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Jan Willem Rustenburg, Geert-Jan van Houtum, Henk Zijm

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Chapter

EXACT AND APPROXIMATE ANALYSIS OF MULTI-ECHELON, MULTI-INDENTURE SPARE PARTS SYSTEMS WITH COMMONALITY

Jan Willem Rustenburg
Districon Management Consultants
P.O. Box 188, 3600 AD Maarssen, The Netherlands
jw.rustenburg@districon.nl

Geert-Jan van Houtum
Dept. of Technology Management, Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven, The Netherlands
g.j.v.houtum@tm.tue.nl

Henk Zijm
Dept. of Applied Mathematics, University of Twente
P.O. Box 217, 7500 AE Enschede, The Netherlands
w.h.m.zijm@math.utwente.nl

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1. Introduction

General spare parts networks, in which items are both repaired and stocked for future use have received considerable attention since the pathbreaking work of Sherbrooke [20] in 1968. In this chapter, we present both an exact and, under somewhat relaxed assumptions, a fast approximate evaluation method of fairly general multi-echelon, multi-indenture spare parts networks that serve to support the operation of technologically highly involved field systems. These evaluation methods provide the basis for a procedure to optimize the availability of these systems, given limited spare parts budgets. In this introductory section, we first describe the general structure of the networks we wish to study, and we motivate their importance. Next, we discuss some key references and finally summarize the main contributions of the current chapter.

The model studied in this chapter generalizes the VARI-METRIC model as analyzed by Sherbrooke [22] in 1986 for two-echelon, two-indenture spare parts networks. We consider identical, or almost identical, technical systems that operate at or are supported by various *base stations*. The base stations are supported by other, *supporting stations.* The supporting and base stations together constitute a general multi-echelon network with a pure distribution structure. For the technical systems, a general multi-indenture structure is assumed. Each technical system is built up from several assemblies whose complete material breakdown structure is given. We allow that technical systems at different bases consist of slightly different sets of assemblies. One can consider this as a form of *commonality* at the level of assemblies. Commonality may also occur in the assembly structures, i.e. subassemblies may occur in different assemblies, and similarly for components at lower levels. Spare parts can be stocked at each of the base and supporting stations, and repair facilities may be present at each of the stations (e.g., limited repair facilities at base stations and advanced repair facilities at supporting stations). The spare parts stocks are controlled by basestock policies.

The network of supporting and base stations and the configuration of the technical systems are described by graphs. As a result, also basic systems such as the two-echelon, single-indenture system and the single-echelon, two-indenture system fit in our model. Also, for the assemblies and all other components we assume repair probabilities. This allows to model both repairable items and consumable items (for which repair probabilities equal 0 at all stations), and we can deal with *condemnation.*

Logistic support systems such as those described here involve a large number of decisions that have to be taken in order to optimize the field system's operations: what parts (at what level of the material breakdown structures) should be kept on stock, and at which stations (at the bases, or more central, or both)? Also, how should repair facilities be equipped and manpowered, in order to enable certain repairs at certain stations (e.g. simple repairs at the bases, more involved restorations only at support stations)? In this chapter, we do not consider the modeling of the repair facilities explicitly but instead concentrate on the questions related to inventory management. We will come back to the other questions in the final section.

The importance of the possibility to analyze models such as described here (and in more detail in the next section) is that it allows the evaluation of rather complex systems and complex logistic support (supply ánd repair) structures. In particular, the techniques developed enable decision makers to relate overall system availability to available budgets via optimal or close-to-optimal stock allocation policies, that prescribe how much money should be invested in each part, at each location. In other words: what is the optimal system availability that can be achieved, given a prescribed available budget? Or, vice versa: how much inventory investment is minimally needed to achieve a target overall system availability? These questions are highly relevant for many capital-intensive equipment installations such as military weapon systems, medical equipment, aircraft, nuclear power plant installations, and computer systems and infrastructures. In fact, the current study was motivated by problems encountered at the ship maintenance facilities of the Royal Netherlands Navy.

Models for spare parts networks have initially been considered by Sherbrooke [20] and have become known as METRIC models. Following an earlier paper by Feeney and Sherbrooke [9] on an exact analysis for a single-echelon, single-indenture model, Sherbrooke [20] presented an approximate analysis for a divergent two-echelon, single-indenture model controlled by basestock policies. Assuming ample repair capacities, he focuses on the determination of optimal basestock levels at both the bases and the central stocking center, for technical systems composed of multiple items (each item may fail, leading to a replacement and a repair of the broken item, either at the local base or at the central facility). Although, under a basestock policy, the total number of rotating items of each type is fixed, the number of items operating in the field is assumed to be sufficiently large, or down-times of technical systems are assumed to be sufficiently large, to justify the modeling of each assembly's failure process as a Poisson process with a constant rate. The assumption of ample repair capacities and the assumption of Poisson failure processes are typical for METRIC type models, and they constitute the key to come to relatively simple analyses for complicated systems.

An *exact analysis* of a two-echelon, single-indenture model has been presented by Simon [24]. This work has been extended by Kruse [13] to multi-echelon systems and by Shanker [19] to compound Poisson demand processes.

In Sherbrooke [21], an *approximate procedure* for a single-site, two-indenture model has been discussed. Muckstadt [14] extended the existing METRIC model to a two-echelon, two-indenture model, which is also referred to as MOD-METRIC. Another variant of METRIC is VARI-METRIC, a two-echelon, single-indenture model developed by Slay [25]. In the core part of the analysis of the initial METRIC and MOD-METRIC model, it is assumed that, for each product, the number of items in the repair pipeline (i.e., in repair or waiting for components needed for the repair) follows a Poisson distribution (of which the variance equals the mean). In his VARI-METRIC method, Slay derives an approximate expression for the variance of the number of items in the pipeline. Next, for each product, he fits a negative binomial distribution on the first two moments of these items in order to obtain a more accurate approximation. Graves [10] independently developed a slightly simpler approximation for the variance of the number of items in the repair pipeline. Next, he also continues with fitting a negative binomial distribution on the first two moments. Sherbrooke [22] extended the original VARI-METRIC method to a version for two-indenture, two-echelon systems. By simulation, he has verified that the results produced by this method are fairly accurate. An overview of METRIC type models is given in Sherbrooke [23]; see also Guide Jr. and Srivastava [12]. In Rustenburg [16] further generalizations and a unifying framework for the approximate analysis of general multi-echelon, multi-indenture spare parts systems with commonality and condemnation have been presented (see also Rustenburg et al. [18]). Two extensions that were developed recently are by Rustenburg et al. [17] and Caggiano et al. [7]. Rustenburg et al. [17] studied so-called resupply problems, in which an annual budget is available each year to replace condemned repairable and consumable parts. In case of low budgets the question then arises how to spend the remaining budget optimally. Caggiano et al. [7] studied multi-echelon, single-indenture systems with so-called time-based fill rate constraints, which are quite common in practice.

A line of research that is closely related to the research on METRIC type models concerns models with limited repair capacities. Since this is not the focus of the current chapter, we only mention some references: Gross et al. [11] , Albright [2], Diaz and Fu [8], Avsar and Zijm [3, 32], and Sleptchenko et al. [26, 27]. Another related line of research can be found in the work of Axsäter and subsequent authors. They present exact methods for the analysis of classical, continuous-review, single-item, multi-echelon inventory systems for consumable products. The difference with the METRIC models is the absence of any notion of repair or production centers. These authors are capable to analyze models with compound Poisson demand processes and more general (R, Q) policies. See e.g. Axsäter [4, 5] and the references therein for this line of research. Exact methods for classical, periodic-review, multi-echelon models are available too; see e.g. Van Houtum and Zijm [30, 31] and the references therein. Finally, the METRIC type models are related to models for assemble-to-order and assemble-to-stock systems with similar assumptions, such as Poisson demand processes, ample capacities, and basestock control. A main difference is that in the latter models a demand for an end-product decomposes into coupled demands for underlying components. These coupled demands for components complicate the analysis considerably. For references on this type of research, see e.g. Song and Yao [29] and the references therein.

As stated above, we present both an exact and an approximate method for the evaluation of basestock policies in a general multi-echelon, multi-indenture model with commonality. These methods constitute the *main contribution* of this chapter. The exact method generalizes previous work in this area for single-indenture systems without commonality (see Simon [24] and Kruse [13]). The key to the analysis, i.e. the development of recursive expressions, has to the best of our knowledge not been presented earlier. The approximate method extends previous approximate methods for general systems without commonality (see Sherbrooke [22, 23]). Based on the recursive expressions developed for the exact method, the derivation of the approximate method is rather straightforward and we obtain a simple formal procedure (which reduces the complexity for implementations). The approximate method uses two-moment fits of pipeline distribution functions, and is clearly much more efficient than the exact method. This is useful for large systems (many items, many indenture levels and/or many locations), and in particular for procedures for the optimization of basestock levels for such systems, for which usually many evaluations are needed. In addition, the approximate method is quite flexible and allows the relaxation of some assumptions (in particular, deterministic repair and order and ship times, see Section 4) and extensions (e.g., to compound Poisson demand processes).

The *organization* of this chapter is as follows. In Section 2, we describe the model in detail. Next, the exact and approximate evaluation method are described in Section 3 and Section 4, respectively. Subsequently, in

Section 5, we report on the use of the model in two field tests at the Royal Netherlands Navy. This includes a presentation of an optimization procedure for the generation of *efficient solutions* with respect to the inventory investment and average availability. Finally, conclusions and suggestions for further research are given in Section 6.

2. Modeling general spare parts support systems

In this section, the multi-echelon, multi-indenture model with commonality is described. In particular, we discuss the technical system's structure, the material breakdown of the assemblies that constitute each technical system, and the divergent multi-echelon network of stations where spare parts are stocked and repaired. After that, we describe the repair and distribution process in more detail and give an overview of assumptions and notations.

