Modelling and control of product life-cycles

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Modelling and Control of Product Life-Cycles

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Modelling and Control of Product Life-Cycles

PROEFSCHRIFT

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in memoriam
Hartmut Kleineidam

Contents

Summary

In the last decades governments as well as industry have taken measures to achieve sustainability. One major opportunity for sustainable production is the reuse of waste streams, because recycling of discarded products is an effective way of diminishing the discharge of wastes and decreasing the depletion of resources.

Since measures taken to influence the recycling of a product have effects on the production and consumption processes linked to the product, entire chains of such production and consumption processes are considered. A product-process chain describes such a complete chain in a standard way. Product-process chains are used in the framework of industrial metabolism to map the flow of materials and products through the industrial system by applying the laws of mass and energy conservation. In this study, the whole life-cycle of products is examined, from the initial extraction of raw materials to the reintegration, at some point, of their wastes into the ecosystem.

Chains are influenced by the decisions of producers and consumers regarding product flows. These flows can be altered by the substitution of materials and products, the recycling of materials and the reuse of products. Governmental measures are external influences: regulatory influences, for example by the prescription of maximum emission levels, economic influences, for example by taxes and subsidies, and social influences, for example by the stimulation of environmental research and the stimulation of self-regulation of the chains.

This study presents a method for modelling and controlling product-process chains, which is of interest for governmental authorities as well as for industry. The chain models consist of elementary company models, which are connected by market models. Dynamic processes, such as market and storage processes, are included. By combining the company and market models, a large variety of product chains can be modelled, with the option to include recycle loops. Companies are conceived as consisting of resource and product inventories and of a transformation process. By applying the law of mass conservation to these inventories, two coupled dynamic equations that describe a company are obtained. Production decisions and decisions on supply and

demand of products are substituted into these equations. The decisions are based on market prices. The price development is determined by the difference between demand and supply.

The contribution of control theory to chain analysis and chain control is demonstrated. The chain model can be put into the standard state space description used in control theory, thereby making it possible to use various methods for system analysis and control that are available in control theory, such as stability, controllability and observability analysis. These properties are prerequisites for effective chain management, such as in the evaluation of recycling policy.

To illustrate the modelling and analysis techniques, the Dutch paper chain is examined. This chain is chosen because quite extensive historical data of prices, imports to and exports from the chain have been documented. It consists of paper producers, paper consumers and waste-paper recyclers, as well as of a pulp market, a paper market, and a waste-paper market. The historical prices are in good agreement with those calculated by the model provided that the imports and exports have the observed values.

In the model analysis, the stability of the paper chain is demonstrated first. Stability means that the product flows and prices in the chain do not grow unboundedly. Second, the controllability of the chain is analysed. The measures expected to influence the chain are examined to see whether they are indeed able to guide the chain in the desired way. Controllability is shown with respect to taxes and subsidies on products, taxes and subsidies on activities, the regulation of imports and exports, and an adjustment of the price for final treatment. Moreover, it is demonstrated that the chain is observable by the measurement of the product flows, the product prices or the recycling rate. This indicates that these quantities contain enough information for effective chain regulation.

Some examples of regulations of the paper chain are modelled to show the value of the model for evaluating influences on the chain. The regulation of the paper flow by a tax on pulp illustrates a measure that leads to the attainment of the control objective. The regulation of the recycling rate by an increase of recycling activities is modelled as a second example. It turns out that this is an ineffective measure. Furthermore, it is shown that the recycling rate can be controlled by adjusting the price for final treatment. This recycling rate can also be controlled through a combination of the final treatment price with a subsidy for waste-paper, which is a politically attractive regulation.

Finally two measures aimed at reducing price fluctuations are modelled: a waste-paper subsidy and some kind of self-regulation, which has recently been introduced in the paper chain. This so-called paper fibre convenant provides mutual sale and purchase agreements aimed at a more secure throughput of paper through the chain and an increase of household waste-paper collection. It is demonstrated that the two influences can indeed improve the chain behaviour.

The paper chain is an example of an existing chain in which recycling has been applied for a long time. Therefore it serves the purpose of examining the modelling and analysis techniques described in this thesis. The case study demonstrates the usefulness of these techniques for the analysis and control of product chains.

List of Symbols

Symbol	**Unit**	**Description**	**page**
A		system matrix	42
A_{A_i}		system matrix of actor i	54
B		control input matrix	42
B_{A_i}		input matrix of actor i	54
C		output matrix	42
$\mathcal{C}$		controllability matrix	96
$c_1,\ c_2$	kg/year	autonomous demand, supply	54
c_q	kg/year	autonomous production	54
D		direct feedthrough matrix	42
E		disturbance input matrix	42
e	kg/year	total trade	58
e		error	106
e_s	kg/year	trade by supplier	58
e_d	kg/year	trade by demander	58
F		disturbance feedthrough matrix	42
f		state function	42
g		output function	42
g_i	kg	inventory i	53
g_f	kg	inventory of removal fund	119
H_{A_i}		matrix of actor i	54
K_P, K_I, K_D		constants in PID controller	106
M_i		market i	70
m		measurement vector	80
$\mathcal{O}$		observability matrix	99
P		matrix in quadratic Lyapunov function	94
p	€/kg	price	53

continued on next page

continued from previous page			
Symbol	**Unit**	**Description**	**page**
p_m		minimal price in regulation by subsidy	116
p_w	€/kg	final treatment price	78
q_d	kg/year	demanded products	54
q_p	kg/year	produced products	54
q_r	kg/year	supply of primary resources	73
q_s	kg/year	supplied products	54
q_w	kg/year	additional waste to be recycled	78
u		input vector	37
u_{tax}	€/kg	product tax	76
u_{s}	€/kg	product subsidy	77
u_{subs}	€/year	subsidy	77
$u_{\max}$	kg/year	maximal product flow	78
V		weight matrix	80
$V(x)$		Lyapunov function	91
v		disturbance vector	37
w		measurement error	37
x		state vector	37
y		output vector	37
α	$\mathrm{kg}^2/(€\mathrm{year})$	production constant	54
β	$\mathrm{kg}^2/(€\mathrm{year})$	supply/demand constant	54
γ	$\mathrm{kg}^2/(€\mathrm{year})$	production constant	66
γ_0	kg/year	autonomous demand	60
γ_1	kg/(€year)	price dependent demand	60
δ	$\mathrm{kg}^2/(€\mathrm{year})$	production constant	66
ϵ	1/year	production time constant	66
$\varepsilon(x)$		Heaviside function	86
η	-	efficiency	53
λ	1/year	production time constant	54
μ	1/year	demand/supply time constant	54
ν_0	kg/year	supply constant	73
ν_1	$\mathrm{kg}^2/(€\mathrm{year})$	supply constant	73
ξ	1/year	production time constant	66
ρ_M	$\mathrm{kg}^2/(€\mathrm{year}^2)$	Marshallian adjustment constant	56
ρ_W	$€/\mathrm{kg}^2$	Walrasian adjustment constant	57

Chapter 1

Introduction

This chapter outlines the major themes to be explored and developed in subsequent chapters, and it describes how the study is organized. It provides some background to the subject of sustainable development in general and to product chains in particular. The objective and the framework of the study are introduced.

1.1 Sustainable Development

Environmental protection has had a significant place on the political and scientific agenda in the recent decades. One milestone in the process of becoming aware of the environmental problem in a wide political and public arena was the publication of the Club of Rome's study *The Limits to Growth* in 1972 [106]. It presented the results of a computer model of the world economy, which predicted the world's future. According to this model, economic growth would come to an end within the foreseeable future because of environmental limits: a limit to the amount of food and a limit to the amount of natural resources. Although the predictions proved to be too pessimistic, the report had a great impact on public awareness concerning environmental problems.

A subsequent milestone — fifteen years after *The Limits to Growth* — was the publication of the so-called Brundtland Report, which gives the definition of 'sustainable development' that is generally used nowadays [152]:

> 'Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs.'

This report, entitled *Our Common Future* and produced by the World Commission on Environment and Development (WCED) in 1987, is one of the

Period	**Focus of Environmental Policy**
Seventies	Focus on shortage of resources as a reaction to the energy crisis; local problems, mainly air and water pollution, and noise.
Eighties	Focus on emissions as a reaction to acid rain, greenhouse effect; policy evolution towards prevention, mainly with regional and international orientation.
Nineties	Focus on integral chain management; development of more specific policy instruments, global orientation.

Table 1.1: *Development of Dutch environmental policy*

landmarks in the increasing concern about the earth's future. It derives its popular title from the name of the Chairman of the Commission, Gro Harlem Brundtland, a former Minister for the Environment, and Prime Minister, of Norway. In the sequel, sustainable development has often been interpreted as *ecologically* sustainable *economic* development, although the original definition is much broader. 'Sustainable' then refers to the natural environmental basis of development; and development is restricted to economic development [22, 43].

The Brundtland Report is based on an understanding of the nature of economy-environment interactions that is similar to that of *The Limits to Growth*, but draws the conclusion that growth can and should continue. However, this growth would take a different form and would actually be sustainable development.

Now, nearly three decades after the publication of the Club of Rome's report, a broad consensus on the need for environmentally friendly behaviour has emerged. Attention is shifting towards the concrete measures that can be taken to achieve sustainable development both by governments and industry, as well as by consumers.

In Table 1.1 the development of the Dutch environmental policy is summarized for example. In the Netherlands, as in most industrialized countries, the initial response to environmental problems were measures on the effect side. In due course, it was recognized that preventive measures were more effective [46]. The environmental problems that are recognized as most urgent, have changed over the years. In the seventies, as a reaction to the energy crisis, the shortage of resources was one of the main problems. Later on, as a response to acid rain, the greenhouse effect, the ozone layer depletion, etc., the emphasis shifted to emissions. Instead of end-of-pipe measures, such as filters, more integrated approaches, such as integral chain management have gained ground.

1.2 Background

A major opportunity to tackle the environmental problem is to increase the resource and energy efficiencies. Recycling of wastes is one way of increasing resource efficiency since it decreases the use of primary resources and the production of waste. Examples are metals, paper, glass, plastic, batteries and textiles as well as computers, kitchen articles, tyres and cars. Apparently, numerous environmental benefits can be attained by closing substance chains. On the other hand, it is obvious that recycling does not always have an environmental benefit, since the benefits of resource savings have to be compared to resource and energy use for the recycling processes themselves.

Since the closing of substance chains has become increasingly important, complex many-branched, entangled networks of production and recycling processes have developed. Thus, it is becoming increasingly difficult for regional, national, and international governments to direct the producers and consumers in their territories towards the adequate levels of production, consumption and recycling.

Product-process chains constitute a way to follow the flow of materials and products through the economy. They describe the sequence of processes necessary for a product to evolve from raw materials to a finished product and further to a waste product or a recycled product. Relevant processes in this context are, for example, the extraction of raw materials, the production and consumption of products, and the upgrading or discharge of discarded products.

1.3 Objective of the Study

The objective of this research project is the development of a 'method for modelling and controlling a product chain'. It originates from the need of both authorities and industry to gain more insight into product-process chains, particularly when recycling loops are present. Since the complexity of these chains is increasing, it is essential to apply appropriate control measures so as to guide the chain's performance to the desired values.

To that purpose, quantitative models have to be developed with a modular structure so that they can easily be used for a variety of chains. The models have to capture the chain dynamics including, for example, market dynamics. Methods from control theory can be useful for a definite class of problems

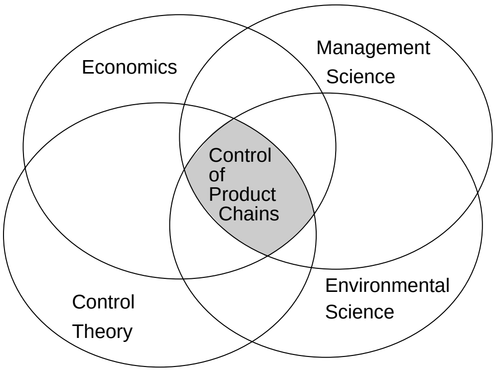

Figure 1.1: *The research context*

because they are particularly suitable for the analysis and control of dynamic systems.

1.4 Framework of the Study

This study concentrates on the intersection of at least four disciplines: environmental science, economics, management science and control theory, see Figure 1.1.

Environmental science

Environmental science can be defined as the experimental and scientific study of the effects of human beings on their abiotic and biotic environment. In this context, the study of material flows through the economic system from cradle (i.e. extraction) to grave (i.e. final discharge) is important, because they determine resource consumption, wastes and emissions. The method that is applied in this study is that of product-process chains. By including production and consumption processes, product-process chains explicitly study the nexus between the economic activities and the impact on the environment. One of the objectives of this study is to minimize negative environmental effects.

Economics

Product chain analysis is an economic problem in so far as economics is the study of the ways in which human beings satisfy their material needs. Economic mechanisms play a role in decision making, such as in the case of the substitution of a product by another one. Moreover, economic instruments are frequently used for control purposes. A crucial factor within the concept of product chains is the market concept. Therefore, the economic theory of markets should be incorporated in the model.

Operational research and management science

Management science aims at providing rational bases for decision making by seeking to understand and structure complex situations and to use this knowledge to predict system behaviour and to improve system performance. It is similar to operational research, which is defined by the Operational Research Society of Great Britain as [118]:

> 'Operational Research is the attack of modern science on complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business, government and defence. Its distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as change and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically.'

Operational research and management science draw upon ideas from engineering, management, mathematics, and psychology, and contributes to a wide variety of application domains; the field has an interdisciplinary character and is closely related to several other fields in the 'decision sciences' – applied mathematics, computer science, economics, industrial engineering, and systems engineering. The mathematical analysis of product chains is part of the field of operational research.

Systems and control theory

Systems theory is concerned with dynamic processes and the description of their input-output relations. The science of control adds a dimension to systems theory as it is concerned with influencing dynamical processes to achieve desired goals. Product chains are dynamical systems, which show similarities to many technical systems that are the traditional objects of control theory. Therefore, the description and modification of product chain behaviour by control theory is obvious. As stated in Section 1.3, the objective of this study is to show how control theory can contribute to product chain control. It will therefore play a substantial role in this study.

1.5 Overview of the Study

Chapter 2 provides essential features of product-process chains. Their definition is given, and system boundaries concerning geographical extent, the considered time horizon and level of aggregation are defined. The necessity of a chain approach is made clear by shedding light on the environmental and economic relationships in such chains.

In Chapter 3, strategies are discussed that can be followed to guide product chains to the desired behaviour. Product and material flows can be changed, for example, by introducing recycling activities or by substituting one material with another. Governmental and chain instruments are reviewed.

In Chapter 4, the basics of control theory are introduced. It is pointed out that the problem of guiding product chains is a typical control problem just like the problem of, for example, controlling the temperature in a room. Relevant methods of modelling and analysing such systems, which are available in control theory, are presented. The value of a model is clarified.

Chapter 5 presents a general modelling technique for product chains. The essential processes are dynamic company and market processes, since a product chain consists of companies with markets on the resource and on the product side, just as it consists of markets with companies on the demand and on the supply side.

In Chapter 6, the company and market models are combined to form a chain model. The paper chain is taken as an example because it is a well-established chain, and quite extensive time series data exist for it. The parameters of the paper chain model are estimated such that the model results agree with the given data.

Chapter 7 shows the application of some of the methods of control theory. The paper chain is analysed with respect to stability, controllability and observability. What is examined, is whether the chain does not collapse, whether the measures taken to influence the chain are able to achieve the desired objectives and whether the observation of certain data provides information about the entire chain.

In Chapter 8, some examples of chain regulations are presented. The regulation of product flows and recycling rates by instruments such as taxes and subsidies is simulated. Among others a measure that has been implemented in the Dutch paper chain as a consequence of agreements between government and industry, the so-called paper fibre convenant, is modelled, and its behaviour is compared to the chain behaviour without convenant.

The final chapter, Chapter 9, contains the main conclusions and some suggestions for further research on product chains.

Chapter 2

Product-Process Chains — Basic Concepts

2.1 Introduction

In this chapter, the systems that will be explored in the following chapters — product-process chains — are introduced. Product-process chains are defined in the context of industrial metabolism. They consist of *processes*, which are linked together by *product* flows and which are associated with a final product. The boundaries of the systems considered in the industrial metabolism framework are the boundaries of the technological system, which is considered as complementary to its ecological counterpart, the ecosystem. Physical laws such as the laws of mass and energy conservation are generally used to describe the material flows through the technological system.

Section 2.2 describes the industrial metabolism framework. In Section 2.3, the definition of product-process chains as used in this thesis is given. Two approaches to these chains, which will be dealt with in the following chapters, are discussed here. The *environmental* viewpoint is presented in Section 2.4, and the *economic* viewpoint is discussed in Section 2.5. In Sections 2.6 and 2.7, some characteristics of product-process chains are treated, such as the level of aggregation and the chain boundaries with respect to the geographical extent, the time horizon and with respect to other chains. In Section 2.8, some motives for cooperation of the members of the entire chain are discussed. In Section 2.9, an example from practice, the example of a reel producer, is presented to illustrate some of the mentioned characteristics. Finally, in Section 2.10, the main points of this chapter are resumed.

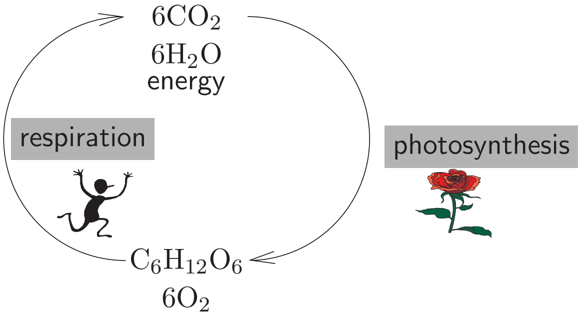

Figure 2.1: *The (idealized) oxygen and carbon dioxide cycle*

2.2 Industrial Metabolism

Before defining product-process chains as they are used in this thesis, we introduce the concept of industrial metabolism. This represents an old idea, which nevertheless was not introduced systematically until the late eighties [15, 16, 61].

Industrial metabolism[1] starts out from the assumption that the industrial system can be seen as analogue to the ecosystem[2]. The concept of industrial metabolism is inspired by the biological concept of metabolism. Like the ecosystem, it can be described by material, energy and information flows. The ultimately sustainable biological ecosystems have evolved over a long term so as to be completely cyclical, which means that 'resources' and 'waste' are undefined: waste of one component of the system represents resources to another [17, 63]. The only resource used in these systems is some kind of energy. One example is the (idealized) oxygen and carbon dioxide cycle [14]: The two main processes here are respiration, which means the transformation of oxygen (O_2) into carbon dioxide (CO_2), water (H_2O) and energy, and the transformation of carbon dioxide into glucose ($C_6H_{12}O_6$) and oxygen by photosynthesis using solar radiation as energy source and producing oxygen as a waste (Figure 2.1).

Industrial metabolism is concerned with the material and energy flows going through the industrial system, the technosystem. Since matter can neither be created nor destroyed by human activity, the law of mass conservation ap-

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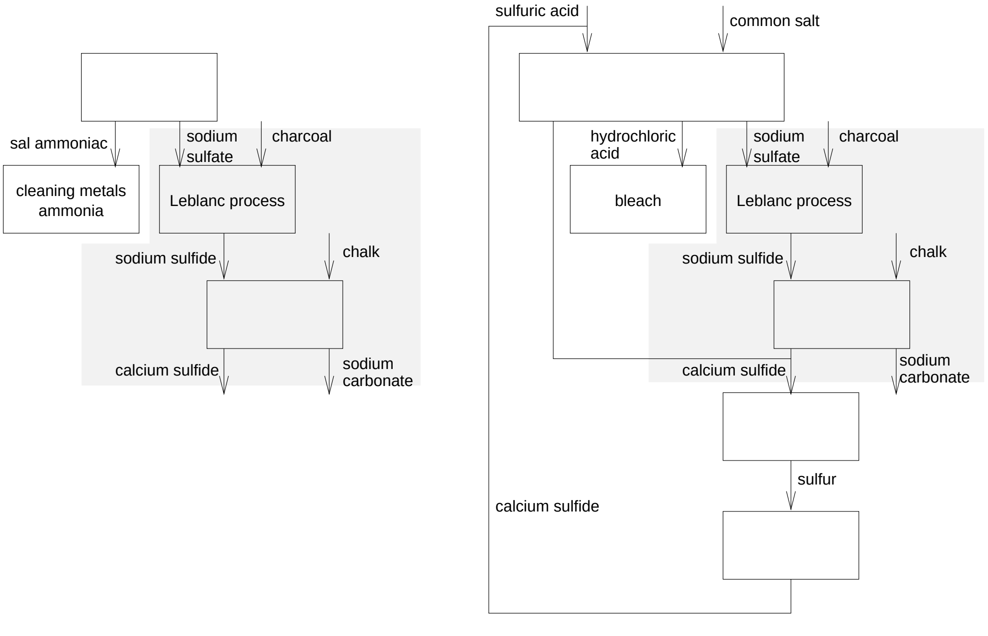
[1] Some authors distinguish industrial *metabolism* from industrial *ecology*. Since the underlying ideas are very similar, the two are considered equivalent here.

[2] The ecosystem can be defined as the natural environment, or, according to [41]: 'The environment of a community of organisms and all the interactions between organisms, and between organisms and their environment'.



(a) circa 1880 **(b) later**

Figure 2.2: *The historical example of the manufacturing process of sodium carbonate and the finding of new uses for former waste products*

plies[3]. Through the application of this law, industrial metabolism aims at understanding the circulation of materials and energy flows linked to human activity, from their initial extraction from the ecosystem to their inevitable reintegration, sooner or later, back into the ecosystem [55]. An industrial metabolism — in analogy to the biological metabolism — consists of closed substance chains and thus results in reduced extraction of primary materials, reduced discharge of waste materials, and increased recycling of useful ones.

In the technological system, for example, the history of the chemical industry is in considerable part one of finding new uses for former waste products. Ayres [15] cites the historical example of the Leblanc process for manufacturing sodium carbonate Na_2CO_3 in circa 1800 (see Figure 2.2a). With the Leblanc process, sodium sulfate Na_2SO_4, a former waste product of the sal ammoniac (ammonium chloride NH_4Cl) making process[4], found a useful destination. The Leblanc process is the reaction of sodium sulfate Na_2SO_4 with

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[3] Nuclear effects are neglected. The law of mass conservation is often called *materials balance* in this context [84].

[4] Sal ammoniac was used, among others, for cleaning metals and for producing ammonia.

charcoal C to sodium sulfide Na_2S, which in turn reacts with chalk $CaCO_3$ to sodium carbonate Na_2CO_3 and calcium sulfide CaS. This sodium carbonate Na_2CO_3 has many useful applications in the chemical industry, for example in the neutralization of inorganic and organic acids, and in the glass industry [142]. Meanwhile, the market for sal ammoniac NH_4Cl failed and sodium sulfate Na_2SO_4 had to be produced by a reaction of sulfuric acid H_2SO_4 with common salt NaCl (Figure 2.2b). Further products of this reaction are hydrochloric acid HCl and calcium sulfide CaS. Hydrochloric acid is one of the most important basic industrial chemicals, and is used, for example, as a strong inorganic acid for the manufacture of chlorides, dissolution of minerals, etc. Calcium sulfide could be used from 1880 on for producing sulfur, which in turn could react to sulfuric acid again — a nice example of a closed chain.

2.3 Product-Process Chains

The material flows through the technological system as studied in the industrial metabolism framework are often represented by product-process chains[5]. In general, a product-process chain can be defined as a chain of flows of materials and products between different transformation processes. One example of a chain is the historical example of the previous section shown in Figure 2.2, where products are denoted by arrows and processes by blocks.

In this thesis, the product-process chain description is used to visualize the material life-cycle of a product. In this context, we define the product-process chain more specifically as the set of producing, stocking, transportation, consumption, disposal, and recycling activities related to one specific product, for example an economically favourable product or an environmentally relevant product. The considered product can be a consumer product such as a television set, or a semi manufactured product used in various final products, such as a capacitor, or anything else. It is evident that the product-process chains corresponding to the different products are different.

In the most elementary appearance, a material life-cycle product-process chain consists of four basic processes. Resources are successively

(i) extracted,

(ii) transformed to products,

(iii) consumed, and

(iv) finally discharged.

This is depicted in the product-process chain diagram of Figure 2.3a. In

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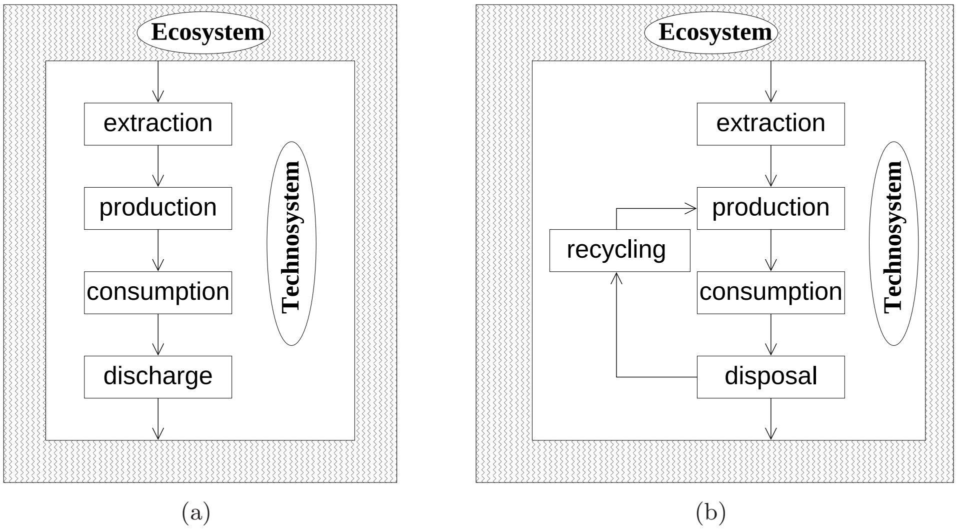
[5] We will also use the term *product chain* as shorthand for *product-process chain*



(a) (b)

Figure 2.3: *The flow of materials from the environment back to the environment, a material life-cycle. A linear chain (a) and a chain including recycling (b).*

practice, the linear chain is extended by recycle loops, for example in the case of recycling and reuse of discarded products. Generally, only part of the discarded products is reusable; the remainder is discharged (see Figure 2.3b). It is also possible that only product parts or materials are recycled. These parts and materials may be reused for the production of the same product or of a different product. In the latter case, they are directed to the product-process chain of another product.

A product chain can be approached in various ways, depending on the scope of interest. The two approaches which will be discussed here, are the environmental and the economic approach. The former approach deals with the effects on the environment due to the various activities in the chain, such as production, transportation etc. The latter approach deals with the optimization of the product chain with respect to the economic costs and revenues of the activities in the chain.

2.4 The Environmental Approach

Production processes in general are not possible without the production of waste and emissions as well. Waste is the flow disposed of in a controlled way, mostly as solid and liquid products. There are, however, some fundamental difficulties in the definition of waste, since from the moment that waste is reused in a useful application, it can no longer be considered as waste. Among the legal definitions of waste are the following [63]

- Wastes are materials — coming from a manufacturing process — that are not directly used in another process
- Wastes are materials — coming from a manufacturing process — for which no further use within the company is foreseen
- Wastes are materials — coming from a manufacturing process — that are marked for disposal
- Wastes are materials — coming from a manufacturing process — that are released into the environment

Emissions are uncontrolled disposals, in general towards water, soil and atmosphere. They can be material (gases, soluble and dispersed matter, liquids) or energetic (radiation, vibrations, etc.).

The environmental problems, which are a consequence of the production and consumption processes, can be divided into three classes [64, 65]: (i) depletion, (ii) pollution and (iii) disturbances. *Depletion* is referred to as all the types of actions related to material inputs from the environment. Some examples are:

- Depletion of ores and fossil fuels due to their extraction
- Depletion of biotic resources, for instance by agricultural production and by fishery

Pollution includes the issues related to outputs to the environment, for example issues related to

- Dispersion of toxic materials such as heavy metals (for example mercury Hg, lead Pb, cadmium Cd)
- Acidification due to sulphur dioxide SO_2 (for example from power plants and petroleum refineries), oxides of nitrogen NO_x (from combustion processes, where nitrogen in the air reacts with oxygen) and ammonia NH_3 (mainly from agriculture, for example from urine vapour)
- Eutrophication caused by ammonia, phosphorus in various compounds, and compounds of nitrogen such as nitrate and nitrite
- Ozone layer depletion, which is caused by emissions such as chlorofluorocarbons (CFCs)

- Global warming due to materials as carbon dioxide CO_2, methane CH_4, laughing gas N_2O and CFCs

Disturbances include the changes of structure within the environment, such as

- Degradation of land-use, for example the degradation of tropical rain forest to agricultural areas and further to industrial areas and urban areas
- Soil degradation

Some of these problems have a local character, some occur at a regional, continental or global level. In many cases, pollution is an international problem, because of natural (rivers, seas, air) or man-made transport across the frontier [132]. The consumption of tropical hardwood in industrialized countries is an example since it aggravates the problem of deforestation in the exporting countries. Another example is the international trade of animal feed, which causes a loss of nutrients in the exporting countries and an excess of nutrients in the importing countries.

Not all the effects are really measurable and comparable. Furthermore, different environmental effects may be different in different regions, because of, for example, a different climate or a different kind of soil. Eutrophication, for example, may be adverse in areas of bio-industry and beneficial in a desert area.

One method of determining a *quantitative* environmental effect score is the weighting of classified effects of the environmental problems with their specific classification factors into effect scores [69, 78]. It is usually used in methods such as life-cycle analysis.

> **An Example:** Life-cycle analysis (LCA)
> A widely used tool for determining the environmental performance of a product is life-cycle analysis or life-cycle assessment (LCA). According to the ISO-14000 norm [79] 'life-cycle assessment is a systematic set of procedures for compiling and examining the inputs and outputs of materials and energy and the associated environmental impacts directly attributable to the functioning of a product or service system throughout its life-cycle.' It should be stressed that LCA is intended for comparative use; so results of LCA studies have a comparative significance rather than providing absolute values on the environmental impact related to a definite product.

The determination of the environmental effects is highly subjective. There is, for example, no standard scale; the greenhouse effect is expressed in different

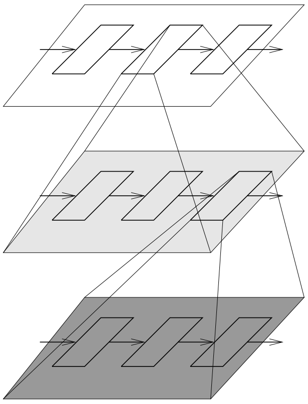


Figure 2.4: *The aggregation of serial processes*

units from those of the depletion of resources. To obtain total environmental effect scores, as is done in life-cycle analysis, a weighted summation of partial environmental effect scores is performed, with subjective weighting functions. Another difficulty is, that research on environmental problems constantly leads to new insights into the severity of these problems and that the effect scores are therefore not constant in time [49, 62].

2.5 The Economic Relationships in the Chain

The driving forces behind many processes in the chain are economic considerations. Producers are willing to produce because they expect profit, and consumers buy products because they consider them useful. The actors in the chain cause costs, such as resource extraction costs, production costs, consumption costs and recycling costs, but also transportation costs, costs for the use of products, and costs made to dispose of discarded products, inventory costs (costs due to the maintenance of the storage), and transaction costs (for example for advertising). By directly relating indirect costs, such as overhead costs, to the flows of materials, the chain costs can be derived from the product flows.

At first sight, two successive actors in the chain have counteracting interests. The vendor of a product is interested in a high price, and the buyer, on the contrary, tries to achieve a low price. From the point of view of the entire chain, this competing behaviour is often not optimal. The additional profit the vendor can obtain by fetching a higher price leads to higher costs and less profit on the side of the buyer. Thus the *chain* performance cannot be expressed by the profit of one single actor. A common interest of all the chain actors is to minimize the costs. This results in a good motivation for

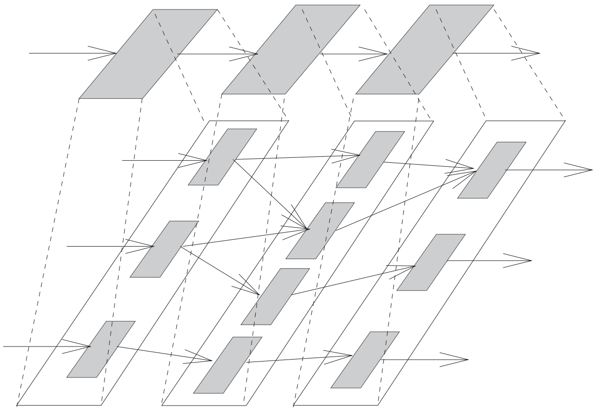


Figure 2.5: *The aggregation of parallel processes*

entering into agreements.

2.6 The Level of Aggregation

It is possible to set up a product chain on different levels of aggregation. The product-process chain of a production process can be established in great detail, which might be interesting for an operational manager. On the other hand, the general director of a company will probably be interested in more aggregated information. Then several successive processes are combined to one process. The joining of several successive (serial) processes, which leads to a higher level of aggregation, is shown in Figure 2.4. By the choice of a high level of aggregation, it is possible to describe the product flows associated with one final product in a clearly structured way.

Besides this horizontal aggregation, processes can be aggregated vertically. In general, there are several vendors of a product or raw material, and several customers. The parallel processes can be combined on a higher level of aggregation, if they contain identical processes. Figure 2.5 shows such an aggregation in a product-process chain with parallel processes. The two kinds of aggregation are often used simultaneously.

2.7 System Boundaries

The boundaries of the considered product chain must be defined in several dimensions, among others: geographical system boundaries, the time horizon and the boundaries between the considered product chain and related product chains of other products [139]. Many chains spread over more than one nation.

This has implications for the regulation on these chains. Since they extend over the borders of the administrative powers of environmental agencies, they can only be regulated by supranational authorities such as the European Union, the United Nations and the managements of multi-national enterprises. Thus, when considering a particular geographical area, the imports and exports of products must be taken into account. As mentioned in Section 2.4, transboundary pollution must be considered as well.

The time horizon that is interesting to be considered depends on the product and on the kind of information the product chain designer is interested in. It depends, for example, on the lifetime of the product and on the time needed for production.

The determination of the boundaries between the considered product chain and related chains of other products is perhaps the most difficult issue. Nowadays, nearly all products are part of an entangled network of production, recycling and waste disposal processes. Only the main processes linked to a product can be studied in order not to end up with a system that would be too large to be handled.

2.8 Chain Management

Chain management is defined as the management of material flows that result from chains of social and economic activities [86]. It is obvious that not the individual interests of the chain actors but those of the *integral chain* should be optimized. Decisions on *environmental* measures, for example, have traditionally been made according to single processes. This approach often leads to suboptimization because positive effects in one part of the chain may lead to even larger negative effects in other parts. In order to overcome the problem of suboptimization, more integrated approaches to chains are becoming increasingly popular. These approaches consider the environmental effects caused by the various processes in the chain simultaneously [85, 109]. It becomes increasingly important for a chain actor to know what influence other actors in the product chain have on his process to reach sustainable production in the future.

With respect to *economic* effects, an integral chain approach is often advantageous, too. Since political and technological changes caused such a speed up in border-crossing processes with the consequence of globalization, the chain actors are forced to cooperate in order to be competitive. The dependence on suppliers is growing and the outsourcing of certain activities to subcontractors makes cooperation indispensable. The form of optimization of the product chain depends on the type of product. A food chain, for example, has to cope

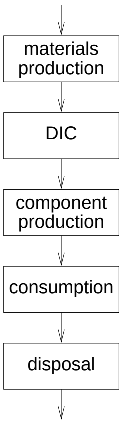


Figure 2.6: *The actual chain of the reels*

with other problems than a chemical product chain. In the first case, the limited storage life is a restrictive factor, whereas in the second case start-up and shutdown problems play a dominant role. In both cases, the individual actors have to cooperate somehow, instead of trying to achieve their different individual goals. In the pig farming industry, for example, it is common practice that the pig breeder and the fattener agree to share in the profits.

The possibility to influence the product chain at different points leads to more degrees of freedom. By considering the entire chain, measures can be taken in a coordinated way. This decreases the risk that undesired effects in other parts of the chain might be overlooked.

2.9 An Example: The Chain of a Reel Producer

In order to illustrate the ideas of chain management, we discuss the example of a reel producer, which we call DIC Plastics here [98]. The chain of DIC, depicted in Figure 2.6, must be seen in close relationship to the production of electronic components. These components are produced in large quantities, mounted to tapes and subsequently wrapped to reels. These reels are sold to the producers of printed circuit boards (PCBs), who unwrap the reels and mount the components on the PCBs by pick and place machines. The empty reels are disposed of. The reels must be seen as sophisticated packaging material.

2.9.1 Environmental aspects

Packaging causes a considerable amount of waste. Therefore, authorities in several countries have taken measures to reduce the amount of packaging waste. Some examples can be found in take back legislation as introduced by the *Kreislaufgesetz* in Germany [60] or the raising of the disposal costs.

In the Netherlands, appointments between government and industry have been made on a voluntary base, so-called convenants. The packaging convenant includes the agreement on take back of used packaging materials and minimal recycling ratios of the various packaging materials. The packager is assigned as the company that has the final responsibility for the compliance with the obligations of the convenant.

2.9.2 Economic aspects

Due to changes in governmental regulations, the component producer (the wrapper) will be forced in the future to take back the used reels. The component producer will probably try to pass the responsibility to the reel producer, DIC. In the context of competitive advantage, DIC will be interested in the possibilities of taking back used reels.

This confronts DIC with the problem of redesigning the product chain. The alternatives are to reuse, to recycle or to finally discharge the used reels. The differences between recycling and reuse will be discussed in the following chapter. Here, we only mention that reuse of reels is the use of the returned reels for the same purpose (after having been cleaned or repaired), whereas recycling is the reuse of the plastic in new products. There are at least ten alternative ways to solve the design problem. The alternatives of treating the used product carriers are (see Figure 2.7):

1. Reuse by DIC
2. Reuse by the customers of DIC
3. Reuse in another chain after preparation by DIC
4. Reuse in another chain after being sold by the customer
5. Recycling by DIC
6. Shredding by DIC, and recycling outside the chain
7. Recycling in another chain
8. Recycling in another chain after collection by DIC

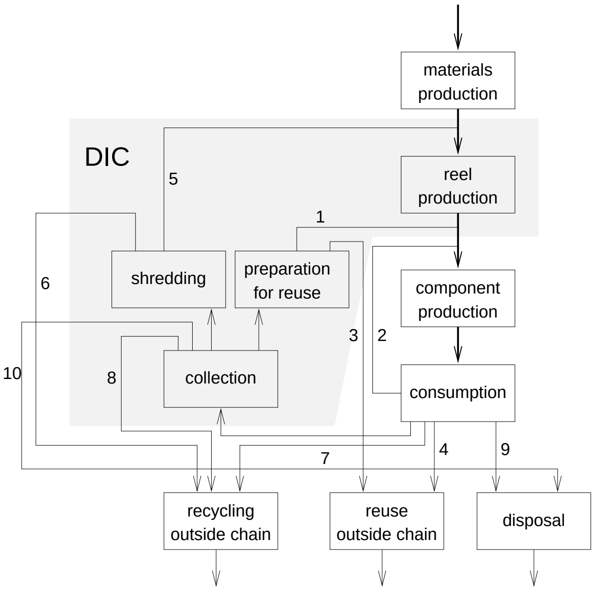


Figure 2.7: *The new chain of DIC. The recycling and reuse options are displayed.*

9. Disposal by the customer
10. Disposal by DIC

For most of these alternatives some processes or products must be redesigned. For the reuse options, for example, it must be found out if it is possible to prepare the used reels in such a way that they meet the quality requirements, such as the antistatic requirements. It must be taken into account that the disposal options 9 and 10 may be forbidden by legislation in the future.

2.10 Summary

Product-process chains describe the products and processes linked to a final product in a 'cradle to grave' approach, i.e. from the initial extraction of resources to the final disposal of product wastes. They increasingly often include material recycling processes. When considering the environmental effects caused by a product, the integral chain should be optimized, not only individual processes. With respect to the economic performance of the chain, an integral chain approach is generally advantageous as well.

For a definite application of product chains, the appropriate level of horizontal and vertical aggregation of processes must be chosen. Since in a complex economy, networks of products and processes are interwoven, the product-process chain boundaries must be clearly defined with respect to the geographic limits, the considered time horizon and with respect to other chains.

Chapter 3

Influences on Product Flows

3.1 Introduction

The preceding chapter dealt with the definition of product-process chains and their system boundaries. In this chapter, the design of the chain, i.e. the direction of product and material (re-)flows is discussed, and some background is provided to external economic and environmental measures that can be taken by governmental authorities. These measures are categorized according to their regulatory, economic or social nature. The design of such measures involves various issues, among which the choice of goals, objectives and targets to be attained, and the determination of specific instruments to attain these targets. In the following chapters, the instruments that can be captured in mathematical models will be modelled and subsequently analysed.

In Section 3.2, the options are discussed to change the material and product flows in the chain with the emphasis put on various recycling and reuse strategies. In Section 3.3, the desirability of governmental intervention is discussed. Some instruments are studied that are supposed to lead to a change in the product chain. We focus on regulatory instruments, such as disposal prohibition, on economic instruments, such as taxes and subsidies, on social instruments, such as advertising campaigns, and on self-regulation by voluntary agreements between the chain actors. Section 3.4 mentions criteria that governments and industry can lay down to evaluate the chosen instruments. In Section 3.5, the main points of this chapter are summarized.

3.2 Changes of Product Flows

The economic and environmental effects caused by a product chain can be modified by changing the product flows between the different processes or by changing the individual processes themselves. Modifications can be driven by political measures (*policy driven* changes) or by technological development and its supposed economic advantages (*technology driven* changes) [119].

The major opportunities for reducing wastes and pollution consist of using by-products with an originally restricted value as raw materials for others. Technical feasibility is a prerequisite for initial consideration by the concerned actors. Economic feasibility is an important criterion for the final implementation. Figure 3.1 shows some possibilities of the prevention of waste and emissions. They include

- Product changes, for example by the substitution of materials or products
- Recycling of materials and reuse of product parts and entire products
- Changes in production, consumption and waste treatment processes

These issues will be discussed more thoroughly in the following sections.

3.2.1 Substitution of materials or products and dematerialization

Substitution of the considered final product with another product constitutes the most obvious change in a product chain. In fact, in this case, the completely different product chain of this new product must be considered. An example is the substitution of the record player with the CD-player because the latter represents an easier to use and better quality final product, serving the same purpose as the record player. An example of the substitution of materials is the substitution of metals for plastics in many applications, such as pipes, where copper is substituted for PVC. A reason for the substitution is that the new materials may be cheaper, or cause less environmental damage. Another reason encouraging substitution is that definite materials are easier to be recycled than others.

The term *dematerialization* is often used to characterize the decline over time in weight of the materials used in industrial final products [70]. This means a more efficient use of materials for a given function. A classical example is to be found in the electronics industry. The functionality per unit of mass has increased enormously since the early days of data processing. A tiny semiconductor chip performs nowadays more functions than a machine for which entire rooms full of electronic equipment were necessary some decades ago. It must be noted that 'less' is not necessarily less from an environmental point of view. If smaller and lighter products are also inferior in quality, more products will be used, which will increase the total amount of utilized materials.

