

ATM virtual connection performance modeling

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ATM Virtual Connection Performance Modeling

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de Rector Magnificus, prof.dr. M. Rem, voor een commissie aangewezen door het College van Dekanen in het openbaar te verdedigen op maandag 26 mei 1997 om 16.00 uur

 door

BART J. VAN RIJNSOEVER

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Dit proefschrift is goedgekeurd door de promotoren: prof.ir. F. van den Dool en prof.ir. J. de Stigter en door de copromotor: dr.ir. J. van der Wal

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Rijnsoever, Bart J. van

ATM virtual connection performance modeling / by Bart J. van Rijnsoever. -Eindhoven : Technische Universiteit Eindhoven, 1997. Proefschrift. - ISBN 90-386-0300-2 NUGI 832 Trefw.: geintegreerde telecommunicatienetwerken / computernetwerken ; packetswitching / wachttijden ; telecommunicatie / telecommunicatie ; verkeerstheorie. Subject headings: asynchronous transfer mode / telecommunication congestion control / queueing theory.

Abstract

The Asynchronous Transfer Mode (ATM) is a multiplexing and switching technique for telecommunication networks. In principle, ATM supports any service (computer data, video, speech, ...). ATM is based on short fixed length packets, called cells, that follow a predefined route through the network. This route is called a virtual connection (VC).

To design and operate an ATM network and the equipment that is connected to it, the quality of the service offered by the network must be known. The quality is expressed in terms of the probability that a cell is lost and the probability distribution of the waiting time of cells. This thesis presents two methods to approximate the end-to-end cell waiting time distribution on a VC through an ATM network.

We model an ATM network as a network of queues. Each queue represents the multiplexing of traffic streams on a single transmission link. Congestion occurs in the multiplexers, because the demand for transmission bandwidth may temporarily exceed the capacity. Congestion is the cause of the cell waiting times that we are interested in.

Almost all existing methods to determine ATM VC performance analyze the queues of the network model in isolation, i.e., they do not take the Queuing Network Phenomena (QNP) into account. In this thesis, we study the QNP by simulation and by numerical analysis. We conclude that (depending on the parameters of the network) they may have a relevant effect on VC performance. The VC performance evaluation methods developed in this thesis take the QNP into account, where appropriate.

The most relevant QNP describes that the waiting times of a cell in the queues of the network are correlated (most often, positively correlated). If this QNP is neglected, the assessment of the cell waiting time distribution on a VC is too optimistic, so it is important that it is taken into account. The other two QNP describe that the characteristics of the traffic stream on a VC change due to queueuing in the network and that VC traffic streams become correlated when they are multiplexed on the same transmission link.

The first VC performance evaluation method concerns smooth VC traffic streams, i.e., VC traffic streams that vary little in time. The second method concerns bursty VC traffic streams, i.e., VC traffic streams that vary considerably in time. Both methods take into account the QNP where relevant. They provide more accurate results than existing methods. The methods are validated by simulation results.

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A number of people have coached me during this project which has lasted a number of years. They are the first promoter, the copromoter and Rinus van Weert, who has been involved from the very start. They have commented on my writings numerous times, and the work has certainly benefitted from it, especially as far as the presentation is concerned. Their attention and efforts are greatly appreciated.

Paul ter Horst has been involved in the project when he worked on his master's thesis. He has written some of the simulation programs that have been used. His contribution is appreciated.

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List of Abbreviations

AAL	:	ATM adaptation layer
ABR	:	arbitrated bit rate
AMS	;	Anick, Mitra, Sondhi
ARQ	:	automatic repeat request
ATM	:	Asynchronous Transfer Mode
В	:	Bernouilli process
B-ISDN	:	Broadband ISDN
BMAP	:	batch Markovian arrival process
CAC	:	connection admission control
CD	:	conditional decomposition
Cor	:	correlation
Cov	:	covariance
D	:	deterministic distribution, deterministic process
D-BMAH):	discrete-time batch Markovian arrival process
D-MAP	:	discrete-time Markovian arrival process
E	:	expectation
FCFS	:	first come first serve
FEC	;	forward error correction
FIFO	:	first in first out
G	:	general distribution
GEO	:	geometric distribution
GI	:	general and independent distribution
HEC	:	header error correction
IBP	:	interrupted Bernoulli process
IDC	:	index of dispersion for counts
IDI	:	index of dispersion for intervals
IDP	:	interrupted deterministic process
IPP	:	interrupted Poisson process
ISDN	:	integrated services digital network

Μ	:	exponential distribution, poisson process
MAP	:	Markovian arrival process
MMPP(n)	:	Markov modulated Poisson process of n states
MPEG	;	moving pictures expert group (video encoding standard)
NNI	:	network node interface
NPC	:	network parameter control
PASTA	:	Poisson arrivals see time averages
PCD	:	practical conditional decomposition
PGF	:	probability generating function
QNA	:	queuing network analyzer
QNP	:	queuing network phenomenon, queuing network phenomena
QOS	:	quality of service
TASI	:	time assigned speech interpolation
UNI	:	user network interface
UPC	:	usage parameter control
Var	:	variance
VC	:	virtual connection
VP	:	virtual path

Chapter 1

Introduction

Due to deregulation and privatisation, the global telecommunications market place is changing rapidly. The increase of competition coincides with and is further fertilized by a demand for new services and rapid technological progress in the areas of integrated electronic circuits and optical communications, see e.g. [White et al., 1987; de Prycker, 1991].

Traditional telecommunications networks are optimized for a specific service (telephony, data, TV distribution, etc.) and can often only inefficiently, if at all, support other or new services. These networks do not allow flexible service provisioning. Further, maintaining networks in parallel is inefficient, because they are each dedicated to a single service and do not share resources. In the now more dynamic market for telecommunication services, this state of affairs was no longer economically acceptable and has motivated the international telecommunication standards institute, the CCITT¹ to define first the Integrated Services Digital Network (ISDN) and later the Broadband Integrated Services Digital Network (B-ISDN). ISDN and B-ISDN allow flexible service provisioning. The concept of B-ISDN is an extension of the concept of ISDN to services that require high bandwidth. In addition, service integration is taken much further in B-ISDN than in ISDN, where it is essentially restricted to the interface between user and network. The switching and multiplexing techniques used in ISDN and B-ISDN are completely different. We will consider only the B-ISDN. The CCITT issued the first and still very rough standards on the B-ISDN in 1988 ([I.113, 1988; I.121, 1988]). Since then, standardization work has been going on.

The ability to provide, at least in principle, any service requires the B-ISDN to support a very wide range of service characteristics and performance requirements. The switching and multiplexing technique the CCITT expects to comply with these requirements most efficiently, is the Asynchronous Transfer Mode (ATM), see [I.121, 1988]. ATM achieves integration of the access to the network, of transmission and of switching. It is a packet switching technique in which as many network functions as possible have been transferred to the edges of the network in order to allow the network speed to be increased, so that high bandwidth and low delay services can be supported. The reduction of the network

¹After it started working on B-ISDN, the CCITT has been renamed ITU-T (International Telecommunication Union, Telecommunication Standardization Sector).

CHAPTER 1. INTRODUCTION



Figure 1.1: Generic network model

functionality is possible because of the low bit error probability in optical transmission systems. The packet switching nature of ATM enables more efficient support of bursty traffic sources than circuit switching.

ATM has gained wide acceptance among vendors of telecommunication equipment. This is illustrated by the success of ATM Forum. ATM Forum is an organization (established in 1991) of mainly vendors that intends to accelerate the use of ATM products and services. It does so by selecting standards, resolving differences among standards, and recommending new standards. ATM has also made its way into local area networks, see [Leslie et al., 1993].

