements have been carried out on two undulating straight roads, viz. a rather strongly undulating road and a mildly undulating road.

Table 1.1 Vehicle specifications (all vehicles are front-wheel-driven)

Vehicle A	small size passenger car without power steer
Vehicle B	medium size passenger car without power steer
Vehicle C	full size passenger car with power steer

During straight line keeping on undulating roads, vehicle motions are small. The occurring phenomena can easily be perturbed by the effect of sidewind. Therefore, all experiments have been carried out without sidewind or with very weak sidewind.

The remainder of this thesis is structured as follows. Chapter 2 describes the deduction of the elementary qualities of vehicle straight line stability on undulating road surfaces. It contains investigations of the subjective judgment, of the driver-vehicle closed loop system, and of the excitation by the road undulations.

Chapter 3 contains a discussion of the preparatory measurements that have been carried out for the investigation of vehicle behavior during straight line keeping. These tests have been performed on flat and undulating road surfaces.

The mathematical vehicle model is described in Chapter 4. It contains dedicated sub-models of the steering system and the tire which are described in the Sections 4.5 and 4.6, respectively.

The results of the validation of the vehicle model are shown in Chapter 5. The validations of the steering responses to medium and small steering wheel angle inputs are discussed in Section 5.2. Section 5.3 contains the validation of the response to road undulations.

The driver model for the simulation of straight line stability is discussed in Chapter 6. It has been copied from literature and has neither been optimized nor validated because priority has been given to the modelling of the vehicle. The only specific experiments which have been carried out were aimed at the determination of the required driver parameters.

Chapter 7 describes the statistical analysis of simulation and test results which has been carried out for the derivation of the estimation algorithm from the elementary straight line stability qualities. The test program for the generation of the experimental results is also discussed.

Finally, in Chapter 8 the research presented in this thesis is summarized and the conclusions of this thesis are given. Suggestions for future research are also formulated.

Chapter 2

Vehicle Straight Line Stability

2.1 Introduction

The method for the numerical design of vehicles with optimal straight line stability on undulating road surfaces is based on simulations of elementary straight line stability qualities. In the present chapter, the elementary qualities are derived by means of analyses of the phenomenon of vehicle straight line stability on undulating road surfaces and of its subjective judgment by a driver.

2.2 Subjective judgment

Subjective judgment is the only suitable criterion for assessing the straight line stability qualities of a vehicle. It cannot be replaced by measurements because reliable measurement criteria are not available. In literature, only experimental criteria have been presented (cf. Tanaka [46]). Nevertheless, the subjective judgment has deficiencies. It has a mediocre reproducibility and it depends on the specific driver. In the present research, these deficiencies have been overcome by a suitable design of the subjective test program.

Experience shows that the subjective judgment of vehicle straight line stability is based on several basic judgments. These basic judgments can be divided into three classes:

- Global judgments, which are averaged roadholding observations over a certain time interval. Global judgments can be based on the steering effort but also on the course of the vehicle as an assessment of, for instance, the yaw angle or the lateral displacement.
- Local judgments, in which the response to one particular obstacle or to a critical series of obstacles is of major importance.
- Judgments of the steering feeling, in which the steering wheel couple plays the important role.

Local judgments are the most difficult to evaluate by numerical simulation because the critical obstacles or series of obstacles depend on the priorities of the driver and on the characteristics of the vehicle. Judgments of the steering feeling are also rather difficult to simulate because the accurate simulation of the steering wheel couple is very complicated, especially when driving more or less in a straight line. Global judgments, on the contrary, are well suited for numerical simulation because all simulated state variables are available for inclusion in an averaging criterion. In case of a closed loop simulation, even the steering wheel angle can be employed for evaluation.

The conclusion that the numerical design of vehicles with optimal straight line stability is not feasible because some elementary judgments are unsuitable for numerical simulation cannot be drawn. The reason is that the importance of these judgments is unknown. This applies especially to the local judgments. An indication of the importance of the different classes of judgments can be found from the statistical analysis of test and simulation results.

2.3 Elementary straight line stability qualities

Global judgment of vehicle straight line stability on undulating road surfaces is performed by evaluating the vehicle's motion or the required steering effort. The vehicle's straight line stability is good if little vehicle motion occurs or if only small steering efforts are required. Hence, a positive judgment corresponds to little response of the driver-vehicle closed loop system to the road undulations. Therefore, the sensitivity of the driver-vehicle closed loop system to road undulations is the most straightforward elementary straight line stability quality. This sensitivity can be assessed with the aid of closed loop simulations that yield all state variables, including the steering wheel angle α_{sw} (see Figure 2.1).

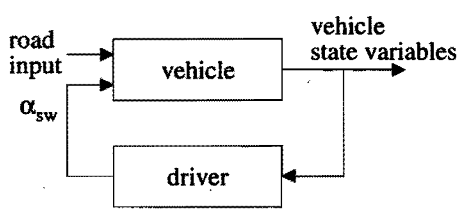


Figure 2.1 Driver-vehicle closed loop system

Closed loop simulation, however, has several disadvantages. It yields little understanding of the vehicle characteristics that are important for vehicle straight line stability. Moreover, it is very sensitive to errors in the driver and vehicle models. Even small modelling inaccuracies may yield erroneous closed loop behavior. Finally, the most serious disadvantage of closed loop simulation is the fact that it requires an accurate representation of the driver which is difficult to model. For these reasons, alternative elementary straight line stability qualities are required. These qualities should be based on the vehicle instead of on the driver-vehicle closed loop system.

The closed loop system shown in Figure 2.1 may yield large steering angles or large motion variables due to two causes. Either the vehicle is rather sensitive to the road undulations or the closed loop stability is poor. The vehicle's sensitivity to road undulations is an elementary quality which is very appropriate for numerical simulation. The closed loop stability also yields suitable elementary qualities. According to basic control theory, vehicle characteristics which yield a poor closed loop stability are a large phase delay or an inappropriate gain of the directional response to steering wheel angle inputs.

Judgments of the steering feeling yield one specific elementary straight line stability quality. Some parts of the steering feeling are already included in the above-mentioned elementary qualities because they also influence the global judgments, for instance the phase delay of the vehicle's response to steering inputs. The steering wheel couple, however, has not yet been included. During straight line keeping, the most important task of the steering wheel couple is the information to the driver on the state of the vehicle. Therefore, the transfer of information through the steering wheel couple is added as elementary straight line stability quality.

Accordingly, the analysis of vehicle straight line stability on undulating road surfaces yields five elementary qualities:

- The sensitivity of the driver-vehicle closed loop system to road undulations.
- The vehicle's open loop sensitivity to road undulations.
- The phase delay of the vehicle's directional response to steering wheel angle inputs.
- The gain of the vehicle's directional response to steering wheel angle inputs.
- The transfer of information through the steering wheel couple.

A mathematical description of the elementary straight line stability qualities will be given in Section 7.2.

2.4 Sources of excitation

This section contains a discussion of the components of the undulating road which excite the passing vehicle. First, the components are defined. Then, the importances of the excitations by the different components are estimated with the aid of measurement and simulation results, and, finally, the excitation mechanisms are analyzed.

Throughout this thesis, one basic assumption is used in describing the components of the undulating road: the road surface is supposed to be flat in the tire-road contact patch. This assumption is allowed because vehicle straight line stability on undulating road surfaces is a low frequency phenomenon at relatively high driving speeds. Moreover, longitudinal grooves in the road surface and their influence on vehicle behavior are not part of this study. Vehicle straight line stability on undulating road surfaces is a low frequency phenomenon because both the control actions by the driver and the reactions of the vehicle have a limited bandwidth.

The road surface under the wheels of one axle can be fully described using six variables. This is explained with the aid of the example of the front axle with Wheels 1 and 2 shown in Figure 2.2. The road height in the center of a contact patch is represented by $q_v(i)$ and the lateral inclination of the road surface in the contact center is represented by $\sigma(i)$. The longitudinal inclination of the road surface in the contact center depends on the time derivative $\dot{q}_v(i)$ of the local road height and on the driving velocity V. In addition to the six primary variables, two dependent variables, viz. the global road inclination θ_{gl} and the mean road height $q_{v,m}$, are employed because they yield a good understanding of the occurring phenomena. Both variables depend on the two road heights $q_v(1)$ and $q_v(2)$, while the global road inclination also depends on the track of the vehicle:

$$\theta_{gl} = \frac{q_v(1) - q_v(2)}{track} \tag{2.1}$$

and

$$q_{v,m} = \frac{q_v(1) + q_v(2)}{2} \; . \tag{2.2}$$

When a vehicle is driven on a straight road, its yaw angle remains rather small. The wheels of the front and rear axles pass at almost the same lateral positions at a certain road cross section. Hence, the front axle's input, after a time delay, will also be the rear axle's input. Therefore, only six signals are required for the description of the complete vehicle input. The time delay between the front and the rear axles depends on the wheelbase of the vehicle and on the driving velocity.

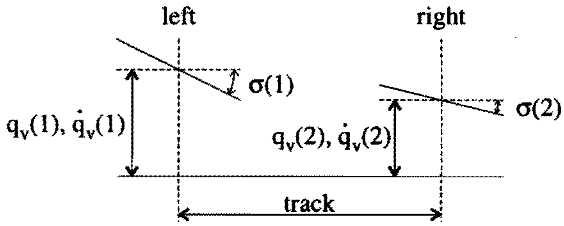


Figure 2.2 Road inputs at the front axle

Only four of the six variables which are required for the description of the road surface under the wheels of an axle have a direct influence on the vehicle's behavior, viz. the lateral inclinations of the road surface in the contact patches, the global road inclination, and the mean road height. The longitudinal inclinations in the contact patches are just required for the mathematical description of the road surface in the vehicle model. Therefore, they will be ignored in the analysis of the sources of excitation. Moreover, the term "local road inclination" will be used for the lateral inclination of the road surface in a tire-road contact patch in order to emphasize the contrast with the global road inclination.

For the experimental analysis of vehicle behavior on undulating road surfaces, it is essential that all measurements, including the measurements of the road inputs, are carried out simultaneously. A combination of two separate measurements, one of the road surface and one of the driver-vehicle behavior on the same road surface, cannot be used because the actual road input depends on the specific course of the vehicle. In the case of the global road inclination and of the mean road height, real time measurement can be carried out with sufficient accuracy. Both variables can be estimated by measuring two suspension deflections, the vehicle height, and the roll angle. Measurement of the local road inclinations, on the other hand, is much more complicated, especially if the measurement has to be carried out at more than 100 km/h without influencing the vehicle's behavior. For this reason, the local road inclinations are not included in the vehicle measurements on undulating road surfaces that are reported in this thesis.

Figures 2.3 and 2.4 show the results of a closed loop measurement of Vehicle B1 at 110 km/h on the rather strongly undulating road. The spectral densities of the two estimated road inputs, the steering wheel angle, the yaw velocity, the lateral acceleration, and the roll angle are presented in Figure 2.3. The low spectral density of the mean road height at frequencies below 0.3 Hz is related to the method of estimation which will be discussed in Section 3.2. Apart from this side effect of the estimation method, almost all spectral densities decrease with increasing frequency. Only the lateral acceleration, which has been measured on the car bottom in front of the passenger seat, contains a maximum around 2 Hz as a result of the influence of the roll acceleration.

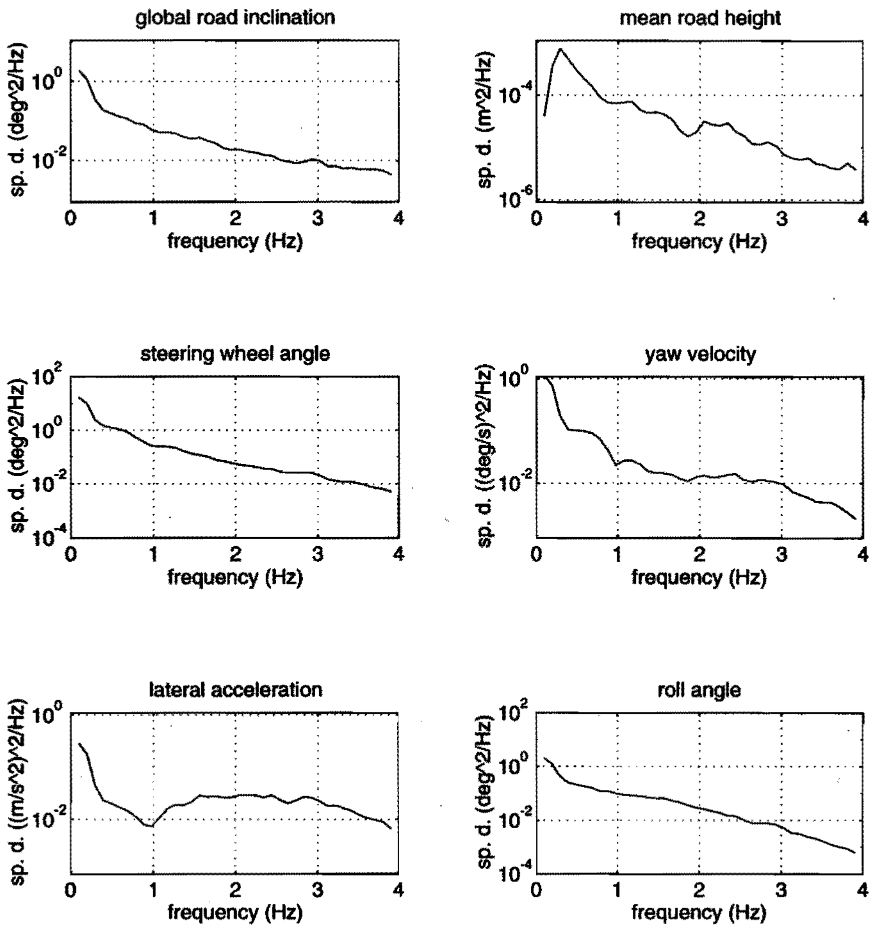
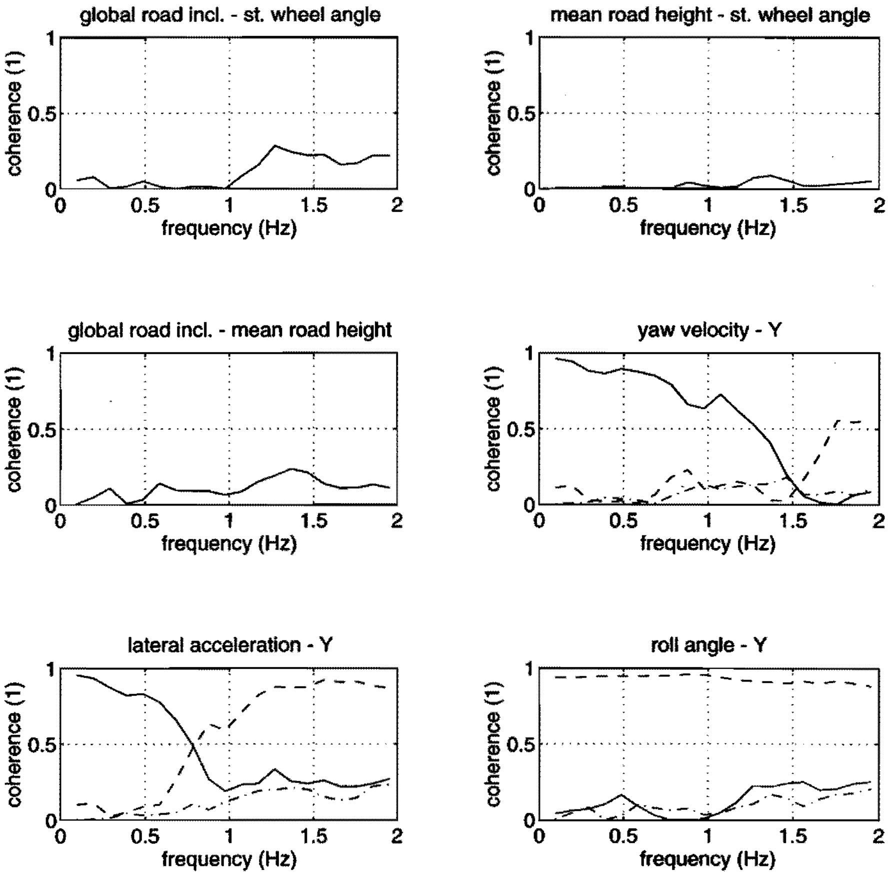


Figure 2.3 *Power spectral densities during closed loop drive on the rather strongly undulating road (Vehicle B1, Driver 1, V = 110 km/h)*

The coherences between the three measured inputs and the vehicle outputs are shown in Figure 2.4. One of the interesting results that is contained in this figure is the low level of the coherences between the steering wheel angle and the estimated road inputs, despite the rather accurate measurement of these three signals. The fact that the sum of these coherences is much smaller that unity either implies that the other two road inputs, the local road inclinations, yield a much bigger vehicle response than the global road inclination and the mean road height or that at least one of the elements in the closed loop system of Figure 2.1 is not completely linear.



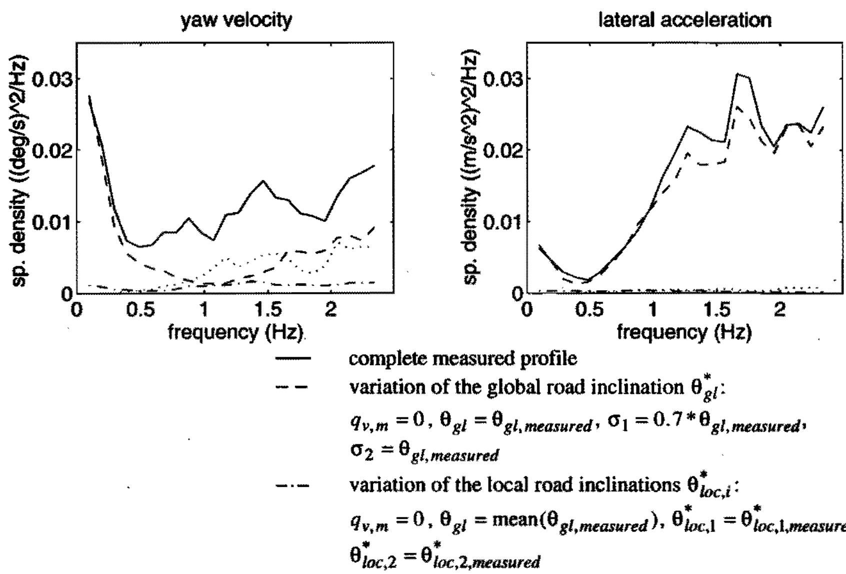
—— Y = steering wheel angle
– – Y = global road inclination
–·– Y = mean road height

Figure 2.4 Coherence functions during closed loop drive on the rather strongly undulating road (Vehicle B1, Driver 1, V = 110 km/h)

Hereafter, an open loop simulation of the behavior of Vehicle B1 on the rather strongly undulating road will be described. This simulation shows that the responses to the local road inclinations are much smaller than the response to the global road inclination. Therefore, the coherences between the local road inclinations and the steering wheel angle will be small too because the driver's corrections are based on the course of the vehicle. Hence, at least one of the elements of the driver-vehicle closed loop system is not completely linear.

The coherences between the vehicle inputs on the one hand and the yaw velocity and the lateral acceleration on the other hand show, especially for low frequencies, that the major part of the vehicle's directional response is induced by the steering wheel angle. Vehicle roll, on the contrary, is mainly caused by the global road inclination. Above 1 Hz, the relationship between the global road inclination and the roll angle also yields a high coherence between the global road inclination and the lateral acceleration through the roll acceleration.

Local road inclinations are very difficult to measure. Therefore, the importance of their contribution to the excitation of the vehicle is assessed with the aid of simulations. Figure 2.5 shows the model's responses to the complete profile of the rather strongly undulating road and to several components of this profile. The profile of the rather strongly undulating road is available from a measurement with special equipment (cf. Appendix A). During this measurement, all road variables, including the local road inclinations, were recorded.



—— complete measured profile

– – variation of the global road inclination θ^*_{gl}:

$q_{v,m} = 0$, $\theta_{gl} = \theta_{gl,measured}$, $\sigma_1 = 0.7 * \theta_{gl,measured}$, $\sigma_2 = \theta_{gl,measured}$

–·– variation of the local road inclinations $\theta^*_{loc,i}$:

$q_{v,m} = 0$, $\theta_{gl} = \text{mean}(\theta_{gl,measured})$, $\theta^*_{loc,1} = \theta^*_{loc,1,measured}$, $\theta^*_{loc,2} = \theta^*_{loc,2,measured}$

····· variation of the mean road height $q_{v,m}$:

$q_{v,m} = q_{v,m,measured}$, $\theta_{gl} = \text{mean}(\theta_{gl,measured})$, $\sigma_1 = \text{mean}(\sigma_{1,measured})$, $\sigma_2 = \text{mean}(\sigma_{2,measured})$

Figure 2.5 Simulated open loop responses to (parts of) the profile of the rather strongly undulating road (fixed steering wheel angle, Vehicle B1, V = 110 km/h)

The division of the profile of the rather strongly undulating road into its components is based on special definitions for the road inclinations. These special definitions yield negligible coherences between the road inputs (cf. Appendix A) and, therefore, facilitate the analysis of a vehicle's response to the undulating road. For instance, in Figure 2.5, they permit the comparison of the response to the complete profile to the sum of the other responses. The mean road height is not touched by the special definitions because its correlation to the other road inputs is already very small under the standard definitions, see for instance the coherence between the global road inclination and the mean road height in Figure 2.4.

The newly defined local road inclinations $\theta^*_{loc,i}$ are the relative inclinations with respect to the special global inclination θ^*_{gl} (see Figure 2.6). This special global road inclination consists of a difference in track height and of accompanying local inclinations that are 70 and 100% at left and right hand side, respectively, of the global road inclination that corresponds to the track height difference (see Figure 2.6 and Appendix A). It does not have a physical background but it has been found by an analysis of the measured road profile.

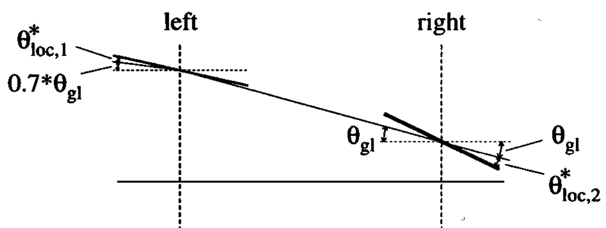


Figure 2.6 Definition of road inclinations without coherence

Hence, the response to global road inclination which is shown in Figure 2.5 is a pure response to θ^*_{gl} without mean road height variations or additional local inclinations. For the simulation of the responses to the local road inclinations, the local inclinations $\theta^*_{loc,i}$, at left and right hand side simultaneously, are combined with the mean value of θ^*_{gl}. The response to the mean road height $q_{v,m}$ is simulated in combination with the mean values of the global and local road inclinations.

Figure 2.5 shows that the independent excitation by the two local road inclinations is rather small. The most important excitation is produced by the global inclination. In spite of the use of special definitions for the road inclinations, the sum of the separated excitations does not match the excitation by the complete profile. This effect is probably due to non-linearities, especially in the vehicle's response to the mean road height.

The course variations that are produced by the mean road height can arise in two ways. First, they can be the product of a geometrical asymmetry within the vehicle's suspension, which, in combination with vehicle bump or pitch, yields changes in the steering or camber angles. The other possible phenomenon is a destabilization of the vehicle through a variation of the vertical wheel load. The wheel load can destabilize the vehicle through its influence on the lateral forces that are produced by the tire, even when driving in a straight line. When driving in a straight line, lateral forces can be produced by, for instance, toe in or toe out (see Mitschke [23]) or by local road inclinations. Hence, if the wheel loads change, the lateral forces change and a resulting force may act on the vehicle. The destabilization of the vehicle becomes more important on an inclined road surface because of the much bigger lateral forces. Therefore, the mean road height yields a strongly non-linear vehicle response. In Figure 2.5, the directional response to the mean road height is only produced by the wheel load variations because suspension asymmetry is not included in the vehicle model.

Variation of a local road inclination can yield a directional response because it yields a camber force and a horizontal component of the normal force in the tire-road contact patch (cf. Figure 2.7). Since the camber force and the horizontal component of the normal force act in opposite directions, the response to the local road inclination will be very small if the tire's camber stiffness (per radian) is close to the absolute value of the wheel load.

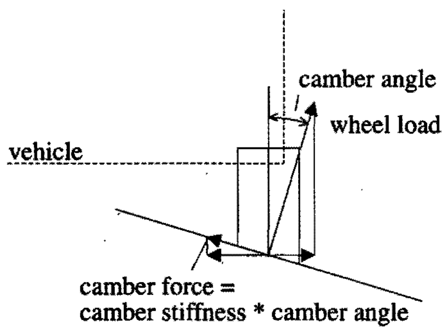


Figure 2.7 Forces between a tire and a locally inclined road surface

Wheel load variations, camber forces and horizontal components of the normal forces also contribute to the vehicle excitation by the global road inclination θ^*_{gl}. The only specific excitation by the global road inclination is the excitation through the so-called "roll steer" effect. Roll steer is the contribution to the steering angles of the wheels which is produced by the suspension geometry if the car body rolls with respect to the unsprung masses.

The contribution of the wheel load variations to the excitation by the global road inclination yields a non-linear influence on the vehicle's response to this input because the response to the wheel load variations depends on the local road inclination. However, this non-linear influence is assumed to be insignificant. First, the global road inclination θ^*_{gl} determines the major part of the local road inclinations $\sigma(i)$. Therefore, the non-linearity of the excitation through the wheel load variations is limited. Second, the contribution of the wheel load variations is only one out of four excitation mechanisms. Finally, the wheel load variations are small at the low frequencies which are the most important for straight line stability.

The phenomena which have been described above are important for the excitation of the vehicle by the road surface. However, the response of the vehicle to road excitation is not completely determined by these phenomena. It depends on all vehicle parameters that control the free damped vibration of the vehicle.

Chapter 3

Vehicle Behavior on Undulating Straight Roads

3.1 Introduction

The present chapter contains a discussion of the preparatory measurements that have been carried out in order to obtain knowledge of vehicle behavior on undulating straight roads. The required information could not be obtained from literature because measurements of vehicle behavior on undulating straight roads have, to the knowledge of the author, not been reported in literature.

3.2 Test method

A vehicle which drives on an undulating straight road has four road inputs (see Figure 3.1). In addition, the inputs to the front axle are supposed to excite the rear axle after a certain time delay which depends on the wheelbase and on the driving velocity. Therefore, the two road inputs that have been measured during the vehicle tests on undulating roads, the global road inclination and the mean road height, have been measured at one axle only. In case the measurement was carried out at the rear axle, the measured signals were transformed into front axle inputs in order to maintain a single standard.

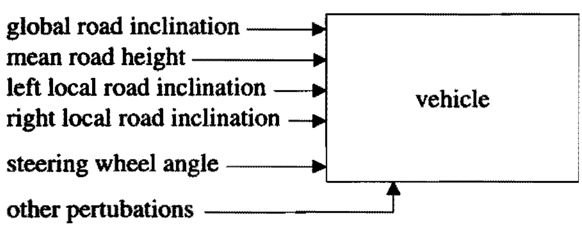


Figure 3.1 Inputs to a vehicle on an undulating road

The global road inclination is calculated from the vehicle roll angle, from the suspension deflections, and from estimations of the tire deflections. The estimations of the tire deflections are based on a computation of the vertical wheel loads from the suspension deflections. It uses the stiffness of the tire, the stiffness and damping of the suspension and the stiffness of the anti-roll bar as parameters.

Likewise, the mean road height estimation is based on the tire and suspension deflections. Here, the position of the vehicle body is obtained by integrating the vertical body acceleration above the relevant axle. As a result of the biases that are introduced by the numerical integration of the body acceleration, the mean road height has little value in the time domain. It has to be used in the frequency domain.

Simulations of Vehicle B have shown that the other two road inputs, the local road inclinations, yield responses that are less important than the responses to the mean road height and the global road inclination (cf. Section 2.4). In addition, it is very difficult to measure these inclinations at high vehicle test speed, especially if vehicle behavior must not be influenced. For these reasons, measurement of local road inclinations has been omitted from the project reported in this thesis.

Most of the test results of vehicle behavior on undulating straight roads reported in this chapter are analyzed with the aid of transfer functions, i.e. after Fourier Transformation into the frequency domain. Frequency domain analyses of the small measurement signals with their rather unfavorable ratios to the measurement noise are much more practical than time domain analyses. Of course, to allow these analyses with the aid of transfer functions, a predominant linear system behavior must be demonstrated.

On undulating roads, the derivation of transfer functions from measurement results is complicated by the fact that the vehicle inputs are correlated (see Figure 2.4). The correlations between the inputs yield mutual influences on the transfer functions if a conventional computation is used. This can be illustrated with the aid of an imaginary linear system with inputs *X* and *Y* and output *Z* (see Figure 3.2).