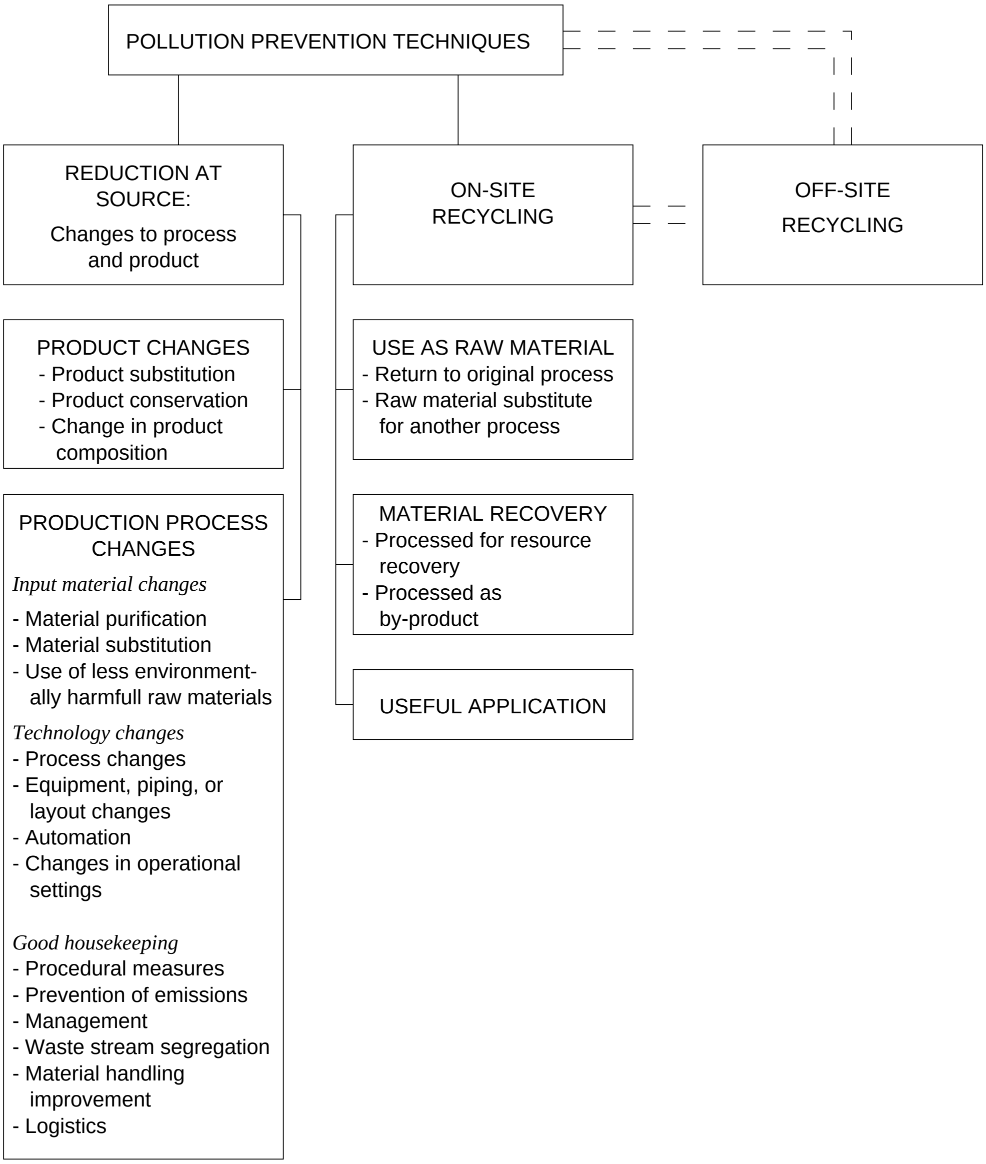


Figure 3.1: *Techniques for the prevention of waste and emissions ([51, 87])*

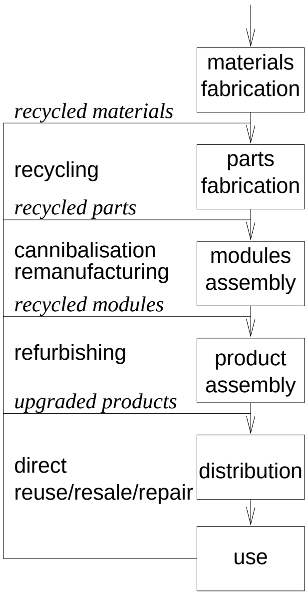


Figure 3.2: *Recycling and reuse options in a simple product chain*

It is also thinkable that smaller equipment requires extended environmentally harmful processing.

3.2.2 Recycling of materials and reuse of products

Recycling is an approach to waste reduction that has been advocated by environmentalists for decades. The literal meaning of recycling is: *To return to the previous stage of the cyclic process.* The common use of the word recycling, however, includes all actions by which existing products or materials are processed for a new application. In this thesis, we discriminate between recycling and reuse: the word recycling is restricted to the processing of materials, and the word reuse is used for parts and products (product reuse and parts reuse).

Figure 3.2 shows the possible recycling and reuse options in a simple product chain [137]. They include direct reuse, resale and repair, refurbishing, cannibalization, remanufacturing, and recycling. Each of these options involves the collection of used products and components, their reprocessing, and their redistribution. The main difference is in the depth of reprocessing: Repair, refurbishing and remanufacturing refer to the 'upgrading' of used products and differ with respect to the degree of upgrading.

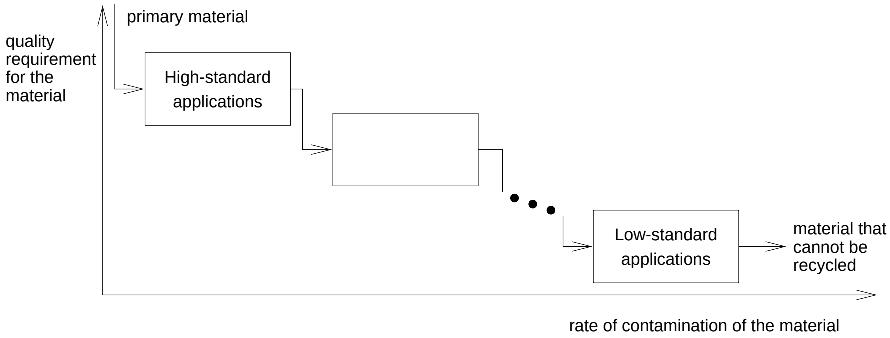


Figure 3.3: *Downcycling of materials (cascading)*

Recycling

The purpose of recycling, as it is used in this thesis, is the reuse of materials from used products and components. It implies the disassembly of the product into parts and the subsequent separation of the parts into different material categories. Often the waste residuals of production processes are directly redirected to the same process as raw materials. This is referred to as *primary recycling*, microrecycling, or waste-mining. *Secondary recycling* is then defined as the recycling of materials after the product has been used[1].

The recycled materials can be reused for the same purpose as the original product (*direct recycling*) or for other purposes (*indirect recycling*). Depending on the type of materials, they can be reused for an equivalent application or for a lower grade application. The latter case — the so-called *downcycling* or *cascading* (see Figure 3.3) — holds, for instance, for plastics. Here, no mechanical technology is available, which would be able to recycle used plastics to plastics with the same quality as primary plastics. This is due to the inhomogeneity of the wasted plastics, to the presence of additives that are used in the production process, to the contamination by other materials, to the degradation of polymers, and others. In these cases, cascading is a way to optimally use the material [133]. Primary plastics are, for example, an imperative for applications such as food and drink packaging and high-tech applications. First order regranulate can be used for high standard applications such as non-food packaging, foils, and plastic car parts. In these applications regranulate and primary material are perfect substitutes. Low quality regranulate can be used for low standard applications such as garden furniture and poles for fences.

The possibilities of recycling a product highly depend on the variety of materials used in this product. The use of less types of materials usually

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[1] Sometimes, the incineration of waste is referred to as *tertiary recycling*, because energy is recovered, but it is doubted if this treatment can really be called recycling [86].

results in a better recyclability. Therefore, the possibilities of recycling must already be taken into account in the design stage of a product. This is referred to as *design for recycling*. A simple example of a product that is *not* designed for recycling is a glass bottle with a screw cap that leaves a metal ring affixed when the bottle is opened.

Since disassembly is a necessary step in the recycling process of most products, the disassembility should also already be taken into account in the design stage, for instance, by using a modular structure of the product. By a good design for recycling and for disassembly, the costs for recycling can be reduced, which can lead to higher recycling rates [131]. Recyclability and disassembility are restricted by product and process specifications.

It must be noted that, from an environmental point of view, recycling is not necessarily 'better' than the incineration of waste because recycling involves the use of energy (for the transportation and processing of the materials), whereas the incineration of waste generally generates energy. The environmental desirability depends on the energy content of the material [68].

Although recycling is often rejected by economic arguments, practice demonstrates that, in many cases, new opportunities arise for economically feasible recycling options. The paper industry, for example, has for long been recycling waste-paper because pulp from waste-paper is a relatively cheap substitute for primary pulp [6]. Therefore, waste-paper recycling rates attain nowadays 25 to 55 percent in the EU countries, for example.

Cannibalization

Cannibalization is a product recovery option in which the product is selectively disassembled and only valuable parts or parts which are necessary for the repair of other products are reused, commonly as spare parts. The optimal disassembly procedures can be calculated in order to realize maximal profit [95, 96]. Old timers may serve as an example. They are disassembled to use the parts for other old timers. Another example is the use of important parts of a machine or factory in the case of an embargo, or of military equipment in the case of wartime. These parts need not necessarily have a high value themselves but they are necessary for the working of the equipment.

Remanufacturing and refurbishing

By remanufacturing a product, one seeks to bring it into a state comparable to a new product. Therefore, it is disassembled and all parts are extensively inspected. Worn out and outdated parts are replaced by new ones, others are repaired, and afterwards the product is remanufactured, leading to a product which is ’as new’. Examples of remanufacturing can be found among many high value products; standard examples are the remanufacturing of aircrafts, the remanufacturing of engines, and the remanufacturing of personal computers. Remanufacturing is gaining importance since it offers the option to upgrade products by keeping a number of original parts. Refurbishing resembles remanufacturing. Here, quality standards are less strict than for remanufactured products.

Repair

The purpose of repair is to bring used products back to ’working products’. It involves the upgrading and/or replacement of broken parts. The quality of repaired products might be less than the quality of new products. Buildings and automobiles have been repaired for longer use since a long time ago. Note, however, that modern technology tends to make repair more complicated, although products might last longer without repair being necessary. Currently, often modules are exchanged instead of the broken parts.

3.2.3 Changes in production, consumption and waste treatment processes

The rising environmental awareness brought about changes in many of the processes associated with a product. An obvious change occurred in the treatment of waste. The initial societal response to problems of pollution was an attempt to redirect or sequester the wastes in order to prevent diseases and smell. Since this traditional way of waste treatment occurs at the end of the chain, it is called end-of-pipe treatment. Examples are the installation of filters, the incineration of wastes and the installation of wastewater treatment plants. Initially these techniques were designed to only redirect the waste and emission flows, but not to reduce or eliminate them. Later it was recognized that, due to the concentration of certain contents of the waste (for example in filters), these become tractable for further treatment or for useful applications.

A more integrated approach consists of changing the production process or the recycling process of a product. Technological development made the production of many products more efficient so that less raw materials are needed for a product and that less waste is produced. However, thermodynamic constraints must be taken into account [125].

The consumption process is also subject to change. The pattern of consumption is continually changing, so that some products are consumed instead of others. The reasons for these changes are, for instance, that a particular product provides the same service but causes less environmental damage, or that it is cheaper. This substitution process has already been discussed in Section 3.2.1. It must be noted that substitution may have secondary effects. The substitution of a washing-up brush with a dishwasher, for example, may lead to a higher energy consumption in the washing process *and* in the production process.

The consumers' waste supply decisions have considerable influence on the product flows. By sorting different kinds of waste from discarded products, for example, subsequent waste treatment can be made more efficient.

3.3 External Influences on the Chain

Once the goals concerning, for instance, the flows in the chain have been set, the question remains how to achieve these goals. Economists have generally classified the instruments that can be applied from outside the chain into *quantity*, *price* and *social* instruments, and in *ex ante* versus *ex post* instruments.

Quantity instruments are the traditionally used regulatory instruments, which prescribe certain levels of emissions directly. An example is the disposal prohibition of certain substances such as chlorofluorocarbons (CFCs) as a measure against ozone layer depletion. Price instruments — also called economic incentive instruments in economics — aim at controlling environmental effects by putting a price on the use of environment, thereby influencing the market mechanism. Examples of such instruments are emission taxes, or a deposit on returnable items [121, 129]. Social instruments are, for example, advertising and the stimulation of environmental research.

The mentioned examples are all *ex ante* instruments, which means that they are active at the time the polluting activity occurs. In contrast, *ex post* instruments only become operative at the time that damages from a certain activity occur. A classical example is legal liability, which makes the polluter responsible for some or all of the damages that result from his activity.

In cases where the use of one single instrument does not lead to the achievement of the objective, the simultaneous use of various instruments may well result in the achievement of the objective.

3.3.1 Externalities and public goods

One of the basic challenges of environmental policy is to find out whether governmental intervention on markets is desirable. In general, markets will cause goods to be allocated efficiently, given certain conditions (such as perfect competition and complete information) and rational and selfish individual behaviour: there will be a complete set of market equilibria determining the quantities produced, the prices at which trade takes place, and the distribution of goods across individuals. In such an allocatively efficient state, it is not possible to make one individual better off by her own assessment without making some other individual worse off by her own assessment. The market outcome, however, will not necessarily be desirable in regard to justice or equity between individuals.

The complete markets condition implies that there exists competitive markets for everything that is of interest to anybody. It needs to be subject to enforceable individual private property rights since one can only sell (legally) what one owns. Two conditions of breakdown of the complete market hypothesis are *externalities* and *public goods* [45]. An externality exists when the activities of a firm or individual give rise to unintended effects on other firms or individuals and these effects do not figure in the costs and benefits associated with the activity of the firm or individual responsible for it. A standard example of an externality is the release by a firm of emissions giving rise to atmospheric pollution. The release involves no costs to the firm since nobody owns the atmosphere, so that the firm cannot be charged for its use.

A public good is something that is *non-rival* and *non-excludable* in use, i.e. individuals cannot be excluded from enjoyment of the services provided by these goods and one individual's enjoyment of those services is not rival to other individuals' enjoyment of those services. The services provided by many environmental assets have the characteristic of public goods. Economists recognise the non-existence of private property rights as a source of market failure in relation to environmental services. The overuse of environmental inputs due to incomplete private property rights is often referred to as the *tragedy of the commons* [67].

One goal for government intervention in the operation of actual markets is to correct market failure so as to facilitate the attainment of allocative efficiency.

3.3.2 Regulatory instruments

Regulatory instruments directly influence the behaviour of the economic agents. They prescribe definite standards with which the polluters have to comply. If these standards are not met, penalties will be imposed. Regulatory instruments either prescribe *objectives* to be attained or the *means* that must be used. The objective of such an instrument is, for example, a product standard such as the maximum amount of pollutants contained in a product. An example of a prescribed means is a process standard such as the installation of filters. The advantage of a product standard is that some flexibility is left to the polluters: When confronted with the standard, the company can, on

	advantages	disadvantages
taxes and charges	-conformity with 'polluter pays' principle, -stimulus for environmental friendliness	-difficulty to translate the environmental objective into the amount of the tax or charge
subsidies	-stimulus for environmental friendliness	-nonconformity with 'polluter pays' principle
tradeable permits	-knowledge of the amount of environmental damage -stimulus for environmental friendliness	-difficulties in initial allocation -disadvantages for newcomers

Table 3.1: *Advantages and disadvantages of economic instruments*

the basis of the specific knowledge of its process, adapt to the standard by making use of its specific possibilities, for example, by recycling process waste but also by reducing its amount. It is free to make the choice.

Furthermore, the amount of a certain pollutant may be regulated (for example the maximum amount of cadmium in the soil), or a certain maximum quantity of emissions or waste can be prescribed (for example the maximum quantity of SO_2 emitted by industry). Examples of regulatory instruments applied to product chains are minimum quota of recycled materials to be used in the production process, prohibition of disposal, etc.

Regulatory instruments are inflexible, because, in comparison to other instruments, they leave little choice to the polluters. Furthermore, they do not provide incentives to continually improve pollution abatement technologies. Once the standard imposed by regulatory instruments is met, further improvement will not be beneficial for polluters [19]. On the other hand, these instruments have the advantage that — at least in principle — the total environmental effect is known in advance. Additionally, regulatory instruments have no positive or negative financial consequences for the authorities, which makes them very attractive.

3.3.3 Economic instruments

Economic instruments offer financial incentives to guide the economic agents to the desired environmental actions. The instruments mostly discussed in literature are charges, subsidies and tradeable (emission) permits [18, 77, 94]. Economic instruments leave the agents more freedom than regulatory instruments. The advantages and drawbacks of these instruments are briefly discussed here. They are summarized in Table 3.1.

Charges and Taxes

Charges and taxes constitute a payment for each unit of environmental damage. *Charges* have several advantages compared to regulatory instruments: They provide a stimulus for continuous search for environmentally friendlier technologies and they charge polluters for polluting activities. However, charges have some disadvantages as well. The main drawback is that the desired environmental objective is difficult to be translated into the amount of the charge; the effect of a chosen charge is uncertain. It depends on some — not exactly known — coefficient of elasticity.

Taxes are comparable to charges. The difference between taxes and charges is that charges are directly related to a service, for instance, a waste collection charge, whereas taxes are not linked to a service by the authorities, for instance, a tax on a product. A tax instrument that is tailored to product chains is the installation of a deposit refund system. By this instrument, a tax (the deposit) is imposed on a unit of product or service. If the producers and consumers fulfil certain conditions, it is refunded as a subsidy, which makes the system financially neutral for the authorities [31]. This makes such a system politically attainable.

Subsidies

Subsidies are comparable to taxes in that they create incentives for diminishing pollution and leave some freedom to the economic agents. The same disadvantages as for charges hold as well. The main difference is that subsidies are not conform to the 'polluter pays' principle, i.e. even for polluting activities subsidies can be obtained. Since subsidies have a cost-decreasing effect, they can lead to more production and thus even to more pollution.

Permits

Permits are environmental quota that are distributed by the authorities. Permits that have received much attention are tradeable permits [18, 42]. After their initial distribution they can be traded between the polluters. Each permit allows a polluter to cause a definite amount of pollution. The environmental authority distributes a number of permits according to the environmental standard agreed upon. Since the permits may be traded freely, a permit market develops and a market clearing price results. Tradeable permits have the same advantages as charges that environmentally friendly behaviour and innovation are stimulated. They do not have the disadvantage of charges: The total level of pollution is determined by the number of emitted permits. However, there are some drawbacks. The main difficulty is the initial allocation of permits to polluters. Above this, the system puts 'new' polluters in a disadvantage. Moreover, high transaction costs restrict the trade, and the application of permits becomes extremely complicated — perhaps even impossible — if the location of the emission is relevant [94].

3.3.4 Social and organizational instruments

Social instruments are used to influence producer and consumer behaviour. This can be done, for example, by internal and external training. Such training is given, not only to educate individual employees in companies towards environmentally friendly behaviour, but also to educate the companies themselves.

Other social instruments are propaganda, negotiation and the encouragement of environmental research. An example of propaganda is the influence exerted on consumers' purchasing behaviour, their use behaviour, and their waste handling behaviour by advertising campaigns. As a consequence of such influences, producers, on the other hand, take back products (used cars, for example) to provide themselves a 'green' image.

An example of negotiation is the self-regulation stimulating policy which is becoming increasingly popular. Self-regulation of a product chain means voluntary agreements between the chain actors and government, which implies that the chain actors agree upon targets to be attained. The government's main task in self-regulation is not to regulate but rather to create conditions that foster the emergence of self-regulation. It involves influencing attitudes and activities regarding the environmental threats or goals, bringing parties — the members of a product chain — together for discussions, and distributing expertise in chain management through investigation of best-practices in self-regulation. Relaxation of legislative instruments is often offered when a functioning self-regulation is present. It must be noted, however, that government has the possibilities to enforce a definite behaviour, if self-regulation fails. This puts pressure on the parties involved to achieve agreements.

From an organizational point of view, governments can help companies to behave in an environmentally friendly way by developing (international) standards, which contain procedures that form a good starting point for the desired behaviour. An example is the series of environmental management standards known as the ISO 14000 series [122, 124, 138], which has recently been set in operation. It is a series of voluntary environmental standards that is likely to have a significant impact on companies around the world. The standards contain several areas including environmental management systems, environmental auditing, environmental labelling, environmental performance evaluation and environmental life-cycle assessment (see Section 2.4). The ISO

14001 is designed to assist organizations in implementing and maintaining an environmental management system, verifying conformance with their environmental policies and objectives and demonstrating such conformance to other organizations via certification by a third party or via self-declaration. ISO 14000 is intended to be the environmental counterpart of the ISO 9000 product-related standard. Conformance to ISO 9000 has become a condition for doing business in certain markets.

3.4 Evaluation of Instruments

For evaluating instruments, there are three major conceptual issues to be taken into account:

(i) The existence of a policy which achieves the chosen objective. If no policy exists that is able to achieve the objective, further analysis is quite useless. In this case, the objectives must be adapted or new instruments must be found.

(ii) The uniqueness of such a policy if it exists. The uniqueness of the policy implies that there is no instrument choice. In order to attain the objective, the unique instrument must be applied.

(iii) The design of policy

The first author who studied those three issues systematically for a static system was Tinbergen [140, 141]. In a *dynamic* situation, these issues play an important role as well. Here, the stability of the system is an additional issue, i.e. it must be investigated, if the application of the instruments does not lead to instability of the total system.

A very popular criterion, among economists, for determining the desirability of alternative governmental policy instruments is economic efficiency: At pollution target levels that satisfy this criterion, the benefits of additional pollution reduction equal the costs of achieving these reductions. Further criteria are [29], for example

(i) Information intensity: The amount of data that must be available to implement an efficient pollution control system

(ii) Ease of monitoring and enforcement: The possibilities of making and interpreting the measurements which are necessary to judge the compliance with standards, to prepare bills, etc.

(iii) Flexibility in the face of exogenous change: The possibilities of adjusting the system when changes in taste, technology, etc. occur

(iv) Political considerations: Considerations of various kinds, for example distributional, and ethical considerations.

The major criterion for enterprises, on the other hand, is the influence of the chosen instrument on their profit.

3.5 Summary

The influences on the product flows in chains are manifold. Governmental authorities and the chain actors themselves are involved in influencing chains so that these behave in the desired way. Their objectives are related to the chain's economic and environmental performances. These performances can be influenced by rearranging product chains, for example by recycling materials, by reusing former waste products in new applications and by the cannibalization, remanufacturing and repair of products. Other changes in product chains are caused by changes in product design, dematerialization, substitution of definite materials or products with others, and changes in production, consumption and waste treatment processes.

Authorities' instruments are imposed on the chain actors. Chain instruments result from self-regulation between these chain actors. Authorities' instruments aimed at influencing product chains can be categorized according to their regulatory, economic, or social nature. The major instruments are the regulation of wastes and emissions, charges and taxes on polluting behaviour and products, subsidies for environmentally friendly behaviour and the installation of tradeable emission permit systems as well as the stimulation of environmentally friendly behaviour. Instruments that are gaining importance are voluntary agreements between government and industry on the guidance of product flows. The main evaluation criteria are the economic efficiency for authorities' instruments and the costs for the chain instruments.

Chapter 4

Systems and Control Theory

4.1 Introduction

In the previous chapters, the systems under study — product-process chains — were defined, and influences on them were discussed. In this chapter, a general framework of modelling and controlling systems is presented. It appears that standard control theory is applicable to product-process chains.

The term 'control' is generally understood in different ways within different branches of science. In management science, for example, it usually means the control of quality or the control of a part of a production plant, or most commonly it refers to the use of accounting or accounting-related data in order to give a stewardship report of a firm, and to account for either poor or exemplary performance. Many firms even have a special staff function that is called controllership [21].

In Section 4.2, we introduce the meaning of control in the sense of control theory. In Section 4.3, we discuss the abstract elements of a control system, such as the plant and the controller and the signals — inputs and outputs — between them. In Section 4.4, we motivate how product chain analysis fits into this context. Section 4.5 is concerned with a mathematical standard description of dynamic systems, the state space description. In Section 4.6, we introduce some basic concepts of control theory such as stability, controllability and observability, and in Section 4.7 we discuss the performance of control systems. In Section 4.8, we deal with constraints on such systems, and in Section 4.9 we give a summary of the concepts of this chapter.

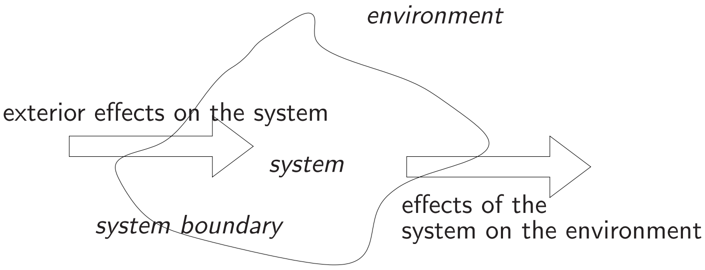


Figure 4.1: *A system and its environment*

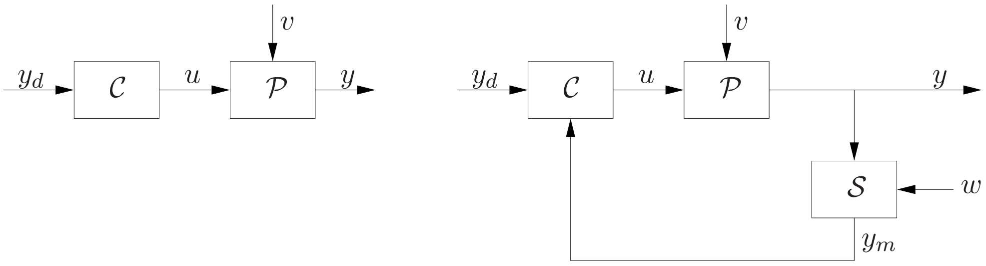
4.2 The Definition of Control

Before discussing applications of control theory in chain models, we have to define the meaning of control. Control as used in control theory can be defined as follows:

> The objective of control is to cause specific system variables to conform to some reference values. This can be done by steering the system, or by modifying the dynamic behaviour by means of feedforward and/or feedback.

As stated in Section 1.4, product chain analysis belongs to the field of operational research. It is sometimes difficult to distinguish control methods from other methods used in operational research. Mathematical programming techniques, such as linear and dynamic programming, are important in control theory, but they are also important in other fields. Since such mathematical tools are not limited to dynamical systems, it seems natural in this context not to number them among standard control theory methods [47, 48]. We shall also exclude system dynamics techniques as introduced by Forrester [57]. 'System dynamics' usually means that the dynamic properties of a system are analysed with the aid of simulation. This is often a very fruitful approach for gaining insight into the behaviour of a system. Simulation, however, cannot be regarded as a typical control theory method, but more as a useful tool if needed.

Besides specific control theoretical methods, there are also some basic concepts that obviously have an impact on related areas. One such concept, which is important, is feedback control, i.e. a control policy which is a function of the present state or system output. The advantage of feedback control is its ability to cushion the effects of, for example, modelling errors or stochastic disturbances.



(a) open loop control (b) feedback control

Figure 4.2: *The abstract elements of an open loop control system (a) and of a feedback control system (b)*

4.3 Basic Elements of a Control System

The power of control theory derives from its ability to fit a multitude of different problems into a single abstract, mathematical framework. It translates the relevant properties of a system, such as depicted in Figure 4.1, into a general framework. The system is defined by the boundary, which separates it from the environment, and the interactions between the system and the environment. These interactions are exterior effects influencing the system (the input signals) and the effects of the system on its environment (the output signals). It is of interest to know the inputs, outputs and disturbances and the relations between them.

Block diagrams such as depicted in Figure 4.2a and 4.2b usually constitute a very efficient way of illustrating how a control system works. They show the abstract elements of a control system. The target variable y_d represents the control goal and is the input to the system. It is defined by the system designer and is chosen in relation with the overall goals or objectives of the controlled system. It may be kept constant, in which case the objective is *regulation* (for example in the case of regulation of the temperature in a room), or it may vary, and the problem becomes one of *tracking* (for example in the case of the control of a robot arm along a preset trajectory). The problem is to select targets which are manageable and which substitute as a measure for what one would like to achieve.

The plant[1], represented by $\mathcal{P}$ in Figures 4.2a and 4.2b, generates the output y from the inputs u and v. We distinguish two kinds of inputs to the plant:

(i) Inputs that can be manipulated. They are denoted by u.

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[1] The word plant has become a generic term in control theory. It may be used in its wider sense to mean some body or system of components which has one or more inputs and outputs and which may be subject to outside influences or disturbances.

(ii) Other input variables, which are beyond the control of the controller, such as disturbances. They are denoted by v.

In open loop systems as indicated in Figure 4.2a, on the basis of knowledge about the system, the controller $\mathcal{C}$ makes a prediction of what the input u should be to give the desired output y_d. The real output y may be equal to y_d but it may also be different, if the disturbances v influence the system in an unpredicted way.

> **An example of open loop control:** The cooking of an egg
> An example of an open loop controlled system is the cooking of an egg: The target y_d here is to obtain an egg of a definite predefined hardness. This is the input to the controller, which tries to achieve this target by choosing the cooking time (based on experience) at the beginning of the cooking process. The hardness of the egg is not measured during the cooking. The real hardness of the egg can deviate from the desired hardness due to disturbances such as variations in ambient temperature or variations in the size and composition of the egg.

Open loop control is frequently unsatisfactory because any unexpected disturbances to the system can cause a deviation — in the output — from the desired value. It is used, for instance, if there are no output measurements available. Although a single-input-single-output case has been discussed, the general case of multiple inputs and multiple outputs may be represented similarly. In this general case, all the inputs and outputs are vector valued variables.

The basis of a *feedback* control system (see Figure 4.2b) is the measurement of the system output. This measured output y_m is fed back to the plant via the controller $\mathcal{C}$. It is obtained from the output y by the sensor $\mathcal{S}$ and includes a possible measurement error or uncertainty w. The controller $\mathcal{C}$ generates the actuator commands u from the difference between the measurements y_m and the target y_d.

The usefulness of feedback is given by its capability to stabilize unstable systems, to compensate for errors, to balance model uncertainties, to reduce disturbances and to achieve robustness. It should be mentioned, however, that feedback — if it is badly realized — can also make the overall system unstable, and it can lead to extremely large inputs, possibly destroying the system.

The final objective of control system design is to find the mathematical operator $\mathcal{C}$ which generates the appropriate actuator signal u. In general, the 'form' of the controller depends on the properties of the plant. In many cases, a model of the plant is used, which describes the interrelationship of the inputs to the system and its outputs. Such a controller is called a model

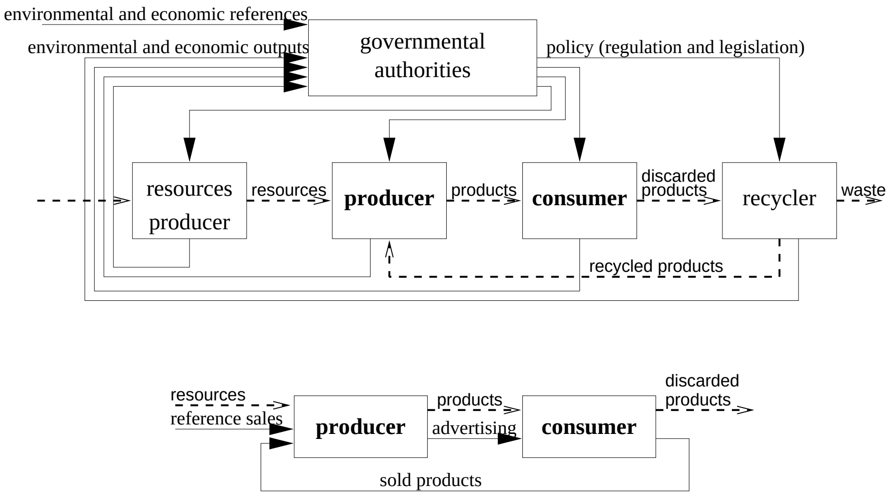


Figure 4.3: *Two different control loops in a product chain. Dashed lines represent product flows, and solid lines represent flows of information (signals). The governmental authorities act as a global controller of the product chain. The producer inside the chain tries to control the consumer by choosing his advertising strategy.*

based controller. In feedback systems, controllers based on a black-box model of the plant are often used, too. The most famous example is the well-proven Proportional Integral Differential (PID) Controller.

4.4 Control in Product Chains

The block diagram structure as developed in the previous section is used to represent some control structures in product chains. Since information is exchanged between the chain actors, on the one hand, and between the chain actors and governmental authorities, on the other hand, a large amount of signals (or information flows) occurs in these chains. Many different control structures are present on different hierarchical levels [11].

Figure 4.3 shows some of the feedback control loops. In this figure, the solid arrows represent signals and the dashed arrows represent the product flows. The authorities act as a global controller, which influences the individual actors in the chain by regulation and legislation. In an ideal case, the actual environmental and economic performances of each actor are fed back to the authorities, which determine the policy inputs on the basis of this information in order to achieve the reference values. A control loop *inside* the chain is shown as well: The producer tries to influence the consumer by marketing activities in such a way that his sales correspond to his reference sales. The difference between the actual sales and the reference determines the advertising instruments which are supposed to take the sales to the desired value in an optimal way. Other control loops, for instance the management of a producer, which can be seen as the controller of the production process (a local controller) are present, but not represented here.

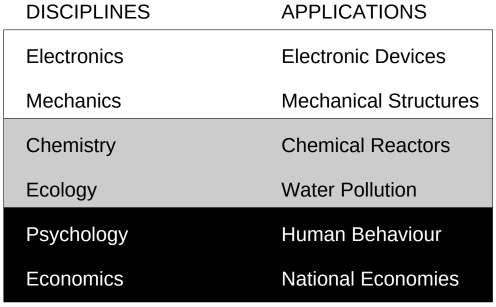
In the sequel, the global control of the entire product chain will be considered. The ultimate control goal of sustainable development can be translated into the objective of bringing under control the environmental damage without impairing the economic interests of the chain. Thus, the variables to be controlled are related to both the environmental damage caused by the chain and its economic performance. Some instruments for influencing the chain (policy inputs) were discussed in Section 3.3, for example economic instruments such as pollution charges and subsidies, regulatory instruments such as pollution standards, and instruments aiming at voluntary participation such as propaganda, education, and negotiation. The fact that this kind of problems actually *is* considered a problem of control also reveals itself in frequently used terms such as *pollution control* and *environmental control.* However, the control is usually not performed by active feedback.

Once the target variables are defined, the desired target *value* or the desired target *path* for the controlled variables must be defined. The question remains how such reference paths are chosen and how they should be chosen. A reference path usually represents a compromise among competing or conflicting goals of the policymakers [9]. In the case of environmental control, the determination of this target is usually the result of a political process, and can be expressed in a reduction of definite emissions or in the prescription of recycling rates to be attained. When the target values or paths are known, the next question is, if the desired values can be attained, and which inputs must or should be chosen. If it is possible to achieve the targets, feedback control can be used to determine how the targets can be achieved in the best way.

4.5 Models of the Plant

When the inputs and outputs of a given system are known, knowledge about its dynamical behaviour must be gathered so that the system can be analysed. This should lead to the translation of the properties of the plant into mathematical formulae that represent the relationship between the inputs and the outputs of the system. These formulae form the basis of a system model.

A general distinction that can be made is between *white-* and *black-box* models. White-box modelling refers to the development of models from the



DISCIPLINES	APPLICATIONS
Electronics	Electronic Devices
Mechanics	Mechanical Structures
Chemistry	Chemical Reactors
Ecology	Water Pollution
Psychology	Human Behaviour
Economics	National Economies

Figure 4.4: *Black and white modelling in different disciplines (see [30])*

knowledge of the basic laws and constitutive equations underlying the system, such as mass, momentum and energy conservation laws in physical systems. Black-box modelling techniques try to estimate a system model from input and output time series, not requiring any prior knowledge of the system inside the box. Identification theory can show if a unique solution to the problem exists and can give an indication of remaining model uncertainties. In the subsequent controller design these uncertainties can be taken into account. *Grey-box* models are in between white- and black-box models, i.e. they represent systems that are partially known.

Since the basic laws of economic theory are still more ambiguous than their physical counterparts, the concept of white-box modelling, which is frequently encountered in physical systems and control engineering, is difficult to apply to economic systems. A first-principles economic model will generally be more 'black' than 'white' (see Figure 4.4). However, to be able to estimate all the parameters in the chosen model structure, the external influences that are used for the estimation need to sufficiently excite the system such that the entire dynamics become perceptible. In the case of product chain systems, it is doubtful whether enough data can be obtained for the considered system. Moreover, it is not possible to carry out experiments on the real product chain system to obtain the data. Furthermore, little insight can be gained from a black-box model [59]. Therefore, a more white-box oriented technique will be used in the following.

The reaction of the output of a system to a particular input is typically reflected by a mathematical model. The output of a system may be instantaneously related to the input, leading to a model in the form of an algebraic equation $y = f(u)$. However, more frequently, the output is coupled dynamically to the input. A large class of dynamical models can be expressed in a standard way, as sets of first-order differential equations as follows (see Figure 4.5):

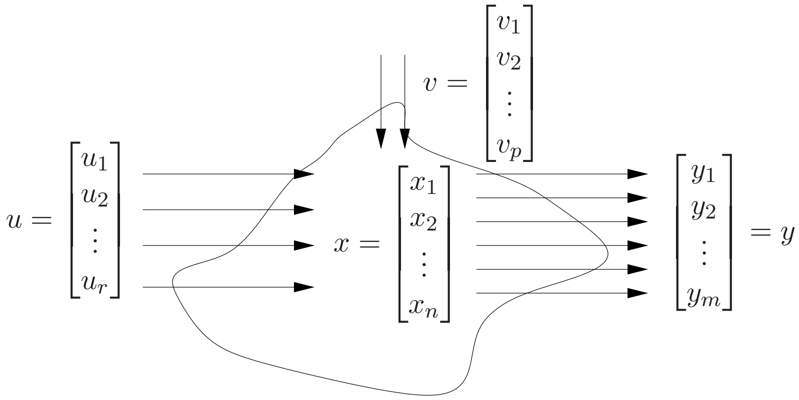


Figure 4.5: *The state space description of a system*

$$\dot{x} = f(x, u, v) \tag{4.1}$$

$$y = g(x, u, v) \tag{4.2}$$

In this state space description, the vector $u = [u_1, u_2, \ldots, u_r]^T$ contains the r input variables, the vector $y = [y_1, y_2, \ldots, y_m]^T$ the m output variables, and $v = [v_1, v_2, \ldots, v_p]^T$ the p disturbance variables. The state vector $x = [x_1, x_2, \ldots, x_n]^T$ is the minimal set of variables required to fully describe the system state[2]. In the state variables, the effects of former inputs are stored. The system dynamics are described by (4.1). The output equation (4.2) depends on the choice of the output the designer is interested in. When the outputs and inputs are defined, the corresponding states are determined.

In a *linear* system, f and g are linear functions of x, u, v. The state space equations then have the form

$$\dot{x} = Ax + Bu + Ev \tag{4.3}$$

$$y = Cx + Du + Fv \tag{4.4}$$

with A, B, C, D, E and F matrices of appropriate dimensions.

The product chain systems that we will consider in the following are *nonlinear*, which means that f and g are nonlinear functions of x, u, v. The theory of nonlinear systems is not yet as developed as the theory of linear systems, for which a variety of powerful analysis methods exists. Although very few systems, in practice, are *exactly* linear, they often are adequately approximated by linear models. The assumption made in these cases is that the system is indeed linearizable for small variations around a nominal trajectory.

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[2] In econometric theory, the state variables are usually called *endogenous* variables and the input variables *exogenous* variables.

4.6 Properties of a System

For a model of the system in the state space form (4.1,4.2), standard methods for analysing it can be used. For instance, the stability of the system can be examined. Other important properties of dynamic systems are controllability and observability. These properties of a system determine the principal possibilities of the input to influence the system state (controllability) and to deduce the system state from the measured output (observability).

4.6.1 Stability

A system is said to be *stable*, if the state evolves into a certain constant value for a situation where all inputs are zero. Stability of the closed loop system is always an absolute requirement since its absence causes signals to grow without bound, eventually destroying and breaking down the system. This is what happens, for example, when a nuclear reactor heats up uncontrollably and melts down. In many practical applications, the open loop plant is unstable, and the task of feedback control is to stabilize the system. Comprehensive knowledge about stability is available for linear systems as well as for nonlinear systems, and about the possibilities of influencing it by control.

Within the framework of product chain analysis, an unstable chain means, for example, that the flows keep growing unboundedly. An infinite increase of the flows would mean that the chain would collapse, which would be an undesired situation. The chain actors are interested in a stable chain, because this guarantees the predictability and the continuity of the chain in the future.

4.6.2 Controllability

A system is said to be *controllable* if, for a given initial situation, each value of the state x can be reached at a chosen time T by a proper choice of the input u. When the input variables are given, controllability is a property of the system and cannot be influenced. It is determined by its internal structure and by the way in which the inputs can influence the states.

Based on this concept, various other related concepts play a role: Often it is, for example, not necessary to be able to control the entire state, but it is sufficient to aim at the controllability with respect to the chosen output. The same value of the output variables may be achieved by various different values of the state variables.

In the theory of policy making a stricter kind of controllability is of special interest, the *perfect output controllability* (*target path controllability*) [150]. A system having this property is able to make the output vector follow any prescribed *trajectory* over a definite time interval[3]. The difference to the former kind of controllability lies in the fact that in case of simple controllability the system is in general not capable to guide the output vector along any arbitrary trajectory. It is only guaranteed that the system passes through the desired target point; it will not necessarily stay there if this point is not an equilibrium point. Thus, if the policy makers desire perfect control outside the steady state, target path controllability is necessary [8, 103, 116].

Having defined the principle controllability of the system, we have to realize that in practice the effort needed to reach the targets is also important. A system may be controllable in principle but the inputs to achieve a desired objective may be very large or may require a large amount of energy. The aspect of practical control energy needed to reach certain targets is addressed by the controllability Gramian [110].

Since controllability is concerned with the operation of the policy inputs on the states, it is desired to have a product chain model with this characteristic. In the case of an uncontrollable chain, it may be desirable to find new policy inputs, which make the chain controllable.

4.6.3 Observability

Another standard problem of control theory is observability. A system is said to be *observable* if the system state can be uniquely deduced from the knowledge of the input and the output at all times. The concept of observability determines if the sensors are able to capture the entire dynamic behaviour of the system. It refers to the possibility of determining the current state from observation data.

A close connection exists between controllability and observability concerning theory and tests. Observability is to sensors what controllability is to actuators. Observability is determined by the internal structure of the system and the way the sensors couple into the states.

The question to be studied is not, as for controllability, whether the policy instruments can be adjusted intertemporally to bring about an arbitrary change in the state; but rather, given that a particular intertemporal output change has occurred, is there a *unique* intertemporal state change responsible for this change in the output? The failure of the observability property has, for example, the consequence that unobservable unstable modes of the system cannot be detected.

