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by

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Abstract

In order to control and optimize the production process in the glass packaging industry, simulation tools are needed. In this article we describe the main phases in bottle and jar manufacturing, give the mathematical model and present the simulations. Glass can be considered an incompressible Newtonian fluid. Approximation of Navier-Stokes equations leads to treating the problem as a pseudo-time-dependent free (moving) boundary problem. The problem of mass conservation arising in numerical implementation will be also discussed. The standard numerical schemes for time integration can not be performed in this case.

1 Introduction

Glass has been an important material for packing and storing goods a long time. Due to competition from other materials like polymers on one hand, and cost regulation, like the obligation to lower energy consumption, on the other hand there is a growing need for mathematical modeling and numerical simulation. Realizing that glass needs to be produced in an oven which takes about $24h$ to transform the raw material (like siliciumdioxide and soda) to a liquid of about 1200 °C, it is clear that less material per unit product would require less energy and thus will be cheaper, i.e. more competitive at the same time. Therefore much attention is focussed on optimizing forming process. The requirements for a product like a bottle or a jar relate to quality, like strength, weight and residual stress, however, also the process itself put up some constraints like the thickness of the form in order to let a flow take place at all. This calls for sophisticated analysis of the process.

This paper is devoted to a small part of the morphology process in producing packing glass. From the oven a gob of glass is being transported to a mould and there it is pressed into a preform by a plunger which is moving upward. The result of this is a so called *parison* (see fig.1), which is an intermediate product only. The next and final step is the blowing phase where this parison is brought to its final shape by air pressure (see fig.2).

We shall only consider the pressing phase here.

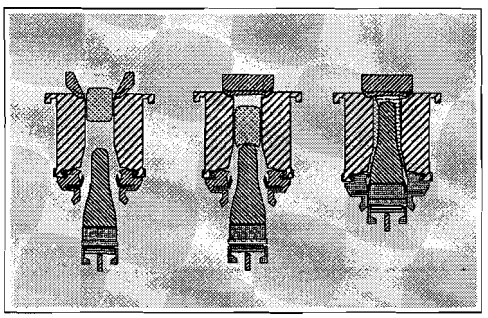


Figure 1: The pressing phase.

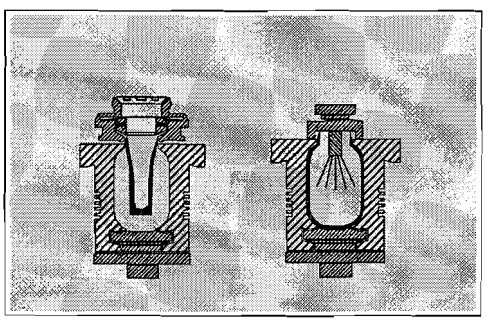


Figure 2: The blowing phase.

2 Modelling the problem

The pressing of glass in a mould is still a complicated process. In order to simplify our discussion below we will neglect the influence of the temperature on the flow. Since the viscosity of the material is strongly dependent on the temperature, one may wonder whether this is realistic.One can show, however, because of the low conducting of glass the actual flow is nearly isothermal in most cases, so that heat exchange effectively takes places only after the motion has stopped. Actually, in the present problem that kind of a decoupling is a bit more complicated. One can assume if the viscous forces are high, heat may arise because of friction. But it is not the case here (see [1]). Hence, in order to describe the morphology of the glass we consider only the motion equations with corresponding boundary conditions, which define the velocity field and the pressure. Moreover we will assume the glass to be an incompressible Newtonian fluid.

Typical values for the problem under consideration are:

$$
\begin{array}{lcl}
\mu = \mu(T_g) = 10^4 \text{ kgs/m} & - & \text{the dynamic viscosity of the glass} \\
\rho = 2500 \text{ kg/m}^3 & - & \text{the density of glass} \\
U = 10^{-1} \text{ m/s} & - & \text{the typical velocity of the plunger} \\
L = 10^{-2} \text{ m} & - & \text{the typical scale for the parison}
\end{array} \tag{1}
$$