2.1. Model description

We assume the existence of a set N_{ba} of base stations or *bases*, each of which serves a number of technical systems. Technical systems at the same base are assumed to be *identical*. Let $Z_n \in \mathbb{N}$ ($\mathbb{N} := \{1, 2, \ldots\}$) denote the number of technical systems at base n ($n \in N_{ba}$). Technical systems at different bases may be *different* (although in many real-life situations they are identical or at least similar). A technical system consists of several *assemblies*, each of which may fail incidentally. Let I_{as} denote the set of all assemblies that may occur in the configurations of the technical systems, and let z_{in} denote the number of assemblies i in each technical system served at base n ($i \in I_{as}$, $n \in N_{ba}$, $z_{in} \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$). We assume that the total stream of failures of assemblies i as observed by base n constitutes a Poisson process with a constant rate m_{in}. This assumption is standard in METRIC type models (and a key factor for obtaining a tractable model). For many real-life systems, lifetimes of assemblies are (close-to-)exponential, or lifetimes are not precisely exponential but the total stream of failures is a composition of subprocesses coming from relatively many technical systems that are supported by a base. In those cases it is reasonable to assume a Poisson failure process. Further, in practice, one does not allow long down-times of technical systems, and thus, for relevant situations, it is reasonable to assume constant failure rates.

For each assembly, a complete material breakdown structure is given, through subassemblies, sub-subassemblies, and so on. We use the word *part* to indicate any item in the material breakdown structure, i.e., parts indicate assemblies, subassemblies, sub-subassemblies, until basic com-

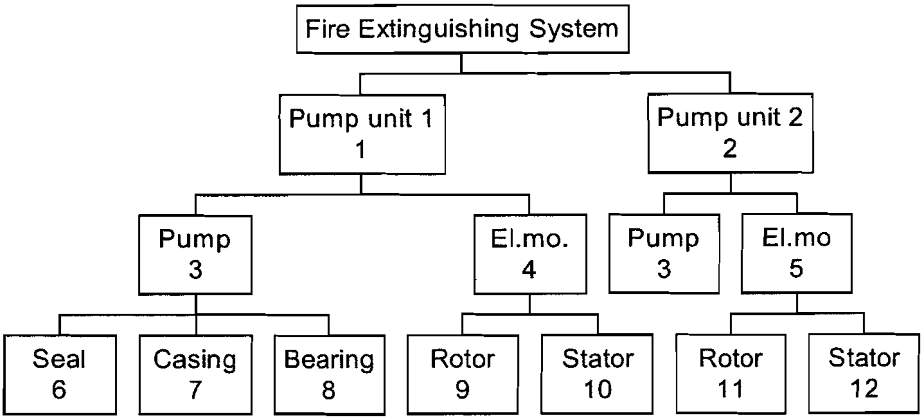

Figure 1.1. The material breakdown structure of a fire extinguishing system. This system consists of two pump units with a pump as a common item.

ponents. Let I denote the set of all parts. Hence, $I \supset I_{as}$. The parts are numbered $1, \ldots, |I|$. Each part $i \in I \setminus I_{as}$ has a set of parents $P_{mb}(i)$. We allow *commonality*, i.e., subassemblies may occur in the configurations of two or more assemblies, and similarly for lower-level items. (That an assembly occurs in the configuration of another assembly is not allowed in the current model. But this may be relaxed if needed.) So, we may have $|P_{mb}(i)| > 1$ for several parts $i \in I \setminus I_{as}$. For each part $i \in I$, $C_{mb}(i) = \{j \in I | i \in P_{mb}(j)\}$ denotes the set of children of i. The set $I_{cl} = \{i \in I | C_{mb}(i) = \emptyset\}$ consists of all childless parts. We assume that the directed graph with the parts as nodes and the parent-child relations as directed arcs contains no circles. For computational convenience, we assume that the assemblies and underlying parts are numbered such that $j < i$ for all $j \in P_{mb}(i)$, $i \in I \setminus I_{as}$.

Notice that very general material breakdown structures can be modeled. Also single-indenture systems fit. For those systems, we have $I_{as} = I_{cl} = I$. For an example of a material breakdown structure of a complete technical system, see Figure 1.1.

Each base station may hold stock of any part and in addition may have repair facilities. Apart from the base stations, there exists a number of support stations with the same possibilities. These stations in turn may be supported by a few other stations, etc. The network of all stations is supposed to constitute a tree. Hence, there is a single root station in the network. This station is supported by external suppliers. Each of the other station has a *unique* supporting station. Let N denote the set of all stations. Hence, $N \supset N_{ba}$. The root station, also called *central depot*, is denoted as station 0 and the other stations are numbered $1, \ldots, |N| - 1$.

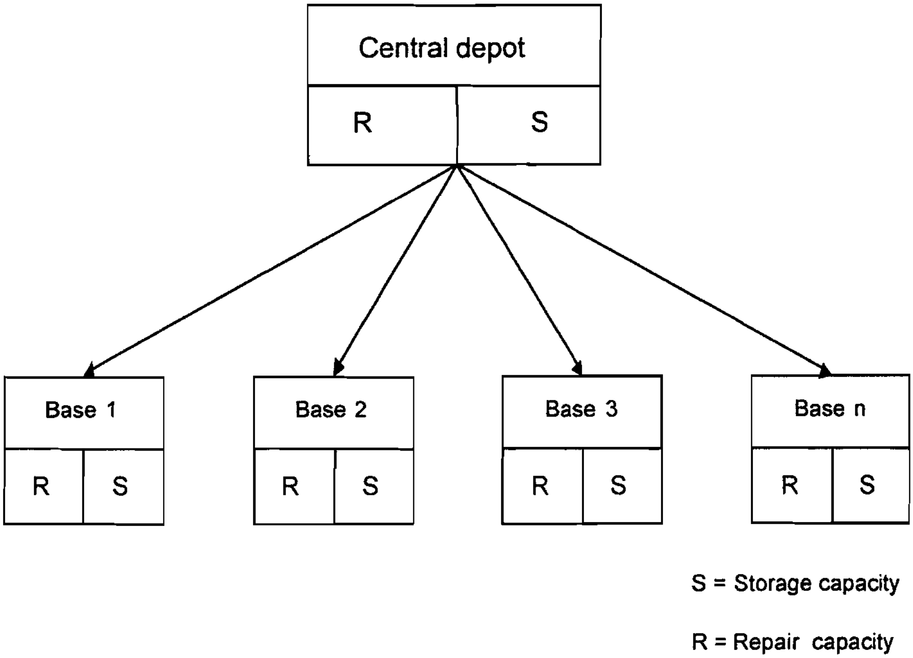

Figure 1.2. A distribution network consisting of a central depot and an arbitrary number of bases.

For each station $n \geq 1$, the unique supporting station, called the parent, is denoted by $p_{dn}(n)$. For computational convenience, we assume that the locations are numbered such that $p_{dn}(n) < n$, $n \in N \setminus \{0\}$. For each station n, $C_{dn}(n) = \{k \in N | p_{dn}(k) = n\}$ denotes the set of children. Hence, the set of base stations N_{ba} satisfies $N_{ba} = \{n \in N | C_{dn}(n) = \emptyset\}$. Notice that the distribution network may consist of any number of echelon levels. Also the extreme case with only one echelon level is allowed. In that case, we have one single location only, which is a base and central depot at the same time. Then $N = N_{ba} = \{0\}$. For an example of a distribution network, see Figure 1.2.

We now describe the operational process, which involves the failures, repairs and distribution of parts. A series of actions starts with each failure of a technical system. We assume that failures only occur (or, are observed) at the assembly level. Therefore, the description starts with a failure of an assembly.

Suppose an assembly i of a technical system at some base station n fails. Then the technical system goes down. To keep the down time of the system as short as possible, the complete assembly is replaced by a ready-for-use one from the base stock, if available, while the malfunctioning assembly is sent into repair. With probability r_{in} assembly i can be repaired at base n, and then the part is sent to the repair shop at the base. With probability $1 - r_{in}$ the assembly cannot be repaired at base n. In the latter case, the malfunctioning assembly is sent to the parent station $p_{dn}(n)$, while at the same time a request for a ready-for-use assembly of type i is placed at station $p_{dn}(n)$. The order and ship time for a part i from the parent station $p_{dn}(n)$ to n is denoted by O_{in}. This time is excluding a possible waiting at station $p_{dn}(n)$ in case a ready-for-use assembly is not immediately available there. In case the malfunctioning assembly can be repaired at base n, the repair action involves the possible detection of a subassembly j that causes the problem. Let q_{ijn} denote the probability that the failure of assembly i is due to subassembly j. With probability $1 - \sum_{j \in C_{mb}(i)} q_{ijn}$ the failure is not due to one of the children $j \in C_{mb}(i)$. Then the failure may be due to a component that is not considered in our model and of which always sufficient spare parts are available, or the failure is due to environmental conditions (for instance, dust), or a real repair of the assembly itself is needed. Once assembly i is sent into repair at base n (i.e., *after* a possible delay because a requested subassembly is not available immediately), it takes a repair leadtime T_{in} until the assembly is returned to the spare parts stock as a ready-for-use item. If the repair of assembly i is outsourced to parent station $p_{dn}(n)$, then the same actions take place at station $p_{dn}(n)$.