Before observing product chains, the sensors must be defined. In a product chain, the quantities to be observed are, for example, the material and product

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[3] Perfect output controllability is also called *output functional reproducibility* or *functional controllability* in the control literature [36].

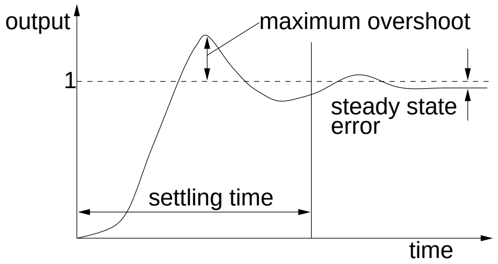

Figure 4.6: *Some system response specifications with respect to a unit step response: the steady state error, the settling time and the maximum overshoot*

flows and the prices in the chain. When the sensors are defined, observability with the help of these sensors can be analysed. The presence of unobservable unstable modes indicates that additional sensors must be placed to make the state observable.

4.7 System Performance

Before we can make the system behave in a proper way, we have to formulate its desired properties. One absolute requirement, for example, which must be satisfied before all other considerations, is internal stability. Furthermore, robustness concerning modelling errors must also be assured. This means that the system has to remain internally stable even in the presence of these errors. Once these requirements are met, attention can be focused on the system response. Usually an accurate and rapid achievement of y_d with minimal variations around the reference is desired. These properties can usually not be satisfied for all y_d and for all disturbances and measurement noise. Therefore, the response specifications are generally given with respect to specific signals, for example, to a step input. The specifications are then given in terms of, among others (see Figure 4.6)

- Steady state error: the difference between the desired final value and the actual final value
- Settling time: the time taken until the output falls and remains within, for example ±5% of the steady state value, and
- Maximum overshoot: the maximum value of the response as a percentage of the final value

The aim of an *optimal* policy of operation may be expressed in a 'performance criterion' which is a scalar measure of performance. By definition, the 'optimal policy' is the input for which the criterion is either maximal or minimal. The performance index J may be expressed as follows:

$$J = k_T(y(T), u(T)) + \int_0^T k(y(t), u(t))dt \tag{4.5}$$

where $k(y, u)$ is the so-called cost function, which has to be chosen by the designer. k_T is a different function, which represents the costs at the end of the optimization horizon T. In general, one is interested in a rapid achievement of the desired output. This, however, usually means that large inputs must be generated, which in turn is often expensive. Therefore, the integral criterion is often chosen so that the accumulated deviation of the control result y from the nominal result y_d is weighted against the accumulated deviation of the control effort u from the nominal effort u_d. Then k is a function of the differences $y - y_d$ and $u - u_d$ [4].

The performance index is a *scalar*[4], and the actual value of J is usually not meaningful; control system performance is really too complex and multifaceted to be expressed in a single number. We cast the control problem as an optimization problem not because we particularly wish to minimize J, but because the optimal solution is mathematically tractable, and has desirable properties. In order to achieve this scalar, some trade-off between the magnitude of the inputs and the rapid achievement of the desired output is performed. In the case of multiple inputs or outputs, the relative importance of these inputs or outputs must be chosen as well. These weighting procedures introduce subjectivity into the problem.

As far as the optimization horizon T is concerned, the case $T \to \infty$ is important in many cases in which the achievement of a constant output is desired, and it usually simplifies the search for a solution to the problem.

In the case of environmental control, the performance criterion includes the economic and environmental performances and the various policy inputs. Measures for the economic performance are the profit or the costs caused by the chain actors. An indication of the environmental damage may be the recycling rate or the total energy consumption. The establishment of proper measurement standards and the development of economic and environmental standard figures is a subject of ongoing research [80]. One has, for example, to decide what is preferable: a lower energy requirement or a lower ozone layer depletion. Here characteristic figures representing the weight of the environmental problems can be used, as discussed in the framework of life-cycle

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[4] For the sake of completeness, we mention here that multiobjective criteria can be treated by considering pareto-optimality [56]. However, this kind of problem is beyond the scope of this discussion, and will not be further considered here.

analysis in Section 2.4. The policy inputs can be weighted by comparing the costs of the different measures.

4.8 Constraints

The various concepts discussed above do not take into account constraints on the input and state vectors. However, in reality these constraints do exist and must be taken into consideration.

Hard constraints (such as saturation of the control input) are the most commonly encountered constraints and occur due to actuator saturation and the like. In the product chain, an example of a constrained input is a tax on a product. If government imposes a tax, this must be greater or equal to zero. So-called *soft constraints* (constraints on the outputs), on the other hand, are often imposed by safety or performance necessities. For some environmental effects, such as toxicity, definite output levels must not be exceeded because they cause irreversible effects that cannot be undone by lower levels at other times.

Control methods exist, which are able to handle both input and output constraints by incorporating them into the optimization process. One such method, which is specially designed for explicitly dealing with constraints, is model predictive control, also known as receding horizon or moving control.

4.9 Summary

This chapter introduces some basic topics of control theory that are relevant to the analysis of product chains. The information flows inside the chain, and between the chain and a regulating authority can be represented by the well-known block diagrams from control theory. For gaining insight into the structure of the chain, a mathematical model in state space form is useful because, for such models, comprehensive knowledge about determining stability, controllability, and observability exists. The determination of these properties helps in the design of a controller for product chains.

Chapter 5

Company and Market Modelling

5.1 Introduction

In the previous chapters, product chains and their environment were introduced, and some background was provided to control theoretical methods for modelling and analysing such dynamic systems. In this chapter, we illustrate how these methods can be used for modelling the companies in a chain and the markets between them. In the following chapters, the company and market models will be combined to a chain model that will subsequently be analysed by the methods presented in Chapter 4.

Earlier studies on product chains were often restricted to descriptive ones [13, 139], or, in a few cases, made use of mathematical programming techniques, for example involving technology choice, or the optimal design of production and distribution locations [27, 74, 151]. These studies were usually based on static and dynamic linear programming and related optimizing techniques. Strangely enough, the methods available in systems and control theory are rarely encountered in studies on the meso-, i.e. product chain, level. In economics, the main applications of systems and control theory concern the optimal control of macroeconomic systems [3, 5, 9, 40, 107, 113]. Optimal control is also used in some problems related to logistic systems [7, 12, 33, 34, 72]. Another application can be found in the dynamic investment policy of a firm [28, 44, 73, 145].

In Sections 5.2 and 5.3, general company and market models are presented. In Section 5.4, the models are illustrated by two examples of company market combinations. Some possibilities of model analysis are shown. In Section 5.5, options to extend the company model are mentioned. Finally, the main con-

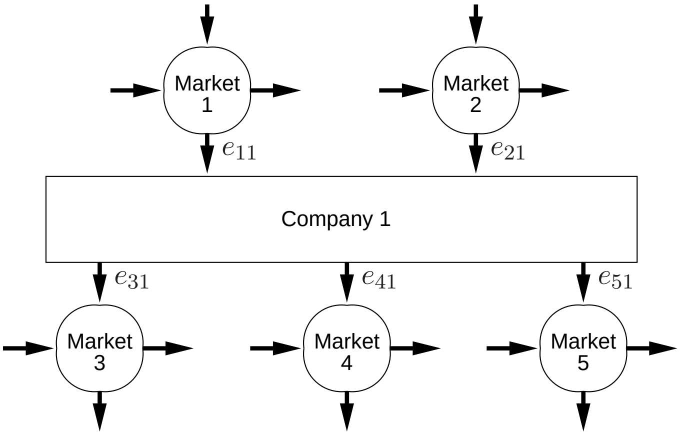

Figure 5.1: *The flowchart of a generalized company representing the material flows. The information flows are* not *depicted here. The product flow between Market i and Company j is denoted by e_{ij}.*

clusions of this chapter are drawn.

5.2 The Company Model

The building blocks of a product chain are the chain actors: the various companies involved and the consumers of finished products. These actors mirror the transformation processes of materials. Although each process (resource extraction, production, etc.) in the product chain may be executed by several different companies, these are often combined to a single aggregated 'company' (as *pars pro toto*). A process-product chain is then considered as a network of basic units, each represented by a generalized company model.

Figure 5.1 shows a flowchart of a generalized company. The company uses two kinds of resources, which it buys on Markets 1 and 2, and produces three kinds of products, which it sells on Markets 3,4 and 5. The resource inflows consist of energy, raw materials, semi manufactured products, etc. The outflows consist of products and wastes. The resource and product markets constitute the link to other companies in the chain.

In the following, it will be assumed for the sake of simplicity that one main resource and one main product are present, as represented in Figure 5.2. The description focuses on these two markets because it is assumed that the company's decision is based on these markets, although others may be present, too. Obviously, the number of markets to be considered depends on the de-

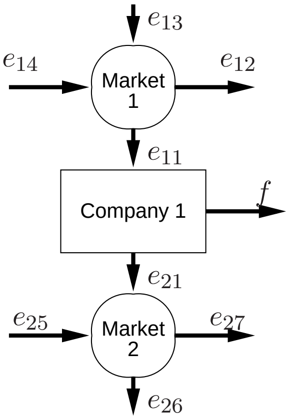

Figure 5.2: *The flowchart of a company. The resource and product flows are denoted by* e_{ij}*, and the waste flow is denoted by* f *and considered as a loss.*

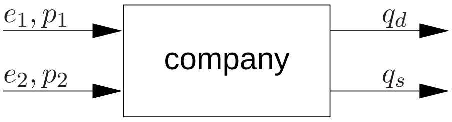

Figure 5.3: *The inputs and outputs of the company system*

sired level of aggregation. Instead of considering the copper, nickel and cobalt market of a mineral extractor, for example, the metal market can be considered on a higher level of aggregation [97]. The extension to the case of multiple resources and products is straightforward. It will briefly be discussed in Section 5.5.

The corresponding signals, which link the company to the markets, are depicted in Figure 5.3[1]. The company receives information about the amount e_1 of resources that it obtains from the resource market (Market 1) and about their price p_1. Moreover, the price of finished products (p_2) and the amount of products traded on the product market (e_2) influence the company's decisions. On the basis of this knowledge, the company determines the amount of resources it wants to purchase on the resource market — its demand q_d — and the amount of finished products it is willing to sell — its supply q_s.

The markets will be discussed in detail in Section 5.3. Here we only mention that a market is considered a virtual place where supply and demand of a

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[1] In order to prevent confusion between diagrams representing material flows and diagrams representing signals, the latter are drawn horizontally with thin arrows whereas the first are drawn vertically with thick arrows.

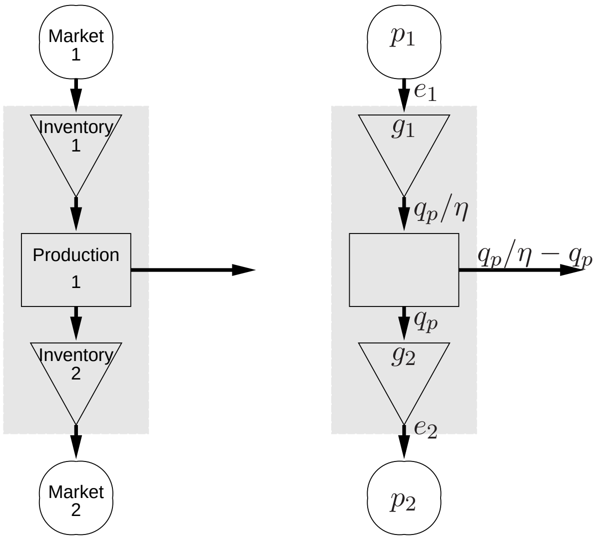

Figure 5.4: *The product flows in a company with two inventories. The left figure shows the transformation processes and the product flows. The right figure shows the variables corresponding to the processes and product flows.*

product meet each other. On the market, the product price is determined as well as the quantities each supplier can sell and the quantities each demander can purchase. Note that due to possible differences in the amount of products supplied to a market and the amount of products demanded on the market, the amount of traded products is the minimum of supply and demand (see also Section 5.3 and Figure 5.5). A supplier cannot sell more than he supplies and a demander will not purchase more than he needs.

The physical basis of the company model that is derived here, is the general setup of Figure 5.4. The company is characterized by two inventories (for resources and finished products) and a production process. The inventories are represented by triangles, and the production process by a rectangle. Inventories can be seen as a stabilizing factor. They are considered as something that a cost-minimizing firm can use to smooth production in view of fluctuating sales. Due to inventories, production does not need to respond fully to changes in sales. Rather, inventories should rise and fall to buffer production from the vagaries of demand. Moreover, inventories can be kept, for example, as unavoidable 'pipeline' inventories, to improve production scheduling, to minimize stockout costs, to speculate on or hedge against price movements, to reduce purchasing costs by buying in quantity and to shorten delivery lags [26, 123]. Note that, in the limiting case, one or both inventory levels in Figure 5.4 may constantly be zero. The decision to hold inventories or not depends on the kind of product the company produces. It is a consequence of its production decision system (production to stock, production to order, etc.) [24, 89, 149]. If both inventories are zero, for example, a 'Just In Time' system is represented.

We now consider the coupling between production and inventories in more detail. In order to simplify notation, the resource inventory is denoted as Inventory 1, the resource market as Market 1 and, correspondingly, the product inventory and market are denoted as Inventory 2 and Market 2. Abiding by the law of mass conservation, the in- and outflows of mass to the inventories need to balance their accumulation. The mass-balance applied to the second inventory, for example, states that the accumulation of products in stock ($\dot{g}_2$ in kg/year, g_2 in kg) equals the inflow into the inventory (the production q_p in kg/year) minus the outflow, which equals the flow of products sold on Market 2 (e_2 in kg/year)

$$\dot{g}_2 = q_p - e_2 \tag{5.1}$$

Accordingly, the accumulation in Inventory 1 equals the flow of purchased resources e_1 minus the amount of resources needed for production. This is given by q_p/η, if we take the production efficiency η (in kg product/kg resource) into account. We obtain the balance equation for Inventory 1

$$\eta\dot{g}_1 = \eta e_1 - q_p \tag{5.2}$$

In the model developed here, the state variables of a company are the two inventory levels g_1 and g_2. The company state vector consists of these two inventory levels:

$$x = \begin{bmatrix} \eta g_1 \\ g_2 \end{bmatrix} \tag{5.3}$$

In order to express the company model in the standard state space representation (4.1,4.2), the time-derivative of the state vector is expressed as a function of the state variables and the input variables. The input variables are the prices p_1, p_2 (in €/kg) and the trades e_1, e_2, as shown in Figure 5.3:

$$u = [u_1\ u_2\ u_3\ u_4]^T = [p_1\ p_2\ e_1\ e_2]^T \tag{5.4}$$

The main decision in a company, its production decision, must be included in the model. The production decision is translated into a mathematical formula by expressing the produced quantity of products q_p as a function of the company state x, and its input u

$$q_p = q_p(x, u) \tag{5.5}$$

Assuming that production depends linearly on the product and on the resource prices as well as on the inventories it can be expressed by

$$q_p = c_q - \alpha_1 p_1 + \alpha_2 p_2 + \eta\lambda_1 g_1 - \lambda_2 g_2 \tag{5.6}$$

where c_q is the production constant and the constants $\alpha_1, \alpha_2, \lambda_1, \lambda_2$ are chosen such that a positive value corresponds to a 'normal' business strategy. In most cases, λ_1 will be zero because decisions on production virtually do not depend on the resource stock.

The company can then be described by the state equation (4.1)

$$\dot{x} = Ax + B_1 u_1 + B_2 u_2 + B_3 u_3 + B_4 u_4 + Hc_q \tag{5.7}$$

with the two-dimensional state vector x (5.3), the four-dimensional input vector u (5.4), and with[2]

$$A = \begin{bmatrix} -\lambda_1 & \lambda_2 \\ \lambda_1 & -\lambda_2 \end{bmatrix}, \; B_1 = \begin{bmatrix} \alpha_1 \\ -\alpha_1 \end{bmatrix}, \; B_2 = \begin{bmatrix} -\alpha_2 \\ \alpha_2 \end{bmatrix}, \; B_3 = \begin{bmatrix} \eta \\ 0 \end{bmatrix}, \; B_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$H = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The company's output vector consists of its demand for resources and its supply of finished products, as shown in Figure 5.3

$$y = \begin{bmatrix} q_d \\ q_s \end{bmatrix} \tag{5.8}$$

The decisions on demand and supply depend on the production as well as on the resource and product stocks. The company is willing to purchase the quantity of resources q_d (in kg/year) on Market 1. A linear demand function is assumed as is common in microeconomic theory [104, 143]

$$\eta q_d = c_1 - \eta\beta_1 p_1 - \eta\mu_1 g_1 \tag{5.9}$$

with the constants c_1, β_1 and μ_1. Abstracting from the inventory g_1 this is a standard demand function of a firm. The supply of finished products q_s (in kg/year) on Market 2 is, in general, a function of the stock of finished products g_2 and the price p_2 the company can obtain on Market 2. In a linear setting, it has the form

$$q_s = c_2 + \beta_2 p_2 + \mu_2 g_2 \tag{5.10}$$

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[2] Note the constraints that the demand, supply and product stock are always greater than or equal to zero.

with the constants c_2, β_2 and μ_2. The output equations (4.2) are then given by

$$\begin{aligned} y_1 &= q_d = C_1 x + D_1 u_1 + c_1 \\ y_2 &= q_s = C_2 x + D_2 u_2 + c_2 \end{aligned} \tag{5.11}$$

with

$$C_1 = \begin{bmatrix} -\mu_1/\eta & 0 \end{bmatrix},\ C_2 = \begin{bmatrix} 0 & \mu_2 \end{bmatrix},\ D_1 = -\beta_1,\ D_2 = \beta_2 \tag{5.12}$$

The equations (5.7) and (5.11) constitute the state space model of the company in the form (4.1,4.2).

In the above equations, c_1, c_2 and c_q are not necessarily constant in time. They may be time-varying functions. Parameters (which are constant in time) are denoted by Greek letters; functions and variables are denoted by Roman letters.

The values of α_i in (5.6) and of β_i in (5.9,5.10) reflect the company's business decision. If $\beta_1 \gg \alpha_1$, for example, the demand will vary heavily with price fluctuations, and the production will remain relatively constant. In general, a large resource inventory will be necessary in this case to avoid an empty inventory. On the other hand, if $\alpha_1 \gg \beta_1$, production fluctuations will be large, but the resource inventory will fluctuate less so that it can be kept smaller.

5.3 The Market Model

In economic science, markets are crucial. They are considered as virtual places in which a trade-off between demand and supply takes place, and they are governed by a price mechanism. The description of markets has received much attention [71, 127]. The development of quantitative models, however, has proven to be difficult [10, 120, 128]. The markets that we consider here are so-called competitive markets. In a perfectly competitive market suppliers as well as demanders take the price as given. The price adjustment is not limited, for example, because of a monopoly situation.

In standard economic theory, a demand curve is assumed that decreases with increasing price, and a supply curve that increases with increasing price, as depicted in Figure 5.5. The intersection of these two curves is the market equilibrium. The shape of the curves is a result of different mechanisms (substitution, change of real income) that cannot be analysed individually. As an extra complication, the dynamics of the system play a role. Change in supply is often not instantaneously possible, but requires preparations, such as investments in capital products, or the recruitment of appropriately skilled labour

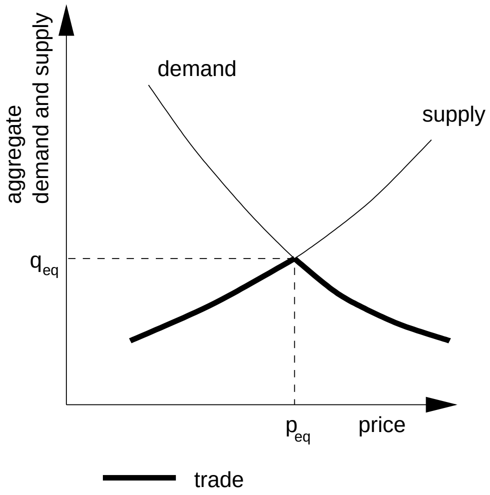

Figure 5.5: *Supply and demand on a market. The intersection of the supply and the demand curve is the market equilibrium. The thick line is the trade.*

force. On the other hand, in the case that demand and supply are in disequilibrium on a market, the price adjustment will not occur instantaneously but rather in a dynamic way. Two theories about adjusting the market variables will be discussed here: The *Marshallian* theory of quantity adjustment and the *Walrasian* theory of price adjustment [2, 20, 66].

5.3.1 Marshallian quantity adjustment

The Marshallian theory of *quantity* adjustment is based on two price functions: the price $p_d(q)$ (in €/kg) is the price that demanders are willing to pay, and $p_s(q)$ is the price that sellers are charging for a given quantity q (in kg/year) of products. The dynamic adjustment of the traded quantity is then described by

$$\dot{q} = \rho_M \big(p_d(q) - p_s(q) \big) \tag{5.13}$$

where $\rho_M > 0$ (in kg^2/(€year^2)) is the speed of adjustment of the market. This reflects the fact that if $p_d(q) > p_s(q)$, the suppliers can profitably increase the quantity supplied and if $p_d(q) < p_s(q)$, the traded quantity will decrease.

5.3.2 Walrasian price adjustment

The *price* adjustment as originally proposed by Walras is based on the simple statement that the time derivative of the market price equals some adjustment parameter ρ_W (in €/kg^2) times the excess demand [50]. The totals of market

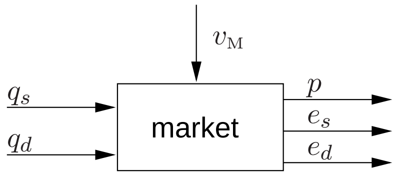

Figure 5.6: *The signals entering and leaving the Walrasian market*

demand q_d and supply q_s are obtained by summing the supply and demand of the individual actors

$$\dot{p} = \rho_W \big(q_d - q_s\big) \tag{5.14}$$

The Walrasian price adjustment assumes that neither demanders nor suppliers can affect the price on a market but they take it as given. This is the premise of a perfectly competitive market. The second premise is that the price is the *only* adjusting parameter of the market. Demanders and suppliers, respectively, adjust the quantities they wish to buy and supply, only on the basis of the information on the market price.

The Walrasian and Marshallian adjustment mechanisms obviously do not coincide. The Marshallian mechanism is characteristic of the theory of production, and the Walrasian mechanism is characteristic of the theory of exchange [115, 136]. The difference can be illustrated by the idealized example of the market for apples. When apples are harvested in the fall, the total amount of apples a producer supplies is fixed. The market is characterized as an exchange market and can be described by the Walrasian mechanism. Once an equilibrium price is determined, the producers determine the quantity of apples they will produce the next year. This can be described by the Marshallian quantity adjustment. Here the emphasis lies on the production. Since in our description of the market, exchange is crucial, the Walrasian description is adopted here.

The signals entering and leaving the Walrasian market are depicted in Figure 5.6. The inputs are the supplied and demanded quantities, and the outputs are the price as well as the trade of the demanders and suppliers. In the model that is used here, supply originates from both inside and outside the system boundaries. Similarly, demand is the sum of demand from inside and outside the system. Trade from and to the actors outside the considered system constitutes a disturbance of the market and is denoted by v_{M}. We use positive values of v_{M} to express imports to the system, and negative values of v_{M} to express exports. The price adjustment can then be approximated by

$$\dot{p} = \rho_W (q_d - q_s - v_{\mathrm{M}}) \tag{5.15}$$

This is the state equation of the Walrasian market with the state vector $x = p$, the input vector $u = [q_d \; q_s]^T$ and the disturbance vector $v = v_{\mathrm{M}}$.

5.3.3 Trade: the exchange of products

It is obvious that demand and supply do not always coincide. The real product flow, or trade e, is given by the minimum of demand and supply. A supplier cannot sell more than he supplies and a demander will not purchase more than he needs. If we take imports and exports into account, the total trade on the market is described by

$$\begin{aligned} e &= \begin{cases} \min(q_s + v_{\mathrm{M}}, q_d) & \text{if } v_{\mathrm{M}} \geq 0 \\ \min(q_s, q_d - v_{\mathrm{M}}) & \text{if } v_{\mathrm{M}} < 0 \end{cases} && (5.16) \\ &= \min(q_s + v_{\mathrm{M}}, q_d) - \min(0, v_{\mathrm{M}}) && (5.17) \end{aligned}$$

For calculating the quantity e_s traded by the supplier *inside* the considered system, it must be noted that in the case of imports to the system ($v_{\mathrm{M}} \geq 0$) only part of the trade is for the account of the supplier inside the system. The amount v_{M} is supplied from outside. The trade from the supplier inside the system is given by[3]

$$\begin{aligned} e_s &= \begin{cases} \min(q_s + v_{\mathrm{M}}, q_d) - v_{\mathrm{M}} & \text{if } v_{\mathrm{M}} \geq 0 \\ \min(q_s, q_d - v_{\mathrm{M}}) & \text{if } v_{\mathrm{M}} < 0 \end{cases} && (5.18) \\ &= \min(q_s, q_d - v_{\mathrm{M}}) && (5.19) \end{aligned}$$

Accordingly, the demander inside the system trades

$$\begin{aligned} e_d &= \min(q_s + v_{\mathrm{M}}, q_d) && (5.20) \\ &= e_s + v_{\mathrm{M}} && (5.21) \end{aligned}$$

5.4 Two Examples

In this section, we illustrate how the company and market models can be combined to describe a company together with its resource or product markets.

5.4.1 A company in ideal circumstances

As an example, we study a company that receives the resources it wants to purchase on the resource market and that is able to sell all the products it

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[3] Note that v_{M} is the *trade* of external actors and not their demand or supply.

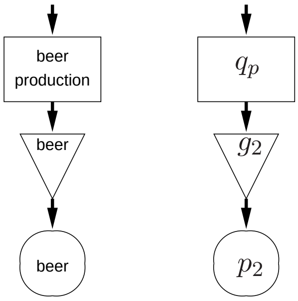

Figure 5.7: *The mass flows of the company market system of the beer producer Thirst*

supplies. For this company operating in ideal circumstances, we have $u_3 = e_1 = q_d$ and $u_4 = e_2 = q_s$. With (5.11), the state equation (5.7) can be written as

$$\dot{x} = A_s x + B_s u + H_s c_s \tag{5.22}$$

with the state vector (5.3) $x = [\eta g_1 \; g_2]^T$, the input vector $u = [p_1 \; p_2]^T$, and $c_s = [c_q \; c_1 \; c_2]^T$. The matrices A_s, B_s and H_s are given by

$$A_s = A + B_3 C_1 + B_4 C_2 = \begin{bmatrix} -\lambda_1 - \mu_1 & \lambda_2 \\ \lambda_1 & -\lambda_2 - \mu_2 \end{bmatrix} \tag{5.23}$$

$$B_s = \begin{bmatrix} B_1 + B_3 D_1 & B_2 + B_4 D_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 - \eta\beta_1 & -\alpha_2 \\ -\alpha_1 & \alpha_2 - \beta_2 \end{bmatrix} \tag{5.24}$$

$$H_s = \begin{bmatrix} H & B_3 & B_4 \end{bmatrix} = \begin{bmatrix} -1 & \eta & 0 \\ 1 & 0 & -1 \end{bmatrix} \tag{5.25}$$

This system constitutes a linear system (see (4.3,4.4)), and can therefore be analysed by the methods of standard linear systems theory. Stability of the system can, for example, be determined by examining the eigenvalues of the system matrix A_s, as discussed in Appendix C.1. We consider the case $\lambda_1 = 0$. Then the eigenvalues of A_s are given by

$$-\lambda_2 - \mu_2, \; -\mu_1 \tag{5.26}$$

Since, in a realistic business strategy, all the parameters are positive, the eigenvalues are always negative. According to linear systems theory, this means that the system is stable, see Appendix C.1.

5.4.2 The company market model of the beer producer Thirst

The second example discussed here is the model of a typical chain element: a company with its product market as depicted in Figure 5.7. First we derive the

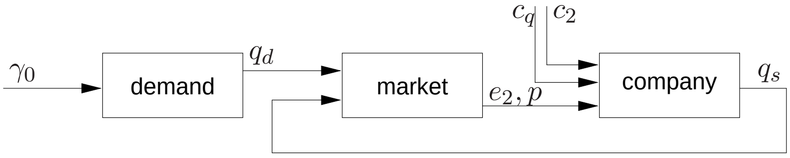

Figure 5.8: *Thirst: the inputs and outputs*

model corresponding to this element. Then we present some results concerning its stability, and we investigate the influence of external changes on the system behaviour. Consider, for example, a beer producing company, which we call Thirst here. Thirst produces to stock and experiences the price dependent external demand $q_d = \gamma_0 - \gamma_1 p$. Thirst can be described by the differential equations, see (5.1,5.6,5.17) and (5.10,5.14)

$$\dot{g}_2 = q_p - e_2 \quad = c_q + \alpha_2 p - \lambda_2 g_2 - \min(\gamma_0 - \gamma_1 p, c_2 + \beta_2 p + \mu_2 g_2) \tag{5.27}$$

$$\dot{p} = \rho(q_d - q_s) = \rho(\gamma_0 - \gamma_1 p - c_2 - \beta_2 p - \mu_2 g_2) \tag{5.28}$$

with the parameter values

parameter	μ_2=2, λ_2=1	α_2=0.1, β_2=0.2	γ_0=0.15, c_q=0.1, c_2=-0.3
unit	1/year	10^9kg^2/(year€)	10^9kg/year

parameter	γ_1=0.05	ρ=1
unit	10^9kg/(year€)	10^{-9}€/kg^2

With the definition of the state vector x as consisting of the beer inventory and the beer price, and the definition of the input vector u as consisting of the production constant c_q, the supply constant c_2 and the autonomous demand γ_0 (see Figure 5.8):

$$x = [g_2 \; p]^T \qquad\qquad u = [c_q \; c_2 \; \gamma_0]^T \tag{5.29}$$

the model can be written in the state space form (4.1,4.2) $\dot{x} = f_T(x, u)$. The definition of the abovementioned inputs u allows the analysis of a change in these inputs later.

The line $\gamma_0 - \gamma_1 p = c_2 + \beta_2 p + \mu_2 g_2$ splits the system into the two linear systems

$$\dot{x} = A_i x + B_i u \tag{5.30}$$

with the matrices A_i and B_i

$$A_1 = \begin{bmatrix} -\lambda_2 & \alpha_2 + \gamma_1 \\ -\rho\mu_2 & -\rho(\gamma_1 + \beta_2) \end{bmatrix} \qquad B_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -\rho & \rho \end{bmatrix} \tag{5.31}$$

$$A_2 = \begin{bmatrix} -\mu_2 - \lambda_2 & \alpha_2 - \beta_2 \\ -\rho\mu_2 & -\rho(\gamma_1 + \beta_2) \end{bmatrix} \qquad B_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -\rho & \rho \end{bmatrix} \tag{5.32}$$

If the beer supply is larger than the demand ($c_2 + \beta_2 p + \mu_2 g_2 \geq \gamma_0 - \gamma_1 p$), the system behaves as system (A_1, B_1). Otherwise, if the beer supply is smaller than the demand ($c_2 + \beta_2 p + \mu_2 g_2 < \gamma_0 - \gamma_1 p$), the system behaves as system (A_2, B_2).

First we investigate the stability of this system. The interest of stability analysis is to find out if the price or the inventory can grow without bounds. For an indication on stability, we consider the eigenvalues of the system matrices A_1 and A_2 as it is usual in linear systems analysis. Due to the switching between the two models, stability of the two individual systems does not necessarily mean stability of the switched system as will be discussed in detail in Section 7.3.2. However, an indication of the system behaviour may be found for this example. A linear system is stable if and only if the eigenvalues of the system matrix have a negative real part, see Appendix C.1. In the case of Thirst, the eigenvalues of A_1 are $\lambda_{1,2} \approx -0.63 \pm i0.40$ and those of A_2 are $\lambda_{3,4} \approx -3.1, -0.18$.

When it is known that a system is stable, the question remains whether it always evolves into one equilibrium situation[4] or whether there are several equilibria and the system evolves into one of them depending on the initial situation. A simple example of a system with two stable equilibria is a bipolar multivibrator (flip flop). The equilibrium points of a given system can easily be calculated. Since an equilibrium means that the state does not change, it is given by $\dot{x} = 0$. The equilibrium points are thus given by the solutions of $f_T(x, u) = 0$. In the case of Thirst, there is only one equilibrium point

$$x_{\text{ss}} = -A_1^{-1} B_1 u = -A_2^{-1} B_2 u = [10^8 \; 1]^T \tag{5.33}$$

The behaviour of Thirst that is described above can be illustrated by simulations of the system behaviour. Figure 5.9a shows the evolution of the two state variables for a given initial situation $x_0 = [0.9\,10^8 \;\; 1.05]^T$ as a function of time. The price falls towards the equilibrium price; the inventory first rises and, after having reached its maximum value at $t \approx 2$ years, evolves towards its equilibrium value. Figure 5.9b shows the results of the same simulation once again in another kind of plot. Here the plane having the two state variables as coordinates (the so-called phase plane) is drawn. With t varied from zero to infinity, the solution $x(t)$ of system (5.27,5.28) is represented as a curve, the so-called phase trajectory. The time corresponding to the points along the curve is written as a parameter. The arrows, the isoclines, show the direction of the derivative of the trajectory in each point, thus indicating the direction in which the system develops. The power of this kind of phase-plot lies in the

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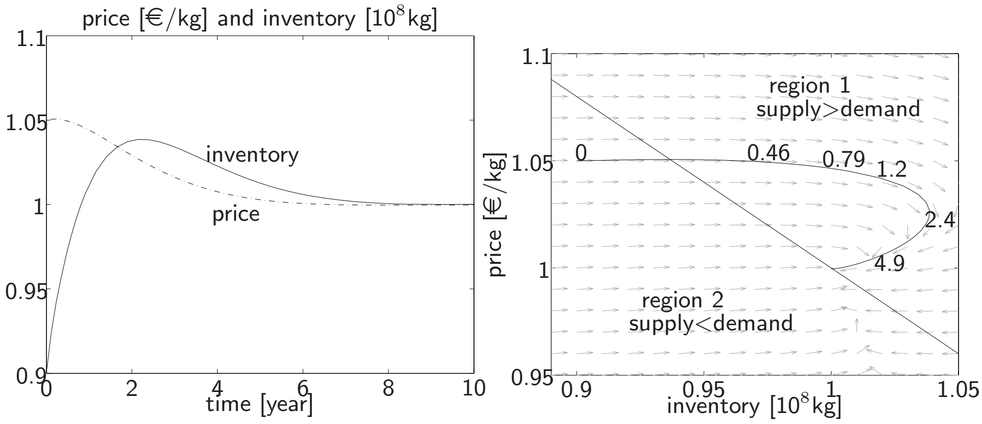
[4] A point x^* is called an equilibrium point, if it has the property that whenever the state of the isolated system starts at x^* it will remain at x^* for all future time.



(a) (b)

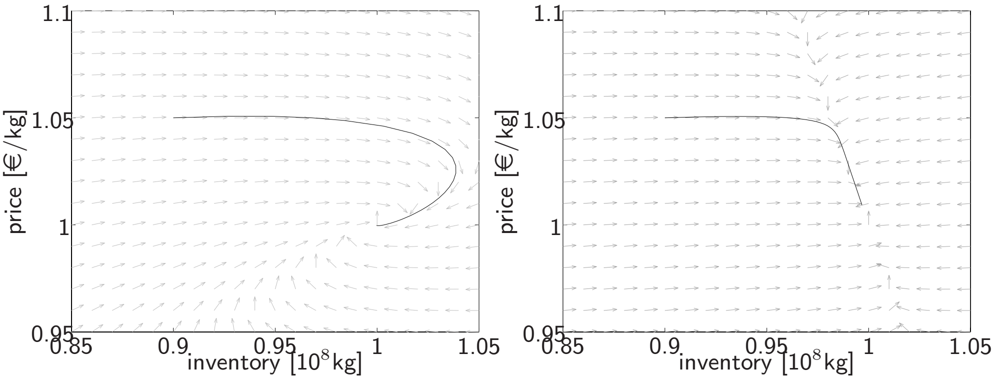
Figure 5.9: *The company market model: evolution in time of the price and the inventory (a), and phaseplot (b)*

fact that the nature of the system evolution corresponding to various initial conditions is directly displayed. It can be seen, for example, that the system of Thirst starts in region 2, passes to region 1 and will end up in the equilibrium state on the switching line. The trajectory corresponding to other initial conditions can be found by following the arrows.

Figure 5.10 shows the evolution of the two individual systems (A_1, B_1) and (A_2, B_2) in the phase plane. The model of region 1 has a so-called stable focus as can be seen in Figure 5.10a. This corresponds to the two negative complex conjugated eigenvalues $\lambda_{1,2}$. The system of region 2 has a stable node, which corresponds to the two real negative eigenvalues of Thirst. The total system is a combination of the two systems.

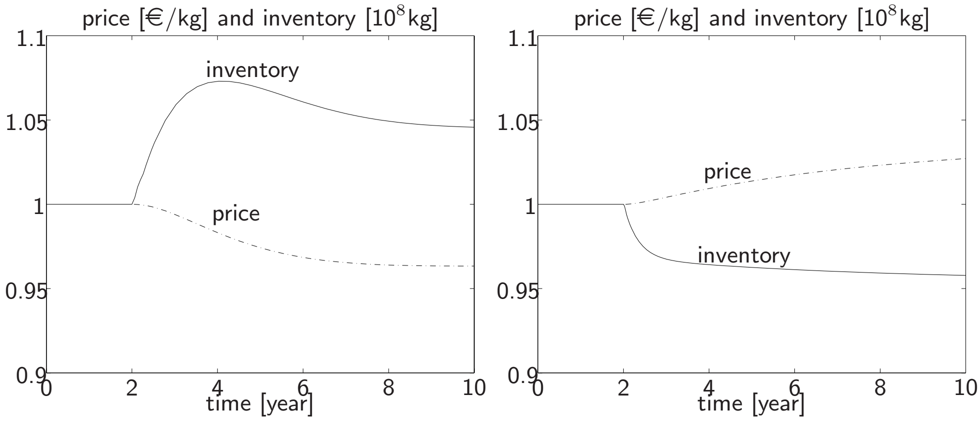
The simulations carried out so far concern the evolution of Thirst from a given initial situation into the equilibrium. Now we discuss several illustrative situations in which Thirst starts in its equilibrium and in which changes of the system are modelled. We examine changes in the input vector u. Note that the system after the change is stable. Stability is only concerned with the matrix A, which is left unchanged.

We examine successively the effect of a sudden change in production, in demand, and in supply. This corresponds to a step input in the input variables c_q, γ_0 and c_2 respectively. Figure 5.11 shows the reaction of Thirst to a positive and a negative step input on production. The system starts in the equilibrium state. After t=2 years production is suddenly increased or decreased by a step of ± 0.01 10^9kg/year. An increase in production (Figure 5.11a) causes the inventory (and consequently the supply) to rise. The excess supply causes



(a) (b)

Figure 5.10: *Phaseplot of the two regions*



(a) (b)

Figure 5.11: *The reaction to a positive (a) and a negative (b) step input of* $0.01\ 10^9 kg/year$ *on the production* c_q

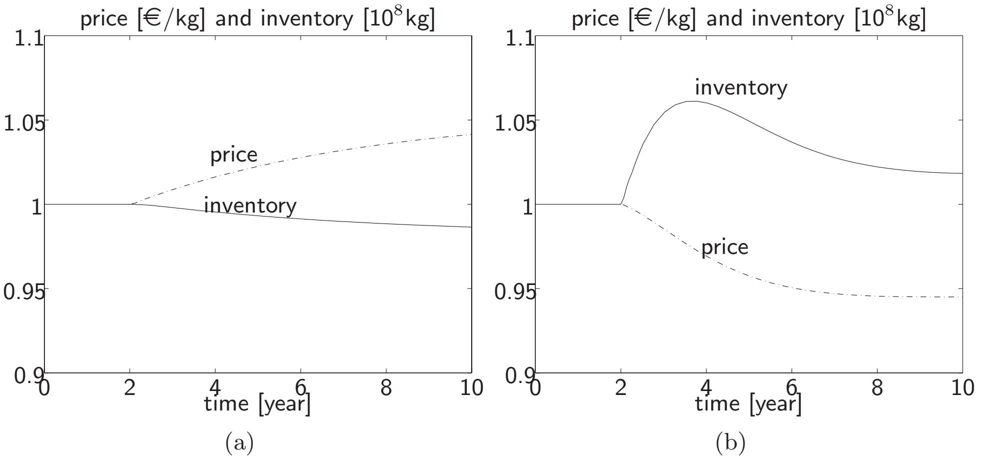


Figure 5.12: *The reaction to a positive (a) and a negative (b) step input of* 0.01 10^9*kg/year on the autonomous demand* γ_0

the price to decrease. As a consequence of the lower price, demand increases. More beer can be sold, which decreases the inventory. The system evolves into a new equilibrium situation with a lower price and a higher inventory than in the original situation. This equilibrium point can be calculated by (5.33). A decrease in production, on the other hand, (Figure 5.11b) causes a decrease of the beer inventory because less beer is produced and demand is still high. Since beer supply decreases, the beer price rises and the system reaches a new equilibrium state as expected.

The reaction to a sudden increase and decrease of the demand is depicted in Figure 5.12. An increase of the demand may be caused, for example, by advertising, good weather, etc. A decrease may, for example, be caused by anti-alcohol campaigns, bad weather, etc. A sudden increase in demand (Figure 5.12a) causes the price to rise and the inventory to fall because of the excess demand. The price and inventory evolve into a new equilibrium state. A negative step on demand (Figure 5.12b) causes an excess beer supply. The inventory rises and the price decreases. The decreasing price causes the demand to increase again so that the inventory decreases. A new equilibrium state is reached in the end.

An increase in supply (Figure 5.13a) causes the price to decrease. This causes the demand to increase so that more products can be sold. Thus the inventory decreases. A sudden decrease in supply (Figure 5.13b) causes an excess demand, and this causes the price to rise, which causes the demand to decrease and the inventory to increase.

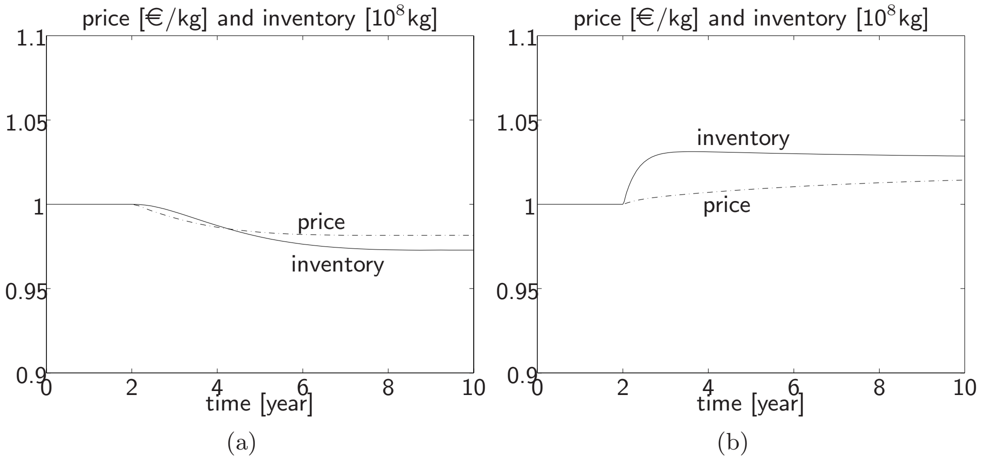


Figure 5.13: *The reaction to a positive (a) and a negative (b) step input of* 0.01 10^9*kg/year on the supply* c_2

5.5 A company with several resource and product markets

In general, a company buys various resources from various resource markets and supplies various products to various product markets. In this section, we discuss how to incorporate several resource and product markets into the presented model. First we show how to model a company using several resources. Next we model a company that furnishes several product markets.