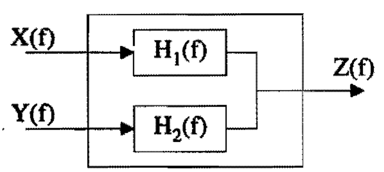


Figure 3.2 Imaginary linear system

In the frequency domain, the output of the imaginary linear system can be calculated with the aid of the transfer functions $H_1(f)$ and $H_2(f)$:

$$Z(f) = H_1(f)X(f) + H_2(f)Y(f) \ . \tag{3.1}$$

Calculation of the cross spectral density $S_{xz}(f)$ of X and Z yields

$$S_{xz}(f) = H_1(f) S_{xx}(f) + H_2(f) S_{xy}(f) \,, \tag{3.2}$$

while the conventional estimation of the transfer function $H_{xz}(f)$ is given by:

$$H_{xz}(f) = \frac{S_{xz}(f)}{S_{xx}(f)} = H_1(f) + H_2(f) \frac{S_{xy}(f)}{S_{xx}(f)} \,. \tag{3.3}$$

Hence, the conventional estimation does not yield the desired result $H_1(f)$ if X and Y are correlated ($S_{xy}(f) \neq 0$).

For the elimination of the influence of the correlations between system inputs, special analysis techniques have to be used (see for instance Bendat [6]). A demonstration of these techniques can be found in Appendix B which describes the derivation, from a measurement, of the transfer function between the steering wheel angle and the yaw velocity.

The special analysis techniques not only yield an estimate of the transfer function, they also yield a partial coherence function which indicates the linearity between the input and the output after elimination of the unwanted influences. The partial coherence function is the equivalent of the normal coherence function in systems without correlation between the input signals. Hence, a partial coherence close to unity indicates a strong linear relationship between the input and the output, whereas a vanishing partial coherence indicates that either the system between the input and the output is non-linear or that the signals are contaminated.

The influences of the correlations between the local road inclinations and the other inputs cannot be eliminated because the local road inclinations are not measured. In the case of the response to the global road inclination, the correlations with the local road inclinations are rather important (see Section 2.4 and Appendix A). Therefore, they have to be taken into account during the validation of the vehicle's response to this input. However, in the case of the vehicle's response to a steering wheel angle input, simulations and measurements have shown that the coherences between the local road inclinations and the steering wheel angle are very small (see Section 2.4). For that reason, the influences of the correlations between the steering wheel angle and the unknown local road inclinations are disregarded.

3.3 Response to road undulations

The test method discussed in the preceding section yields estimates of the transfer functions and of the partial coherences for the responses to the steering wheel angle, to the global road inclination, and to the mean road height. Figure 3.3 shows the response to the global road inclination which has been derived from a measurement of Vehicle B2 on the rather strongly undulating road. The linear part of the response to the other measured road input, the mean road height, has not been plotted because this response is known to be non-linear (cf. Section 2.4). Moreover, the mean road height yields only a small contribution to the excitation of Vehicle B (see Figures 2.4 and 2.5). The response to the steering wheel angle will be discussed in the following section.

The vehicle's response to the global road inclination is assumed to be sufficiently linear to allow an analysis with the aid of transfer functions. The variations of the global road inclination are small and there are no physical reasons to expect a significant non-linearity in this response (cf. Section 2.4). Evidently, a sufficient level of partial coherence is required for the justification of the hypothesis of linearity and as a guarantee for a reasonable accuracy of the estimate of the corresponding transfer function. Estimates of transfer functions that are accompanied by low (partial) coherence functions are of little precision (cf. Bendat [6]). A low estimate of a (partial) coherence function is not always the result of non-linear behavior. It can also be caused by measurement noise. This is rather likely to occur in measurements of vehicle behavior on undulating straight roads because many signals have small signal to noise ratios.

The roll angle response to the global road inclination resembles the behavior of a simple damped system with one degree of freedom. The partial coherence is close to unity in the important frequency range from 0 to 2 Hz, indicating a predominant linear behavior in this frequency range. The relationship between the global road inclination and the steering wheel couple, on the contrary, is probably non-linear. The relevant partial coherence function is close to 0.5. This non-linearity can be explained by the fact that the steering wheel couple is transmitted by the steering system which always contains friction.

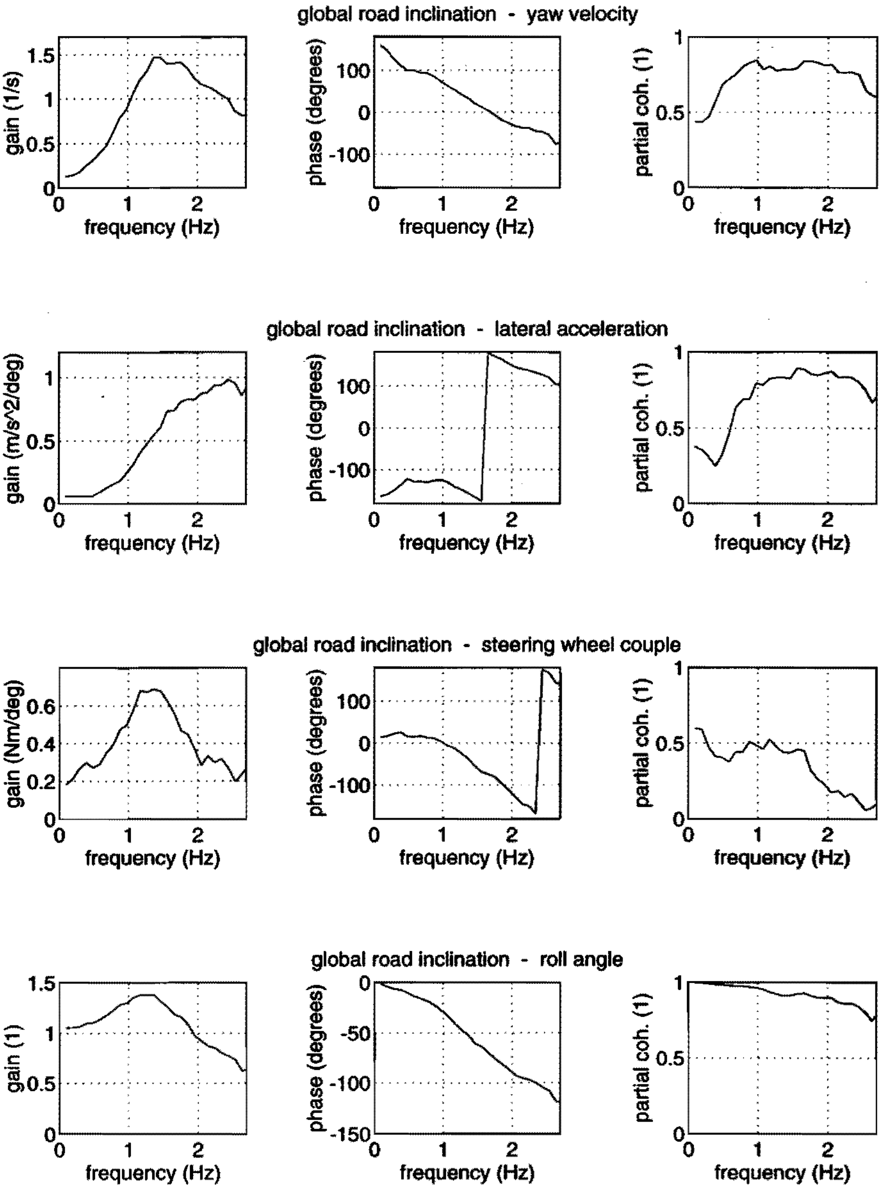


Figure 3.3 *Transfer functions between the global road inclination and four vehicle outputs (measurement on the rather strongly undulating road, Vehicle B2, V = 110 km/h; the influences of the correlations between the global road inclination, steering wheel angle, and mean road height inputs have been eliminated)*

The other two responses which have been plotted in Figure 3.3 have small partial coherences in the frequency range from 0 up to 0.8 Hz and rather high partial coherences in the range from 0.8 to 2.0 Hz. In both cases, the lack of coherence below 0.8 Hz can be explained by the fact that, at low frequencies, the major contribution to the yaw velocity and to the lateral acceleration is produced by the steering wheel angle (see Figure 2.4). Therefore, the responses to the global road inclination are relatively small. Even very small non-linearities in the steering responses will cause significant reductions in the partial coherences related to the global road inclination. Moreover, at low frequencies, measurement noise is relatively large because the responses to the global road inclination are also very small on an absolute scale.

Under the unfavorable measurement conditions during the measurement of road responses on undulating straight roads, a partial coherence of 0.7 must be considered as a rather high value. Therefore, the transfer functions between the global road inclination as input and the roll angle, the yaw velocity, and the lateral acceleration of the vehicle as outputs are supposed to be valid in the frequency ranges 0 - 2.0 Hz, 0.8 - 2.0 Hz, and 0.8 - 2.0 Hz, respectively. This conclusion is especially important for the validation of the vehicle model. With the aid of the transfer functions, the model's response to the global road inclination can be validated under practical circumstances: on undulating straight roads.

3.4 Steering response

The steering response is the most important factor in the roadholding of a vehicle. It is not only important for the vehicle's straight line stability. It influences almost every area of its roadholding. In analyses, often two types of steering responses are distinguished, viz. the steering responses that remain in the linear domain of vehicle behavior and the responses that exceed this domain and become non-linear. A widely accepted criterion indicating linear vehicle behavior is a maximum value of lateral acceleration of about 3 m/s^2. Beyond this limit, vehicle behavior becomes non-linear, principally as a result of non-linear tire behavior.

In steering response analyses within the linear domain, the area on and around the straight ahead driving position, the so-called "close to zero" area, is normally disregarded. In this area, however, the steering response of the vehicle is likely to be non-linear. The steering system which transmits the input from the driver to the front wheels contains non-linearities such as friction. Probably, these non-linearities are more important at small lateral accelerations than they are in steering responses where the lateral acceleration reaches 2 or 3 m/s^2.

The non-linear area beyond the lateral acceleration limit of 3 m/s^2 is irrelevant for vehicle straight line stability because, on straight undulating roads, the lateral acceleration never reaches 3 m/s^2. A possible non-linear behavior around zero, on the contrary, is expected to be important. Therefore, during the preparatory investigation of the steering response, both medium and small steering wheel angle inputs have been applied.

Figure 3.4 shows an example of a steering response to a medium steering wheel angle input. This response of Vehicle B1 was measured at 110 km/h on a wide even road. It was obtained with an approximately harmonic excitation. The steering wheel angle amplitude was about 15 degrees which corresponds to about 2.5 m/s^2 lateral acceleration. The coherence functions in Figure 3.4 show that the important relationships between the steering wheel angle as input and the yaw velocity and the lateral acceleration as outputs are linear. Hence, under these circumstances, the influence of the non-linearities in the steering system on both relationships is negligible.

The transfer function between the steering wheel angle and the yaw velocity contains some resonance close to 1 Hz. Conversely, the transfer function between the steering wheel angle and the lateral acceleration does not contain any resonance. It also has a different phase characteristic: the phase delay of the lateral acceleration decreases with frequency between 1.3 and 2.5 Hz.

The response of the steering wheel couple to the steering wheel angle input is significantly non-linear. Despite the favorable measurement conditions with a signal to noise ratio which is comparable to the ratios corresponding to the lateral acceleration and the yaw velocity, the relevant coherence in Figure 3.4 is relatively low. Hence, the relationship between the steering wheel angle and the steering wheel couple cannot be studied in the frequency domain.

During maneuvering on an even road, vehicle roll is caused by the lateral acceleration of the vehicle and not directly by the steering wheel angle input. Therefore, the transfer function between the lateral acceleration and the roll angle is more appropriate for the characterization of a vehicle's roll behavior than the transfer function between the steering wheel angle and the roll angle. Figure 3.4 shows that the coherence between the lateral acceleration and the roll angle is only moderate, especially at frequencies higher than 1.3 Hz. This lack of coherence is probably caused by the fact that the roll angle which is caused by the lateral acceleration and, hence, by the steering wheel angle becomes small with respect to system noise if frequency exceeds 1 Hz. Even a road surface which appears to be even yields a little roll.

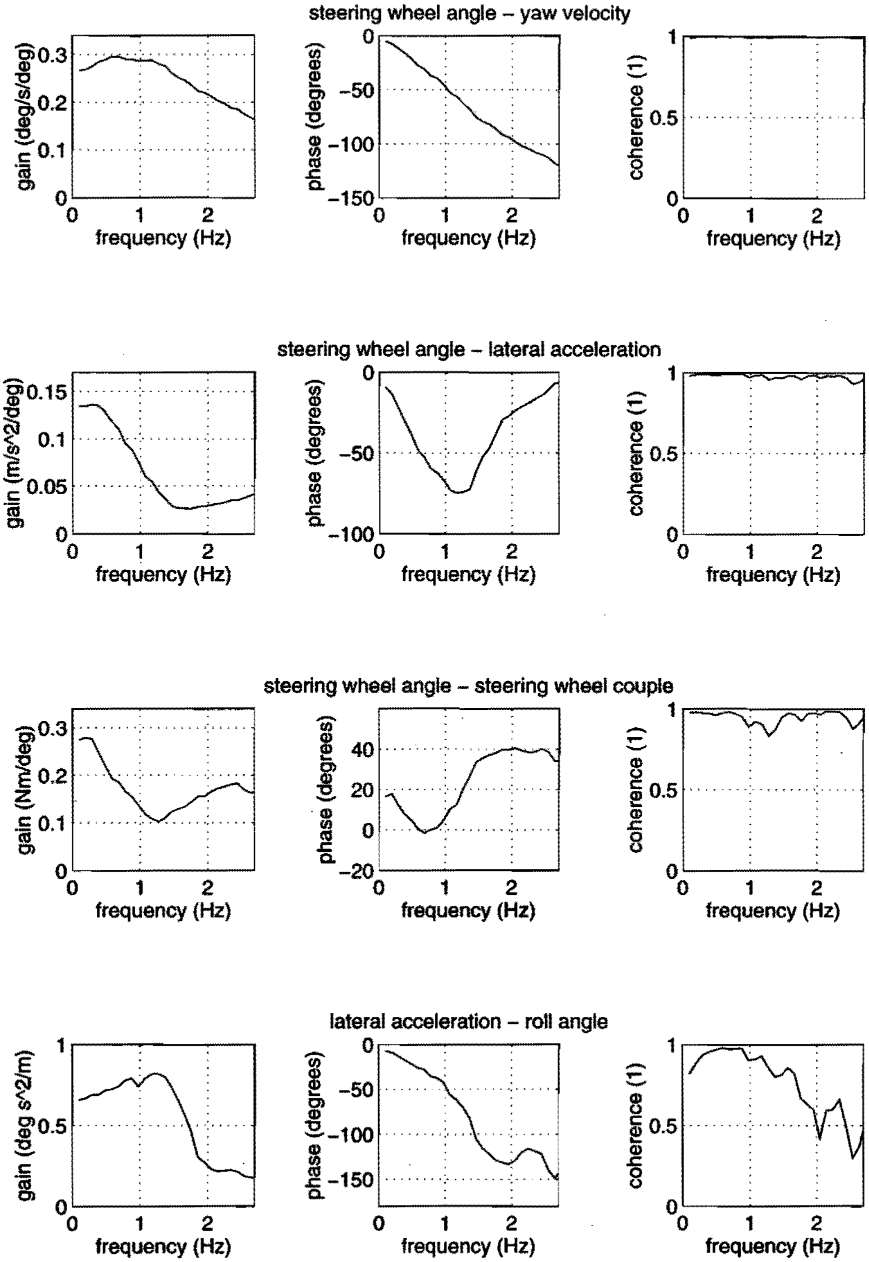


Figure 3.4 Transfer functions between the steering wheel angle and three vehicle outputs as well as between the lateral acceleration and the roll angle (medium steering wheel angle input on even road, Vehicle B1, V = 110 km/h, steering wheel angle amplitude ≈ 15 degrees)

The steering response which is the most important for vehicle straight line stability is the response around zero, hence at small steering wheel angle and lateral acceleration levels. Exploratory measurements of this response have been carried out under practical circumstances, viz. on undulating roads of different qualities (see Figure 3.5). The test method discussed in Section 3.2 was used for the elimination of the influence of the correlations between the vehicle inputs.

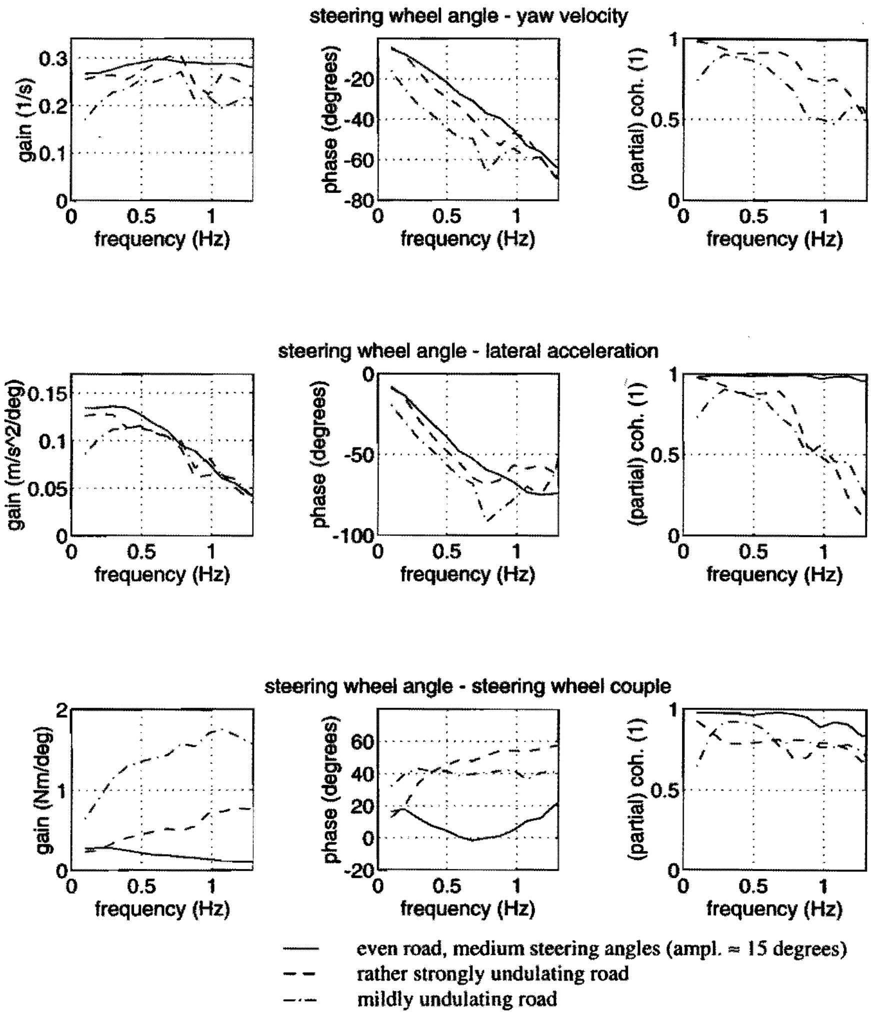


Figure 3.5 Transfer functions between the steering wheel angle and three vehicle outputs (Vehicle B1, V = 110 km/h; the influences of the correlations between the steering wheel angle, global road inclination, and mean road height inputs have been eliminated)

Figure 3.5 shows that the steering responses on the mildly and rather strongly undulating roads are different. On the mildly undulating road, the vehicle's response is probably non-linear; the partial coherences are relatively small and indicate a non-linearity at low frequencies. On the rather strongly undulating road, on the contrary, the partial coherences between the steering wheel angle and the yaw velocity and the lateral acceleration are close to unity in the frequency range from 0 to 0.8 Hz. Above 0.8 Hz, the steering wheel angle's contribution to the excitation of the vehicle becomes less important (see Figure 2.4). The partial coherence between the steering wheel angle and the steering wheel couple is relatively small, even on the rather strongly undulating road.

The differences between the steering responses on the mildly and rather strongly undulating roads can be explained by the influence of the road undulations, through the non-linear steering system, on the vehicle's behavior. In the case of the measurements of Figure 3.5, the rather strongly undulating road yields larger forces on the steering system, for instance as a result of larger wheel load variations. These larger forces could facilitate the transmission, by the steering system, of steering angles from the steering wheel to the front wheels.

Another explanation of the difference between the steering responses on the mildly and rather strongly undulating road surfaces is based on the power spectral densities of the two road inputs and of the steering wheel angle (see Figure 3.6). At low frequencies, the mean road height variations of both undulating roads are almost equal. The rather strongly undulating road especially contains more variations of the global road inclination. As a result of this more severe excitation by the road, the driver has to apply larger corrections on the rather strongly undulating road. Hence, on the rather strongly undulating road, the influence of steering system non-linearities around zero will be less important.

Finally, the differences between the steering responses on the two undulating roads might also be the result of the influence of the road undulations, through the wheel load, on non-linear tire behavior. However, as will be shown in Section 4.2, this influence may be expected to be rather small because tire slip angles are small and because even the load variations are not very large. Moreover, the non-linear influences of the road undulations should be the most important on the rather strongly undulating road.

The partial coherences of the steering response of Vehicle B1 on the mildly undulating road are too small for a study of this response with the aid of transfer functions. An alternative approach for the analysis of the steering response on the mildly undulating road, for instance in the time domain, also yields complications. Most signals are accompanied by high frequency vibrations and measurement noise. In addition, the separation of the effects of the steering wheel angle and of the road inputs is much more complicated in the time domain. Therefore, no alternative characterization of the steering response on the mildly undulating road has been attempted.

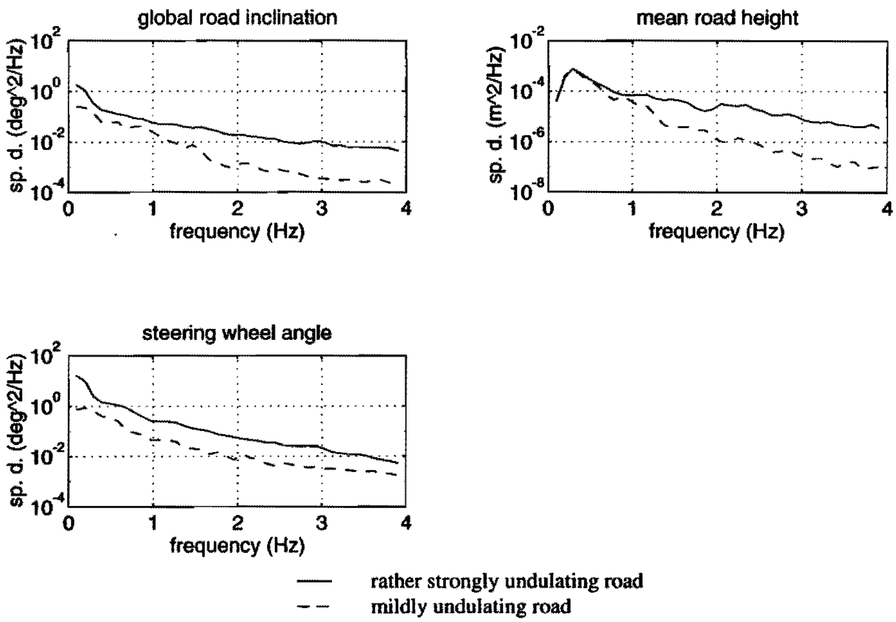


Figure 3.6 Power spectral densities on undulating roads (Vehicle B1, V = 110 km/h)

On the rather strongly undulating road, the partial coherences between the steering wheel angle input and the yaw velocity and lateral acceleration outputs are about 0.9 for frequencies within the range from 0 to 0.75 Hz. Under the relevant measurement conditions with a rather unfavorable signal to noise ratio, this value is assumed to justify the assumption of linear vehicle behavior in the operating range. Hence the transfer functions between the steering wheel angle as input and the yaw velocity and the lateral acceleration as outputs are supposed to be valid up to 0.75 Hz. Comparison of these transfer functions with the transfer functions that were measured with the harmonic excitation shows that the gains are not significantly different but that, on the rather strongly undulating road, the phase delays of both transfer functions are larger. In addition, the phase delay differences between the straight undulating road and the harmonic excitation are almost identical in the two transfer functions and increase with frequency.

Theoretically, the steering responses that were measured on the undulating roads may contain an influence of the vehicle's response to the local road inclinations. This influence cannot be eliminated because the local road inclinations are not measured. However, simulations show that the local road inclinations yield vehicle responses which are much smaller than the response to the global road inclination which only has a small coherence with the steering wheel angle (cf. Section 2.4). Therefore, the coherences between the local road inclinations and the steering wheel angle and the influence of the local road inclinations on the steering responses may be assumed to be very small.

Since the phase delay of the directional response to steering is one of the elementary qualities for the simulation of vehicle straight line stability on undulating road surfaces, the supplementary delay on the rather strongly undulating road is an important result. It should be included in the vehicle modelling.

Chapter 4

Vehicle Model

4.1 Introduction

The vehicle model allowing the design optimization of straight line stability on undulating road surfaces will be applied in the early stages of a vehicle development process. In these stages, many vehicle parameters are still unknown or difficult to obtain. Therefore, the model must be relatively simple and the number of input parameters must be kept at an absolute minimum.

The basic straight line stability qualities that will be simulated by the model have already been defined in Section 2.3. There it was explained that the model should yield accurate simulation results of:

- the vehicle's open loop sensitivity to road undulations,
- the phase delay of the vehicle's directional response to steering wheel angle inputs,
- the gain of the vehicle's directional response to steering wheel angle inputs, and
- the transfer of information through the steering wheel couple.

In addition, vehicle behavior on undulating straight roads has been analyzed in Chapter 3. It was shown that the test vehicle's steering responses on undulating straight roads, hence with small steering wheel angle inputs, differ significantly from its steering response during handling tests with medium steering wheel angle inputs. For straight line stability, vehicle behavior on straight roads is the most important of these two responses. Therefore, the objective with respect to the vehicle's directional steering response was optimum accuracy in the response around zero. This had to be accomplished without violating the basic condition of suitability for the application in the early stages of a vehicle development process.

The objective of the accurate simulation of the vehicle responses to small steering wheel angle inputs cannot be obtained without an accurate steering system model in which the relevant non-linearities are taken into account. Vehicle response to the road undulations input, on the other hand, is most demanding on the modelling of tire behavior. Therefore, specific sub-models have been developed for the steering system and the tire. The sub-models and the overall vehicle model have been programmed in ACSL ("Advanced Continuous Simulation Language") [1]. Matlab [19] is used for post-processing in the time and frequency domains.

4.2 Modelling assumptions

The model which has been developed for the simulation of vehicle straight line stability does not contain all the effects that might influence the vehicle's behavior on undulating straight roads. Some influences that may be expected to be of second order and that require extensive modelling have been omitted in order to obtain a model suitable for application in the early stages of a vehicle development process (see Roos [33, 34]).

First the treatment of non-linear effects will be discussed. This is an important subject for the modelling of vehicle behavior on undulating straight roads. The non-linear behaviors of the tire and the steering system yield interactions between the responses to the separate inputs as well as between the responses to different frequency components of one particular input.

For straight line stability on undulating road surfaces, the most important factor in tire non-linearity is the dependence of a tire's cornering stiffness on its vertical load (cf. the sample characteristic in Figure 4.1). The other major factor in tire non-linearity is the saturation of lateral force with increasing slip angle. This factor is irrelevant for straight line stability because steering and slip angles remain very small.

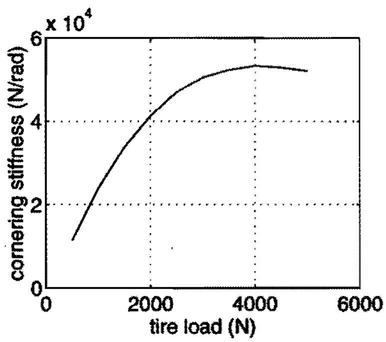


Figure 4.1 *Cornering stiffness of a 165/70 R13 tire as a function of the tire load*

The tire characteristics related to self-aligning torque and camber, which also contain non-linearities, will not be discussed in relation to the modelling assumptions because they have a smaller influence on vehicle behavior than the cornering stiffness. Moreover, they yield effects that are similar to the non-linear effects caused by the cornering stiffness.

The kinds of non-linearities that can be found in a steering system partially depend on its particular design (e.g. a rack and pinion system with servo-assistance or a system containing a steering gearbox without servo-assistance). Friction is the only non-linearity occurring in all steering systems. It cannot be avoided. Moreover, friction is desired because it yields an additional firmness in the steering feeling. A hydraulic servo-assistance adds supplementary non-linearities, for instance as a result of its U-shaped couple-pressure relation. In addition, free play may exist in some types of steering systems, for instance in systems containing a steering gearbox. In this thesis, however, the analysis is limited to rack and pinion steering systems only. In such systems, free play may be neglected.

Vehicle straight line stability on undulating road surfaces is a low frequency vehicle characteristic. Experience has shown that the frequency range between 0 and 1 Hz is very important for straight line stability and that variations above 2 Hz have no direct influence. Therefore, two types of inputs are distinguished in the frequency domain, viz. the inputs between 0 and 2 Hz and the inputs above 2 Hz which only have an indirect influence through the non-linearities. Combination of these two types of inputs with the distinction between the inputs induced by the road and those induced by the driver finally yields three input sections to be discussed in relation to the treatment of the vehicle non-linearities:

- low frequency driver inputs (0 - 2 Hz),
- low frequency road inputs (0 - 2 Hz), and
- high frequency road inputs (from 2 Hz).