Fig. 1.1 shows a generic model of a telecommunication network. The customer's view of the network is described by the Quality of Service (QOS). The QOS comprises the performance of the terminal equipment, of the customer premises network and of the public network. The network provider has to guarantee the QOS in the public part of the network. He does so by traffic engineering (when designing the network) and by traffic control (when operating the network). The subscriber may enhance the QOS by implementing end-to-end protocols and end-terminal functions.

In an ATM network, all information that is transmitted from one customer to another customer follows the same path through the network during the entire session. This path is called a Virtual Connection (VC). This thesis is about the quality of information trans-

1.1. SERVICES

mission on VCs. It intends to provide a set of methods to evaluate VC performance. Applications of these VC performance evaluation methods include traffic engineering and the design of traffic control rules, of end-to-end protocols and of terminal functions.

This chapter introduces and motivates the problem addressed in this thesis: VC performance evaluation. The first two sections give a more detailed description of telecommunication services and of ATM. Sect. 1.3 shows possible applications of VC performance evaluation methods. Sect. 1.4 describes the approach towards VC performance evaluation that we have taken. The last section gives an outline of the thesis.

1.1 Services

B-ISDN supports a very wide range of services. The services differ with respect to their traffic characteristics and QOS requirements. In this section, we first make some general remarks on service characteristics and QOS, and then discuss the most important types of service (namely, speech, date and video) in more detail. The purpose is to illustrate the requirements on an ATM network.

Traffic characteristics may be divided into characteristics associated with the establishment and release of connections and characteristics during the information transfer phase of a connection. The VC performance evaluation methods devised in this thesis concern the information transfer phase. The main characteristics in the connection establishment/release phase are the connection request rate and the holding time. The main characteristic in the information transfer phase is the bit rate at which a source transmits. If the bit rate is variable, the variation of the rate should also be described.

Ref. [E.800, 1988] provides a formal framework to discuss QOS. For our purposes, the relevant aspects of QOS are *trafficability performance* and *transmission performance*. Trafficability performance addresses performance in the connection establishment/release phase of a connection (like e.g. the connection blocking probability). Transmission performance addresses performance in the information transfer phase of a connection. Transmission performance and the information delay and delay variation.

1.1.1 Speech services

The common code rate for telephone speech is 64 Kbit/s, although excellent quality can also be achieved at 16 Kbit/s, see [Keshav, 1992]. Due to the alternation between speakers and short periods of silence during speech, telephone speech shows an on-off behavior that can be exploited in the network. A speaker is typically active only 40 % of the time, and speech bursts typically last 1 second on the average.

Telephony is particularly sensitive to delay. Without echo cancellation², an end-to-end

 $^{^{2}}$ A telephone set is connected to the network by two wires. This set of wires carries two signals, one in each direction. At the receiving (analog) telephone set, the arriving signal is retransmitted to the sending telephone set in attenuated form. This effect occurs because the receiving set does not perfectly separate

delay of less than 25 ms is required; with echo cancellation, it should not exceed 400 ms (CCITT recommendation G.164). On the other hand, speech is rather insensitive to bit errors: a bit error rate of up to 1 % can be tolerated.

1.1.2 Data services

Data traffic may take many different forms. Often, data traffic shows on-off behavior. The fraction of time a source is active, the activity factor, may be very small, but also very high. The bit rate during on-periods may be low, but also high. The main distinction is between interactive data (either person-to-machine or machine-to-machine) and bulk data (machine-to-machine), see [Chen et al., 1988]. Interactive data is bursty and asymmetric of character; bulk data is continuous, unidirectional and high speed. Images form a special type of data. They are coded at 96 Kbit to 300 Kbit for facsimile to 64 Mbit for X-ray images ([Hluchyj et al., 1992]).

In general, data is intolerant to loss. Bit error probabilities down to 10^{-12} are required, depending on the application, see e.g. [Armbruester et al., 1992]. Interactive data is (by definition) relatively intolerant to delay, although the delay and delay jitter requirements for data communication are less severe than for telephone speech, see e.g. [Armbruester et al., 1992].

1.1.3 Video services

In order to reduce the transmission bandwidth of a video signal, one can exploit the spatial and temporal correlation in a video sequence and the way in which the human perception of video works. Among others, entropy coding³ may be applied. The bit rate of the resulting signal varies continuously.

The characteristics and the required QOS of a video signal depend on the application and the applied coding scheme. CCITT standards exist for reduced quality video telephony and video conferencing at 64-128 Kbit/s and 384 Kbit/s - 2 Mbit/s, respectively (recommendation H.261). Video standards from the Moving Pictures Expert Group (MPEG) exist at 1.5 Mbit/s for video cassette recorder quality and at 4 Mbit/s for standard TV quality, see [Armbruester et al., 1992]. According to [Keshav, 1992] the minimum rate required is 1.5 Mbit/s. For High Definition TV distribution at most 50 Mbit/s is required ([Armbruester et al., 1992]), and 20 Mbit/s is required using state-of-the-art coding ([Keshav, 1992]).

The bit error requirements for video are inversely proportional to the degree of compression and vary between 10^{-6} and 10^{-12} , see [Armbruester et al., 1992]. The delay and

the two signals carried by the wires that connect it to the network. As a result, a speaker hears an echo of its own speech.

³The best known example of entropy coding is Morse. Symbols that occur often are represented by efficient symbols in the code. Reversely, symbols that occur seldom are represented by inefficient symbols in the code.

delay jitter requirements for video telephony and video conferencing are the same as for telephony. For video distribution delay may be much higher.

Finally, several services may be combined into a multi-media service, see e.g. [Armbruester et al., 1992]. These services may be carried on parallel connections or on a single connection in the network (i.e. the media are multiplexed at the terminal). If parallel connections are used, the traffic streams on these connections are correlated (e.g. image and sound of a single scene) and at the receiving end the streams have to be synchronized (e.g. the image of a speaking person and the sound of his speech may be shifted in time by at most 50 ms).

1.2 The Asynchronous Transfer Mode

In this section, we provide a brief introduction to ATM networks. A basic understanding of ATM is required before the problem addressed in this thesis can be described. We successively discuss the basic ideas behind ATM, the ATM protocol reference model and the causes of performance degradation in an ATM network.

1.2.1 ATM

The essential structure and working of ATM was agreed upon within the CCITT in 1990, see [de Prycker, 1991; Anderson, 1991; Kano et al., 1991; Boudec, 1992]. In essence, ATM is a packet switching technique that is enhanced with some circuit switching-like features, so that it can provide services that are sensitive to delay. ATM can be summarized as follows:

- Packet routing is based on virtual connections. This allows reservation of network resources and thus alleviates the problem of flow control. As a consequence, special measures have to be taken to support connectionless services.
- Error and flow control are not performed link-by-link, like in a traditional network as X.25. Error control intends to mask transmission errors by retransmission of packets in which an error is detected. Flow control throttles the flow of packets into a network node in order to avoid buffer overflow. Due to the increased quality of optical fiber networks, adequate error performance can be achieved without link-by-link error control. End-to-end error performance may be enhanced by forward error correction or an end-to-end automatic repeat request protocol. Flow control can essentially be achieved by a preventive traffic control scheme, see 1.3.2.
- The packets have a fixed, small length; they are called cells. Fixed packet length is slightly less bandwidth efficient than variable packet length, but gives more time for header processing, facilitates buffer management, and eases buffer dimensioning. Small size packets have been chosen, because of the reduced packetization and queuing delay. Cells consist of a 5 byte header and a 48 byte payload field. The reduction



Figure 1.2: B-ISDN/ATM Protocol Reference Model

of the network functionality (i.e., no link-by-link error and flow control) has allowed reduction of the header size to essentially identification of the virtual connection to which the cell belongs. Small header size provides high bandwidth efficiency.

