Modelling glass parisons

Modelling Glass Parisons

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Abstract

In order to design better glass products, numerical modelling is essential. In this paper we focus, more specifically, on pressing forms, so called parison, appearing in the production of packing glass. We model the flow of the molten glass during this process as well as the cooling of the parison during and after pressing. Both problems lead to boundary value formulations . Due to the fact that we are mainly interested in what happens on the (free) boundary these problems are best solved by Boundary Element Techniques.

1 Description of the problem

Glass is an interesting material, having a multiple applications on one hand and being available in unlimited quantities on the other hand. Indeed, it is transparent, making it indispensable for applications like pane (windows), but at the same time, its constitutive properties are such that it appears to be flexible when used as fibre, making it a favorite material for data transport by light. The raw material is mainly silicium–dioxide, i.e. ordinary sand.

Production of glass goes more or less along the following lines: First grains and additives, like soda, are being heated in a tank. This can be a device several tens of metres long and a few metres high and wide (width being larger than height). Here gas burners or electric heaters provide for the heat necessary to warm up the material till some $1600°C$. At one end the liquid glass comes out and is e.g. led to a pressing or blowing machine, or it ends up on a bed of liquid tin, where it spreads out to become float glass (for pane, wind shields etc.). In the latter case the major problems are to have a smooth flow from the oven on the bed and to control the spreading and flattening. An essential part then is the cooling of the product. Hence the production involves a high energy cost factor, inducing a continuous search for more efficient techniques. For one part these are to be found in a better control of the combustion (or more general) heating process and the design of the oven. For another they can be formed by a better control of the end product. Indeed, by e.g. better monitoring the cooling one may reduce residual stresses in the material to allow for thinner glass (thus reducing the actual material costs), another example is the actual morphology phase, where a piece of hot glass is formed into the desired shape; in fact, this may be even more important for obtaining thinner glass products.

Although glass technology has long been based on expertise and experimental knowledge, it turns out that this is no longer sufficient to improve the de-

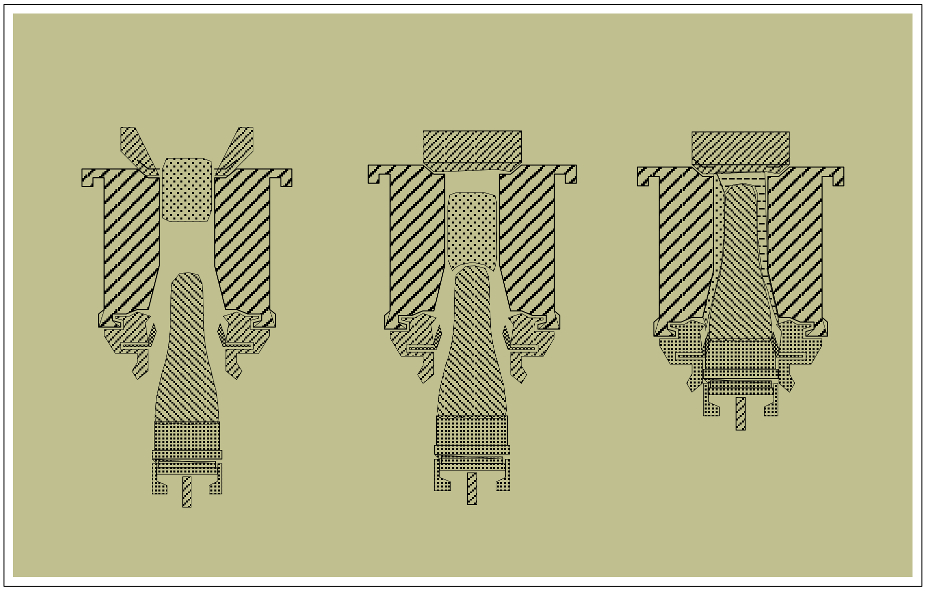

Figure 1: The pressing phase

signs. The need for such improvement does not only come from governmental requirements but also from fierce competition by materials like polymers. Hence mathematical modelling and numerical simulation are needed.

Here we shall consider a typical example of a morphology problem, the production of a parison. This is a preform that occurs in the production of packing glass, like bottles and jars (where the above mentioned problems, like thickness and strength make sense, obviously).

