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B. R. Pearson[a)]
School of Mechanical, Materials, Manufacturing Engineering and Management, University of Nottingham, Nottingham NG7 2RD, United Kingdom

P.-Å. Krogstad
Department of Applied Mechanics, Thermodynamics and Fluid Dynamics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

W. van de Water
Fluid Dynamics Laboratory, Department of Applied Physics, University of Technology, Eindhoven, 5600MB Eindhoven, The Netherlands

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The one-dimensional surrogate for the dimensionless energy dissipation rate C_ϵ is measured in shear flows over a range of the Taylor microscale Reynolds number R_λ, $70 \lesssim R_\lambda \lesssim 1217$. We recommend that C_ϵ should be defined with respect to an energy length scale derived from the turbulent energy spectrum. For $R_\lambda \gtrsim 300$, a value of $C_\epsilon \approx 0.5$ appears to be a good universal approximation for flow regions free of strong mean shear. The present results for C_ϵ support a key assumption of turbulence—the mean turbulent energy dissipation rate is finite in the limit of zero viscosity.
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The mechanism of the dissipation of turbulent energy is probably one of the most fundamental aspects of turbulence. In 1922, Richardson[1] proposed his phenomenological forward cascade model, whereby the rate of turbulent energy dissipation is determined by the rate in which the large-scale eddies pass energy downward to the small-scale eddies. A key consequence of the forward energy cascade in strong turbulence is that the nondimensional mean energy dissipation rate C_ϵ is independent of viscosity.[2,3] Thus, C_ϵ defined as

$$C_\epsilon = \langle \epsilon \rangle L / \langle u^2 \rangle^{3/2}, \tag{1}$$

(here, $\langle \epsilon \rangle$ is the mean energy dissipation rate per unit mass, L and u are characteristic large length and velocity scales, respectively) should be independent of the Reynolds number and be of order unity.[4] The order of magnitude assumption has been rigorously demonstrated by Lohse.[5] He derived, starting with a mean-field theoretical result for the Navier–Stokes equations and ignoring the possibility of inertial range intermittency effects, C_ϵ as a function of the Reynolds number. In summary, Lohse showed the asymptotic form for C_ϵ to be

$$C_\epsilon \simeq (a/C)^{3/2}; \tag{2}$$

here, C is the Kolmogorov constant for the second-order structure function $D(r) = C\langle \epsilon \rangle^{2/3} r^{2/3}$, and a is estimated at $r = L$, as $D(r=L) = a\langle u^2 \rangle$. The latest direct numerical simulations (DNS) results for forced isotropic box turbulence[6] suggest $a \approx 1.25$ and $C \approx 2.05$ giving $C_\epsilon \approx 0.48$. Note that, irrespective of the structure function scaling deviating slightly from 2/3 due to small-scale intermittency corrections, the magnitude of C_ϵ is $\sim O(1)$.

The first experimental evidence for the satisfaction of a one-dimensional equivalent for relation (1), in quasi-homogeneous grid turbulence, was given by Batchelor.[3] Saffman[7] remarked that the evidence was not quite convincing and the possibility of a dependence on viscosity could not be ruled out. Sreenivasan[4] provided an update on the experimental grid turbulence evidence to support relation (1), over the Taylor microscale Reynolds number R_λ [$R_\lambda \equiv \langle u^2 \rangle^{1/2} \lambda / \nu$, where λ is the Taylor microscale $= \langle u^2 \rangle^{1/2} / \langle (\partial u / \partial x)^2 \rangle^{1/2}$, x is the streamwise direction and u is the streamwise velocity fluctuation] range of $50 \lesssim R_\lambda \lesssim 500$. Roshko[8] noted that there were simpler ways of acquiring direct experimental evidence for the constancy of C_ϵ than relation (1), e.g., the "universal" spreading rate of axisymmetric jets, observed under many different types of laboratory conditions and over considerable Reynolds number ranges was considered by him to be proof enough. Later, Sreenivasan[9] showed that $C_\epsilon \sim O(1)$ was well obeyed for a number of nonhomogeneous "canonical" turbulent flows and for an ensemble[10] of DNS of forced and decaying isotropic box turbulence.