Let Ω_t be the region occupied by the liquid at time $t \in [0, T]$. Denote the velocity and the stress tensor by $\mathbf{v}$ and σ respectively. Consider then the Navier-Stokes equations for incompressible fluids in the time dependent domain Ω_t:

$$
\begin{aligned}
\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) - \operatorname{div}\sigma &= \rho \mathrm{f}, && \text{in } \Omega_t, \\
\operatorname{div}\mathbf{v} &= 0, && \text{in } \Omega_t,
\end{aligned} \tag{2}
$$

where ρ is the mass density and f are the volume forces. The stress tensor σ is related to velocity gradient $\nabla\mathbf{v}$, pressure p and dynamic viscosity μ as follows:

$$
\sigma = -p\mathrm{I} + \mu\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right) \tag{3}
$$

Substitution of (3) into (2) gives us the following equations

$$
\begin{aligned}
\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) &= \rho \mathrm{f} + \nabla p - \mu\nabla^2\mathbf{v} && \text{in } \Omega_t, \\
\operatorname{div}\mathbf{v} &= 0, && \text{in } \Omega_t.
\end{aligned} \tag{4}
$$

The same equations can be rewritten in dimensionless form. Using the values in (1) we have:

$$
\mathbf{v} = U\mathbf{v}', \quad p = \frac{1}{Re}\rho U^2 p', \quad Re = \frac{\rho U L}{\mu},
$$

$$
\mathbf{x} = L\mathbf{x}', \quad t = \frac{L}{U}t',
$$

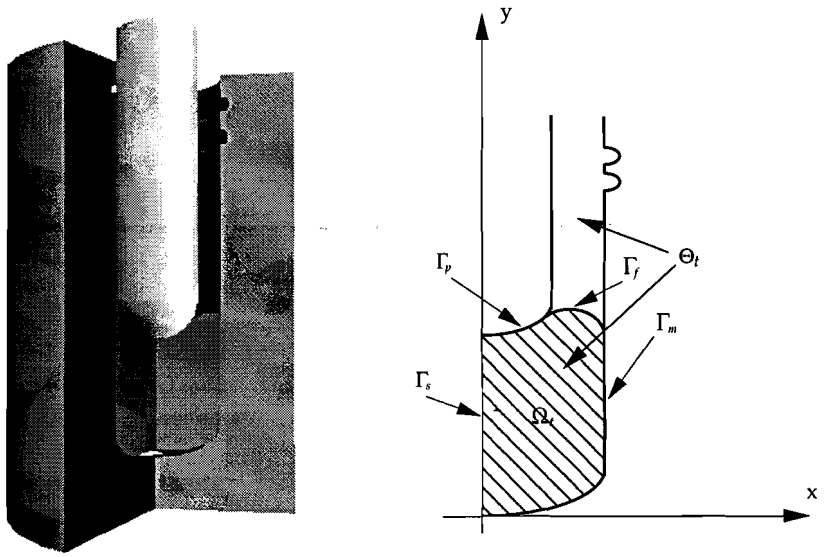
where Re is the *Reynolds number* and $\mathbf{v}'$, p', $\mathbf{x}'$, t' are dimensionless variables. Then the previous system of equations (4) reads as follows:

$$
\begin{aligned}
Re\left(\frac{\partial \mathbf{v}'}{\partial t'} + \mathbf{v}'\cdot\nabla\mathbf{v}'\right) &= Re\frac{L}{U^2}\mathrm{f} + \nabla p' - \mu\nabla^2\mathbf{v}' && \text{in } \Omega_t, \\
\operatorname{div}\mathbf{v}' &= 0, && \text{in } \Omega_t.
\end{aligned} \tag{5}
$$

The volume forces consist of the force of gravity only, i.e.

$$
\|\mathrm{f}\| \approx 10 \text{ kg m/s}^2.
$$

According to (1) the Reynolds number for the problem is of order 10^{-4}; hence the left-hand side is sufficiently small and can thus effectively be skipped from the equation.



a) b)

Figure 3: Problem domain.

According to the previous remark the term which includes the volume forces is of order 10^{-3} which is also negligible. This all means that the viscous forces dominate the volume forces.

As a result we obtain the Stokes equations for an incompressible fluid. Ommiting the prime they read

$$\begin{aligned} \mu\nabla^2\mathbf{v} - \nabla p &= 0 \qquad \text{in } \Omega_t, \\ \operatorname{div}\mathbf{v} &= 0, \qquad \text{in } \Omega_t. \end{aligned} \tag{6}$$

This system of equations together with boundary conditions define the velocity field and the pressure.