The complete process now follows: First a piece of hot glass, the *gob*, coming directly from the oven, drops in a mould. This mould consist of a fixed part and a so called plunger, which moves up quickly (say 0.5 seconds) after the mould has closed (see Figure 1). In the final stage, a bell–like shape is formed, the *parison*, which is then taken out of the mould by laying hold of the lower part of the parison; the latter part is sufficiently cooled such that it behaves like a solid. After a small stay, the so called *reheating phase*, the parison is put in a second mould and blown into a final shape (see Figure 2).

This seemingly simple problem is actually quite sophisticated from a modelling point of view. We shall consider two specific questions arising here. First we investigate the morphology of the parison process in section 2. Then we model the temperature, both at the pressing and during the reheating process in section 3.

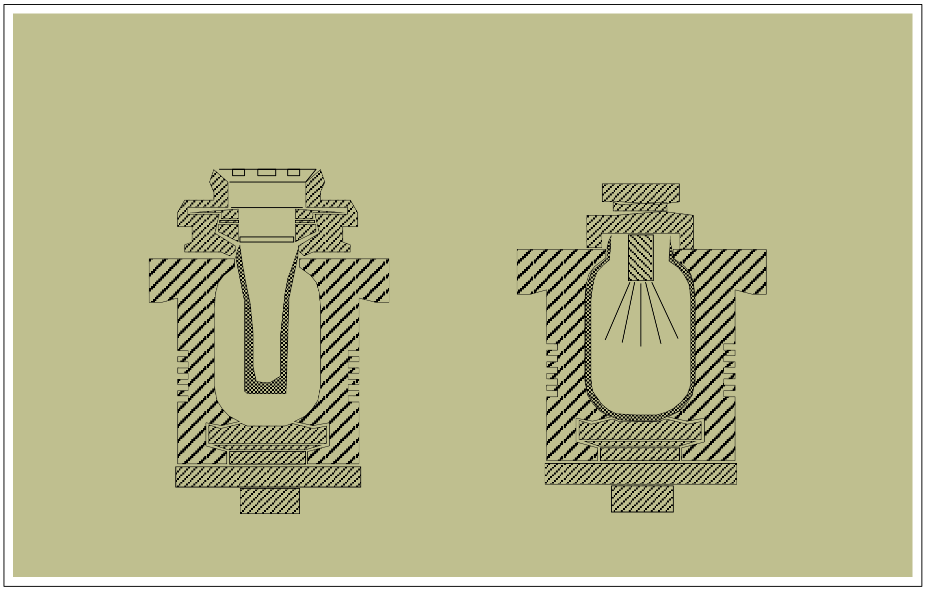

Figure 2: The blowing phase

2 Mathematical modelling of glass flows

We first study the flow of the glass in the mould during the pressing.

If we let ρ be the density of the fluid, $\mathbf{g} = (g_i)$ the body forces (here only gravitational force is considered), $\sigma = (\sigma_{ij})$ stress tensor, $\mathbf{v} = (v_i)$ the velocity vector and p the pressure. Then flow of the glass in the pressing phase can be seen to be governed by the following two equations. The first one deals with the *conservation of mass*, i.e. the *continuity equation*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \tag{1}$$

The second one deals with the *conservation of momentum* and leads to the *equation of motion*

$$\rho(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}) = \nabla \cdot \sigma + \rho \mathbf{g}. \tag{2}$$

We may assume the glass to be an incompressible Newtonian fluid. This yields the following constitutive equation for stress tensor σ and the rate of deformation tensor $E = (\mathcal{E}_{ij})$

$$\sigma = -p\mathbf{I} + 2\eta E, \tag{3}$$

where $\mathbf{I}$ is the unit matrix, η is the dynamic viscosity and

$$E = \frac{1}{2}(\nabla \mathbf{v} + (\nabla \mathbf{v})^T).$$

By using the incompressibility in the continuity equation and substituting the constitutive equation and the continuity equation into the equation of motion, we have the so-called *Navier-Stokes equations*

$$\begin{cases} \nabla \cdot \mathbf{v} = 0, \\ \rho(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}) = -\nabla p + \eta \Delta \mathbf{v} + \rho \mathbf{g}. \end{cases} \tag{4}$$