While the order of magnitude of C_ϵ is found to be unity, its precise value is thought to depend on the forcing scheme in DNS[10] and on the macroscale for the case of experiments.[4,9–12] To date, the experimental evidence for the independence of C_ϵ from R_λ can be considered scant and the issue unresolved.[13] The aim of the present work is to test the constancy of one-dimensional surrogates for relation (1) in shear flows over a R_λ range which is considerably larger than that in Refs. 4, 9–12. Such information will be a useful

[a)]Author to whom correspondence should be addressed. Telephone: +44 (0)115 95 13789; fax: +44 (0)115 95 13800; electronic mail: bruce.pearson@nottingham.ac.uk

addition to the results already obtained in quasi-homogeneous grid turbulence[3,4] and forced/decaying isotropic DNS of box turbulence.[10]

The majority of data is acquired in a simple inexpensive geometry, which we call a NORMAN grid, that "stirs" vigorously on large scales. The geometry is composed of a perforated plate superimposed over a bi-plane grid of square rods. Further details of the geometry and the resulting flow will be described elsewhere and only a brief description of the experimental setup is given here. In order to span a large R_λ range, two wind tunnels are used. The first grid, hereafter $N1$, is located in a blow-down wind tunnel[11] of test section dimensions $35\times 35\ \mathrm{cm}^2$ and 2 m length. The second grid, hereafter $N2$, is located in a recirculating wind tunnel with a test section of $2.7\times 1.8\ \mathrm{m}^2$ cross section and length 11 m. For $N1$, the central three rows of the original bi-plane grid (mesh size $M=50$ mm, original solidity $\sigma=33\%$) have alternate meshes blocked (final $\sigma=46\%$). For $N2$ ($M=240$ mm) the original $\sigma=28\%$ and the final $\sigma=42\%$. As well as the NORMAN grid geometries, normal plate wake data and centerline pipe measurements are re-evaluated here and details can be found in Refs. 11 and 12, respectively. Also, measurements are made on the centerline of a wake formed behind a circular disk of 40 mm diameter in the same facility as that for $N1$, the normal plate wake and the circular cylinder wake. For the disk flow the measurement station is located at $x/d\approx 45$. For all flows, signals of u are acquired, for the most part, on the mean shear profile centerline. For $N2$, data are also obtained slightly off the centerline at a transverse distance of one mesh height. All data are acquired using the constant temperature anemometry (CTA) hot-wire technique with a single-wire probe made of 1.27 μm diameter Wollaston (Pt-10% Rh) wire. The instantaneous bridge voltage is buck-and-gained and the amplified signals are low-pass filtered f_{lp} with the sampling frequency f_s always at least twice f_{lp}. The resulting signal is recorded with 12-bit resolution and for the $N1$ data reduced velocities are saved with 13-bit resolution. Throughout this work, time differences τ and frequencies f are converted to streamwise distance ($\equiv \tau U$) and one-dimensional longitudinal wave number k_1 ($\equiv 2\pi f/U$), respectively, using Taylor's hypothesis. The mean dissipation rate $\langle\epsilon\rangle$ is estimated assuming isotropy of the velocity derivatives, i.e., $\langle\epsilon\rangle \equiv \epsilon_{\mathrm{iso}} = 15\nu\langle(\partial u/\partial x)^2\rangle$. We estimate $\langle(\partial u/\partial x)^2\rangle$ from $\phi_u(k_1)$ [the one-dimensional energy spectrum of u such that $\langle u^2\rangle = \int_0^\infty \phi_u(k_1)dk_1$ and $\langle(\partial u/\partial x)^2\rangle = \int_0^\infty k_1^2\phi_u(k_1)dk_1$]. We have chosen not to correct for the decrease in wire resolution that is associated with an increase in R_λ, since all methods known to us rely on an assumed distribution for the three-dimensional energy spectrum. For most of the data, the worst wire resolution is $\approx 2\eta$ where η is the dissipative length scale$\equiv(\nu^3/\epsilon_{\mathrm{iso}})^{1/4}$. For $N1$, the worst wire resolution is $\approx 4\eta$. Finally, we also consider the moderately high R_λ data obtained in "active" grid flows (see Refs. 13–16 for further experimental details).