It is noted that the fourth combination is not taken into account because a driver hardly yields any input above 2 Hz.

Tire non-linearity is assumed not to yield any significant interactions between the various input sections or within one particular input section. In the case of the two low frequency input sections, this assumption is justified by the fact that, at frequencies below 2 Hz, load variations and, consequently, cornering stiffness variations are relatively small. The disregard of the non-linear tire response to high frequency road inputs will explained below. In spite of the fact that the interactions through non-linear tire behavior are not taken into account, the load dependency of the cornering stiffness has been included in the model because it influences some (linear) responses, for instance the response to global road angle variations (cf. Section 2.4).

The tire related non-linear influence of high frequency road inputs on low frequency behavior is a reduction of the effective cornering stiffness. This reduction has a stationary portion through the shape of the cornering stiffness as a function of vertical load (see Figure 4.1), and a dynamic portion through the loss of contact in the contact patch. It is disregarded because the tire slip angle remains very small and because even the load variations are rather small. The influence of both parameters on the loss of cornering stiffness has been studied frequently (see for instance Laermann [15], Mühlmeier [24], Pacejka [32], and Takahashi [45]). Mühlmeier [24] has reported simulation results of the loss of cornering stiffness as a function of the slip angle and the standard deviation of the load variations. These results allow the assessment of the non-linear influence of the high frequency road inputs through the tire. They have been obtained with the aid of a highly detailed tire model not suited for implementation into a vehicle design process.

In order to assess the loss of effective cornering stiffness during driving on undulating straight roads, the standard deviation of the vertical tire load variations on the rather strongly undulating road has been estimated. Simulations with the measured road profiles reported in Appendix A showed that the standard deviation of the tire load is about 15 to 20% of its stationary value. For this standard deviation and a slip angle close to zero, the loss of effective cornering stiffness will probably be less that 2% [24]. Such a reduction may be neglected under practical circumstances. The differences between tires and the inaccuracy of measurement equipment are at least as important.

The fact that the steering system contains friction makes all interactions important. Even a small variation of the global road inclination at 15 Hz influences the response to a steering manoeuvre at 0.2 Hz if it yields a force on the rack. However, a physical description of the non-linearities in the steering system is very complicated. Therefore, only the interactions within the two low frequency input sections have been modelled. The interactions between the various input sections are taken into account by a description of their importance as part of the test or simulation circumstances. For instance, a test can be carried out on a bad undulating straight road. This means that the steering wheel angle input is small and that the road input is large at all frequencies.

In conclusion, the influences of the high frequency road inputs through non-linear tire and steering system behavior have been excluded from the modelling. The model has been aimed at the accurate reproduction of the responses to the low frequency inputs. Since the frequency range between 0 and 1 Hz is the most important for straight line stability, the model should yield accurate responses at least up to 1 Hz. The objective of the modelling was accuracy up to 2 Hz.

A limitation of the vehicle model which is not related to the above-mentioned disregard of some non-linear influences is the omission of certain aerodynamic effects. Dynamic models that yield an accurate representation of the aerodynamic forces and moments acting on a vehicle with varying yaw angle and that can be used in vehicle dynamics simulations are not available. Accordingly, only the stationary aerodynamic influences are taken into account.

The second group of assumptions made in defining the model is related to the interpretation of the test results of Chapter 3. In Section 3.4, measured transfer functions of the steering response of Vehicle B1 during normal driving on the rather strongly undulating road have been presented. The differences between this response on the rather strongly undulating road and the response to the medium steering wheel angle input are attributed to the steering system. There are no reasons to expect significant changes of behavior of the vehicle, of its suspension, or of the tires between the responses to medium and small steering wheel angle inputs. The steering system non-linearities which are taken into account are friction and power steering.

4.3. Inputs

On an undulating road, a vehicle has two types of inputs. It is perturbed by the road input and controlled by the driver through the steering wheel angle.

4.3.1 Road input

In the model, the road surface is described by the six variables which have been defined in Section 2.4. The road height in the center of a tire-road contact patch is represented by the symbol q_v, the lateral inclination by the symbol σ. The longitudinal inclination of the road surface depends on the time derivative of q_v, i.e. on $\dot{q}_v$, and on the driving velocity V. This description of the road input is based on two assumptions:

- The road surface is supposed to be flat in the tire-road contact patch.
- The wheels of the front and rear axle are supposed to pass at the same location in a cross section of the road.

4.3.2 Steering wheel angle

The model offers two possibilities for the driver's control of the course of the vehicle, viz. open loop and closed loop control. During open loop simulations, the model can use a steering pattern supplied in an ASCII-file or it can use one of the functions which have been defined within the program. In closed loop, a driver model controls the vehicle on a straight road (see Chapter 6).

The chosen approach of steering wheel angle input and steering wheel couple output is not the only one possible. The inverse approach, i.e. with steering wheel couple input and steering wheel angle output, could also be used. However, this approach makes vehicle and driver modelling much more difficult. Vehicle response to steering wheel couple inputs is more complicated to model than vehicle response to steering wheel angle inputs because the steering wheel couple is a dynamic instead of a kinematic input whereas most of the relevant outputs are kinematic (yaw velocity, lateral acceleration, etc.).

4.4 Model

The vehicle model which has been developed for the simulation of vehicle behavior on undulating road surfaces is a lumped parameter model. In contrast with models based on the alternative multi-body modelling method, it does not contain a complete description of all the parts of the suspension. Examples of important lumped parameters are the suspension compliances (e.g. steering under self-aligning torque) and the suspension derivatives (e.g. roll steer).

The lumped parameter modelling approach has been chosen because it offers several advantages. First, a description of the suspension with the aid of lumped parameters requires less simulation time than a multi-body description because it only contains the characteristics that are required to obtain accurate simulation results. A multi-body model is usually more detailed and contains more masses, flexibilities and geometrical transformations which demand additional simulation time.

Second, the lumped parameter modelling approach produces a model with a robust simulation quality because the relevant behavior of the suspension is described in a direct manner. In a multi-body model, the elements of the suspension first have to yield an accurate suspension functioning before they can yield an accurate contribution to the resulting vehicle behavior.

Finally, the lumped parameter modelling approach is well suited for simulation in early stages of vehicle development because it offers two possibilities. If detailed information on the suspension of the vehicle under development is available, the required parameters can be computed with the aid of specialized simulation programs or measured on prototypes. In case the detailed information is not available yet, estimated or objective parameter values can be used.

4.4.1 Degrees of freedom

The vehicle model has ten degrees of freedom (see Figure 4.2). The motion of the car body is completely defined by six degrees of freedom. Four vertical degrees of freedom $z_w(i)$, $i = 1,..,4$, have been added for the unsprung masses. These are required for an accurate representation of the vertical tire loads, which are important for tire modelling, and for the acquisition of accurate tire and suspension deflections. The unsprung masses are referred to by the numbers 1 to 4 for front left, front right, rear left and rear right, respectively.

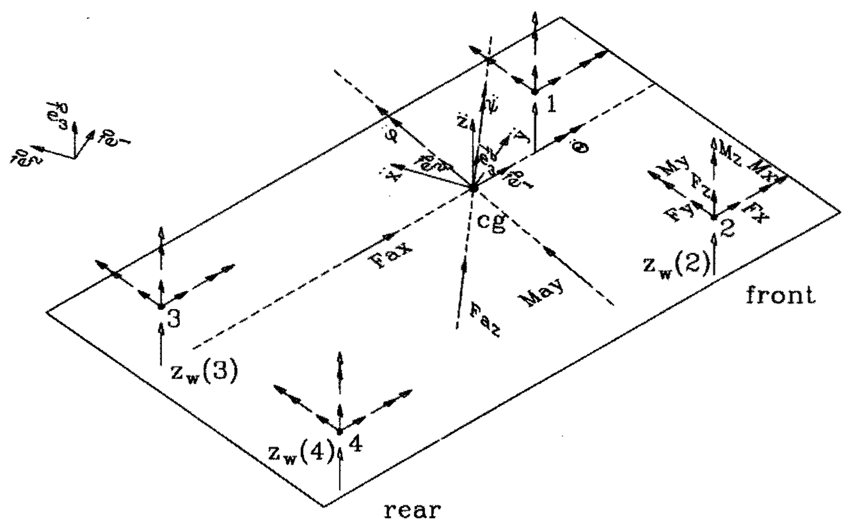


Figure 4.2 Vehicle model (cg denotes the vehicle's center of gravity)

Coordinate systems

The accelerations $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ of the center of gravity of the complete vehicle have been defined in the global coordinate system $\underline{\vec{e}}^0$. They are directed in longitudinal, lateral, and vertical directions, respectively. On the other hand, the roll, pitch, and yaw angular accelerations, $\ddot{\theta}$, $\ddot{\varphi}$, and $\ddot{\psi}$, respectively, have been defined in the local coordinate system $\underline{\vec{e}}^b$ which is fixed to the car body. The tensor [1] $\boldsymbol{R}$ and the corresponding matrix [2] $\underline{R}$ relate the two coordinate systems:

$$\underline{\vec{e}}^{b^T} = \boldsymbol{R}\underline{\vec{e}}^{0^T} \quad (4.1)$$

1 Tensors are depicted with bold face characters.

2 Underlined variables are matrices or columns.

and

$$\underline{\vec{e}}^b = \underline{R}^T \underline{\vec{e}}^0 \ . \tag{4.2}$$

Cardan angles γ_i, i = 1,2,3, are used for the computation of the transformation matrix (cf. Schiehlen [40]):

$$\underline{R} = \begin{bmatrix} \cos\gamma_2\cos\gamma_3 & -\cos\gamma_2\sin\gamma_3 & \sin\gamma_2 \\ \cos\gamma_1\sin\gamma_3+\sin\gamma_1\sin\gamma_2\cos\gamma_3 & \cos\gamma_1\cos\gamma_3-\sin\gamma_1\sin\gamma_2\sin\gamma_3 & -\sin\gamma_1\cos\gamma_2 \\ \sin\gamma_1\sin\gamma_3-\cos\gamma_1\sin\gamma_2\cos\gamma_3 & \sin\gamma_1\cos\gamma_3+\cos\gamma_1\sin\gamma_2\sin\gamma_3 & \cos\gamma_1\cos\gamma_2 \end{bmatrix} . \tag{4.3}$$

They are calculated from the angular velocity vector $\vec{\omega}$,

$$\begin{aligned} \vec{\omega} &= \dot{\theta}\vec{e}_1^{\,b} + \dot{\varphi}\vec{e}_2^{\,b} + \dot{\psi}\vec{e}_3^{\,b} \\ &= \begin{bmatrix} \dot{\theta} & \dot{\varphi} & \dot{\psi} \end{bmatrix} \underline{\vec{e}}^b : \end{aligned} \tag{4.4}$$

$$\begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \dot{\gamma}_3 \end{bmatrix} = \begin{bmatrix} \cos\gamma_3/\cos\gamma_2 & -\sin\gamma_3/\cos\gamma_2 & 0 \\ \sin\gamma_3 & \cos\gamma_3 & 0 \\ -\cos\gamma_3\tan\gamma_2 & \sin\gamma_3\tan\gamma_2 & 1 \end{bmatrix} \underline{\omega} \ . \tag{4.5}$$

Dynamic equations

The four unsprung masses each have a vertical degree of freedom. Their other motions are guided by the motion of the car body through the suspension. Figure 4.3 shows the unsprung mass $m_w(i)$ with the force $F_z(i)$ from the suspension, the force $F_{z,ti}(i)$ from the tire, and the gravitational force $m_w(i)g$. The vertical motion of the unsprung mass i is described by:

$$m_w(i)\ddot{z}_w(i) = F_{z,ti}(i) - F_z(i) - m_w(i)g \ . \tag{4.6}$$

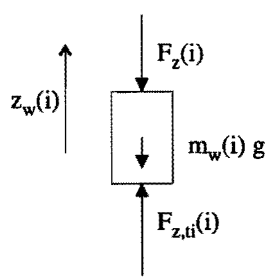


Figure 4.3 Forces acting on an unsprung mass

The car body has six degrees of freedom. Only one of these degrees of freedom, the vertical translation, is treated as an exclusive motion of the car body itself. All other translations and rotations are complete vehicle motions where the inertia of the unsprung masses are included. Of course, the fact that the unsprung masses are linked to the car through the suspension with its flexibility in vertical direction has to be taken into account when the moments of inertia are supplied to the model. The distinction between sprung and unsprung masses with respect to the generation of roll by lateral acceleration or by roll itself is taken into account in the model of the suspension to be discussed in Section 4.4.2. The unsprung masses are located under the suspension and do not contribute to the above mentioned roll between car body and suspension.

The forces on the vehicle are introduced in the tire-road contact patches, at the center of gravity of the car body, and at the center of gravity of the complete vehicle (see Figure 4.2). In each contact patch center i, the forces $F_x(i)$, $F_y(i)$, and $F_z(i)$ act on the vehicle in longitudinal, lateral, and vertical directions of the local car body coordinate system, respectively. The gravitational force of the vehicle body is introduced at its center of gravity whereas the stationary aerodynamic forces F_{ax} and F_{az} in longitudinal and vertical directions, respectively, act at the center of gravity of the complete vehicle. Together, the forces constitute the resulting forces $\Sigma F_{x,b}$, $\Sigma F_{y,b}$, and $\Sigma F_{z,b}$ in the local coordinate system which is fixed to the car body:

$$\begin{bmatrix} \sum F_{x,b} \\ \sum F_{y,b} \\ \sum F_{z,b} \end{bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} F_x(i) \\ F_y(i) \\ F_z(i) \end{bmatrix} + \begin{bmatrix} F_{ax} \\ 0 \\ F_{az} \end{bmatrix} - \underline{R}^T \begin{bmatrix} 0 \\ 0 \\ (m - \sum_{i=1}^{4} m_w(i))g \end{bmatrix}. \tag{4.7}$$

A mass matrix is used for the adaptation of the vehicle mass in vertical direction:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m - \sum_{i=1}^{4} m_w(i) \end{bmatrix} \underline{R}^T \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \sum F_{x,b} \\ \sum F_{y,b} \\ \sum F_{z,b} \end{bmatrix}. \tag{4.8}$$

The description of the rotations around the center of gravity of the complete vehicle is based on the assumption that the vehicle is symmetric about the x-z center plane. This assumption yields the symmetrical moments of inertia matrix $\underline{J}$:

$$\underline{J} = \begin{bmatrix} J_{xx} & 0 & -J_{xz} \\ 0 & J_{yy} & 0 \\ -J_{xz} & 0 & J_{zz} \end{bmatrix}. \tag{4.9}$$

In addition, local coordinates $l_x(i)$ and $l_y(i)$ are used to describe the locations of the contact patch centers with respect to the vehicle center of gravity in longitudinal and in lateral directions, respectively. They depend on the suspension deflections and are obtained by integration of the relative contact velocities that will be discussed in Section 4.4.2. In vertical direction, the position of the contact center i is defined by the initial height h_{cg} of the center of gravity and by the deflections $\Delta z(i)$ and $\Delta z_{ti}(i)$ of the suspension and the tire, respectively.

The local coordinates $\Sigma M_{x,b}$, $\Sigma M_{y,b}$, and $\Sigma M_{z,b}$ of the resulting moment $\Sigma \vec{M}$ on the vehicle contain contributions of the forces and moments in the tire-road contact patches, of the aerodynamic couple M_{ay} in lateral direction, and of the gravitational force on the car body. This latter contribution originates from the fact that the centers of gravity of vehicle and car body normally do not coincide. The location of the center of gravity of the car body is described by the local coordinates $x_{cb,b}$, $y_{cb,b}$, and $z_{cb,b}$:

$$\begin{bmatrix} \sum M_{x,b} \\ \sum M_{y,b} \\ \sum M_{z,b} \end{bmatrix} = \sum_{i=1}^{4} \left(\begin{bmatrix} M_x(i) \\ M_y(i) \\ M_z(i) \end{bmatrix} + \begin{bmatrix} l_x(i) \\ l_y(i) \\ -h_{cg} + \Delta z(i) + \Delta z_{ti}(i) \end{bmatrix} \times \begin{bmatrix} F_x(i) \\ F_y(i) \\ F_z(i) \end{bmatrix} \right) + \\ \begin{bmatrix} 0 \\ M_{ay} \\ 0 \end{bmatrix} - \begin{bmatrix} x_{cb,b} \\ y_{cb,b} \\ z_{cb,b} \end{bmatrix} \times \left(\underline{R}^T \begin{bmatrix} 0 \\ 0 \\ (m - \sum_{i=1}^{4} m_w(i)) g \end{bmatrix} \right) . \tag{4.10}$$

Substitution of Equations (4.9) and (4.10) into the equation for the angular momentum,

$$J \cdot \dot{\vec{\omega}} + \vec{\omega} \times (J \cdot \vec{\omega}) = \sum \vec{M} , \tag{4.11}$$

yields the equations for the computation of the rotational accelerations $\ddot{\theta}$, $\ddot{\varphi}$, and $\ddot{\psi}$:

$$J_{xx}\ddot{\theta} - J_{xz}\ddot{\psi} + (J_{zz} - J_{yy})\dot{\varphi}\dot{\psi} - J_{xz}\dot{\theta}\dot{\varphi} = \sum M_{x,b} , \tag{4.12}$$

$$J_{yy}\ddot{\varphi} + (J_{xx} - J_{zz})\dot{\theta}\dot{\psi} + J_{xz}(\dot{\theta}^2 - \dot{\psi}^2) = \sum M_{y,b} , \tag{4.13}$$

and

$$J_{zz}\ddot{\psi} - J_{xz}\ddot{\theta} + (J_{yy} - J_{xx})\dot{\theta}\dot{\varphi} + J_{xz}\dot{\varphi}\dot{\psi} = \sum M_{z,b} . \tag{4.14}$$

4.4.2 Suspension

The suspension model is based on lumped parameters that can be derived from measurements or from calculations with specialized programs. For instance, the effects of the suspension geometry, such as load transfer and tire-road contact velocities and displacements, have been modelled directly with the aid of coefficients.

Vertical forces

The vertical forces between the unsprung mass i and the car body and between the unsprung mass i and the road surface are denoted by $F_z(i)$ and $F_{z,ti}(i)$, respectively. They have been modelled in body vertical direction above the contact patch center (see Figure 4.4). The implementation of these forces into the model will be discussed for the rear axle. The implementation of the forces at the front wheels is analogous.

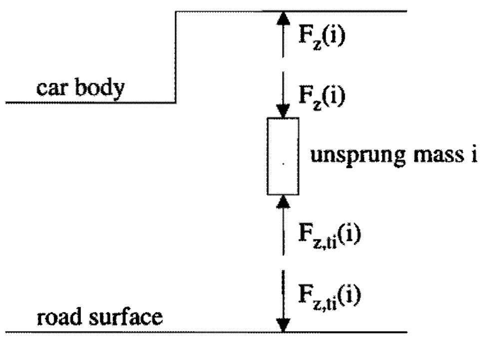


Figure 4.4 Vertical forces

The vertical forces $F_z(i)$ contain contributions of the vertical suspension stiffnesses, the dampers, the anti-roll bars, the suspension induced load transfers and the imaginary forces that eliminate the contribution of the unsprung masses to the roll between the suspension and the car body under lateral acceleration and roll. If $h_{cg,r}$ is the height of the center of gravity of the rear unsprung masses and $\ddot{y}_y$ the lateral acceleration of the center of gravity of the complete vehicle, these imaginary forces $F_{z,cg}(i)$ are described by:

$$F_{z,cg}(3) = -(m_w(3)+m_w(4))(\ddot{y}_y + g\sin(\theta))\frac{h_{cg,r}}{l_y(3)-l_y(4)} \tag{4.15}$$

and

$$F_{z,cg}(4) = (m_w(3)+m_w(4))(\ddot{y}_y + g\sin(\theta))\frac{h_{cg,r}}{l_y(3)-l_y(4)}. \tag{4.16}$$

The forces $F_{z,b}(i)$ which result from the vertical suspension stiffnesses are supplied to the model in a table $F_{z,b,r}(\Delta z)$ (subscript r denoting the rear axle) as a function of the suspension deflection:

$$F_{z,b}(i) = F_{z,b,r}(\Delta z(i)), \quad i = 3,4 \ . \tag{4.17}$$

The damper forces $F_{z,bd}(i)$ are calculated from the rear damper characteristic $F_{z,bd,r}$ and from the coefficient λ_r, the transfer ratio between the velocity in the damper and the vertical velocity in the contact patch center:

$$F_{z,bd}(i) = \lambda_r F_{z,bd,r}(\lambda_r \, \Delta\dot{z}(i)), \quad i = 3,4 \ . \tag{4.18}$$

The anti-roll bar with stiffness $k_{r,r}$ yields the forces $F_{z,r}(i)$:

$$F_{z,r}(3) = k_{r,r} \frac{\Delta z(3) - \Delta z(4)}{(l_y(3) - l_y(4))^2} \tag{4.19}$$

and

$$F_{z,r}(4) = -k_{r,r} \frac{\Delta z(3) - \Delta z(4)}{(l_y(3) - l_y(4))^2} \ . \tag{4.20}$$

Load transfer has been modelled for the general case of an interdependent suspension, hence with cross-coupling between the two wheels on the same axle. The lateral load transfer forces $F_{z,rc}(i)$ are described with the aid of the rear axle direct and indirect roll center coefficients $c_{rc,d,r}$ and $c_{rc,i,r}$, respectively, yielding:

$$F_{z,rc}(3) = c_{rc,d,r} F_y(3) - c_{rc,i,r} F_y(4) \tag{4.21}$$

and

$$F_{z,rc}(4) = c_{rc,i,r} F_y(3) - c_{rc,d,r} F_y(4) \ . \tag{4.22}$$

An independent suspension is treated as a special case where cross-coupling coefficients equal zero. The pitch center coefficient $c_{pc,r}$ is used for the description of the longitudinal load transfer forces $F_{z,pc}(i)$:

$$F_{z,pc}(i) = c_{pc,r} \, F_x(i), \quad i = 3,4 \ . \tag{4.23}$$

The total forces $F_z(3)$ and $F_z(4)$ are given by:

$$F_z(i) = F_{z,cg}(i) + F_{z,b}(i) + F_{z,bd}(i) + F_{z,r}(i) + F_{z,rc}(i) + F_{z,pc}(i), \quad i = 3,4 \ . \tag{4.24}$$

The vertical tire model contains a linear spring with stiffness $k_{ti,r}$ and a linear damper with coefficient $c_{ti,r}$. In addition, the tire deflection and the tire deflection velocity are denoted by $\Delta z_{ti}(i)$ and $\Delta \dot{z}_{ti}(i)$:

$$F_{z,ti}(i) = k_{ti,r}(i)\,\Delta z_{ti}(i) + c_{ti,r}(i)\,\Delta \dot{z}_{ti}(i)\,, \quad i = 3,4 \tag{4.25}$$

Velocities

Suspension deflection produces horizontal velocities of the contact patch centers relative to the vehicle center of gravity, thereby affecting the tire slip angles. The longitudinal components of these velocities may be disregarded because they are small with respect to the driving velocity. Only the lateral velocities are important. These lateral velocities $\dot{y}_{le}$ and $\dot{y}_{ri}$ of the left and right wheel of an axle with respect to the center of gravity have been modelled as a linear function of the vertical suspension deflection velocities $\Delta \dot{z}_{le}$ and $\Delta \dot{z}_{ri}$ of the same wheels (see Figure 4.5). They depend on the roll center coefficients of the relevant axle:

$$\begin{bmatrix} \dot{y}_{le} \\ \dot{y}_{ri} \end{bmatrix} = \begin{bmatrix} -c_{rc,d,f/r} & -c_{rc,i,f/r} \\ c_{rc,i,f/r} & c_{rc,d,f/r} \end{bmatrix} \begin{bmatrix} \Delta \dot{z}_{le} \\ \Delta \dot{z}_{ri} \end{bmatrix} . \tag{4.26}$$

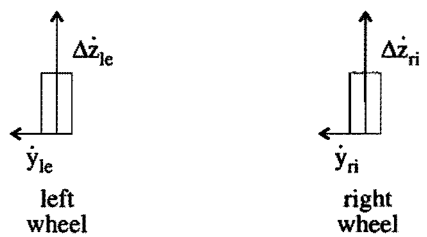


Figure 4.5 Relative velocities of the contact patch centers

Elasto-kinematic steering and camber angles

The steering angles of the rear wheels and the camber angles of all wheels are described by elasto-kinematic models. The front wheel steering angles, on the other hand, have been integrated in the steering system model (cf. Section 4.5).

The steering angle $\alpha_t(i)$ of a rear wheel contains the initial steering angle $\alpha_0(i)$ and a so-called elasto-kinematic angle $\alpha_s(i)$:

$$\alpha_t(i) = \alpha_0(i) + \alpha_s(i)\,, \quad i = 3,4\ . \tag{4.27}$$

The elasto-kinematic steering angle $\alpha_s(i)$ depends on the mean suspension deflection, on the roll angle between the suspension and the body, and on the longitudinal and lateral forces and the self-aligning torque in the tire-road contact patch. The lateral force and the self aligning torque directly influence the steering angle at the wheel at which they are applied, but also influence the steering angle of the other wheel on the same axle. All influences are supposed to be linear when driving on an undulating straight road. This yields, for instance, for the elasto-kinematic steering angle of the left rear wheel:

$$\begin{aligned}\alpha_s(3) = \; & \alpha_{sb,r} \frac{\Delta z(3)+\Delta z(4)}{2} + \alpha_{sr,r} \frac{\Delta z(3)-\Delta z(4)}{l_3(3)-l_4(4)} + \alpha_{sy,d,r} F_y(3) + \\ & \alpha_{sx,r} F_x(3) + \alpha_{sm,d,r} M_z(3) + \alpha_{sy,i,r} F_y(4) + \alpha_{sm,i,r} M_z(4) \quad .\end{aligned} \tag{4.28}$$

A similar approach with linear influences is used for the description of the camber angles. The camber angle $\beta_t(i)$ of a front or rear wheel contains the initial camber angle $\beta_0(i)$ and an elasto-kinematic camber angle variation $\beta_s(i)$:

$$\beta_t(i) = \beta_0(i) + \beta_s(i), \quad i = 1,2,3,4 \; . \tag{4.29}$$

The elasto-kinematic camber angle depends on the mean suspension deflection, on the roll angle between the suspension and the car body, and on the lateral force. At the left rear wheel, this yields :

$$\beta_s(3) = \beta_{sb,r} \frac{\Delta z(3)+\Delta z(4)}{2} + \beta_{sr,r} \frac{\Delta z(3)-\Delta z(4)}{l_y(3)-l_y(4)} + \beta_{sy,r} F_y(3) \; . \tag{4.30}$$

4.4.3 Geometrical tire-road interface

A description of the velocities in the contact patch center is required for the computations of the slip angle and of the tire deflection. At the vehicle side and the road side of the tire-road contact, some velocities are known. The remaining velocities can be obtained with the aid of coordinate transformations.