Although the initial form of the gob may not necessarily be axisymmetric, it is reasonable to employ the axisymmetric geometry of mould and plunger and assume the flow as such to be axisymmetric as well; see fig.3a,b. Hence it can be reduced to a two dimensional case. The domain Ω_t will be associated now with the configuration in fig.3b.

Let $\Gamma_t = \partial\Omega_t$ be the boundary of Ω_t. It is easy to see that Γ_t consists of four parts:

$$\Gamma_t = \Gamma_m \cup \Gamma_f \cup \Gamma_p \cup \Gamma_s,$$

corresponding to the mould, free boundary, plunger, and symmetric part respectively.

As glass during the process is assumed to be a fluid, no-slip boundary conditions can be assumed for correspondent parts of boundary Γ_t:

$$\begin{aligned} \mathbf{v} &= \mathbf{v}_p, \\ \mathbf{v} &= 0, \end{aligned} \tag{7}$$

where $\mathbf{v}_p$ is the velocity of the plunger.

A symmetry boundary condition is required for Γ_s:

$$\begin{aligned} \mathbf{v}_n &= 0, \\ \frac{\partial \mathbf{v}_s}{\partial n} &= 0, \\ \frac{\partial p}{\partial n} &= 0. \end{aligned} \tag{8}$$

At the free boundary normal stress must be equal to external pressure p_0, which is assumed to be constant. The tangential stress must be equal to zero. Hence:

$$\begin{aligned} p - 2\mu \frac{\partial \mathbf{v}_n}{\partial n} &= p_0, \\ \frac{\partial \mathbf{v}_n}{\partial t} + \frac{\partial \mathbf{v}_s}{\partial n} &= 0. \end{aligned} \tag{9}$$

One showed note that (6) is not a stationary problem since we have kinematic boundary conditions (7). Indeed, the domain Ω_t, corresponding to the region occupied with glass at time t, is time-dependent and changes during the process. In the next section we will discuss how we deal with this problem numerically.

3 Time integration

Let $\mathbf{x} : [0, T] \times \Omega_0 \to \mathbb{R}^2$ be such mapping that:

$$\mathbf{x}(0) = \Omega_0, \quad \mathbf{x}(t) = \Omega_t,$$

where Ω_t is the problem domain as defined before. Then the relation between velocity field and the domain geometry can be described by initial value problem:

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &= \mathbf{v}(\mathbf{x}(t)), \quad t \in [0, T], \\ \mathbf{x}(0) &= \Omega_0 \end{aligned} \tag{10}$$

The velocity field $\mathbf{v}(\mathbf{x}(t))$ can be obtained by solving Stokes equations in Ω_t. However, one should realize that the geometry of Ω_t depends on that velocity field.

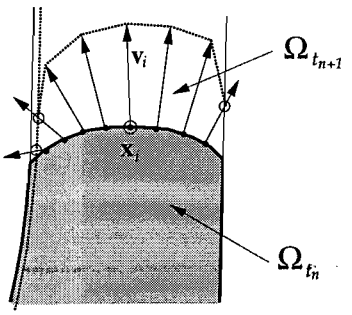


Figure 4: Clip algorithm.

In order to overcome this problem we will use the following strategy. Let us define

$$t_n = n\Delta t, \quad n = 1, \ldots, N,$$

such that $t_0 = 0$, $t_N = T$. After discretization and solving Stokes equations with correspondent boundary conditions in Ω_{t_n} (which are assumed to be defined) we obtain the velocity field $\mathbf{v}^n$. Instead of (10) we solve the initial value problem (11):

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &= \mathbf{v}^n, \quad t \in [t_n, t_{n+1}], \\ \mathbf{x}(t_n) &= \Omega_{t_n}. \end{aligned} \tag{11}$$

In particular for any point $\mathbf{x}_i^n$ at the free boundary Γ_f we may consider this as a Lagrangian displacement, which we may e.g. discretize by the explicit Euler method:

$$\mathbf{x}_i^{n+1} = \mathbf{x}_i^n + \Delta t \mathbf{v}_i^n. \tag{12}$$

One can recognise here the well known Euler explicit integration scheme. The local error for this algorithm is of the first order.

The geometry of $\Omega_{t_{n+1}}$ can be obtained now, and hence the boundary conditions required for solving the flow equations at t_{n+1} can be defined. The same procedure is repeated then until the final geometry Ω_{t_N} and correspondent flow quantities will be computed. Instead of Euler explicit it is possible to use more sophisticated integration schemes. For our problem it turns out to be one of the most important aspects.

