

# Modelling as a foundation for academic forming in mathematics education

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# Modelling as a Foundation for Academic Reflection in the Mathematics Curriculum

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*The Bachelor curriculum of Applied Mathematics in Eindhoven includes a series of modelling projects where pairs of students solve mathematical problems posed in non-mathematical language. Communication skills training is integrated with this track. Recently a new course has been added. The students build a portfolio during the track and in the new course they reflect on their former modelling activities, especially from a mathematical and a social perspective. Essay writing on these subjects is followed by peer assessment and peer feedback.*

## 1 Introduction

In 1995, the curriculum of Applied Mathematics at the Technische Universiteit Eindhoven grew from four to five years and introduced the Modelling Track as a reaction to the employers' complaint about graduates' poor ability to apply their theoretical knowledge. Graduates did possess enough knowledge of various subjects, but could neither apply nor integrate this knowledge in an industrial or a social context. During the Modelling Track, in the courses Modelling 1 to Modelling 4, the students collaborate in pairs on non-mathematically defined problems using mathematical methods. They are also trained on giving a presentation and writing a project report.

Recently the Bachelor-Master system has been introduced in Dutch higher education. Therefore the distinction between university education and other forms of higher education has become more important. The Technische Universiteit Eindhoven decided to strengthen the aspect of academic education in its study programmes. In 2002 several changes were carried out in the mathematics curriculum in this direction, such as the implementation of a course on mathematics and its usage in historical perspective and the adding of an interdisciplinary project. In this paper we will focus on another new course, Modelling 5, meant to complete the Modelling Track at the end of the Bachelor with all kinds of reflective activities. In Modelling 1 to 4 the students construct a portfolio, to be used in Modelling 5 for academic reflection.

In this paper, we start with a description of the original Modelling Track within the context of the Bachelor programme and we will compare it with some other modelling courses elsewhere. Then an overview of the learning goals and the content of Modelling 5 is presented. We conclude with an evaluation of the first implementation of this new course.

## 2 The original Modelling Track

### 2.1 The Bachelor Curriculum

In the Bachelor of mathematics in Eindhoven about 30 students are starting every year. In the first year, a series of obliged courses is given: analysis, algebra, probabilities and statistics, computer science, computer algebra, mechanics, business administration and modelling. In the second year there are obliged courses on abstract analysis, numerical linear algebra, mathematical statistics, optimisation, stochastic processes and modelling again. In addition there are some courses on applied mathematics to choose from. In the third year, the students choose out of three specialisations: Statistics, Probability and Operations research, Discrete Mathematics and Applications, or Computational Science and Engineering, and follow courses in their specialisation.

The regular didactical forms are lectures, practicals and a mix of the two. However, the modelling courses Modelling 1 to 4 have another form: the students perform a series of projects in pairs.

### 2.2 The Implementation of the original Modelling Track

The overall learning goal of the Modelling Track is learning how to solve mathematical problems posed in a non-mathematical language.

Firstly, it means the ability to complete the so-called model-cycle that consists of the following steps:

- Problem analysis; mapping out the problem using common sense.
- Problem translation; constructing a mathematical model for the problem.
- Mathematical analysis; mathematical elaboration of (sub-) problems.
- Implementation; implementation of the solution into a computer program.
- Retranslation; translation of the solution back to the original problem.

Secondly, it means the ability to work along project lines, with the following characteristics:

- Working according to a plan.
- Collaborating in a small team.
- Communicating verbally with colleagues and interested laymen.
- Giving a verbal presentation to colleagues and interesting laymen.
- Reporting on paper to colleagues and interested laymen.
- Critically following fellow students' mathematical activities.