1.2.2 ATM protocol reference model

The CCITT recommendations on B-ISDN relate to two interfaces (see Fig. 1.1):

- UNI: the user-network interface between terminal and network.
- NNI: the network node interface between nodes in the network.

It was attempted to develop identical interfaces, and this has largely been achieved. The protocol reference model (see Fig. 1.2) is a way to describe the protocols across these interfaces.

The protocol reference model is structured into (vertical) planes and (horizontal) layers. The planes are used to distinguish between:

- User functions, that take care of the information transfer phase of a virtual connection.
- Control functions, that set up and break down a virtual connection.
- Network management functions, that we will not consider further.

The layers divide the protocols into independent sets. Similar to the OSI-model, the reference model distinguishes between the physical layer, the ATM layer, the ATM adaptation layer, and higher layers. In the physical layer and the ATM layer, there is no distinction

1.2. THE ASYNCHRONOUS TRANSFER MODE

between the user plane and the control plane. In the ATM adaptation layer, the user plane and the control plane may use different protocols (out of the same set of protocols defined by the layer). The higher layer protocols are different in the user plane and the control plane.

The user plane of the NNI comprises only the physical layer and the ATM layer. The user plane of the UNI however comprises all layers. So, in the user plane the protocols operate end-to-end (i.e., between the terminals) in the ATM adaptation layer and in the higher layers.

Next, we discuss the three lower layers of the reference model in more detail. The VC performance evaluation methods developed in this thesis pertain to the ATM layer. The ATM layer relies upon the physical layer for the transport of cells and provides a service to the adaptation layer.

Physical layer

The physical layer provides to the ATM layer the transport of valid cells and timing information. Standardized transmission rates are 155.520 Mbit/s and 622.080 Mbit/s.

The header of ATM cells is protected by header error control (HEC). HEC operates in one of two modes. In the first mode it corrects single header bit errors and detects multiple header bit errors. After HEC has detected a header containing bit errors, it switches to the second mode. In the second mode, HEC only detects header bit errors. After a number of correct headers have been received, HEC again switches to the first mode. Apart from error detection and correction, HEC is also used for synchronization at the cell level.

As the ATM cell stream carries its own synchronization information, in principle any transmission system can be used in the physical layer, subject to QOS requirements. At the UNI, two transmission systems have been standardized: a system based on the Synchronous Digital Hierarchy and a cell based system. The former is a standardized transmission system for digital networks and is chosen to achieve identity with the NNI. In case of a cell based interface, the ATM is also used as a transmission technique. The transmission overhead is then carried in ATM cells.

ATM layer

In the ATM layer, cell multiplexing and switching are performed. ATM is connection oriented. Before cells are transmitted a Virtual Connection (VC) between source and destination is established. All cells of a connection pass through the network via the same route. The VC of a cell is identified by the label field in the cell header.

The header of an ATM cell comprises several fields:

- the label,
- the HEC field previously discussed,
- a single bit to indicate the cell loss priority,

- a payload type field,
- the Generic Flow Control field (only at the UNI).

The priority bit indicates which cells on a VC are less eligible to drop in case of network congestion. The payload type distinguishes between user and network internal cells on a VC. Network internal cells may for example be used for performance monitoring. The Generic Flow Control field can be used to implement the medium access control mechanism of the customer premises network.

ATM adaptation layer

Several protocols have been defined for the ATM adaptation layer (AAL), and the definition of this layer is still under discussion. The AAL provides at least segmentation and reassembly of the higher layer information units into ATM cells. Further it may (see also section 1.3.3):

- detect loss and insertion of cells.
- recover lost cells by retransmission or forward error correction.
- provide flow control between source and destination.
- recover the timing of the cell stream by time stamping of cells or a smoothing buffer.
- multiplex higher layer traffic streams into a single ATM stream.

Most of these functions require exchange of information between the source and destination AALs. Together with the higher layer information units that information forms the cell payload, thus reducing the effective bandwidth.

1.2.3 ATM network performance

The performance of a VC in an ATM network is essentially determined by the ATM layer. VC performance is described in terms of loss and delay of cells (see [Takahashi et al., 1989; de Prycker, 1991; Nagarajan et al., 1992; Yokoi et al., 1992; Murakami et al., 1992]). It indicates the quality of the service that the ATM layer offers to the ATM adaptation layer. Performance on a VC is of course also determined by the physical layer, but the physical layer performs much better than the ATM layer and does not play an essential role in the application of the VC performance evaluation methods developed in this thesis. These methods apply to design of the network in the ATM layer.

Next, we first consider the causes of performance degradation in the three lower layers of the protocol reference model, then describe measures of performance, and finally indicate values of some performance measures.

1.2. THE ASYNCHRONOUS TRANSFER MODE

Causes of performance degradation

In the physical layer, bit errors cause performance degradation. They may occur randomly (mainly due to noise) or in bursts (in optical fiber networks, mainly due to maintenance actions). The bit error rate in optical fiber networks is less than 10^{-8} ([de Prycker, 1991, p. 42]). Further, the physical layer introduces fixed delays due to propagation, transmission and processing.

Cell headers are protected against bit errors by header error correction (HEC), because cell header bit errors would cause error multiplication (an entire cell would be lost instead of one bit). As HEC may correct single bit errors, the transmission system should have a low burst error rate. The net cell loss rate due to header errors is then very low.

In the ATM layer, cell buffers in switches and multiplexers may overflow due to congestion, and thus cause cell loss. The buffers also introduce variable queuing delay. Loss and especially delay in cell buffers is the main topic of this thesis. Further, switches also introduce a fixed processing delay.

In the adaptation layer, additional delay is introduced by cell segmentation and reassembly of higher layer protocol units into ATM cells. If variable cell delay is smoothed at the receiving side by a smoothing buffer, overflow or underflow of this buffer also results in cell loss. (In case of underflow, a cell arrives too late and is no longer relevant.)

Performance measures

ATM layer network performance measures can be categorized into the classes speed, accuracy and dependability.

As far as speed is concerned, cell transfer capacity and cell delay are important. Cell transfer capacity is the maximum mean cell transfer rate that the network supports for a specific service. In principle, a user can transmit cells at a rate up to the transmission capacity of the UNI. However, traffic control (see 1.3.2) restricts the cell transfer rate. Cell transfer delay is a random variable. Not only the mean cell transfer delay is relevant, but also cell delay variation or jitter. Several measures of jitter can be envisioned, see [Anagnostou et al., 1991]: cell delay variance, a percentile of the cell delay distribution, or a percentile of (the distribution of) the difference between the delays of consecutive cells.

Accuracy performance is measured by the errored cell ratio, i.e. the fraction of cells that arrives at the destination with a bit error in the cell payload.

Dependability is determined by the cell loss ratio (i.e., the fraction of transmitted cells that does not reach the destination) and by the cell insertion rate (i.e., the rate at which cells not intended for a destination reach that destination). Next to mean values of these performance measures, the distribution of impairments over cells or in time is also relevant, because the user or the AAL may depend on it.⁴

 $^{^{4}}$ In [Noorchahm et al., 1992], the multiply-errored-cell-block ratio partly covers this need. A cell block is defined as a set of cells that are consecutively transmitted by a source. A multiply-errored-cell-block occurs when at the receiver more than a given number of errored, lost, or misinserted cells are observed in a cell block. The multiply-errored-cell-block ratio denotes the fraction of such blocks.