Consider first in more detail the deformation of free boundary during a time step. Applying formula (12) for a point $\mathbf{x}_i^n$ at the boundary Γ_f^n (i.e. the boundary Γ_f at time t_n) with corresponding velocities $\mathbf{v}_i^n$, we see that some of the points $\mathbf{x}_i^{n+1}$ don't belong to the domain as defined by the mould and the plunger. Let us denote the letter by $\Theta_{t_{n+1}}$. This configuration is changed explicitly by moving the plunger at each time iteration. We now simply clip displacement outside this $\Theta_{t_{n+1}}$, see fig.4. So the position of $\mathbf{x}_i^{n+1}$ is defined now by intersection of $\mathbf{x}_i^{n+1} + span\{\mathbf{v}_i^n\}$ and $\Theta_{t_{n+1}}$

$$\mathbf{x}_i^{n+1} = \mathbf{x}_i^n + \alpha_i \delta t \mathbf{v}_i^n, \quad \alpha_i \in (0, 1]. \tag{13}$$

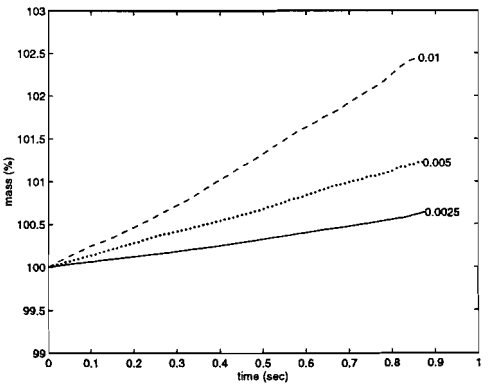


Figure 5: Volume graph for different time steps.

Where α_i is chose such that

$$\Omega_{t_{n+1}} \subset \Theta_{t_{n+1}}.$$

We shall call this algorithm the "clip" algorithm. Note that for the local error in (13) we have

$$\frac{\|\mathbf{x}_i(t_{n+1}) - \mathbf{x}_i^{n+1}\|}{\Delta t} = \frac{\|\mathbf{x}_i(t_n) + \Delta t\dot{\mathbf{x}}_i(t_n) - \mathbf{x}_i^n - \alpha_i \Delta t \mathbf{v}_i^n\|}{\Delta t} + O(\Delta t) =$$

$$\frac{\|\mathbf{x}_i(t_n) + \Delta t\dot{\mathbf{x}}_i(t_n) - \alpha_i \mathbf{x}_i(t_n) - \alpha_i \Delta t\dot{\mathbf{x}}_i(t_n)\|}{\Delta t} + O(\Delta t) =$$

$$(1 - \alpha_i)\|\dot{\mathbf{x}}_i(t_n)\| + O(\Delta t).$$

For the velocities that must be clipped ($\alpha_i < 1$) the error is apparently $O(1)$! The actual values of α_i depend on the characteristics of the process, Δt and the mesh size h. In a practical implementation the term $(1 - \alpha_i)\|\dot{\mathbf{x}}_i(t_n)\|$ should be of order Δt.

4 Mass conservation

In the previous section we described one step of the actual solution process, i.e. solving Stokes, doing one step Euler, and clipping "non physical" values. Clearly the latter procedure leads to the question whether mass is still conserved. The finite volume of glass (which can be associated with the mass because of incompressibility) is given a priori and equal to Θ_{t_N}, i.e. the volume of mould-plunger system in it final position, when the final domain is filled with glass. Numerically we may find the mass decreasing or increasing. If this is significant (say more than 1 %) the simulation process is useless. For example, in the case of mass decreasing we can see that there is space left in Θ_{t_N} and $\Theta_{t_N} \setminus \Omega_{t_N} \neq \{\emptyset\}$.

In order to solve this problem we can perform the process with a smaller time step, which requires more computational time for solving flow equations, or increase accuracy of numerical integration by using scheme of higher order. Increasing of mass (Fig.