The vertical contact velocity $\dot{z}_{trc,0}(i)$ in global coordinates is known from the road input model:

$$\dot{z}_{trc,0}(i) = \dot{q}_v(i), \quad i = 1,2,3,4 \; . \tag{4.31}$$

The horizontal velocities, however, are not known from the road input model. They have to be calculated from the vehicle motion. At the vehicle side, the velocities of the center of gravity and the angular velocity vector are known. Hence, the velocities $\dot{x}'_{trc,b}(i)$, $\dot{y}'_{trc,b}(i)$, and $\dot{z}'_{trc,b}(i)$ of the contact patch center $\underline{b(i)}$ as a point rigidly connected to the car body can be calculated from:

$$\begin{bmatrix} \dot{x}'_{trc,b}(i) \\ \dot{y}'_{trc,b}(i) \\ \dot{z}'_{trc,b}(i) \end{bmatrix} = \underline{R}^T \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \underline{\omega} \times \underline{b(i)}, \quad i = 1,2,3,4 . \tag{4.32}$$

Substitution of the suspension deflection velocity $\Delta\dot{z}(i)$,

$$\Delta\dot{z}(i) = \dot{z}_w(i) - \dot{z}'_{trc,b}(i), \quad i = 1,2,3,4 , \tag{4.33}$$

into Equation (4.26) yields the suspension induced lateral velocity which can be added to $\dot{y}'_{trc,b}(i)$ in order to obtain the true relative lateral velocity of the contact patch center in the local car body coordinate system:

$$\dot{y}_{trc,b}(i) = \dot{y}'_{trc,b}(i) + \dot{y}(i), \quad i = 1,2,3,4 . \tag{4.34}$$

The longitudinal suspension induced velocity is neglected because it is much smaller than the driving velocity:

$$\dot{x}_{trc,b}(i) = \dot{x}'_{trc,b}(i), \quad i = 1,2,3,4 . \tag{4.35}$$

The three unknown velocities, the true relative vertical velocity $\dot{z}_{trc,b}(i)$ in the local car body coordinate system and the longitudinal and lateral velocities $\dot{x}_{trc,0}(i)$ and $\dot{y}_{trc,0}(i)$ in the global coordinate system, can be obtained from:

$$\begin{bmatrix} \dot{x}_{trc,0}(i) \\ \dot{y}_{trc,0}(i) \\ \dot{z}_{trc,0}(i) \end{bmatrix} = \underline{R} \begin{bmatrix} \dot{x}_{trc,b}(i) \\ \dot{y}_{trc,b}(i) \\ \dot{z}_{trc,b}(i) \end{bmatrix}, \quad i = 1,2,3,4 . \tag{4.36}$$

After resolution of Equation (4.36), the vertical contact velocity $\dot{z}_{trc,b}(i)$ and the vertical velocity of the unsprung mass i yield the tire deflection velocity:

$$\Delta\dot{z}_{ti}(i) = \dot{z}_{trc,b}(i) - \dot{z}_w(i), \quad i = 1,2,3,4 . \tag{4.37}$$

Integration of the suspension and tire deflection velocities yields the deflections:

$$\Delta z(i) = \int_{\tau=0}^{t} \Delta\dot{z}(i,\tau)\, d\tau , \quad i = 1,2,3,4 \tag{4.38}$$

and

$$\Delta z_{ti}(i) = \int_{\tau=0}^{t} \Delta \dot{z}_{ti}(i,\tau)\, d\tau \,, \quad i = 1,2,3,4 \,. \tag{4.39}$$

For the computation of the slip angle, the velocities $\dot{x}_{trc,r}(i)$ and $\dot{y}_{trc,r}(i)$ parallel to the inclined road surface are required. The requisite transformation matrix $\underline{R_{br}(i)}$ between body coordinates and inclined road coordinates can be obtained by substitution of the Cardan angles

$$\gamma_1 = \sigma(i) - \theta \,, \tag{4.40}$$

$$\gamma_2 = -\frac{dq_v}{dx} - \varphi \,, \tag{4.41}$$

and

$$\gamma_3 = 0 \tag{4.42}$$

into an equation similar to Equation (4.3). The transformation then yields:

$$\begin{bmatrix} \dot{x}_{trc,r}(i) \\ \dot{y}_{trc,r}(i) \\ \dot{z}_{trc,r}(i) \end{bmatrix} = \underline{R_{br}(i)}^T \begin{bmatrix} \dot{x}_{trc,b}(i) \\ \dot{y}_{trc,b}(i) \\ \dot{z}_{trc,b}(i) \end{bmatrix}, \quad i = 1,2,3,4 \,. \tag{4.43}$$

4.5 Steering system sub-model

The steering system has an important influence on vehicle straight line stability. It affects the vehicle's steering response because it transmits the steering wheel angle to the front wheels. Furthermore, it has a supplementary influence on the subjective judgment of vehicle straight line stability through the steering wheel couple.

In vehicle modelling, the steering system is a difficult subject because it has a pronounced non-linear behavior. This non-linear behavior explains the differences which have been measured between the vehicle responses to small and medium steering wheel angle inputs (cf. Section 3.4). Steering systems contain friction, and, in addition, free play and non-linear power steering can also be present. Free play is ignored in the model because only rack and pinion steering systems are taken into account. Such steering systems hardly ever contain significant free play. Servo-assisted steering, on the contrary, can be represented by the model.

The objective of the vehicle modelling was optimum accuracy under the relevant circumstances. Therefore, the steering system model should contribute to the accurate simulation of the vehicle's steering response to small steering angles. However, the modelling of the behavior of the steering system and of its friction is particularly complicated if steering angles remain small. The friction yields a complete interaction between the responses to different inputs and to different frequency components of one input. Moreover, it has several origins within the steering system. Friction can be found in the bearings of the steering column, on the rack, around the steering axes of the front wheels, etc.

The steering system sub-model which has been developed is a rather simple model in which the steering angles of the front wheels are composed of an elasto-kinematic angle and of an angle produced by rack displacement. Friction is taken into account in the computation of the rack displacement. It is supposed to be concentrated on the rack and has been modelled by a simple empirical model.

The alternative approach, the development of a complete physical model of the steering system has not been chosen because it yields a model with little practical value, especially in early stages of vehicle development. A physical model of the steering system must be very sophisticated if steering input remains small. It should contain accurate descriptions of all steering system frictions. Moreover, it should contain a dynamic model of the steering system which is accurate up to the natural frequency of the unsprung masses on the tires. It may be expected that a physical model of the steering system even requires a tire model which yields accurate lateral forces and self-aligning torques up to this frequency of about 15 Hz.

One of the disadvantages of the empirical model which has been developed is the fact that it requires parameter data from complete vehicle tests. Another disadvantage is that, due to its simple nature, the model cannot be used to represent the rather non-linear steering response on the mildly undulating road.

The model of the friction on the rack will be discussed in Section 4.5.1 which describes the modelling of a steering system without power steering. The elements of the model that are related to the servo-assistance are described in Section 4.5.2. Section 4.5.3 discusses the interface between the rack and the front wheels. Finally, in Section 4.5.4, the model's dynamic behavior around zero will be compared with the test results described in Section 3.4.

4.5.1 Rack and pinion without servo-assistance

In the steering system sub-model, the steering angle $\alpha_t(i)$ of a front wheel contains the initial steering angle $\alpha_0(i)$, the elasto-kinematic steering angle $\alpha_s(i)$, and the steering angle $\alpha_d(i)$ caused by the rack displacement:

$$\alpha_t(i) = \alpha_0(i) + \alpha_s(i) + \alpha_d(i), \quad i = 1,2 \ . \tag{4.44}$$

The elasto-kinematic steering angles of the front wheels are calculated in the same way as at the rear axle (see Equation (4.28)). They are a function of the mean suspension deflection at the front axle, of the roll angle between the car body and the front suspension, and of the lateral and longitudinal forces as well as the self-aligning torques at both front wheels.

Figure 4.6 shows the model of the rack which generates the lateral rack displacement y_{rack} as a product of the steering wheel angle α_{sw} and of the forces F_1 and F_2 from the front wheels on the rack. The forces F_1 and F_2 are produced by the interface between the rack and the wheels which will be discussed in Section 4.5.3. They do not depend on the lateral displacement of the rack because the dynamic tire model does not yield an instantaneous response to a steering input (cf. Section 4.6). The interface between the rack and the wheels also transforms the lateral rack displacement into the steering angles $\alpha_d(i)$.

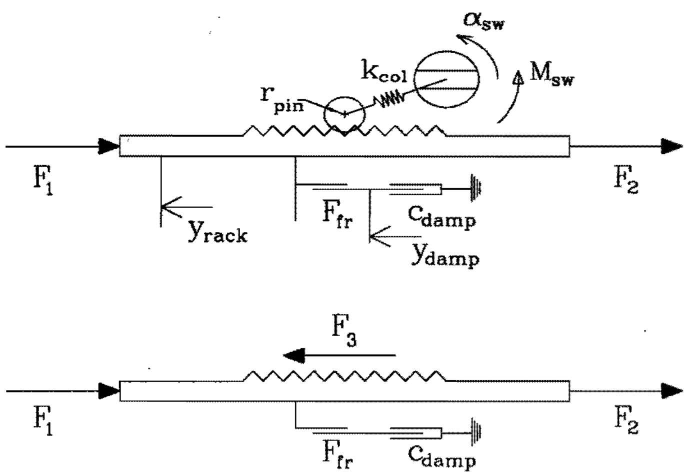


Figure 4.6 Model of the rack in a rack and pinion steering system without power steering

Experience has shown that, under stationary circumstances, rack and pinion steering systems contain Coulomb friction. The resulting friction force can take any value up to a limiting value. Friction in a steering system is technically inevitable and a certain level of friction is desired for its contribution to the steering feeling. However, when driving on a straight road, a driver can control the course of his vehicle with small steering wheel angles and with small steering wheel couples. This would not be possible with a pure Coulomb friction between the steering wheel and the front wheels.

The different behavior of the steering system under driving conditions can be explained by the vibrations that are caused by the road surface and by the engine. These vibrations yield lateral forces on the rack which may assist the driver in overriding the friction. The vibrations can also directly influence the friction through the modification of contact conditions. Both effects may be expected to have a larger influence during small slowly executed steering actions than during large quickly executed steering actions. Therefore, the resulting steering system behavior has been modelled by the addition of an imaginary damper with damping coefficient c_{damp} to the concentrated Coulomb friction with maximum force F_{fr} which acts on the rack. Other important parameters of the steering system sub-model are the radius r_{pin} of the pinion and the rotational stiffness k_{col} of the steering column.

The steering system shown in the scheme of Figure 4.6 has been implemented in the overall vehicle model. At every time step t during a simulation, the force F_3 from the pinion on the rack is first calculated without sliding in the friction element:

$$F_3 = \frac{k_{col}}{r_{pin}}\left(-\alpha_{sw}(t) - \frac{y_{rack}(t)}{r_{pin}}\right) . \tag{4.45}$$

Without sliding, the rack follows the damper. Hence, under these circumstances, the present rack displacement $y_{rack}(t)$ depends on the present damper position $y_{damp}(t)$ and on the positions $y_{rack}(t-\Delta t)$ and $y_{damp}(t-\Delta t)$ of the rack and the damper, respectively, at the previous time step $t-\Delta t$:

$$y_{rack}(t) = y_{rack}(t-\Delta t) + y_{damp}(t) - y_{damp}(t-\Delta t) . \tag{4.46}$$

The position of the damper is obtained by integration of the damper velocity $\dot{y}_{damp}(t)$,

$$\dot{y}_{damp}(t) = \frac{F_3 - F_1 - F_2}{c_{damp}} ; \tag{4.47}$$

$$y_{damp}(t) = \int \dot{y}_{damp}(\tau)\, d\tau . \tag{4.48}$$

Subtracting the force F_3 from F_1 and F_2 provides a check if sliding occurs. For sliding not to occur, the following condition must be met:

$$|F_1 + F_2 - F_3| \le F_{fr} . \tag{4.49}$$

In this case, the calculated rack displacement is correct and the steering wheel couple M_{sw} is given by:

$$M_{sw}(t) = k_{col}\left(\alpha_{sw}(t) + \frac{y_{rack}(t)}{r_{pin}}\right). \qquad (4.50)$$

In case sliding occurs, there are two possibilities. If

$$F_1 + F_2 - F_3 > F_{fr} \qquad (4.51)$$

sliding stops when a new equilibrium is attained at:

$$F_3 = F_1 + F_2 - F_{fr}\ . \qquad (4.52)$$

If

$$F_1 + F_2 - F_3 < -F_{fr}\ , \qquad (4.53)$$

the new equilibrium is

$$F_3 = F_1 + F_2 + F_{fr}\ . \qquad (4.54)$$

The position of the rack after sliding can be calculated from the actual value of F_3 obtained from Equation (4.52) or Equation (4.54):

$$y_{rack}(t) = -\left(\frac{F_3 r_{pin}}{k_{col}} + \alpha_{sw}(t)\right) r_{pin}\ . \qquad (4.55)$$

Equation (4.50) remains valid for the computation of the steering wheel couple.

4.5.2 Rack and pinion with hydraulic servo-assistance

Figure 4.7 shows a schematic presentation of the model of a rack and pinion steering system with servo-assistance. The hydraulic servo-assistance has two important characteristics. First, it applies a force F_{as} on the rack. This force depends on the twist θ_{as} of the torsion bar between the pinion and the steering column:

$$F_{as} = F_{as}(\theta_{as})\ . \qquad (4.56)$$

The function of the torsion bar is the estimation of the steering wheel couple. Therefore, the second characteristic of the hydraulic servo-assistance is the relation between the twist θ_{as} and the steering wheel couple:

$$M_{sw} = M_{sw}(\theta_{as})\ . \qquad (4.57)$$

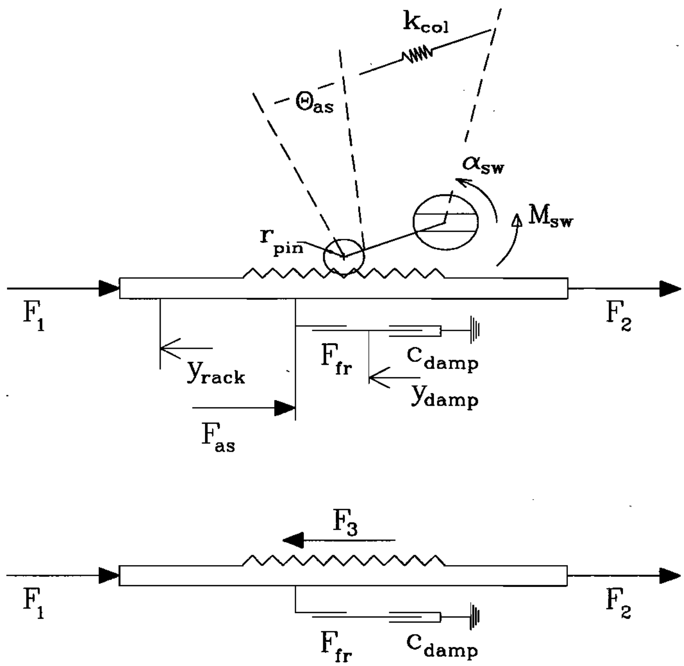


Figure 4.7 Model of a rack and pinion steering system with hydraulic servo-assistance

The relationships of Equations (4.56) and (4.57) are likely to be non-linear. As a result, the model of the servo-assisted steering system contains some iterative computations. As described for the case without power steering, the model first computes the new position of the rack without sliding from Equations (4.46), (4.47), and (4.48). Then the torsion angle θ_{as} which yields identical couples in the torsion bar and in the steering column is determined by an iterative calculation process:

$$M_{sw}(\theta_{as}) = k_{col}\left(\alpha_{sw}(t) + \frac{y_{rack}(t)}{r_{pin}} - \theta_{as}\right) . \tag{4.58}$$

This torsion angle yields the force F_3 on the rack,

$$F_3 = -\frac{M_{sw}(\theta_{as})}{r_{pin}} - F_{as}(\theta_{as}) , \tag{4.59}$$

which is substituted into Equation (4.49) in order to verify the absence of sliding. In case sliding occurs, Equations (4.51) to (4.54) still yield the actual value of F_3. The torsion angle which matches this value of F_3 is determined by an iterative solution of Equation

(4.59). The new position of the rack depends on the steering wheel angle and on the torsion angles of the steering column and the torsion bar:

$$y_{rack}(t) = r_{pin}\left(\theta_{as} - \alpha_{sw}(t) - \frac{(F_3 + F_{as}(\theta_{as}))\, r_{pin}}{k_{col}}\right). \tag{4.60}$$

In a steering system with hydraulic servo-assistance, the steering wheel couple can be obtained from Equation (4.57).

4.5.3 Rack-wheel interface

The computation of the forces F_1 and F_2 on the rack from the forces and moments in the tire-road contact patches and the transfer of the rack displacement into the steering angles at the front wheels require a rack-wheel interface. In accordance with the nature of the model, the interface which has been developed is based on simple equations with coefficients (see Roos [33]). The required coefficients can be obtained from other proprietary simulation programs, whereas the inclusion of an accurate geometric model of the steering system would significantly complicate the model. The coefficients are supplied to the model in tables as functions of the suspension deflection. Since steering angles remain very small, the dependence of the coefficients on the steering wheel angle is disregarded.

The vertical moment from wheel i on the steering system and on the vehicle is denoted by $M_z(i)$. In addition to the self-aligning moment $M_{z,ti}(i)$ from the tire, it also contains a gyroscopic couple caused by the rotating wheel with polar inertial moment $I_{p,w}(i)$ and radius $R_{ti}(i)$:

$$M_z(i) = M_{z,ti}(i) - I_{p,w}(i)(\dot{\theta} + \dot{\beta}_t(i))\frac{V}{R_{ti}(i)}, \quad i = 1,2,3,4 , \tag{4.61}$$

in which $\dot{\beta}_t(i)$ is the relative camber velocity of the wheel with respect to the vehicle. This velocity is computed with the elasto-kinematic suspension model with coefficients β_{sb}, β_{sr}, and β_{sy} which has been described in Section 4.4.2. It is a function of the time derivatives $\Delta\dot{z}$ and $\dot{F}_y$ of the suspension deflection and the lateral force, respectively. For example, the relative camber velocity of the left front wheel is represented by:

$$\dot{\beta}_t(1) = \beta_{sb,f}\frac{\Delta\dot{z}(1) + \Delta\dot{z}(2)}{2} + \beta_{sr,f}\frac{\Delta\dot{z}(1) - \Delta\dot{z}(2)}{l_y(1) - l_y(2)} + \beta_{sy,f}\dot{F}_y(1) . \tag{4.62}$$

The time derivative of the lateral force is approximated by the time derivative $\dot{F}_{y,ti}$ of the lateral force $F_{y,ti}$ which is produced by the tire (cf. Section 4.6):

$$\dot{F}_y(i) = \dot{F}_{y,ti}(i), \quad i = 1,2,3,4 . \tag{4.63}$$

The coefficients in the rack-wheel interface are valid for a vehicle without pitch on an even road. Therefore, the actual longitudinal displacement of the contact patch center on undulating road surfaces has to be taken into account when the vertical moment in the imaginary contact center Q is computed (cf. Figure 4.8). A correct force on the rack is obtained by introducing a supplementary moment

$$F_y(i)\,R_{ti}(i)(\varphi+\frac{\dot{q}_v(i)}{V}), \quad i=1,2 \ , \tag{4.64}$$

on the steering system in addition to the self-aligning moment $M_z(i)$.

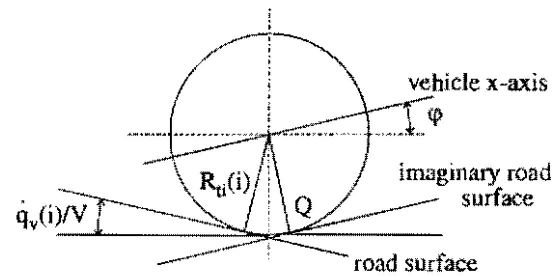


Figure 4.8 Center of the tire-road contact patch on undulating road

4.5.4 Dynamic behavior at small steering angles

The dynamic behavior of the steering system at small steering angles and the influence of the imaginary damper on this behavior will be analyzed for the example of a steering system without servo-assistance. Since servo-assistance is relatively unimportant at small steering angles, the behavior of a steering system with power steering will be rather similar.

If steering angles remain small, no sliding occurs in the friction element and the raek will have the same displacement as the damper:

$$y_{rack}(t)=y_{damp}(t) \ . \tag{4.65}$$

Substitution of the above equation and of Equation (4.45) into Equation (4.47) yields:

$$\dot{y}_{rack}(t)=\frac{-1}{c_{damp}}\left(F_1+F_2+\frac{k_{cal}}{r_{pin}}(\alpha_{sw}(t)+\frac{y_{rack}(t)}{r_{pin}})\right) \tag{4.66}$$

in the time domain and

$$y_{rack}(s) = -\frac{r_{pin}}{1+\frac{r_{pin}^2 c_{damp}}{k_{col}}s}\alpha(s) - \frac{r_{pin}^2}{k_{col} + r_{pin}^2 c_{damp} s}(F_1(s)+F_2(s)) \tag{4.67}$$

in the domain of the Laplace variable s.

Hence, the imaginary damper introduces a first order filter between the steering wheel angle and the front wheels. The most important effect of this filter is a supplementary phase delay between the steering wheel input and the vehicle outputs. Like the difference between the phase delays of the vehicle's directional responses to medium steering wheel angle inputs and on the rather strongly undulating road, this supplementary phase delay increases with frequency. Therefore, the empirical friction model seems capable of simulating the vehicle's steering response on the rather strongly undulating road.

4.6 Tire sub-model

From empirical experience it is known that the tires have an important influence on a vehicle's straight line stability on undulating road surfaces. In addition, tire modelling is difficult because the tire is a rather complicated product with a behavior which is hard to characterize experimentally. For these reasons, a special tire sub-model has been developed.

Tire modelling for the simulation of vehicle straight line stability cannot be carried out with standard tire models. The specific conditions during straight line stability tests require the inclusion of some components which are normally neglected in tire models for roadholding simulations. For instance, on undulating straight roads, driving velocity is generally rather high and slip angles remain very small. Furthermore, vehicle straight line stability is a dynamic vehicle characteristic whereas most tire models for roadholding simulations ignore the dynamic aspects of tire behavior.

In tire modelling, several methods can be used which yield different types of models. The type of model which has been selected for the simulation of vehicle straight line stability is the hybrid one. If possible, physical equations are used in combination with parameters that have been obtained from actual tire behavior. A hybrid tire model can yield accurate simulation results and is much faster than a completely physical tire model. As a result of the complicated composition of the tire, a physical tire model, for instance a finite element model, can hardly be used for complete vehicle simulations. In addition, a completely empirical approach is not very suitable for the modelling of dynamic tire behavior; the information which is required for a reliable model selection is often not available.

The input to the tire model is composed of the tire load $F_{z,ti}(i)$ and of several geometric variables such as the horizontal velocities in the contact patch center and the camber angle between the wheel and the road surface. At the output side, the model yields three variables: the lateral force $F_{y,ti}(i)$ produced by the tire, its time derivative $\dot{F}_{y,ti}(i)$, and the tire's self-aligning torque $M_{z,ti}(i)$. The longitudinal force in the tire is neither computed by the model nor entered as an input. Since this force is small and (almost) constant, its influences on the lateral force and on the self-aligning torque are disregarded.

Figure 4.9 shows the local coordinate system which is used by the tire model. The x-axis of the tire coordinate system is parallel to the wheel mid-plane and to the road surface and is directed in driving direction. The y-axis is parallel to the road surface and directed to the left. Hence, the tire coordinate system is different from the vehicle coordinate system. Nevertheless, the z-components of the force and the moment between the vehicle and the tire are not modified between both coordinate systems. The tire vertical load $F_{z,ti}(i)$ and the self-aligning torque $M_{z,ti}(i)$ are much larger than the other forces and moments in the tire and all angles remain small. The tire lateral force $F_{y,ti}(i)$, on the other hand, has to be transformed before it can be substituted into the vehicle model. It has to be corrected for the road and vehicle inclinations. In this way, the lateral force $F_y(i)$ in vehicle coordinates contains the influence of the tire load which is very important for the modelling of vehicle behavior on undulating road surfaces (cf. Section 2.4).

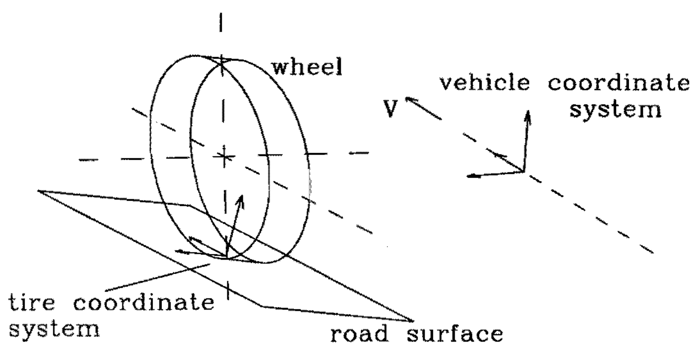


Figure 4.9 Tire coordinate system

In the tire model, the lateral force $F_{y,ti}(i)$ and the self-aligning torque $M_{z,ti}(i)$ are composed of two components. The most important contributions are the lateral force $F_{y,\delta}(i)$ and the self-aligning torque $M_{z,\delta}(i)$ that are generated by the effective slip angle in the contact patch. This effective slip angle $\delta_e(i)$ contains the slip angle $\delta(i)$ which can be computed from the lateral and longitudinal velocities in the tire-road contact patch (cf. Section 4.4.3):

$$\delta(i) = \arctan\left(\frac{\dot{x}_{trc,r}(i)\sin(\alpha_t(i)) - \dot{y}_{trc,r}(i)\cos(\alpha_t(i))}{\dot{x}_{trc,r}(i)\cos(\alpha_t(i)) + \dot{y}_{trc,r}(i)\sin(\alpha_t(i))}\right). \tag{4.68}$$

In addition, the effective slip angle contains a contribution from a gyroscopic effect. For the modelling of this gyroscopic effect, the tire has been replaced by a rigid belt with a flexible foundation on the rim (see Figure 4.10). The polar inertial moment of the belt is denoted by $I_{ti,p}$ and the resulting rotational stiffness between the belt and the rim under relative rotation around the vertical axis is denoted by $k_{ti,\delta}$.

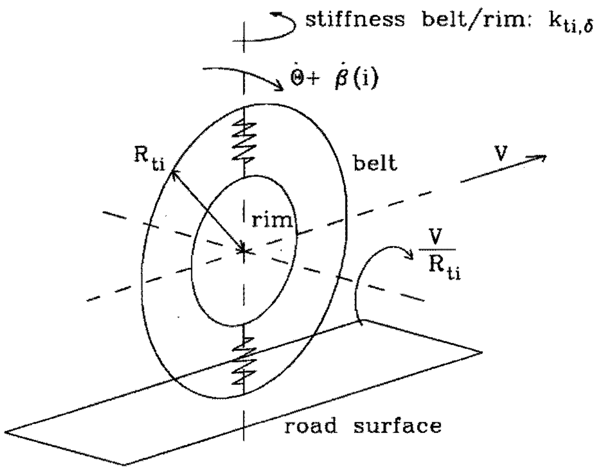


Figure 4.10 Rigid belt model of the gyroscopic effect

If the wheel rotates around the longitudinal and lateral axes, the polar inertial moment of the belt yields a gyroscopic couple. This couple is transmitted to the rim by the flexible zone between the rim and the tire belt. It therefore yields a rotation of the tire belt with respect to the rim which, in the contact patch, acts as a supplementary slip angle. This so-called "inertial" slip angle $\delta_{in}(i)$ depends on the driving velocity V, the tire radius R_{ti}, the stiffness $k_{ti,\delta}$, the polar inertial moment $I_{ti,p}$, and the roll velocities $\dot{\theta}$ and $\dot{\beta}_t(i)$ of the car body and of the wheel, respectively:

$$\delta_{in}(i) = -\frac{I_{ti,p}}{k_{ti,\delta}}\frac{V}{R_{ti}}(\dot{\theta}+\dot{\beta}_t(i)). \tag{4.69}$$

The effective slip angle $\delta_e(i)$ is given by:

$$\delta_e(i) = \delta(i) + \delta_{in}(i) \ . \tag{4.70}$$

The gyroscopic contribution to the slip angle is significant because, during straight line keeping on undulating roads, driving velocity is relatively high and steering angles remain very small. In addition, the undulating roads generate rather important roll velocities.

Physical models for the description of the dynamic tire response to slip angle inputs have been presented by several authors. One of the models which can be used is the stretched string model (see for instance Pacejka [27, 30]). In this model, the tire is represented by a stretched string which, in lateral direction, is flexibility connected to the rim. Simplification of the mathematical description of the stretched string model yields two first order differential equations [27]:

$$\frac{\sigma_\delta}{V} \frac{dF_{y,\delta}(i)}{dt} + F_{y,\delta}(i) = C_{f,\delta}\, \delta_e(i) \tag{4.71}$$

and

$$\frac{\sigma_\delta}{V} \frac{dM_{z,\delta}(i)}{dt} + M_{z,\delta}(i) = C_{m,\delta}\, \delta_e(i) \ , \tag{4.72}$$

where σ_δ denotes the relaxation length related to the slip angle and $C_{f,\delta}$ and $C_{m,\delta}$ denote the cornering stiffness and the self-aligning stiffness, respectively. The tire parameters in Equations (4.71) and (4.72) are functions of the tire load: $\sigma_\delta = \sigma_\delta(F_{z,ti})$, $C_{f,\delta} = C_{f,\delta}(F_{z,ti})$ and $C_{m,\delta} = C_{m,\delta}(F_{z,ti})$.

In the tire sub-model, the lateral force and the self-aligning torque are treated as linear functions of the slip angle. This disregard of the tire's saturation with increasing slip angle is allowed because the slip angle remains very small during straight line stability tests on undulating road surfaces. In fact, Equations (4.71) and (4.72) can only be derived for the tire's response to (small) slip angle variations. However, these equations can also be used for the simulation of the response to load variations if the load variations and the slip angle remain small. Hence, if no slip occurs in the contact patch. This has been shown by Takahashi [45] and Pacejka [32] who successfully applied similar equations for the estimation of the effective cornering stiffness on uneven roads.

The second contribution to the lateral force and to the self-aligning torque is produced by the camber angle $\gamma(i)$:

$$\gamma(i) = \sigma(i) - \theta - \beta_t(i) \ . \tag{4.73}$$

Dynamic camber response cannot be modelled by equations having a physical origin such as Equations (4.71) and (4.72) because suitable camber models are not available. As a result of one of the few investigations of dynamic camber that have been carried out, Segel [41] has found that if the camber angle of a tire is increased in a stepwise fashion, 20% of the camber force is immediately available and that the remaining part is related to a relaxation length.