In the beginning of the track, the students do small projects with intensive coaching. Gradually, the projects become larger and more complex; at the other hand, the students work more independently. Each time each pair chooses a project assignment out of a set of possibilities. Most of the time two different pairs do not do the same project. We will give some examples of project assignments:

- 1) An example from Statistics, Probability and Operations research: *Buffer design*.  
On a machine in a sweet factory marshmallows are sealed in fixed batches. The marshmallows are transported to this machine on a conveyor belt. The arrival process is irregular, i.e. the inter-arrival times of the marshmallows vary. However, the machine works continuously at constant speed. To prevent the machine from sealing to many 'empty positions', a vertical buffer is positioned between the conveyor belt and the sealing machine. *Develop a model to determine the optimal buffer size.*

- 2) An example from Discrete Mathematics and Applications: *Teletext*.  
Teletext is an on-line information medium available on television. It usually takes some time before a selected information page appears on the screen. *Develop a model to explain the delay, and, if possible, propose a method to reduce the delay.*
- 3) An example from Computational Science and Engineering: *Noise reduction by wings*.  
For the design of a new aeroplane one has to take in account international regulations for the production of noise. In these regulations it is stated that the amount of noise produced by an aeroplane landing or taking off should not exceed certain thresholds. Most of the noise produced by aeroplanes comes from their engines; their location within the aircraft is possibly a significant factor in the noise production: above the wings at the tail, or below them. The claim is that in the first case the wings yield a considerable reduction of noise. *Develop a model to estimate the reduction of noise.*

Each pair has its own Web site where it presents its plans and products. Integrated with the project work in the first and second year is a course on verbal presentation, report writing and planning.

The modelling programme in the Bachelor curriculum consisted of two courses in the first year, taking up a modest part of 10 percent of the curriculum, and two courses in the second year for about 15 percent.

The experience of almost ten years tells that most students like to do these courses. The majority of faculty has accepted them as a normal part of the curriculum, although there is some criticism about the fact that, contrary to the subject exams, almost every student passes for the Modelling Track. Formulating more detailed rules for assessment has been given attention lately. A minority of the faculty still has the opinion that mathematics has to be taught only using lectures and practicals, while project work is 'for kindergarten'.

### 2.3 Comparison with other modelling programmes

Gray (1998) distinguishes three ways of presenting mathematical modelling: 1) various mathematical systems applied in various areas, 2) several mathematical systems, each applied in a single area, and 3) a single area of application. Our approach is nearest to the first way; sometimes even a mathematical system to be applied has to be studied by the students themselves as part of the project. In the track, aside from an introductory lecture about the modelling process, not much modelling theory is presented beforehand. The emphasis is upon creative and empirical model construction coached by a staff member (compare with Giordano & Weir, 1987).

When we look at other modelling courses, we conclude that integration with the training of presentation and report writing skills is not unusual (see for instance Edwards & Morton, 1987, and Usher & Earl, 1987). Contrary to some other programmes, our students work in pairs, as the overall staff opinion is that the development of mathematical modelling skills should get priority over the development of skills to work in larger groups. In the interdisciplinary project, mentioned before, the students experience the work in a larger and inhomogeneous group. In Burton (1997) and Houston (1998) the importance is stressed of peer assessment and peer tutoring. In the original Modelling Track as well as in the Bachelor curriculum as a whole, such activities are scarce. In the new course of Modelling 5 their role is important. The recent completion of our Modelling Track with a substantial set of reflective activities, to be described in the next paragraph, makes it different from many other modelling courses.

For more details on the original Modelling Track see Perrenet and Adan (2002). These authors also discuss the Modelling Track from the perspective of the general educational model of the Technische Universiteit Eindhoven: Design Based Learning (DBL). For more detailed information about DBL, see Wijnen (2000).

### 3 Modelling 5: Various reflective activities

#### 3.1 The Portfolio in the Modelling Track

As a preparation for the reflective activities in Modelling 5, students have to build up a portfolio during the Modelling 1 to 4 courses. They have to store their project reports and products together with notes and remarks about the discussions with staff or peers. Also they have to reflect on their work after completing each modelling project by answering questions like “When looking back, had it been possible to handle the assignment in a better way, especially from the mathematical point of view?” and “What is the social importance of the assignment?”

#### 3.2 Overview of reflective activities

Important aspects of academic education are the competence to criticise and improve ones own methods and products, to be aware of the social impact of ones own work and to be able to reflect on ones own educational progress. For Modelling 5, a number of assignments were constructed to make these learning goals operational. These assignments generally consist of writing a short essay according to certain guidelines. Concepts are read and generally commented by the teacher<sup>1</sup> as well as read and specifically commented by two peers. After feedback a final version has to be delivered. At every assignment the students choose suited modelling projects from their portfolio.