5.5.1 A company using several resources

We consider a production process in which several resources are used to produce one final product, a so-called convergent process. A typical example of such a production process is an assembly process.

In economic theory, resources are characterized as *complements* or as *substitutes*. Complementary products are products that are consumed together, for example, two products that are both needed in a production process such as coke and ore in the blast furnace process or a bolt and a mutter in an assembly process. Substitutes are products that can be used for the same purpose. For example, copper and aluminium for electricity cables.

In order to show the modelling method we consider the case of just two kinds of resources which are used for the production of one final product as represented in Figure 5.14. For substitutes as well as for complements, the

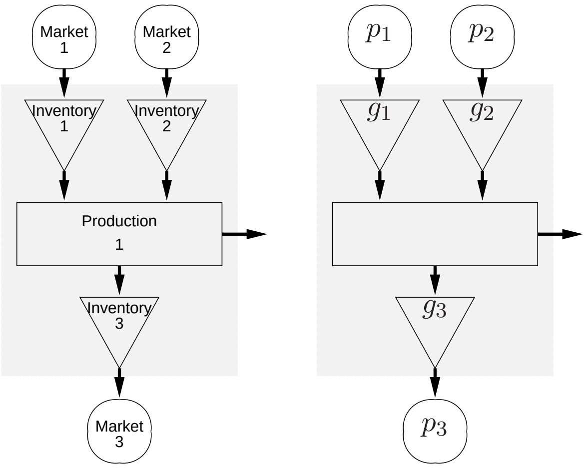


Figure 5.14: *A company using two resources for the production of one final product*

usual balance equations hold for the three inventories shown in Figure 5.14

$$\dot{g}_i = e_i - q_i \text{ for } i = 1,2 \qquad \text{and} \qquad \dot{g}_3 = q_3 - e_3 \tag{5.34}$$

where q_1 and q_2 represent the quantities of Resource 1 and Resource 2 respectively needed for the production q_3 of products. The relationship between q_3, and q_1 and q_2 is different in the case of complements and of substitutes.

For *complementary* products, the three quantities can easily be linked in the same way as in the case of a single resource, see (5.6)

$$q_3 = c_{q3} - \alpha_1 p_1 - \alpha_2 p_2 + \alpha_3 p_3 + \eta_1 \lambda_1 g_1 + \eta_2 \lambda_2 g_2 - \lambda_3 g_3 \tag{5.35}$$

Moreover, q_1 and q_2 are needed for production in a constant ratio. For the production of one kg of q_3, η_1 kg of q_1 and η_2 kg of q_2 are needed

$$q_1 = q_3/\eta_1 \qquad q_2 = q_3/\eta_2 \tag{5.36}$$

In the case that Resource 1 and 2 are *substitutes*, the situation is more complex. In a linear setting, the quantities q_1 and q_2 needed for production depend on their respective prices[5] and inventories

$$q_1 = c_{q1} + \gamma_1 p_1 - \delta_1 p_2 + \epsilon_1 g_1 - \xi_1 g_2 \tag{5.37}$$

$$q_2 = c_{q2} - \gamma_2 p_1 + \delta_2 p_2 - \epsilon_2 g_1 + \xi_2 g_2 \tag{5.38}$$

---

[5] The cross price elasticity $\frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i}$, $i \neq j$ is not zero.

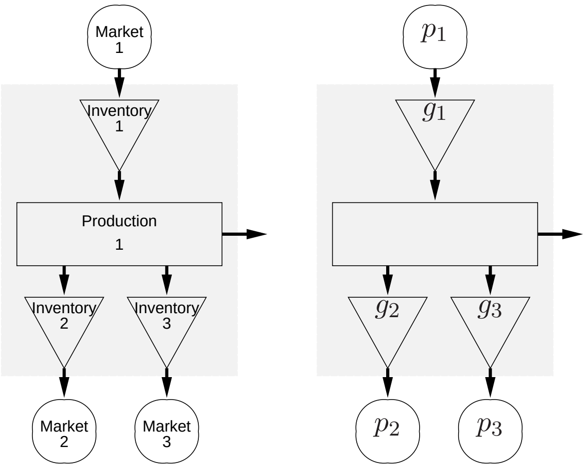


Figure 5.15: *A company producing two products from one resource*

with $\gamma_i, \delta_i, \epsilon_i, \xi_i$ constants of appropriate dimension. The production is given by

$$q_3 = \eta_1 q_1 + \eta_2 q_2 \tag{5.39}$$

By these additions, a company using two resources and producing one final product can be modelled.

5.5.2 A company producing several products

An example of a company that produces several products and supplies these products to different markets, i.e. a company using a so-called divergent process, is depicted in Figure 5.15. In a refinery, for example, crude oil is transformed into a range of products such as propane, naphtha, gasoline, fuel oil, etc. By definition, the products supplied to different markets are not substitutable.

We consider the case of two product markets as an example. The quantity q_1 of resources needed for production is given by

$$q_1 = c_{q1} - \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \lambda_1 g_1 - \frac{\lambda_2 g_2}{\eta_2} - \frac{\lambda_3 g_3}{\eta_3} \tag{5.40}$$

The produced quantities of product 2 and 3, q_2 and q_3, respectively, are given by

$$q_2 = \eta_2 q_1 \qquad q_3 = \eta_3 q_1 \tag{5.41}$$

These are the equations describing a company with one resource market and two product markets. The extension to a company with several resource and several product markets is straightforward and consists of a combination of the company models described in Section 5.5.1 and in Section 5.5.2.

5.6 Conclusions and Discussion

The building blocks of a mathematical product-process chain model are company and market models. A company can be modelled by inventories for resources and for products that are linked to each other by a production process. By taking mass conservation into account, the company can then be represented in state space form by a two-dimensional state vector, which is defined by the inventory levels. The limiting case in which a company does not hold inventories can be included. The companies are modelled on a rather high level of aggregation: only the resource inflows and product outflows are considered. The underlying production processes are not modelled individually; they are aggregated to one overall process. The market dynamics are described by Walrasian price adjustment with the market price as the only state variable. A perfectly competitive market is assumed.

The example of a beer producing company and a beer market shows how to use this modelling method. The model consists of a company and its product market. With such a model, stability of the system can be analysed and typical situations can be simulated, for example the influence of advertising and the effect of changes in demand and supply on the company's behaviour.

Chapter 6

Chain Modelling

6.1 Introduction

The previous chapter dealt with a method of modelling companies and markets. This chapter describes how the presented models can be combined to form a model of a complete product chain. As an example of a product chain including recycling, the paper chain is modelled. This chain will be analysed in the following chapter. Next simulations of some typical chain regulations will be carried out.

In Section 6.2, a way to combine the company and market models is described, taking into account the possibility of product recycling. Section 6.3 presents a model of the Dutch paper chain, which will serve as an example in the following chapters of this thesis. In Section 6.4, some characteristic influences on the chain and some important measurements are discussed. In Section 6.5, the parameters in the paper chain model are determined in such a way that the model results agree with observed time series. In Section 6.6, the chain model results are discussed.

6.2 Combination of Company and Market Models

A model of a complete product chain is built up by combining the company and market models. The outputs of the market models constitute inputs to the company models and vice versa. In the following, we will consider a chain which consists of companies that have just one market on the resource side and one market on the product side, i.e. companies that use one kind of resource to produce one kind of product. Since the consumption of products is a transformation process just as the production processes, the consumers in the chain can be modelled in the same way as the other actors. We use the

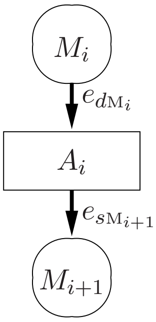


Figure 6.1: *The actor A_i and his resource and product markets*

indices $M_1, M_2 \dots$ for variables of the markets and the indices $A_1, A_2 \dots$ for variables of the companies.

When an actor in the chain is connected to one resource market and one product market as depicted in Figure 6.1, the following quantities occurring in the company (5.7,5.11) and market (5.15,5.19,5.21) sub-models equal each other:

$p_{1\mathrm{A}_i} = p_{\mathrm{M}_i}$	The price the company has to pay equals the company's resource market price
$e_{1\mathrm{A}_i} = e_{d\mathrm{M}_i}$	The amount of resources the company buys equals the trade of the demander on the company's resource market
$q_{d\mathrm{A}_i} = q_{d\mathrm{M}_i}$	The company's demand equals the demand on the company's resource market
$p_{2\mathrm{A}_i} = p_{\mathrm{M}_{i+1}}$	The price the company receives for its products equals the company's product market price
$e_{2\mathrm{A}_i} = e_{s\mathrm{M}_{i+1}}$	The amount of products the company sells equals the trade of the supplier on the company's product market
$q_{s\mathrm{A}_i} = q_{s\mathrm{M}_{i+1}}$	The company's supply equals the supply on the company's product market

For a closed chain consisting of n actors and n markets, the index $n+1$ must be replaced by 1 to close the chain. In the following, we will use the notation on the right of the above equations. In the simplest case, a closed chain consists of two companies and two markets. The first company is a producer who buys discarded products and sells finished products, and the second 'company' is a producer of discarded products, who buys products and produces discarded products to be sold to the first company.

6.3 The Paper Chain

Paper can be defined as a substance made from fibrous cellulose material [148]. The cellulose pulp used in the paper manufacturing process is either a primary raw material such as pulp from wood or a secondary raw material such as pulp from waste-paper. Depending on the further use of the secondary pulp, the waste-paper must be de-inked and prepared for the pulp production. Paper and paper products are used in various products and groups of products in the whole economy.

In the following, the Dutch paper chain will be modelled. The flowchart of the paper sector as it is currently encountered in the Netherlands is depicted in Figure 6.2. The paper and cardboard industry either buys primary pulp or pulp from waste-paper (or a combination of both) on the pulp market (Market 1 in Figure 6.2). Subsequently, it supplies the paper it has produced to the paper and cardboard market. The paper and cardboard market also experiences supply from outside the chain by foreign suppliers. The crude paper is bought by the paper and cardboard product industry, which converts it into products of paper and cardboard (printed paper, cardboard boxes for packaging, etc.). These products are bought by offices, shops, services and companies (OSSC) and by households. Currently, wastes from the OSSC-sector are supposed to be transferred to the waste-paper market. Household wastes are collected by the municipality services and by a large and entangled network of associations. Both flows form a supply to the waste-paper market where the waste-paper industry buys its resources. The sorted and processed waste-paper is supplied to the pulp market [146].

As discussed above, the paper chain, as it is modelled here, consists of three markets and three chain actors. The state vector of this chain consists of nine variables (one state variable for each market and two state variables for each company) and is given by, see (5.3,5.15)

$$x = [p_{\mathrm{M}_1} \; x_{\mathrm{A}_1} \; p_{\mathrm{M}_2} \; x_{\mathrm{A}_2} \; p_{\mathrm{M}_3} \; x_{\mathrm{A}_3}]^T \tag{6.1}$$

The state equation is, see (5.7,5.15)

$$\dot{x} = \begin{bmatrix} \rho_1(q_{d\mathrm{M}_1} - q_{s\mathrm{M}_1} - q_r - q_{e1}) \\ A_{\mathrm{A}_1}x_{\mathrm{A}_1} + B_{1\mathrm{A}_1}p_{\mathrm{M}_1} + B_{2\mathrm{A}_1}p_{\mathrm{M}_2} + B_{3\mathrm{A}_1}e_{d\mathrm{M}_1} + B_{4\mathrm{A}_1}e_{s\mathrm{M}_2} + H_{\mathrm{A}_1}c_{q\mathrm{A}_1} \\ \rho_2(q_{d\mathrm{M}_2} - q_{s\mathrm{M}_2} - q_{e2}) \\ A_{\mathrm{A}_2}x_{\mathrm{A}_2} + B_{1\mathrm{A}_2}p_{\mathrm{M}_2} + B_{2\mathrm{A}_2}p_{\mathrm{M}_2} + B_{3\mathrm{A}_2}e_{d\mathrm{M}_2} + B_{4\mathrm{A}_2}e_{s\mathrm{M}_3} + H_{\mathrm{A}_2}c_{q\mathrm{A}_2} \\ \rho_3(q_{d\mathrm{M}_3} - q_{s\mathrm{M}_3} - q_{e3}) \\ A_{\mathrm{A}_3}x_{\mathrm{A}_3} + B_{1\mathrm{A}_3}p_{\mathrm{M}_3} + B_{2\mathrm{A}_3}p_{\mathrm{M}_1} + B_{3\mathrm{A}_3}e_{d\mathrm{M}_3} + B_{4\mathrm{A}_3}e_{s\mathrm{M}_1} + H_{\mathrm{A}_3}c_{q\mathrm{A}_3} \end{bmatrix} \tag{6.2}$$

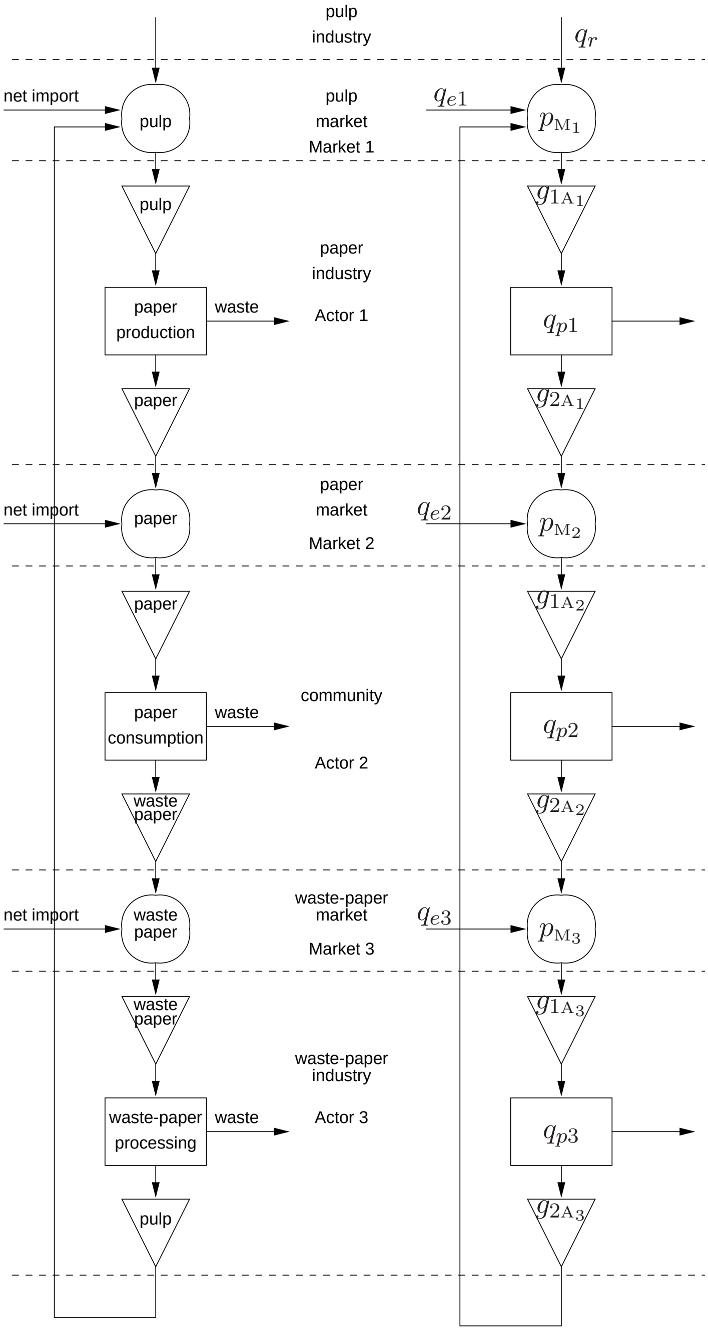


Figure 6.2: *The flowchart of the paper chain. The left figure shows the various markets and companies, and the material flows. The right figure shows the corresponding variables.*

The raw material suppliers, who supply pulp to Market 1, are regarded as having an infinite materials inventory. Thus, only their decisions on supply are relevant. Their (price dependent) supply is described by

$$q_r = \nu_0 + \nu_1 p_{\mathrm{M}_1} \tag{6.3}$$

with ν_0 (in kg/year) and ν_1 (in kg^2/(year€)) being time invariant. This means that they are stimulated to supply more raw materials, when the price increases.

The physical flows crossing the system boundaries can be interpreted as disturbance inputs. Since the complete branch is considered here, they equal the imports and exports across the international boundary. They are represented by

$$q = [q_{e1}\ q_{e2}\ q_{e3}]^T \tag{6.4}$$

and act on the markets as described in Section 5.3.3. In Figure 6.2, the net imports q_{ei} to the chain are indicated by arrows. In the case of a net export, q_{ei} is negative, and the direction of the arrow corresponding to the product flow is reversed. For the pulp market M_1, we take into account that v_{M_1} does not only consist of imports and exports q_{e1}, but also of raw materials supply q_r (6.3). Thus

$$v = [v_{\mathrm{M}_1}\ v_{\mathrm{M}_2}\ v_{\mathrm{M}_3}]^T = [q_{e1} + q_r\ q_{e2}\ q_{e3}]^T \tag{6.5}$$

The chain model can be written as

$$\dot{x} = A_c x + B_c q + H_c c + f_c \tag{6.6}$$

The matrices A_c, B_c and H_c, and the vector of constants c are given in Appendix B.1. The nonlinearities resulting from the nonlinear market models are included in the vector f_c

$$f_c = \begin{bmatrix} 0 \\ B_{3\mathrm{A}_1} e_{d\mathrm{M}_1} + B_{4\mathrm{A}_1} e_{s\mathrm{M}_2} \\ 0 \\ B_{3\mathrm{A}_2} e_{d\mathrm{M}_2} + B_{4\mathrm{A}_2} e_{s\mathrm{M}_3} \\ 0 \\ B_{3\mathrm{A}_3} e_{d\mathrm{M}_3} + B_{4\mathrm{A}_3} e_{s\mathrm{M}_1} \end{bmatrix} \quad \begin{array}{l} \text{with} \\ e_{s\mathrm{M}_i} = \min(q_{s\mathrm{M}_i}, q_{d\mathrm{M}_i} - v_{\mathrm{M}_i}) \\ e_{d\mathrm{M}_i} = \min(q_{s\mathrm{M}_i} + v_{\mathrm{M}_i}, q_{d\mathrm{M}_i}) \\ \text{from (5.19,5.21)} \end{array} \tag{6.7}$$

Due to the nonlinearities in f_c, the chain system is nonlinear. To be more precise, it is piecewise linear. It can be divided into eight linear models, which are valid in the corresponding regions of the state space. These regions are defined in Table 6.1. Since systems like this product chain system, switch between different behaviours, they are also called *switched* or *switching systems.*

	Market 1		Market 2		Market 3	
	$q_{s\mathrm{A}_3} + v_{\mathrm{M}_1}$	$q_{d\mathrm{A}_1}$	$q_{s\mathrm{A}_1} + v_{\mathrm{M}_2}$	$q_{d\mathrm{A}_2}$	$q_{s\mathrm{A}_2} + v_{\mathrm{M}_3}$	$q_{d\mathrm{A}_3}$
Region 1	●		●		●	
Region 2	●		●			●
Region 3	●			●	●	
Region 4	●			●		●
Region 5		●	●		●	
Region 6		●	●			●
Region 7		●		●	●	
Region 8		●		●		●

Table 6.1: *The regions of the state space of the paper chain model. The dots show the smaller argument in the trade function (6.7).*

6.4 Measured Quantities and Influences on the Chain

In the following, not only the state variables themselves will be considered but also outputs that are combinations of the states. There are various reasons for calculating these outputs. The output consisting of the prices in the chain is used in the parameter estimation procedure: The prices calculated by the model are compared to the observed prices. Other outputs, such as the product flows and the recycling rate contain the interesting variables to be controlled.

Moreover, various variables are considered that influence the chain, for example taxes and subsidies. We show how they can be incorporated as inputs into the chain model.

6.4.1 Measured quantities

The outputs discussed in this section are the prices on the markets, the product flows and the recycling rate.

The prices

The prices are part of the state vector and can therefore easily be calculated. The output equation for the prices y_{prices} is of the linear form (4.4)

$$y = Cx + Du \qquad \text{with} \tag{6.8}$$

$$y_{\text{prices}} = \begin{bmatrix} p_{\mathrm{M}_1} \\ p_{\mathrm{M}_2} \\ p_{\mathrm{M}_3} \end{bmatrix} \qquad C_{\text{prices}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad D_{\text{prices}} = 0 \tag{6.9}$$

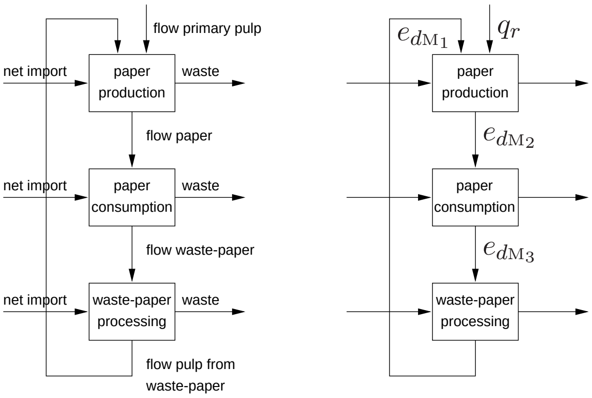


Figure 6.3: *The material flows in the paper chain*

The product flows

The product flows output consists of four variables: the flow of primary material, the flow of finished products to the consumers, the flow of consumed products to the recyclers and the flow of products from the recyclers back to the producers. They are depicted in Figure 6.3 and can be calculated by

$$y_{\text{flows}} = \begin{bmatrix} q_r \\ e_{d\mathrm{M}_1} \\ e_{d\mathrm{M}_2} \\ e_{d\mathrm{M}_3} \end{bmatrix} = \begin{bmatrix} \nu_0 + \nu_1 p_{\mathrm{M}_1} \\ \min(q_{s\mathrm{M}_1} + v_{\mathrm{M}_1}, q_{d\mathrm{M}_1}) \\ \min(q_{s\mathrm{M}_2} + v_{\mathrm{M}_2}, q_{d\mathrm{M}_2}) \\ \min(q_{s\mathrm{M}_3} + v_{\mathrm{M}_3}, q_{d\mathrm{M}_3}) \end{bmatrix} \tag{6.10}$$

The equation corresponding to this output is piecewise linear due to the minimum functions that define the trades $e_{d\mathrm{M}_i}$.

The recycling rate

The recycling rate is considered as an output because it mirrors the environmental performance of the chain. Although it can be doubted if this is an unequivocal indicator, as stated in Section 3.2.2, it is often used as such in practice.

The definition of the recycling rate is ambiguous, because it can be calculated between different points in a chain. The *technical* recycling rate, for example, can be defined as the ratio of recycled resources and the total amount of resources that enter the production process. In this study, the recycling rate is defined as the ratio of products bought by the recyclers and the amount of products purchased by the consumers at the same instant. This is the definition which is also handled by the paper recycling industry [134], for example.

In a dynamic context, however, this can lead to recycling rates of more than 100%, if production decreases.

A further difficulty is due to export and import of waste products. Do imported waste products that are recycled in the considered chain increase the recycling rate and do exported products decrease it? In the definition which is used here, imported and exported waste products do influence the recycling rate. It is assumed that waste products that are exported will be recycled in the country of their destination. Thus they contribute to the recycling rate. Imported waste products, on the other hand, do not contribute to the recycling rate of the considered chain but only to the recycling rate of the exporting chain [23]. Consequently, the recycling rate can be expressed as

$$y_{\text{rec}} = \frac{e_{d\text{M}_3} - \min(0, v_{\text{M}_3})}{e_{d\text{M}_2}} \tag{6.11}$$

Note that this definition of the recycling rate slightly differs from the definition of the waste-paper industry [134]. This industry does not consider all the consumed products $e_{d\text{M}_2}$ but only 'recyclable' products and thus obtains higher recycling rates. If a constant rate of the consumed paper is considered unrecyclable, the two recycling rates just differ in a constant factor.

In contrast to the system and output equations discussed before, this output equation is neither linear nor piecewise linear.

6.4.2 Influences on the chain

The stimuli to change the system state constitute inputs to the chain. In this section, the modelling of some governmental incentives for an environmentally friendlier behaviour of the product chain are presented. As discussed in Section 3.3, various policy instruments might be enforced. As for those which are promising to fit for inclusion into the model, it is tried to capture them mathematically.

Economic instruments

Taxes u_{tax} on *products* or raw materials as discussed in Section 3.3.3 can easily be incorporated into the model. The price the buyers are charged for a product is modified by adding the tax u_{tax} (in €/kg) to the actual price the buyers have to pay (see Figure 6.4)

$$p' = p + u_{\text{tax}} \tag{6.12}$$

The sellers of the product, however, only receive p. The remainder is transferred to the authorities. A subsidy for products can be modelled in the same

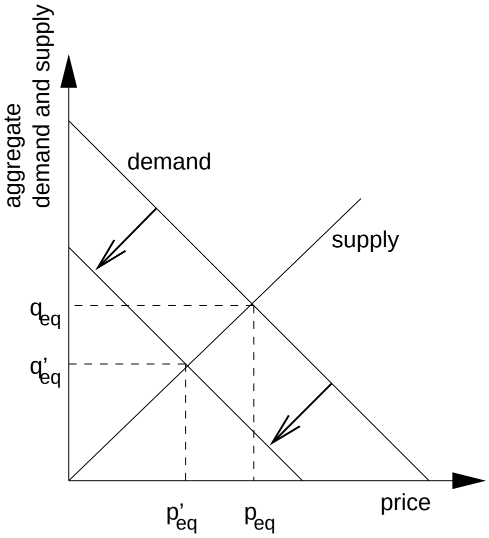


Figure 6.4: *The shift of the demand curve as a consequence of a tax*

way by adding the subsidy u_s (in €/kg) to the price the sellers of a product obtain

$$p' = p + u_\mathrm{s} \tag{6.13}$$

The buyers of the product only have to pay the price p.

Subsidies and taxes on *activities* can be incorporated by increasing or decreasing the production c_q of a company by the quantity u_q (in kg/year), see (5.6). The corresponding demand and supply constants, see (5.9,5.10), are then increased or decreased accordingly

$$c'_q = c_q + u_q \qquad c'_1 = c_1 + u_q/\eta \qquad c'_2 = c_2 + u_q \tag{6.14}$$

If we assume that the increase linearly depends on the amount of a subsidy u_subs (in €/year), the subsidy can be calculated by

$$u_q = \beta_q u_\mathrm{subs} \tag{6.15}$$

with the proportionality constant β_q in kg/€. The value of u_subs is positive in the case of a subsidy and negative in the case of a tax.

Furthermore, the economic measure of an adjustment of the price for final treatment can be included in the model. The final treatment activities can be landfill or incineration. A higher final treatment price p_w will stimulate chain actors to better sort their waste and to supply more waste for recycling. The quantity which is transferred to the waste products inventory is described by the function q_w, which depends on the final treatment price p_w. The state equations (5.1,5.2) of the waste-producing actor are then modified to

$$\dot{g}_1 = e_1 - \frac{q_p}{\eta} \tag{6.16}$$

$$\dot{g}_2 = q_p - e_2 + q_w \tag{6.17}$$

For a linear dependence on p_w, q_w is given by

$$q_w = \beta_w(p_w - p_{w0}) \tag{6.18}$$

with β_w in kg^2/(year€) and p_{w0} the initial final treatment price.

Regulatory instruments

Regulatory instruments in general were discussed in Section 3.3.2. In a product chain, regulatory influences manifest themselves through restrictions in definite material flows. Such a restriction is, for example, the disposal prohibition of several wastes enforced by the Dutch government. Such restrictions on product flows can be incorporated into the chain model by including the maximal flow $u_{\max}$ (in kg/year). The flow $e_{\text{restricted}}$ is then bounded to

$$e_{\text{restricted}} = \max(u_{\max}, e) \tag{6.19}$$

However, such a flow restriction adds a nonlinear term to the model, which impedes the analysis of the model. Therefore, this kind of influence is not studied in the frame of this thesis.

Self-regulation

Self-regulation was mentioned in Section 3.3.4 as a social instrument for influencing product chains. The government's main task in a self-regulation stimulating policy is to create conditions that foster self-regulation in a particular industrial sector. However, the modelling of these conditions in general mathematical equations exceeds the scope of this study. Since it is possible to model an industrial sector operating under some kind of self-regulating regime, the chain model can be used to analyse such a regulated chain. The removal fund, an element of the paper fibre convenant, that is supposed to take some control over the Dutch paper industry, will be modelled and evaluated in the following.

Imports and exports

Since imports and exports influence the markets in the chain, they also constitute inputs. In the previous sections they were considered as disturbance inputs. If it is possible, for example for governmental authorities, to influence imports and exports, they can be used as manipulated inputs to control the chain. This case can be incorporated into the chain model by considering q_{ei} (6.4) as a manipulated input.

6.5 System Parameters

In the model equations (6.6), a number of constants, such as the price adjustment constants ρ_i and the efficiencies η_i, occur as parameters. Some of them have a physical meaning and can be measured directly: In the paper chain the efficiencies, for example, are known. Their values are (see Appendix A)

$$\eta_1 = 0.9,\ \eta_2 = 0.6,\ \eta_3 = 0.8$$

In this chain, 35 unknown parameters are present. Here we describe how their value is obtained from observations of the chain behaviour. For the problem of parameter estimation, the time series of the prices $p_{\mathrm{M}1}, p_{\mathrm{M}2}, p_{\mathrm{M}3}$ and of the net imports q_{e1}, q_{e2}, q_{e3} can be used. The prices, imports and exports have been observed by the Dutch Statistical Office. Here we consider the time span from January 1990 to December 1995. The various data sources and the way of monitoring the data are discussed in detail in Appendix A. In the appendix, the accuracy of these data is also discussed. It must be taken into account that the accuracy is less than usually obtainable in technical systems.

The objective of the parameter estimation is to find parameters such that the prices y_{prices} (6.9) in the model show approximately the same behaviour as the observed prices, given that the inputs q (6.4) behave as measured. Since the initial state x_0 in January 1990 is not known (no data exist on the inventories, and the observed prices are subject to measurement errors, as discussed in Appendix A[1]), the initial state is estimated as well.

The parameters are calculated by minimizing the least squares difference between the model outputs and the given data [117]. If we denote the vector of parameter values and initial states x_0 to be determined by θ, the published values for the prices at time instant t_i by the vectors $m(t_i)$ (with $i = 1 \ldots n$), and the values of the prices determined by the model with parameters θ by

---

[1] It is assumed that — like the other observed prices — the initial prices in January 1990 need not be exactly met by the model.

constants	A_1		A_2		A_3	
	value	s	value	s	value	s
μ_1[1/year]	2.8	0.4	0.40	0.04	2.8	0.3
μ_2[1/year]	2.6	0.3	0.50	0.06	2.7	0.2
λ_1[1/year]	0		0		0	
λ_2[1/year]	0.07	0.01	0.037	0.006	0.03	0.02
α_1[10^9kg^2/(€ year)]	5.8	0.6	0.59	0.06	5.9	0.9
α_2[10^9kg^2/(€ year)]	5.9	0.6	0.69	0.02	5.9	0.5
β_1[10^9kg^2/(€ year)]	5.9	0.8	0.55	0.01	5.9	0.9
β_2[10^9kg^2/(€ year)]	5.8	0.6	5.9	0.8	5.9	0.7
c_1[10^9kg/year]	19	2	8.0	0.4	16	3
c_2[10^9kg/year]	-7.8	0.8	1.5	0.3	-10	2
c_q[10^9kg/year]	4.4	0.3	3.8	0.3	-0.40	0.07

price adjustment parameters	value	s
ρ_1 [10^{-9} €/kg^2]	0.022	0.001
ρ_2 [10^{-9} €/kg^2]	0.32	0.05
ρ_3 [10^{-9} €/kg^2]	0.0293	0.0007

supply function $\nu_0 + \nu_1 p_{\mathrm{M1}}$	value	s
ν_0 [10^9 kg/year]	0.41	0.06
ν_1 [10^9 kg^2/(€year)]	4.6	0.6

Table 6.2: *The parameters of the paper chain. The parameter values and the standard deviations s are reported. The parameter λ_1 is not estimated.*

the vector $y(t_i, \theta)$, we solve the optimization problem of minimizing

$$h(\theta) = \frac{1}{2}\sum_{i=1}^{n} \big[y(t_i,\theta) - m(t_i)\big]^T V \big[y(t_i,\theta) - m(t_i)\big] \tag{6.20}$$

with respect to θ and with the condition (6.2,6.9)

$$\begin{aligned} \dot{x} &= f(x,q,\theta) \\ y &= g(x,q,\theta) \end{aligned} \qquad \text{with} \qquad q = [q_{e1}\; q_{e2}\; q_{e3}]^T \tag{6.21}$$

By the choice of the weight matrix V, the reliability of the measurements can be incorporated. We choose V as the unity matrix, because none of the measurements is more reliable than the others. It is obvious that this problem is a nonlinear optimization problem, which requires careful evaluation. The details of the solution to this problem are described in Appendix D.

The parameters that are found as a result of the parameter estimation procedure are shown in Table 6.2. The development of the prices in the paper chain is depicted in Figure 6.5. The lines represent the prices of pulp, paper and waste-paper calculated by the model with the estimated parameters, and

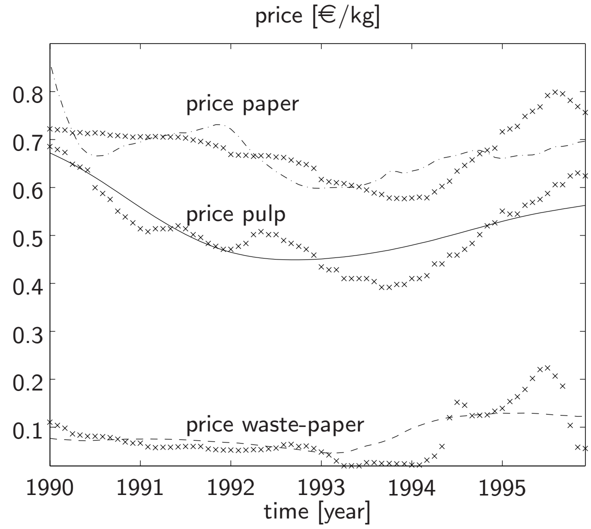
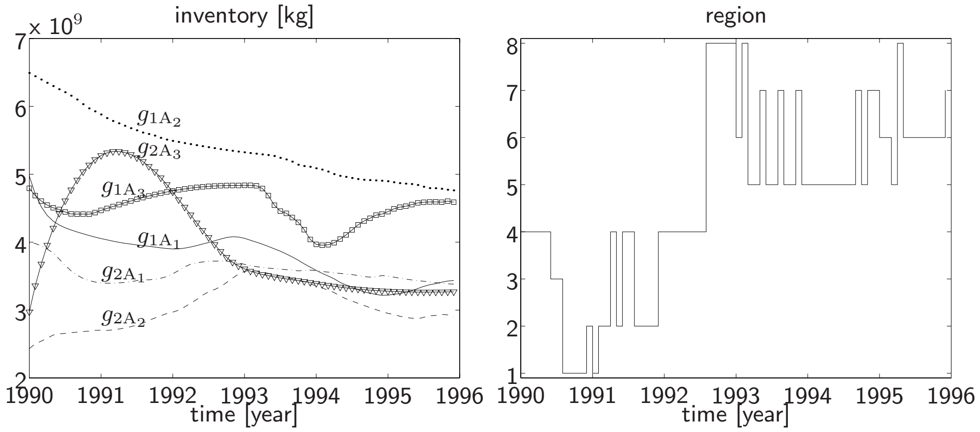


Figure 6.5: *A comparison of prices in the model with observations. The crosses represent the observations and the lines represent the model results.*

the crosses represent the prices published by the Dutch Statistical Office. It can be seen that the two curves are in quite good agreement. Especially the striking behaviour of the waste-paper price is reflected by the model. It is approximately constant in the first years and increases in the last years of the considered time span.

It must be noted that, compared to technical systems, the number of measurements (there are 216 data points) is very small in relation to the 44 parameters (35 parameters and nine initial values) to be estimated. Some aspects of the quality of the estimate are discussed in Appendix D.2, and some of the results are presented in this section. The standard deviation s of the estimated parameters is shown in Table 6.2. It must be noted that these quantities are based on a linear approximation of the output function $y(\theta)$, evaluated for the optimal parameters $\theta_{\min}$.

Another test of the quality of the estimate is the autocorrelation of the residuals $y(t_i) - m(t_i)$, i.e. the correlation of successive values of the residuals. Although the number of observations is rather small, Figure 6.5 suggests that successive values of the residuals are correlated. The analysis in Appendix D.2 confirms autocorrelation of the residuals. Autocorrelated residuals indicate that an important variable may have been omitted from the model. In the case of the product chain model, autocorrelation may be due to, for example, economic trends not included in the model. The problem can be solved by adding state variables to the model. However, this would introduce extra



(a) (b)

Figure 6.6: *The model results with estimated parameters: inventories (a) and regions of the state space(b)*

parameters and the physical meaning of the added variables is not clear.

Furthermore, heteroscedasticity of the residuals is tested. It measures the dependence of the variance of the residuals on the value of the output. An example of a heteroscedastic distribution is the yearly household expenditure for vacations as a function of household income. There will tend to be more variation for high-income households than for low-income households, who tend to consistently spend much less for vacation. In the presence of heteroscedasticity, the standard deviation of the parameters will be understated [105]. In the present case, heteroscedasticity does not seem to occur (see Appendix D.2).

In Figure 6.6, further results of the chain model with the estimated parameters are depicted. Figure 6.6a shows the development of the inventories in the chain. It can be seen that the inventories of the paper producers (g_{1A_1},g_{2A_1}) show a smooth behaviour. The consumers' paper inventory g_{1A_2} decreases in the course of the indicated six years. Their waste-paper inventory g_{2A_2} first increases and decreases subsequently. The recyclers' waste-paper inventory g_{1A_3} is approximately constant and their pulp inventory g_{2A_3} shows the largest fluctuations of all the inventories. Figure 6.6b shows the regions in the state space according to Table 6.1. It can seen that the state of the paper chain switches between the regions, which means that the trade on the three markets switches between demand and supply. There is no market with a structural excess supply or demand.

6.6 Conclusions and Discussion

The company and market models can be combined to form a model of an entire product chain, including the possibility of inserting recycle loops. Due to the modular structure, a large variety of chains can be modelled in this way, and existing chain models can be extended by adding company and market modules. With respect to the paper chain, a chain model, which consists of three chain actors — paper producers, paper consumers, and waste-paper recyclers — can be composed in such a standard way. The parameters in the model can be estimated so as to well approximate the paper chain behaviour. Here we summarize some of the limitations of the model and mention some points of particular interest for further examination.

The market dynamics, and the dynamics of production and consumption are assumed to occur on a comparable time scale. However, the case that the dynamics of one module are much slower than the dynamics of the others can easily be incorporated. It is a limiting case of the presented model. The dynamics of the fast modules may then be neglected. In the steel chain, for instance, the time span of consumption is of the order of twenty years in many applications such as buildings, cars, and ships, whereas the production time is of the order of magnitude of a month. Production can be seen as immediate here.

As already stated before, by modelling all the actors in one life-phase of a product as an aggregated company, it is not possible to *explicitly* model the competition between the companies in one life-phase. However, the behaviour of the aggregated companies is modelled. This is the behaviour *including* competition.

Another aggregation applied here is in the modelling of producers and consumers of only one product. Most materials, however, are used in a large variety of products. It must be decided from case to case, if it is possible to aggregate the producers and consumers of these products or if a differentiation of product categories is necessary. Note that, in principle, it is possible to make such a differentiation by using the presented modelling technique.

From a systems and control engineering point of view, the chain model constitutes a rather hard to handle nonlinear model. In retrospect, it seems by large this is due to the fact that the foundations of the economic theory are not developed extensively and in an unambiguous way as they are, for instance, in physics — it seems that the economic sciences cannot yet oversee the field so as the physical sciences can. This does not necessarily have to come as a surprise since in comparison with physics, in economics '...*we* are the molecules...' [54]. The studied systems are complex systems that include many unknown psychological and sociological mechanisms.

Chapter 7

Chain Analysis

7.1 Introduction

In the previous chapter, a model of the Dutch paper chain was presented, based on the company and market models developed in Chapter 5. Chapter 4 provided some background on the concepts for system analysis that are available in control theory. Some of these concepts are used in this chapter as a basis to examine stability, controllability and observability of the product chain model. The nonlinearities in the considered system, which result from the switching of the market trade between supply and demand, make an in-depth analysis of the chain difficult but not impossible.

The local-state-space-models-approach, which involves the introduction of different local models in different regions of the state space, can be used. This approach is described in Section 7.2. The stability of the linearized chain as well as of the nonlinear chain is examined in Section 7.3. The nonlinear chain system is analysed by Lyapunov methods. The further insights with respect to the system characteristics, for instance controllability and observability, are gained from a linearized model as is often the case for nonlinear systems. Controllability and observability analysis are carried out in Sections 7.4 and 7.5. Finally, in Section 7.6, the main conclusions are drawn.

7.2 System Representation

One of the nonlinearities arising in the product-process chain model is due to the trade of products (5.17) being bounded to the minimum of products supplied to a market or demanded on a market. The trade function of the form $e = \min(q_s, q_d)$ is a continuous function in its arguments q_s, q_d, but its derivative is discontinuous, where the two arguments equal each other. To avoid this discontinuous derivative, two possible approaches will be followed here:

(i) The minimum function is approximated by a differentiable function, and

(ii) The minimum function is represented by two different continuous functions in two different regions of the state space (local models). Although the discontinuity persists, this representation offers possibilities for analysis.

7.2.1 Approximation by a differentiable function

The trade function $e = \min(q_s, q_d)$ can be written with the help of the Heaviside step function[1] ε as

$$e = q_d + (q_s - q_d)\varepsilon(q_d - q_s) \qquad \text{with} \qquad \varepsilon(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{else} \end{cases} \tag{7.1}$$

The Heaviside function can be approximated by a differentiable function — for example a tanh-like function $\tilde{\varepsilon}$ — that has a continuous derivative at the point $x = 0$. One such function, $\tilde{\varepsilon}(x) = \frac{1}{2} + \frac{1}{2}\tanh(cx)$, is depicted in Figure 7.1a for different values of the parameter c. The approximation of the minimum function is shown in Figure 7.1b.