However, the lateral force and the self-aligning torque that are produced by camber are small when compared to the force and moment that are produced by the slip angle. Therefore, the immediate response to camber is ignored and the following approximations are used for the lateral force and self-aligning torque components $F_{y,\gamma}(i)$ and $M_{z,\gamma}(i)$, respectively, that are produced by camber:

$$\frac{\sigma_\gamma}{V}\frac{dF_{y,\gamma}(i)}{dt} + F_{y,\gamma}(i) = C_{f,\gamma}\,\gamma(i) \tag{4.74}$$

and

$$\frac{\sigma_\gamma}{V}\frac{dM_{z,\gamma}(i)}{dt} + M_{z,\gamma}(i) = C_{m,\gamma}\,\gamma(i)\ , \tag{4.75}$$

where $\sigma_\gamma = \sigma_\gamma(F_{z,ti})$, $C_{f,\gamma} = C_{f,\gamma}(F_{z,ti})$ and $C_{m,\gamma} = C_{m,\gamma}(F_{z,ti})$ are the relaxation length and the stiffnesses related to camber, respectively. Since the measurement of dynamic camber response cannot be carried out with the aid of conventional laboratory equipment, the relaxation length σ_γ is approximated by the one related to the slip angle:

$$\sigma_\gamma = \sigma_\delta\ . \tag{4.76}$$

The complete lateral force $F_{y,ti}(i)$, its time derivative $\dot{F}_{y,ti}(i)$, and the self-aligning torque $M_{z,ti}(i)$ that are produced by tire i yield:

$$F_{y,ti}(i) = F_{y,\delta}(i) + F_{y,\gamma}(i)\ , \tag{4.77}$$

$$\dot{F}_{y,ti}(i) = \dot{F}_{y,\delta}(i) + \dot{F}_{y,\gamma}(i)\ , \tag{4.78}$$

and

$$M_{z,ti}(i) = M_{z,\delta}(i) + M_{z,\gamma}(i)\ . \tag{4.79}$$

The gyroscopic couple which produces the inertial slip angle δ_{in} is not included in the tire's self-aligning torque because it is taken into account in Equation (4.61).

Chapter 5

Vehicle Model Validation

5.1 Introduction

The vehicle model for the simulation of straight line stability has been validated in three stages (Roos [34]). First, the model's response to medium steering wheel angle inputs has been compared to measurement results. The second vehicle characteristic which has been validated is the steering response which, in measurements, can be derived from vehicle straight line stability tests on the rather strongly undulating road. Finally, the model's response to variations of the global road inclination has been compared to measurement results. Other road responses have not been validated. The reason is that, in the case of excitation by the local road inclinations, the input signals are difficult to measure and, in the case of excitation by the mean road height, examination of the small and non-linear response is very complicated. In addition, the responses to the local road inclinations, the mean road height, and the global road inclination are based on similar phenomena.

The response to small steering wheel angle inputs has only been validated on the rather strongly undulating road because, on this undulating road, the partial coherence is sufficiently high for a validation with the aid of transfer functions. On the mildly undulating straight road, on the contrary, the vehicle's steering response is non-linear which necessitates a validation in the time domain (see Section 3.4). Such a validation method cannot be employed because of the unfavorable signal to noise ratio during straight line stability tests. Moreover, the non-linear response to the small steering wheel angle input on the mildly undulating road has been ignored in the steering system model.

The objective of the modelling, the accurate simulation of the basic qualities of vehicle straight line stability on undulating road surfaces under the relevant circumstances, has already been stated in Chapter 1. An assessment of the acceptable inaccuracies, however, is difficult. The reason is that the vehicle responses to road inputs and to small steering wheel angle inputs have hardly been discussed in literature. Simulations of vehicle responses to medium steering wheel angle inputs, on the other hand, have been reported, but only rarely in combination with measurement results.

5.2 Steering response

5.2.1 Medium steering wheel angles

The vehicle responses to medium steering wheel angle inputs which have been measured and simulated are responses to pseudo-harmonical inputs applied by a test driver. The amplitude of the steering wheel angle was about 15 degrees which corresponds to a lateral acceleration amplitude of 2 to 3 m/s^2. For the simulation of this kind of steering responses, a simple friction in the steering system suffices. Therefore, a very large value has been assumed for the damping c_{damp} in the steering system:

$$c_{damp} \rightarrow \infty \ . \tag{5.1}$$

Figure 5.1 shows the measured and simulated responses of Vehicle A1. It shows that the measured coherences between the steering wheel angle input and the yaw velocity and lateral acceleration outputs are close to unity. Only for frequencies above 1.5 Hz, the coherence between the steering wheel angle and the lateral acceleration is a little lower. This phenomenon can be explained by the difficulty to supply sufficient excitation amplitudes in this frequency range and by the decreasing gain of the transfer function between the steering wheel angle and the lateral acceleration. However, frequencies higher than 1.5 Hz are of little interest for vehicle straight line stability.

As stated in Section 3.4, the transfer function between the steering wheel angle and the steering wheel couple does not suffice for the characterization of the relationship between these two variables. Despite similar measurement conditions, the measured coherence between the steering wheel angle and the steering wheel couple is significantly smaller, especially at low frequencies, than the coherences between the steering wheel angle as input and the yaw velocity and the lateral acceleration as outputs. Moreover, the steering wheel couple response may be expected to contain a large non-linear influence of the friction in the steering system. Therefore, the relationship between the steering wheel angle and the steering wheel couple has been validated in the time domain (cf. Figures 5.2 and 5.3).

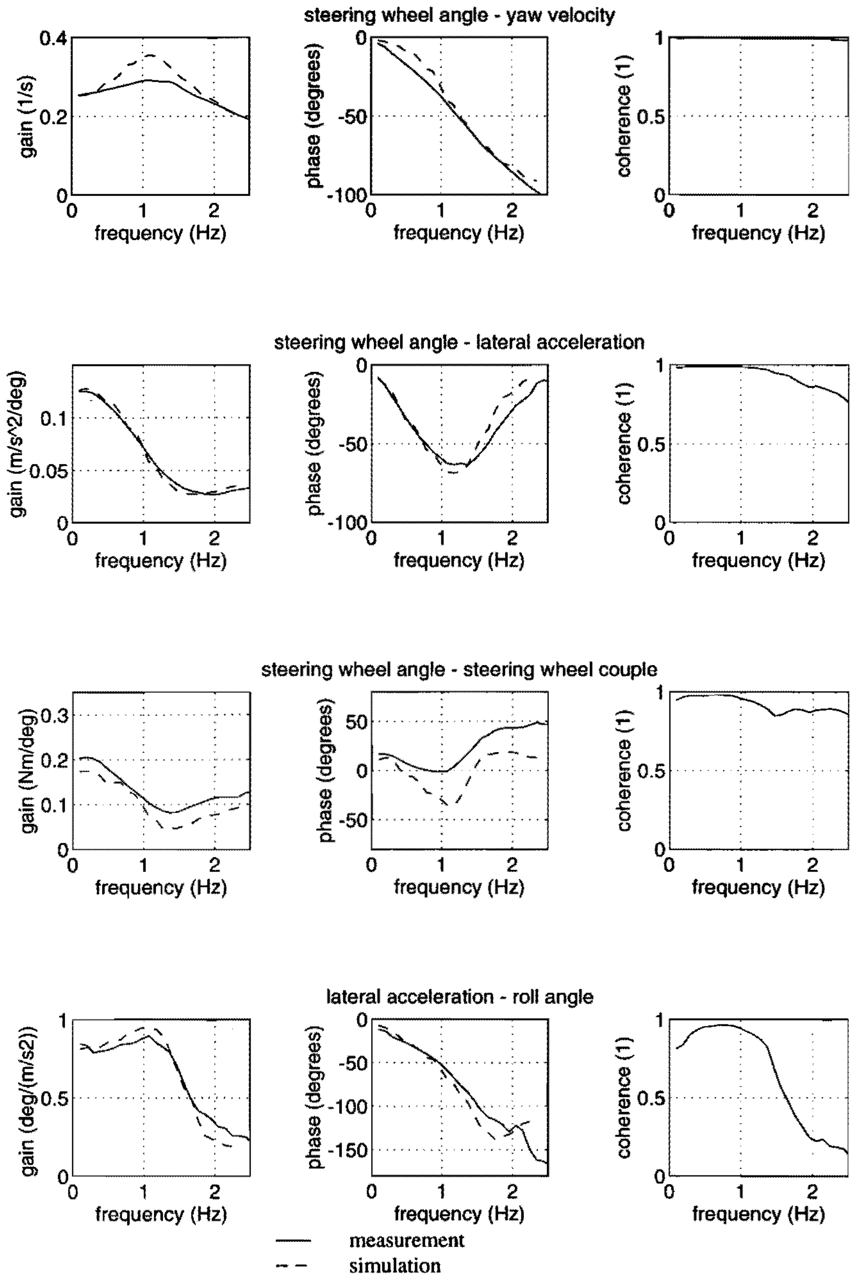


Figure 5.1 *Transfer functions between the steering wheel angle and three vehicle outputs as well as between the lateral acceleration and the roll angle (medium steering wheel angle input on even road, Vehicle A1, V = 110 km/h, steering wheel angle amplitude ≈ 15 degrees)*

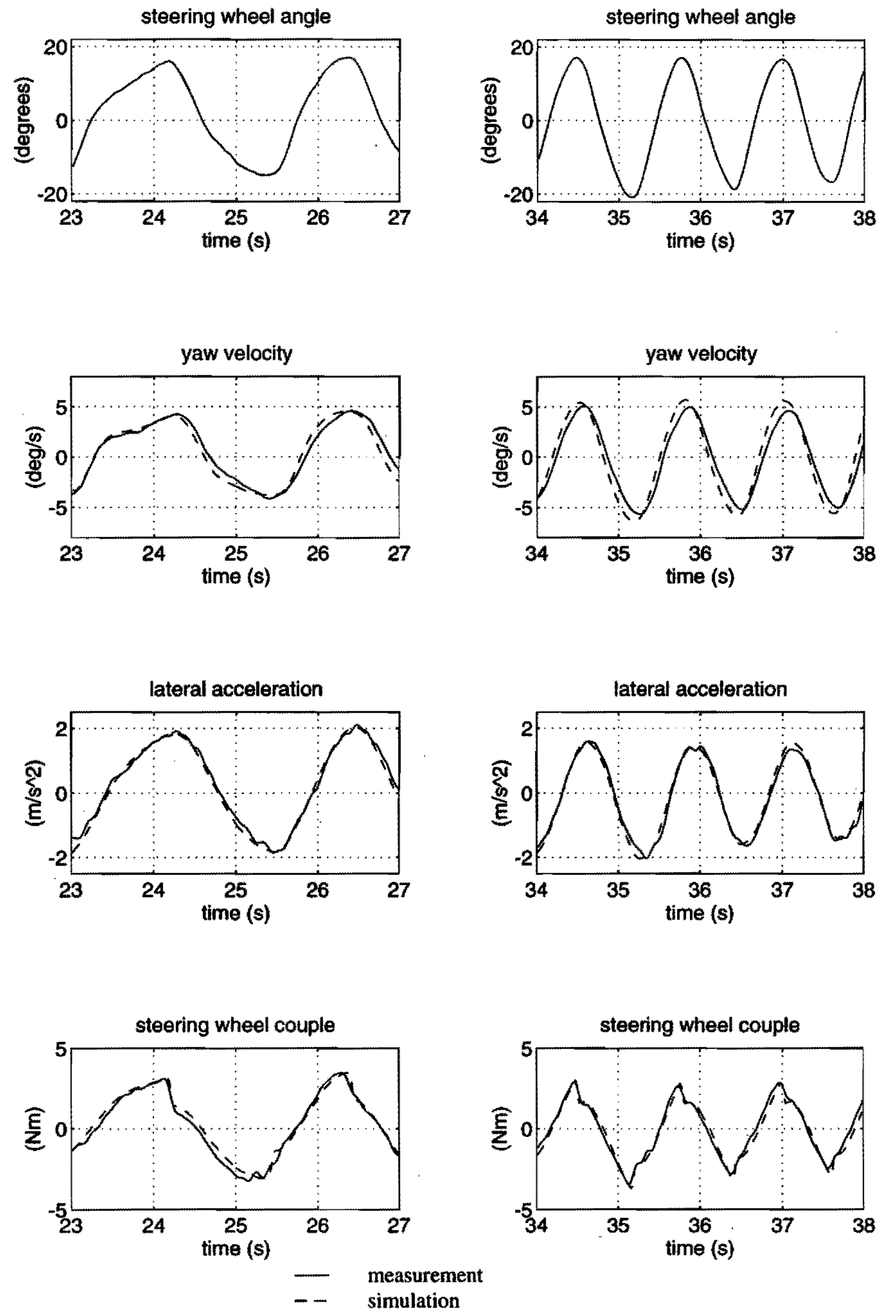


Figure 5.2 Steering response to medium steering wheel angle input (even road, Vehicle A1, V = 110 km/h, frequencies: approximately 0.4 and 0.8 Hz)

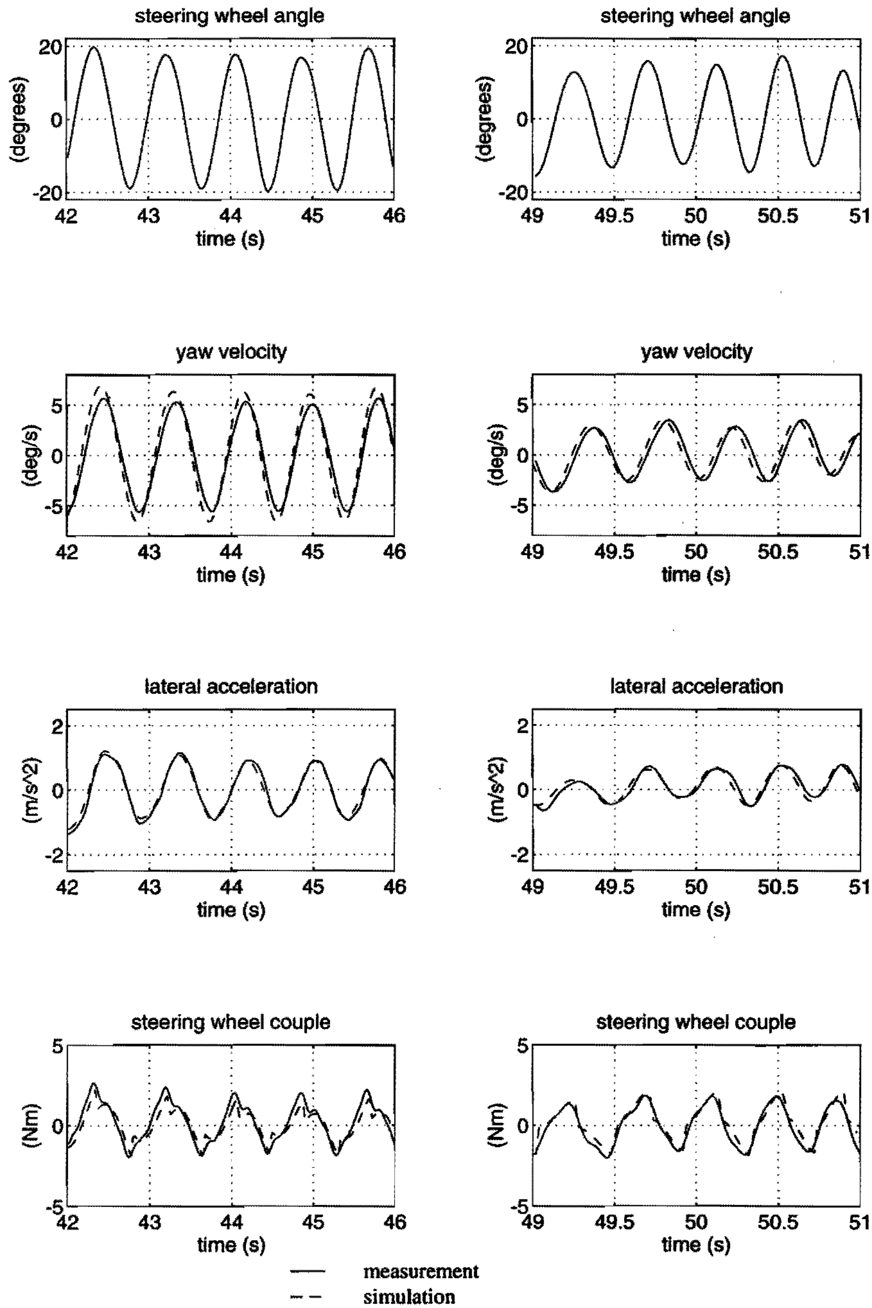


Figure 5.3 Steering response to medium steering wheel angle input (even road, Vehicle A1, V = 110 km/h, frequencies: approximately 1.2 and 2.2 Hz)

The coherence between the lateral acceleration and the roll angle is rather small for frequencies between 0 and 0.4 Hz and above 1.3 Hz. The lack of coherence above 1.3 Hz is caused by the vanishing ratio between the driver induced vehicle roll and the global road inclinations. The relatively small coherence below 0.4 Hz is caused by the limited length of the test track which necessitates interruptions during the measurements. These interruptions affect the coherence at low frequencies because, in combination with variations in the mean global road inclination, they produce leaps in the roll signal. Hence, the local minima in the coherence function between the lateral acceleration and the roll angle can be explained by the measurement conditions. In addition, it may be expected that the vehicle roll motion has no direct influence on (the judgment of) vehicle straight line stability. Therefore, the transfer function between the lateral acceleration and the roll angle is accepted as the base for the validation of the vehicle roll motion under excitation at the steering wheel.

Comparison of the measured and simulated transfer functions between the steering wheel angle and the yaw velocity shows that the model's damping of the yaw motion is too weak. This phenomenon can be observed in the gain around the resonance frequency and in the phase between 0 and 1 Hz. Conversely, the simulated transfer functions between the steering wheel angle and the lateral acceleration as well as between the lateral acceleration and the roll angle yield a very good correlation with the measurements.

The Figures 5.2 and 5.3 show the measured and simulated steering responses in the time domain. The frequency sweep which has been used for the excitation of the vehicle and the model has been cut into pieces for the accurate validation at different frequencies. The frequencies which have been plotted are about 0.4, 0.8, 1.2, and 2.2 Hz. The lack of yaw damping which has been found in the frequency domain can also be recognized in the time domain. The model yields a yaw response which is too large around 1 Hz and which, especially at low frequencies, is in advance with respect to the measured response. Again, the correlation between the simulated and measured lateral acceleration responses is rather good.

In the time domain, the steering wheel couple which is produced by the model also corresponds very well to the measured couple. Only the amplitude of the simulated response at about 1.2 Hz is slightly too small. This result is rather surprising because the transfer functions between the steering wheel angle input and the steering wheel couple output do not completely correspond. The difference between the correlations of the measured and simulated steering wheel couple responses in the time domain and in the frequency domain is probably the result of the friction in the steering system. In the model, this friction has been modelled by one simple Coulomb friction, whereas, in reality, it might have a different frequency content. Nevertheless, in the case of the non-linear steering wheel couple response, the time domain is of overriding importance.

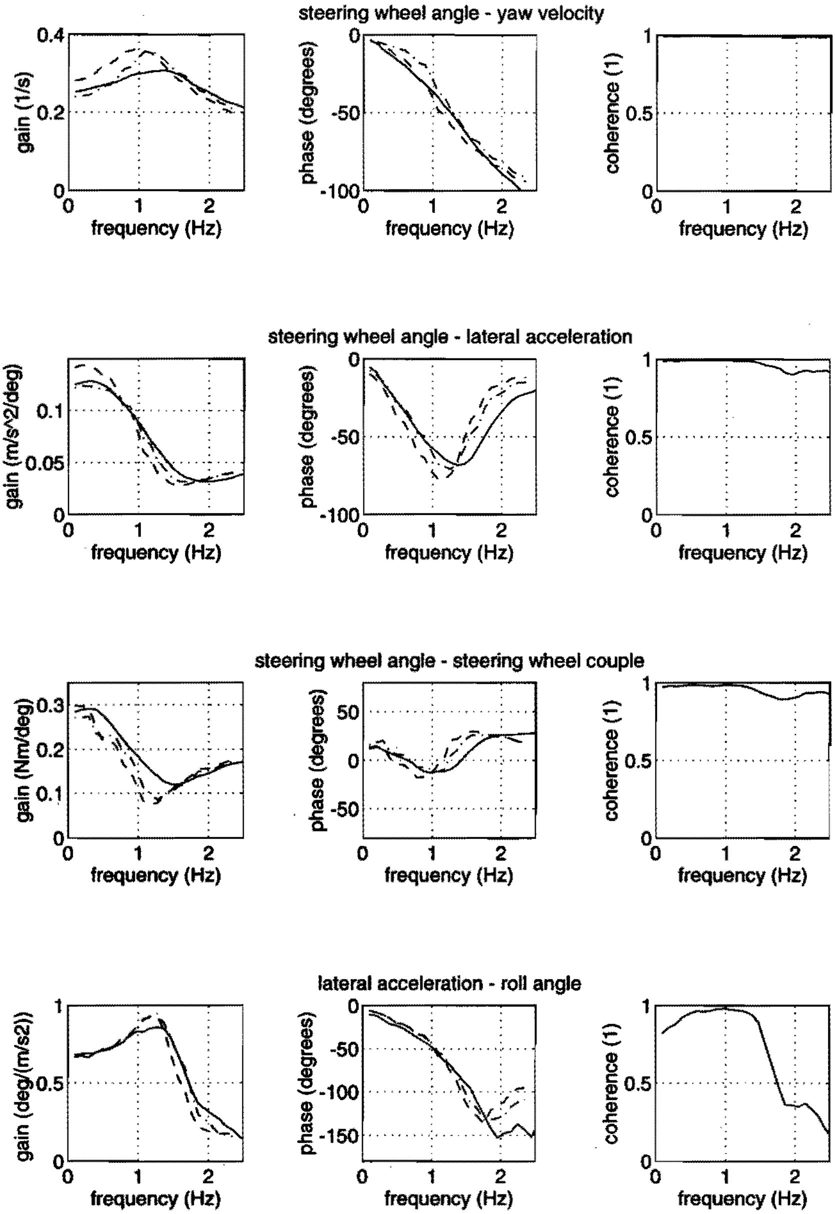
Therefore, it is concluded that the model of Vehicle A1 yields an accurate steering wheel couple response.

The second vehicle which has been used for the validation of the response to medium steering wheel angle inputs is Vehicle B3 (see Figure 5.4). Comparison of the simulation result on the base of the original tire data with the measured transfer functions shows the major problem which has been encountered during the validation. The simulations sometimes yield an incorrect amount of understeer. In the case of Vehicle B3, the model yields responses that are too large and it yields a shift of the phase and gain characteristics towards lower frequencies. Both phenomena correspond to a lack of understeer in the simulation.

The moderate accuracy of the simulated understeer characteristics is probably caused by the tire data. These data are obtained by laboratory measurements of tire behavior on a drum. Simulations of the steering responses of one particular vehicle with various tire sets show that the model can either yield a lack or a surplus of understeer, depending on the specific tire set.

In addition, two explanations can be given for the inaccuracy of the tire data. Tires, like all industrial products, are subject to production variations. Therefore, one or more vehicle tires may be significantly different from the characterization used in the model. The second explanation is based on the fact that a tire's behavior depends on the particular operating conditions such as the roughness of the road surface as well as the road, air, and tire temperatures. Accurate reproduction, during tire measurements in a laboratory, of actual vehicle measurement conditions is infeasible.

The inaccuracy of the tire data not only influences the simulation of vehicle straight line stability. It is a problem for roadholding simulations in general and it must be thoroughly analyzed. Within the present research project, an extensive analysis of the measurement of tire characteristics was infeasible. Nevertheless, a good correlation between simulations and actual vehicle behavior is indispensable for the development of a method for the numerical design of vehicles with optimal straight line stability. Therefore, it has been anticipated that, in the future, accurate tire data will become available. In case the original tire data did not yield the accurate amount of understeer, the most relevant tire parameters, the cornering stiffnesses, have been slightly corrected (cf. Roos [34]).



—— measurement
– – simulation (original tire data)
–·– simulation (optimized cornering stiffnesses)

Figure 5.4 ***Transfer functions between the steering wheel angle and three vehicle outputs as well as between the lateral acceleration and the roll angle (medium steering wheel angle input on even road, Vehicle B3, V = 110 km/h, steering wheel angle amplitude ≈ 15 degrees)***

In the case of Vehicle B3, the optimization of the cornering stiffnesses yields good results. It yields accurate transfer functions between the steering wheel angle and the lateral acceleration as well as between the lateral acceleration and the roll angle. However, the damping of the yaw motion remains too weak. The model's steering wheel couple response correlates rather well with real vehicle behavior. This has been verified with the aid of time domain plots similar to Figures 5.2 and 5.3.

Figure 5.5 shows the steering responses of Vehicle C1. This vehicle yields results that are similar to the results of Vehicle B3. The steering response based on the original tire data contains less understeer than the measured response. Again, a slight optimization of the cornering stiffnesses has been carried out in order to obtain a better correlation. After this correction, the match between the measured and the simulated responses is rather close, except for the damping of the yaw motion which remains too weak.

5.2.2 Small steering wheel angles

The model's response to small steering wheel angle inputs has been validated with the aid of measurements of normal driving on the rather strongly undulating straight road. During these measurements, the two measurable road inputs, the global road inclination and the mean road height, were also determined. The desired transfer functions between the steering wheel angle as input and the yaw velocity, the lateral acceleration and the steering wheel couple as outputs have been obtained with the aid of the data analysis techniques which have been discussed in Section 3.2 and which are demonstrated in Appendix B.

Figure 5.6 shows one measured and two simulated responses of Vehicle A1 to small steering wheel angle inputs. In the measured response, the partial coherences between the steering wheel angle as input and the yaw velocity and the lateral acceleration as outputs are in the order of 0.9 up to 0.7 Hz. Hence the corresponding transfer functions are reliable in this frequency range which covers most of the frequency interval important for vehicle straight line stability. As in the response to medium steering wheel angle inputs, the partial coherence between the steering wheel angle and the steering wheel couple is insufficient, even at low frequencies. Therefore, the measured transfer function between the steering wheel angle and the steering wheel couple has limited value. An alternative validation of the steering wheel couple response in the time domain has not been attempted because the signal to noise ratio is rather low. Moreover, the influence of the road inputs cannot be eliminated in the time domain.