The present investigation is limited to one-dimensional measurements and suitable surrogates for relation (1). For the mean energy dissipation rate $\langle\epsilon\rangle$ we use ϵ_{iso}. There are two convenient possibilities for L, the characteristic length scale of the large-scale motions. The first is L_u, the stream-

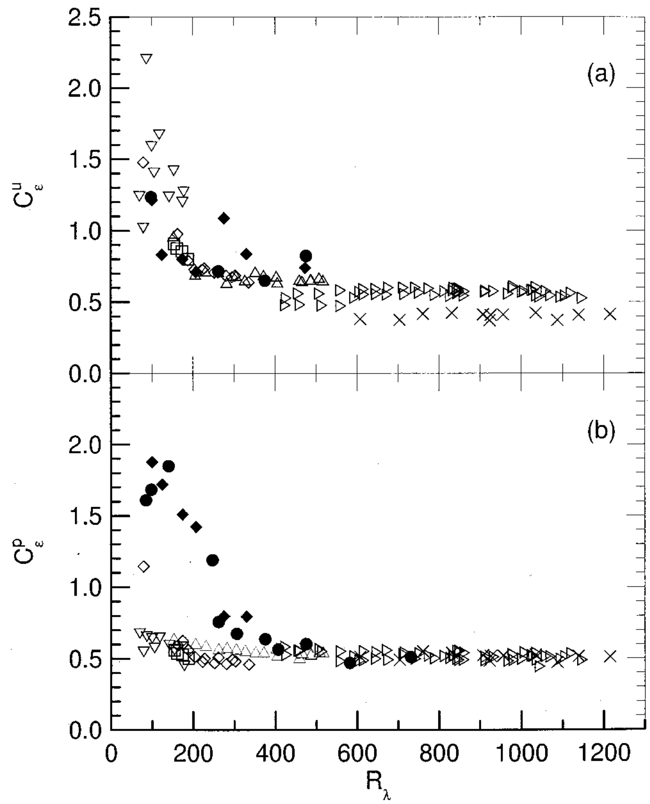

FIG. 1. Normalized dissipation rate for a number of shear flows. Details as found in this work and Refs. 14–16. (a) C^u_ϵ [Eq. (3)]; (b) C^p_ϵ [Eq. (4)]. □, circular disk, $154\lesssim R_\lambda\lesssim 188$; ▽, pipe, $70\lesssim R_\lambda\lesssim 178$; ◇, normal plate, $79\lesssim R_\lambda\lesssim 335$; △, NORMAN grid, $174\lesssim R_\lambda\lesssim 516$; × NORMAN grid (slight mean shear, $dU/dy\approx dU/dy|_{\max}/2$), $607\lesssim R_\lambda\lesssim 1217$; ▷, NORMAN grid (zero mean shear), $425\lesssim R_\lambda\lesssim 1120$; ●, "active" grid Refs. 14, 15, $100\lesssim R_\lambda\lesssim 731$; ◆, "active" grid, with L_u estimated by Ref. 16. For Ref. 14 data, we estimate $L_p\approx 0.1$ m and for Ref. 15 data we estimate $L_p\approx 0.225$ m.

wise integral length scale, computed from the streamwise autocorrelation function $\rho_{uu}(\tau)$ [L_u is defined as the area under the corresponding autocorrelation function $\rho_{uu}(\tau)$ such that $L_u = U\int_0^{\tau_0}\rho_{uu}(\tau)d\tau$ with time τ_0 chosen as the first zero-crossing] and a plausible surrogate for relation (1) is

$$C^u_\epsilon = \epsilon_{\mathrm{iso}} L_u/\langle u^2\rangle^{3/2}. \tag{3}$$

The second possibility for L is L_p, the predominant energy scale that follows directly from the spectrum $\phi_u(k_1)$. The length scale L_p is estimated from the wave number $k_{1,p}$ at which a peak in the compensated spectrum $k_1\phi_u(k_1)$ occurs, i.e., $L_p = 1/k_{1,p}$. A second suitable surrogate for relation (1) is

$$C^p_\epsilon = \epsilon_{\mathrm{iso}} L_p/\langle u^2\rangle^{3/2}. \tag{4}$$

Since the majority of flows investigated in the present work are wake flows it is useful to recall that all wakes form some semblance of a vortex street and the governing parameter of a vortex street is the Strouhal number St. For the flows considered here, it is simple to show, noting that $k_{1,p}=1/L_p$, that St can be defined as

$$\mathrm{St} = L_u/2\pi L_p, \tag{5}$$

which, when rearranging relations (3) and (4), gives

$$\mathrm{St}=C_\epsilon^u/2\pi C_\epsilon^p. \tag{6}$$

Relation (6) indicates that St accounts for the difference between relations (3) and (4). It is interesting to note that Refs. 4, 9–12 observed a distinction between different flows or different large-scale forcing schemes for C_ϵ^u. It may very well be that these flows are distinguishable on the basis of the Strouhal number, and that possible differences in C_ϵ^u can be reconciled using relation (6). Uncovering possible differences between the result of relations (3) and (4) is, therefore, another aim of the present work.