A broken subassembly j undergoes the same routine, i.e. after inspection a possible malfunctioning component is detected and replaced. Again, repair of the subassembly may not be possible at the station itself in which case the parent station becomes involved. In this way, we may proceed along the material breakdown structure of any assembly until its lowest level is reached. In principle at each level, except the lowest one, a repair action basically consists of a disassembly, component replacement and finally an assembly action. Only at the lowest level repair indeed means an actual repair action. All underlying items correspond to lower *indenture levels* in the material breakdown structure. However, at any level it may be that repair is not possible at the base in which case the next higher station is asked for support, i.e. the broken item is sent there and a ready-for-use item is shipped downstream to replenish the stock. The higher station in turn may need help as well from its supporting station as was described above, etc. However, if a part i appears to be irreparable at the central depot, which happens with probability $1 - r_{i0}$, then it is disposed of. Then the part is called *condemned*, and immediately a new part is ordered at an external supplier in order to replace this condemned part. We assume that the external supplier of

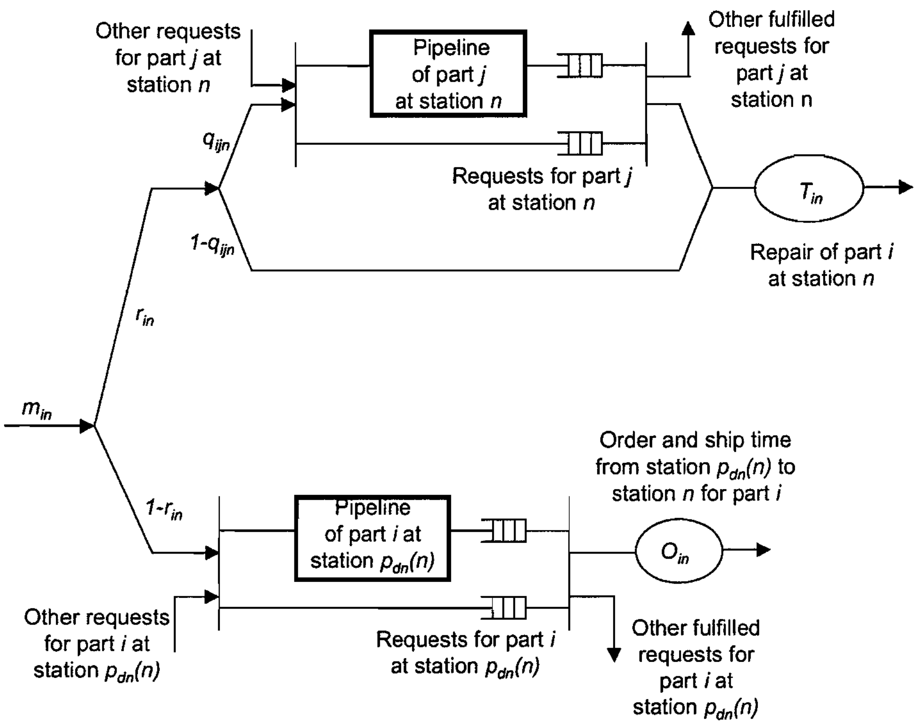

Figure 1.3. Possible actions for a broken part $i \in I$ with one child $j \in C_{mb}(i)$ at a station $n \in N \setminus \{0\}$.

part i can always deliver. The total procurement time is denoted by the random variable O_{i0}.

Notice that implicitly a *one-for-one procurement/replenishment policy* has been assumed for all items at all stations. I.e., each part i at each station n is controlled according to a *basestock policy*. The corresponding basestock level is denoted by S_{in}. Since we often have to deal with expensive slow moving items this seems to be a reasonable assumption. Also note that consumable items are included in our model. Consumables items are irreparable at any station (and thus they are disposed of at the central depot), i.e. for a consumable item i we simply have $r_{in} = 0$ for all $n \in N$.

Obviously, for each part $i \in I$ at each station $n \in N$ backordered demands may occur. In all cases these backorders are treated in FCFS order. Possible actions for a broken part $i \in I$ with one child $j \in C_{mb}(i)$ at a station $n \in N \setminus \{0\}$ are visualized in Figure 1.3. Similar pictures are obtained for other parts and for possible actions at the central depot $(n = 0)$.

With respect to the repair leadtimes T_{in}, the order and ship times O_{in}, $n \in N \setminus \{0\}$, and procurement leadtimes O_{i0}, we assume the following. We assume that the expected values of all these times are given. These expected values are needed in the analysis below. For all parts i, the procurement leadtimes O_{i0} may be stochastic. Further, for all childless parts $i \in I_{cl}$, the repair leadtimes T_{in} may be stochastic. All other repair leadtimes and all order and ship times are assumed to be *deterministic*, and hence can be viewed as nominal leadtimes as used in e.g. ERP systems. Under these assumptions, an exact evaluation is possible as we will see in Section 3. The approximate evaluation as presented in Section 4 also allows stochastic order and ship times and stochastic leadtimes for all parts.

The objective of both the exact (Section 3) and the approximate evaluation (Section 4) of the above model is to determine its overall performance, given the basestock levels S_{in} of all items at all stations. We distinguish the following performance measures. We define β_n as the fraction of all demands for assemblies at base $n \in N_{ba}$ which are immediately fulfilled from stock. This measure is called the *fill rate at base n.* The *overall fill rate* β denotes the fill rate for all demands for assemblies at all bases together. Next, we define A_n as the average fraction of all Z_n technical systems at base $n \in N_{ba}$ which are not down because of a lack of assemblies. This measure is called the *average availability at base n.* The *overall average availability* A denotes the average availability for all technical systems at all bases together. The higher all these measures the better the performance, but also the higher the *total investment in spare parts* will be. The total investment in spare parts is denoted by C and equals

$$C = \sum_{i \in I} c_i \sum_{n \in N} S_{in} , \tag{1.1}$$

where c_i denotes the price of a part i.

2.2. Overview of assumptions and notations

The *main assumptions* of the model are as follows:

1 At each of the bases, the failures for the different assemblies occur according to independent Poisson processes;

2 Each failure of a part with one or more children is due to at most one child;

3 The directed graph with the parts as nodes and the parent-child relations as directed arcs contains no circles. Further, no assembly occurs in the material breakdown structure of another assembly;

4 For each childless part and each stations, the repair leadtimes of all items of that part at that station are independent and identically distributed random variables, while for the exact analysis (Section 3) the repair leadtimes for all other parts are assumed to be deterministic. The approximate evaluation method (Section 4) allows for random repair leadtimes of all parts;

5 For each part, the procurement leadtimes of all items of that part are independent and identically distributed random variables;

6 For the exact analysis, for all parts the order and ship times are assumed to be deterministic (Section 3). The approximate evaluation method (Section 4) allows for stochastic order and ship times;

7 A one-for-one replenishment/procurement policy is applied for all parts at all stations;

8 There is no lateral supply in the distribution network.

The *input variables* of the model are as follows:

N: Set of all stations in the distribution network. The central depot has index 0 and the other stations are numbered $1, \ldots, |N| - 1$.

$p_{dn}(n)$, $n \in N \setminus \{0\}$: Unique parent station of station n. The stations are numbered such that $p_{dn}(n) < n$ for all $n \in N \setminus \{0\}$.

$C_{dn}(n)$, $n \in N$: Set of children of station n.

N_{ba}: Set of bases, i.e. set of stations n with $C_{dn}(n) = \emptyset$.

Z_n, $n \in N_{ba}$: Number of identical technical systems installed at base n ($Z_n \in \mathbb{N}$).

I: Set of all parts (assemblies and underlying parts) that occur in the configurations of the technical systems.

I_{as}: Set of all assemblies ($I_{as} \subset I$).

$P_{mb}(i)$, $i \in I \setminus I_{as}$: Set of parents of part i. The items are numbered such that $j < i$ for all $j \in P_{mb}(i)$, $i \in I \setminus I_{as}$.

$C_{mb}(i)$, $i \in I$: Set of children of item i.

I_{cl}: Set of childless items, i.e. set of items i with $C_{mb}(i) = \emptyset$.

z_{in}, $i \in I_{as}$, $n \in N_{ba}$: Number of occurrences of assembly i in the technical systems installed at base n. If assembly i does not occur in the configuration of the technical systems at base n, then $z_{in} =$

0, otherwise $z_{in} \in \mathbb{N}$. We require that $\sum_{n \in N_{ba}} z_{in} > 0$ for each assembly $i \in I_{as}$.

m_{in}, $i \in I_{as}$, $n \in N_{ba}$: Total failure rate (in failures per year) for assembly i at base n. We require that $m_{in} = 0$ if $z_{in} = 0$, and $\sum_{i \in I_{as}} m_{in} > 0$ for each $n \in N_{ba}$.

r_{in}, $i \in I$, $n \in N$: Probability that a failed part i at station n is repairable at station n itself ($0 \leq r_{in} \leq 1$).

q_{ijn}, $i \in I$, $j \in C_{mb}(i)$, $n \in N$: For a part i, being repaired at station n, each q_{ijn} with $j \in C_{mb}(i)$ denotes the probability that part j is the cause of the failure of this part i. We require that $q_{ijn} \geq 0$ for all $j \in C_{mb}(i)$ and $\sum_{j \in C_{mb}(i)} q_{ijn} \leq 1$.

$\mathsf{E}T_{in}$, $i \in I$, $n \in N$: Mean repair leadtime (in years) for a part i being repaired at station n ($\mathsf{E}T_{in} > 0$).

$\mathsf{E}O_{in}$, $i \in I$, $n \in N \setminus \{0\}$: Mean order and ship time (in years) for a part i being sent from station $p_{dn}(n)$ to station n ($\mathsf{E}O_{in} > 0$).

$\mathsf{E}O_{i0}$, $i \in I$: Mean procurement leadtime (in years) for a part i procured by the central depot at the external supplier ($\mathsf{E}O_{i0} > 0$).

S_{in}, $i \in I$, $n \in N$: Basestock level of part i at station n ($S_{in} \in \mathbb{N}_0$).

c_i, $i \in I$: Price of part i.