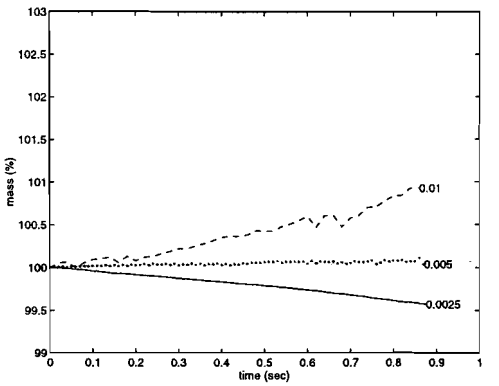


Figure 6: Volume graph for different time steps.

5) arises because of Euler explicit, as it is not conservative scheme (see [2]). Instead of (9) we shall use the following algorithm:

$$\begin{aligned} \mathbf{y}_i^0 &= \mathbf{x}_i^0, \\ \mathbf{y}_i^{n+1} &= \mathbf{x}_i^n + \Delta t \mathbf{v}_i^n, \\ \mathbf{x}_i^{n+1} &= \mathbf{x}_i^n + \frac{\Delta t}{2}(\mathbf{v}_i^n + \mathbf{v}_i^{n+1}), \end{aligned} \tag{14}$$

where $\mathbf{v}_i^n$ now is the velocity in $\mathbf{y}_i^n$. The advantage of this explicit predictor-corrector scheme is that the velocity field has to be calculated only once each time step. With respect to mass conservation, simulation using the combination of the clip algorithm and (11) is illustrated by Figure 6. The mass still increases for $\Delta t = 0.01$ because of Euler explicit used in (11) for predicting $\mathbf{y}_i^{n+1}$. The local error in (11) is of second order with respect to Δt, the discretization error in clip algorithm is of smaller order, hence, for a smaller time step ($\Delta t = 0.0025$) we can see that the clip algorithm, which decreases mass by clipping the velocities, dominates. The graph for $\Delta t = 0.005$, which gives almost mass conservation, is a case where Euler explicit and clip algorithms errors, cancel by another.

Consider the mass conservation from the theoretical point of view:

$$\int_{\Omega_t} \operatorname{div} \mathbf{v} d\Omega = \int_{\Gamma_t} \mathbf{v} \cdot n ds = 0,$$

which means that the mass entering Ω_t is equal to the mass that leaves Ω_t. The formula above leads to another algorithm instead of clip algorithm in order to preserve the area. As it shown in Figure 7 we clip now only the tangential component of the velocity vector. The new position of $\mathbf{x}_i^n$, as it shown on Figure 7 is the point $\mathbf{x}_i^{n+1} \in \Theta_{t_{n+1}}$, such that

$$\mathbf{x}_i^{n+1} = \mathbf{x}_i^n + \Delta t \tilde{\mathbf{v}}_i^n,$$

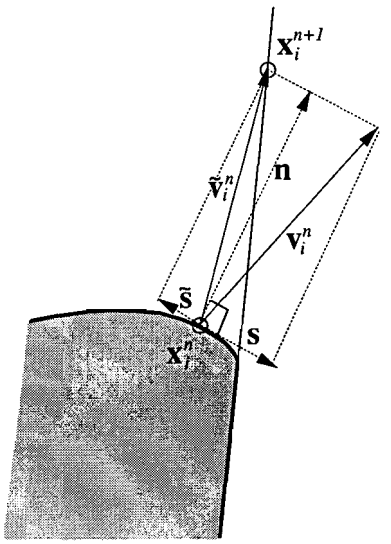


Figure 7: Clip algorithm

where

$$\tilde{\mathbf{v}}_i^n = \begin{pmatrix} \mathbf{n} \\ \tilde{\mathbf{s}} \end{pmatrix}.$$

This algorithm of clipping the velocities will be called tangential clipping. It's easy to see that

$$\int_{\Omega_t} \operatorname{div} \mathbf{v} d\Omega = \int_{\Gamma_t} \mathbf{v} \cdot nds = \int_{\Gamma_t} \tilde{\mathbf{v}} \cdot nds = 0,$$

so modified velocity field is divergence free.

References

[1] T.D. Chandra and S.W.Rienstra, *Analytical Approximations to the Viscous Glass Flow Problem in the Mould-Plunger Pressing Process*, RANA 97-08, Technical University of Eindhoven,1997.

[2] Sanz-Serna, J.M., M.P. Calvo, *Numerical Hamiltonian Problems*, Chapman Hall, London, 1994.