Below, only the assignments directed related to modelling are elaborated. Besides those there was (a) an assignment asking for reflection on the students’ own learning attitude and study methods as a freshman compared to as a near Bachelor, and (b) an assignment about making choices in ones own educational career. The following three assignments aiming for reflection on former modelling activities form the main part: (1) reflection on the modelling process in general, (2) mathematical reflection on former modelling activities from a more sophisticated position, (3) reflection on the social importance of certain modelling activities. In general each kind of reflection has to be applied to several modelling projects performed before by the student. We will describe the various types of reflection in more detail by giving the assignments as presented to the students.

- 1) *Reflection on the activity of modelling*  
 Comment on the following statement: “Modelling is essentially a three step process, consisting of a language step (from problem to model), a calculation step (from model to solution) and an interpretation step (from solution back to the problem).” Construct a more detailed scheme and confront this scheme with your own former modelling work from your portfolio.
  
- 2) *Mathematical reflection*  
 If you had to do the modelling project concerned again, would you do it in a mathematically different way? Would you use other methods or the same method

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<sup>1</sup> The first author

more thoroughly? Did you learn anything since that is useful for this project? What new aspects you would take into account now, which you did not think of earlier?

3) *Social reflection*<sup>2</sup>

Analyse your modelling project on the two following social aspects:

Firstly, the *actors* aspect: Which actors (groups, organisations, institutions, persons) have an interest in the issue of the problem and its solution? What is each actor aiming for? What interests are at stake for the various actors? Are the various actors satisfied or not with the present situation? In what way can the various actors influence the situation? Answer these questions at least for the following four types of actors (but do not leave out other important actors)

- Technology producers, such as universities or production companies
- Technology users, such as consumers or companies
- Technology regulators, such as governments or standardisation organisations
- Technology advisors, such as engineering- or designing offices, commissions and policy staff members.

Secondly, the aspect of *history and future*: Does the problem possess a history? Are former solutions tried out? Is it possible to predict certain short-term or long-term consequences of the solution, intended or unintended?

Conclude your analysis by arguing which social aspects are the most important and whether it would be possible take into account these aspects in the modelling process.

Find at least one scientific article or policy memorandum on your subject, summarise it and incorporate it in your essay.

In the next paragraph we will look back on the first realisation of the course.

## 4 Evaluation

A total number of 16 students took part this first time. Because the decision to construct the course was taken in 2002, the students taking Modelling 5 in 2003 did not purposefully build up a portfolio, as they were not told to do so. This was not a problem: most of them had stored enough products and reports to work with and generally they had quite a good memory for their former modelling work.

The students were almost always present at the meetings; generally they delivered their assignments in time and gave the assignments enough attention. The mark for the course depended on the quality of the series of assignments. Peer assessment played an important role in the marking process, the amount of comments by peers on a concept version as well as the improvements as a reaction to these comments in the final version. Most students passed the course with marks varying from 'just acceptable' to 'splendid'; one of them had to complete some extra assignments.

The data from a questionnaire about the course showed that the students in general were satisfied with the course. There were some complaints about the time they had to invest, but at the other hand they found most of the assignments interesting. They liked the activity of assessing other students' essays and giving feedback, which was quite new for them. Note that most of the time the project assignment they had to comment on was not an assignment

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<sup>2</sup> The questions are inspired on the interactive study package STUDIO, in use at the Technische Universiteit Eindhoven and the Delft University of Technology in the study programme of Society and Technology. See <http://www.studio.tbm.tudelft.nl/studio/>.

they had performed themselves, as each student had a more or less personal assignment history.

The teachers' own evaluation of the course is in the first place, that students were not only trained in various kinds of academic reflection but that they were trained in essay writing as well. Secondly, the subject that will need extra attention next time is the history-and-future aspect of the social reflection, as many essays were too superficial concerning this aspect. For the history aspect this can be done by making a stronger link with the course on mathematics and its usage in historical perspective mentioned before. For the future aspect the following question could be added to the assignment concerned: „What will be the reactions of the various actors to the solution?“. Thirdly and finally, it is advisable to explain first and second year students more about the goal of building up the portfolio and to explain other staff members more about the meaning of the course.

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