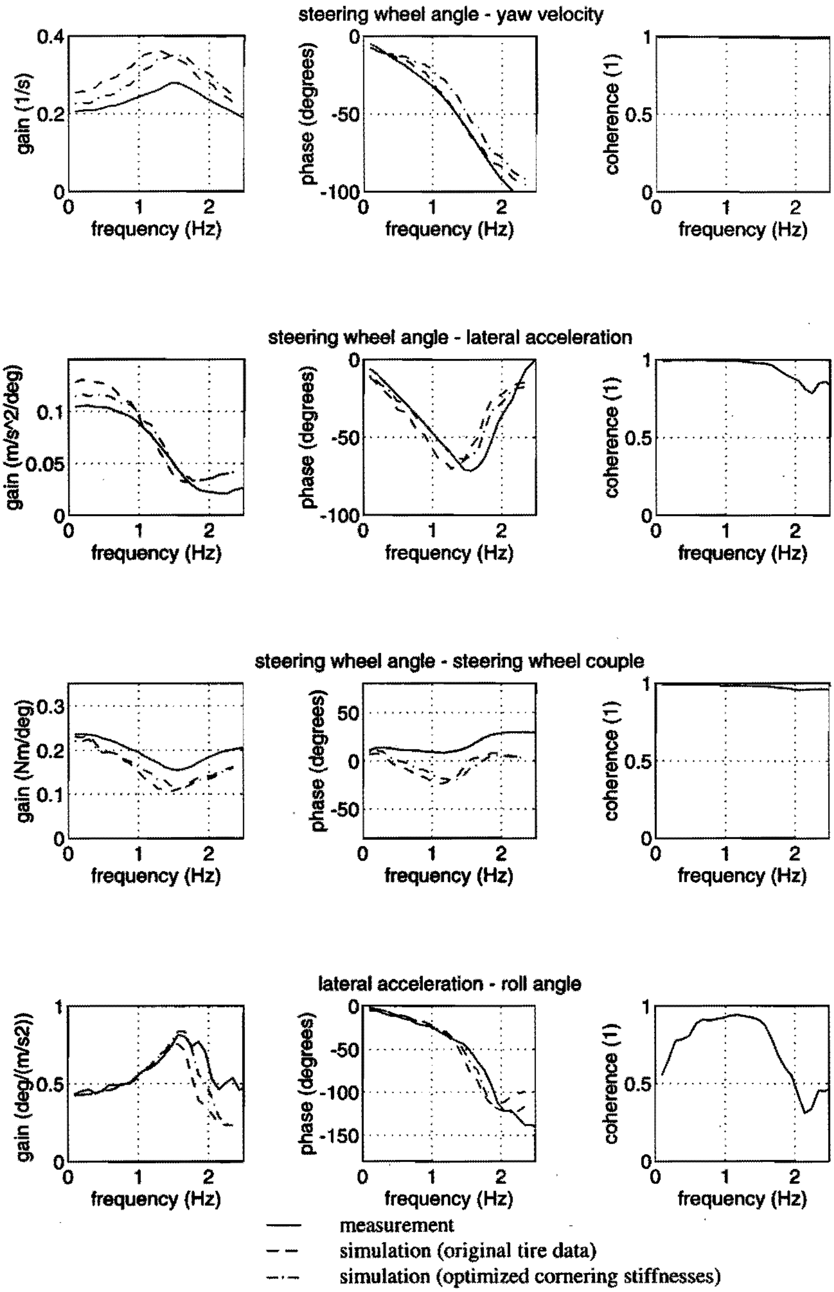
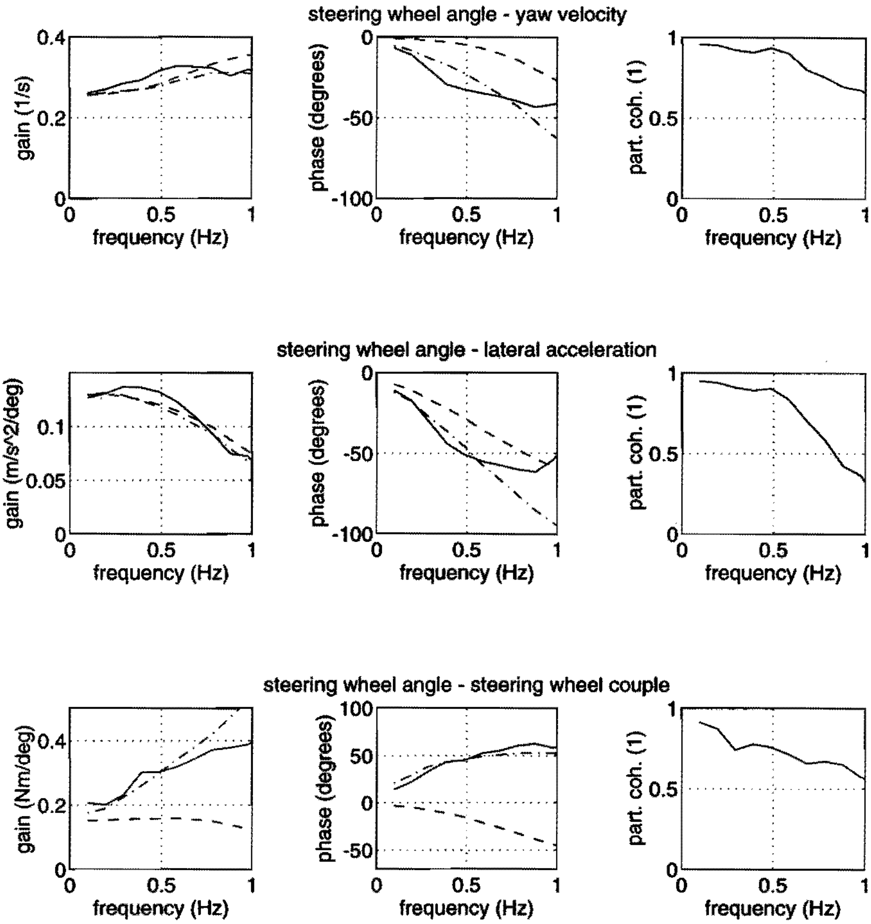


Figure 5.5 Transfer functions between the steering wheel angle and three vehicle outputs as well as between the lateral acceleration and the roll angle (medium steering wheel angle input on even road, Vehicle C1, V = 110 km/h, steering wheel angle amplitude ≈ 15 degrees)



——	measurement
– –	simulation (original tire data, without friction in the steering system model)
–·–	simulation (original tire data, with friction and damping in the steering system model)

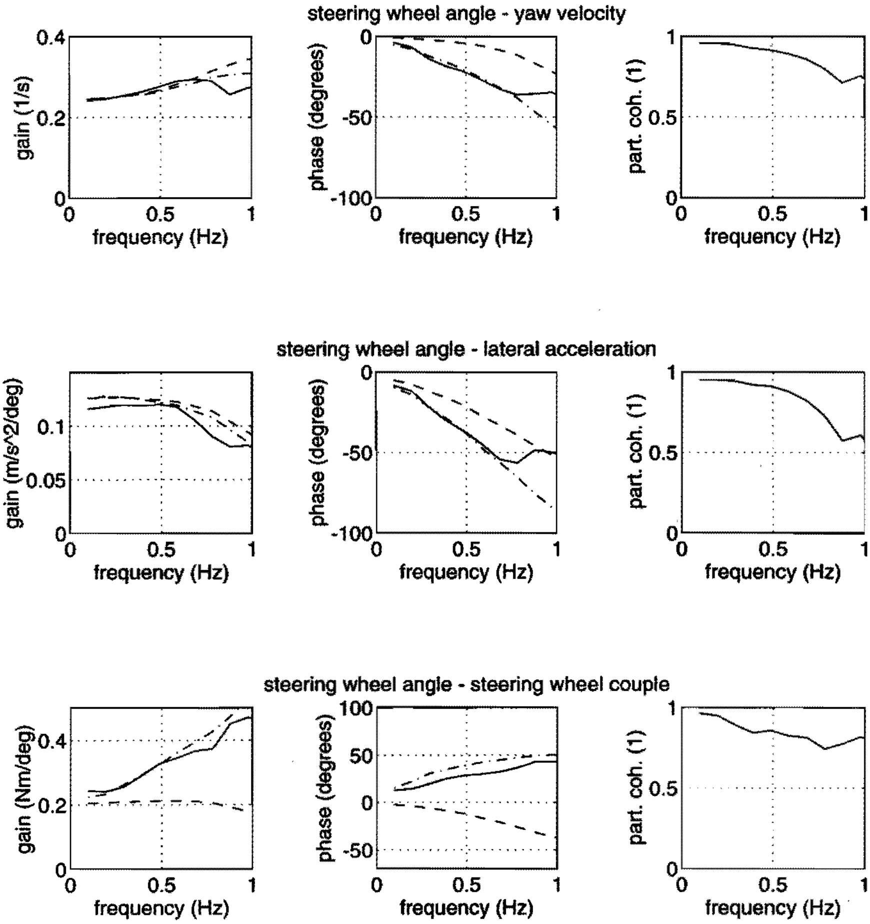
Figure 5.6 Transfer functions between the steering wheel angle and three vehicle outputs (normal driving on the rather strongly undulating road, Vehicle A1, V = 110 km/h; the influences of the correlations between the steering wheel angle, global road inclination, and mean road height inputs have been eliminated from the measurement)

The difference between the two simulated responses of Vehicle A1 is located in the steering system model. One of the results has been obtained without friction in the steering system, whereas the other has been obtained with friction and with optimized values of the damping c_{damp} in the steering system and the stiffness k_{col} of the steering column. Both parameters influence the supplementary phase lag in the steering system (cf. Section 4.5.4).

The gains of the simulated transfer functions between the steering wheel angle and the yaw velocity as well as between the steering wheel angle and the lateral acceleration correlate very well with the measurements. The damping in the steering system is only required for the accurate simulation of the phase characteristics. With damping in the steering system, the model also yields accurate simulations of the phases of the above-mentioned transfer functions.

In spite of the limited value of the measured transfer function between the steering wheel angle and the steering wheel couple, two remarks can be made with respect to this response. First, the good resemblance between the measurements and the model with damping in the steering system increases the confidence in this model. Second, comparison of both simulated transfer functions between the steering wheel angle and the steering wheel couple shows the diminishing influence of the damping with decreasing frequency.

The steering responses of Vehicles B3 and C1 to small steering wheel angle inputs have been plotted in the Figures 5.7 and 5.8, respectively. In both cases, the measured transfer function between the steering wheel angle and the yaw velocity as well as that between the steering wheel angle and the lateral acceleration are valid up to 0.7 Hz. In this interval, the model with damping in the steering system yields accurate simulation results of the gains and phases of the transfer functions mentioned above.

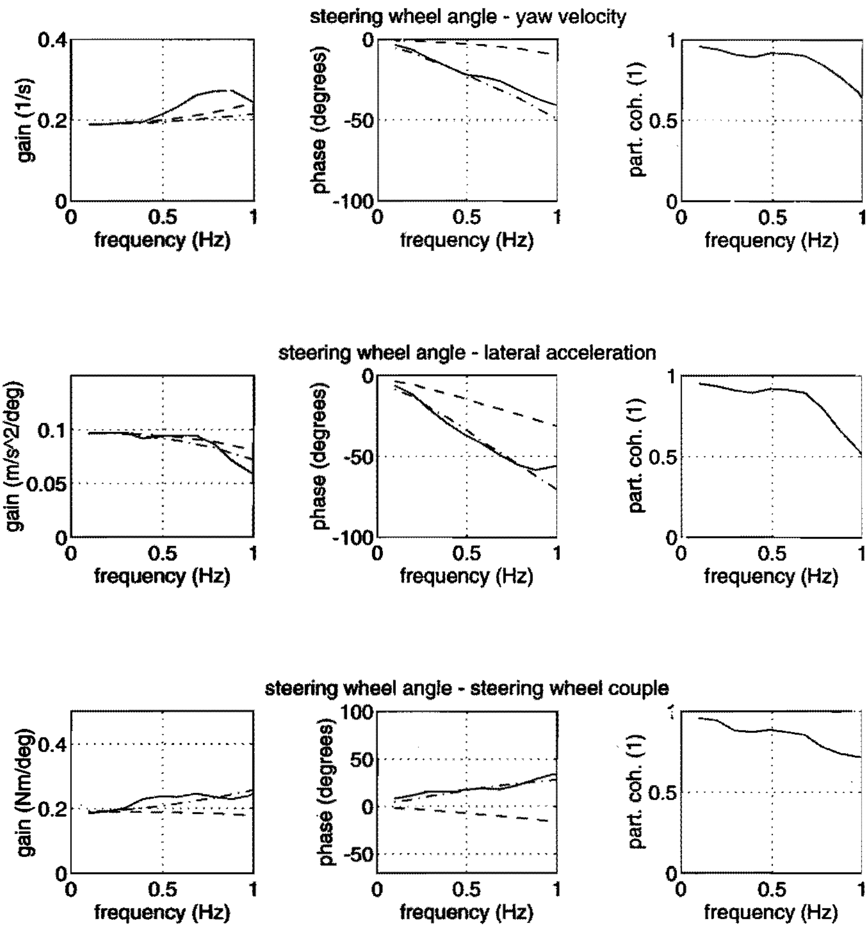


—— measurement

– – simulation (optimized cornering stiffnesses, without friction in the steering system model)

–·– simulation (optimized cornering stiffnesses, with friction and damping in the steering system model)

Figure 5.7 Transfer functions between the steering wheel angle and three vehicle outputs (normal driving on the rather strongly undulating road, Vehicle B3, V = 110 km/h; the influences of the correlations between the steering wheel angle, global road inclination, and mean road height inputs have been eliminated from the measurement)



— measurement

– – simulation (optimized cornering stiffnesses, without friction in the steering system model)

–·– simulation (optimized cornering stiffnesses, with friction and damping in the steering system model)

Figure 5.8 Transfer functions between the steering wheel angle and three vehicle outputs (normal driving on the rather strongly undulating road, Vehicle C1, V = 110 km/h; the influences of the correlations between the steering wheel angle, global road inclination, and mean road height inputs have been eliminated from the measurement)

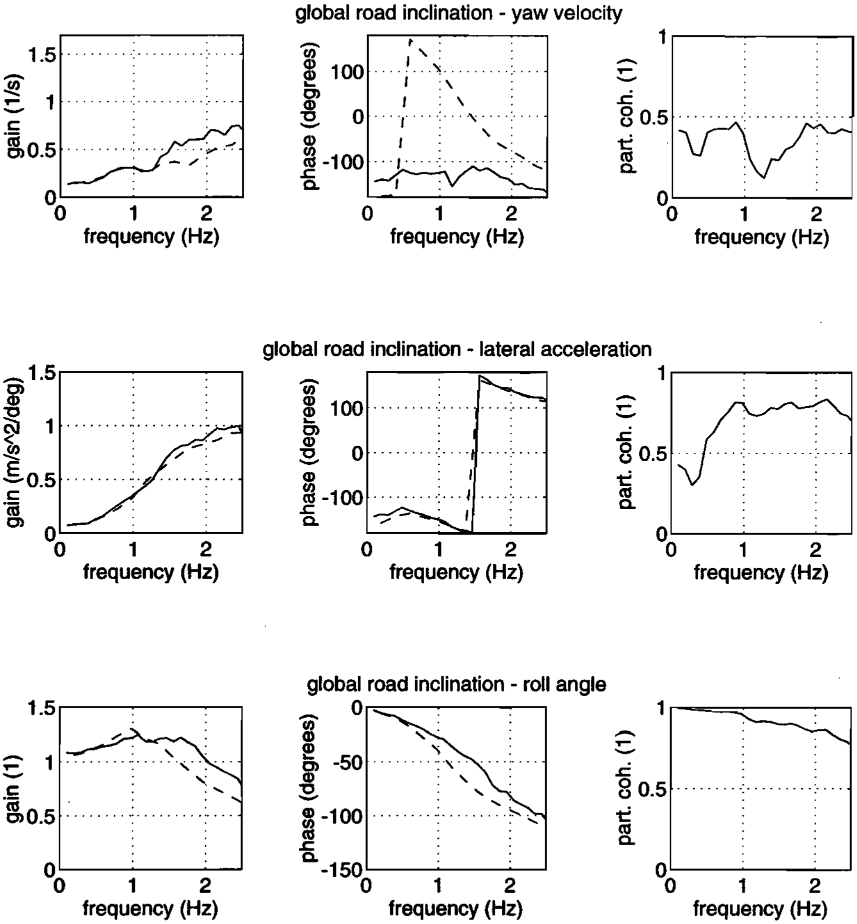
5.3 Response to road undulations

The second vehicle characteristic which is very important for the simulation of vehicle straight line stability on undulating road surfaces is the vehicle's response to road undulations. This response has been validated with the aid of transfer functions between the global road inclination, the difference between the left and right road heights divided by the track width, as input, and the yaw velocity, the lateral acceleration, and the roll angle as outputs.

The measured transfer functions of the response to the global road inclination have been derived from the measurements of normal driving on the rather strongly undulating straight road which have also been used for the validation of the response to small steering wheel angle inputs. Again, the data analysis techniques which have been discussed in Section 3.2 were used for the elimination of the influences from the other measured inputs, i.e. the mean road height and the steering wheel angle.

The simulations of the vehicle responses to the global road inclination have been carried out with the aid of the measured profile of the test road (cf. Appendix A). In this way, the coherences between the global road inclination and the local road inclinations are the same and have identical influences in the simulations and in the measurements. Therefore, the measured and simulated transfer functions of the responses to the global road inclination can be compared. Moreover, the simulations of the responses to the global road inclination have been carried out without friction in the steering system model. At fixed steering wheel angle, this choice yields the best correlation between measurement and simulation.

Figure 5.9 shows the responses of Vehicle A1. The partial coherence between the global road inclination and the yaw velocity is rather low. Nevertheless, there are no reasons to expect significant non-linearities in the relationships of which the transfer functions have been plotted. In Section 4.2, it has been shown that non-linear tire behavior does not influence the relationships between the global road inclination as input and the yaw velocity, the lateral acceleration, and the roll angle as outputs. The influence of non-linear steering system behavior is also of second order because the steering system is not directly implicated in the vehicle's directional response to road inputs. Moreover, the other friction which might yield a non-linear influence on the road responses, the friction in the suspension, is rather small. Therefore, this non-linear influence may be neglected.



—— measurement

– – simulation (original tire data)

Figure 5.9 Transfer functions between the global road inclination and three vehicle outputs (rather strongly undulating road, Vehicle A1, V = 110 km/h; the influences of the correlations between the global road inclination, steering wheel angle and mean road height inputs have been eliminated from the measurement)

The low partial coherence between the global road inclination and the yaw velocity is assumed to be caused by the relatively important measurement noise and by the unfavorable ratio between the vehicle motions that are induced by the global road inclination on the one hand and the motions that are induced by the vehicle inputs which have not been measured on the other hand. Examples of vehicle inputs which have not been measured are the uncorrelated parts of the local road inclinations and the side wind.

In contrast with the low partial coherence between the global road inclination and the yaw velocity, the partial coherence between the global road inclination and the lateral acceleration as well as that between the global road inclination and the roll angle are both relatively high. Therefore, it may be concluded that the corresponding transfer functions are valid for frequencies from 0.8 to 2.0 Hz and up to 1.5 Hz, respectively.

The measured transfer function between the global road inclination and the lateral acceleration is valid from 0.8 to 2.0 Hz. In this frequency range, the correlation between measurement and simulation is rather good. The transfer function between the global road inclination and the roll angle yields a fairly good correlation in the frequency range with sufficient partial coherence (0-1.5 Hz).

Figure 5.10 shows the response of Vehicle C1 to the global road inclination. This vehicle yields a slightly better partial coherence between the global road inclination and the yaw velocity but the limit of 0.7 is hardly reached. However, since the low partial coherence can be explained by the adverse measurement conditions with a lot of (uncorrelated) noise, the measured transfer function is assumed to be reasonably accurate in the frequency ranges where the coherence is higher than 0.5. In addition, the measured and simulated transfer functions between the global road inclination and the yaw velocity correlate well. Therefore, it is concluded that the model yields an accurate simulation of the yaw response of Vehicle C1 to the global road inclination. Again, the measured and simulated transfer functions between the global road inclination and the lateral acceleration as well as between the global road inclination and the roll angle correlate rather well, even outside the intervals with a partial coherence close to or higher than 0.7 (1.4-2.2 Hz and 0-1.8 Hz, respectively).

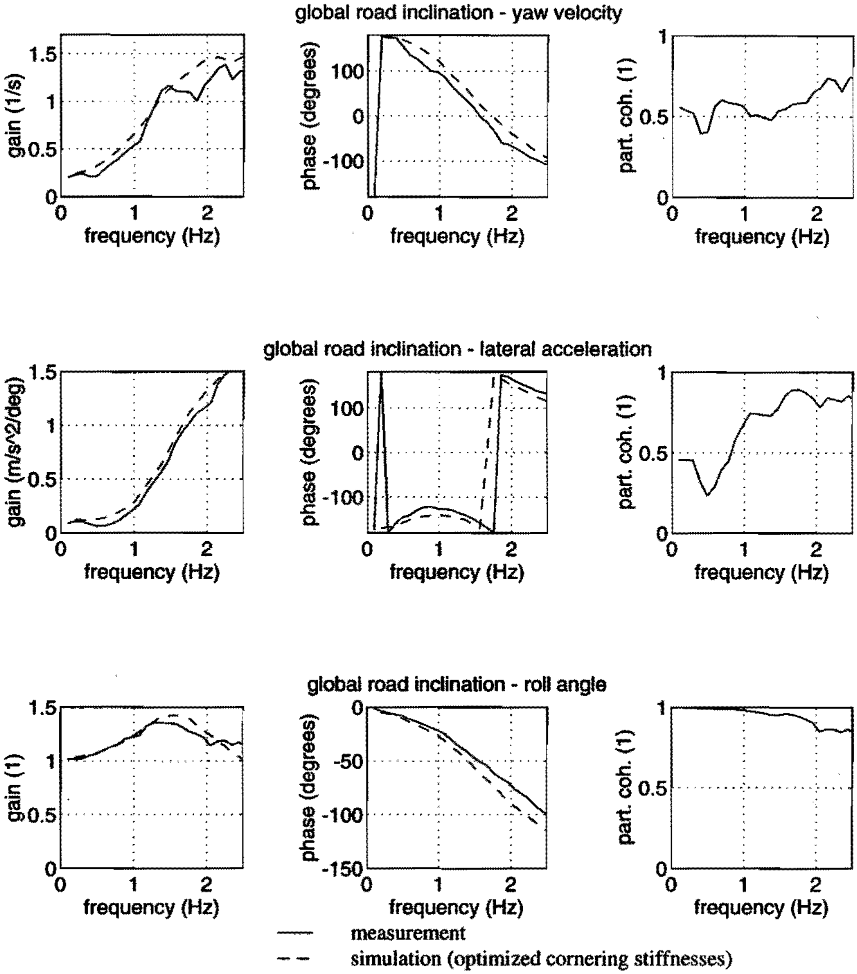


Figure 5.10 Transfer functions between the global road inclination and three vehicle outputs (rather strongly undulating road, Vehicle C1, V = 110 km/h; the influences of the correlations between the global road inclination, steering wheel angle, and mean road height inputs have been eliminated from the measurement)

5.4 Discussion

The major simulation problem which has been encountered during the model validation is the limited precision in predicting the amount of understeer. This problem, which is probably caused by a moderate accuracy of the tire data, requires extensive analysis. Moreover, it is not only a problem for the simulation of vehicle straight line stability; it affects roadholding simulations in general. Therefore, it is suggested as an important topic for additional investigation.

In this thesis, the tire data inaccuracies were compensated by slight corrections of the cornering stiffnesses of the tire sets that, in simulation, yield erroneous amounts of understeer. In this way, a rather good correlation has been obtained between measurements and simulations of the three responses that have been taken into account:

- the vehicle's response to medium steering wheel angle inputs,
- the vehicle's response to (a specific) small steering wheel angle input, and
- the vehicle's response to the global road inclination.

The comparison of measured and simulated responses to pseudo-harmonical steering wheel angle inputs with an amplitude of about 15 degrees (medium steering wheel angle input) has shown that, in general, the simulations are accurate if appropriate tire data are used. However, it has also revealed one shortcoming of the model: its damping of the yaw motion is too weak. This phenomenon can be observed in the simulated transfer functions between the steering wheel angle and the yaw velocity. The gains in these transfer functions are too high around the resonance frequency and their phase lags are too small at low frequencies (cf. Figures 5.1 to 5.5).

The other responses to medium steering wheel angle inputs yield good to very good correlations. The simulated transfer functions between the steering wheel angle and the lateral acceleration as well as between the lateral acceleration and the roll angle correspond very well to the measured ones. Moreover, time domain comparisons have shown that the model produces steering wheel couple responses which correlate well with the experimental results.

The validation of the model's response to small steering wheel angle inputs is based on transfer functions that have been derived from measurements of normal driving on the rather strongly undulating road. Under these circumstances, application of the steering system model with the imaginary damper yields accurate simulations of the vehicle transfer functions between the steering wheel angle as input and the yaw velocity and the lateral acceleration as outputs.

Nevertheless, the steering system model which has been used for the simulation of the responses to the small steering wheel angle inputs has an important drawback. The model's parameters, the stiffness k_{col} of the steering column and, especially, the damping c_{damp}, cannot be derived from the characteristics of steering system components. They have to be identified from measurements of the complete vehicle. Therefore, the value of this part of the steering system model for the prediction of vehicle straight line stability is limited. In addition, the steering system model with the imaginary damper can only be used for the simulation of one particular type of steering response, the response which can be derived from measurements of normal driving on the rather strongly undulating road.

For these reasons, the steering system's specific responses to small steering wheel angle inputs have been disregarded in the prediction of vehicle straight line stability on undulating road surfaces. The steering responses that are computed within the design method are simulated without friction in the steering system model.

Of course, the disregard of the steering system's specific behavior around zero reduces the value of the method for the numerical design of vehicles with optimal straight line stability. However, this behavior can be regarded as a preliminary treatment on the steering wheel angle before transformation, by a system representing the vehicle without friction in the steering system, into the actual vehicle outputs (cf. Figure 5.11). The most important assumption of this representation is the disregard of the vehicle's influence on the steering system's behavior around zero. In the steering system model with damping, this assumption does not introduce large errors because the rack displacements which are produced by the forces from the wheels on the rack are small with respect to the rack displacement produced by the steering wheel angle (cf. Equation (4.67)).

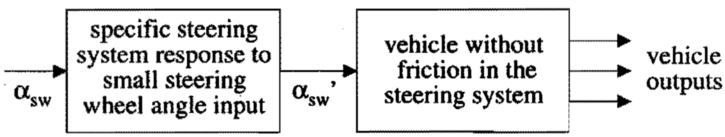


Figure 5.11 Simplified representation of a vehicle's response to a small steering wheel angle input

Hence, the straight line stability simulations without friction in the steering system model can be used for the development and optimization of a fairly large part of the vehicle. Only the parts of the steering system which influence the specific behavior of this system around zero cannot be elaborated by computer simulation. They have to be optimized experimentally with the aid of prototypes.

In spite of its limitations, the numerical design of vehicle straight line stability on undulating road surfaces still means an important improvement upon the actual situation. Generally, the vehicle design parameters that influence the part of the steering response which is taken into account have to be fixed in an early stage of vehicle development. Examples of such parameters are the parameters that describe the suspension geometry and the ratio between the masses on the front and rear axles. The experimental optimization of the specific steering system behavior around zero is less troublesome than the experimental optimization of the complete vehicle because it mostly concerns parameters that can be modified rather easily. Parameters that might influence this behavior are, for instance, the flexibilities of the joints in the steering system and the prestress between the push rod and the steering rack.

The disregard of the steering system's specific behavior around zero also influences the value of the estimation algorithm which will be discussed in Chapter 7. The predicted subjective judgment of a vehicle's straight line stability on undulating road surfaces is based on the assumption that the vehicle's steering system has the same quality as the steering systems of the vehicles which have been used for the development of the simulation method. This assumption is not valid if the steering systems are based on different designs.

The validation of the model's response to the global road inclination is also based on measurements of normal driving on the rather strongly undulating road. These measurements yield reliable estimates of the transfer functions between the global road inclination as input and the lateral acceleration and the roll angle as outputs. However, in the case of the relationship between the global road inclination and the yaw velocity, the partial coherence is relatively small.

Hence, the validation, on actual undulating road, of the vehicle's yaw response to the global road inclination is difficult. Nevertheless, the estimates of the measured transfer functions between the global road inclination and the yaw velocity are assumed to be rather accurate in the intervals where the partial coherence is close to or greater than 0.5. In these zones, the similarity between the measurements and the simulations is relatively good.

The measured and simulated transfer functions between the global road inclination and the lateral acceleration as well as between the global road inclination and the roll angle yield a very good correlation. They can be compared dependably in large intervals of the frequency zone from 0 to 2 Hz which is relevant for vehicle straight line stability on undulating road surfaces. Therefore, it may be concluded that there are no reasons to distrust the model's response to road undulations.

An alternative test method for the validation of the response to the road undulations is using artificial excitation by a translating mass (cf. Roos [36]). This method is not discussed in this thesis. However, it is noted that the advantages of this specific test method, viz. the straightforward interpretation and good reproducibility of the test results, might compensate the apparent disadvantage that it seems less realistic than the validation on actual undulating road.

To summarize, the validation has shown that the model yields acceptable responses to the steering wheel angle input and to the input from the road undulations. The two major failings which have been encountered, the inadequate yaw damping of the model and the difficulty to predict the amount of understeer are not prohibitive for the development of a method for the numerical design of vehicles with optimal straight line stability on undulating road surfaces. The insufficient yaw damping has only limited influence on the transfer function between the steering wheel angle and the yaw velocity. The difficulty to predict the amount of understeer is more significant but it has been circumvented by slight corrections of the inaccurate tire data. Analysis of this problem is suggested as an important subject for additional investigation.

Chapter 6

Driver Model

6.1 Introduction

Modelling of driver behavior is complicated and arduous. Each driver has his own particular behavior. Moreover, the same driver behaves differently in different cars. Nevertheless, a driver model is wanted because the closed loop simulation of vehicle behavior on an undulating road is the most natural way of predicting a vehicle's straight line stability on undulating road surfaces. Therefore, a relatively simple model has been adopted from literature [17, 18, 47] and its parameters have been identified (cf. Roos [37]). It is probably not the most accurate model which can be developed for straight line keeping but the development or evolution of a more sophisticated model is a research topic on its own. The development of an accurate driver model is particularly arduous because the vehicle dependence of driver behavior and, hence, of driver parameters has to be taken into account.

The experimental determination of the parameters of the driver model will be described in Section 6.3. This determination has been carried out with experiments that are inappropriate for a proper model validation. However, these experiments do yield interesting information on the accuracy of model.

Figure 6.1 shows a block diagram of the driver-vehicle system on curved roads. On curved roads, a driver has three major tasks: course decision, anticipatory control, and compensatory control. Course decision refers to the selection of a desired course on the base of visual information, knowledge of the destination, etc. Anticipatory control is the driver's "open loop" transformation of the desired course into a basic steering wheel pattern. Commonly this pattern does not yield the desired course. Therefore, a driver also exercises compensatory control which is aimed at the reduction of the difference between the desired and the actual course of the vehicle to an acceptable level.