7.2.2 Approximation by local models

The second approach used here to deal with the nonlinearity consists of expressing the trade function $e = \min(q_s, q_d)$ by a piecewise continuous function

Region 1: $e = q_s$ if $q_d > q_s$
Region 2: $e = q_d$ if $q_d < q_s$

The dynamic model (6.6) $\dot{x} = \min(f_1(x,u,v), f_2(x,u,v))$ can then be described by the local models

Region 1: $\dot{x} = f_1(x,u,v)$ if $f_2(x,u,v) > f_1(x,u,v)$
Region 2: $\dot{x} = f_2(x,u,v)$ if $f_2(x,u,v) < f_1(x,u,v)$

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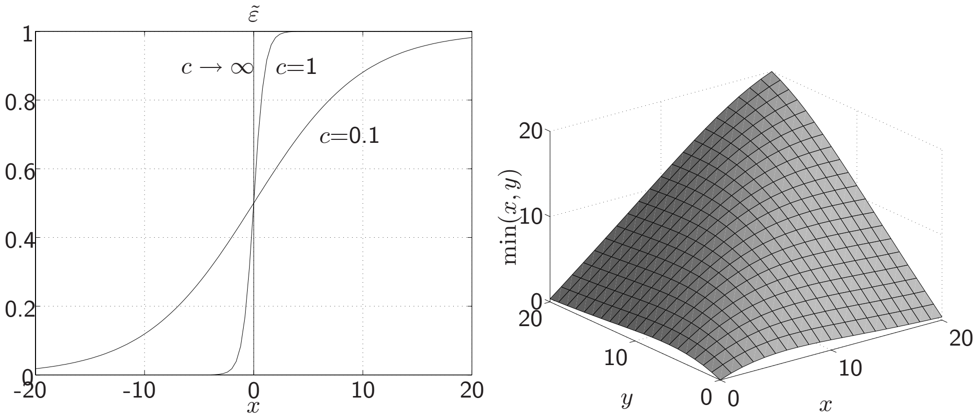
[1] For the sake of completeness, we mention here that the Heaviside function is not a function in the strict sense of the term, but a so-called generalized function, see for example [100].



(a) (b)

Figure 7.1: *The approximation of the Heaviside function (a) and the minimum function (b) with different values of c*

If the local models $f_i(x, u, v)$ are linear and their system matrices are denoted by A_i, and if the autonomous case with u=0,v=0 is considered, it is possible to use the following state space representation for the piecewise linear functions

$$\dot{x} = A_{i(x)}x \tag{7.2}$$

The index $i(x)$ of the matrix $A_{i(x)}$ depends on the state x. Depending on the actual state, it can take the value $1, 2, \ldots$ Then the corresponding system matrix $A_1, A_2, \ldots$ is valid. The state space can be divided into regions where the local models $A_{i(x)}$ are valid.

7.3 Stability

The two representations of the previous section are used to examine the stability of the product chain model. Among the various ways of analysing stability, two approaches are chosen. In the *first* approach, the chain model is approximated by a differentiable model as described in Section 7.2.1. Starting out from this, it is linearized and the stability of the linearized system is analysed. In the *second* approach, another kind of stability, stability according to Lyapunov, is analysed.

7.3.1 The linearized chain

A standard approach of analysing nonlinear models consists of linearizing them along the system trajectory and of analysing the corresponding linearized model. The linearized model approximates the nonlinear model for small deviations from the reference trajectory. Since the product chain system is not differentiable in the whole state space, it is not linearizable in each point of the state space. The approximation proposed in Section 7.2.1, however, allows linearization. The linearized system has the state space representation (4.1,4.2)

$$\begin{aligned} \Delta\dot{x} &= A\Delta x + B\Delta u + E\Delta v \\ \Delta y &= C\Delta x + D\Delta u + F\Delta v \end{aligned} \qquad \text{with} \qquad \begin{aligned} \Delta x &= x - x_0 \\ \Delta u &= u - u_0 \\ \Delta v &= v - v_0 \end{aligned} \tag{7.3}$$

with the reference trajectory (x_0, u_0, v_0) and the matrices describing the system given by

$$A = \left.\frac{\partial f}{\partial x}\right|_{x_0,u_0,v_0} \qquad B = \left.\frac{\partial f}{\partial u}\right|_{x_0,u_0,v_0} \qquad E = \left.\frac{\partial f}{\partial v}\right|_{x_0,u_0,v_0} \tag{7.4}$$

$$C = \left.\frac{\partial g}{\partial x}\right|_{x_0,u_0,v_0} \qquad D = \left.\frac{\partial g}{\partial u}\right|_{x_0,u_0,v_0} \qquad F = \left.\frac{\partial g}{\partial v}\right|_{x_0,u_0,v_0} \tag{7.5}$$

The matrices A, B, C, D, E, F are time dependent, if the reference trajectory (x_0, u_0, v_0) is so.

The stability of the linearized chain can be demonstrated by investigating the eigenvalues of the system matrix A. If they have a negative real part, the linearized system is stable (see Appendix C.1).

For the paper chain system, linearized along the trajectory of Figures 6.5 and 6.6, the eigenvalues of the system matrix A lie in the left complex half plane. This indicates that the linearized system is stable. A proof of global stability of the nonlinear chain will be given in Section 7.3.2.

7.3.2 Lyapunov stability of the piecewise linear chain

The analysis of the eigenvalues of the system matrix for proving stability — as done in the previous section — is only possible for linear systems. For nonlinear systems, such as systems with different behaviour in different regions of the state space (e.g. the product chain system), an analysis of the eigenvalues is not always appropriate for proving stability. This is illustrated by the following example.

> **An Example:** switched systems
> This example motivates that a mere analysis of the eigenvalues of the system matrix is not always appropriate. It shows the stability analysis of two linear time invariant systems and of one switched

system. Here no inputs are present and the outputs are the states of the system.

System 1:

$$\dot{x} = A_1 x \qquad \text{with } A_1 = \begin{bmatrix} -0.1 & 5 \\ -1 & -0.1 \end{bmatrix} \text{ and } x(0) = x_0 \tag{7.6}$$

System 2:

$$\dot{x} = A_2 x \qquad \text{with } A_2 = \begin{bmatrix} -0.1 & 1 \\ -5 & -0.1 \end{bmatrix} \text{ and } x(0) = x_0 \tag{7.7}$$

System 3:

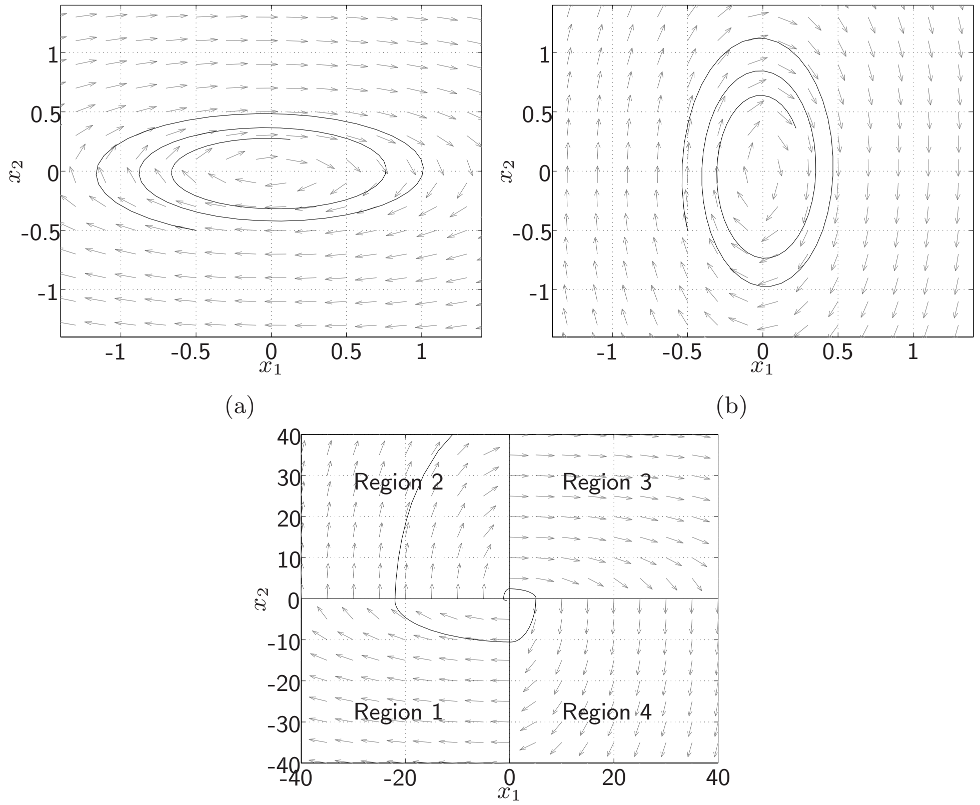
$$\dot{x} = A_i x \qquad \text{with } \begin{array}{l} A_i = A_1 \text{ if } x_1 x_2 > 0 \\ A_i = A_2 \text{ if } x_1 x_2 < 0 \end{array} \text{ and } x(0) = x_0 \tag{7.8}$$

with A_1 and A_2 from System 1 and System 2.

System 3 can be divided into four regions:

	$x_1 < 0$	$x_1 > 0$
$x_2 < 0$	Region 1	Region 4
$x_2 > 0$	Region 2	Region 3

The system matrices of System 1 and 2 have the same eigenvalues: $\lambda_{1,2} = -0.1 \pm 2.24i$. They have a negative real part showing that the systems are (exponentially) stable. The eigenvalues of System 3 are the same as for System 1 and System 2. This system, however, is not stable in contrast to what the eigenvalues suggest. In Figure 7.2, the trajectories in the phase plane of the three systems are represented with the initial condition $x_0 = [-0.5 \ -0.5]^T$. Figure 7.2a and 7.2b show the phase trajectories of System 1 and 2 respectively. It can be seen that the trajectories evolve to the origin along a vertical and a horizontal ellipse-like trajectory. The two systems are stable, which corresponds to the eigenvalues of the system matrices of these linear systems. The motion of System 3 (Figure 7.2c) is directed to infinity. The system is unstable because the switching between System 1 and System 2 is such that the trajectory moves away from the origin. It must be noted that, if in System 3, the switching conditions are changed, for instance, if A_1 and A_2 are interchanged, the system will become stable.



(a)

(b)

(c)

Figure 7.2: *The trajectories of System 1, 2 and 3 (7.6),(7.7),(7.8), all with the same eigenvalues*

Lyapunov functions

A method to analyse the stability of nonlinear systems is by Lyapunov functions. The origin of *Lyapunov's direct method of stability* is based on the idea that the total energy of an unforced dissipative mechanical or electrical system decreases as the state of the system evolves in time [102, 130]. It can be used for linear as well as for nonlinear systems. The system will eventually settle down to its equilibrium point. Therefore, the state vector approaches a constant value corresponding to the minimum energy level.

To analyse the energy, a scalar function of the state x, the so-called Lyapunov function $V(x)$ is introduced. If such a function exists with continuous first order derivatives[2] such that

- $V(x)$ is positive definite[3]
- $\dot{V}(x)$ is negative definite
- $V(x) \to \infty$ as $\|x\| \to \infty$

the equilibrium at the origin is globally (asymptotically) stable.

The finding of a Lyapunov function is sufficient for proving stability, but not necessary. As long as no Lyapunov function that fulfils these conditions is found, no conjecture can be made as to stability or instability. Only when a function is found, the system is proven to be stable. There is no systematic method for finding Lyapunov functions. Scalar functions for which the sign definiteness can be easily checked and which are therefore often used as Lyapunov function candidates are of the quadratic form

$$V(x) = x^T P x \tag{7.9}$$

where P is a real symmetric matrix[4]. In this case, $V(x)$ is positive definite if and only if all the eigenvalues of P are positive.

An example: Lyapunov function of the switched system
For the first two (stable) systems in the example (7.6) through

---

[2] Note that stability can be proved for weaker conditions on the differentiability. For the piecewise quadratic Lyapunov functions considered in the following, for example, continuity is sufficient [35].

[3] A scalar continuous function $V(x)$ is said to be (globally) *positive definite*, if $V(0) = 0$ and if for $x \neq 0$, $V(x) > 0$. It is *negative definite*, if $-V(x)$ is positive definite.

[4] In a real matrix P, all the entries p_{ij} are real, and in a symmetric matrix p_{ij} equals p_{ji}.

(7.8), quadratic Lyapunov functions can be found, for example

$$V_1(x) = x^T P_1 x \text{ with } P_1 = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \text{ for System 1, and}$$

$$V_2(x) = x^T P_2 x \text{ with } P_2 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \text{ for System 2}$$

It can easily be verified that these functions satisfy the Lyapunov conditions.

Why does this approach fail for the third system? An obvious Lyapunov function candidate is constructed by combining the two abovementioned Lyapunov functions to a piecewise quadratic function:

$$V_3(x) = \begin{cases} V_1(x) & \text{in Regions 1 and 3} \\ V_2(x) & \text{in Regions 2 and 4} \end{cases} \tag{7.10}$$

This function satisfies the Lyapunov conditions inside each region but not at the boundaries. If we consider, for example, the time derivative of the Lyapunov function candidate at the boundary between Region 1 and Region 2, we obtain (since $x_2 = 0$)

$$\begin{aligned} V_3(x) &= V_1(x) = x_1^2 && \text{when approaching the boundary from Region 1, and} \\ V_3(x) &= V_2(x) = 5x_1^2 > V_1(x) && \text{when approaching the boundary from Region 2} \end{aligned}$$

Since the system trajectory passes from Region 1 to Region 2 with increasing time, the Lyapunov function candidate increases at the region boundary and thus violates the second Lyapunov condition.

A more elaborated approach, which is particularly suited for piecewise linear systems, is the search for piecewise quadratic Lyapunov functions [81, 82]. Here, a computational approach for stability analysis of nonlinear and switched systems is followed, as presented in [81]. It is sketched in Appendix B.3. The search for piecewise quadratic Lyapunov functions is formulated as a convex optimization problem in terms of Linear Matrix Inequalities. For this search, the so-called $\mathcal{S}$-procedure is introduced in the stability conditions [1, 32]. Compared to a quadratic Lyapunov function this introduces extra degrees of freedom, which, on the one hand, make the search more difficult inside a particular region, but which, on the other hand, may be simpler to be satisfied outside the region. An example — the flower system from [81] — will be presented to illustrate this approach. In this example, a piecewise linear system is discussed.

An example: The flower system
The system is a piecewise linear system defined by

$$\dot{x} = A_i x \tag{7.11}$$

The matrices A_i are the same matrices as in the previous example. A_1 is equal to A_3 and given by (7.6), and A_2 equals A_4 and is given by (7.7). The only difference to the previous example is the region partition:

	$x_1 < x_2$	$x_1 > x_2$
$x_1 < -x_2$	Region 1	Region 4
$x_1 > -x_2$	Region 2	Region 3

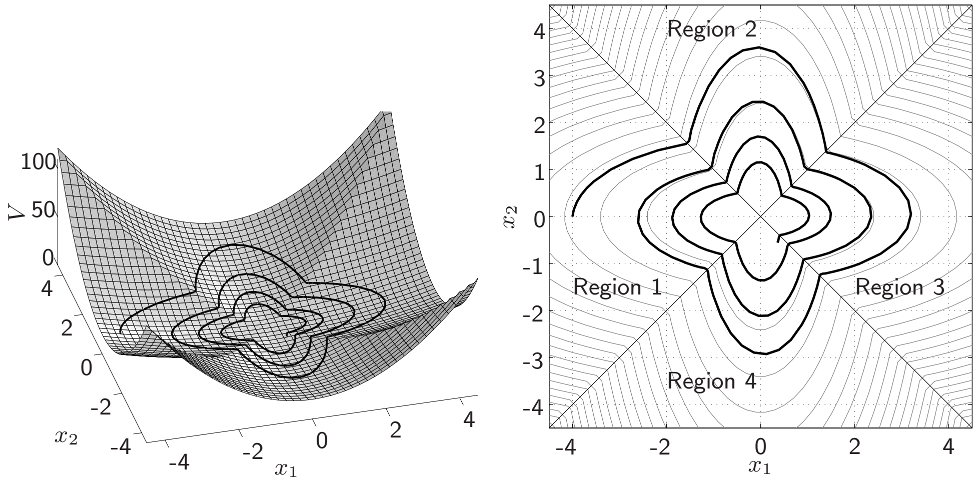
To prove stability we try to find a Lyapunov function. With the computational approach mentioned above, a piecewise quadratic Lyapunov function is found. It is given by $V(x) = x^T P_i x$ with

$$P_1 = P_3 = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \qquad P_2 = P_4 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \tag{7.12}$$

This proves global stability for each initial condition. Figure 7.3 shows the system trajectory for the initial condition $x_0 = [-4\ 0]^T$, and the Lyapunov function. Figure 7.3a shows a three-dimensional plot and Figure 7.3b a two-dimensional projection with the levels of the Lyapunov function indicated by grey lines. It can be seen that, along the trajectory of the simulation, the state moves towards lower levels and will end up in the origin.

Lyapunov stability of the chain

We now apply the Lyapunov approach to the product chain system. In principle, the same approach as in the example can be followed here. We consider the paper chain model (6.6) with the parameters of Table 6.2 around its (unique) equilibrium point, which is calculated in Appendix B.2. The imports to the chain are chosen to be constant and equal to the average of the period 1990 to 1995: q_{e1}=0.7 10^9kg/year, q_{e2}=0.25 10^9kg/year, and q_{e3}=−0.88 10^9kg/year. The product chain model consists of eight continuous sub-models, which are



(a) (b)

Figure 7.3: *The trajectory of the flower system together with its associated Lyapunov function in a three-dimensional plot (a) and in a two-dimensional projection (b). The flower like curves indicate levels of energy. The energy levels decrease as they move towards the origin.*

valid in eight distinct regions as mentioned in Table 6.1. By the same approach as used in the flower system above, a piecewise quadratic Lyapunov function

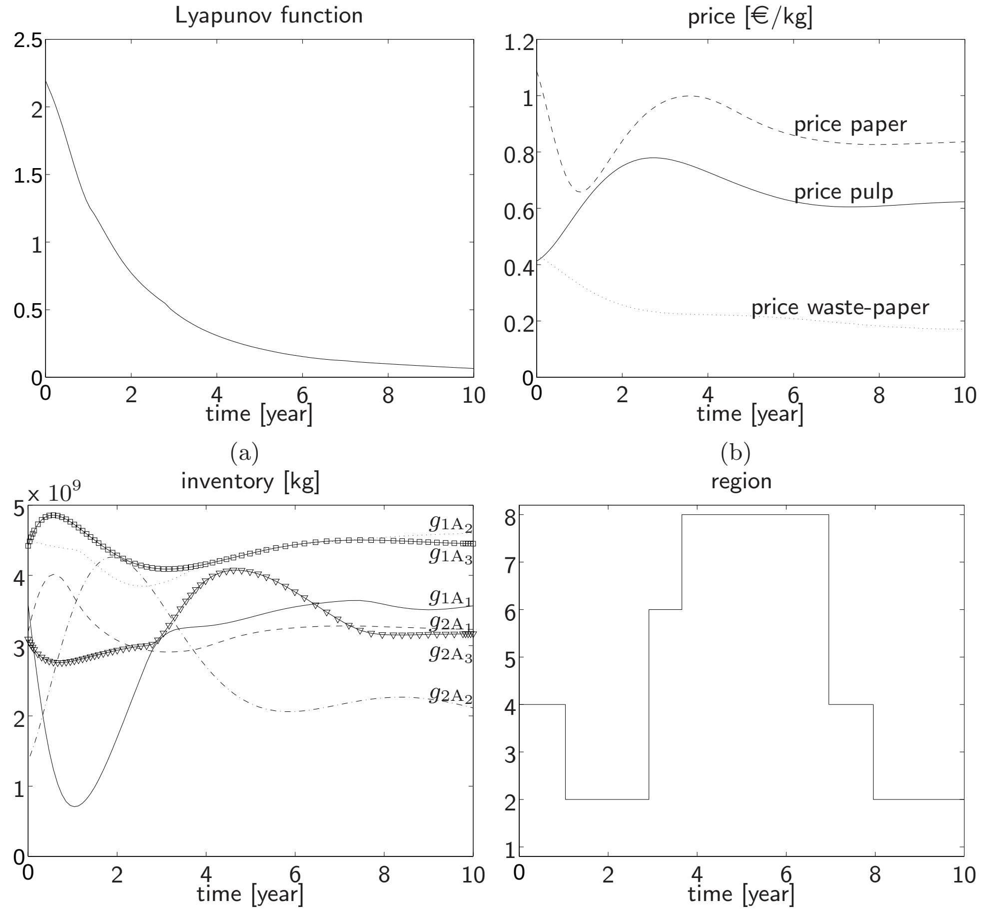
$$V(x) = x^T P_i x \tag{7.13}$$

is found. The corresponding matrices P_i are given in Appendix B.4. This proves global (asymptotic) stability for each initial condition. To illustrate the Lyapunov function we consider an example. Figure 7.4 shows the behaviour of the paper chain with the parameters of Table 6.2 and with the initial condition

	$g_{1\mathrm{A}_1}$	$g_{2\mathrm{A}_1}$	$g_{1\mathrm{A}_2}$	$g_{2\mathrm{A}_2}$	$g_{1\mathrm{A}_3}$	$g_{2\mathrm{A}_3}$
[10^9kg]	3.2	3.2	2.7	1.4	3.5	3.1

	p_{M_1}	p_{M_2}	p_{M_3}
[€/kg]	0.41	1.1	0.43

The time evolution of the Lyapunov function is shown in Figure 7.4a. The Lyapunov function decreases as can be seen from the figure. The corresponding time evolution of the prices is shown in Figure 7.4b. After some initial oscillations they evolve towards their equilibrium values. The evolution of the other state variables — the inventories of the chain actors — is depicted in Figure 7.4c. It can be seen that they move from the initial values to their



(a) (b) (c) (d)

Figure 7.4: *The paper chain system with constant imports: Lyapunov function (a), prices (b), inventories (c) and regions in the state space (d)*

equilibrium values as well. Figure 7.4d shows the corresponding regions of the state space as defined in Table 6.1. By comparing Figure 7.4d and Figure 7.4a, the switching of the Lyapunov function can be seen.

It can be concluded from this that the nonlinear paper chain model is globally (asymptotically) stable for each initial state and for the chosen parameters. Note, however, that stability is only proved for this particular set of parameters.

7.4 Controllability

As discussed in Section 4.6.2, controllability of the product chain is desired for various reasons. The chain actors are interested in the control of the product flows, for example, in order to achieve smooth product flows. The authorities are interested in controllability by, for example, economic policy inputs [88]. In this section, controllability of the paper chain is analysed with respect to those inputs which can be included in the model as described in Section 6.4.2.

The piecewise linear approach, used in the previous section for stability analysis, is not yet as developed for controllability as for stability. Moreover, a change in the inputs also changes the region boundaries. Therefore, the piecewise linear approach is not followed here. The examination of controllability is restricted to the *linearized* chain. To that order, the linearized description (7.3) is used, and the corresponding linear controllability tests are carried out. These are quite easy to perform, since they amount to compute the rank of the so-called *controllability matrix* (see Appendix C.2)

$$\mathcal{C} = [B\ AB\ \cdots\ A^{n-1}B] \tag{7.14}$$

If the controllability matrix has full rank, i.e. if the rank of $\mathcal{C}$ is equal to the number n of states, the system state is controllable; otherwise it is not. It can be seen that controllability only depends on the system matrix A and the input matrix B.

> **An example:** controllability of a simple product chain
> A simple product chain will serve as an example to illustrate the controllability analysis of a system. Consider a chain consisting of two companies and two markets as depicted in Figure 7.5. We assume, for simplicity of the example, that the companies' inventories play no role and that their supply and demand decisions only depend on the prices of resources and products. Company 1 supplies the quantity $q_{s\mathrm{A}_1}$ and demands the quantity $q_{d\mathrm{A}_1}$ (see (5.6)

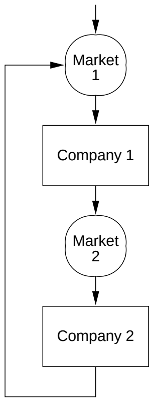


Figure 7.5: *Example of a simple product chain with two markets and two companies*

and Figure 5.4)

$$q_{s\mathrm{A}_1} = \alpha_{2\mathrm{A}_1} p_{\mathrm{M}_2} - \alpha_{1\mathrm{A}_1} p_{\mathrm{M}_1} + c_{q1} \qquad q_{d\mathrm{A}_1} = \frac{q_{s\mathrm{A}_1}}{\eta_1} \tag{7.15}$$

and Company 2 supplies and demands

$$q_{s\mathrm{A}_2} = \alpha_{2\mathrm{A}_2} p_{\mathrm{M}_1} - \alpha_{1\mathrm{A}_2} p_{\mathrm{M}_2} + c_{q2} \qquad q_{d\mathrm{A}_2} = \frac{q_{s\mathrm{A}_2}}{\eta_2} \tag{7.16}$$

We consider a chain with an external supplier to Market 1. This supplier has to pay the product tax u (see (6.12)). The external supply q_r is then given by (6.3)

$$q_r = \nu_0 + \nu_1 (p_{\mathrm{M}_1} + u) \tag{7.17}$$

The only dynamic variables are the market prices p_{M_1} and p_{M_2}. According to the Walrasian price adjustment (5.14), we obtain

$$\dot{p}_{\mathrm{M}_1} = \rho_1 (q_{d\mathrm{A}_1} - q_{s\mathrm{A}_2} - \nu_0 - \nu_1 p_{\mathrm{M}_1} - \nu_1 u) \tag{7.18}$$

$$\dot{p}_{\mathrm{M}_2} = \rho_2 (q_{d\mathrm{A}_2} - q_{s\mathrm{A}_1}) \tag{7.19}$$

The output y is chosen to be the price on Market 2

$$y = p_{\mathrm{M}_2} \tag{7.20}$$

This system can be written in state space representation (4.1,4.2). Choosing the state vector $x = [p_{\mathrm{M}_1} \; p_{\mathrm{M}_2}]^T$, (7.15) through (7.20) can be collected into

$$\dot{x} = Ax + Bu + c \tag{7.21}$$

$$y = Cx \tag{7.22}$$

with

$$A = \begin{bmatrix} \rho_1(-\frac{\alpha_{1\mathrm{A}_1}}{\eta_1} - \alpha_{2\mathrm{A}_2} - \nu_1) & \rho_1(\frac{\alpha_{2\mathrm{A}_1}}{\eta_1} + \alpha_{1\mathrm{A}_2}) \\ \rho_2(\frac{\alpha_{2\mathrm{A}_2}}{\eta_2} + \alpha_{1\mathrm{A}_1}) & \rho_2(-\frac{\alpha_{1\mathrm{A}_2}}{\eta_2} - \alpha_{2\mathrm{A}_1}) \end{bmatrix} \tag{7.23}$$

$$B = \begin{bmatrix} -\rho_1\nu_1 \\ 0 \end{bmatrix} \qquad c = \begin{bmatrix} \rho_1(\frac{c_{q1}}{\eta_1} - c_{q2} - \nu_0) \\ \rho_2(\frac{c_{q2}}{\eta_2} - c_{q1}) \end{bmatrix} \tag{7.24}$$

$$C = [0 \quad 1] \tag{7.25}$$

Due to presence of the vector c, this is an affine system and does not have the standard linear form (4.3,4.4). However, by following the same approach as in Appendix C.2 it is found that the standard controllability test can be performed for showing controllability. The controllability matrix of this system is

$$\mathcal{C} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} -\rho_1\nu_1 & \rho_1(-\frac{\alpha_{1\mathrm{A}_1}}{\eta_1} - \alpha_{2\mathrm{A}_2} - \nu_1)(-\rho_1\nu_1) \\ 0 & \rho_2(\frac{\alpha_{2\mathrm{A}_2}}{\eta_2} + \alpha_{1\mathrm{A}_1})(-\rho_1\nu_1) \end{bmatrix} \tag{7.26}$$

The rank of $\mathcal{C}$ is 2 for nearly all realistic parameters — i.e. all parameters greater than or equal to zero. It is only less than 2 in the following cases

rank $\mathcal{C}$ <2	This means
$\alpha_{1\mathrm{A}_1}$=0 and $\alpha_{2\mathrm{A}_2}$=0	supply and demand decisions of the two companies are independent of the first price p_{M_1}
ρ_1=0	first price adjustment mechanism does not work
ρ_2=0	second price adjustment mechanism does not work

Thus the chain is controllable except if the supply and demand decisions of the two companies are independent of the first price p_{M_1}, or if one or both price adjustments do not work. The uncontrollability can be intuitively explained by the fact that if none of the demand and supply decisions depend on p_{M_1} but the input

> only influences p_{M_1}, the chain behaviour cannot be completely controlled by such an input. If ρ_2=0, the input has no influence on the second price; if ρ_1=0, it has no influence on the system, since the input matrix B is zero.

We examine controllability of the linearized paper chain model along the trajectory of Figures 6.5 and 6.6 [92]. First we consider a single input to the chain. For taxes on products u_{tax} (6.12), the controllability matrix has rank nine all the time. This holds for a tax on pulp as well as for a tax on paper as for a tax on waste-paper. Since the system order is nine, the system is controllable by each of the mentioned inputs. Obviously, it is also controllable by a combination of these inputs. The chain is also controllable by each of the other inputs described in Section 6.4.2. It is controllable by subsidies for products (6.13), by the final treatment price (6.16,6.17), by subsidies and taxes on activities (6.14), and by the regulation of imports and exports (6.4).

The analysis of the controllability *inside* the eight regions of the chain model shows that the chain model is controllable inside each region by each of the inputs mentioned above.

7.5 Observability

In order to determine observability (see Section 4.6.3), we use the linearized description as we did for controllability analysis, and we apply the corresponding linear observability tests. Like the controllability tests, these tests are quite easy to be carried out, since they amount to compute the rank of the so-called observability matrix

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{7.27}$$

If the observability matrix has full rank, the system state is observable; otherwise it is not, see Appendix C.3. It can be seen that observability only depends on the system matrix A and the output matrix C.

> **An example:** observability of a simple product chain
> We calculate the observability matrix of the example chain (7.21,7.22). It is given by
>
> $$\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \rho_2(\frac{\alpha_{2\mathrm{A}_2}}{\eta_2} + \alpha_{1\mathrm{A}_1}) & \rho_2(-\frac{\alpha_{1\mathrm{A}_2}}{\eta_2} - \alpha_{2\mathrm{A}_1}) \end{bmatrix} \tag{7.28}$$

The rank of the observability matrix is only less than two in the following cases — for all parameters greater than or equal to zero

rank $\mathcal{O}$ <2	This means
$\alpha_{1\mathrm{A}_1}$=0 and $\alpha_{2\mathrm{A}_2}$=0	all supply and demand decisions are independent of p_{M_1}
ρ_2=0	second price adjustment mechanism does not work

This means that the chain is observable except if all supply and demand decisions are independent of p_{M_1}, or if the second price adjustment mechanism does not work. In these cases, p_{M_2} is independent of p_{M_1}, and the measurement of p_{M_2}, which is done here, does not contain information on p_{M_1}. Thus p_{M_1} is not observable in this case.

For the linearized paper chain model, the observability matrix is calculated along the system trajectory of Figures 6.5 and 6.6. For the outputs defined in Section 6.4.1 — the prices, the linearized recycling rate and the product flows — the matrix has full rank. The system is observable with respect to each of these outputs. An analysis of the observability of the chain model inside each region shows observability of the chain inside the regions by each of the mentioned outputs as well.

7.6 Conclusions

For the chain model in state space representation, standard control theoretical methods can be used to determine stability, controllability and observability. Stability of the piecewise linear chain model can be analysed by a Lyapunov approach. For the paper chain model, a piecewise quadratic Lyapunov function is found. This is the proof for stability of the unregulated chain. Another aspect of stability is the stability of the regulated chain. This is not considered here because it depends on the kind of regulation. However, the same kind of analysis can be performed in the latter case.

Controllability and observability analyses are performed on the basis of a linearized chain model. It is demonstrated that the linearized paper chain is controllable by taxes and subsidies on products and activities, by the final treatment price and by influences on imports and exports. The state of the linearized model is observable, for example, by the measurement of one of the prices in the chain. Since a linearized model is analysed, the results on controllability and observability only hold for small deviations from the reference trajectory.

The example of a simplified chain indicates some cases in which a chain may not be controllable or observable. The simplified chain is *uncontrollable* in the case that the input influences a market and the price adjustments do not work or the company's decisions are independent of the prices. It is *unobservable* if, for example, the output is a market price and the corresponding price adjustment mechanism does not work.

Chapter 8

Chain Regulation

8.1 Introduction

In the previous chapters, the theoretical background for chain regulation was discussed. A general model of product-process chains was derived, and the example of the paper chain was discussed in detail. Stability of the paper chain model was proven, and controllability and observability of the linearized model were demonstrated.

In this chapter, some examples of regulation are described and subsequently simulated with the same paper chain model. It is demonstrated that the model can be used to study the results of a great number of different regulation strategies applied to product-process chains. As stated in Section 6.4.1, possible objectives of these regulations are the control of

- Definite material flows
- Product prices
- Recycling rates

In Section 6.4.2, measures that are available to influence the chain were discussed. Some of these measures are

- The direct regulation of definite product flows or of the recycling rate
- The stimulation of import to or export from the chain
- The adjustment of the waste incineration or final disposal price
- The levying of taxes on products and the granting of subsidies for products

- The levying of taxes on companies' activities and the granting of subsidies for their activities
- The establishment of actors which demand products and supply products to the markets

A combination of such measures is obviously possible as well, for example a combination of taxes and subsidies. This can be very attractive because in such a way instruments that are financially neutral for the authorities can be constructed. An example is a tax on paper which generates the income that can be used for a subsidy for waste-paper.

When implementing control structures, it must be taken into account that the measures can often not be adjusted continually. It is obviously not practically applicable to change the amount of a tax or subsidy on a day-to-day basis. Most governmental measures will therefore be designed in such a way that they are constant over a period of at least one year. The chain must have some certainty that the general conditions are constant for a not too short period of time. This adds difficulty to the choice of the right measure because corrections can only be made after a lapse of time.

Another difficulty, which occurs in the regulation of systems such as product chains, is due to the unavailability of current data. Often, decisions must be taken on the basis of data that originate from the preceding year or even earlier. Then, decisions are not based on the actual situation and may therefore be inaccurate.

In this chapter, some of the measures and objectives mentioned above are simulated to illustrate the possibilities of the model developed earlier. These simulations give the chain behaviour for a particular initial situation and a particular time span. Unlike the results of stability, controllability and observability analysis discussed in the previous chapter, simulations do not allow any conclusions for other initial situations, or for times outside the considered time frame. Measures taken by the authorities as well as measures taken by the chain actors are modelled. In all, six different kinds of regulation are illustrated:

(i) The control of the paper flow by a tax imposed on pulp (Section 8.2). A proportional-integral controller is used. It is illustrated by an example, how the objective can be attained.

(ii) The control of the recycling rate via an increase of recycling activities (Section 8.3). This example demonstrates that measures taken on the chain must be carefully evaluated. It is shown that the obvious measure of increasing recycling activities does not lead to a clear increase of the recycling rate in the long term.

(iii) The control of the recycling rate by the adjustment of the price for final treatment, for example, the incineration price or the landfill price (Section 8.4). This is an example of a controller that achieves the objective.

(iv) The control of the chain through a combination of option (iii) and a waste-paper subsidy as a 'politically' attractive option (Section 8.5).

(v) The direct control of the waste-paper price by means of a subsidy for waste-paper, if the waste-paper price drops below a definite critical level (Section 8.6). It is shown that this measure can make the waste-paper price behave smoother.

(vi) A more indirect control of the waste-paper price by a regulation scheme as it is actually introduced by the paper fibre convenant in the Netherlands (Section 8.7). This voluntary agreement between government and industry involves the installation of a so-called removal fund, an additional actor on the waste-paper market. By the means of the removal fund, it is possible to obtain smoother product flows and less fluctuating prices. This kind of regulation is compared to the previous one.

In Section 8.8, the influence of external disturbances on a chain with waste-paper subsidy and on a chain with removal fund is examined. In Section 8.9, the main conclusions of this chapter are drawn.

8.2 A Tax on Pulp

The first measure we discuss here is a tax on pulp that is imposed to control the paper flow. Although the most natural measure to control the paper flow seems to be a tax on paper, in a situation, in which a tax on paper is practically not possible, a tax on pulp may be a reasonable measure. As an example, we model regulation by a PI-controller, a special case of the well-known PID controller. Here we briefly review the essentials of PID control. For extensive discussions see one of the numerous textbooks, for example [58].

> **An Example:** PID control
> This example shows some characteristics of each of the proportional (P), the integral (I), and the derivative (D) controls, and how to use them to obtain a desired system behaviour. We consider the system shown in Figure 8.1, which corresponds to the system in Figure 4.2b with a special kind of controller and an ideal sensor. On the basis of this figure, we discuss how the PID controller works in a closed-loop system. The variable e represents

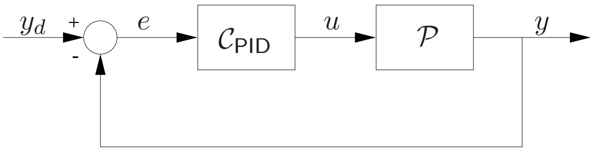


Figure 8.1: *The block diagram of a PID controlled system*

the tracking error, the difference between the desired output value y_d and the actual output y. The simplest form of control is one in which the error signal e is multiplied by a constant K_p to yield the input signal u to the plant $\mathcal{P}$. The magnitude of K_p determines the amount of corrective effort which is applied for a given value of the error e. This arrangement is called *proportional* control. For low values of K_p, the corrective effort is small and, hence, the regulation does not have great impact. For larger values, the system reaction will become more rapid for the same error e, and for large values of K_p instability is likely to result. In general, the final value of the proportionally controlled system will not be equal to the desired value y_d; the overall system will have a steady-state error.

A principal requirement of many control systems is that there should be *no* error in the steady state. This can be achieved for constant y_d by introducing *integral* action within the controller. A signal proportional to the time integral of the error is added to the proportional term, i.e.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau \tag{8.1}$$

Since the error signal is integrated within the controller, even a very small error eventually produces a corrective signal of sufficient magnitude to actuate the system with the aim to eliminate the error. The system will, theoretically, only come to rest when the error has been reduced to zero.

The control can be made more effective by adding a *derivative* action to the controller. The derivative of the error is added to the normal control action

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e}(t) \tag{8.2}$$

The derivative term contributes to an anticipatory type of control action, where the input u to the plant is modified when the error e is changing rapidly, thus anticipating on future values of e.

	A_1	A_2
$\alpha_1[10^9\text{kg}^2/(\text{€year})]$	3	3
$\alpha_2[10^9\text{kg}^2/(\text{€year})]$	5	5
$c_q[10^9\text{kg/year}]$	0.5	0.5
η [-]	0.9	0.8

$\nu_0 = 0$ kg/year
$\nu_1 = 3\ 10^9\ \text{kg}^2/(\text{€year})$
$\rho_1 = 0.09\ 10^{-9}\text{€/kg}^2$
$\rho_2 = 0.05\ 10^{-9}\text{€/kg}^2$

Table 8.1: *The parameters of the example chain*

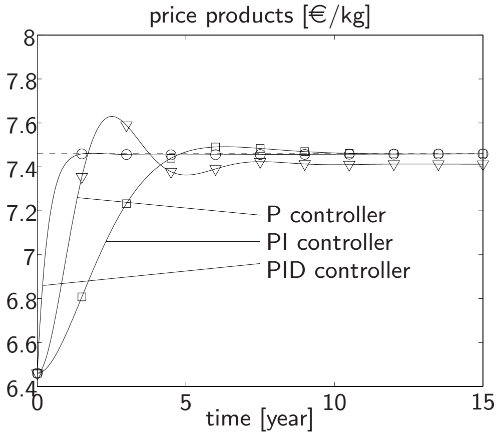


Figure 8.2: *Example of a PID controlled system: The simple product chain of Figure 7.5 with two markets and two companies. The behaviour of the controlled chain is shown with a proportional controller (K_p=20), a PI controller (K_p=6, K_i=0.5/year), and a PID controller (K_p=60, K_i=8/year, K_d=40 year).*

The simple product chain, defined in Section 7.4, will serve as an example to demonstrate the PID control of a system. The parameter values are chosen such as depicted in Table 8.1. We consider the problem of controlling this chain so that the price y is brought from the initial value with input zero y=6.46 €/kg to y_d=7.46 €/kg. Figure 8.2 shows the system behaviour with P-control, PI-control and PID-control respectively. It can be seen that the proportionally controlled chain oscillates and does not reach the desired value. In a PI-controlled chain, the objective is achieved after a small overshoot. With the PID controller, it is possible to achieve the objective more rapidly. In principle, the controller should be kept as simple as possible. Often, for instance, a derivative action is not necessary for achieving the desired per-

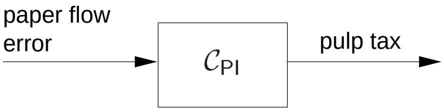


Figure 8.3: *The PI controller of the paper flow*

formance.

Note that the values of K_p, K_i and K_d are interrelated. In fact, changing one of these variables can change the effect of the other two. Therefore they should be chosen carefully.

For the regulation of the paper chain, we consider a tax on pulp. It can be modelled by adding the tax u_{tax} to the pulp price $p_{\mathrm{M}1}$ the paper producers have to pay, see (6.12)

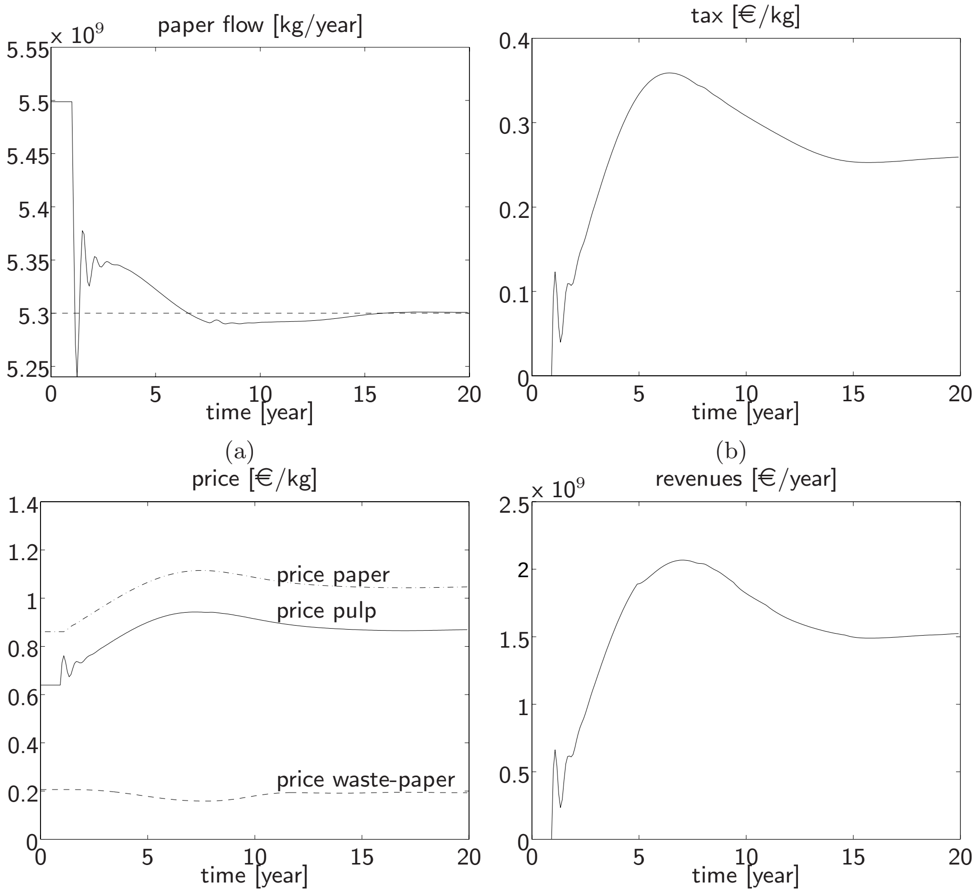
$$p'_{\mathrm{M}_1} = p_{\mathrm{M}1} + u_{\mathrm{tax}} \tag{8.3}$$

The pulp producers, on the other hand, only obtain the price p_{M_1}.