As *output variables*, we distinguish:

β_n, $n \in N_{ba}$: Fill rate at base n.

β: Overall fill rate for all bases together.

A_n, $n \in N_{ba}$: Average availability at base n, i.e., the average fraction of all technical systems installed at base n which are not down because of a lack of assemblies.

A: Overall average availability for all technical systems at all bases together.

C: Total investment in spare parts.

Example 1 To highlight the concepts introduced so far, we introduce an example taken from a small case study carried out at the Royal Netherlands Navy; see Rustenburg [16]. We consider the fire extinguishing system whose material breakdown structure has been depicted in Figure 1.1. An identical fire extinguishing system is installed at each of

5 different bases. So, $Z_n = 1$ for each base n. Each fire extinguishing system consists of 2 different pump units. The pump units are the assemblies. They are numbered 1 and 2, and $z_{1n} = z_{2n} = 1$ for $n = 1, \ldots, 5$. Both pump units consist of a pump and an electromotor. The electromotors are of different types, but the two pumps in the units are identical. Hence, the pumps and all their components are common (the structure of the pump in the second pump unit is not given in more detail in Figure 1.1 because it is identical to the pump in the first pump unit). The bases are supported by one central depot. So, we have a distribution network as depicted in Figure 1.2. The bases are numbered $1, \ldots, 5$.

Also the circumstances at the different bases are similar and hence identical values are assumed for the m_{in}, r_{in}, $\mathsf{E}T_{in}$, $\mathsf{E}O_{in}$, and q_{ijn}. The input data are given in Tables 1.1 and 1.2. The q_{ijn} are given in the latter table. The q_{ij0} at the central depot are assumed to be the same as the q_{ijn} at the bases. Notice that the repair probability r_{i0} at the central depot equals 0 for the parts $6, 7, \ldots, 12$; for that reason no repair times $\mathsf{E}T_{i0}$ are given for these parts. The prices are given in Netherlands Guilders (NLG); 1 NLG is equal to 0.4538 EURO. All repair leadtimes, order and ship times and procurement leadtimes are assumed to be deterministic in this example.

Part no.	Name	z_{in}	m_{in} (p.yr)	r_{in}	r_{i0}	$\mathsf{E}T_{in}$ (yrs)	$\mathsf{E}T_{i0}$ (yrs)	$\mathsf{E}O_{in}$ (yrs)	$\mathsf{E}O_{i0}$ (yrs)	c_i (NLG)
1	p.unit 1	1	20.4	0.8	0.95	0.01	0.1	0.2	0.75	11000
3	pump	-	-	0.2	0.7	0.03	0.2	0.2	0.5	1980
6	bearing	-	-	0.2	0	0.1	-	0.2	0.3	330
7	seal	-	-	0.2	0	0.1	-	0.2	0.3	450
8	casing	-	-	0.2	0	0.1	-	0.2	0.3	440
4	elmo	-	-	0.2	0.75	0.03	0.2	0.2	0.5	5080
9	rotor	-	-	0.2	0	0.1	-	0.2	0.3	150
10	stator	-	-	0.2	0	0.1	-	0.2	0.3	480
2	p.unit 2	1	13.6	0.8	0.95	0.01	0.1	0.2	0.75	10000
5	elmo	-	-	0.2	0.75	0.03	0.2	0.2	0.5	3300
11	rotor	-	-	0.2	0	0.1	-	0.2	0.3	450
12	stator	-	-	0.2	0	0.1	-	0.2	0.3	440

Table 1.1. Input data for Example 1

3. Exact Analysis

In this section we present a complete exact performance analysis. First we give some preliminary results. After that we derive recursive expressions for the determination of the pipeline distribution functions.

$i \backslash j$	3	4	5	6	7	8	9	10	11	12
1	0.55	0.45								
2	0.38		0.62							
3				0.32	0.47	0.21				
4							0.29	0.71		
5									0.37	0.63

Table 1.2. The failure probabilities q_{ijn} for Example 1 (the q_{ijn} are the same for all $n = 0, 1, \ldots, 5$).

Based on these recursive procedures, the evaluation procedure is easily deduced.

3.1. Preliminary results

In the initial state of the spare parts network we have an initial number of S_{in} (possibly zero) spare parts on stock of each part $i \in I$ at each station $n \in N$. These parts are demanded according to some demand processes and the stock is replenished according to a basestock policy with the S_{in} as basestock levels. The latter means that the inventory position of each part $i \in I$ at each station $n \in N$ is kept at a constant level S_{in}.

First, we look at the demand processes. The demands for an assembly $i \in I_{as}$ at a base $n \in N_{ba}$ occur according to a Poisson process with a given rate m_{in}. Each demand may immediately result in a demand for a subassembly at base n or a demand for an assembly i at the parent station $p_{dn}(n)$. A demand for assembly i at base n immediately results in a demand for a subassembly $j \in C_{mb}(i)$ with probability $r_{in}q_{ijn}$ and in a demand for an assembly i at station $p_{dn}(n)$ with probability $1 - r_{in}$. As a result, the Poisson demand process for assembly i at base n splits into independent Poisson demand processes for subassemblies at the same base and the same assembly at the parent station. These subprocesses join with independent other subprocesses and thus constitute Poisson demand processes for the subassemblies $j \in C_{mb}(i)$ at base n and assembly i at station $p_{dn}(n)$. In their turn these Poisson processes are split into Poisson processes in a similar way as for assembly i at base n, and so on. Ultimately we obtain a Poisson demand process for each part $i \in I$ at each station $n \in N$. This is stated in the following lemma, where also the rates are given.

Lemma 2 For each part $i \in I$ and station $n \in N$, the demand process of part i at station n is a Poisson process with rate m_{in}, where the rates

m_{in}, $i \in I_{as}$, $n \in N_{ba}$, are given and

$$m_{in} = \begin{cases} \sum_{j \in P_{mb}(i)} m_{jn} r_{jn} q_{jin} & \text{if } i \in I \setminus I_{as},\ n \in N_{ba}; \\ \sum_{k \in C_{dn}(n)} m_{ik}(1 - r_{ik}) & \text{if } i \in I_{as},\ n \in N \setminus N_{ba}; \\ \sum_{j \in P_{mb}(i)} m_{jn} r_{jn} q_{jin} & \\ \quad + \sum_{k \in C_{dn}(n)} m_{ik}(1 - r_{ik}) & \text{if } i \in I \setminus I_{as},\ n \in N \setminus N_{ba}. \end{cases} \tag{1.2}$$

We now consider the inventory of a part $i \in I$ at a station $n \in N$. The inventory position is kept at a constant level S_{in}. Hence,

$$OH_{in}(t) + X_{in}(t) - BO_{in}(t) = S_{in} \ , \qquad t \geq 0, \tag{1.3}$$

where $OH_{in}(t)$, $X_{in}(t)$, and $BO_{in}(t)$ denote the physical stock on hand, the number of parts in repair and on order, and the number of backorders at time instant t. $X_{in}(t)$ is also called the *pipeline stock*. Obviously, at each time instant $t \geq 0$, either $OH_{in}(t) = 0$ or $BO_{in}(t) = 0$, or both. Therefore, if $X_{in}(t)$ is known, then $OH_{in}(t)$ and $BO_{in}(t)$ are known. Equation (1.3) is known as the *stock balance equation.*

Let OH_{in}, X_{in}, and BO_{in} be random variables which denote the physical stock on hand, the number of parts in repair and on order, and the number of backorders in steady state. Then

$$OH_{in} + X_{in} - BO_{in} = S_{in} \ , \qquad i \in I,\ n \in N.$$

By this equation the distribution of BO_{in} and OH_{in} can be determined from the distribution of X_{in}. For the backorder distribution, this is stated in the following lemma.

Lemma 3 For all $i \in I$ and $n \in N$, the distribution of the number of backorders BO_{in} is given by:

$$\mathsf{P}\{BO_{in} = x\} = \begin{cases} \sum_{y=0}^{S_{in}} \mathsf{P}\{X_{in} = y\} & \text{if } x = 0; \\ \mathsf{P}\{X_{in} = x + S_{in}\} & \text{if } x > 0. \end{cases} \tag{1.4}$$

By Lemma 3 we can compute backorder distributions from pipeline distributions. The other way around appears to be possible too, as we will see in the next subsection. There we will derive a recursive procedure for the computation of all pipeline and backorder distributions. That procedure starts with the pipeline distributions of childless parts at the central depot and ends with the distributions for the pipelines X_{in} and backorders BO_{in} of all assemblies $i \in I_{as}$ at all bases $n \in N_{ba}$. Once we have the latter distributions, we can easily determine the steady-state fill rates and availabilities. For the fill rate at a base, we find:

$$\beta_n = \sum_{i \in I_{as}} \frac{m_{in}}{m_{0n}} \mathsf{P}\{X_{in} < S_{in}\} \ , \qquad n \in N_{ba}, \tag{1.5}$$

with $m_{0n} := \sum_{i \in I_{as}} m_{in}$ for all $n \in N_{ba}$. The overall fill rate equals

$$\beta = \sum_{n \in N_{ba}} \frac{m_{0n}}{m_{00}} \beta_n \ , \tag{1.6}$$

with $m_{00} := \sum_{n \in N_{ba}} m_{0n}$. For the average availability of the technical systems at a base $n \in N$, we find (cf. [16] and [23])

$$\begin{aligned} A_n &= \prod_{i \in I_{as}} 1_{\{z_{in}>0\}} \mathsf{P}\{X_{in} \leq S_{in}\} \\ &= \prod_{i \in I_{as}} 1_{\{z_{in}>0\}} (1 - \mathsf{P}\{BO_{in} > 0\}) \qquad \text{if } Z_n = 1, \qquad (1.7) \\ A_n &\approx \prod_{i \in I_{as}} 1_{\{z_{in}>0\}} \left(1 - \frac{\mathsf{E}\{BO_{in}\}}{Z_n z_{in}}\right)^{z_{in}} \qquad \text{if } Z_n > 1. \qquad (1.8) \end{aligned}$$

The average availability for all technical systems together equals

$$A = \sum_{n \in N_{ba}} \frac{Z_n}{Z_{tot}} A_n \ , \tag{1.9}$$

with $Z_{tot} := \sum_{n \in N_{ba}} Z_n$.