Now we derive a dimensionless form for these Navier-Stokes equations. For this we need some characteristic quantities. We first replace $\mathbf{g}$ by $g\tilde{\mathbf{g}}$ with $g := \|\mathbf{g}\| \approx 10m/s^2$ the *acceleration of gravity*. At this stage, the viscosity η is assumed to be constant, a typical value is $\eta_0 \approx 10^4 kg/ms$ which is defined as the characteristic viscosity. A typical average velocity of the plunger, say $V_0 \approx 10^{-1} m/s$, can be used as a characteristic velocity. A characteristic length is taken as the average thickness of the parison, say $L_0 \approx 10^{-2} m$. Consequently the characteristic pressure and the characteristic time can be defined by

$$P_0 = \eta_0 V_0 / L_0,$$

and

$$T_0 = L_0 / V_0,$$

respectively.

Substituting the following dimensionless quantities

$$\tilde{\mathbf{x}} = \frac{\mathbf{x}}{L_0},$$

$$\tilde{t} = \frac{t}{T_0},$$

$$\tilde{\mathbf{v}} = \frac{\mathbf{v}}{V_0},$$

$$\tilde{p} = \frac{p}{P_0}.$$

into the Navier-Stokes equations yields

$$\begin{cases} \nabla \cdot \tilde{\mathbf{v}} = 0, \\ Re(\frac{\partial \tilde{\mathbf{v}}}{\partial \tilde{t}} + (\tilde{\mathbf{v}} \cdot \nabla)\tilde{\mathbf{v}}) = -\nabla \tilde{p} + \Delta \tilde{\mathbf{v}} + \frac{Re}{Fr}\tilde{\mathbf{g}}. \end{cases} \tag{5}$$

Here two dimensionless numbers, the *Reynolds number* Re and the *Froude number* Fr, are defined by

$$Re := \frac{\rho V_0 L_0}{\eta_0} \quad \text{and} \quad Fr := \frac{V_0^2}{g L_0},$$

respectively. The Reynolds number indicates the ratio between inertia forces and viscous forces, whereas the quotient of the Reynolds number and the Froude number indicates the ratio between volume forces and viscous forces. Using the values above, the two numbers are approximately equal to

$$Re \approx 10^{-4} \quad \text{and} \quad Fr \approx 10^{-1}.$$

From this we conclude that the viscous forces dominate the Navier–Stokes equations. Thus the flow can be described by the continuity equation

$$\nabla \cdot \mathbf{v} = 0, \tag{6}$$

the momentum equation

$$\Delta \mathbf{v} - \nabla p = 0. \tag{7}$$

These equations are the dimensionless *Stokes equations*. Moreover the constitutive equation reads

$$\sigma = -p\mathbf{I} + (\nabla \mathbf{v} + (\nabla \mathbf{v})^T). \tag{8}$$

Here we have omitted the tilde for ease of writing.

In order to find a unique velocity $\mathbf{v}$ and a pressure p, at time t, a set of boundary conditions has to be imposed (see Figure 3). Note that, in Figure 3 we have turned over the geometry by 180° for convenience.

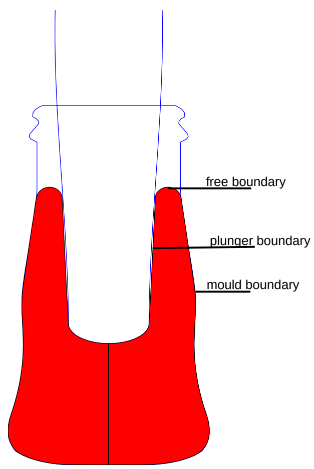

Figure 3: The boundaries

We first consider the free boundary. Surface tension is a force acting on the free boundaries of the glass drop. It is usually denoted by γ and one can find that for glass γ approximately equals $0.3N/m$. The surface tension, as well as the plunger motion, plays its role in the system only in terms of boundary conditions. A typical velocity for a flow driven by surface tension is given by γ/η_0. Thus, to investigate whether surface tension is of any significance, we have to compare this typical velocity with the one induced by the plunger motion, e.g. V_0. It is clear that the plunger velocity is much larger than γ/η_0 during most of the pressing phase. From this observation one can derive that the surface tension will not be significant. Free boundaries are in contact with the surrounding, thus we assume the atmospheric pressure, say p_0, to prevail everywhere. For the description of the atmospheric pressure in terms of boundary condition the stress vector σn has to be prescribed on the free boundaries. To be precise, the boundary condition on the free boundary is given by

$$\sigma n = -p_0 \mathbf{n}, \tag{9}$$

where $\mathbf{n} = (n_i)$ is the outward unit normal.