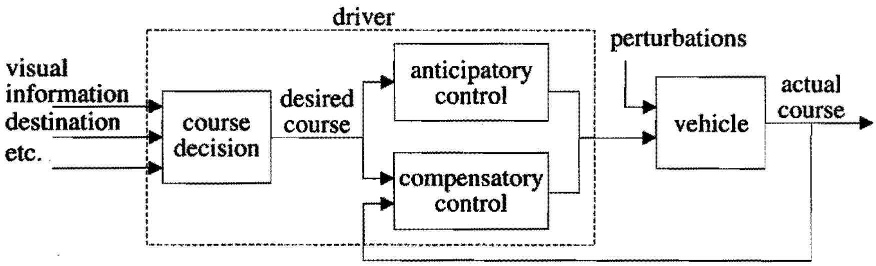


Figure 6.1 Driver-vehicle system on curved road

When driving on a straight road, a driver has only one task, viz. compensatory control (see Figure 6.2). This task can be modelled in two ways. In the driver model, the driver can apply a steering angle or a steering couple to the steering wheel. Perhaps the second approach, the application of a steering couple to the steering wheel in order to obtain a desired steering angle, is the most realistic, but it is also the most complicated. Therefore, a driver model which applies a steering angle is used for the simulation of straight line stability.

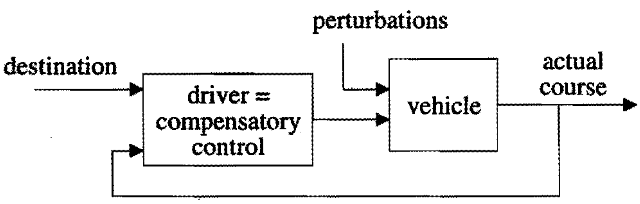


Figure 6.2 Driver-vehicle system on straight road

A driver model which applies a steering couple at the steering wheel is more complicated because it needs more parameters. Generally, such a model first computes the desired steering wheel angle and then calculates the couple as a function of the difference between the actual and the desired steering angle (cf. for instance Nagai [25]). Moreover, a steering couple based driver model only yields an accurate closed loop driver-vehicle behavior if the vehicle model yields an accurate response to couples that are applied at the steering wheel. Such a response is much more difficult to obtain and to validate than an accurate vehicle response to a steering wheel angle input (cf. Section 4.3.2).

6.2 Model selection

The simulations of the driver-vehicle closed loop system will be used for the judgment and comparison of vehicle straight line stability qualities. It is important that the simulations of different vehicles yield the correct order of merit without distortion by the driver model. Therefore, accurate modelling of the relationship between vehicle characteristics and driver behavior is more important than optimization of the driver model for a single vehicle. In literature, even non-linear driver models have been reported but the dependence on vehicle characteristics is hardly ever included in such models. For that reason, only linear models are taken into account in this evaluation of existing models.

McRuer [20, 21, 22] and Allen [2] have done a lot of research in the field of compensatory control. They apply the cross-over model from manual control theory to the system of Figure 6.3 with loops for the yaw angle error ψ_e and for the lateral displacement error y_e. The cross-over model is founded on the hypothesis that the overall transfer function in a control loop has a constant cross-over frequency, i.e. a constant frequency at which its gain equals unity. Hence, in the case of the double loop driver-vehicle system, the driver is supposed to maintain the cross-over frequencies of both loops. This hypothesis was validated by McRuer [21].

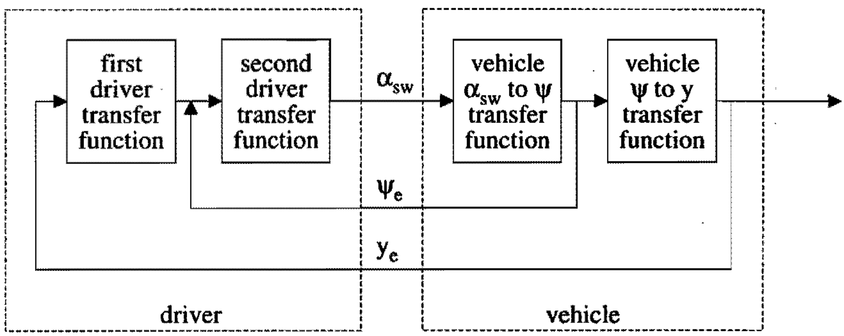


Figure 6.3 Double loop representation of the driver-vehicle system

The steering control law which corresponds to the model of Figure 6.3 contains a time delay τ and uses the lateral displacement error and the yaw angle error as well as their time derivatives for the computation of the steering wheel angle α_{sw} (cf. McRuer [22] and Garrot [11]):

$$\alpha_{sw}(t) = -K_y\, y_e(t-\tau) - K_{\dot{y}}\, \dot{y}_e(t-\tau) - K_\psi\, \psi_e(t-\tau) - K_{\dot{\psi}}\, \dot{\psi}_e(t-\tau)\ . \qquad (6.1)$$

The coefficients K_y, $K_{\dot{y}}$, K_ψ, and $K_{\dot{\psi}}$ depend on the cross-over frequencies of the two loops and on the vehicle's behavior. McRuer proposes a simple, two degree of freedom model for the representation of vehicle behavior and supposes neutral steering behavior.

The model of Equation (6.1) is not used for the simulation of vehicle straight line stability because the corresponding cross-over frequencies were measured in a 1974 Chevrolet Nova which differs too much from modern cars to maintain the hypothesis of their invariance and because the cross-over frequencies' experimental determination is rather elaborate [21]. Moreover, the two degree of freedom model which is used by McRuer for the computation of the coefficients and the assumption of neutral steering behavior may introduce inaccuracies.

Many studies of the driver-vehicle system have been carried out with a rather simple driver model. Examples have been reported by Legouis [17, 18] and Tousi [47]. This model contains a gain K, a time delay τ, and a preview time t_p:

$$\alpha_{sw}(t) = K(y_e(t-\tau) + t_p\,\dot{y}_e(t-\tau)) \,. \tag{6.2}$$

Laplace's transformation yields the equivalent:

$$\frac{\alpha_{sw}(s)}{y_e(s)} = K(1 + t_p\,s)e^{-\tau s} \,. \tag{6.3}$$

Figure 6.4 shows the underlying idea of the model which can be regarded as a mathematical simplification of the model of McRuer. The driver looks a distance $t_p V$ ahead for the estimation of the future error which is equivalent to the future lateral displacement of the center of gravity.

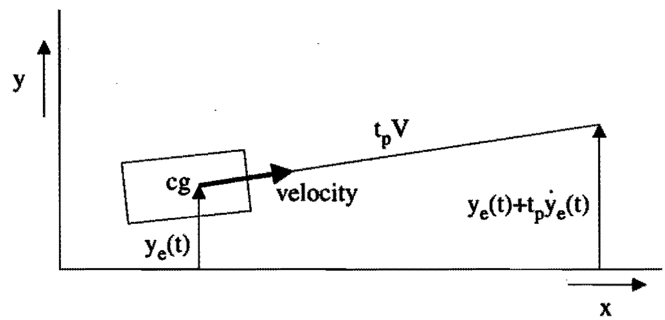


Figure 6.4 Simple driver model (cg = center of gravity)

The simple driver model has been selected for the simulation of vehicle straight line stability on undulating road surfaces because it contains only few parameters. Moreover, these parameters are easy to determine. Vehicle dependence of the driver has been approximated by gain dependence. In accordance with the idea of McRuer, the total gain of the driver-vehicle system is kept constant whereas the driver's time delay and the preview time are also supposed to be constant.

6.3 Experimental parameter determination

The parameters K, t_p and τ of the simple driver model have been determined for two drivers driving in Vehicle B1 on the rather strongly undulating road (cf. Roos [37]). On an undulating road, the driver almost continuously corrects the course of the vehicle. Hence, the transfer function between the lateral displacement of the vehicle's center of gravity and the steering wheel angle can be used for a fit of Equation (6.3). During the experiments, the lateral acceleration and the steering wheel angle were measured. The lateral displacement of the center of gravity was obtained from the lateral acceleration by double integration.

Figure 6.5 shows a block diagram of the driver-vehicle system on an undulating road. It is obvious that the tests on an undulating road cannot be used for the validation of the driver model. A relationship between the steering wheel angle and the lateral displacement of the center of gravity will be found even if it does not exist within the driver because it exists within the vehicle.

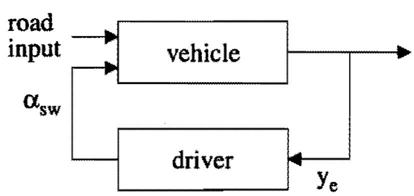
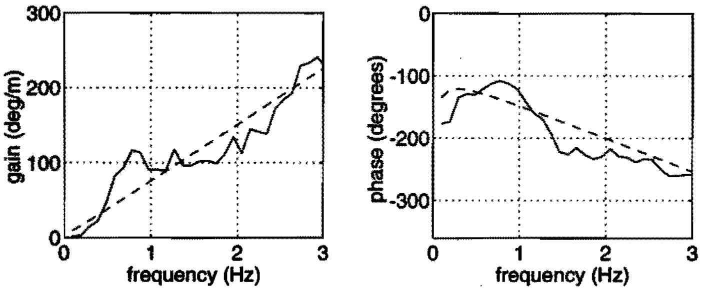


Figure 6.5 Driver-vehicle system on undulating road

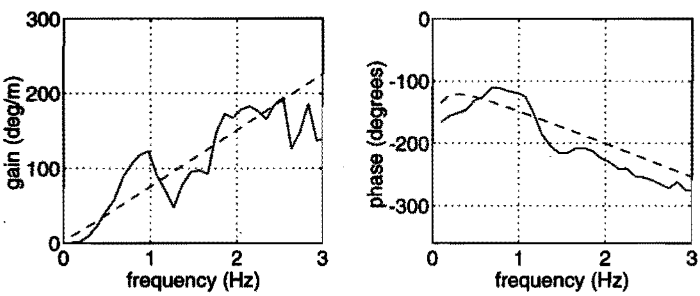
The Figures 6.6 and 6.7 present comparisons between test results and model simulations for two drivers. In both figures, the parameters K = 6 deg./m, τ = 0.15 s, and t_p = 2.0 s yield a reasonable fit of the model. Therefore, these parameter values are used in the simulations of Vehicle B1.



— test result

-- model (K = 6.0 deg./m, τ = 0.15 s, t_p = 2.0 s)

Figure 6.6 Transfer functions between the lateral displacement of the center of gravity and the steering wheel angle (Vehicle B1, Driver 1, V = 110 km/h)



— test result

-- model (K = 6.0 deg./m, τ = 0.15 s, t_p = 2.0 s)

Figure 6.7 Transfer functions between the lateral displacement of the center of gravity and the steering wheel angle (Vehicle B1, Driver 2, V = 110 km/h)

Even though validation of the model is not possible with the measurements on the undulating road, the gains of the transfer functions in Figures 6.6 and 6.7 indicate the limited accuracy of the model. At some frequencies, the model yields a gain which is much too small whereas at other frequencies the model yields a gain which is significantly larger than the measured one.

Chapter 7

Correlation Analysis

7.1 Introduction

The derivation of an estimation algorithm was the final stage in the development of the method for the numerical design of vehicles with optimal straight line stability on undulating road surfaces. It has been carried out by a statistical analysis using the results of experiments with many different vehicle configurations (cf. Roos [35]).

7.2 Elementary straight line stability qualities

The elementary qualities which will be used for the simulation of vehicle straight line stability on undulating road surfaces have been derived in Section 2.3. In the following sub-sections, they will be described in more detail. Moreover, their suitability for numerical simulation will be investigated with the aid of the modelling results of the vehicle and the driver.

7.2.1 Closed loop

The simulation of driver-vehicle closed loop behavior on an undulating road is the most straightforward method for predicting vehicle straight line stability on undulating road surfaces. Therefore, the closed loop sensitivity to road undulations is the first elementary straight line stability quality which has been mentioned in Section 2.3. However, the simulation of the driver-vehicle closed loop system has several drawbacks that are related to the fact that it requires models of the driver and the vehicle that are accurate under the relevant circumstances. First, driver modelling is rather complicated. The accuracy of the driver model which has been described in Chapter 6 is limited (cf. Section 6.3). Second, the vehicle model's accuracy under the relevant circumstances is adversely affected by the disregard of the steering system's specific behavior around zero (cf. Section 5.4). In addition, closed loop simulation is very sensitive to modelling errors. Minor modelling inaccuracies in one of the elements of a closed loop system can yield large errors in the closed loop behavior because of amplification within the loop.

The limited value of closed loop simulations is illustrated with the aid of Figures 7.1 and 7.2 which show measured and simulated closed loop responses of Driver 3 in combination with Vehicles A1 and C1. Both the measured and the simulated responses are responses to (the profile of) the rather strongly undulating straight road. In addition, the steering system model with damping was used for the computation of the simulated results. Hence, these simulations are more accurate than the simulations that can be used for the estimation of vehicle straight line stability qualities when a simplified steering system model has to be applied.

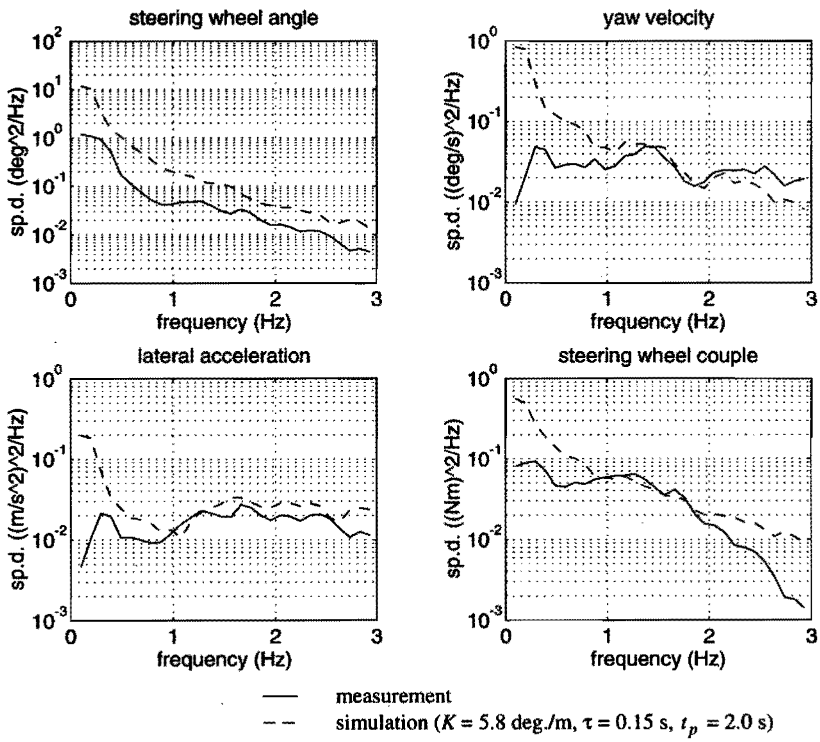


Figure 7.1 *Measured and simulated spectral densities of a driver-vehicle closed loop system on the rather strongly undulating straight road (Vehicle A1, Driver 3, V = 110 km/h; the simulated response was obtained with damping in the steering system model)*

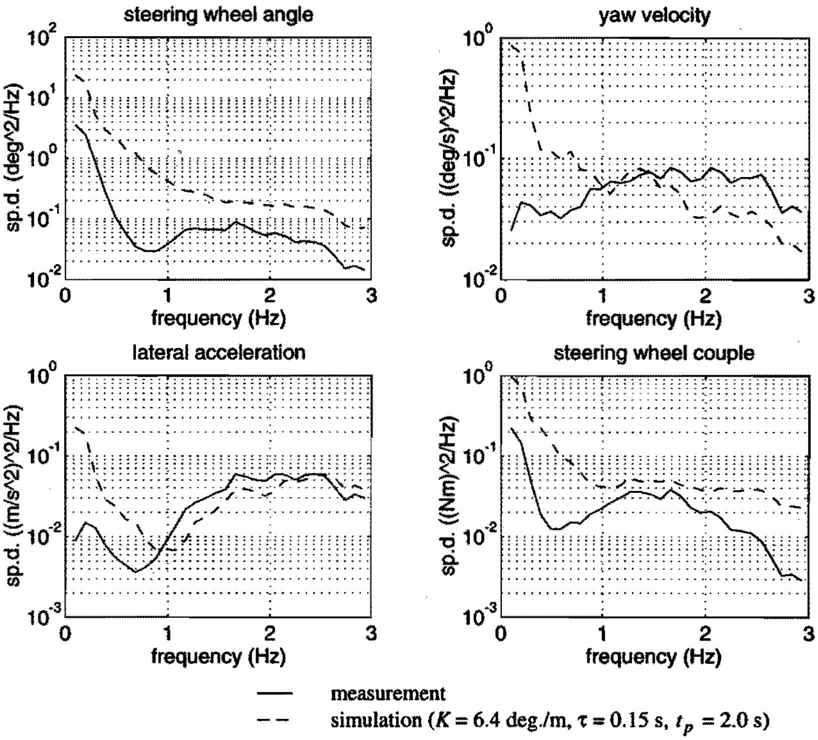


Figure 7.2 Measured and simulated spectral densities of a driver-vehicle closed loop system on the rather strongly undulating straight road (Vehicle C1, Driver 3, V = 110 km/h, the simulated response was obtained with damping in the steering system model)

Comparison of the measured and simulated spectral densities shows that the simulated steering wheel angle spectral density is significantly larger. This surplus of steering corrections also influences the other simulated spectral densities which are too large in the frequency range from 0 to 1 Hz where the steering wheel angle input is more important than the road input. Above 1 Hz, on the contrary, the steering wheel angle input is relatively small and the similarity between the measured and simulated spectral densities of the yaw velocity, the lateral acceleration and the steering wheel couple is much better.

The most important frequency range for vehicle straight line stability is the range between 0 and 1 Hz. In this range, the correlation between the simulated and measured spectral densities is poor. Therefore, it may be concluded that closed loop simulation is inappropriate for the prediction of vehicle straight line stability. The other elementary straight line stability qualities which have been proposed in Section 2.3 concern open loop vehicle characteristics. They will be discussed in the next section.

7.2.2 Open loop

The second elementary straight line stability quality which has been proposed in Section 2.3 is the vehicle's open loop sensitivity to road undulations. This sensitivity is suitable for numerical simulation. At fixed steering wheel angle, the vehicle model yields an accurate response to the input from the road undulations (cf. Section 5.4) and the simulation of this elementary quality can be carried out by using a measured road profile.

A vehicle's open loop sensitivity to road undulations can be assessed with the aid of the spectral densities of, for instance, the yaw velocity, the lateral acceleration, or the vehicle's slip angle. In addition, several methods can be used for the evaluation of these spectral densities. They can be evaluated at a single frequency, or they can be evaluated over a frequency range with the aid of weighing functions. An example of the latter technique is the computation of the mean spectral density over a frequency interval.

The third and the fourth elementary straight line stability qualities, the phase delay of the vehicle's directional response to steering wheel angle inputs and the gain of the vehicle's directional response to steering wheel angle inputs, respectively, can be obtained from an open loop simulation of the vehicle's response to a small steering wheel angle input. This simulation should be carried out on even road and without friction in the steering system. In addition, the steering input has to remain small in order to obtain correct results for vehicles with servo-assisted steering systems.

Examples of transfer functions which can be used for the evaluation of the third and fourth elementary qualities are the transfer function between the steering wheel angle and the yaw velocity, the transfer function between the steering wheel angle and the lateral acceleration, as well as the transfer function between the steering wheel angle and the vehicle's slip angle. The frequencies at which the phase delay and the gain should be estimated cannot be fixed in advance. They must be optimized in order to obtain a good correlation with the subjective judgment.

The phase delay and the gain of the vehicle's directional response to a steering wheel angle input are well suited for numerical simulation. Their only drawback is the fact that the specific behavior of the steering system around zero cannot be taken into account (see Section 5.4). The validation discussed in Chapter 5 has also shown that, in simulations, the transfer function between the steering wheel angle and the yaw velocity is less accurate than the transfer function between the steering wheel angle and the lateral acceleration. Therefore, an evaluation of the vehicle's response to steering wheel angle inputs with the aid of the lateral acceleration has to be preferred to one based on the yaw velocity.

The fifth and final elementary quality, the transfer of information through the steering wheel couple, is the open loop quality which is the least suitable for numerical simulation. The relationship between the global road inclination and the steering wheel couple has not been validated and the simulation of the steering wheel couple response to a small steering wheel angle input is strongly affected by the disregard of the specific behavior of the steering system around zero.

The only simulation result which might be relevant for the transfer of information through the steering wheel couple is the gain of the transfer function between the steering wheel angle and the steering wheel couple under small steering wheel angle excitation. This gain is mainly a function of the self-aligning torques of the front tires, of the geometry of the steering system, and of the characteristics of a possible servo-assistance. Presumably, in order to obtain a good compromise between steering feeling and steering ease, the gain should lie within certain limits.

Accordingly, two separate simulations are required for the computation of the elementary straight line stability qualities. An open loop simulation on an undulating road is required for the estimation of the vehicle's sensitivity to road undulations. In addition, the gain and the phase of the vehicle's directional response to the steering input and the transfer of information through the steering wheel couple have to be simulated on even road with small steering wheel angle excitation.

7.3 Subjective test program

The correlation study which has been carried out for the development of the simulation method required the subjective judgments of many different vehicles. These data have been obtained by an extensive test program with three vehicles which have been tested in various configurations.

For the judgment of the straight line stability qualities of the vehicle configurations, a special questionnaire has been developed (cf. Verdet [49]). This questionnaire was used for the collection of the relevant information on the nature of the vehicle configuration, the weather conditions, the particular characteristics of the configuration's behavior, etc. It also contains specific questions on elements of vehicle straight line stability on undulating road surfaces, for instance on the vehicle's sensitivity to road undulations, on its controllability, and on the so-called on center feeling. The questionnaire is completed by giving the mark of the specific configuration. This mark may vary between 0 (very dangerous) and 10 (excellent).

The straight line stability qualities of the vehicles have been measured by the subjective judgment because reliable alternative measuring standards were not available (cf. Section 2.2). The subjective judgment has been selected in spite of its two major disadvantages. First, it yields results that depend on the driver. This disadvantage has been circumvented by using a single experienced test driver to judge all vehicle configurations. The importance of the second disadvantage of the subjective judgment, viz. its mediocre reproducibility, has been minimized during the test program. During the test program, the testing of new configurations has been alternated with the testing of reference vehicles in order to prevent variation in the assessment standards. Moreover, special attention has been paid to the weather conditions.

Correct reproducibility of the subjective judgment can also be supported by the judgment of vehicles with significantly different straight line stability qualities. In addition, the distribution of the qualities of the vehicle configurations over the judgment range had to be balanced for the development of the estimation algorithm. Therefore, a limited number of vehicle configurations has been tested under various circumstances in order to find the optimal test conditions. These tests have shown that the judgment of vehicle straight line stability on the mildly undulating straight road and at a driving velocity of 140 km/h yields an adequate reproducibility and a suitable distribution. Furthermore, it yields results that correlate well with the opinion of ordinary drivers.

The different vehicle configurations have been created by modifications of vehicle characteristics that are properly modelled in the vehicle model. Modification of vehicle characteristics that have been disregarded in the vehicle modelling does not add any information. It only makes the derivation of the estimation algorithm more complicated. The most important vehicle characteristics which have not been modelled or which have been modelled only partially are the aerodynamic forces and moments on the vehicle and the behavior of the steering system. Therefore, the vehicle heights, which influence the air flows, and the compositions and adjustments of the steering systems have not been changed. Moreover, no spoilers have been employed in the subjective test program.

Modifications which have been applied very frequently are changes of tires and of suspension elements, variations of tire pressures, and modifications of suspension characteristics such as toe in or toe out. Sometimes different tire designs have been used on the front and rear axles of a vehicle in order to obtain results with more significant differences. On the whole, the range of the subjective judgments of the vehicle configurations is rather large (see Figure 7.3). The most interesting results have been obtained with Vehicle A. The straight line stability judgments of this vehicle vary between "very dangerous" and "very good". The experiments with Vehicle C, on the contrary, did not yield any bad configurations.

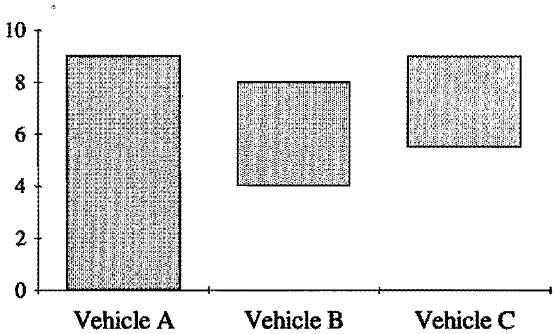


Figure 7.3 Ranges of subjective judgments

7.4 Analysis

The correlation analysis between the subjective judgments and the simulated results of the elementary straight line stability qualities has been carried out in two stages (cf. Roos [35]). First, a visual analysis has been carried out in order to find parts of the simulated elementary qualities that correlate with the subjective judgments. Second, the estimation algorithm has been constituted from the selected parts by regression.

The resulting estimation algorithm is based on the yaw velocity and steering wheel couple spectral densities from the open loop simulation on the undulating road and on the phase lag of the transfer function between steering wheel angle input and lateral acceleration output (cf. Roos [35]). It yields a good correlation with the subjective judgments of Vehicles A and B, see Figure 7.4. The correlation with the judgments of Vehicle C is poor. However, this can be explained by the small differences in the straight line stability qualities of the various configurations of Vehicle C and by the fact that, in the case of Vehicle C, only a few vehicle measurements were available. Hence, most of Vehicle C's tire parameter corrections have been carried out by extrapolation from other tire-vehicle combinations.

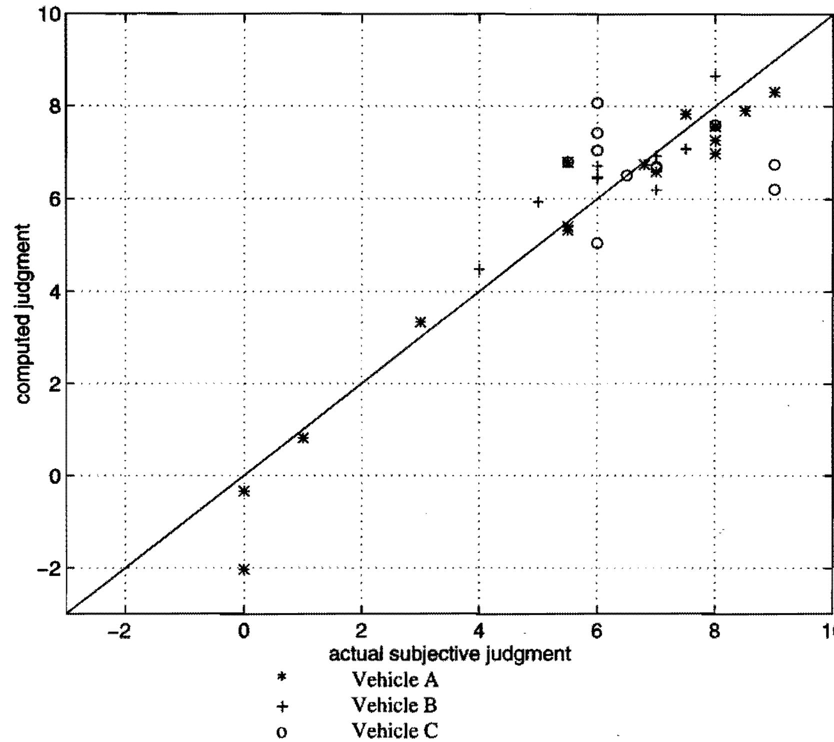


Figure 7.4 Subjective judgments

Vehicle A yields one configuration with a mediocre correlation: the configuration with the estimation -2.0 of the actual subjective judgment 0.0. This mediocre correlation is probably due to the fact that, with respect to the other configurations, this configuration actually did not deserve the minimum judgment of the fixed judgment scale.

7.5 Discussion

Figure 7.4 shows that the estimation algorithm yields very promising results. The computed judgments yield a rather good correlation with the test results, especially if the restricted reproducibility of the subjective judgment and the limited accuracy of the tire data are taken into account. Therefore, it has been proven that the prediction of the subjective judgment of a vehicle's straight line stability qualities is feasible. Moreover, since the proposed vehicle model requires only few vehicle and tire data, it may be concluded that this prediction can be carried out from early stages of vehicle development.

Nevertheless, the new method for the numerical design of vehicles with optimal straight line stability on undulating road surfaces is not completely finished yet. The complete validation of the very important estimation algorithm requires a larger analysis by means of more vehicle tests. Especially the consequences of the choices that have been made in the steering system modelling should be further examined. There, the specific behavior around zero had to be omitted from the design method (cf. Section 5.4). This omission might necessitate different parameters in the estimation algorithm or different algorithms for dissimilar steering system constructions. For instance, it might necessitate a different algorithm for vehicles with servo-assisted steering.

The current estimation algorithm for the subjective judgment of vehicle straight line stability can be used for vehicles with steering system designs similar to the steering system designs of Vehicles A and B. Nevertheless, the limited degree of validation of the algorithm has to be taken into account during the interpretation of the simulation results.

Application of the current estimation algorithm has two advantages. First, it enables the numerical design of vehicles with optimal straight line stability on undulating road surfaces. Hence, the possibility to take this driving quality into account from early stages of vehicle development. Second, application of the estimation algorithm generates experience with the numerical design method. For instance, it will demonstrate the limits of validity of the algorithm.