3.2. Recursive expressions for pipelines

In this subsection, we describe the recursive procedure for the computation of all pipeline and backorder distributions. By Lemma 3, we can compute the distribution of the number of backorders BO_{in} of a part i at a station n from the pipeline distribution of the same part at the same station. It is also possible to compute pipeline distributions from backorder distributions. However, the distribution of the pipeline stock X_{in} of a part i at a station n is not computed from the backorder distribution of the same part at the same station, but from the backorder distribution of the same part i at the parent station $p_{dn}(n)$ (if applicable) and the backorder distributions of parts $j \in C_{mb}(i)$ (if applicable) at the same station n. To explain this in more detail, let X_{in}^{rep} and X_{in}^{res} be random variables which denote the number of parts i in repair at station n and the number of parts i on order by station n (= being resupplied to station n) in steady state. Then

$$X_{in} = X_{in}^{rep} + X_{in}^{res} \ , \qquad i \in I, \ n \in N, \tag{1.10}$$

and X_{in}^{rep} and X_{in}^{res} are mutually independent. So, the distribution of X_{in} can be obtained by convoluting the distributions of X_{in}^{rep} and X_{in}^{res}.

For both the *repair pipeline stocks* X_{in}^{rep} and the *resupply pipeline stocks* X_{in}^{res}, we derive recursive expressions below. The distribution of X_{in}^{rep} can be computed directly if part i is childless and from the backorder distributions of the children $j \in C_{mb}(i)$ at station n otherwise; see Lemma 4. The distribution of X_{in}^{res} can be computed directly at the central depot (i.e., for $n = 0$) and from the backorder distribution of part i at the parent station $p_{dn}(n)$ otherwise; see Lemma 5. These recursive expressions, together with the expressions (1.4) and (1.10), result directly in an exact, recursive evaluation procedure. The procedure starts with the pipeline distributions of childless parts at the central depot and ends with pipeline and backorder distributions of assemblies at all bases; see the algorithm formulated at the end of this subsection.

Lemma 4 Let $i \in I$ and $n \in N$.

(i) If $i \in I_{cl}$, then the repair pipeline X_{in}^{rep} is Poisson distributed with parameter $m_{in} r_{in} \mathsf{E} T_{in}$;

(ii) If $i \in I \setminus I_{cl}$, then

$$X_{in}^{rep} = Y_0 + \sum_{j \in C_{mb}(i)} Y_j ,$$

where Y_0 is a Poisson distributed random variable with parameter $m_{in} r_{in} \mathsf{E} T_{in}$, Y_j is a random variable with

$$\begin{aligned} \mathsf{P}\{Y_j = y\} &= \sum_{x=y}^{\infty} \binom{x}{y} \left(\frac{m_{in} r_{in} q_{ijn}}{m_{jn}} \right)^y \\ &\quad \left(1 - \frac{m_{in} r_{in} q_{ijn}}{m_{jn}} \right)^{x-y} \mathsf{P}\{BO_{jn} = x\}, \; y \in \mathbb{N}_0, \end{aligned}$$

for each $j \in C_{mb}(i)$ (with the convention that $\mathsf{P}\{Y_j = 0\} = 1$ and $\mathsf{P}\{Y_j = y\} = 0$ for all $y \geq 1$ when $m_{jn} = 0$), and Y_0 and all Y_j are independent.

Proof. Let $i \in I$ and $n \in N$. Demands for part i at station n occur according to a Poisson process with rate m_{in}. Each demand is accompanied by a failed part that is returned. Each failed part is sent into repair with probability r_{in}. So, parts enter the repair pipeline of part i according to a Poisson process with rate $m_{in} r_{in}$. For each part, the repair leadtime is given by the generic random variable T_{in} and this time is independent of other parts being sent into repair.

If $i \in I_{cl}$, i.e., if part i has no children, then the repair of a failed part i cannot be delayed because some underlying part is not immediately available. In that case we can apply *Palm's theorem* (see [15]), and we find that X_{in}^{rep} is Poisson distributed with parameter $m_{in}r_{in}\mathsf{E}T_{in}$. This completes the proof of part (i).

We now consider the case $i \in I \setminus I_{cl}$, i.e., part i has one or more children $j \in C_{mb}(i)$. Then each failed part has a deterministic repair leadtime $T_{in} = \mathsf{E}T_{in}$, but the start of this deterministic repair leadtime may be delayed because an underlying part is required for the repair while that part is not immediately available.

Let $X_{in}^{rep}(t)$ denote the number of parts i in repair at station n at some time instant t (assume $t \geq T_{in}$). Then

$$\begin{aligned} X_{in}^{rep}(t) &= \text{Parts in repair which arrived in the interval } (t - T_{in}, t) \\ &+ \sum_{j \in C_{mb}(i)} [\text{Parts in repair which arrived prior to } t - T_{in} \text{ and which} \\ &\qquad \text{are waiting for a backordered part } j \text{ at time instant } t - T_{in}] \,. \end{aligned}$$

Note that the terms in the latter sum have the same distributions as the numbers of parts i waiting for a backordered part j, $j \in C_{mb}(i)$, at an arbitrary time instant. Now, let random variable Y_0 denote the number of parts i sent into repair at station n in an interval of length T_{in}, and let Y_j, $j \in C_{mb}(i)$, denote the number of parts i which are waiting for an underlying part j at an arbitrary instant. Then $X_{in}^{rep} = Y_0 + \sum_{j \in C_{mb}(i)} Y_j$. Further, Y_0 is Poisson distributed with parameter $m_{in}r_{in}T_{in} = m_{in}r_{in}\mathsf{E}T_{in}$, and Y_0 is independent of the Y_j, $j \in C_{mb}(i)$, due to the non-overlapping time intervals and the fact that Poisson processes have independent increments. For the random variable Y_j we obtain a binomial distribution if we condition to the total number of backorders for part j at station n at an arbitrary time instant:

$$\mathsf{P}\{Y_j = y | BO_{jn} = x\} = \binom{x}{y} \left(\frac{m_{in}r_{in}q_{ijn}}{m_{jn}} \right)^y \left(1 - \frac{m_{in}r_{in}q_{ijn}}{m_{jn}} \right)^{x-y}$$

for all $0 \leq y \leq x$. The explanation for this result is as follows. Demands for part j at station n arrive according to a Poisson process with rate m_{jn}. A fraction $m_{in}r_{in}q_{ijn}$ of this stream comes from parts i that need a part j for their repair at station n. Hence, each demand, and thus also each backordered request comes from parts i being repaired at station n with probability $(m_{in}r_{in}q_{ijn})/m_{jn}$. Finally, it is easily seen that the Y_j, $j \in C_{mb}(i)$, are independent of each other. This completes the proof of part (ii). □

Lemma 5 Let $i \in I$ and $n \in N$.

(i) If $n = 0$, then the resupply pipeline $X_{in}^{res} = X_{i0}^{res}$ is Poisson distributed with parameter $m_{i0}(1 - r_{i0})\mathsf{E}O_{i0}$;

(ii) If $n \in N\backslash\{0\}$, then $X_{in}^{res} = Y_0 + Y$, where Y_0 is a Poisson distributed random variable with parameter $m_{in}(1 - r_{in})\mathsf{E}O_{in}$, Y is a random variable with

$$\mathsf{P}\{Y = y\} = \sum_{x=y}^{\infty} \binom{x}{y} \left(\frac{m_{in}(1 - r_{in})}{m_{ik}}\right)^{y} \left(1 - \frac{m_{in}(1 - r_{in})}{m_{ik}}\right)^{x-y} \mathsf{P}\{BO_{ik} = x\}, \ y \in \mathbb{N}_0,$$

where $k = p_{dn}(n)$ (with the convention that $\mathsf{P}\{Y = 0\} = 1$ and $\mathsf{P}\{Y = y\} = 0$ for all $y \geq 1$ when $m_{ik} = 0$), and Y_0 and Y are mutually independent.

Proof. The proof of this lemma is along the same lines as the proof of the preceding one. Note that each term of the sum in the equation in the second part of the lemma denotes the probability that out of a total of x backlogged items at station k, y items can be attributed to demands from station n. □

Algorithm 6 - Exact evaluation (of a given basestock policy)

Step 1. Read all input variables (see Subsection 2.2); this includes basestock levels S_{in} for all items $i \in I$ and all stations $n \in N$. Apply Equation (1.2) to determine all m_{in} for all i and n with $i \in I \setminus I_{as}$ or $n \in N \setminus N_{ba}$.

Step 2. For all $i = |I|, |I| - 1, \ldots, 1$ and $n = 0, 1, \ldots, |N| - 1$ do:
(i) Determine the distribution of X_{in}^{rep} by Lemma 4;
(ii) Determine the distribution of X_{in}^{res} by Lemma 5;
(iii) Determine the distribution of X_{in} (cf. Equation (1.10));
(iv) Determine the distribution of BO_{in} by Lemma 3.

Step 3. Compute the relevant performance measures cf. the Equations (1.1) and (1.5)-(1.9).