Performance measure values

After the network performance measures have been established, values have to be attributed to them. The end-to-end cell delay on a VC is for long connections dominated by the propagation delay ($\approx 5\mu s/km$) and for low bit rate services by the cell assembly delay (6 ms at 64 Kbit/s). The delays in the ATM layer are small due to high transmission rate and small cell length. A buffer of 100 cells gives a maximum delay of 0.3 ms at 150 Mbit/s transmission rate and 53 byte cell length. Note, however, that variable cell delays are entirely due to queuing in buffers.

The errored cell ratio is directly determined by the performance of the physical layer. This also holds for the probability of cell loss due to header errors. The probability of cell loss due to a header error is determined by the probability of more than one bit error in the header (HEC restores a single bit error in the header, if it is the first header received in error) and is usually very small.

The errored cell ratio and the header error probability give a lower bound on the design goal for the probability of cell loss due to buffer overflow, see [Anagnostou et al., 1991]. It is useless to have a probability of cell loss due to congestion that is considerably lower than the cell loss probability in the physical layer.

1.3 Applications of VC performance models

The subject of this thesis is ATM virtual connection (VC) performance modeling. As described earlier, a VC indicates the route that the cells of a connection between source and destination take through the network. In this section, we show where VC performance models can be applied: network design, traffic control, and design of end-to-end protocols and end-terminal functions.

1.3.1 Network Design

Network design intends to choose the topological structure (i.e. geographical locations of nodes and their interconnections) and capacity of the network such that the costs are minimized while the required QOS is still achieved. It takes long term decisions based on forecasts of traffic load and traffic characteristics. Design of ATM networks differs considerably from the design of circuit switched networks. In ATM networks, there is a very complex relationship between traffic load on the one hand and bandwidth and buffer size required to support that load on the other hand.

As noted in [Roosma, 1991], dimensioning methods (i.e., choosing capacities) for circuit switched networks are based on four models:

- A network traffic model that determines the traffic load on each link based on the end-to-end traffic load.
- A network performance model that determines the required performance on each link based on the end-to-end performance.

1.3. APPLICATIONS OF VC PERFORMANCE MODELS

- A link model that relates traffic load, capacity, and performance at the call level.
- An optimization model that describes how to optimize the network by using the other three models.

These models provide a convenient way of network dimensioning. They allow to separately consider each link in the network.

Application of these methods to ATM networks would require extension of the four models from the circuit-level (or, in case of ATM, rather the VC-level) to the cell-level, see [Roosma, 1991]. This entails several problems that are the subject of VC performance modeling:

- When determining the traffic load on each link, should it be neglected that traffic characteristics change due to multiplexing in the network ?
- When distributing the end-to-end performance requirements over the nodes in the network, should the performance in each node be assumed to be independent ?
- How should the appropriate cell-level capacity of a link be determined given the traffic load ?

The relationship between traffic load and link capacity is structured in [Uose et al., 1992]. It comprises two levels⁵:

- VC-level dimensioning determines the required capacity in terms of the number of VCs. Much in the same way as the number of circuits required in circuit switching.
- Cell-level dimensioning determines the required capacity in terms of transmission bandwidth.

The complexity of these dimensioning procedures depends on the degree to which statistical multiplexing is applied in the network and on the number of service classes that is supported. At the VC-level, capacity may be dedicated to service classes or it may be shared between service classes. At the cell-level, capacity may be dedicated to VCs, it may be shared between VCs in the same service class, or it may be shared between VCs in different service classes. The larger the extent to which resources are shared, the higher resource utilization becomes, however, at expense of increasingly complex dimensioning and traffic control.⁶

 $^{{}^{5}}A$ third level could be distinguished in between the VC-level and the cell-level, namely the Virtual Path-level. A Virtual Path (VP) is a clustering of a number of VCs that collectively pass through a part of the ATM network. Inside the network, the VCs that make up a single VP need not be addressed individually. VPs facilitate the management of the network. We do not consider VPs any further, because they do not require special attention with respect to VC performance models.

⁶There are two alternatives to full resource sharing (see [Lea, 1992; Leslie et al., 1993]). The first alternative assumes that resources are dedicated to service classes (at the VC-level) and to VCs (at the cell-level) Any free capacity that is observed is again handed out, however, without performance guarantee. In the second alternative, resources that are dedicated to service classes are reallocated at a slow rate.

1.3.2 Traffic Control

Traffic control intends to guarantee the QOS on all VCs and simultaneously to achieve high utilization of network resources. Unlike design, traffic control is a real-time function.

Traffic control in ATM networks is the subject of intense study, see e.g. the survey articles [Bae et al., 1991; Burgin et al., 1991; Cooper et al., 1990; Doshi et al., 1991; Eckberg et al., 1989; Eckberg et al., 1990; Eckberg et al., 1991; Eckberg, 1992; Gilbert et al., 1991; Guen et al., 1992; Habib et al., 1991; Roberts, 1991b; Saito et al., 1991; Uose et al., 1992; Wernik et al., 1992; Woodruff et al., 1988; Woodruff et al., 1990; Yazid et al., 1992]. Traffic control entails a trade-off between bandwidth efficiency and buffer sizes on the one hand and processing complexity and signaling complexity on the other hand. Further, it should not present a bottleneck to the versatility of the network, and thus be flexible with respect to traffic characteristics and performance requirements. In comparison with traffic control in packet switched networks, traffic control in ATM networks presents many new problems:

- Traffic characteristics and quality requirements differ widely between the services that are offered by a single network.
- Many services concern real-time traffic that is less controllable than traditional data traffic. (It is no use to delay the output of a video source, because information that arrives too late is worthless.)
- Due to the increased transmission rate, the bandwidth-delay product (i.e. the amount of data in transit) is very high. As a result, the data flow is very inert, and reactive control schemes are often not fast enough to prevent oncoming congestion.⁷ So, traffic control in ATM networks mainly consists of preventive control that intends to avoid congestion. As a result, bandwidth utilization will be lower.
- The high cell transmission rate requires processing associated with traffic control at the cell level to be simple.
- In contrast to most data networks, an ATM network is a public network, in which the cooperation of network users is not guaranteed.

Traffic control in ATM networks may be subdivided according to the subject of control, or, equivalently, according to the time scales in which control actions occur: traffic control at the virtual connection, burst, and cell levels, respectively. The burst level is the level that describes the on-off behavior of voice and data sources and the frame-level of video sources, see 1.1 and 2.1.1. This categorization is used subsequently to briefly discuss the traffic control functions that have been proposed in the literature. Note that no agreement exists on the necessity and feasibility of some of these functions.

⁷Note, however, that the effect of increased transmission rates might be offset by also increasing memory chip size and processor speed, see [Fraser, 1991].

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Virtual connection level

VC level traffic control is applied at connection set-up time in each network node through which a projected VC passes. The main element of VC level traffic control is the connection admission control (CAC) algorithm. It decides whether a new connection can be carried on a transmission link without violating the QOS of existing VCs and the new VC. If VCs of different service classes are multiplexed on the same link (which significantly adds to the complexity of the CAC algorithm), CAC should also guarantee the VC blocking probabilities in the different classes.

Inputs to the CAC-algorithm are a description of the traffic stream on the requested VC and the state of the link. The traffic description should be sufficiently detailed to allow evaluation of the QOS of the multiplexed VCs. On the other hand, it should be controllable at the UNI (see cell level control) and the user-terminal should be able to present it.