Chapter 8

Conclusions and Recommendations

8.1 Overview of the research

Vehicle straight line stability on undulating road surfaces is an important driving quality. It influences driver satisfaction, driver comfort, and even road safety. Nevertheless, vehicle straight line stability could, until now, hardly be taken into account in the early stages of vehicle development when many important vehicle parameters are fixed. In current industrial practice, it is optimized by tests with prototypes on actual undulating roads.

Therefore, a method has been developed for the numerical design of vehicles with optimal straight line stability on undulating road surfaces. This method is based on an estimation algorithm for the subjective judgment. With the aid of the algorithm, design alternatives can be compared with each other and with existing car models. Hence, it can be used as a safeguard against a bad straight line stability as well as for optimization purposes.

The design method has been developed by a combined physical and statistical approach. First, the behavior of the driver-vehicle closed loop system on undulating roads has been analyzed in order to deduce elementary straight line stability qualities. Second, the models of the vehicle and the driver that are required for the simulation of the elementary qualities have been developed. Finally, the estimation algorithm has been derived through a statistical analysis using the results of experiments with many different vehicle configurations.

The subjective judgment has been used as reference for the simulation of vehicle straight line stability. It has been selected because no reliable alternative measurements were available. The importance of the two disadvantages of the subjective judgment, its driver dependency and its mediocre reproducibility, has been minimized throughout this research.

Problem analysis

The sensitivity of the driver-vehicle system to road undulations is the most straightforward elementary quality for the simulation of vehicle straight line stability on undulating road surfaces. However, the simulation of the driver-vehicle closed loop system requires an accurate driver model. Such a model is very difficult to obtain. Therefore, the driver-vehicle system has been analyzed in more detail in order to obtain elementary straight line stability qualities that are based on the vehicle instead of on the closed loop system.

In total, five elementary straight line stability qualities have been derived:

- the sensitivity of the driver-vehicle closed loop system to road undulations,
- the vehicle's open loop sensitivity to road undulations,
- the phase delay of the vehicle's directional response to steering wheel angle inputs,
- the gain of the vehicle's directional response to steering wheel angle inputs, and
- the transfer of information through the steering wheel couple.

In this research, the road surface in the tire-road contact patch is assumed to be flat. Therefore, only four road inputs have been taken into account: the global road inclination, the local road inclinations in the centers of the left and right contacts patches, and the mean road height. Among these road inputs, the global road inclination yields the largest vehicle response. From the problem analysis it has also been found that the vehicle response to the excitation by the mean road height is significantly non-linear and that, at low frequencies, the largest part of the vehicle motions is caused by the driver.

Modelling

Vehicle behavior on undulating straight roads has hardly been investigated before. Therefore, prior to the vehicle modelling, preparatory measurements have been carried out of normal driving on mildly and rather strongly undulating roads. During these tests, three vehicle inputs were measured: the steering wheel angle, the global road inclination, and the mean road height. In addition, special analysis techniques have been used for the elimination of the influences of the correlations between the inputs. These techniques yield transfer functions for the two most important vehicle responses, viz. the responses to the global road inclination and to the steering wheel angle.

The preparatory measurements of a vehicle's response to the global road inclination show that the partial coherences between the global road inclination and the vehicle outputs are rather small, especially in the case of the yaw velocity. This effect can be explained by the fact that measurement noise is relatively important and by the unfavorable ratio between the (yaw) motions produced by the global road inclinations on the one hand and the (yaw) motions produced by the vehicle inputs which have not been measured on the other hand.

For the vehicle's response to small steering wheel angle inputs, two different results have been found. On the rather strongly undulating straight road, the partial coherences between the steering wheel angle as input and the yaw velocity and lateral acceleration as outputs are sufficiently high for the validation of the corresponding measured transfer functions. On the mildly undulating straight road, on the contrary, the measured vehicle response is significantly non-linear. The difference between the vehicle steering responses on the mildly and rather strongly undulating road can be explained by the fact that, on the rather strongly undulating road, the forces on the steering system and the steering wheel angles are larger.

The vehicle model which has been developed for the simulation of vehicle straight line stability on undulating road surfaces is a lumped parameter model. It includes specific sub-models of the tire and the steering system. The steering system sub-model contains an imaginary damper in series with the friction on the rack. This damper permits the reproduction of the steering response which has been measured on the rather strongly undulating road. However, the steering system sub-model with the damper cannot be used for the prediction of a new vehicle's straight line stability because the damping value has to be obtained from complete vehicle measurements. In addition, modelling of the steering response which has been measured on the mildly undulating road has not been attempted because this non-linear response requires a very complicated model with little practical value, especially in early stages of vehicle development.

Accordingly, the specific behavior of the steering system around zero is disregarded in the simulation of the elementary straight line stability qualities. These qualities are simulated without friction in the steering system. Therefore, the vehicle elements which influence the specific steering system behavior around zero have to be optimized by tests with prototypes. Nevertheless, the proposed numerical design method still means a large improvement with respect to the actual experimental optimization of the entire vehicle because the elements of the steering system can be modified rather easily in the late stages of vehicle development when prototypes are available.

The validation of the vehicle's response to medium steering wheel angle inputs has demonstrated a rather important simulation problem. If measured tire data are used, the simulations sometimes yield an erroneous amount of understeer. Nevertheless, this problem is a problem for roadholding simulations in general. Therefore, it is recommended as a subject for additional investigation. In this research, the problem has been circumvented by slight corrections of the cornering stiffnesses. If the appropriate tire data are used, the model yields only one minor inaccuracy in the responses to steering wheel angle inputs: viz. its yaw damping is a little too weak. The validation of the vehicle's response to the global road inclination has also shown a rather good correlation between measurements and simulations.

For the driver, a simple linear model has been copied from literature. This model has not been validated. Nevertheless, the measurement results of the experimental parameter determination show that the correlation between the model and actual driver behavior is only mediocre.

Correlation analysis

The estimation algorithm for the subjective straight line stability judgment has been derived by a correlation analysis. This analysis required the subjective judgments of many different vehicles. Therefore, an extensive test program has been carried out with three vehicles which have been tested in various configurations. The subjective judgments for the correlation analysis were carried out at 140 km/h on the mildly undulating road.

Simulations of the driver-vehicle closed loop system on the rather strongly undulating road yield a bad correlation with measurements on the same road. Therefore, only open loop elementary straight line stability qualities have been taken into account in the derivation of the estimation algorithm.

The estimation algorithm which has been derived is based on the vehicle's yaw velocity and steering wheel couple responses during open loop drive on the rather strongly undulating road and on the phase lag of the lateral acceleration response to the steering wheel angle input. It yields a rather good correlation with the subjective judgment.

8.2 Conclusions

From the presented research of the numerical design of vehicles with optimal straight line stability on undulating road surfaces, several conclusions can be drawn:

- The standard for vehicle straight line stability on undulating road surfaces, being the subjective judgment by a professional test driver, can be predicted from early stages of a vehicle development process.
- The proposed estimation algorithm yields a good correlation with the subjective judgments of the tested Vehicles A and B. Therefore, it can be applied for the development of vehicles with steering system designs that are similar to the steering system designs of Vehicles A and B.

- The vehicle model which has been developed yields accurate simulations of the vehicle responses to medium steering wheel angle inputs and to the input from the global road inclination. It also represents the vehicle's response to the small steering wheel angle input which can be derived from measurements of normal driving on the rather strongly undulating straight road. Nevertheless, some observations have to be made. The accuracy of the model's simulation results strongly depends on the reliability of the tire data. In addition, the damping of the model's yaw response to medium steering wheel angle inputs is a little too weak. Finally, the significance of the validation of the model's response to the global road inclination is limited by the fact that the partial coherences between the global road inclination input and the vehicle outputs are rather small.
- A vehicle's response to a small steering wheel angle inputs differs from its response to a medium steering wheel angle input. Moreover, the response to a small steering wheel angle input depends on the measurement conditions. The steering responses which can be derived from measurements with medium steering wheel angle inputs and from measurements of normal driving on the rather strongly undulating straight road can, to a large extent, be analyzed with the aid of transfer functions. They may be linearized in the relevant operating ranges. The steering response during normal driving on the mildly undulating straight road, on the contrary, is significantly non-linear.
- The analysis of a vehicle's directional response to variations of the global road inclination is complicated by the low levels of the partial coherences between the global road inclination as input and the lateral acceleration and, especially, the yaw velocity as outputs.
- During normal driving on undulating straight roads, the largest part of the vehicle's deviations from the straight line is caused by the driver.
- The correlation between a simple driver model and actual driver behavior during straight line keeping on undulating road surfaces is only mediocre.

8.3 Recommendations

The numerical design method which has been developed means a large improvement in the managing, during vehicle development, of vehicle straight line stability on undulating road surfaces. However, its value could still be increased by some additional investigations:

- First, the estimation algorithm's domain of validity should be determined by an extension of the correlation analysis to more vehicle configurations. It is especially important to assess the influence of the disregard of the specific steering system behavior around zero. In addition, the accuracy of the estimation algorithm could be improved by the supplementary vehicle configurations.

- Another important subject for additional investigation is the correlation between tire behavior during the measurement of tire data with the aid of specialized equipment and tire behavior during vehicle testing on an actual road surface. Inaccurate tire data are supposed to be the cause of the limited accuracy of the simulated understeer characteristics. This problem has been encountered during the vehicle model validation. It severely restricts the value of any roadholding simulations, including the simulations of vehicle straight line stability on undulating road surfaces.
- The damping of the vehicle model's yaw response to medium steering wheel angle inputs is too weak. Therefore, this yaw damping is an interesting topic for further investigation.
- Transient aerodynamic effects have been disregarded in the present research. Analysis and modelling of these effects can extend the range of the numerical design of vehicles with optimal straight line stability.
- The used tire sub-model is probably sufficiently accurate for the numerical design of vehicles with optimal straight line stability. However, a more detailed analysis of tire behavior might yield a better representation of the tire responses which have been modelled by approximation, such as the dynamic response to camber inputs.
- The tire and vehicle responses to road undulations could be further investigated with the aid of an artificial excitation on even road, for instance with the aid of a translating mass. Experiments with an artificial excitation are more feasible for these detailed experimental analyses than measurements on actual undulating roads. Measurement of the responses to an artificial excitation yields reproducible results, whereas the analysis of road responses on actual undulating road surfaces is complicated by the low levels of the partial coherences between the road inputs and the vehicle outputs.
- Inclusion of (parts of) the specific steering system responses to small steering wheel angle inputs into the simulation model would also extend the ranges of the numerical design method. Nevertheless, the representation of these specific steering responses requires very detailed models which are unsuitable for application in the early stages of a vehicle development process.
- The correlation between the driver model and actual driver behavior during straight line keeping can be ameliorated. However, the development of an accurate driver model with a realistic adaptation to vehicle characteristics will be very difficult.

Appendix A

Road Profiles

The road surface of the rather strongly undulating road has been measured with a specially equipped trailer (see Figure A.1). During this measurement at low velocity, three road variables were measured: the local road inclinations $\sigma(i)$ and the global road inclination θ_{gl}. The mean road height was not measured.

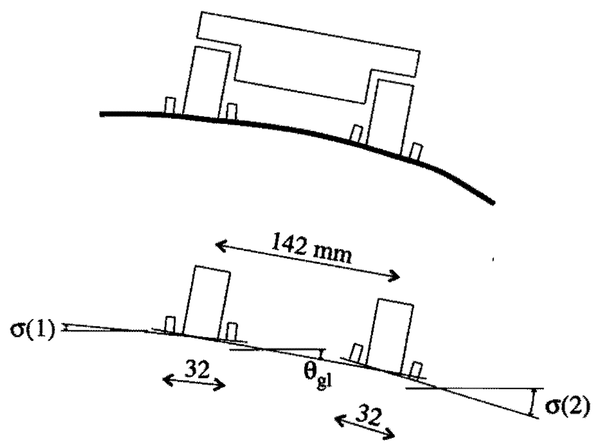


Figure A.1 Device for the measurement of road inclinations

Figure A.2 shows the spectral densities of the three road inclinations after transformation to the standard test velocity (V = 110 km/h). The coherences between the inclinations have been plotted in Figure A.3. Both local road inclinations are partially correlated to the global road inclination. The coherence between the left and right local road inclinations, on the contrary, is very small.

The correlation between the road inclinations makes the analysis of vehicle behavior more complicated. Therefore, special definitions have been introduced in Chapter 2 (cf. Figure 2.6). These definitions yield very small coherences between the road inclinations (see Figure A.5). Figure A.4 shows the spectral densities of the special local road inclinations.

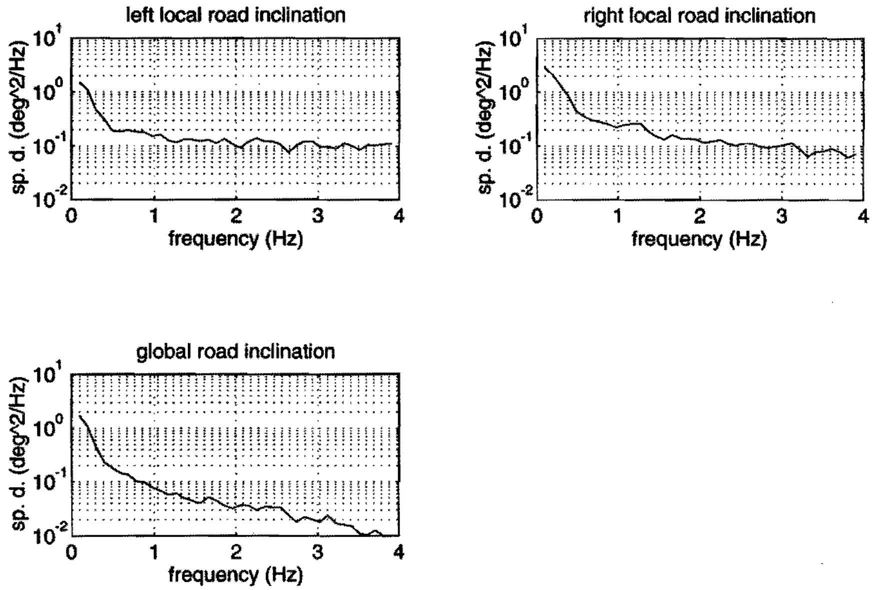


Figure A.2 Spectral densities of the inclinations of the rather strongly undulating road (V = 110 km/h)

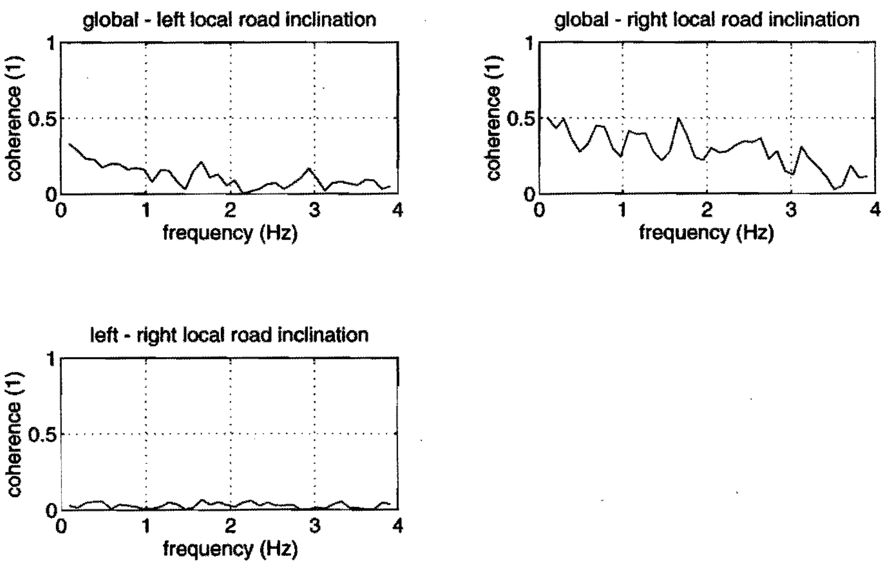


Figure A.3 Coherences between the inclinations of the rather strongly undulating road (V = 110 km/h)

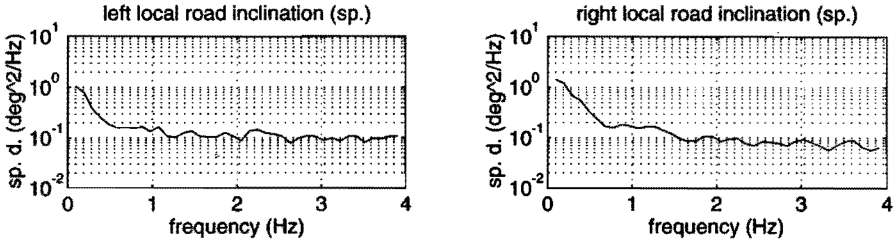


Figure A.4 Spectral densities of local inclinations of the rather strongly undulating road (special definitions, V = 110 km/h)

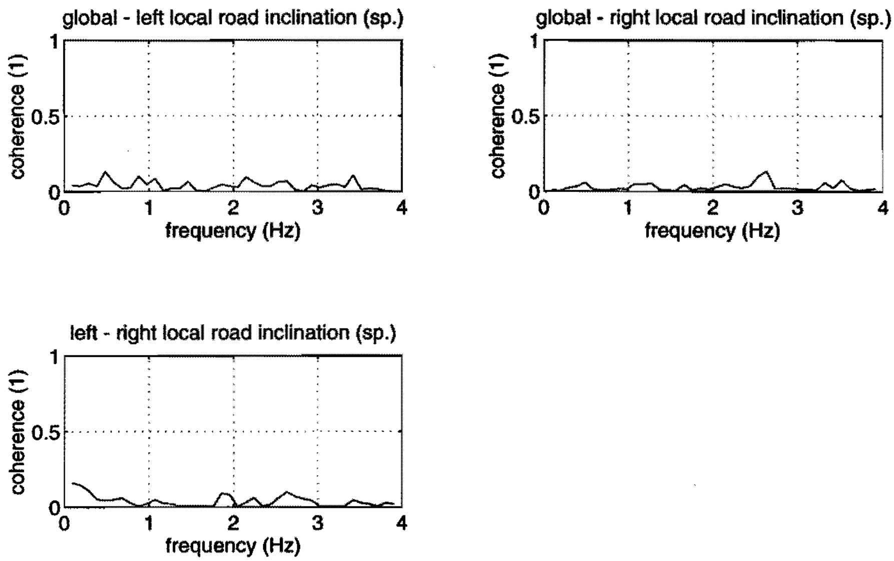


Figure A.5 Coherences between the inclinations of the rather strongly undulating road (special definitions, V = 110 km/h)

The three measured road inclinations do not yield a complete road profile. Such a profile is yet required for the simulation of vehicle behavior on the rather strongly undulating road. Therefore, the three road inclinations from the special measurement have been combined with the mean road height from a vehicle test. In the resulting profile, the mean road height is not correlated to the road inclinations. This situation is probably rather close to reality, see for instance the small coherence between the global road inclination and the mean road height in Figure 2.4.

Appendix B

Data Analysis

The inputs to a vehicle on an undulating road are (moderately) correlated. These correlations have to be taken into account when transfer functions between an input and an output are computed because they make a standard computation unsuitable (cf. Section 3.2). Instead of the standard computation, a more sophisticated method has to be used.

In this appendix, only an example of the application of the analysis method will be presented; the background of the method will not be discussed. It can be found in publications on data analysis, for instance in Bendat [6], Chapter 7 and 11. The equations that are used in this example are based on Bendat's example 11.5 (pages 419-422).

Figure B.1 shows the vehicle on the undulating road with its three measured inputs and with the outputs $\dot{\psi}$, $\ddot{y}$, θ, etc. The transfer function which will be determined in this example is the transfer function between the steering wheel angle and the yaw velocity $\dot{\psi}$.

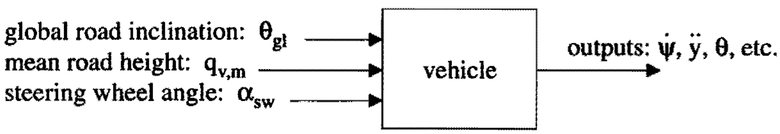


Figure B.1 Three measured inputs to a vehicle on an undulating road

The signals $\theta_{gl}(t)$, $q_{v,m}(t)$, $\alpha_{sw}(t)$, and $\dot{\psi}(t)$ are known from the measurement. By Fast Fourier Transformation, they can be transformed into the spectral densities $S_{ij}(f)$ that are required for the analysis. The first step in the analysis is the elimination from $q_{v,m}$, α_{sw}, and $\dot{\psi}$ of the parts that are correlated to θ_{gl}. This yields conditioned signals and spectral densities that are indicated by the additional subscript $_{\_\theta_{gl}}$:

$$S_{q_{v,m}q_{v,m}\_\theta_{gl}}(f) = S_{q_{v,m}q_{v,m}}(f) - \frac{S_{\theta_{gl}q_{v,m}}(f)\,S_{q_{v,m}\theta_{gl}}(f)}{S_{\theta_{gl}\theta_{gl}}(f)}\,, \tag{B.1}$$

$$S_{q_{v,m}\alpha_{sw}\_\theta_{gl}}(f) = S_{q_{v,m}\alpha_{sw}}(f) - \frac{S_{\theta_{gl}\alpha_{sw}}(f)\,S_{q_{v,m}\theta_{gl}}(f)}{S_{\theta_{gl}\theta_{gl}}(f)}\,, \tag{B.2}$$

$$S_{q_{v,m}\dot{\psi}\_\theta_{gl}}(f) = S_{q_{v,m}\dot{\psi}}(f) - \frac{S_{\theta_{gl}\dot{\psi}}(f)\, S_{q_{v,m}\theta_{gl}}(f)}{S_{\theta_{gl}\theta_{gl}}(f)} , \tag{B.3}$$

$$S_{\alpha_{sw}\alpha_{sw}\_\theta_{gl}}(f) = S_{\alpha_{sw}\alpha_{sw}}(f) - \frac{S_{\theta_{gl}\alpha_{sw}}(f)\, S_{\alpha_{sw}\theta_{gl}}(f)}{S_{\theta_{gl}\theta_{gl}}(f)} , \tag{B.4}$$

$$S_{\alpha_{sw}\dot{\psi}\_\theta_{gl}}(f) = S_{\alpha_{sw}\dot{\psi}}(f) - \frac{S_{\theta_{gl}\dot{\psi}}(f)\, S_{\alpha_{sw}\theta_{gl}}(f)}{S_{\theta_{gl}\theta_{gl}}(f)} , \tag{B.5}$$

and

$$S_{\dot{\psi}\dot{\psi}\_\theta_{gl}}(f) = S_{\dot{\psi}\dot{\psi}}(f) - \frac{S_{\theta_{gl}\dot{\psi}}(f)\, S_{\dot{\psi}\theta_{gl}}(f)}{S_{\theta_{gl}\theta_{gl}}(f)} . \tag{B.6}$$

Conjugation yields:

$$S_{\alpha_{sw}q_{v,m}\_\theta_{gl}}(f) = S^{*}_{q_{v,m}\alpha_{sw}\_\theta_{gl}}(f) , \tag{B.7}$$

and

$$S_{\dot{\psi}q_{v,m}\_\theta_{gl}}(f) = S^{*}_{q_{v,m}\dot{\psi}\_\theta_{gl}}(f) . \tag{B.8}$$

In the second step, the parts that are correlated to $q_{v,m}$ are removed from $\alpha_{sw\_\theta_{gl}}$ and $\dot{\psi}_{\_\theta_{gl}}$. This yields variables with the subscript $_{\_\theta_{gl}q_{v,m}}$:

$$S_{\alpha_{sw}\alpha_{sw}\_\theta_{gl}q_{v,m}}(f) = S_{\alpha_{sw}\alpha_{sw}\_\theta_{gl}}(f) - \frac{S_{q_{v,m}\alpha_{sw}\_\theta_{gl}}(f)\, S_{\alpha_{sw}q_{v,m}\_\theta_{gl}}(f)}{S_{q_{v,m}q_{v,m}\_\theta_{gl}}(f)} , \tag{B.9}$$

$$S_{\alpha_{sw}\dot{\psi}\_\theta_{gl}q_{v,m}}(f) = S_{\alpha_{sw}\dot{\psi}\_\theta_{gl}}(f) - \frac{S_{q_{v,m}\dot{\psi}\_\theta_{gl}}(f)\, S_{\alpha_{sw}q_{v,m}\_\theta_{gl}}(f)}{S_{q_{v,m}q_{v,m}\_\theta_{gl}}(f)} , \tag{B.10}$$

and

$$S_{\dot{\psi}\dot{\psi}\_\theta_{gl}q_{v,m}}(f) = S_{\dot{\psi}\dot{\psi}\_\theta_{gl}}(f) - \frac{S_{q_{v,m}\dot{\psi}\_\theta_{gl}}(f)\, S_{\dot{\psi}q_{v,m}\_\theta_{gl}}(f)}{S_{q_{v,m}q_{v,m}\_\theta_{gl}}(f)} . \tag{B.11}$$

Finally, the transfer function $H_{\alpha_{sw}\dot{\psi}\_\theta_{gl}q_{v,m}}(f)$ between α_{sw} and $\dot{\psi}$ and the partial coherence function $\gamma^2_{\alpha_{sw}\dot{\psi}\_\theta_{gl}q_{v,m}}(f)$ between these two variables can be calculated:

$$H_{\alpha_{sw}\dot{\psi}\_\theta_{gl}q_{v,m}}(f) = \frac{S_{\alpha_{sw}\dot{\psi}\_\theta_{gl}q_{v,m}}(f)}{S_{\alpha_{sw}\alpha_{sw}\_\theta_{gl}q_{v,m}}(f)} \tag{B.12}$$

and

$$\gamma^2_{\alpha_{sw}\,\dot{\psi}\_\theta_{gl}\,q_{v,m}}(f) = \frac{\left|S_{\alpha_{sw}\,\dot{\psi}\_\theta_{gl}\,q_{v,m}}(f)\right|^2}{S_{\alpha_{sw}\,\alpha_{sw}\_\theta_{gl}\,q_{v,m}}(f)\,S_{\dot{\psi}\,\dot{\psi}\_\theta_{gl}\,q_{v,m}}(f)}\,. \tag{B.13}$$

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Gijsbert

Curriculum vitae

Gijsbert Roos was born on May 30, 1967 in Leiden, the Netherlands. Between 1979 and 1985, he attended the Elzendaal College in Boxmeer. Subsequently, he started studying Mechanical Engineering at Eindhoven University of Technology. In 1990, he received his M.Sc. degree in Mechanical Engineering after a final project on four-wheel-steering systems in passenger cars. This final project established his interest in vehicle dynamics. Therefore, in December 1990, he started a biennial postgraduate education surrounding a major project on vehicle straight line stability. The title of this education at the Institute for Continuing Education of Eindhoven University of Technology was "Computational Mechanics". In December 1992, he continued his work as a doctoral research within a joint project of RENAULT S.A. and the Laboratory for Automotive Engineering of Eindhoven University of Technology. The scientific results of both projects on vehicle straight line stability are reported in this thesis.

STELLINGEN

behorende bij het proefschrift

Numerical design of vehicles with optimal straight line stability on undulating road surfaces

1 Op golvende rechte wegen veroorzaakt de bestuurder veel meer voertuigbewegingen dan het wegdek *(dit proefschrift).*

2 In de voertuigdynamica worden op grote schaal eenvoudige bestuurdersmodellen gebruikt. Deze modellen komen slechts in beperkte mate overeen met de gedragingen van werkelijke bestuurders *(dit proefschrift).*

3 De matige correlatie tussen bandgegevens die in laboratoria zijn bepaald en werkelijk bandgedrag op de weg is een belangrijke beperking van de waarde van wegliggingssimulaties. *(dit proefschrift).*

4 De lineariteit van de stuurresponsie van een personenauto hangt, zelfs in het als lineair aangeduide domein van lage dwarsversnellingen, sterk af van de grootte van de stuurwielhoek-input *(dit proefschrift).*

5 De door autojournalisten gehanteerde beoordelingscriteria voor het weggedrag van personenauto's komen vaak niet overeen met de wensen van de gemiddelde automobilist.

6 Een goed beheer van de nationale cultuur vergt een overheidsbeleid dat het midden houdt tussen de huidige Franse en Nederlandse benaderingen.

7 De toenemende studentvriendelijkheid van de Nederlandse universiteiten vormt een gevaar voor het niveau van de ingenieurs en doctorandi die in de toekomst af zullen studeren.

8 De computerindustrie zal pas volwassen zijn als men de klant gaat leveren wat hij werkelijk nodig heeft.

9 Een van de lessen die men uit de Dutchbat-aftocht in het voormalige Joegoslavië kan leren is dat men, in navolging van de heiligverklaring in de katholieke kerk, beter enige tijd kan wachten voordat men mensen tot helden uitroept.

10 Milieubewegingen houden bij de keuze van hun akties meer rekening met de bijbehorende marketing-mogelijkheden dan met de ernst van de eventuele bedreigingen.

Gijsbert Roos
Eindhoven, 19 september 1995