4. An approximate evaluation procedure

From a computational point of view, the calculation of complete distribution functions may be less attractive. An often applied procedure then is to calculate the first two moments of backorder quantities and pipeline stocks. Below we derive recursive expressions to determine the first two moments of pipeline stocks from first two moments of backorder quantities (see Lemmas 8 and 9). For the other way around, we first have to fit a distribution function to the first two moments of a pipeline stock X_{in}, $i \in I$, $n \in N$, in order to determine the first two moments of the backorder quantity BO_{in}. This follows from the expressions in Lemma 7 below. Lemma 7 follows directly from Lemma 3. Lemmas 8 and 9 follow after some tedious but straightforward calculations based on the expressions given in Lemmas 4 and 5.

Lemma 7 For all $i \in I$ and $n \in N$, the first two moments of the number of backorders BO_{in} are given by:

$$\begin{aligned} \mathsf{E}\{BO_{in}\} &= \mathsf{E}\{X_{in}\} - S_{in} + \sum_{x=0}^{S_{in}} (S_{in} - x)\mathsf{P}\{X_{in} = x\} , \\ \mathsf{E}\{BO_{in}^2\} &= \mathsf{E}\{X_{in}^2\} - 2S_{in}\mathsf{E}\{X_{in}\} + S_{in}^2 - \sum_{x=0}^{S_{in}} (S_{in} - x)^2\mathsf{P}\{X_{in} = x\}. \end{aligned}$$

Lemma 8 Let $i \in I$ and $n \in N$.

(i) If $i \in I_{cl}$, then

$$\mathsf{E}\{X_{in}^{rep}\} = \mathsf{Var}\{X_{in}^{rep}\} = m_{in}r_{in}\mathsf{E}T_{in} ;$$

(ii) If $i \in I \setminus I_{cl}$, then

$$\begin{aligned} \mathsf{E}\{X_{in}^{rep}\} &= m_{in}r_{in}\mathsf{E}T_{in} + \sum_{j \in C_{mb}(i)} h_j\mathsf{E}\{BO_{jn}\} , \\ \mathsf{Var}\{X_{in}^{rep}\} &= m_{in}r_{in}\mathsf{E}T_{in} \\ &\quad + \sum_{j \in C_{mb}(i)} \left[f_j(1 - f_j)\mathsf{E}\{BO_{jn}\} + f_j^2\mathsf{Var}\{BO_{jn}\}\right] , \end{aligned}$$

where $f_j := (m_{in}r_{in}q_{ijn})/m_{jn}$ for all $j \in C_{mb}(i)$.

Lemma 9 Let $i \in I$ and $n \in N$.

(i) If $n = 0$, then $X_{in}^{res} = X_{i0}^{res}$ and

$$\mathsf{E}\{X_{i0}^{res}\} = \mathsf{Var}\{X_{i0}^{res}\} = m_{i0}(1 - r_{i0})\mathsf{E}O_{i0} ;$$

(ii) If $n \in N \setminus \{0\}$, then

$$\begin{aligned} \mathsf{E}\{X_{in}^{res}\} &= m_{in}(1 - r_{in})\mathsf{E}O_{in} + f\mathsf{E}\{BO_{ik}\} , \\ \mathsf{Var}\{X_{in}^{res}\} &= m_{in}(1 - r_{in})\mathsf{E}O_{in} \\ &\quad + f(1 - f)\mathsf{E}\{BO_{ik}\} + f^2\mathsf{Var}\{BO_{ik}\} , \end{aligned}$$

where $k = p_{dn}(n)$ and $f := (m_{in}(1 - r_{in}))/m_{ik}$.

Note that, as long as the repair leadtimes are deterministic, we may replace $\mathsf{E}T_{in}$ by T_{in} in Lemma 8. Similarly, in Lemma 9, $\mathsf{E}O_{in}$ can be replaced by O_{in} if the order and ship times are deterministic. The results for the first moments of X_{in}^{rep} and X_{in}^{res} are exact anyhow, while the results for the variances are exact for deterministic repair leadtimes and deterministic order and ship times. Experiments performed in Avsar and Zijm [3] however show that the expressions for the variances of X_{in}^{rep} and X_{in}^{res} are very accurate in case the repair leadtimes and the order and ship times are random variables. Based on these results, from now on we allow all these times to be stochastic, and use the above results as the basis for fitting procedures in these more general cases as well. Note that from Equation (1.10) and the independence of $X_{in}^{rep}$and X_{in}^{res} the mean and variance of X_{in} follow immediately.

We now discuss the fit of a distribution function to the first two moments of a pipeline stock X_{in}. Since X_{in} is a discrete random variable we use discrete probability distribution functions for the fitting. Suppose that a random variable X has a mean $\mathsf{E}\{X\}$ and a variance-to-mean ratio V_x, then it can be shown that, for all possible values of $\mathsf{E}\{X\}$ and V_x, one of the distributions listed in Table 1.3 may be fitted on these values of $\mathsf{E}\{X\}$ and V_x. In Table 1.3 it is also shown which distribution has to be used for each combinations of $\mathsf{E}\{X\}$ and V_x ($V_x < 1 - \mathsf{E}\{X\}$ is not possible; see Adan et al. [1]). For more details on the fitting procedure and the required parameter setting, the reader is referred to Adan et al. [1] and Rustenburg [16].

Combinations of $\mathsf{E}\{X\}$ and V_x	Type of distribution
$1 - \mathsf{E}\{X\} \leq V_x < 1$	mixture of two binomial distributions
$V_x = 1$	Poisson distribution
$1 < V_x \leq 1 + \mathsf{E}\{X\}$	negative binomial distribution
$V_x > 1 + \mathsf{E}\{X\}$	mixture of two geometric distributions

Table 1.3. Four types of distributions related to V_x and $\mathsf{E}\{X\}$

The question now is which class of distributions may be appropriate for the pipeline distributions. Recall that demand is assumed to follow a Poisson process. Combining this with positive basestock levels, it can be shown that the variance of X_{in} always exceeds its mean. So we either have to use the negative binomial distributions or mixtures of two geometric distributions. There is one special case, viz. the case with all stock levels equal to zero. From Lemma 5 we learn that in that particular situation the expected number of backorders of a certain product equals the number of items of that particular product in the pipeline. Moreover the variance of the number of backorders equals the expected number of backorders of these products. So in this situation we arrive at $\mathsf{E}\{X_{in}\} = \mathsf{Var}\{X_{in}\}$. As a consequence the variance-to-mean ratio equals 1, and in that case we have to use a Poisson distribution.

We conclude with a description of the approximate evaluation algorithm, based on moment fitting procedures, and suited for the case in which all repair leadtimes, as well as all order and ship times are random variables.

Algorithm 10 - Approximate evaluation of a basestock policy

Step 1. Read all input variables (see Subsection 2.2). Apply Equation (1.2) to determine all m_{in} for all i and n with $i \in I \setminus I_{as}$ or $n \in N \setminus N_{ba}$. Let basestock levels S_{in} for all items $i \in I$ and all stations $n \in N$ be given.

Step 2. For all $i = |I|, |I|-1, \ldots, 1$ and $n = 0, 1, \ldots, |N|-1$ do:
(i) Determine the mean and variance of X_{in}^{rep} by Lemma 8;
(ii) Determine the mean and variance of X_{in}^{res} by Lemma 9;
(iii) Determine the mean and variance of X_{in} (cf. Equation (1.10));
(iv) Fit an appropriate probability distribution function to the mean and variance of X_{in};
(iv) Determine the mean and variance of BO_{in} by Lemma 7.

Step 3. Compute the relevant performance measures cf. the Equations (1.1) and (1.5)-(1.9).

Example 1 (continued) We apply both the exact and approximate algorithm to evaluate the basestock policy with the following basestock levels:

$$\begin{aligned}(S_{1,0}, \ldots, S_{12,0}) &= (3, 3, 23, 12, 13, 9, 11, 6, 7, 12, 6, 9), \\ (S_{1,n}, \ldots, S_{12,n}) &= (3, 2, 5, 2, 3, 1, 1, 1, 1, 1, 1, 1), \quad n = 1, \ldots, 5.\end{aligned}$$

This basestock policy is one of the solutions generated by the optimization procedure that is described in Subsection 5.1 and that leads to the availability versus investment curve depicted in Figure 1.4. Under this basestock policy, the total investment of spare parts $C = 664930$ NLG. Application of the exact evaluation procedure, Algorithm 6, shows that the overall average availability under this policy equals $A = 89.71\%$. The percentage found by the approximate evaluation procedure, Algorithm 10, is 89.87 %, which is very close. Tests by e.g. Sherbrooke [22] and Rustenburg [16] have shown that the approximate procedure is rather accurate in general. (The computation times of both procedures were low in this case; less than 0.01 seconds on a standard PC.)

5. Field tests at the Royal Netherlands Navy

The evaluation algorithms of Sections 3 and 4 may be used in optimization procedures for the generation of basestock policies under which an optimal tradeoff is obtained for total inventory investment on one hand and e.g. overall average availability on the other hand. Such basestock policies are efficient solutions and by generating a whole series of them one obtains an *efficient frontier* for total inventory investment versus average availability. In the literature, *greedy algorithms* are proposed for the generation of these efficient frontiers; see e.g. Sherbrooke [23] and Rustenburg [16]. In Subsection 5.1, we derive a greedy algorithm for the two-echelon, multi-indenture case, and we justify the use of the greedy algorithm in this case. For the evaluations of basestock policies that are needed in this algorithm, one may use either the exact or the approximate evaluation procedure. The approximate evaluation procedure is advised for somewhat larger systems. That procedure is sufficient accurate and leads to much smaller computation times. (Explicit results on the differences in accuracy and computation time when using the approximate instead of the exact evaluation in the greedy algorithms have not been generated however.)

The greedy algorithm as presented in Subsection 5.1 has been applied to Example 1 and in field tests at the Royal Netherlands Navy. This is reported in Subsection 5.2.