There are two approaches to describing a VC traffic stream:

• Stochastic description:

A VC traffic stream is described by a stochastic model that is characterized by a few parameters (e.g., the mean burst length). The performance guarantees obtained by the CAC-algorithm are stochastic (e.g., the probability that a cell has to wait longer than a given time).

• Deterministic description:

A VC traffic stream is characterized by its worst case behavior (e.g., the maximum burst length). The performance guarantees obtained by the CAC-algorithm are deterministic (e.g., a cell does not wait longer than a given time).

Due to multiplexing, traffic characteristics change at each network node. Whether and how the change of traffic characteristics should be considered at connection set-up is an open issue. Also, the distribution of end-to-end performance requirements over the network nodes is to be studied. In the same way as for design, this requires a VC performance model.

If connection set-up is successful, bandwidth and possibly also buffer space is allocated to the VC. If it is not, other routes will be tried according to the routing algorithm. If none of the routes is successful, the connection is blocked.

Burst level

(The VC performance models developed in this thesis are not designed to evaluate burst level traffic control. This section is included for reasons of completeness.) The combination of VC and cell level control is adequate for traffic sources with known burst characteristics and a peak source rate that is low-relative to the transmission rate. Many such sources may share the transmission bandwidth, so a reasonable utilization of that bandwidth can be combined with a low probability of burst blocking. In other cases, a traffic control scheme at the burst level is required, see [Doshi et al., 1991; Roberts, 1991b]. Two different concepts may be considered:

- either the traffic source requests additional allocation of resources before each burst (This is called in-call parameter negotiations or fast reservation protocol),
- or the traffic source is permanently aware of the network congestion state and decides for itself when a new burst can be transmitted. (The mechanism Available Bit Rate (ABR) ([Saunders, 1994]) is an example of this method.)

Burst level traffic control is not very efficient due to the large bandwidth-delay product in ATM networks. The bandwidth-delay product is a measure for the number of cells in transit between the sources connected to the network and a (congested) node in the network. (The delay is the time it takes for a cell to reach the congested node from the source.) The large bandwidth-delay product makes that a congestion situation has to be predicted long before it (possibly) occurs for reactive measures taken by the sources to be effective. All cells in transit will arrive at the congested node before any measures taken by the sources will become noticeable. Further, the state of the node may have changed in the meantime.

Cell level

An important traffic control function at the cell level is usage parameter control (UPC) (or policing). At the access to the network, the VC cell stream into the network is monitored to detect incompatibility between the observed traffic characteristics and the characteristics agreed upon at connection set-up. UPC is required to protect the network from malicious customers that could send too many cells into the network and thus endanger the QOS.

The UPC algorithm has to compromise between speed with which it responds to incompatibilities and the probability of undeservedly indicating violations. Obviously, the balance has to be on the side of the user. Violating cells may be discarded or, preferably, tagged and discarded in the network only in case of congestion.

UPC should be performed as close to the source as possible, but inside the public part of the network. It should be taken into account that traffic characteristics may change before the traffic reaches the UPC unit.

A second traffic control function at the cell level is performed in the network nodes. Resource utilization may be enhanced by buffer management and scheduling of cells while still meeting the QOS requirements. Loss priority may be given to individual cells or to all cells of a VC. Delay priority may be given to all cells of a VC. (The ATM cell header does not provide for delay priorities for individual cells.)

1.3.3 Enhancing ATM End-To-End Performance

In this section, we consider the enhancement of VC performance by the receiving terminal. We especially consider error correction and restoration of the time relation between the cells of a VC.

1.3. APPLICATIONS OF VC PERFORMANCE MODELS

An ATM network provides the in-sequence transport of cells. Although the network is very reliable, the cells of a VC may be mutilated, lost, or inserted into other VCs. Further the time relation among the cells of a VC is disturbed by the network, because in general cells endure different delays. For services like video and speech it is important that this time relation is maintained.

If the service offered by the ATM layer does not meet the requirements of the application, it may be enhanced by AAL or higher layer protocols and by functions in the terminal of the user. We will consider two examples: error correction and jitter control. These protocols and functions may work on ATM cells but also on (higher layer protocol) packets that comprise several ATM cells, see [Boudec, 1992].

Error correction

(The VC performance models developed in this thesis are not designed to evaluate error correction. This section is included for reasons of completeness.) Error detection is performed by sequence numbers (detecting missing, duplicated, or out-of-sequence items), length fields (detecting incomplete delivery), and checksums (detecting transmission errors). Some applications, like e.g. voice, allow replacement of detected errors by a fixed bit pattern. Other applications require correction. Two schemes are available for error correction: automatic repeat request (ARQ) and forward error correction (FEC).

In ARQ (see e.g. [Doeringer et al., 1990; Bae et al., 1991; Doshi et al., 1992]), the receiver requests the transmitter to retransmit items that have been received in error or have not been received at all. Two variants of ARQ exist:

- In the go-back-n protocol, all items from the requested item onwards are repeated.
- In the selective repeat protocol, only the requested item is retransmitted and the order of the sequence is restored at the receiver.

The go-back-n protocol uses bandwidth inefficiently. The selective repeat protocol requires a resequencing buffer at the receiver. Both disadvantages aggravate if the bandwidth-delay product increases. Decreasing loss rate, however, works in the advantage of go-back-n. Which ARQ protocol to choose and on which item (cell or packet) it is to operate depends on the relative costs of bandwidth and memory and on the error characteristics of the VC. The more errors are correlated the less bandwidth inefficient go-back-n is. Most papers ([Doeringer et al., 1990; Doshi et al., 1992]) plead for protocols that can work in either way.

The second error correction scheme, FEC, adds redundancy to the information transmitted, so that lost information can to some extent be restored at the receiver (see e.g. [Biersack, 1992]). FEC is the only solution if the application requires low latency. It requires processing at the sender and at the receiver, and it decreases bandwidth efficiency. The error correction capability depends on the error characteristics of the VC: the less errors are correlated the higher the probability that they can be corrected by FEC.

The efficiency of error correction schemes depends on the pattern in which errors occur on VCs. It is a task of VC performance evaluation methods to determine this pattern.

Jitter control

Some services (e.g. voice and video) require the time intervals between cells to be preserved by the network. This is called jitter control, pacing, or intra stream synchronization⁸.

Jitter control is performed at the receiver by buffering cells before they are played out to the application. This buffer does not only have the task of compensating for variable network delay, but should also compensate for imperfect transmitter clock recovery at the receiver and, if applicable, restore the variable bit rate signals that have been (partly) smoothed at the sender, see [Lau et al., 1992]. If the buffer overflows, cells are lost. If it underflows (i.e., a cell is not available in the buffer at the moment it should have been played out), cells will have to be repeated or dummy cells have to be inserted. Dimensioning and initializing this buffer requires knowledge of the end-to-end cell delay distribution. A VC performance model provides this distribution.

Especially if the service rate is high, differences between the transmitter clock and the receiver clock may cause significant slip. Cells are played out too fast (so that the buffer underflows) or too slow (so that the buffer overflows). Independent clocks of nominally equal frequency in transmitter and receiver would require expensive clock circuits to achieve the required accuracy. If both source and destination are connected to the same synchronous network (in general, they are not), the terminal clocks could be derived from the network clock. The most universal solution is therefore to derive the receiver clock from the transmitted signal (see e.g. [Boudec, 1992; de Prycker, 1991]). This can be achieved by a phase locked loop fed by the (low pass filtered) filling level of the dejittering buffer or by time stamps transmitted by the sender. Design of the clock recovery circuit again requires knowledge of the cell delay process, that a VC performance evaluation method could provide.