We consider the objective of reducing the paper flow, see (6.10), from the initial equilibrium flow of 5.5 10^9 kg/year to 5.3 10^9 kg/year. The height of the tax is continually adjusted, and a proportional-integral controller is used (see Figure 8.3). In Figure 8.4, some results of the controlled paper chain are depicted. Figure 8.4a shows the evolution in time of the paper flow. It can be seen that the objective is attained. Figure 8.4b shows the corresponding pulp tax, and Figure 8.4c shows the corresponding evolution of the pulp price including the tax, the paper price, and the waste-paper price. The tax that must be imposed is rather high. It amounts to 0.3 €/kg, which is about a third of the pulp price. The price behaviour is relatively smooth. As a consequence of the tax, the pulp and paper prices increase and the waste-paper price decreases. The revenues generated for the authorities by the tax are shown in Figure 8.4d. They are considerable, ca 1.5 10^9 €/year.

As mentioned in the previous section, difficulties in chain regulation may occur due to a lack of recent data and due to a discontinuous adjustment of the input. To illustrate these situations we consider the case that the pulp tax is only adjusted once in twelve months and the case that the information on the paper flow only becomes available after twelve months. The results of the paper flow and of the corresponding tax are depicted for specific examples in Figures 8.5 and 8.6 respectively.

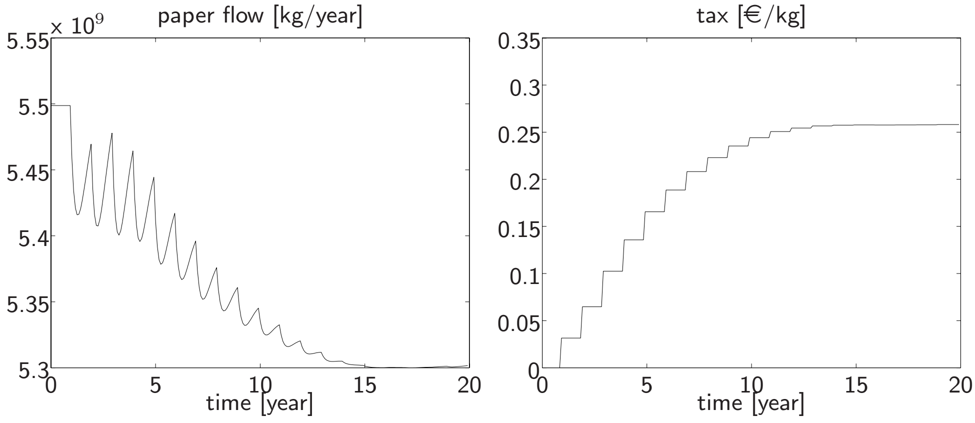
The three examples depicted in Figures 8.4, 8.5, and 8.6 are difficult to be compared because different controllers are used. With the yearly adjustment of the tax, it can be seen that, for this example, the paper flow first decreases



(a) (b)

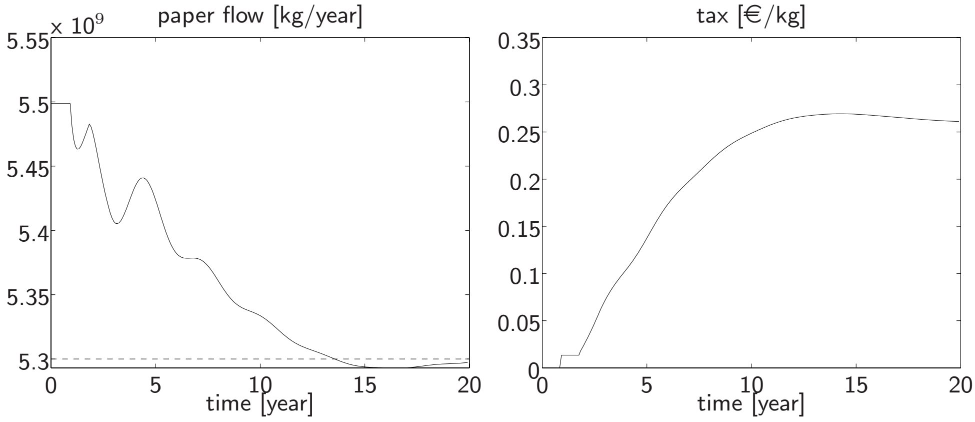
(c) (d)

Figure 8.4: *The paper flow (a), the pulp tax (b), the prices (c) and the revenues (d) in the PI-regulated paper chain with* K_p=0.7 10^{-9} *(€year)/kg*^{2} *and* K_i=0.33 10^{-9} *€/(kg*^{2}*year)*



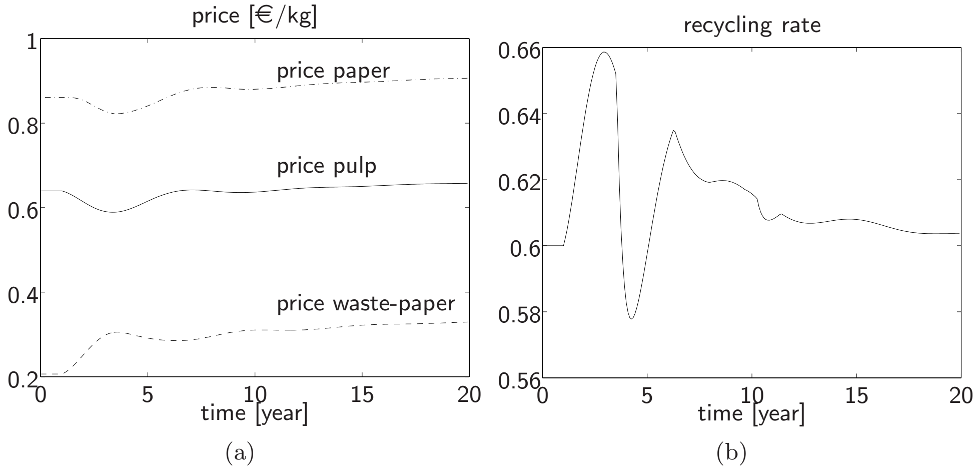
(a) (b)

Figure 8.5: *The paper flow (a) and the pulp tax (b) in the PI-regulated paper chain with a yearly adjustment of the tax and with K_p=0.3 10^{-9} (€year)/kg^2 and K_i=0.05 10^{-9} €/(kg^{2}year)*



(a) (b)

Figure 8.6: *The paper flow (a) and the pulp tax (b) in the PI-regulated paper chain in which the paper flow becomes known after one year. The parameters of the PI controller are K_p=0.1 10^{-9} (€year)/kg^2 and K_i=0.05 10^{-9} €/(kg^{2}year).*



(a) (b)

Figure 8.7: *The prices (a) and recycling rates (b) of the paper chain with a constant recycling capacity increase of* $0.7\ 10^9$ *kg/year*

and subsequently rises between the instants of adjustment. In the case of the time lag between the paper flow and the knowledge of its height, the regulation seems to be slower than in the other cases.

It is possible to regulate the paper flow by a pulp tax. However a rather large effort, i.e. a high tax, is required.

8.3 A Subsidy for Recycling Activities

In the following, we will discuss some measures aimed at the regulation of the recycling rate (6.11). A measure that immediately comes to mind when thinking about an increase of the recycling rate is an increase of recycling activities. An increase of recycling is modelled by increasing the pulp production and supply, and the waste-paper demand, as described by (6.14).

Figure 8.7 shows the effect of a capacity increase on the behaviour of the paper chain. Initially, the chain is in its equilibrium state. After 1 year, a constant increase of $u_q = 0.7\ 10^9$ kg/year is modelled. Figure 8.7a shows the effect on the prices. Since the increased pulp production causes an increase of the demand for waste-paper and an increase of the pulp supply, the waste-paper price increases and the pulp price decreases. Obviously, these price changes have an effect on the recycling rate. As can be seen in Figure 8.7b, the recycling rate first rises due to the increased demand of the recyclers. However, since the waste-paper price increases, the recyclers' waste-paper demand and pulp production decrease. This results in a lower trade on the waste-paper market. The paper price and the paper trade roughly stay at their initial levels. This has the consequence that, in the end, the recycling rate is approximately the same as in the initial situation[1].

The reason for the lack of influence on the final recycling rate can be found by calculating the recycling rate in the equilibrium situation. The equilibrium recycling rate can be calculated from the equilibrium state (B.5) by taking the effect of u_q into account. It appears that the equilibrium recycling rate is given by

$$y_{\mathrm{rec,ss}} = \eta_{\mathrm{A}_2} \tag{8.4}$$

It is independent of the increased pulp production. The simulations show that an increase of recycling activities only temporarily increases recycling but that it has no effect on the final recycling rate, thus confirming the equilibrium analysis.

This example shows that influences on a chain must be carefully evaluated. As already stated in Section 2.8 the reaction of the entire chain, not only of some parts, must be taken into account. An increase of recycling activities alone is not an effective measure for increasing the long term recycling rate.

8.4 The Final Treatment Price

In this section, we discuss a measure that influences the *consumer* behaviour. We consider a change in the quantity of waste-paper that is collected and supplied to the waste-paper industry. Such a change in the collected quantity can be achieved, for example, by adjusting the price for final treatment, i.e. the price for incineration or landfill. A higher final treatment price p_w will stimulate consumers to better sort their waste and to supply more waste-paper to the waste-paper market. When taking the final treatment price into account, the model of the consumers must be slightly modified as described by (6.16) to (6.18). The constant β_w in (6.18) is of the order of magnitude of $\alpha_{2\mathrm{A}_2} \approx 0.7\ 10^9 \mathrm{kg}^2/(\mathrm{year}€)$ in (5.6) and Table 6.2.

The additional disposal costs (in €/year) for the chain, caused by the increase of the final treatment price can be calculated. With the initial waste

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[1] Note that this behaviour does not contradict the controllability of the chain. Controllability only guarantees the ability to reach a particular point but not necessarily to stay there. Moreover, it does not take the magnitude of the control effort into account.

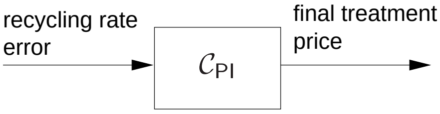


Figure 8.8: *The PI controller of the recycling rate*

stream $w_0 = \big(\frac{1}{\eta_{\mathrm{A}_2}} - 1\big) q_{p20}$ (see Figure 5.4), the disposal costs are

$$\begin{aligned} \text{costs} &= \underbrace{p_w(w_0 - q_w)}_{\text{actual costs}} - \underbrace{p_{w0} q_{w0}}_{\text{initial costs}} \qquad (8.5) \\ &= (p_w - p_{w0})\Big(\frac{1}{\eta_{\mathrm{A}_2}} - 1\Big) q_{p20} - p_w q_w \qquad (8.6) \end{aligned}$$

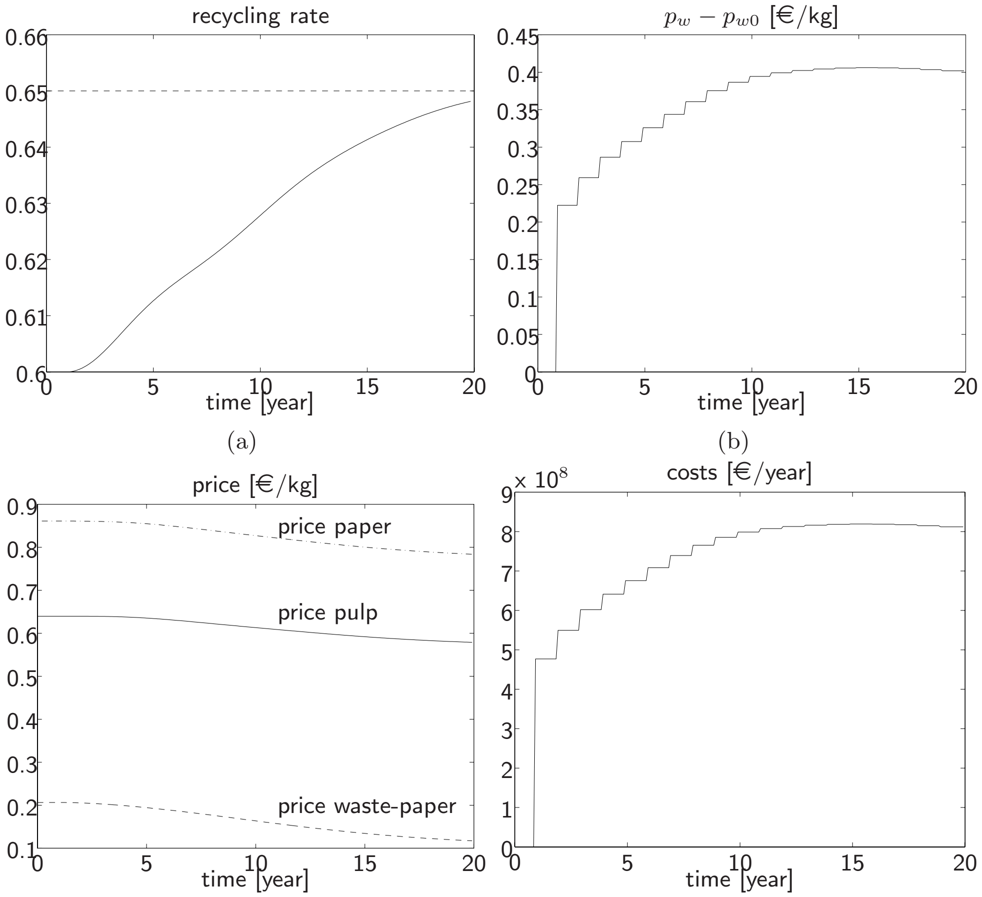
The regulation is illustrated by the example of a proportional-integral controller as depicted in Figure 8.8. Its objective is the adjustment of the recycling rate. In the initial equilibrium situation, the recycling rate is 60%. The objective is chosen to be 65%. Note that not only recycling rates that are lower than the objective value are penalized but also higher values. This is reasonable because a higher recycling rate means extremely high costs, which are certainly undesired.

The results are depicted in Figure 8.9 with a yearly adjustment of the final treatment price. Figure 8.9a shows how the recycling rate approaches the objective value. The corresponding final treatment price is shown in Figure 8.9b. It must be increased by 0.4 €/kg to achieve the objective. The development of the prices in the chain is shown in Figure 8.9c. It can be seen that the waste-paper price drops. This is due to the fact that the waste-paper supply increases. The pulp price and the paper price drop as well. This is due to a rising supply of pulp from waste-paper and subsequently a rising supply of paper, and a nearly constant paper demand. The additional final treatment costs for the chain are depicted in Figure 8.9d. They amount to 0.8 10^9 €/year.

An increase in the recycling rate can be achieved by a change in the final treatment price. However, the increase of this price must be considerable, and as a consequence the final treatment costs increase significantly.

8.5 A Combination of Instruments: Raising the Final Treatment Price and Granting Subsidies for Waste-Paper

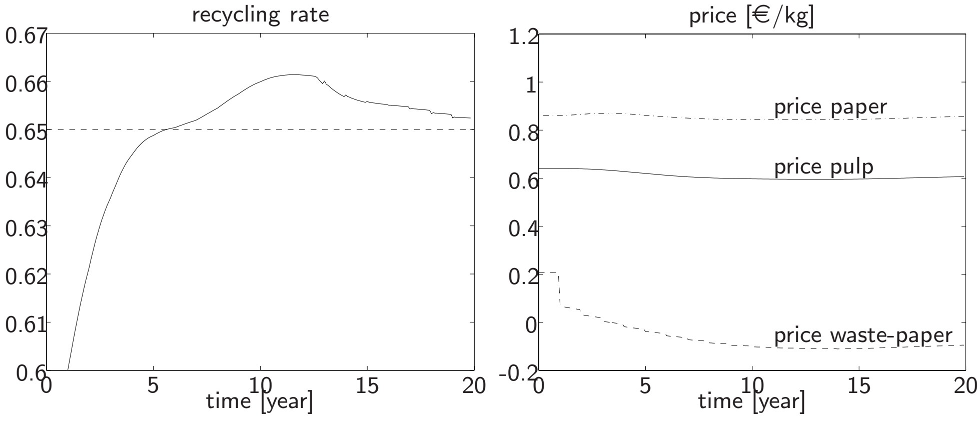
A combination of measures can be applied to better attain the control objective. The control can be made more rapid or more accurate. Moreover, by



(a) (b)

(c) (d)

Figure 8.9: *The recycling rate (a), the final treatment price (b), the product prices (c) and the additional costs for the chain (d) with a PI-regulation of the final treatment price, with* $K_p = -8$ *€/kg,* $K_i = -0.15$ *€/(kg year)*



(a) (b)

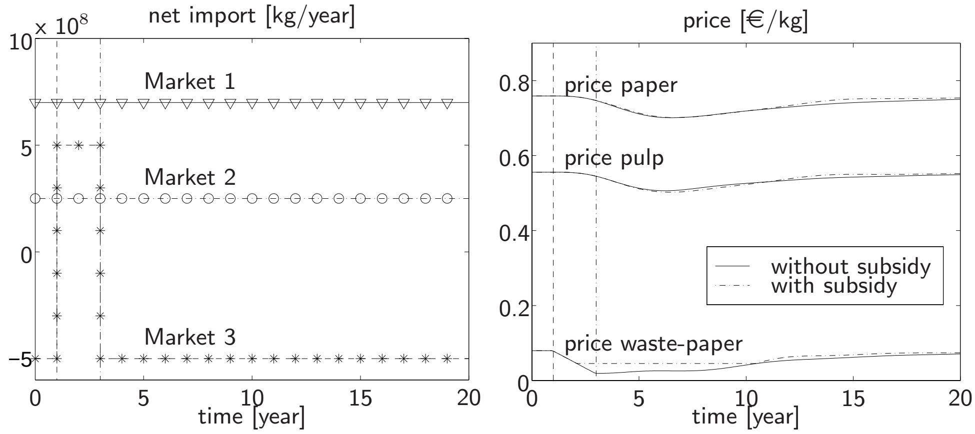
Figure 8.10: *The recycling rate (a) and the prices (b) for a combination of instruments*

a combination of instruments, interesting additional effects can be obtained. If desired, a combination of governmental instruments can, for example, be designed in such a way that it is financially neutral for the authorities.

As an example, we model a chain which is regulated by a combination of the price for final waste treatment and a subsidy for waste-paper. The revenues that the authorities generate by increasing the final treatment price are granted as subsidies for the waste-paper price. This instrument is a combination of the instruments described in Section 8.4 and the subsidy described by (6.13).

In the simulation, the same revenues as calculated in Section 8.4 (Figure 8.9d) are granted as subsidies. The results are shown in Figure 8.10. Figure 8.10a shows the evolution of the recycling rate. It shows a clearly different behaviour than in the case of a mere adjustment of the final treatment price. The recycling rate first rise sharply and exceeds 65%. Then it approaches the objective value from above. The corresponding prices are depicted in Figure 8.10b. The waste-paper price the recyclers have to pay is quite low and even becomes negative. It must be noted that the waste-paper price the collectors obtain is significantly higher. This is due to the relatively high subsidy. The other prices do not show a striking behaviour. They are approximately constant.

By combining the adjustment of the final treatment price and a subsidy for waste-paper, the recycling rate can be increased more rapidly than by a mere adjustment of the final treatment price. Due to the high subsidy for waste-paper, the waste-paper price the recyclers have to pay becomes negative.



(a) (b)

Figure 8.11: *The imports (a) and the prices (b) in the chain without and with a waste-paper subsidy*

8.6 Regulation of the Waste-Paper Price

In this section, we present a measure which is intended to reduce fluctuations of the prices, above all of the waste-paper price. In the past, the waste-paper price was subject to heavy fluctuations, which had consequences for the whole chain. Due to low waste-paper prices the waste-paper collectors were no longer willing to collect waste-paper. This led to a blocking of the chain. A measure that can be taken to prevent the waste-paper price from dropping below a definite level p_m is the installation of a fund which pays the difference between the actual waste-paper price and this level when the waste-paper price drops below it. Such a fund can be described by the subsidy (6.13) $p'_{\mathrm{M3}} = p_{\mathrm{M3}} + u_{\mathrm{s}}$ with

$$u_{\mathrm{s}} = \begin{cases} p_m - p_{\mathrm{M3}} & \text{if } p_{\mathrm{M3}} \leq p_m \\ 0 & \text{else} \end{cases} \tag{8.7}$$

The price p_m is the standard cost price for waste-paper; in the Netherlands it amounts to $p_m \approx 0.04$ €/kg [134].

The behaviour of a chain with such a fund, as well as the behaviour without the fund is shown in Figure 8.11. Figure 8.11a shows the imports and exports that are applied to the chain in this simulation. In the initial situation, there is a net waste-paper export (q_{e3} is negative). After one year, it is drastically changed to a situation with a significant waste-paper import. In practice, such a situation occurred, for example, in 1992 in the Netherlands as a consequence

	average recycling rate	$\|\frac{dp_{\mathrm{M}_1}}{dt}\|_2$	$\|\frac{dp_{\mathrm{M}_2}}{dt}\|_2$	$\|\frac{dp_{\mathrm{M}_3}}{dt}\|_2$
unregulated	0.61	0.029	0.033	0.045
with subsidy	0.61	0.032	0.034	0.035

Table 8.2: *The average recycling rate and the price fluctuations for a chain without fund and with a waste-paper price subsidy*

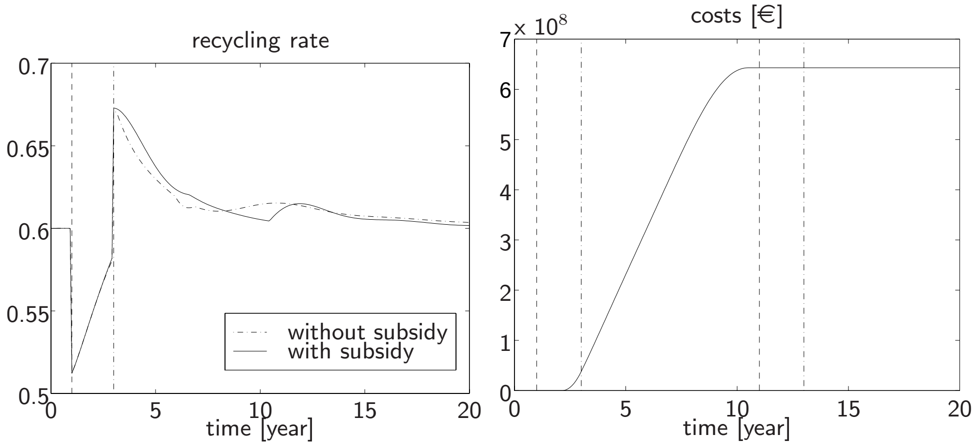
of the so-called *Verpackungsverordnung* of the *Abfallgesetz* in Germany (see [60]). This law prohibited the incineration of packaging waste. Since the German recycling capacity was insufficient, waste that could not be recycled in Germany was exported to the neighbouring countries. The large quantities supplied to the Dutch market caused a decrease in the waste-paper price, which even became negative for some grades of waste-paper. In the simulation, two years after the change, the initial export situation is applied again so that the chain can return to the initial situation.

Figure 8.11b shows the price development with and without subsidy. It can be seen that the subsidy is granted after 2 1/2 years. It prevents the waste-paper price p_{M_3} from dropping below p_m. The other prices behave similarly in the case with subsidy and in the case without subsidy. Since the objective of the subsidy is a damping of the price fluctuations, we examine these fluctuations in more detail. A measure for the fluctuations is the norm of the derivative with respect to time

$$\left\| \frac{dp_{\mathrm{M}_i}}{dt} \right\| \quad \text{for example the quadratic norm } \|x\|_2 = \left[\int_0^{20} |x(t)|^2 dt \right]^{\frac{1}{2}} \tag{8.8}$$

In Table 8.2, the price fluctuations in the chain without fund and with a waste-paper price subsidy are compared for the net import to the waste-paper market depicted in Figure 8.11a. The influence of various imports and exports will be considered in more detail in Section 8.8. It can be seen that, for this situation, the subsidy clearly reduces the fluctuations of the waste-paper price. The fluctuations of the other prices slightly increase as a consequence of the subsidy.

Figure 8.12a shows the behaviour of the recycling rate for a situation with a waste-paper subsidy and without subsidy. The recycling rate is not significantly increased by the subsidy; the average is the same as for the unregulated chain (see also Table 8.2). The costs for the fund are depicted in Figure 8.12b. They amount to nearly 0.65 10^9 €, which is quite considerable.



(a) (b)

Figure 8.12: *The recycling rates (a), and costs (b) of the chain without and with a waste-paper subsidy*

8.7 Regulation by Convenants

This section deals with the installation of a self-regulation scheme as it was introduced by the so-called paper fibre convenant in the Netherlands. The paper fibre convenant was introduced as a voluntary agreement between the paper chain actors and the government. It was a consequence of the governmental 'Note on Prevention and Recycling' of 1989 and later additions of the Dutch National Environmental Policy Plan [108], which cites paper waste[2] as a principal waste stream, and demands the entire chain to work on prevention and recycling of this stream. The most important measures introduced in the paper chain by the paper fibre convenant are [134]

- The intensification of waste-paper collection in households and industry.
- A purchase guarantee to communities and waste-paper collectors for all collected or supplied waste-paper. The costs that arise from this guarantee are at the expense of the paper and cardboard industry.
- A participation binding on all paper and cardboard producers and consumers in order to prevent free ridership of single companies.

It must be noted that these agreements are not only interesting from an *environmental* point of view, but as mentioned in the previous section, they are also in the *economic* interest of the chain. In the past, waste-paper supply and

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[2] For the definition of waste, see Section 2.4.

demand fluctuated heavily [76]. In the 80s the fluctuations were due to general economic trends. Later they were related to the development of a world market. Presently, supply and demand depend on many international factors. Demand for pulp rises stepwise due to investments in big new paper mills whereas supply grows gradually due to national recycling regulations. These factors together with the international character of the waste-paper market led to extreme price fluctuations in the past. It is expected that the agreements of the convenant will be able to moderate these fluctuations. Moreover, a less fluctuating waste-paper supply means that recycling capacity can be better used, which is economically advantageous.

The agreements on purchase guarantees, included in the convenant can be put into practice by installing a so-called *removal fund*. If the waste-paper price drops below the standard cost price, this fund will buy the excess waste-paper and will pay the cost price. The costs of the removal fund are to be paid by the first paper users: printers, publishers, producers of packaging materials, etc.

When modelling the paper fibre convenant, not all the aspects can easily be captured in mathematical formulae. The intensification of collection, for example, is difficult to quantify. The removal fund is modelled by adding an inventory g_f to the paper chain system at the location of the waste-paper market as shown in Figure 8.13. It acts as a bypass of waste-paper: If the waste-paper price drops below the standard cost price p_m, the removal fund will come into action. It will buy the excess waste-paper supply. When demand is larger than supply and the waste-paper price is above a definite reasonable price p_r, the removal fund can be emptied by selling waste-paper to the recyclers. In any case, the removal fund does not (directly) influence the waste-paper price. This behaviour can be modelled by adding an external actor to Market 3 which acts as an importing and exporting actor according to (see 5.17)

$$q_{\mathrm{ef}} = \begin{cases} \max(q_{s\mathrm{M}_3} - q_{d\mathrm{M}_3}, 0) & \text{if } p_{\mathrm{M}_3} \le p_m \\ \max(q_{d\mathrm{M}_3} - q_{s\mathrm{M}_3}, 0) & \text{if } p_{\mathrm{M}_3} > p_r \text{ and } g_f > 0 \end{cases} \tag{8.9}$$

The inventory g_f is filled if $q_{\mathrm{ef}} > 0$ and emptied if $q_{\mathrm{ef}} < 0$.

Here, the system after the implementation of the removal fund is compared to the system without fund [91, 90]. In this example, the equilibrium situation is taken as initial situation. The net imports to the chain used in the simulation of the paper chain model are shown in Figure 8.14a. In the beginning, they are the same as in Section 8.6 (Figure 8.11a). After 11 years an inverse situation is created with an increase of waste-paper export. This increase of the export makes it possible to empty the removal fund inventory. The reactions of the prices in a chain with removal fund and of a chain without removal fund are

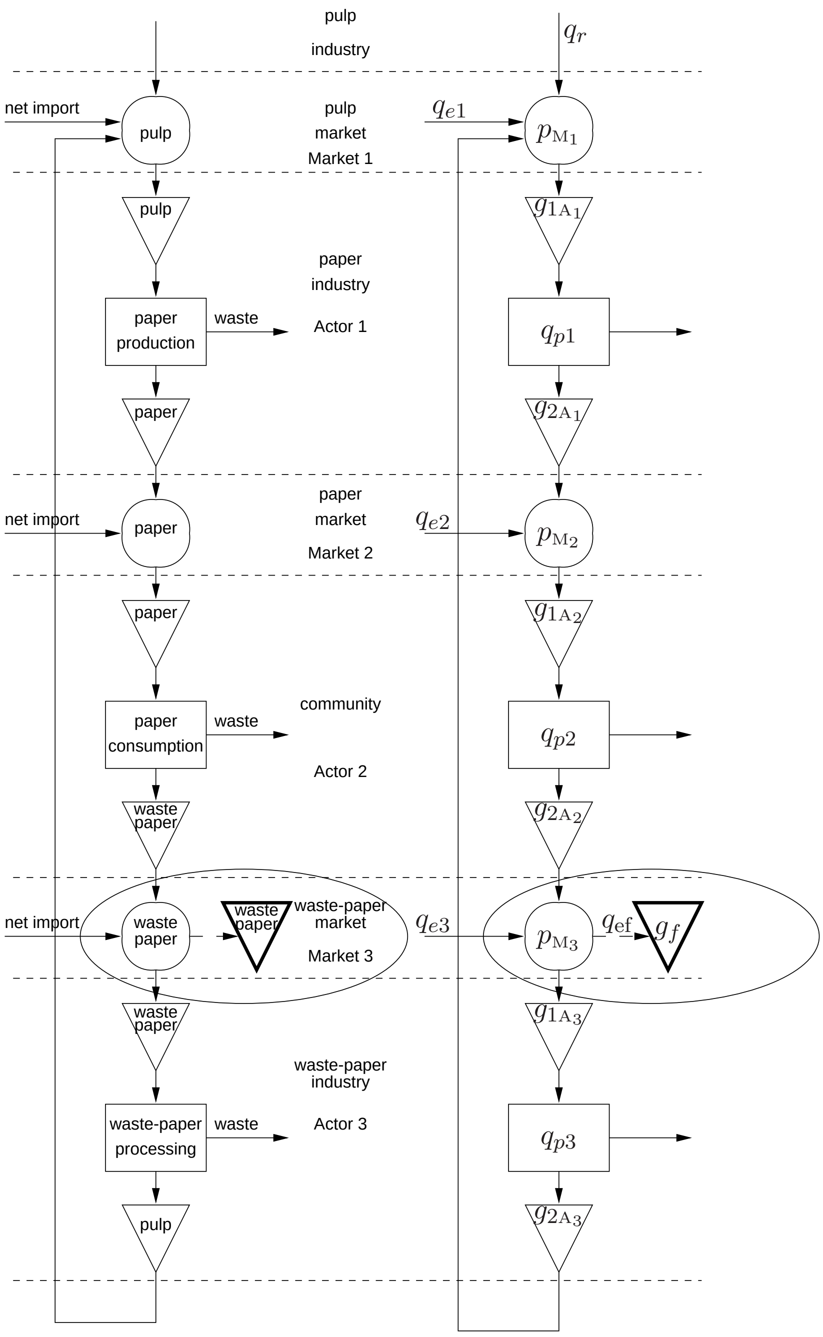
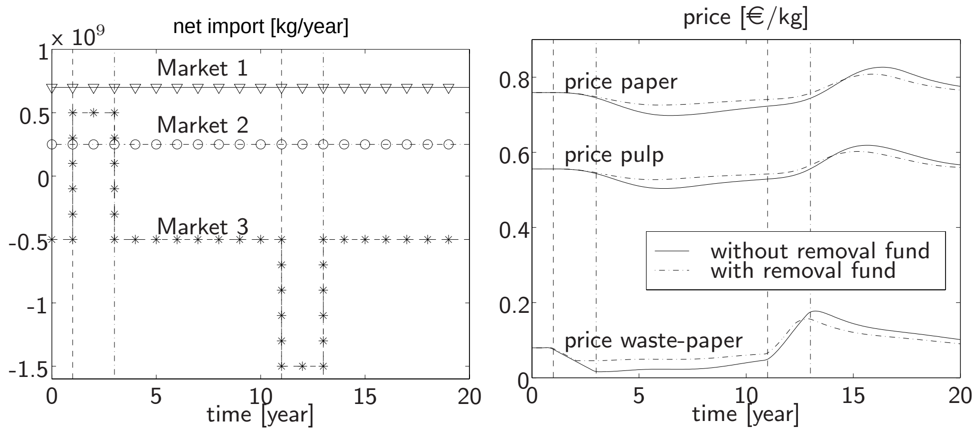


Figure 8.13: *The flowchart of the paper chain with removal fund. The left figure shows the various markets and companies, and the material flows. The right figure shows the corresponding variables. The waste-paper inventory of the removal fund is shown inside the ellipse.*



(a) (b)

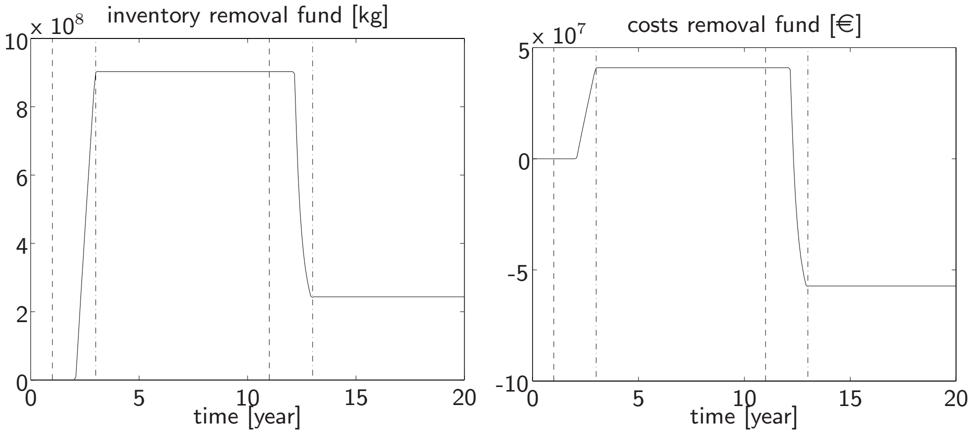
Figure 8.14: *The net imports (a) and the prices (b) of the paper chain with and without paper convenant, with p_r=0.14€/kg*

	$\|\frac{dp_{M_1}}{dt}\|_2$	$\|\frac{dp_{M_2}}{dt}\|_2$	$\|\frac{dp_{M_3}}{dt}\|_2$
unregulated	0.062	0.067	0.107
with removal fund	0.043	0.046	0.085

Table 8.3: *The price fluctuations for a chain without fund and with a removal fund*

depicted in Figure 8.14b. In both situations all the prices drop after one year due to the excess waste-paper supply. The increase of export after 11 years, on the other hand, causes the prices to rise. It can be seen that in the case of the removal fund, the prices fluctuate less than without removal fund. The norm of the price derivatives (8.8) is depicted in Table 8.3. The fluctuations of the three prices are clearly smaller in the chain with removal fund. They are reduced by 20 to 30%.

The evolution of the removal fund inventory is shown in Figure 8.15a. Initially, it rises due to the low waste-paper price and the excess waste-paper supply. When the net waste-paper import stops, the excess supply also stops and the removal fund inventory remains unchanged. When the situation is changed into the situation of excess waste-paper demand and the waste-paper price rises above the reasonable price p_r, the removal fund can sell some of the stored waste-paper, which results in a decrease of the removal fund inventory. When the export is changed to the initial level again, the excess demand situation stops and the removal fund inventory stays at a constant level.



(a) (b)

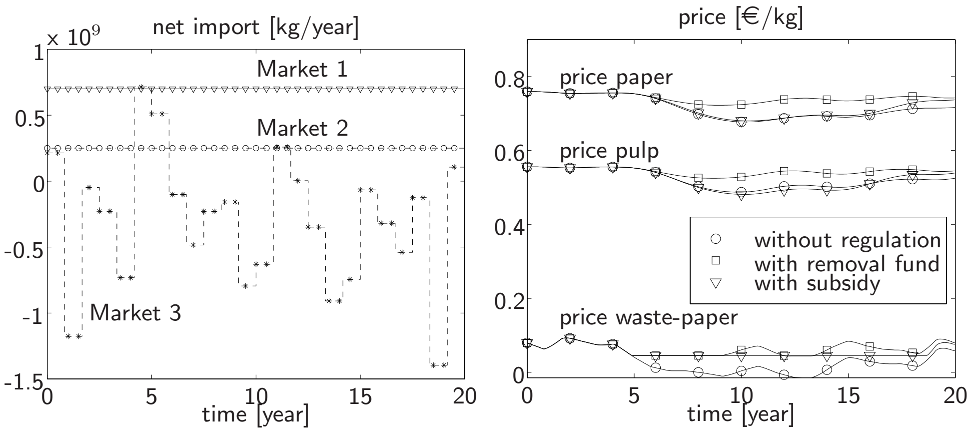
Figure 8.15: *The inventory of the removal fund (a) and the costs (b) associated with it*

The costs associated with the removal fund are shown in Figure 8.15b. The costs for purchasing waste-paper and the revenues from selling waste-paper are considered. First, the costs increase because the removal fund has to buy waste-paper. When this waste-paper can be sold, the removal fund generates revenues. For one kg of waste-paper these revenues are larger than the costs made to purchase the waste-paper, because the removal fund sells at a higher price than it purchases. In the example situation modelled here, even net revenues are achieved in the end. In sum, a maximal expense of $40\ 10^6$ € is involved, and in the end revenues of $60\ 10^6$ € are obtained. For the considered situation, the costs of the removal fund system are much lower than the costs of the subsidy system, as can be seen from a comparison of Figures 8.12b and 8.15b.

8.8 Disturbances

A product chain is subject to disturbances of various kinds. The markets in the chain, for example, are influenced by external factors such as external demand and supply. The companies are subject to changes in resource quality, supply of skilled workers, general economic trends and crises, etc.

We already considered a disturbance of the waste-paper market in the previous sections. This disturbance — due to changes of imports and exports — had the shape of a step function. In this section, we consider the example of a *random* disturbance on the waste-paper market. We evaluate the effect



(a) (b)

Figure 8.16: *The influence of a randomly disturbed waste-paper market (a) on the prices (b) in the chain*

of such a disturbance on an unregulated chain, on a chain with a subsidy for waste-paper as discussed in Section 8.6, and on a chain with removal fund as discussed in Section 8.7.

Figure 8.16a shows an example of such a disturbance. The net waste-paper import q_{e3} consists of a constant part and of a random part, which changes every 10 months. The imports are $q_{e1} = 0.7\,10^9$kg/year, $q_{e2} = 0.25\,10^9$kg/year, $q_{e3} = -0.5\,10^9$kg/year$+q_d$ with q_d a normally distributed random input with mean 0 and variance σ^2=$(0.75\,10^9$kg/year$)^2$. The reaction of the prices to the example disturbance is shown in Figure 8.16b. In the beginning, the prices in the regulated chains behave similarly to the prices in the unregulated chain. After five years, a clearly different behaviour can be observed.

This kind of random disturbance is applied to the chain models — the unregulated chain, the chain with waste-paper subsidy and the chain with removal fund — and averaged over 1000 runs in which at least one of the regulations becomes active. The result for the price fluctuations is shown in Table 8.4. The two regulations achieve the objective of damping the waste-paper price fluctuations $\|\frac{dp_{\mathrm{M}_3}}{dt}\|_2$. The waste-paper subsidy performs better than the removal fund. However, it also causes the pulp price and the paper price to fluctuate more heavily than in the unregulated chain, whereas the removal fund damps their fluctuations.

	$\lVert \frac{dp_{\mathrm{M}_1}}{dt} \rVert_2$	$\lVert \frac{dp_{\mathrm{M}_2}}{dt} \rVert_2$	$\lVert \frac{dp_{\mathrm{M}_3}}{dt} \rVert_2$
unregulated	0.059	0.062	0.141
with subsidy	0.062	0.063	0.122
with removal fund	0.051	0.053	0.129

Table 8.4: *The fluctuations of the prices due to a random disturbance on the waste-paper market*

8.9 Conclusions

With the chain model, various external influences can be simulated and evaluated. The simulations describe the evolution in time of the chain system for a particular initial situation. Unlike the analysis of stability, controllability and observability described in the previous chapter, they do not allow conclusions for other initial conditions or conclusions outside the considered time frame. Examples of such simulations are the control of the paper flow by the adjustment of a pulp tax according to a proportional-integral controller, and the control of the recycling rate by an increase of recycling activities. This increase of recycling activities is an example of a measure that does not have the desired effect. It does not increase the long-term recycling rate.

The recycling rate can be controlled by the adjustment of the price for final waste treatment (for example final discharge or incineration). It can also be controlled through a combination of measures in which the final treatment price is raised and the revenues are granted as subsidies for waste-paper. The effect of the additional subsidy is considerable. It causes the recycling rate to increase much faster.

Price fluctuations, for instance of the waste-paper price, caused an undesired chain behaviour in the past. Two measures aimed at reducing price fluctuations are examined and compared: (i) a waste-paper subsidy and (ii) a removal fund, which buys waste-paper in the case of an undesired low waste-paper price and sells at a high price — a measure taken recently by the actors of the paper chain in the framework of the paper fibre convenant. It is demonstrated that the two measures do damp the waste-paper price fluctuations. The waste-paper subsidy leads to a more effective damping of the waste-paper price fluctuations but it does not reduce fluctuations of the other prices, whereas the removal fund reduces the fluctuations of all the prices.

Chapter 9

Conclusions and Suggestions for further Research

In the previous chapters, essential properties of product chains were discussed. They include environmental as well as economic aspects. Subsequently, a method was developed to model such chains. The possibilities of analysing and controlling product chains were illustrated for the example of the paper chain.

In this chapter, the main conclusions are drawn with respect to the chain model (Section 9.1.1) and with respect to chain control (Section 9.1.2), and suggestions are given for further research.

9.1 Main Conclusions

This study examined how control theory can contribute to the analysis and control of product chains. The study of chain dynamics is relevant for the appropriate control of product chains, which is of considerable economic, societal, and environmental interest. The increasing importance of recycling manifests itself in the great variety of products that are being recycled, ranging from metals, paper, glass, plastic, batteries and textiles to computers, kitchen articles, tyres and cars. Therefore, product chain complexity is increasing. It is interesting for governments as well as for industry to know how to control such chains.