5.1. Optimizing availability under a given budget

As stated above, in this subsection we derive an optimization procedure for the *two-echelon, multi-indenture case (with commonality)*. The echelon structure then is represented by one central depot, denoted by index 0, that supplies a number of local bases, referred to by indices $1, \ldots, |N| - 1$. For notational convenience, we introduce $\hat{N} := |N| - 1$ to denote the number of bases. As before, let I denote the set of all possible items, and I_{as} the set of assemblies as they appear in the technical systems. For ease of presentation we assume that one technical system is present at each base, i.e. $Z_n = 1$ for $n = 1, \ldots, \hat{N}$, and that the technical systems at the various bases are identical. In this identical systems case we can limit ourselves to the consideration of present assemblies, i.e. $z_{in} > 0$ for all $i \in I_{as}$ and $n = 1, \ldots, \hat{N}$. Let $\underline{S}_n$ denote the vector that describes the basestock levels at station n of all items $i \in I$, $n = 0, 1, \ldots, \hat{N}$. Then we may express the availability of the technical system at base n by rewriting (1.7) as

$$A_n(\underline{S}_0, \underline{S}_n) = \prod_{i \in I_{as}} (1 - \mathsf{P}\{BO_{in}(\underline{S}_0, \underline{S}_n) > 0\}), \tag{1.11}$$

where the arguments $\underline{S}_0$ and $\underline{S}_n$ indicate the dependence of availability and backorder probabilities of the basestock levels of all parts, at both base n and the central depot (notice that, due to the FCFS allocation rule at the central depot, the BO_{in} at a given base n do not depend on the basestock levels at other bases; they only depend on the rates with which the other bases place orders at the central depot). There is some redundancy in the notation here, since clearly $BO_{in}(\underline{S}_0, \underline{S}_n)$ depends only on the basestock levels of those parts that appear in the product structure of the assembly i.

The average availability over all technical systems at the respective bases now follows from (1.9) as

$$A(\underline{S}) = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} A_n(\underline{S}_0, \underline{S}_n), \tag{1.12}$$

where $\underline{S} = (\underline{S}_0, \underline{S}_1, \ldots, \underline{S}_N)$. The objective is to determine a set of stock levels $\underline{S}$ such that the availability $A(\underline{S})$ is maximized given a limited budget $\hat{C}$. The objective function (1.12) however is not a very convenient one. Therefore, we first rewrite (1.12) as follows:

$$A(\underline{S}) = 1 - \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} (1 - A_n(\underline{S}_0, \underline{S}_n)). \tag{1.13}$$

When $A_n(\underline{S}_0, \underline{S}_n)$ is sufficiently close to 1, then, according to (1.11), all $\mathsf{P}\{BO_{in}(\underline{S}_0, \underline{S}_n) > 0\}$ must be small and thus

$$A_n(\underline{S}_0, \underline{S}_n) \approx 1 - \sum_{i \in I_{as}} \mathsf{P}\{BO_{in}(\underline{S}_0, \underline{S}_n) > 0\}.$$

Then $1 - A_n(\underline{S}_0, \underline{S}_n)$ is approximately equal to the sum of backorder probabilities on the righthand side of this expression, and substituting that result into (1.13) gives:

$$A(\underline{S}) \approx 1 - \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} \sum_{i \in I_{as}} \mathsf{P}\{BO_{in}(\underline{S}_0, \underline{S}_n) > 0\}. \tag{1.14}$$

We may conclude from Equation (1.14) that, within the area of relevant choices for all basestock levels, maximizing availability is equivalent to minimizing the sum of all the backorder probabilities.

We can now formulate the following nonlinear integer programming problem:

$$\min \sum_{n=1}^{\hat{N}} \sum_{i \in I_{as}} \mathsf{P}\{BO_{in}(\underline{S}_0, \underline{S}_n) > 0\} \tag{1.15}$$

subject to:

$$\sum_{n=1}^{\hat{N}} \sum_{i \in I} c_i S_{in} \leq \hat{C},$$

$$S_{in} \in \mathbb{N}_0 \text{ for all } i \in I \text{ and } n = 0, 1, \ldots, \hat{N}.$$

We apply a marginal analysis approach to solve the latter problem. However, such an algorithm only provides optimal solutions if, for each assembly $i \in I_{as}$ and each base $n = 1, \ldots, \hat{N}$, the backorder probabilities $\mathsf{P}\{BO_{in}(\underline{S}_0, \underline{S}_n) > 0\}$ are convex as a function of the basestock levels S_{in}. We will not extensively discuss solution procedures for this; the reader is referred to Rustenburg [16] for further details. For simple single-echelon, single-indenture systems we can use simple arguments to derive lower bounds above which the backorder probability are indeed convex. Here we only remark that extensive tests have revealed that in our more general case the functions $\mathsf{P}\{BO_{in}(\underline{S}_0, \underline{S}_n) > 0\}$ fulfill convexity requirements if the arguments exceed a certain minimum value, indicating that we should start the marginal analysis with not too small values of the basestock levels. From the experiments, we obtained the following initial basestock levels for the two-echelon, multi-indenture case as a starting point for the marginal analysis:

$$\begin{aligned} S_{in} &:= round(m_{in}(r_{in}\mathsf{E}T_{in} + (1 - r_{in})\mathsf{E}O_{in})); \\ S_{i0} &:= round(\tfrac{1}{2}m_{i0}(r_{i0}\mathsf{E}T_{i0} + (1 - r_{in})\mathsf{E}O_{i0})). \end{aligned} \tag{1.16}$$

Notice that these initial levels are related to the expected values of the pipeline inventories. They are still relatively low, and thus associated with high backorder probabilities.

The marginal analysis now boils down to a procedure where in each step an item i and a station n is selected such that adding that item to that station yields the largest decrease in the overall objective function per unit of money invested (sometimes referred to as "the biggest bang for the buck" approach, and similar to some well-known knapsack problem heuristics). The formal procedure is described in the algorithm below. In this procedure, $\underline{e}_j$ is an $|I|$-dimensional vector with a 1 on the j-th position and zeros on all other positions.

Algorithm 11 - Greedy procedure for the two-echelon, multi-indenture model

Step 1. Set $S_{in} := round(m_{in}(r_{in}\mathsf{E}T_{in} + (1 - r_{in})\mathsf{E}O_{in}))$ for each $i \in I$ and each $n = 1, \ldots, \hat{N}$.
Set $S_{i0} := round(\frac{1}{2}m_{i0}(r_{i0}\mathsf{E}T_{i0} + (1 - r_{in})\mathsf{E}O_{i0}))$ for each $i \in I$.
$C := \sum_{i \in I} \sum_{n=0}^{\hat{N}} c_i S_{in}$.

Step 2. $\Delta_{i0} := \Big[\sum_{n=1}^{\hat{N}} \sum_{j \in I_{as}} \mathsf{P}\{BO_{jn}(\underline{S}_0, \underline{S}_n) > 0\}$
$- \sum_{n=1}^{\hat{N}} \sum_{j \in I_{as}} \mathsf{P}\{BO_{jn}(\underline{S}_0 + \underline{e}_i, \underline{S}_n) > 0\}\Big] / c_i$ for all $i \in I$.
$\Delta_{in} := \Big[\sum_{j \in I_{as}} \mathsf{P}\{BO_{jn}(\underline{S}_0, \underline{S}_n) > 0\}$
$- \sum_{j \in I_{as}} \mathsf{P}\{BO_{jn}(\underline{S}_0, \underline{S}_n + \underline{e}_i) > 0\}\Big] / c_i$ for all $i \in I$ and $n = 1, \ldots, \hat{N}$.
$k, l := \arg\max\{\Delta_{in} | i \in I; n = 0, 1, \ldots, \hat{N}\}$

Step 3. If $C + c_k \le \hat{C}$, then $C := C + c_k$, $S_{kl} := S_{kl} + 1$ and return to step 2, else stop.

In Algorithm 11, in each step the basestock vector $\underline{S}$ constituted by all current basestock levels can be stored. Then, at the end of the algorithm, one has a series of solutions $\underline{S}$ under which one has an optimal combination for the average availability $A(\underline{S})$ and the inventory investment C. The tuples $(C, A(\underline{S}))$ constitute the efficient frontier for the average availability versus inventory investment. Procedures for the generation of the efficient frontier of inventory investment versus overall fill rate, or other performance measures, can be derived in a similar way.

5.2. Field tests

In this subsection, we present numerical results for Example 1 and field tests carried out at the Royal Netherlands Navy.

Consider the fire extinguishing system of Example 1. The resulting availability-investment curve is depicted in Figure 1.4, for both the situation in which we do account for commonality effects and in which we do not (in the latter case, the two pumps are treated as different subassemblies, with different components). Both curves have been generated by Algorithm 11, where the approximate evaluation procedure was used for all evaluations. Figure 1.4 indicates that small savings are possible when commonality is taken into account. For example the 'commonality curve' reaches 95.0 % availability at an inventory investment of $7.43 \cdot 10^6$ NLG. When commonality is not taken into account, 95.0 % availability

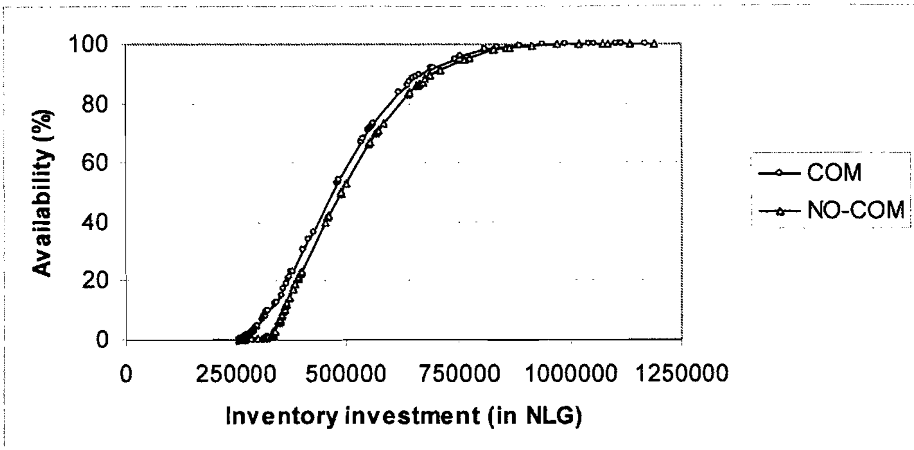

Figure 1.4. Availability versus investment curve for Example 1, for both the situation in which commonality is taken into account and the situation where common parts are treated as different parts.

is reached at an inventory investment of approximately $7.63 \cdot 10^6$ NLG. In this example, the savings are small because the common parts (pump, seal, casing, bearing) are rather cheap in comparison to the other parts in the material breakdown structure. Much higher savings are obtained when common parts are expensive.