1.4 The modeling technique

The performance evaluation methods that are used for more traditional networks do not apply to ATM, because ATM differs from traditional networks in many essential respects. In this section, we describe our approach to ATM VC performance modeling.

ATM VC performance modeling is motivated by its application in the design of the network, of traffic control rules and of end-terminal functions. For network design and traffic control rule design, a VC performance model should show the trade-off between QOS on the one hand and bandwidth and buffer size on the other hand. For end-terminal design, a VC performance model should provide the performance characteristics of the VC.

The QOS presents the user perspective of a service. The for the network provider more useful measures are collectively referred to as network performance⁹. Generic network

⁸Intra stream synchronization is to be contrasted with inter stream synchronization that refers to the time relation between different streams that together form a single service, e.g. a multi-media service.

 $^{{}^{9}}$ In [1.350, 1988; E.800, 1988], network performance is defined as the ability of a network (or network portion) to provide the functions related to communications between users

performance parameters (see [1.350, 1988]) can be categorized according to two dimensions: the three phases of switched connections (i.e. establishment, information transfer and release) on the one hand and the performance aspects of service speed, accuracy and dependability on the other hand. The parameters are specialized to ATM in [Anagnostou et al., 1991; Noorchahm et al., 1992], see Sect. 1.2.3. Network performance parameters should relate to individual VCs as it is the performance on VCs that is to be guaranteed by the network provider.

We restrict ourselves to the information transfer phase of a connection. Far more than during connection establishment or connection release, it is during this phase that the special characteristics of an ATM network show. So, new techniques for design and control are especially required for this phase. The VC models we present concern the user plane of the ATM protocol reference model.

1.4.1 ATM is different

Although an ATM network resembles a packet switched network, it differs from it in several respects. The applicability of existing models for packet switched networks to the analysis of ATM networks is hampered by the following differences between them:

- ATM cells have short, fixed length. As a consequence, the transmission time of a cell is equal for each cell. In more traditional networks, packets have in general arbitrary length and thus also different transmission times. The transmission time of packets is almost universally assumed to be an independent and exponentially distributed random variable. This assumption is obviously inaccurate for ATM cells.
- ATM is a loss system: a cell that arrives at a full buffer is lost. In more traditional networks, link level flow control protocols throttle the arrival of new packets when buffers have become full.
- The characteristics of the traffic to be expected in an ATM network differ considerably from the commonly assumed Poisson process. Often, the traffic stream on a VC is bursty.
- Much more detailed performance measures and much more severe performance measure values are required: cell loss probability, distribution of the cell loss period, distribution of end-to-end delay, Throughput and mean delay are no longer sufficient, but accurate estimates of the tails of buffer occupancies distributions are required.
- An ATM network is synchronous (slotted) at the cell level. So, it is more accurately modeled by a discrete time model (with the slot size as basic interval) than by a continuous time model.

1.4.2 Approach towards VC performance modeling

Distinction should be made between deterministic and statistical performance guarantees, see [Nagarajan et al., 1992; Kurose, 1993]. A deterministic guarantee ensures the performance of all cells. A statistical guarantee, on the other hand, ensures the performance for a given fraction of the cells. Statistical guarantees may be further divided into steady-state guarantees, that average over an infinite period of time, and guarantees on intervals of time, that guarantee that in at least a given fraction of these time intervals the performance will be better than a reference value. The latter measure is more appropriate because connections are normally established for finite duration and temporal degradation during the connection should also be considered.

There are three approaches to providing performance guarantees in ATM networks (see also Kurose [1993]):

• Deterministic guarantees:

Deterministic guarantees for the network are obtained by adding the deterministic guarantees for the nodes (queues) that make up the network. In order to be able to calculate the guarantees for the individual nodes, two approaches can be taken:

- The first approach is to explicitly account for the change, due to queuing, of the (rather generally described) traffic characteristics.
- The second approach is to make sure that the traffic characteristics do not change by the introduction of a special queuing discipline. Such a queuing discipline is complex and uses transmission bandwidth inefficiently.

Deterministic guarantees mostly guarantee performance at a level that is much worse than the performance achieved with statistical guarantees.

• Statistical guarantees:

The traffic streams are characterized by mathematically tractable models that allow statistical guarantees to be determined for the nodes of the network. In order to achieve guarantees for the network, the guarantees for the individual nodes cannot be accumulated in a straightforward way. It should be accounted for that the traffic characteristics change in the network, that traffic streams become dependent, and that the guarantees for the individual nodes are dependent. Most methods do not account for these phenomena, however, so that they do not provide guarantees, strictly speaking.

The advantages of these guarantees is that they are relatively simple and that they allow the application of statistical multiplexing, which increases bandwidth efficiency (see chapter 3).

• Observation-based "guarantees": Traffic is characterized by measurements of the traffic stream itself or of previous

1.5. OUTLINE OF THE THESIS

streams of the same type. Connection admission control is based on these measurements and on the measured behavior of the other traffic. This approach allows a high network utilization. The QOS will, however, be subject to deviations in the traffic characteristics from the measured values and thus cannot be guaranteed firmly.

We consider the statistical guarantees superior to the other guarantees. They have the potential to provide accurate performance estimates leading to high bandwidth efficiency. Further, they do not require the network operation to be adapted like some of the deterministic guarantees. The disadvantage of existing statistical methods is that they are not very accurate. In this thesis, we intend to increase their accuracy by incorporating where required the phenomena described (i.e., dependence between the guarantees in different nodes, changing traffic characteristics, and dependence between traffic).

A statistical approach can be based on either simulation or on mathematical analysis. Both require the development of a model that captures the essential part of the system behavior. If the model is to be solved analytically, system behavior is captured in a set of equations. This requires a mathematically tractable modeling technique. Especially the size of the model state space is a limiting factor. The simplification required to achieve mathematical tractability may introduce inaccuracies. On the other hand, the need to reduce the model to the bare necessities may provide valuable insight into the problem. If the model is to be solved by simulation, system evolution in time is imitated by a computer. In general simulation allows a higher level of detail than mathematical analysis, but requires more computer time.

We choose an analytical approach to statistical performance guarantees. The approximations required in the model are validated by simulation. Relying entirely on simulation impedes an extensive design study, in which wide ranges of parameter values are considered. Single simulation runs take much time even if fast computers and advanced simulation techniques are used, because of the rarity of the events involved (typically a probability of 10^{-9} of loosing a cell). Further, the insight in the problem provided by the analytical approach is also valuable.

In summary, we identified the need for an ATM VC performance evaluation method. Performance evaluation methods for traditional networks do not apply to ATM. Performance evaluation in ATM can be addressed in three different ways, of which we consider the statistical approaches most promising. Existing statistical approaches are however rather rough. In this thesis, we will improve these approaches by approximately incorporating the effects previously neglected.

1.5 Outline of the Thesis

In this chapter, we have established the subject of the thesis, namely ATM VC performance modeling, and the approach that we take to it. We will develop a model or, rather, two models that allow accurate VC performance evaluation. The models are stochastic descriptions of ATM VCs, and they are mathematically analyzed. The models differ from existing models by the incorporation of phenomena previously neglected.



Figure 1.3: Thesis outline

The outline of the thesis is as follows (see Fig. 1.3):

• Chapter 2 provides a survey of the literature on traffic modeling in ATM networks. Its contribution to the development of the field is to categorize and assess the work by others. Traffic models are a basic building block of the ATM VC model. This chapter provides a basis for the ensuing chapters.