Control theory offers tools and concepts to model dynamic systems and to analyse them with respect to essential properties, such as stability, controllability, and observability. These tools have long been used in the analysis of, especially, mechanical, electrical and chemical systems. The aim of this study was to illustrate the usefulness of these tools and concepts for chain control.

They appear to be particularly suitable for the analysis of product chains, because product chains can be described in the same way as many technical systems.

9.1.1 Chain modelling

Product chains are combinations of standard production systems and of product markets between them. Consequently, by combining company and market models, models of product chains can be obtained. Since companies and markets are dynamic systems, they can be described by dynamic mathematical models in state space form.

The company models describe the resource and product inventory behaviour. By applying the law of mass conservation to the inventories, two coupled dynamic equations can be derived. A company's production decision, and its decisions on the supply and demand of products are substituted into these equations. These decisions are based on the market prices. All the actors in a chain can be modelled in the same standard way, because various production decisions, such as production to stock and just-in-time production, can be included. The actors are modelled on a rather high level of aggregation. They do not include specific processes *inside* a company, but only one aggregated process that links the company resources to its products. For a model on the chain level, this is the appropriate level of aggregation.

The market models are based on the Walrasian theory of exchange. They are based on the idea that the derivative of the price on a market is a linear function of the excess demand. It turns out that such a simple market model is an adequate description of markets on the product chain level. Since the amount of products traded on a market is bounded to the minimum of products supplied or demanded, the chain models are nonlinear and therefore mathematically complicated.

Influences on product chains include regulatory, economic, and social governmental influences, such as taxes, subsidies and the stimulation of environmental research, and influences of the chain actors, such as agreements concerning product flows and on the introduction of recycle loops. Influences such as product taxes and subsidies can be incorporated straightforwardly as inputs into the chain model. Other influences, such as the stimulation of environmental research, however, are beyond the scope of this study, and have not been considered.

The chain models contain various — unknown — parameters. In principle, these parameters can be estimated by standard parameter estimation methods, such as least squares methods. However, the collection of relevant time series for the estimation proved to be quite involved. The amount of data material is less extensive than in most technical systems. As an example the Dutch paper chain was examined. For this chain, limited time series of pulp, paper, and waste-paper prices have been monitored, as well as time series of imported and exported mass flows. With these data, the parameters in the paper chain model can be estimated such that a good agreement between the prices calculated by the model and the observed prices is obtained, given that the imports and exports behave as monitored. Although the amount of time series data is rather small for an accurate parameter estimation, it can be concluded that the model reflects the observed price behaviour.

9.1.2 Chain analysis and control

The main conclusions obtained on the basis of the simple chain model are given here. The paper chain was analysed with respect to stability, controllability and observability. The stability of the nonlinear chain model was analysed by means of a Lyapunov approach. It has been shown that the paper chain is stable; the prices and product flows do not grow without bounds. Controllability and observability results were obtained from a linearized model. It was made clear that controllability analysis can help us to identify appropriate ways to influence product chains. In principle, the linearized product chain is controllable by governmental instruments (taxes and subsidies on products, taxes and subsidies on activities, the regulation of imports and exports, or an adjustment of the price for final treatment), as well as by measures taken by the chain actors themselves, such as those laid down in convenants. The observability of the chain by the measurement of the product flows, the product prices and the recycling rate was analysed. The chain appears to be observable by the measurement of any of these output variables.

Via simulations, the effectiveness of various regulations was examined. Simulations describe the evolution in time of the chain system for a *particular* initial situation. Unlike the analysis of stability, controllability and observability mentioned above, they do not allow conclusions for other initial conditions or conclusions outside the considered time frame. A number of objectives and instruments were considered. Difficulties in chain regulation are due to the fact that measures cannot be taken on a day-to-day basis and that the data, which form the basis of the regulations, are often outdated. The model nonlinearities add to the regulation difficulties. It appears that the long-term recycling rate cannot be regulated by means of an increase of recycling activities alone. However, other objectives can be reached. The paper flow, for example can be controlled by a tax on pulp, and the recycling rate can be regulated by adjusting the price for final waste treatment. By a combination of instruments — the adjustment of the final treatment price and a simultaneous subsidy for waste-paper — the desired recycling rate can be achieved more rapidly. Since this kind of regulation has neither positive nor negative financial effects for the authorities, such measures are politically interesting.

Finally, it was demonstrated that the model can be used to compare the effect of various regulations. For the objective of damping price fluctuations — due to exterior influences — a waste-paper subsidy was compared to the self-regulating regime, agreed upon in the paper fibre convenant. This convenant foresees mutual sale and purchase agreements, aimed at a more secure throughput of paper through the chain and at less fluctuating prices. This can be achieved by the installation of a removal fund. The removal fund is modelled by the addition of an actor who supplies and demands on the waste-paper market. Simulations show that, for a number of example situations, both the waste-paper subsidy and the removal fund are indeed able to reduce price fluctuations. The waste-paper subsidy performs better for the waste-paper price, but it does not damp the fluctuations of the other prices. The removal fund reduces the fluctuations of all the prices.

9.2 Suggestions for further Research

The techniques that were presented in this study for the modelling, analysis and control of product chains show some options for effective chain control. For better insight into product chains the following suggestions for future research are made.

The parameters in the paper chain model are estimated on the basis of the observed time series of prices, imports and exports in this chain. The paper chain was chosen as an example chain because a relatively large amount of data is available. However, compared to the usual parameter estimation of technical systems, the amount of data is rather small. Furthermore, the data are often less accurate than those usually obtained in technical systems. Moreover, the number of parameters that must be estimated is considerable. Therefore, more data should be gathered to improve the model validation. Data for other chains, such as the steel chain seem to be available. Modelling of these chains and estimating the parameters can contribute to the validation of the modelling technique.

The chain model has a piecewise linear structure. For controllability and observability analysis, a linear approximation of this model is applied. To obtain more general results, further analysis with inclusion of the nonlinear model structure is desirable.

The chain control strategies simulated in this study aim to regulate the considered output to a desired *value*. In some cases, regulation along a *trajec-*

tory may be desired. The possibilities of such control can be analysed using the model presented in this study. Further research on this topic should be performed.

In this study, the recycling rate is considered to be an indicator of the environmental performance of the chain. It is doubtful whether the recycling rate is a good indicator of the environmental performance of a product chain, but other indicators of environmental stress may have even larger shortcomings. Research, aimed at the establishment of a good environmental indicator for product chains, is required.

In this study all the chain actors are modelled in the same standard way. The opposite interests of consumers and producers in the chain are not explicitly taken into account. It is useful to also incorporate the opposite interests into a chain model by explicitly modelling the profit maximization of producers and the utility maximization of consumers. The question then arises whether, in that case, it is possible to establish a state space description of the entire product chain. It should be investigated to what extent control theory can be used to analyse such a system.

Chain regulation by convenants as well as quality and economic requirements stimulate the cooperation between chain actors. This has major consequences. Enterprise information systems, for example, should span the entire chain, and investments, recompenses and fines should be evenly distributed in the chain. These trends demonstrate the relevance of further research concerning chain optimization. Suboptimization of parts of chains should be avoided. This need for change can be supported by a governmental policy that focuses on it. Systems that support decisions of government and industry in this domain are advisable and will gain relevance in the future.

Appendix A

Data

This appendix gives an overview of the consulted data sources, and of the assumptions and calculations performed to estimate unavailable data.

A.1 Sources of Data

For obtaining the data of the paper chain, various sources have been consulted. The most important sources of information in the Netherlands with respect to this chain are the following:

- *The Dutch Statistical Office (CBS)* collects data of, among others, the various branches of industry. Supply and consumption charts contain the quantities of produced products on a yearly basis, expressed in *monetary units*: in the actual prices and in so-called constant prices, i.e. the prices of the preceding year. For paper and paper products, the domestic production, consumption, as well as imported and exported quantities have been monitored in *mass units* in an extensive study in 1990 [25]. This study was carried out in the framework of a national programme of an input-output table [93, 99] in physical and monetary units for all branches. Furthermore the Statistical Office monitors price indices. Price indices pretend to represent the price development of a product, a service, or a group of products. They are published on a monthly basis for many products. For others they are published on a yearly basis.

  Supply and consumption are grouped in categories according to a standard classification of companies (SBI) or a classification of products (SGN). Unfortunately, when there are three or less companies active in one category, the data are not allowed to be published because of an active pledge of secrecy. A company would otherwise be able to calculate relevant data of its competitors. The import and export data

		pulp	waste-paper and cardboard	paper and and cardboard	paper and cardboard products
National Statistics	import	639	866	2499	810
	domestic production	0	1605	2757	3798
	export	9	640	1891	845
VNP	import	ca 662	ca 1039	-	-
	domestic production	ca 158	1820	2742	-
	export	ca 15	ca 815	-	-
RIVM	import	-	-	-	-
	domestic production	-	1455	-	3685
	export	-	-	-	-

Table A.1: *The imported, produced and exported paper quantities in 1990 (in [10^6 kg/year]) according to different sources*

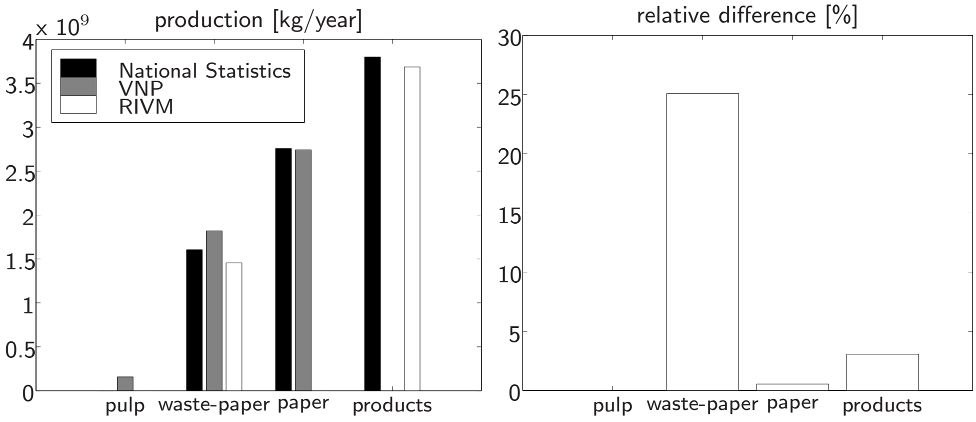
are classified according to Standard International Trade Classification (SITC).

- The *Netherlands' Paper and Board Association (VNP)* monitors the pulp, paper and cardboard sales of Dutch companies, the apparent pulp, paper, and cardboard consumption, as well as the waste-paper collection and consumption in mass units on a yearly basis [144].

- In 1990, the *National Institute of Public Health and Environmental Protection (RIVM)* monitored the waste flows from households and companies in the framework of the Second National Enquiry on the Environment (NMV2) [37, 111, 112]. The waste flows of numerous materials (among which paper and cardboard, ferro etc.) were monitored according to different categories of waste such as household waste, industrial waste, office waste, waste from hospitals, etc.

The different sources handle slightly different criteria for aggregation into categories, which makes direct comparison difficult. Moreover, as stated before, even the Statistical Office uses different classifications in their various publications.

Another point that makes comparison difficult is that the different sources are not independent. The Statistical Office collects data on the basis of surveys of companies, and these companies are organized in the Netherlands' Paper and Board Association. This association, on the other hand, completes its data with those published by the Statistical Office.



(a) (b)

Figure A.1: *The production according to the different sources and the relative difference between the maximum and minimum published values*

A.2 An Example: The Year 1990

For the year of 1990, we formulate a list of relevant data in order to give an idea of their accuracy. Where necessary, we perform calculations on the original data.

The Statistical Office collects data in various ways: production statistics are obtained by surveys in companies. Prices are monitored directly in production and trade companies. Data on foreign trade (import and export) were formerly obtained from customs administration. Since may 1992, this source has no longer been available for countries in the European Union, because of the Common Market. Since 1992, companies are obliged to directly relay the information to the Statistical Office.

The data published by VNP originate from data of the joined paper- and boardmills. The RIVM data were first published in [37]. The sources used in this publication are data of the Statistical Office, reports of consulting companies on waste analysis, and mainly the (in 1990) most recent stocktaking of incinerated and dumped waste of the *Afval Overleg Orgaan (AOO)*, and the most recent environmental exploration by RIVM.

The import, domestic production and export according to different sources are summarized in Table A.1 and shown in Figure A.1a. Figure A.1b shows the relative difference between the data. As can be seen, it is quite large: Where two or more sources are available, the relative difference lies between 0.5% and 25%.

There are various reasons for the differences in the observed quantities.

The Statistical Office, for example, does not observe pulp production in the Netherlands whereas VNP does. This is due to the fact that there is no single pulp producer in the Netherlands but rather an integrated pulp and paper producer. The Statistical Office does not count this activity among the pulp production activities. Another difficulty lies in the observation process itself. A structural difficulty in observing mass flows (in mass units) arises from the addition of chemicals, etc. in the different processes. The mass of produced paper might therefore be larger than the mass of the resources, because of the addition of filling material, ink, binding agent etc. The mass of waste-paper might be larger than the mass of produced paper.

A.3 Time Series

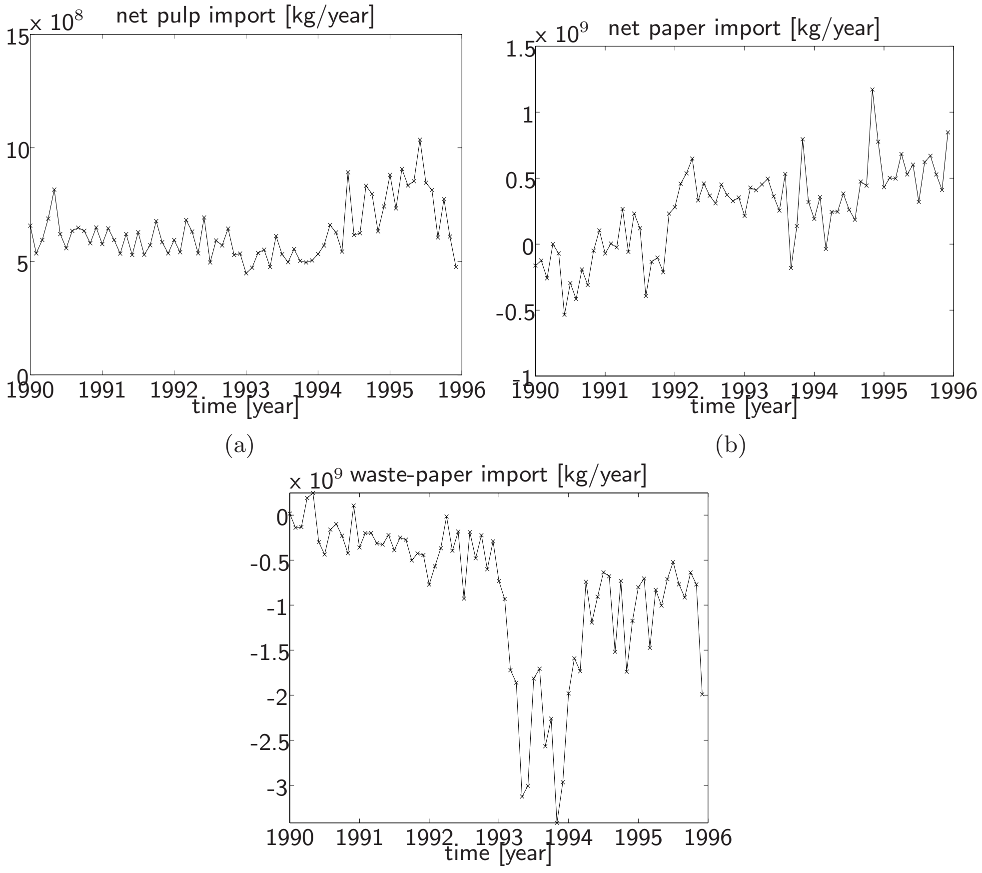
In the following, only the data published by the Statistical Office will be used because this guarantees the most consistent and complete data set. Time series of imported and exported quantities are monitored as well as time series of pulp, paper and waste-paper prices.

The division 'International Trade' of the Dutch Statistical Office publishes monthly data on the value of imported and exported products in different categories, such as pulp and waste-paper, paper and cardboard, and paper and cardboard products [39]. The definitions of import and export used by the Statistical Office are:

- Import: All products that are brought to the Netherlands for use in free trade
- Export: All products that are intended for use outside the Netherlands

Unfortunately, the waste-paper and pulp imports (and waste-paper and pulp exports) are aggregated into one category. The less aggregated data are not published on a monthly basis, but they are only available on a yearly basis. To the purpose of our calculations, it is assumed that the ratio of the amount of imported (respectively exported) waste-paper and the amount of the sum of imported (respectively exported) waste-paper and pulp remains unchanged in the course of one year. The development of imports and exports from 1990 to 1996 is illustrated in Figures A.2a through c. In this period, there is (basically) a net pulp and paper import and a net waste-paper export.

Prices can be calculated on the basis of price indices (published by [38]) and the prices in 1990. The price indices published in [38] represent the price development since 1990. The prices in 1990 can be calculated from [25]. In 1990, the Statistical Office [25] published the imported, produced and exported quantities in monetary units (in actual prices) and in mass units. From this,



(a) (b)

(c)

Figure A.2: *The net pulp (a) and paper (b) imports and the net waste-paper exports (c) between 1990 and 1996. The crosses denote published data.*

	import	domestic production	export	weighted average
pulp	0.61	-	0.55	0.61
waste-paper and cardboard	0.086	0.077	0.127	0.091
paper and cardboard	0.78	0.64	0.74	0.71
paper and cardboard products	2.26	2.41	2.29	2.36

Table A.2: *The prices in 1990 in [€/kg]*

the prices per mass unit can be calculated. They are given in Table A.2 as average prices, weighted according to the mass flows of the sub-categories.

The prices of pulp, paper, and waste-paper are depicted in Figure A.3a through c. For paper, price indices of import, Dutch consumption, and export are published. The average price is depicted in Figure A.3b. For pulp and waste-paper, only price indices for import are published. For the prices, a more recent data set than for import and export is available (up to September 1998), because the prices are published earlier than the import and export data.

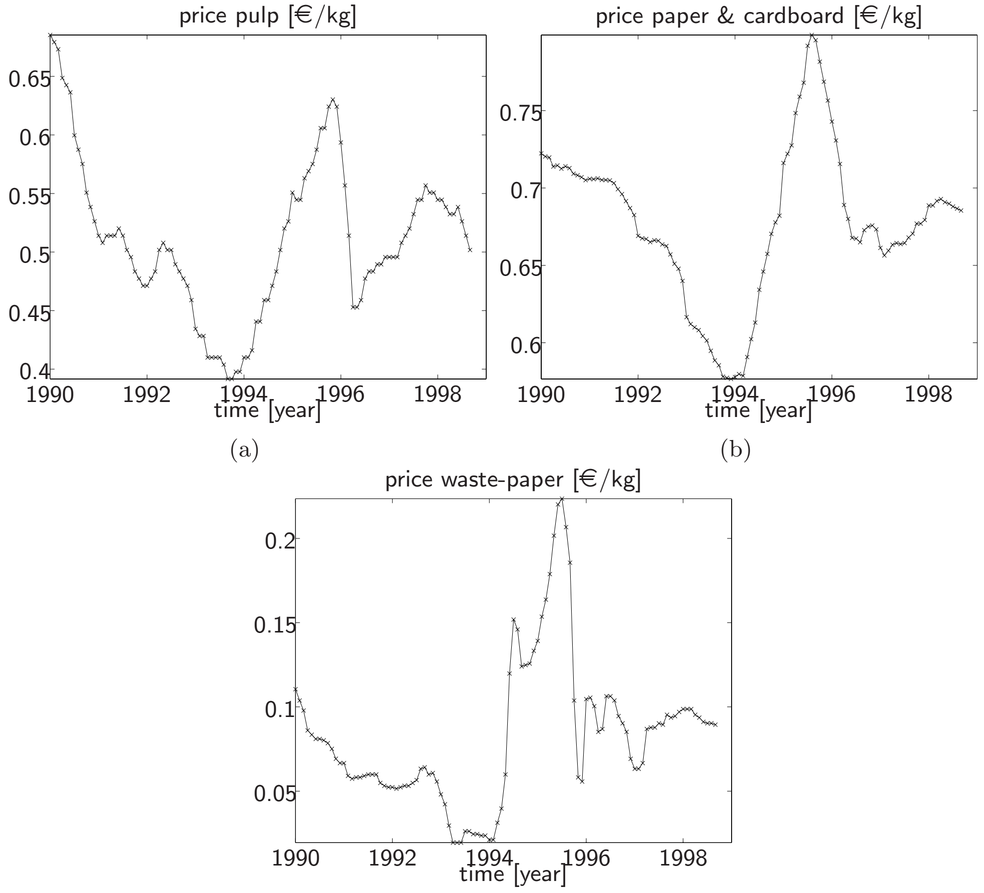
A.4 The Efficiencies

The efficiencies can be estimated from the data of the Statistical Office [25]. They are approximately:

paper production	0.9
waste-paper production	0.6
pulp production from waste-paper	0.8

A.5 Conclusions

Quite extensive data material describing the paper chain is available from various sources. The most complete data sets are collected and published by the Statistical Office. It publishes, for example, prices, imported, and exported quantities of pulp, paper, waste-paper, and paper products on a monthly basis. These data must not be taken as 'hard' data. The comparison with other sources shows significant differences. This indicates the accuracy of the data, which can be explained by fundamental difficulties in collecting data of production systems.



(a) (b)

(c)

Figure A.3: *The pulp (a), paper & cardboard (b), and waste-paper (c) prices. The crosses denote the published data.*

Appendix B

Details of the Chain Model

In this Appendix, the matrices occurring in the equations of the chain model in Chapter 6 and the matrices in the piecewise quadratic Lyapunov function of the system discussed in Chapter 7 are given.

B.1 The Matrices in the System Equations

The paper chain model (6.6) is given by

$$\dot{x} = A_c x + B_c q + H_c c + f_c \tag{B.1}$$

with the matrices A_c, B_c and H_c

$$A_c = \begin{bmatrix} \rho_1(D_{1\mathrm{A}_1} - D_{2\mathrm{A}_3} - \nu_1) & B_{1\mathrm{A}_1} & 0 & 0 & 0 & B_{2\mathrm{A}_3} \\ \rho_1 C_{1\mathrm{A}_1} & A_{\mathrm{A}_1} & -\rho_2 C_{2\mathrm{A}_1} & 0 & 0 & 0 \\ 0 & B_{2\mathrm{A}_1} & \rho_2(D_{1\mathrm{A}_2} - D_{2\mathrm{A}_1}) & B_{1\mathrm{A}_2} & 0 & 0 \\ 0 & 0 & \rho_2 C_{1\mathrm{A}_2} & A_{\mathrm{A}_2} & -\rho_3 C_{2\mathrm{A}_2} & 0 \\ 0 & 0 & 0 & B_{2\mathrm{A}_2} & \rho_3(D_{1\mathrm{A}_3} - D_{2\mathrm{A}_2}) & B_{1\mathrm{A}_3} \\ -\rho_1 C_{2\mathrm{A}_3} & 0 & 0 & 0 & \rho_3 C_{1\mathrm{A}_3} & A_{\mathrm{A}_3} \end{bmatrix}^T$$

$$B_c = \begin{bmatrix} -\rho_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\rho_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\rho_3 \\ 0 & 0 & 0 \end{bmatrix}, \qquad H_c = \begin{bmatrix} \rho_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & H_{\mathrm{A}_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & H_{\mathrm{A}_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & H_{\mathrm{A}_3} \end{bmatrix}$$

The vector c is given by

$$c = \begin{bmatrix} c_{1\mathrm{A}_1} - c_{2\mathrm{A}_2} - \nu_0 & c_{q\mathrm{A}_1} & c_{1\mathrm{A}_2} - c_{2\mathrm{A}_3} & c_{q\mathrm{A}_2} & c_{1\mathrm{A}_3} - c_{2\mathrm{A}_1} & c_{q\mathrm{A}_3} \end{bmatrix}^T$$

and the vector of nonlinearities f_c is (6.7)

$$f_c = \begin{bmatrix} 0 \\ B_{3\mathrm{A}_1} e_{d\mathrm{M}_1} + B_{4\mathrm{A}_1} e_{s\mathrm{M}_1} \\ 0 \\ B_{3\mathrm{A}_2} e_{d\mathrm{M}_2} + B_{4\mathrm{A}_2} e_{s\mathrm{M}_2} \\ 0 \\ B_{3\mathrm{A}_3} e_{d\mathrm{M}_3} + B_{4\mathrm{A}_3} e_{s\mathrm{M}_3} \end{bmatrix} \text{ with } \begin{aligned} e_{s\mathrm{M}_i} &= \min(q_{s\mathrm{M}_i}, q_{d\mathrm{M}_i} - v_{\mathrm{M}_i}) \\ e_{d\mathrm{M}_i} &= \min(q_{s\mathrm{M}_i} + v_{\mathrm{M}_i}, q_{d\mathrm{M}_i}) \end{aligned} \tag{B.2}$$

B.2 The Equilibrium Point of the Chain

The equilibrium points of a system are given by $\dot{x} = 0$, see (5.33). For the nonlinear chain system one equilibrium point is found. It turns out that, in the equilibrium, the arguments of the minimum function in the piecewise linear function f_c (B.2) equal each other. Therefore, f_c can be written for the equilibrium state x_{ss} as

$$f_c(x_{\mathrm{ss}}) = F_c x_{\mathrm{ss}} + c_f \tag{B.3}$$

with

$$F_c = \begin{bmatrix} 0 & B_{3\mathrm{A}_1} D_{1\mathrm{A}_1} + B_{4\mathrm{A}_1} D_{2\mathrm{A}_3} & 0 & 0 & 0 & 0 \\ 0 & B_{3\mathrm{A}_1} C_{1\mathrm{A}_1} & 0 & B_{4\mathrm{A}_2} C_{2\mathrm{A}_1} & 0 & 0 \\ 0 & 0 & 0 & B_{3\mathrm{A}_2} D_{1\mathrm{A}_2} + B_{4\mathrm{A}_2} D_{2\mathrm{A}_1} & 0 & 0 \\ 0 & 0 & 0 & B_{3\mathrm{A}_2} C_{1\mathrm{A}_2} & 0 & B_{4\mathrm{A}_3} C_{2\mathrm{A}_2} \\ 0 & 0 & 0 & 0 & 0 & B_{3\mathrm{A}_3} D_{1\mathrm{A}_3} + B_{4\mathrm{A}_3} D_{2\mathrm{A}_3} \\ 0 & B_{4\mathrm{A}_1} C_{2\mathrm{A}_3} & 0 & 0 & 0 & B_{3\mathrm{A}_3} C_{1\mathrm{A}_3} \end{bmatrix}^T$$

and

$$c_f = \begin{bmatrix} 0 \\ B_{3\mathrm{A}_1} c_{1\mathrm{A}_1} + B_{4\mathrm{A}_1} c_{2\mathrm{A}_3} \\ 0 \\ B_{3\mathrm{A}_2} c_{1\mathrm{A}_2} + B_{4\mathrm{A}_2} c_{2\mathrm{A}_1} \\ 0 \\ B_{3\mathrm{A}_3} c_{1\mathrm{A}_3} + B_{4\mathrm{A}_3} c_{2\mathrm{A}_2} \end{bmatrix} \tag{B.4}$$

The equilibrium point of the chain model (B.1) is then given by

$$x_{\mathrm{ss}} = -(A_c + F_c)^{-1}(B_c v + H_c c + c_f) \tag{B.5}$$

B.3 The Computation of Piecewise Quadratic Lyapunov Functions

Before describing the search for a piecewise quadratic Lyapunov function, which is used in Section 7.3.2, we examine a globally quadratic Lyapunov function for the piecewise linear system (7.2)

$$\dot{x} = A_i x \qquad \text{in region } i \tag{B.6}$$

We consider the Lyapunov function candidate

$$V = x^T P x \tag{B.7}$$

with P a symmetric positive definite matrix. Differentiating V along the system trajectory yields

$$\dot{V} = \dot{x}^T P x + x^T P \dot{x} \tag{B.8}$$

$$= x^T (A_i^T P + P A_i) x \tag{B.9}$$

Thus, if

$$A_i^T P + P A_i < 0 \tag{B.10}$$

then V satisfies the Lyapunov conditions defined in Section 7.3.2 and is a Lyapunov function for the system. This condition is a linear matrix inequality in P. One way of finding such a matrix P is to solve a convex optimization problem for which efficient software is publicly available. Current software is capable of treating several hundreds of variables within seconds. If such a Lyapunov function can be found, stability follows independently of the region definition. On the other hand, the condition (B.10) is unnecessary restrictive for the analysis of piecewise linear systems. In these systems, it is only required that

$$x^T (A_i^T P + P A_i) x < 0 \text{ inside region } i \text{ where the local model } A_i \text{ is valid.}$$

One way to introduce the piecewise analysis is by constructing matrices S_i such that $x^T S_i x \geq 0$ for x inside region i and by considering the stability condition

$$A_i^T P + P A_i + S_i < 0 \tag{B.11}$$

Since $x^T S_i x \geq 0$ within region i, (B.11) clearly implies (B.10). On the other hand, if $x^T S_i x < 0$ outside region i, the inequality (B.11) may be simpler to be satisfied than (B.10).

In order to derive a method for finding piecewise quadratic Lyapunov functions, we define some matrices that will be used in the following. Since the region boundaries are linear, the regions can be described by matrices E_i such that the inequalities

$$E_i x \geq 0 \quad \text{if } x \text{ in region } i \tag{B.12}$$

hold. Additionally, the matrices F_i are constructed with

$$F_i x = F_j x \quad \text{if } x \text{ on the boundary of region } i \text{ and region } j \tag{B.13}$$

The condition $x^T S_i x \geq 0$ can now be incorporated by considering the matrix U_i with nonnegative entries. We then have

$$x^T E_i^T U_i E_i x \geq 0 \tag{B.14}$$

A *continuous* piecewise quadratic Lyapunov function candidate is

$$V = x^T P_i x \quad \text{for } x \text{ in region } i \text{, with} \tag{B.15}$$

$$P_i = F_i^T T F_i \tag{B.16}$$

with the symmetric matrix T. The condition (B.11) and the condition that the Lyapunov function must be positive definite can be expressed by the linear matrix inequalities

$$0 > A_i^T P_i + P_i A_i + E_i^T U_i E_i \tag{B.17}$$

$$0 < P_i - E_i^T W_i E_i \tag{B.18}$$

with the symmetric matrices T, U_i, W_i, where U_i and W_i have nonnegative entries. This is a set of linear matrix inequalities which can be solved by the above-mentioned algorithms. Note that, obviously, a solution not necessarily exists.

B.4 The Matrices of the Lyapunov Function

The matrices P_i in the Lyapunov function (7.13) of the paper chain are given here. The inventories in the chain are expressed in units of 10^9 kg and the prices in €/kg.

$$P_1 = \begin{bmatrix} 3.261 & 0.251 & 0.373 & -0.399 & 0.139 & 0.066 & -0.508 & 0.122 & 0.278 \\ 0.251 & 0.177 & 0.130 & -0.076 & 0.051 & 0.069 & 0.158 & 0.088 & 0.118 \\ 0.373 & 0.130 & 0.306 & -0.045 & 0.166 & 0.072 & 0.196 & 0.095 & 0.109 \\ -0.399 & -0.076 & -0.045 & 0.623 & 0.268 & 0.011 & -0.051 & 0.014 & -0.036 \\ 0.139 & 0.051 & 0.166 & 0.268 & 0.622 & 0.105 & 0.124 & 0.109 & 0.061 \\ 0.066 & 0.069 & 0.072 & 0.011 & 0.105 & 0.108 & 0.257 & 0.111 & 0.061 \\ -0.508 & 0.158 & 0.196 & -0.051 & 0.124 & 0.257 & 2.999 & 0.332 & 0.138 \\ 0.122 & 0.088 & 0.095 & 0.014 & 0.109 & 0.111 & 0.332 & 0.289 & 0.107 \\ 0.278 & 0.118 & 0.109 & -0.036 & 0.061 & 0.061 & 0.138 & 0.107 & 0.180 \end{bmatrix}$$

$$P_2 = \begin{bmatrix}
3.261 & 0.251 & 0.373 & -0.399 & 0.139 & 0.057 & -0.612 & 0.053 & 0.278 \\
0.251 & 0.177 & 0.130 & -0.076 & 0.051 & 0.067 & 0.137 & 0.074 & 0.118 \\
0.373 & 0.130 & 0.306 & -0.045 & 0.166 & 0.056 & 0.024 & -0.019 & 0.109 \\
-0.399 & -0.076 & -0.045 & 0.623 & 0.268 & 0.004 & -0.123 & -0.034 & -0.036 \\
0.139 & 0.051 & 0.166 & 0.268 & 0.622 & 0.105 & 0.123 & 0.108 & 0.061 \\
0.057 & 0.067 & 0.056 & 0.004 & 0.105 & 0.106 & 0.226 & 0.103 & 0.056 \\
-0.612 & 0.137 & 0.024 & -0.123 & 0.123 & 0.226 & 2.660 & 0.228 & 0.083 \\
0.053 & 0.074 & -0.019 & -0.034 & 0.108 & 0.103 & 0.228 & 0.302 & 0.071 \\
0.278 & 0.118 & 0.109 & -0.036 & 0.061 & 0.056 & 0.083 & 0.071 & 0.180
\end{bmatrix}$$

$$P_3 = \begin{bmatrix}
3.261 & 0.251 & 0.313 & -0.466 & 0.124 & 0.066 & -0.508 & 0.122 & 0.278 \\
0.251 & 0.177 & 0.140 & -0.065 & 0.054 & 0.069 & 0.158 & 0.088 & 0.118 \\
0.313 & 0.140 & 0.631 & 0.175 & 0.119 & 0.052 & 0.054 & 0.017 & 0.106 \\
-0.466 & -0.065 & 0.175 & 0.713 & 0.181 & -0.012 & -0.208 & -0.072 & -0.039 \\
0.124 & 0.054 & 0.119 & 0.181 & 0.577 & 0.100 & 0.087 & 0.089 & 0.060 \\
0.066 & 0.069 & 0.052 & -0.012 & 0.100 & 0.108 & 0.257 & 0.111 & 0.061 \\
-0.508 & 0.158 & 0.054 & -0.208 & 0.087 & 0.257 & 2.999 & 0.332 & 0.138 \\
0.122 & 0.088 & 0.017 & -0.072 & 0.089 & 0.111 & 0.332 & 0.289 & 0.107 \\
0.278 & 0.118 & 0.106 & -0.039 & 0.060 & 0.061 & 0.138 & 0.107 & 0.180
\end{bmatrix}$$

$$P_4 = \begin{bmatrix}
3.261 & 0.251 & 0.313 & -0.466 & 0.124 & 0.057 & -0.612 & 0.053 & 0.278 \\
0.251 & 0.177 & 0.140 & -0.065 & 0.054 & 0.067 & 0.137 & 0.074 & 0.118 \\
0.313 & 0.140 & 0.631 & 0.175 & 0.119 & 0.059 & 0.130 & 0.067 & 0.106 \\
-0.466 & -0.065 & 0.175 & 0.713 & 0.181 & 0.007 & -0.007 & 0.062 & -0.039 \\
0.124 & 0.054 & 0.119 & 0.181 & 0.577 & 0.105 & 0.150 & 0.130 & 0.060 \\
0.057 & 0.067 & 0.059 & 0.007 & 0.105 & 0.106 & 0.226 & 0.103 & 0.056 \\
-0.612 & 0.137 & 0.130 & -0.007 & 0.150 & 0.226 & 2.660 & 0.228 & 0.083 \\
0.053 & 0.074 & 0.067 & 0.062 & 0.130 & 0.103 & 0.228 & 0.302 & 0.071 \\
0.278 & 0.118 & 0.106 & -0.039 & 0.060 & 0.056 & 0.083 & 0.071 & 0.180
\end{bmatrix}$$

$$P_5 = \begin{bmatrix}
3.240 & 0.316 & 0.215 & -0.459 & 0.193 & 0.041 & -0.596 & 0.136 & 0.208 \\
0.316 & 0.235 & 0.064 & -0.101 & 0.074 & 0.059 & 0.121 & 0.094 & 0.116 \\
0.215 & 0.064 & 0.306 & -0.045 & 0.166 & 0.072 & 0.196 & 0.095 & 0.053 \\
-0.459 & -0.101 & -0.045 & 0.623 & 0.268 & 0.011 & -0.051 & 0.014 & -0.057 \\
0.193 & 0.074 & 0.166 & 0.268 & 0.622 & 0.105 & 0.124 & 0.109 & 0.080 \\
0.041 & 0.059 & 0.072 & 0.011 & 0.105 & 0.108 & 0.257 & 0.111 & 0.052 \\
-0.596 & 0.121 & 0.196 & -0.051 & 0.124 & 0.257 & 2.999 & 0.332 & 0.106 \\
0.136 & 0.094 & 0.095 & 0.014 & 0.109 & 0.111 & 0.332 & 0.289 & 0.112 \\
0.208 & 0.116 & 0.053 & -0.057 & 0.080 & 0.052 & 0.106 & 0.112 & 0.133
\end{bmatrix}$$

$$P_6 = \begin{bmatrix}
3.240 & 0.316 & 0.215 & -0.459 & 0.193 & 0.041 & -0.605 & 0.130 & 0.208 \\
0.316 & 0.235 & 0.064 & -0.101 & 0.074 & 0.060 & 0.140 & 0.106 & 0.116 \\
0.215 & 0.064 & 0.306 & -0.045 & 0.166 & 0.056 & 0.024 & -0.019 & 0.053 \\
-0.459 & -0.101 & -0.045 & 0.623 & 0.268 & 0.004 & -0.123 & -0.034 & -0.057 \\
0.193 & 0.074 & 0.166 & 0.268 & 0.622 & 0.105 & 0.123 & 0.108 & 0.080 \\
0.041 & 0.060 & 0.056 & 0.004 & 0.105 & 0.106 & 0.226 & 0.103 & 0.050 \\
-0.605 & 0.140 & 0.024 & -0.123 & 0.123 & 0.226 & 2.660 & 0.228 & 0.085 \\
0.130 & 0.106 & -0.019 & -0.034 & 0.108 & 0.103 & 0.228 & 0.302 & 0.098 \\
0.208 & 0.116 & 0.053 & -0.057 & 0.080 & 0.050 & 0.085 & 0.098 & 0.133
\end{bmatrix}$$

$$P_7 = \begin{bmatrix} 3.240 & 0.316 & 0.300 & -0.365 & 0.215 & 0.041 & -0.596 & 0.136 & 0.208 \\ 0.316 & 0.235 & 0.135 & -0.023 & 0.092 & 0.059 & 0.121 & 0.094 & 0.116 \\ 0.300 & 0.135 & 0.631 & 0.175 & 0.119 & 0.052 & 0.054 & 0.017 & 0.101 \\ -0.365 & -0.023 & 0.175 & 0.713 & 0.181 & -0.012 & -0.208 & -0.072 & -0.003 \\ 0.215 & 0.092 & 0.119 & 0.181 & 0.577 & 0.100 & 0.087 & 0.089 & 0.092 \\ 0.041 & 0.059 & 0.052 & -0.012 & 0.100 & 0.108 & 0.257 & 0.111 & 0.052 \\ -0.596 & 0.121 & 0.054 & -0.208 & 0.087 & 0.257 & 2.999 & 0.332 & 0.106 \\ 0.136 & 0.094 & 0.017 & -0.072 & 0.089 & 0.111 & 0.332 & 0.289 & 0.112 \\ 0.208 & 0.116 & 0.101 & -0.003 & 0.092 & 0.052 & 0.106 & 0.112 & 0.133 \end{bmatrix}$$

$$P_8 = \begin{bmatrix} 3.240 & 0.316 & 0.300 & -0.365 & 0.215 & 0.041 & -0.605 & 0.130 & 0.208 \\ 0.316 & 0.235 & 0.135 & -0.023 & 0.092 & 0.060 & 0.140 & 0.106 & 0.116 \\ 0.300 & 0.135 & 0.631 & 0.175 & 0.119 & 0.059 & 0.130 & 0.067 & 0.101 \\ -0.365 & -0.023 & 0.175 & 0.713 & 0.181 & 0.007 & -0.007 & 0.062 & -0.003 \\ 0.215 & 0.092 & 0.119 & 0.181 & 0.577 & 0.105 & 0.150 & 0.130 & 0.092 \\ 0.041 & 0.060 & 0.059 & 0.007 & 0.105 & 0.106 & 0.226 & 0.103 & 0.050 \\ -0.605 & 0.140 & 0.130 & -0.007 & 0.150 & 0.226 & 2.660 & 0.228 & 0.085 \\ 0.130 & 0.106 & 0.067 & 0.062 & 0.130 & 0.103 & 0.228 & 0.302 & 0.098 \\ 0.208 & 0.116 & 0.101 & -0.003 & 0.092 & 0.050 & 0.085 & 0.098 & 0.133 \end{bmatrix}$$

Appendix C

Stability, Controllability and Observability of Linear Systems

In this appendix, some basic properties of linear time-invariant systems are reviewed. Standard stability, controllability and observability tests are presented and the proofs are sketched. For an extensive discussion see one of the many textbooks, for example [101].

C.1 Stability

Among the various definitions of stability we consider the following here.

Definition 1 *A linear time-invariant system*

$$\dot{x} = Ax \tag{C.1}$$

i.e. system (4.3) with zero input, is asymptotically stable if the state vector tends to zero for any initial state.

Because the definition of asymptotic stability is only concerned with the state vector (and not with the output vector) and because the input is zero, the matrices B, C and D are irrelevant to asymptotic stability. The conditions can therefore be formulated with respect to the system matrix A only.

Theorem 1 *A linear time-invariant system is asymptotically stable if, and only if, all eigenvalues of A have a negative real part.*

We sketch the proof of this theorem. Assume that A can be diagonalized. Then there is a matrix M such that

$$A = M^{-1} \Lambda M \tag{C.2}$$

with

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \tag{C.3}$$

and λ_i are the eigenvalues of A. With the definition

$$z = M^{-1} x \tag{C.4}$$

the system can be written as a set of first-order equations

$$\dot{z}_i = \lambda_i z_i \tag{C.5}$$

with the solution

$$z_i = e^{\lambda_i t} z_{i0} \tag{C.6}$$

with the initial condition $z_i(0) = z_{i0}$. It can be seen that z_i tends to 0 if, and only if, the real part of λ_i is negative.

In the presence of multiple eigenvalues λ_i, the matrix A cannot always be diagonalized as in (C.2). However, this does not change the conclusion. Such eigenvalues introduce terms of the form $e^{\lambda_i t}, te^{\lambda_i t}, t^2 e^{\lambda_i t}, \ldots$, which each go to zero for $t \to \infty$ if, and only if, the real part of λ_i is negative.

C.2 Controllability

Definition 2 *A linear time-invariant system (4.3)*

$$\dot{x} = Ax + Bu \tag{C.7}$$

is said to be controllable if, for every given state x_1 and every $T > 0$, there exists an input function $u(t), 0 < t \leq T$, such that the system state is taken from 0 at t=0 to x_1 at $t = T$.

Theorem 2 *The system (C.7) is controllable if, and only if, the controllability matrix*

$$\mathcal{C} = [B \; AB \; \cdots \; A^{n-1}B] \tag{C.8}$$

has rank n, with n the dimension of the state vector x.