Now we consider boundary conditions on the mould and the plunger. They depend on the roughness of the mould and the plunger. We may impose a no-slip condition, i.e. the velocity of the flow equals the velocity of the boundary. The opposite of a no-slip condition is a no-friction condition. i.e. the normal component of the velocity of the flow equals the normal component of the velocity of the boundary and vanishing shear stresses. But it is also possible to impose a boundary condition describing a certain amount of friction. For example, one can impose the following boundary condition

$$\begin{cases} \mathbf{v} \cdot \mathbf{n} = \mathbf{v}_b \cdot \mathbf{n}, \\ \sigma \mathbf{n} \cdot \mathbf{t} = -\beta_b (\mathbf{v} - \mathbf{v}_b) \cdot \mathbf{t}, \end{cases}$$

where $\mathbf{t}$ is the unit tangential direction, $\mathbf{v}_b$ is the velocity of the boundary and β_b is the *slip-parameter* indicating the amount of friction. This boundary condition means that the normal component of the velocity of the flow equals the normal component of the velocity of the boundary and the shear stress is proportional to the tangential velocity difference.

Applying the above boundary conditions to the mould and the plunger yields

$$\begin{cases} \mathbf{v} \cdot \mathbf{n} = 0 \\ \sigma \mathbf{n} \cdot \mathbf{t} = -\beta_m \mathbf{v} \cdot \mathbf{t} \end{cases} \text{on the mould,} \tag{10}$$

and

$$\begin{cases} \mathbf{v} \cdot \mathbf{n} = \mathbf{v}_p \cdot \mathbf{n} \\ \sigma \mathbf{n} \cdot \mathbf{t} = -\beta_p (\mathbf{v} - \mathbf{v}_p) \cdot \mathbf{t} \end{cases} \text{on the plunger,} \tag{11}$$

where $\mathbf{v}_p$ is the velocity of the plunger.

Since at least some part of the boundary is moving, we use a quasi-static approach to describe the movement of a material fluid particle, i.e.

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}(t)). \tag{12}$$

The whole problem (6)–(12) is extremely well suited for a boundary element approach. Indeed, we are only interested in the movement of the free boundary. The modelling problem is then to predict speed and thus position.

Problems like these have been solved before, albeit with different boundary conditions (cf. Kuiken at el. [3] and Mattheij at el. [5]). A nontrivial part of the problem is the actual solution of Eqn. (12), which needs to be done such that mass is being conserved. We shall work this out in a subsequent paper (cf. Wang at el. [13]).

3 Modelling the temperature

As was said in section 1 the temperature plays an important role in the forming process, since only sufficiently hot glass can be deformed. The most important quantity therefore is the viscosity η. The temperature T and the viscosity η are related through the Vogel–Fulcher–Tamman relation(cf. Rawson [9])

$$\eta = Kexp(E_0/(T - T_0)), \tag{13}$$

where E_0 is the *viscosity activation energy*, T_0 a fixed temperature and K a scaling constant.

The temperature follows from the energy equation, which reads, for an incompressible fluid in dimensionless form,

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \frac{1}{Pe} \nabla^2 T + \frac{Ec}{Pe} \Phi. \tag{14}$$

Here Φ is the *dissipation function*, representing the heat equivalent of the rate at which mechanical energy is expanded in the deformation process due to viscosity. E_c is the *Eckhard number*, defined as

$$Ec := \frac{V_0^2}{c_p \Delta T}, \tag{15}$$

where c_p is the *specific heat* and ΔT the *temperature drop* in the relevant area. Moreover Pe is the *Peclet number*, defined as

$$Pe := \frac{\rho V_0 L_0 k}{\eta^2 c_p}, \tag{16}$$

with k the *thermal conductivity*. During pressing we have a temperature difference of about $500°C$, hence we obtain

$$\frac{1}{Pe} = 6.2 \times 10^{-4}, \quad \frac{Ec}{Pe} = 1.2 \times 10^{-4}.$$

This means that we may neglect the right hand side in Eqn. (14).