It introduces the important difference between traffic description at the burst level and at the cell level. The burst level describes the *instantaneous* rate of cell generation and is usually associated with the internal state of the traffic source. If the instantaneous cell generation rate varies considerably, a source is said to be bursty.

There exist models for the traffic generated by a single source and for the traffic generated by a set of sources (i.e., aggregate traffic models). Two important aggregate traffic models are identified, one for bursty traffic sources and one for smooth (i.e., non-bursty) traffic sources.

• Chapter 3 provides a survey of the literature on statistical multiplexing. It reviews the work by others on this subject. Statistical multiplexing is a basic operation in an ATM network. This chapter provides a basis for the ensuing chapters.

The basic operations performed in an ATM network are multiplexing of traffic streams

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on a transmission link and, the other way round, splitting of the traffic stream on a link into separate streams. Chapter 3 focuses on multiplexing, especially on *statistical* multiplexing.

In statistical multiplexing, the instantaneous aggregate cell arrival rate at the multiplexer occasionally exceeds the cell transmission rate of the multiplexer. If such (temporary) overload periods are not allowed to happen, we call the type of multiplexing *deterministic*. Statistical multiplexing only applies to bursty traffic sources. It allows efficient use of the transmission bandwidth, however at the expense of additional buffering in the multiplexer in order to cope with periods of overload.

• Chapter 4 provides a survey of the literature on VC performance evaluation. Performance evaluation methods are categorized and compared. With respect to ATM, VC performance evaluation has received little attention in the literature. This chapter identifies deficiencies in existing models and methods for VC performance evaluation. In the following chapters, these deficiencies are addressed.

From our point of view, an ATM network is a network of multiplexers through which VCs pass. There are three effects (we call them queuing network phenomena, QNP) that describe the influence of one multiplexer in the network on another:

- 1. QNP *waiting time correlation*: correlation between the waiting times of a single cell in subsequent multiplexers.
- 2. QNP traffic characteristics change: change of the characteristics of a VC traffic stream due to multiplexing.
- 3. QNP traffic stream correlation: correlation between VC traffic streams that have been multiplexed on a single transmission link.

Existing VC performance evaluation methods essentially neglect these queuing network phenomena, especially the QNP *waiting time correlation*. There is a need to study the QNP more carefully.

• Chapter 5 describes and assesses the three QNP for both smooth traffic and bursty traffic. The description and analysis of the QNP in the context of ATM is an original contribution. This chapter provides the motivation for the two VC performance evaluation methods that are developed in the following chapters.

The QNPs are relevant to VC performance in certain cases. In general, the QNP waiting time correlation and the QNP traffic stream correlation are relevant if fan out is small (i.e., if the output stream of a multiplexer is divided over a small number of downstream multiplexers); the QNP traffic characteristics change is relevant if the rate of the traffic stream is high. In case of smooth traffic, the QNP that should be accounted for by an ATM VC performance evaluation method is waiting time correlation. In case of bursty traffic, the QNP traffic stream correlation should be accounted for, next to the QNP waiting time correlation.

• Chapter 6 presents a method to assess the end-to-end cell waiting time distribution on an ATM VC. The method applies to smooth VC traffic streams. The method is original and is more accurate than existing methods, because it takes the relevant QNPs into account.

The method is an enhancement of traditional decomposition methods, in which the waiting times of a cell in different multiplexers are assumed to be independent. Correlation between waiting times in the multiplexers through which a cell passes is caused by correlation between the arrival processes at these multiplexers. The enhanced method takes correlation between arrival processes into account by conditioning the waiting time of a cell in a multiplexer on the arrival process at that multiplexer.

• Chapter 7 also presents a new method to assess the end-to-end cell waiting time distribution on an ATM VC, but now for bursty VC traffic streams. Again it takes the relevant QNP into account.

The method is based on the observation that the relevant part of the end-to-end waiting time distribution on a VC is determined by the occurrence of simultaneous overload of 1 or 2 multiplexers. The method exploits this observation and reduces the ATM network to a network of 2 multiplexers in tandem while maintaining the relevant overload behavior of the entire VC. The method accounts for the QNP waiting time correlation and traffic stream correlation.

- Chapter 8 applies the VC performance evaluation methods to switch design, usage parameter control, and smoothing buffer design. The chapter shows that the methods can be applied to realistic design problems and that the results obtained by using the methods provide a valuable contribution to the solution of these problems.
- Chapter 9 provides conclusions. It describes the claims of this thesis. Essentially, the claims are the analysis of the three QNP in chapter 5 and the two VC performance evaluation methods of the chapters 6 and 7. Further, chapter 9 describes the limitations of the two methods and indicates topics for further research.

Chapter 2

Traffic Models

In this thesis, we develop models to evaluate ATM VC performance. Any ATM VC performance evaluation model requires that the process of cell arrivals to the network be described. In fact, this description is part of the model.

In this chapter, we review traffic models for ATM networks presented in the literature. See [Kawashima et al., 1990; Bae et al., 1991; Roberts, 1991a; Roberts et al., 1991a; Habib et al., 1992] for general surveys on traffic modeling in ATM networks. We compare the models and identify models that are particularly useful for our purposes.

We focus on models for traffic entering the network. These models may, however, also be used to describe the traffic processes inside the network, after the influence of the network on the traffic processes has been taken into account, see chapter 5.

Traffic models form the outcome of a compromise between mathematical convenience and accuracy. A model may be more or less appropriate, depending on its application. During the operational phase of an ATM network, traffic models are applied in traffic control. It is required that a user (or his terminal) can in some way specify values for the parameters of the traffic model, so that the traffic control algorithms of the network can (quickly) decide whether or not to support a request by the user. During the design phase of a network, traffic models may be more complex, because no real-time decisions need to be taken and no network users are involved. We focus on more complex and stochastic traffic models.

This chapter is divided into three parts, see Tab. 2.1. In the first part, we describe characteristics of traffic processes. Traffic characteristics form the connection between on the one hand a traffic model and on the other hand a traffic process that can be observed in the real world and that is to be modeled. In ATM, especially important is the distinction between the cell level and the burst level of a traffic description.

In the second part, we consider models for the stream of cells that is generated by a single source. This is the traffic process on a virtual connection (VC). These models are especially applied in the analysis of stand alone multiplexers, see chapter 3. Multiplexers are a basic building block in any ATM network. A multiplexer receives cells from several sources and retransmits these cells (one after the other) on a link in the network. The arriving cell streams and the departing cell stream are slotted and synchronized to each
2.1	Traffic characteristics
2.1.1	Levels of detail
2.1.2	Burstiness
2.1.3	Correlation
2.2	Models for a single traffic source
2.2.1	Two-state Markov chain
2.2.2	Multiple-state Markov chain
2.3	Models for aggregate source traffic
2.3.1	Markovian arrival process
2.3.2	Poisson burstarrival process
2.3.3	Renewal process
2.3.4	Two-state Markov chain

Table 2.1: Structure of chapter 2

other. In each slot, a single cell can be transmitted by the multiplexer. If more than one cell arrives at the multiplexer in a single slot, the number of cells waiting in the multiplexer buffer increases and, if the buffer is full, cells get lost.

In the third part, we study models that describe the aggregate traffic process generated by a set of sources. This is typically the traffic process that arrives at a multiplexer. In this thesis, we consider performance on a VC. This requires analysis of more than one multiplexer, see chapter 5. In order to keep the complexity of the ATM VC model manageable, it is attractive if aggregate cell arrival processes can be described by a simple model. In the ensuing chapters, we will extensively use the techniques for modeling aggregate source traffic, especially Baiocchi's method. This chapter ends with conclusions.