The proof of the ‘only if’-part starts out from the assumption that rank$\mathcal{C} < n$. Then we can find a vector v such that

$$v\mathcal{C} = 0 \text{ or} \tag{C.9}$$

$$vA = vAB = vA^2B = \cdots = vA^{n-1}B = 0 \tag{C.10}$$

To proceed, the Cayley-Hamilton theorem is used. It states that a matrix satisfies its own characteristic equation. If the characteristic equation is

$$s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0 \tag{C.11}$$

then

$$A^n + a_{n-1}A^{n-1} + \cdots + a_1 A + a_0 = 0 \tag{C.12}$$

Therefore

$$-vA^nB = a_{n-1}vA^{n-1}B + \cdots + a_1 vAB + a_0 vB = 0 \tag{C.13}$$

By induction $vA^{n+k}B = 0$ for $k = 0, 1, 2, \ldots$ and thus

$$ve^{At}B = v(1 + At + \frac{A^2}{2!}t^2 + \cdots)B = 0 \tag{C.14}$$

for all t.

Since the system response with initial condition $x_0 = 0$ is given by [101]

$$x(T) = \int_0^T e^{A(T-t)}Bu(t)dt \tag{C.15}$$

$$= e^{AT}\int_0^T e^{-At}Bu(t)dt \tag{C.16}$$

we obtain

$$vx(T) = ve^{AT}\int_0^T e^{-At}Bu(t)dt = 0 \tag{C.17}$$

for all $u(t)$. This implies that all points reachable from the origin are orthogonal to v. This restricts the reachable space and therefore contradicts the definition of controllability.

To show the ’if’-part, we assume that the rank condition holds but that the system is uncontrollable. Then there exists a vector v such that

$$v\int_0^T e^{A(T-t)}Bu(t)dt = 0 \tag{C.18}$$

This implies that

$$ve^{A(T-t)}Bu(t) = 0 \qquad 0 \le t \le T \tag{C.19}$$

If we set $t = T$, we see that $vB = 0$. By differentiating (C.19) and letting $t = T$, we obtain $vAB = 0$. Continuing this process, we obtain

$$vB = vAB = \cdots = vA^{n-1}B = 0 \tag{C.20}$$

which contradicts the assumption that $\mathcal{C}$ has full rank.

C.3 Observability

Definition 3 *A linear time-invariant system*

$$\dot{x} = Ax + Bu \tag{C.21}$$

$$y = Cx + Du \tag{C.22}$$

is observable if the initial state $x(0) = x_0$ *can be uniquely deduced from knowledge of the input* $u(t)$ *and output* $y(t)$ *for all* t *between 0 and any* $T > 0$.

Theorem 3 *The system (C.21,C.22) is observable if, and only if, the observability matrix*

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{C.23}$$

has rank n*, with* n *the dimension of the state vector* x*.*

The proof is similar to the proof of the controllability rank condition. It starts out from the output

$$y(T) = Ce^{AT}x_0 + \int_0^T Ce^{A(T-t)}Bu(t)dt + Du(T) \tag{C.24}$$

Since the input u is given, the second and the third part can be calculated and only the zero input part y_{zi} needs to be considered

$$y_{zi}(T) = Ce^{AT}x_0 \tag{C.25}$$

The rest of the proof is analogous to the controllability proof; the vector $\mathcal{O}v$ is considered.

Appendix D

Details of Parameter Estimation

D.1 Optimization Procedure

The optimization expression (6.20) of chapter 6.5

$$\min_\theta h(\theta) = \min_\theta \frac{1}{2} \sum_{i=1}^{n} [(y(t_i, \theta) - m(t_i))^T (y(t_i, \theta) - m(t_i))] \tag{D.1}$$

with

$$\dot{x} = f(x, q, \theta) \tag{D.2}$$

$$y = g(x, q, \theta) \tag{D.3}$$

can be written in the form

$$\min_\theta \frac{1}{2} k(\theta)^T k(\theta) \tag{D.4}$$

with

$$k(\theta) = \begin{bmatrix} y(t_1, \theta) - m(t_1) \\ y(t_2, \theta) - m(t_2) \\ \vdots \\ y(t_n, \theta) - m(t_n) \end{bmatrix} \tag{D.5}$$

This least square optimization problem can be solved by standard algorithms. The MATLAB function `leastsq`, for example, calculates a solution to this problem. Since it is a nonlinear optimization problem, the solution must be carefully evaluated.

The standard methods use the curvature information of the objective function $h(\theta)$ at each iteration point θ_i to formulate a quadratic optimization problem of the form

$$\min_{\Delta\theta} \frac{1}{2}\Delta\theta^T H \Delta\theta + c^T \Delta\theta + b \tag{D.6}$$

with c the gradient of h at point θ_i: $c = \nabla h$, and H the Hessian of h at point θ_i: $H = \nabla c^T$. The letter b denotes a constant equal to $h(\theta_i)$. The optimal solution $\Delta\theta^*$ of (D.6) is given by

$$\nabla f(\Delta\theta^*) = H\Delta\theta^* + c = 0 \tag{D.7}$$

$$\Delta\theta^* = -H^{-1}c \tag{D.8}$$

Newton-type methods calculate the Hessian directly and proceed the iteration in the direction of descent. However, the calculation of the Hessian involves a large amount of computation. Therefore *Quasi*-Newton methods approximate the Hessian by using the observed behaviour of $h(\theta)$ and $\nabla h(\theta)$ to build up curvature information.

D.2 Quality of the Estimate

In this section some statistical properties of the solution to the least squares optimization are presented [83, 114].

Standard deviation of the parameters. The standard deviation of the estimated parameters θ of a linear system $y = A\theta$ is given by [114]

$$s = \left[(A^T A)^{-1} \frac{(\sum_{i=1}^{n}(y_i - m_i)^2}{n - (k+1)}\right]^{\frac{1}{2}} \tag{D.9}$$

with n the number of observations and k the number of parameters. The meaning of the standard deviation is that the model with $\theta_i \pm 1s$ encompasses the middle 68 percent of the observed values, $\theta_i \pm 2s$ encompasses the middle 95.5 percent of the observed values, etc.

Autocorrelation. A principal aspect of the quality of the estimate is the test for autocorrelation of the residuals, i.e. correlation of successive values of the residuals. Figure D.1 shows the evolution in time of the residuals $y(t_i) - m(t_i)$. It suggests that residuals are autocorrelated.

A simple analytical test for autocorrelation is the Durbin-Watson test [52, 53]. It starts out from the equation

$$e(t) = \rho e(t-1) + u(t) \tag{D.10}$$

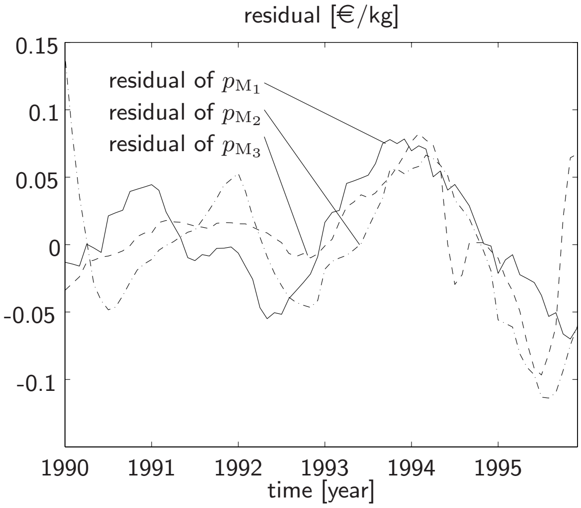


Figure D.1: *The residuals of the prices*

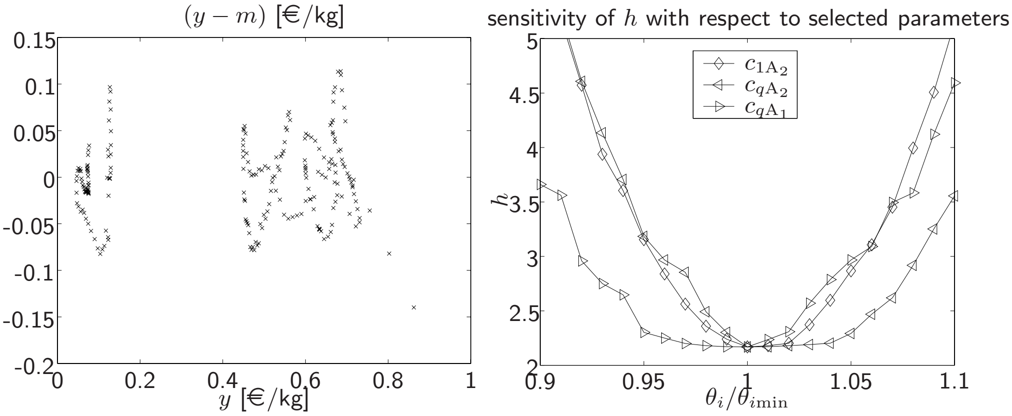
with e the residual and u an independent normally distributed variable. The parameter ρ is called *(first order) autocorrelation parameter*. The Durbin-Watson test checks the null hypothesis of zero autocorrelation ($\rho = 0$) against positive autocorrelation ($\rho > 0$). It involves the calculation of

$$D = \frac{\sum_{t=2}^{n}(e(t) - e(t-1))^2}{\sum_{t=1}^{n} e(t)^2} \tag{D.11}$$

Exact critical values for D are hard to obtain, but Durbin and Watson have obtained lower and upper bounds d_L and d_U such that a value of D outside these bounds leads to a definite decision:

- If $D > d_U$ conclude no autocorrelation
- If $D < d_L$ conclude positive autocorrelation
- If $d_L \leq D \leq d_U$ the test is inconclusive

The value of D is 0.202 for the paper chain model. The most extensive tables of d_L and d_U go up to models with 20 parameters and 200 data points [126]. The value of d_L for 20 parameters and 200 data points is 1.554 for a 5% significance level and 1.462 for 1% significance level, with d_L decreasing by approximately 0.01 for the transition from 19 to 20 parameters. Although the tabulated values do not include the case of 44 parameters, the difference between the tabulated values for d_L and D is so large that the Durbin-Watson test suggests autocorrelation of the residuals.



(a) (b)

Figure D.2: *A plot of the residuals against the output (a). The sensitivity of the objective function with respect to selected parameters of the chain model (b). The parameters are varied between -10% and +10% of the optimal value.*

Heteroscedasticity. In the case of heteroscedasticity the variance of the residuals is not equal for all values of the observations. This indicates that the estimated standard deviations of the parameters are too small. A first impression of the heteroscedasticity can be obtained by plotting the residuals $y(t_i) - m(t_i)$ against the model outputs $y(t_i)$. This kind of plot is shown in Figure D.2a for the paper chain model. It indicates that the variance of the residuals does not depend on the value of the output and thus indicates that the distribution of the residuals is not heteroscedastic.

Sensitivity of the objective function. Furthermore, we calculate the sensitivity of the objective function h with respect to the model parameters. The sensitivity of the objective function with respect to the most sensitive parameters is depicted in Figure D.2b. The value of the objective function $h(\theta)$ that results from a change of the parameters between -10% and +10% of the optimal parameter value is shown. It can be seen that the objective function is particularly sensitive to a change in $c_{1\mathrm{A}_2}$ and $c_{q\mathrm{A}_2}$.

Bibliography

[1] M.A. Aiserman and F. Gantmacher. *Die absolute Stabilität von Regelsystemen (The Absolute Stability of Control Systems).* Oldenbourg Verlag, 1965. in German.

[2] M. Allais. Theories of general economic equilibrium and maximum efficiency. In G. Schwödiauer, editor, *Equilibrium and Disequilibrium in Economic Theory.* Reidel, Dordrecht, 1978.

[3] H. Amman. Numerical methods for linear-quadratic models. In H.M. Amman, D.A. Kendrick, and J. Rust, editors, *Handbook of Computational Economics*, Handbooks in Economics, chapter 13. Elsevier, Amsterdam, 1996.

[4] B.D.O. Anderson and J.B. Moore. *Optimal Control: Linear Quadratic Methods.* Prentice-Hall International, London, 1990.

[5] E.W. Anderson, L.P. Hansen, E.R. McGrattan, and T.J. Sargent. Mechanics of forming and estimating dynamic linear economies. In H.M. Amman, D.A. Kendrick, and J. Rust, editors, *Handbook of Computational Economics*, Handbooks in Economics, chapter 4. Elsevier, Amsterdam, 1996.

[6] C. Andersson. *Reverse Logistic Systems for Wastepaper.* PhD thesis, Lund University, 1995.

[7] A. Andijani and M. Al-Dajani. Analysis of deteriorating inventory/production systems using a linear quadratic regulator. *European Journal of Operational Research*, 106:82–89, 1998.

[8] M. Aoki. On a generalization of Tinbergen's condition in the theory of policy to dynamic models. *Review of Economic Studies*, 42:293–296, 1975.

[9] M. Aoki. *Optimal Control and System Theory in Dynamic Economic Analysis*. North-Holland, Amsterdam, 1976.

[10] K.J. Arrow and F.H. Hahn. *General Competitive Analysis*. Hodden-Day, San Francisco, 1971.

[11] M. Athans and D. Kendrick. Control theory and economics: A survey, forecast and speculations. *IEEE Transactions on Automatic Control*, pages 518–523, october 1974.

[12] S. Axsäter. Control theory concepts in production and inventory control. *International Journal of Systems Science*, 16(2):161–169, 1985.

[13] R.U. Ayres. A materials-process-product model. In A.V. Kneese and B.T. Bower, editors, *Environmental Quality Analysis: Theory and Methods in the Social Sciences*, pages 35–67. Johns Hopkins Press, Baltimore, 1972.

[14] R.U. Ayres. *Resources, Environment, and Economics: Applications of the Materials/Energy Balance Principle*. Wiley, New York, 1978.

[15] R.U. Ayres. Industrial metabolism. In J.H. Ausubel and H.E. Sladovich, editors, *Technology and Environment*, pages 23–49. National Academy Press, Washington, DC, 1989.

[16] R.U. Ayres. Industrial metabolism and global change. *International Social Science Journal*, pages 363–373, 1989.

[17] R.U. Ayres and L.W. Ayres. *Industrial Ecology: Towards Closing the Materials Cycle*. Edward Elgar, Cheltenham, UK, 1996.

[18] W.J. Baumol and W.E. Oates. *The Theory of Environmental Policy*. Cambridge University Press, 1988.

[19] B. Beavis and I.M. Dobbs. The dynamics of optimal environmental regulation. *Journal of Economic Dynamics and Control*, 10:415–423, 1986.

[20] J.P. Benassy. *The Economics of Market Disequilibrium*. Academic Press, London, 1982.

[21] A. Bensoussan, E. Gerald Hurst Jr., and B. Näslund. *Management Applications of Modern Control Theory*. North-Holland, Amsterdam, 1974.

[22] J. van den Bergh. *Dynamic Models for Sustainable Development.* PhD thesis, University of Amsterdam, 1991.

[23] G. Bertolini. Wastepaper cycle management: Incentives and product chain pressure point or 'leverage point' analysis. In J.B. Opschoor and R.K. Turner, editors, *Economic Incentives and Environmental Policies: Principles and Practice*, chapter 11. Kluwer, Dordrecht, 1994.

[24] W.M. Bertrand, J.C. Wortmann, and J. Wijngaard. *Production Control: A Structural and Design Oriented Approach.* Elsevier, Amsterdam, 1990.

[25] F. Blaauwendraat and J. van Dalen. *Papier en Papierprodukten in de Nederlandse Economie, 1990 (Paper and Paper Products in the Dutch Economy, 1990).* Centraal Bureau voor de Statistiek (Dutch Statistical Office), Voorburg, 1993. in Dutch.

[26] A.S. Blinder and L.J. Maccini. Taking stock: a critical assessment of recent research on inventories. *Journal of Economic Perspectives*, 5:73–96, 1991.

[27] J.M. Bloemhof-Ruwaard. *Integration of Operational Research and Environmental Management.* PhD thesis, Wageningen Agricultural University, NL, 1996.

[28] M.W.J. Blok. *Dynamisch Optimaal Investerings-, Produktie- en Dividendbeleid (Dynamic Optimal Investment, Production and Dividend Policy).* PhD thesis, Eindhoven University of Technology, NL, 1994. in Dutch.

[29] P. Bohm and C.S. Russell. Comparative analysis of alternative policy instruments. In *Handbook of Natural Resource and Energy Economics*, volume 1, pages 395–460. Elsevier, Amsterdam, 1985.

[30] P.P.J. van den Bosch and A.C. van der Klauw. *Modeling, Identification and Simulation of Dynamical Systems.* CRC, London, 1994.

[31] A.L. Bovenberg. Dynamic general equilibrium tax models with adjustment costs. *Mathematical Programming Study*, 23:40–55, 1985.

[32] S. Boyd, L. El Ghauoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory.* SIAM, Philadelphia, 1994.

[33] A. Bradshaw and Y. Erol. Control policies for production inventory systems. *International Journal of Systems Science*, 11:947–959, 1980.

[34] A. Bradshaw and B. Porter. Optimal control of production inventory systems. *International Journal of Systems Science*, 4(5):697–706, 1973.

[35] M.S. Branicky. Stability of switched and hybrid systems. In *Proceedings of the 33rd Conference on Decision and Control, Lake Buena Vista*, pages 3498–3502, 1994.

[36] R.W. Brockett and M.D. Mesarovic. The reproducibility of multivariable systems. *Journal of Mathematical Analysis and Applications*, 11:548–563, 1965.

[37] Bureau B & G. Studie naar de invloed van materiaalhergebruik en preventie op omvang en samenstelling van inzamelstromen uitgaande van milieubeleid AOO-achtergronddocument 91-14 (Study of the influence of material recycling and prevention on the amount and composition of collection flows on the basis of environmental policy). Rotterdam, 1991. in Dutch.

[38] Centraal Bureau voor de Statistiek (CBS). *Maandstatistiek van de Prijzen (Monthly Statistics of Prices).* CBS, Voorburg, NL, monthly. in Dutch.

[39] Centraal Bureau voor de Statistiek (CBS). *Jaarstatistiek van de Buitenlandse Handel (Yearly Statistics of Foreign Trade).* CBS, Voorburg, NL, yearly. in Dutch.

[40] M.B. Chiarolla and U.G. Haussmann. Optimal control of inflation: a central bank problem. *SIAM Journal of Control and Optimization*, 3:1099–1132, 1998.

[41] D.D. Chiras. *Environmental Science: Action for a Sustainable Future.* Benjamin/Cummings, Amsterdam, 1994.

[42] R.H. Coase. The problem of social cost. *Journal of Law and Economics*, 3:1–44, 1960.

[43] M. Common. *Sustainability and Policy. Limits to Economics.* Cambridge University Press, 1995.

[44] K. Conrad. Resource and waste taxation in the theory of the firm with recycling activities. *Environmental and Resource Economics*, 14:217–242, 1999.

[45] R. Cornes and T. Sandler. *The Theory of Externalities, Public Goods, and Club Goods.* Cambridge University Press, 1996.

[46] J. Cramer. Milieu als maatschappelijk probleem in Nederland (Environment as societal problem in the Netherlands). In J.J. Boersema et al., editors, *Basisboek Milieukunde*. Boom Meppel, Amsterdam, 1991. in Dutch.

[47] B.D. Craven. *Mathematical Programming and Control Theory*. Chapman and Hall, London, 1978.

[48] B.D. Craven. *Control and Optimization*. Chapman and Hall, London, 1995.

[49] H.E. Daly. Toward some operational principles of sustainable development. *Ecological Economics*, 2:1–6, 1990.

[50] G. Debreu. Excess demand functions. *Journal of Mathematical Economics*, 1:15–21, 1974.

[51] H Dieleman and S. de Hoo. PRISMA: The development of a preventive, multi media strategy for government and industry. In K. Fischer and J. Schot, editors, *Environmental Strategies for Industry: International Perspectives on Research Needs and Policy Implications*, pages 245–275. Island Press, Washington DC, 1993.

[52] J. Durbin and G.S. Watson. Testing for serial regression in least squares regression I. *Biometrika*, 37:409–428, 1950.

[53] J. Durbin and G.S. Watson. Testing for serial regression in least squares regression II. *Biometrika*, 38:159–178, 1951.

[54] J.M. English. Economic theory — new perspectives. In J.J. Dixhoor and F.J. Evans, editors, *Physical Structure in Systems Theory: network approaches to engineering and economics*, pages 279–296. Academic Press, London, 1974.

[55] S. Erkman. Industrial ecology: a historical view. *Journal of Cleaner Production*, 1997.

[56] G. Evans. Overview of techniques for solving multiobjective mathematical programs. *Management Science*, 30:1268–1282, 1984.

[57] J.W. Forrester. *Industrial Dynamics*. M.I.T. Press, Cambridge, Massachusetts, 1961.

[58] G.F. Franklin, J.D. Powell, and A. Emami-Naeini. *Feedback Control of Dynamic Systems*. Addison-Wesley, Amsterdam, 1994.

[59] O.I. Franksen. Basic concepts in engineering and economics. In J.J. Dixhoor and F.J. Evans, editors, *Physical Structure in Systems Theory: network approaches to engineering and economics*, pages 247–278. Academic Press, London, 1974.

[60] W. Frenz. *Verordnungen zum Kreislaufwirtschafts- und Abfallgesetz: Textausgabe und Kommentare (Decrees Concerning the Cycle and Waste Law: Text Edition and Commentaries).* Heymanns, Köln, 1996. in German.

[61] N.A. Gallopoulos and R.A. Frosch. Strategies for manufacturing. *Scientific American*, 261(3):94–103, 1989.

[62] A.J. Gilbert and J.F. Feenstra. A sustainability indicator for the dutch environmental policy theme 'diffusion': cadmium accumulation in soil. *Ecological Economics*, 9:253–265, 1994.

[63] T.E. Graedel and B.R. Allenby. *Industrial Ecology.* Prentice Hall, Englewood Cliffs, 1995.

[64] J.B. Guinée, H.A. Udo de Haes, and G. Huppes. Quantitative life cycle assessment of products, part one: goal definition and inventory. *Journal of Cleaner Production*, 1(1):3–13, 1993.

[65] J.B. Guinée, H.A. Udo de Haes, and G. Huppes. Quantitative life cycle assessment of products, part two: classification, valuation and improvement analysis. *Journal of Cleaner Production*, 1(2):82–91, 1993.

[66] F.H. Hahn and T. Negishi. A theorem on non tâtonnement stability. *Econometrica*, 30(3):463–469, 1962.

[67] G. Hardin. The tragedy of the commons. *Science*, 162:1243–1248, 1968.

[68] R.J.J. van Heijningen, J.F.M. de Castro, E. Worrell, and J.H.O. Hazewinkel. Meer energiekentallen in relatie tot preventie en hergebruik van afvalstromen (More characteristic energy figures related to prevention and recycling of waste streams). Amersfoort, 1992. in Dutch.

[69] R. Heijungs. A generic method for the identification of options for cleaner products. *Ecological Economics*, 10:69–81, 1994.

[70] R. Herman, S.A. Ardekani, and J.H. Ausubel. *Technology and Environment*, chapter Dematerialization. National Academy Press, Washington, 1989.

[71] G. Hewitt. *Economics of the Market.* Fontana/Collins, Glasgow, 1976.

[72] D.P. Heyman. Optimal disposal policies for a single-item inventory system with returns. *Naval Research Logistics Quarterly*, 24:385–405, 1977.

[73] O. van Hilten, P.M. Kort, and P.J.J.M. van Loon. *Dynamic Policies of the Firm: An optimal control approach.* Springer, Berlin, 1993.

[74] T. Hoshino, K. Yura, and K. Hitomi. Optimization analysis for recycle-oriented manufacturing systems. *International Journal of Production Research*, 33(8):2069–2078, 1995.

[75] A. Huhtala. A post-consumer waste management model for determining optimal levels of recycling and landfilling. *Environmental and Resource Economics*, 10:301–314, 1997.

[76] A. Huttunen and T. Pirttila. Price dynamics on the recovered paper market. *International Journal of Production Economics*, 56–57:261–273, 1998.

[77] E.C. van Ierland. *Macroeconomic Analysis of Environmental Policy.* PhD thesis, University of Amsterdam, NL, 1993.

[78] IFIAS. Report of the international federation of institutes for advanced study, workshop no.6 on energy analysis. Guldsmedshyttan, S, 1973.

[79] ISO. *ISO Environmental Management Standardization Efforts.* NISTIR 5638, Brussels, 1995.

[80] M.H. Jansen. *The Environment as a Resource — Developing Environmental Information Systems based on Enterprise Resource Planning Software.* PhD thesis, Eindhoven University of Technology, NL, 1998.

[81] M. Johansson and A. Rantzer. Computation of piecewise quadratic Lyapunov functions for hybrid systems. In *Proceedings of the 1997 European Control Conference*, Brussels, Belgium, July 1997.

[82] M. Johansson and A. Rantzer. Computation of piecewise quadratic Lyapunov functions for hybrid systems. *IEEE Transactions on Automatic Control*, 43(4):555–559, 1998.

[83] G.G. Judge, R.C. Hill, W.E. Griffiths, H. Lütkepohl, and T.-C. Lee. *Introduction to the Theory and Practice of Econometrics.* John Wiley, New York, 1988.

[84] P. Kandelaars, H. Opschoor, and J. van den Bergh. A dynamic simulation model for materials-product-chains: an application to gutters. *Journal of Environmental Systems*, 24(4):345–371, 1996.

[85] P. Kandelaars and J. van den Bergh. Dynamic analysis of materials-product chains: An application to window frames. *Ecological Economics*, 22(1):41–61, 1997.

[86] P.A.A.H. Kandelaars. *Material-Product Chains: Economic Models and Applications.* PhD thesis, Vrije Universiteit Amsterdam, 1998.

[87] R. Kemp. *Environmental Policy and Technical Change: A Comparison of the Technological Impact of Policy Instruments.* PhD thesis, University of Limburg, Maastricht, NL, 1995.

[88] U. Kleineidam. The control of product chains — a model determining effective environmental policy. In *Proceedings of the* 2^{nd} *International Conference on Technology Policy and Innovation*, pages 23.1.1–23.1.8, Lisbon, Portugal, 1998.

[89] U. Kleineidam and F. Eijsink. A dynamic model for determining the environmental performance of a product chain. In M. Muffato and K.S. Pawar, editors, *Proceedings of the* 3^{rd} *International Symposium on Logistics: Enhancing Competitiveness through Logistics Capabilities*, pages 657–662. Servizi Grafici Editoriali, Padua, Italy, 1997.

[90] U. Kleineidam, A.J.D. Lambert, J. Banens, J.J. Kok, and R.J.J. van Heijningen. Modelling of product recycling chains. In I. Troch and F. Breitenecker, editors, 3^{rd} *Mathmod, Vienna*, pages 477–480, 2000.

[91] U. Kleineidam, A.J.D. Lambert, J. Banens, J.J. Kok, and R.J.J. van Heijningen. Controlling product recycling chains — how to achieve a better environment. *Ecological Economics*, submitted for publication.

[92] U. Kleineidam, A.J.D. Lambert, J. Blansjaar, J.J. Kok, and R.J.J. van Heijningen. Optimising product recycling chains by control theory. *International Journal of Production Economics*, 2000. accepted for publication.

[93] P.J.A. Konijn, S. de Boer, and J. van Dalen. *Material Flows and Input-Output Analysis — methodological description and empirical results.* Statistics Netherlands, 1995.

[94] S. Kruitwagen. *Economic Analysis: Tradeable Emission Permits for Sulphur Dioxide Emissions in Europe*. PhD thesis, Wageningen Agricultural University, NL, 1996.

[95] A.J.D. Lambert. Optimal disassembly of complex products. *International Journal of Production Research*, 35(9):2509–2523, 1997.

[96] A.J.D. Lambert. Linear Programming in disassembly/clustering sequence generation. *Computers and Applied Industrial Engineering*, 36:723–738, 1999.

[97] A.J.D. Lambert and J.C.M. Marijnissen. *Het Verband tussen Afvalwarmte en Energiegebruik bij de Winning van Minerale Grondstoffen (The Relationship between Waste Heat and Energy Consumption for the Extraction of Mineral Raw Materials)*. Report EUT/BDK/31, ISBN 90-6757-031-1, Eindhoven University of Technology, 1988. in Dutch.

[98] A.J.D. Lambert and M.A.M. Splinter. Reuse of sophisticated product carriers. *Proceedings of the First International Working Seminar on Reuse, Eindhoven, NL*, pages 213–222, 1996.

[99] W. Leontief. *Input-Output Economics*. Oxford University Press, New York, 1966.

[100] M.J. Lighthill. *Introduction to Fourier Analysis and Generalised Functions*. Cambridge University Press, 1958.

[101] D.G. Luenberger. *Introduction to Dynamic Systems: Theory, Models and Applications*. Wiley, Chichester, 1979.

[102] A.M. Lyapunov. The general problem of motion stability. *reproduced in Ann. Fac. Sci. Toulouse*, 9:203–474, 1907. original (1892) in Russian, reproduction in French.

[103] W.C.A. Maas and H. Nijmeijer. Dynamic path controllability in economic models. *Journal of Economic Dynamics and Control*, 18:781–805, 1994.

[104] E. Mansfield. *Microeconomics: theory and applications*. Norton, London, 1985.

[105] R.D. Mason, D.A. Lind, and W.G. Marchal. *Statistical Techniques in Business and Economics*. Irwin/McGraw-Hill, Boston, 1999.

[106] D.H. Meadows, D.L. Meadows, J. Randers, et al. *The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind*. Universe Books, New York, 1972.

[107] M.D. Merbis. *Optimal Control for Econometric Models*. PhD thesis, Tilburg University, NL, 1986.

[108] Ministerie van Volkshuisvesting, Ruimtelijke Ordening en Milieubeheer. Nationaal milieubeleidsplan plus (National environmental policy plan plus). The Hague, 1990. in Dutch.

[109] H.C. Moll. *Energy Counts and Materials Matter in Models for Sustainable Development*. PhD thesis, Groningen University, NL, 1993.

[110] B.C. Moore. Principle component analysis in linear systems: controllability, observability, and model reduction. *IEEE Transactions on Automatic Control*, 26(1):17–32, 1981.

[111] D. Nagelhout and Z.I. van Lohuizen. Afvalverwijdering 1990-2010 — achtergronddocument bij de nationale milieuverkenning 2 (Waste removal 1990-2010 — background document for the national enquiry on the environment 2). Technical Report 736201010, National Institute of Public Health and Environmental Protection, 1992. in Dutch.

[112] D. Nagelhout, K. Wieringa, and J.M. Joosten. Afval 2000 — een verkenning van de toekomstige afvalverwijderingsstructuur (Waste 2000 — an exploration of the future waste removal structure). Technical Report 738605002, National Institute of Public Health and Environmental Protection, 1989. in Dutch.

[113] R. Neck and S. Karbuz. Optimal control of the Austrian economy under alternative target values. In *Proceedings of the European Control Conference*, Brussels, 1997.

[114] J. Neter, M.H. Kutner, C.J. Nachtsheim, and W. Wasserman. *Applied Linear Statistical Models*. Irwin, Chicago, 1996.

[115] P. Newman. *The Theory of Exchange*. Prentice-Hall, Englewood Cliffs, 1965.

[116] H. Nijmeijer. On dynamic decoupling and dynamic path controllability in economic systems. *Journal of Economic Dynamics and Control*, 13:21–39, 1989.

[117] J.P. Norton. *An Introduction to Identification.* Academic Press, London, 1986.

[118] Operational Research Society of Great Britain. Definition of operational research. *Operational Research Quarterly*, 13(3):282, 1962.

[119] H. Opschoor. Chain management in environmental policy: Analytical and evaluative concepts. In J.B. Opschoor and R.K. Turner, editors, *Economic Incentives and Environmental Policies: Principles and Practice*, pages 197–228. Kluwer Academic Publishers, Dordrecht, 1994.

[120] C.R. Plott and G. George. Marshallian vs. Walrasian stability in an experimental market. *The Economic Journal*, 102:437–460, 1992.

[121] A.J. Preston and A.R. Pagan. *The Theory of Economic Policy.* Cambridge University Press, 1982.

[122] S.C. Puri. *Stepping up to ISO 14000: Integrating Environmental Quality with ISO 9000 and TQM.* Productivity Press, Portland, 1996.

[123] V.A. Ramey. Nonconvex costs and the behavior of inventories. *Journal of Political Economy*, 99:306–334, 1991.

[124] B. Rothery. *Standards in the Services Industry.* Gower, Aldershot, 1997.

[125] M. Ruth. *Integrating Economics, Ecology and Thermodynamics.* Kluwer, Dordrecht, 1993.

[126] N.E. Savin and K.J. White. The Durbin-Watson test for serial correlation with extreme sample sizes or many regressors. *Econometrica*, 45:1989–1996, 1977.

[127] A. Schotter. *Free Market Economics: A Critical Appraisal.* St. Martin's Press, New York, 1985.

[128] G. Schwödiauer, editor. *Equilibrium and Disequilibrium in Economic Theory.* Reidel, Dordrecht, 1978.

[129] K. Segerson. Issues in the choice of environmental policy instruments. In J.B. Braden, H. Folmer, and T.S. Ulen, editors, *Environmental Policy with Political and Economic Integration*, New Horizons in Environmental Economics Series, pages 149–174. Edwar Elgar, Cheltenham, UK, 1996.

[130] J.J.E. Slotine and W. Li. *Applied Nonlinear Control.* Prentice-Hall, Englewood Cliffs, 1991.

[131] T. Spengler. *Industrielle Demontage- und Recyclingkonzepte (Industrial Disassembly and Recycling Concepts).* Erich Schmidt Verlag, Berlin, 1994. in German.

[132] A. Stam, M. Kuula, and H. Cesar. Transboundary air pollution in Europe: An interactive multicriteria tradeoff analysis. *European Journal of Operational Research*, 56:263–277, 1992.

[133] P.F. Starreveld and E.C. van Ierland. Recycling of plastics: a materials balance optimisation model. *Environmental and Resource Economics*, 4:251–264, 1994.

[134] Stichting Integraal Ketenbeheer Papier en Karton (IKP). Milieu actie plan papier en karton (Environmental action plan paper and cardboard). Haarlem, NL, 1997. in Dutch.

[135] Structural change and economic dynamics, special issue: Environmental extensions of national accounts: The NAMEA framework, march 1999.

[136] A. Takayama. *Mathematical Economics.* Cambridge University Press, 1985.

[137] M. Thierry, M. Salomon, J. van Nunen, and L. van Wassenhove. Strategic issues in product recovery management. *California Management Review*, 37(2):114–134, 1995.

[138] T. Tibor. *ISO 14000: a Guide to the new Environmental Management Standards.* Irwin, London, 1996.

[139] A.-M. Tillman, T. Ekvall, H. Baumann, and T. Rydberg. Choice of system boundaries in life cycle assessment. *Journal of Cleaner Production*, 2(1):21–29, 1994.

[140] J. Tinbergen. *On the Theory of Economic Policy.* North-Holland, Amsterdam, 1952.

[141] J. Tinbergen. *Economic Policy: Principles and Design.* North-Holland, Amsterdam, 1956.

[142] Ullmann's encyclopedia of industrial chemistry. VCH, Weinheim, Germany, 1985.

[143] H.R. Varian. *Intermediate Microeconomics.* Norton, New York, 1987.

[144] Vereniging van Nederlandse Papier en Kartonfabrieken (VNP). Key statistics, yearly. Haarlem, NL.

[145] P.A. Verheyen. The economic explanation of the strategy of a self financing firm during the recession period. *European Journal of Operational Research*, 74:344–349, 1994.

[146] Y. Virtanen and S. Nilsson. *Environmental Impacts of Waste Paper Recycling*. Earthscan Publications, London, 1993.

[147] P.M. Weaver, H. Landis Gabel, J.M. Bloemhof-Ruwaard, and L.N. van Wassenhove. Optimizing environmental product life cycles: a case study of the European pulp and paper sector. *Environmental and Resource Economics*, 9:199–224, 1997.

[148] The New International Webster Comprehensive Dictionary of the English Language. Trident Press International, 1996.

[149] W.L. Winston. *Operations Research: Applications and Algorithms*. PWS-Kent, Boston, 1991.

[150] H.-W. Wohltmann. On the controllability of continuous-time macroeconomic models. *Journal of Economics*, 45:47–66, 1985.

[151] W.T.M. Wolters. *Sustainable Industrial Production: An Energy Perspective*. PhD thesis, Eindhoven University of Technology, 1994.

[152] World Commission on Environment and Development (WCED). *Our Common Future*. Oxford University Press, 1987.

Samenvatting

In de laatste decennia hebben regeringen en bedrijfsleven maatregelen genomen om duurzaamheid van productieprocessen te bereiken. Een hele belangrijke manier om duurzame productie te behalen is het sluiten van materiaalketens, omdat recycling van afgedankte producten en procesafval een effectieve methode is om het storten van afval te verminderen en de uitputting van grondstoffen te vertragen.

Omdat maatregelen die genomen worden om de recycling te beïnvloeden, effecten hebben op de productie- en consumptieprocessen die met het product in verband staan, worden de gehele ketens van dergelijke productie- en consumptieprocessen in beschouwing genomen. Een product-proces keten beschrijft zo'n gehele keten op een standaard manier. Product-proces ketens worden in het kader van het industriële metabolisme gebruikt om de materiaal- en productstromen door het industrieel systeem weer te geven, waarbij de wetten van behoud van massa en energie worden toegepast. In deze studie wordt de gehele levenscyclus van producten onderzocht, van de extractie van grondstoffen tot de reïntegratie, vroeger of later, van de afvalstoffen in het ecosysteem.

Ketens worden beïnvloed door beslissingen van producenten en consumenten over productstromen. Deze stromen kunnen worden veranderd door vervanging van stoffen en producten, door recycling van materialen en door hergebruik van producten. Overheidsinvloeden zijn externe invloeden: regulerende invloeden, bijvoorbeeld door het voorschrijven van emissieniveaus, economische invloeden, bijvoorbeeld door belastingen en subsidies en sociale invloeden, bijvoorbeeld door het stimuleren van milieu-onderzoek en het bevorderen van zelfregulatie van ketens.

Deze studie introduceert een methode voor het modelleren en regelen van product-proces ketens wat voor overheden en het bedrijfsleven van belang is. De ketenmodellen bestaan uit elementaire ondernemingsmodellen die door middel van marktmodellen onderling verbonden worden. Dynamische processen zoals markt- en opslagprocessen zijn inbegrepen. Door de ondernemings- en marktmodellen te combineren kan een grote variëteit van productketens

gemodelleerd worden met de mogelijkheid om recycling in te voegen. Ondernemingen worden opgevat als onderdelen bestaande uit grondstof- en productvoorraden en een transformatieproces. Door de wet van behoud van massa op deze voorraden toe te passen worden twee dynamische vergelijkingen verkregen die de onderneming beschrijven. Productiebeslissingen en beslissingen over aanbod en aankoop van producten worden in deze vergelijkingen ingevuld. De beslissingen zijn gebaseerd op marktprijzen. Het gedrag van de prijzen wordt bepaald door het verschil tussen vraag en aanbod.

Het belang van regeltheorie voor ketenanalyse en ketenregeling wordt aangetoond. Het ketenmodel kan in de standaard toestandsrepresentatie gebracht worden die in de regeltheorie gangbaar is en geeft zo de mogelijkheid de verschillende analyse- en regelmethoden uit de regeltheorie te gebruiken, zoals stabiliteits-, regelbaarheids- en observeerbaarheidsanalyse. Deze eigenschappen zijn eerste vereisten voor effectief ketenbeheer, bijvoorbeeld in de evaluatie van recyclingbeleid.

Ter illustratie van de modellerings- en analysetechnieken wordt de Nederlandse papierketen onderzocht. Deze keten is gekozen omdat vrij uitgebreide historische gegevens over prijzen, importen en exporten beschikbaar zijn. De keten bestaat uit papierproducenten, papierconsumenten en oudpapierrecyclers evenals uit een pulpmarkt, een papiermarkt en een oudpapiermarkt. De historische prijzen zijn in goede overeenstemming met de prijzen die met behulp van het model berekend zijn indien de importen en exporten de gemeten waardes aannemen.

In de modelanalyse wordt eerst de stabiliteit van de papierketen aangetoond. Stabiliteit houdt in dat de productstromen en de prijzen niet onbegrensd groeien. Ten tweede wordt de regelbaarheid van de keten geanalyseerd. Er wordt nagegaan of maatregelen die bedoeld zijn om de keten te beïnvloeden daadwerkelijk in staat zijn om de keten op de bedoelde wijze aan te sturen. Regelbaarheid door belastingen en subsidies van producten, door belastingen en subsidies van activiteiten, door de regulering van import en export en door een aanpassing van de eindverwerkingskosten wordt aangetoond. Bovendien wordt bewezen dat de keten observeerbaar is met behulp van de meting van productstromen, productprijzen en het recyclingpercentage. Dit geeft een indicatie dat deze grootheden voldoende informatie bevatten voor effectieve ketenregulering.

Enkele voorbeelden van de regulering van de papierketen worden gemodelleerd om de waarde van het model voor het evalueren van invloeden op de keten te tonen. De regulering van de papierstroom door een heffing op pulp illustreert een maatregel die tot het bereiken van het doel leidt. De regulering van het recyclingpercentage door een vergroting van recycling-activiteiten wordt als tweede voorbeeld gemodelleerd. Het blijkt dat dit een ineffectieve maatregel is. Verder wordt aangetoond dat het recyclingpercentage met behulp van het bepalen van de eindverwerkingsprijs kan worden geregeld. Dit percentage kan ook geregeld worden door een combinatie van de beïnvloeding van de eindverwerkingsprijs en een subsidie op oudpapier, wat een politiek aantrekkelijke regeling is.

Uiteindelijk worden twee ingrepen gemodelleerd met als doel het verminderen van prijsschommelingen: een subsidie van de oudpapierprijs en een soort zelfregulatie, die recentelijk in de papierketen geïntroduceerd werd. Dit zogenaamde papiervezelconvenant voorziet in wederzijdse afname- en verkoopovereenkomsten en heeft als doel een gelijkmatiger papierdoorstroming door de keten en een vergroting van de oudpapierinzameling. Aangetoond wordt dat de twee maatregelen daadwerkelijk het ketengedrag kunnen verbeteren.

De papierketen is een voorbeeld van een bestaande keten waarin al geruime tijd recycling wordt toegepast. Daarom is zij geschikt voor het toetsen van de in dit proefschrift beschreven modellerings- en analysetechnieken. De uitgevoerde case study toont de bruikbaarheid van deze technieken voor de analyse en regeling van productketens aan.

About the Author

Uwe Kleineidam was born in Hattingen/Ruhr (Germany) on January 21, 1969. After his grade from high school he studied Physics at the University of Dortmund (Germany). During his studies he spent a year at the *Université Joseph Fourier* in Grenoble (France) where he took the *Licence de Physique*. He concluded his Physics studies at the University of Dortmund taking a master's degree (*Diplom*) on a research project in the field of theoretical solid state physics on the calculation of transport properties in the Single-Impurity-Anderson-Model. At the same time, he took a *Vordiplom* grade in Management Science and Economics. Subsequently, he joined the energy and environment group and the systems and control group at Eindhoven University of Technology (The Netherlands) where he carried out the research project that is presented in this thesis.

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Uwe Kleineidam
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