Field tests have been executed at the Royal Netherlands Navy, which motivated part of the studies reported here. Below, we briefly present the results for two of these tests, to show the impact of smart spare parts methods on overall inventory investments and system availability; for more results, and further details, see Rustenburg [16]. One system studied is the *Goalkeeper*, a close-in weapon system primarily designed to intercept fast and low incoming missiles, as well as aircrafts, helicopters and surface targets. In the Netherlands Navy, we find these systems on frigates as well as auxiliary ships. All Goalkeepers are more or less identical, hence we consider all ships on which such a system is installed. The second system is the *long range air surveillance radar*, type LW-08/02, which is installed on Multi-Purpose frigates (M-frigates). The primary function of the LW-08/02 is the timely detection of air targets. For that purpose the LW-08/02 is equipped with a long distance radar with a small minimum range and a high resistance to electronic counter measures. A picture of the LW-08/02 is shown in Figure 1.5.

For our studies we selected the parts with a price ≥ 75 NLG (recall that 1 NLG = 0.4538 EURO). Moreover the parts with a registered demand rate equal to zero are excluded. With this selection, we reduced

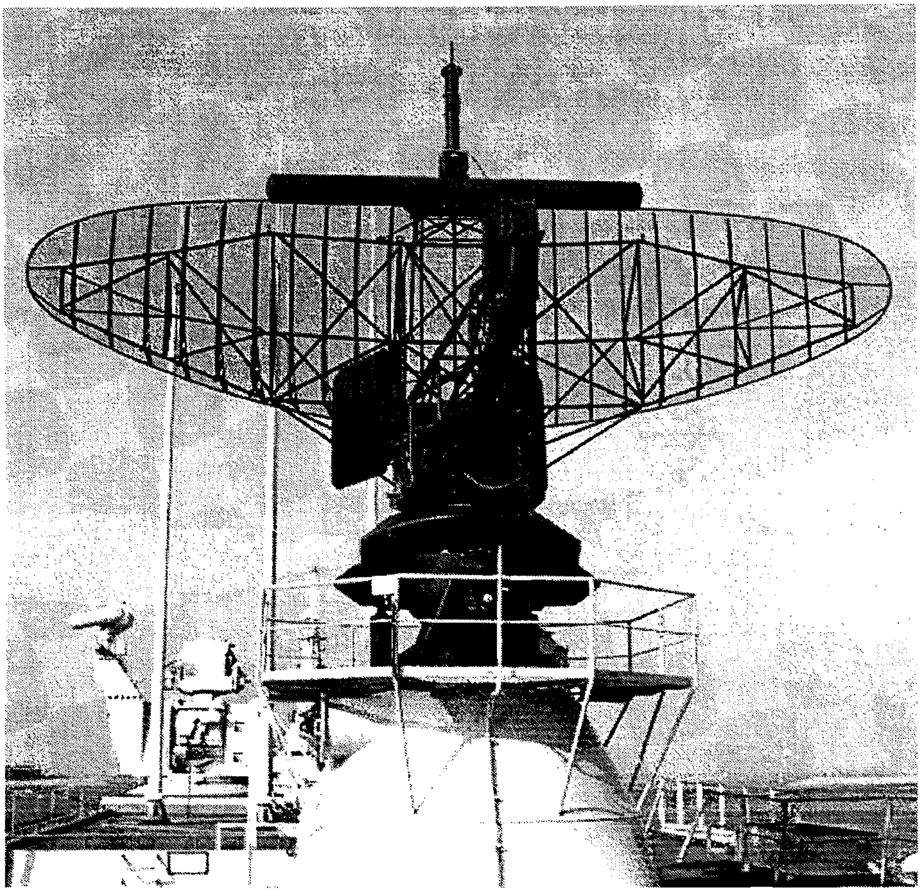

Figure 1.5. A photo of the long range air surveillance radar, type LW-08/02.

the number of products considered in the cases by approximately 12 % while the inventory investment considered was approximately 98 % of the complete investment in spare parts of the systems considered. For the total number of spare parts with corresponding inventory investment and the selected spare parts with corresponding inventory investment, see Table 1.4.

In order to judge the current investments properly, we should note that the Navy Maintenance Company used inventory control policies based on achieving target fill rates that were equal for most items, independent of their level in a product structure as well as their price. Although not emphasized explicitly, the application of the marginal analysis approach typically reveals stock allocations in which the cheaper parts lower in the product breakdown structures obey high fill rates whereas the more expensive higher level parts (assemblies and subassemblies) typically satisfy lower fill rates (one does not want to have a system down due to a shortage on cheap items, while for the more expensive items repair by replacement is a good option).

System	Total no. of parts	Inventory investment (in 10^6 NLG)	Selected nr of parts	Inv. investm. selected parts (in 10^6 NLG)
Goalkeeper	771	114	675	112
LW-08/02	140	26.4	133	26.0

Table 1.4. Selected number of spare parts for the field tests.

In Table 1.5 we compare the current situation with an optimal situation, as obtained by Algorithm 11 (in combination with the approximate evaluation procedure). The optimal situation refers to the combination of the first availability above 90% and the corresponding inventory investment. We learn from Table 1.5 that for both cases the current investments in spare parts are far from optimal with respect to availability. When studying the Goalkeeper we observe that the optimal inventory investment is relatively close to the current inventory investment. However the availability in the current sitation is much lower than in the optimal situation. Hence, in this case we have to do with a substantial *mis*investment. The current availability of the LW-08/02 is rather close to the optimal availability; however the current investment is much higher than the optimal investment. So, here we have to do with a substantial *over*investment.

	Goalkeeper		LW-08/02	
	Current	Optimal	Current	Optimal
Inventory investment (in 10^6 NLG)	112	102	26.0	10.4
Availability (%)	56.3	90.3	88.9	90.7

Table 1.5. Current situation versus an optimal situation.

6. Conclusion

In this chapter, we have presented both an exact and an approximate analysis of general multi-echelon, multi-indenture models, in which the stations are embedded in a divergent (inverse aborescent) structure while in the product material breakdown structure commonality is allowed. In principle, each failed item arriving at a station is replaced by a ready-for-use one from the station stock, while the failed item is attempted to repair in that station's repair facility. At each station repair of items is possible in principle. For all but the lowest items repair means replacement of one of its constituting components, whereas items on the lowest level are either repaired or disposed of, in which case a new one is procured. If repair of some item at a station is not possible it is sent to the next higher station while at the same time an order for a replacing item is issued. For the case where the operation in each repair center is modeled via a (workload independent) leadtime, and order and ship times of items that have to be ordered at higher echelons are deterministic, an exact analysis has been presented. For practical purposes, also an approximate procedure has been developed, based on a moment fitting procedure, that in addition allows us to handle the case in which also order and ship times are random. The approximate procedure has appeared to be rather accurate. An example is used to illustrate its use while results on real life test cases are also briefly reported. We have seen that smart inventory control may lead to considerable savings for the company involved.

This research is currently extended in several directions. First, we mention that, without much difficulty, the approximate method can be extended to handle the case with compound Poisson processes. More important is the extension to capacitated systems. Based on approaches by Buzacott et al. [6], Avsar and Zijm [3] recently developed approximate procedures to handle the case in which each repair center is modeled as a finite capacity queuing network, thus allowing a more careful study of the interplay between available repair capacity, inventory levels and resulting leadtimes. While in this paper they deal with a single item, a second paper [32] discusses the multi-indenture situation. The study of multiple items, each with a material breakdown structure, in a divergent multi-echelon structure is a natural further extension of this line of research.

Quite a different approach can be found in Sleptchenko et al. [28], who study finite capacity serial system where the repair centers are modeled as M/G/c queues (hence allowing for general repair times). In addition, they distinguish between several classes of items and allow for priority setting in the repair centers. They restrict themselves to the single-indenture case.

Still another line of research is explored by the current authors. In [17] they studied the so-called resupply problem that concentrates on repairable items that are disposed of and hence have to be procured again. Often organizations such as the Netherlands Navy only have a limited annual resupply budget available for these procurements; if during the year one foresees a shortage the question arises which items still should be purchased and which not. Again, the efficient frontier curves provide guidelines to answer these questions.

The importance of these models for spare parts logistics is beyond any doubt. They allow us to study spare parts supply chains in relation to product structures and resource availability in repair centers. The exploitation of commonality is just one example of how a smart product structure may influence the costs of logistic support, but more generally the impact of product design on the costs of logistic support during its full lifetime (life cycle analysis) is a topic that has recently attracted much attention in practice, in particular in relation to investments in often expensive equipment in the military industry. A further exploration of models such as the one discussed here can significantly contribute to a thorough basis in such life cycle analysis studies.

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