Figure 1(a) shows C_ϵ^u for each flow calculated by relation (3). After a rapid decrease in C_ϵ^u for $R_\lambda \lesssim 300$, C_ϵ^u tends to a constant value as R_λ increases. However, the apparent magnitude for C_ϵ^u is different for each flow and indicates that L_u, in relation (3), is a sensitive indicator of the particular manner in which each flow is forced at large scales and also may reflect the influence of initial and/or boundary conditions. Figure 1(b) shows C_ϵ^p, based on the spectral energy scale L_p, for all flows. The overall agreement between all data is significant, especially when $R_\lambda \gtrsim 300$. Of particular note is the collapse of the $N1$ data measured at $dU/dy=0$ and $dU/dy=dU/dy|_{\max}/2$ and also the collapse of the active grid data with the present data at high R_λ. Figure 1(b) suggests an asymptotic value of $C_\epsilon^p \approx 0.5$ to be an adequate ''universal'' approximation. Note that all of the measurements are acquired in shear flows in regions of minimal mean shear. It would be fortuitous, indeed, if the estimate of C_ϵ^p in strong mean shear should also be ≈ 0.5. We expect the effect of strong mean shear is to reduce the magnitude of C_ϵ and there is some evidence to suggest this is so.[9] In conclusion, the difference between C_ϵ^p and C_ϵ^u, where C_ϵ^u depends on the realization of the flow field, can be reconciled by accounting for the large-scale forcing in different flows with the respective St number.

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[1] L. F. Richardson, *Weather Prediction by Numerical Process* (Cambridge University Press, Cambridge, 1922).
[2] G. I. Taylor, ''The statistical theory of turbulence: Parts I-IV,'' Proc. R. Soc. London, Ser. A **151**, 421 (1935).
[3] G. K. Batchelor, *The Theory of Homogeneous Turbulence* (Cambridge University Press, Cambridge, 1953).
[4] K. R. Sreenivasan, ''On the scaling of the energy dissipation rate,'' Phys. Fluids **27**, 1048 (1984).
[5] D. Lohse, ''Cross-over from high to low Reynolds number turbulence,'' Phys. Rev. Lett. **73**, 3223 (1994).
[6] D. Fukayama, T. Nakano, A. Bershadskii, and T. Gotoh, ''Local properties of extended self-similarity in three-dimensional turbulence,'' Phys. Rev. E **63**, 012501 (2001).
[7] P. G. Saffman, in *Topics in Non-Linear Physics*, edited by N. Zabusky (Springer, Berlin, 1968), p. 487.
[8] Numerous participants, in *The Global Geometry of Turbulence*, edited by J. Jimenez (Plenum, London, 1991), p. 347.
[9] K. R. Sreenivasan, ''The energy dissipation rate in turbulent shear flows,'' in *Developments in Fluid Dynamics and Aerospace Engineering*, edited by S. M. Deshpande, A. Prabhu, K. R. Sreenivasan, and P. R. Viswanath (Interline, Bangalore, India, 1995), p. 159.
[10] K. R. Sreenivasan, ''An update on the energy dissipation rate in isotropic turbulence,'' Phys. Fluids **10**, 528 (1998).
[11] R. A. Antonia and B. R. Pearson, ''Effect of initial conditions on the mean energy dissipation rate and the scaling exponent,'' Phys. Rev. E **62**, 8086 (2000).
[12] R. A. Antonia and B. R. Pearson, ''Reynolds number dependence of velocity structure functions in a turbulent pipe flow,'' Flow, Turbul. Combust. **64**, 95 (2000).
[13] J. C. R. Hunt, N. D. Sandham, J. C. Vassilicos, B. E. Launder, P. A. Monkewitz, and G. F. Hewitt, ''Developments in turbulence research: A review based on the 1999 programme of the Isaac Newton Institute, Cambridge,'' J. Fluid Mech. **436**, 353 (2001).
[14] L. Mydlarski and Z. Warhaft, ''On the onset of high-Reynolds-number grid-generated wind tunnel turbulence,'' J. Fluid Mech. **320**, 331 (1996).
[15] L. Mydlarski and Z. Warhaft, ''Passive scalar statistics in high-Péclet-number grid turbulence,'' J. Fluid Mech. **358**, 135 (1998).
[16] S. Gamard and W. K. George, ''Reynolds number dependence of energy spectra in the overlap region of isotropic turbulence,'' Flow, Turbul. Combust. **63**, 443 (1999).