Let us now investigate the effect of wall cooling: Let $\Omega_p, \Omega_g, \Omega_m$ denote the plunger (p), glass (g) and mould (m) domains respectively (see Figure 4) and denote by $k_i (i = p, g, m)$ the *thermal conductivity*. One can estimate the numerical

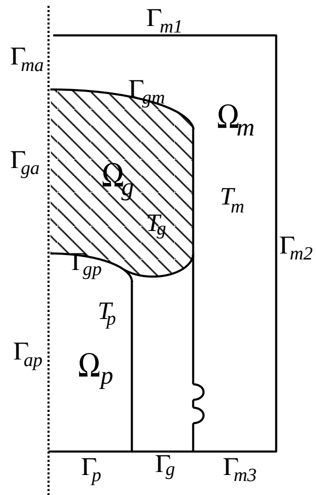

Figure 4: The domains

values of these to find

$$\begin{aligned} k_g &= 6.2 \times 10^{-7}(m^2/s), && \text{thermal diffusivity.} \\ k_p = k_m &= 1.7 \times 10^{-5}(m^2/s), && \text{thermal diffusivity.} \end{aligned}$$

We observe that $k_p = k_m \gg k_g$, and this implies that when the heat process of the glass starts, the heat processes of the plunger and the mould are already in the steady state. This means that the temperature $T_p = T_p(0)$ and $T_m = T_m(0)$. Hence the three heat processes are not coupled.

We conclude that we may consider the heat flow to be quasi stationary, i.e. $\frac{\partial T}{\partial t} = 0$ and so Eqn. (14) becomes

$$\mathbf{v} \cdot \nabla T = 0. \tag{17}$$

If we have e.g. cylindrical coordinates r and z, this means for the velocity $(u, v)^T$

$$u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = 0,$$

having the solution

$$\frac{dz}{dt} = u, \quad \frac{dr}{dt} = v, \quad \frac{dT}{dt} = 0. \tag{18}$$

This implies that the temperature remains constant along streamlines, so a uniform temperature field will remain uniform.

After the plunger has come to a standstill, the parison remains in the mould for some time, thus leading to the problem

$$\frac{1}{k_g}\frac{\partial T}{\partial t} = \Delta T \quad \text{in}\Omega_g. \tag{19}$$

The boundary conditions are given by

$$\begin{cases} \frac{\partial T_g}{\partial n} = 0 & \text{on} \quad \Gamma_{gp} \cup \Gamma_{ga}, \\ k_g \frac{\partial T_g}{\partial n} = h_{gp}(T_g - T_{0p}) & \text{on} \quad \Gamma_{gp}, \\ k_g \frac{\partial T_g}{\partial n} = h_{gm}(T_g - T_{0m}) & \text{on} \quad \Gamma_{gm}. \end{cases} \tag{20}$$

where h_{gp} is the contact conductance between the glass and the plunger and h_{gm} is the contact conductance between the glass and the mould. The contact conductance depends on the surface roughness, the interface pressure and temperature, the thermal conductivities of the contacting materials and the type of fluids or gas in the gap, and is about $h_{gp} = h_{gm} = 2 \times 10^3 [W/m^2/c]$.

On the two boundaries Ω_{gp} and Ω_{gm} we have a temperature drop, depending on the contact conductances, and a boundary layer, depending on the thermal diffusivity of the glass. One can prove that the asymptotic behaviour of the boundary layer is the errorfunction $erfc(r/\sqrt{4k_g t})$.

We would like to remark that the conductivity of the glass is actual a sum of conductivity of diffusive and radiative heat effects. If one uses the Rossland approximation to model the latter (cf. Modest [6]), also during reheating, i.e. after the mould has been removed, and before the blowing phase starts, the heat exchange can be modelled like Eqn. (19) with Neumann boundary conditions (and a given room temperature).

Both the latter and Eqns. (19), (20) lend themselves for a BEM approach, based on dual reciprocity method(cf. Partridge at el. [8] and Simons[11]), as is known the dual reciprocity method uses radial basis functions to deal with the inhomogeneity, in ter Morsche at el. [7] some more results are reported, in particular with respect to the choice of the internal points.

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