2.1 Traffic characteristics

A traffic stream is described by its characteristics. A model of a traffic stream should incorporate those characteristics of the actual traffic stream that have a considerable effect on the performance measure under study. Before we can start modeling traffic streams (in the next two sections), we should first become acquainted with traffic characteristics.

Two approaches to traffic modeling can be taken, see e.g. [Roberts, 1991a, sect. 2.2.3]. In the first, the traffic model tries to capture the behavior of the source. The states of the source itself are represented in the traffic model. Each source state is then associated with a traffic process. In the second, the source is considered a as black box and the cell stream generated by a source is considered as a point process¹. The characteristics of the point process are measured, and the characteristics of the traffic model are chosen in compliance with them.

¹A point process is a stochastic process of which the state changes at discrete points in time.

2.1. TRAFFIC CHARACTERISTICS



Figure 2.1: General ATM source level model

In this section, we consider first the levels of detail at which a source may be modeled (the first approach referred to above) and then two important point process characteristics (the second approach referred to above): burstiness and correlation.

2.1.1 Levels of detail

One may study the behavior of traffic sources at different time scales. The smaller the time scale the more detail a source description shows. It is, however, important to model a source at the right level of detail: enough detail to properly model the effect under study, but not more than that in order not to hamper or even prohibit analysis of the model. In each time scale a set of source states may be distinguished. Often two states are distinguished: an active state and an inactive state. In the next smaller time scale, the source behavior during the active state is refined. Sometimes it is useful to distinguish more states in a given level. These states may then for example represent different level of activity at the next lower level.

It has become very common to model ATM sources at several levels; to the earliest references belong [Hui, 1988; Filipiak, 1988; Schoute, 1988]. The level model of ATM sources is not only important in performance modeling, but also in traffic control, as was already alluded to in section 1.3.2.

Fig. 2.1 shows the general level model of an ATM source. Three levels are distinguished: a connection level, a burst level, and a cell level. The top level describes the presence and absence of a virtual connection. Connection holding times of switched connections may range from hours to seconds. Above the connection level, one might situate a subscription level with the obvious interpretation. In circuit switching, the connection level would be the most detailed level. In ATM switching, we need to consider more levels. The burst level represents the structure of the communication process. It describes the internal source behavior at a high level. In conversational speech, it describes the alternation between speakers and the short silences between words. In video communication, it describes the succession of scenes and, given a scene, the succession of frames. In data communication, it may describe the alternation between partners in interactive communication and the transmission of single packets. As indicated in the examples above, it is often possible to distinguish more than two states at the burst level. The differences between states concern sojourn time distribution and rate. (Alternatively, the burst level might be split into sublevels.)

With each state at the burst level we associate a cell arrival rate, namely the mean cell arrival rate given the source is in that burst level state. We call this rate the *instantaneous* cell arrival rate, as opposed to the cell arrival rate. The cell arrival rate is the mean cell arrival rate measured over the entire duration of the connection.

The burst level is the lowest level of detail in packet switching. Another example of the how the burst level is used to describe traffic streams is TASI (Time Assigned Speech Interpolation) in telephony. In TASI, a number of speech sources share a smaller number of circuits. A circuit is assigned to an active speech source, unless the number of active sources exceeds the number of circuits, in which case 'clipping' occurs.

The cell level constitutes the lowest level. The behavior in the higher levels depends on the individual user; the behavior in this level mostly follows immediately from the burst level: cells are transmitted at deterministic intervals given by the instantaneous transmission rate at the burst level. (An exception form video sources without frame buffer that transmit cells as soon as they are filled without spreading them over the entire frame duration.) Before such a deterministic cell stream reaches the access to the public network, it may have been disturbed due to its transport through the customer premises network, see Fig. 1.1.

A traffic source should be modeled down to and including the level that determines the effect under study. The behavior in lower levels may then be represented by the average of the actual behavior.²

This principle is often applied to statistical multiplexers in ATM networks, see 3.3. The burst level states of the traffic sources being multiplexed determine the (aggregate) instantaneous cell arrival rate at a multiplexer. In a statistical multiplexer, the instantaneous arrival rate occasionally exceeds the output rate of the multiplexer. This is called burst level congestion. If the occurrence of burst level congestion determines the performance of the multiplexer, there is no need to model the cell level of the traffic streams in detail.

²If, in addition, the time scales in successive levels differ several orders of magnitude, the system under study may reach a steady-state in the lower level long before a state change in the higher level occurs. The system is then appropriately modeled applying a *quasi-stationary* approach, in which higher level states are assumed to be fixed. Afterwards, the lower level behaviors at the different higher level states are to be averaged.

2.1. TRAFFIC CHARACTERISTICS

2.1.2 Burstiness

It may be advantageous not to separately describe burst and cell levels, but to merge both levels and describe the cell arrival process as a point process. In this and the next section, we describe two measures that can be used to describe point processes.

Burstiness describes the degree to which the cell interarrival interval length varies over time. The arrival process is said to be more bursty if this variation is larger. Larger burstiness has a detrimental effect on multiplexer performance, hence the need to characterize it in the first place.

When qualitatively assessing the burstiness of a cell stream, the Poisson process is usually taken as a reference process, see e.g. [Habib et al., 1992]. A cell stream is bursty, if the cell interarrival intervals vary more than in a Poisson process. A cell stream is smooth, if the cell interarrival intervals vary less than in a Poisson process. A cell stream with fixed length cell interarrival intervals would be characterized as a process without burstiness.

There is no general agreement on a quantitative measure of burstiness. Possible measures include the coefficient of variation³ of the cell interarrival time and, especially for on-off sources, the ratio of peak and mean cell generation rate, see [Bae et al., 1991]. The coefficient of variation is 1 for a Poisson process. A process would be called bursty if its coefficient exceeds 1.

2.1.3 Correlation

When studying point processes, one may take either of two approaches: count the number of arrivals as a function of time or measure the interval lengths between successive arrivals. The former approach has the considerable advantage over the latter that it can easily be derived for a superposition of traffic sources once it is given for each individual source. For a general ATM source, both the numbers of arrivals in disjoint intervals and the cell interarrival times are correlated. Correlation has a considerable effect on multiplexer performance. Positive correlation (i.e. two random variables (rv's) tend to deviate from their mean values in the same direction) makes multiplexer performance worse; negative correlation (i.e. two rv's tend to deviate from their mean values in opposite directions) improves multiplexer performance. The cell stream generated by an ATM source is mostly positively correlated, because the burst level state fixes the cell level behavior for a relatively long period of time.

Often used second order measures of correlation are *indices of dispersion*, see [Gusella et al., 1991] for a survey. They come in two variants, corresponding to the above noted approaches: the index of dispersion for counts (IDC) and the index of dispersion for interval (IDI). Let X_n denote the *n*-th interarrival time of a weakly stationary stochastic process⁴.

³The coefficient of variation, c_x , of a random variable X is the ratio of the square root of its variance and its mean: $c_x = \frac{\sqrt{Var(x)}}{E(x)}$

⁴A stochastic process $\{X_n\}$ is said to be weakly stationary, if the first two moments, $E(X_n)$ and $E(X_n^2)$, and the autocovariance function, $Cov(X_n, X_{n+m}) = E(X_n \cdot X_{n+m}) - E(X_n) \cdot E(X_{n+m})$, are independent of *n*. Autocorrelation is autocovariance normalized by variance.

The IDI at n is defined as (see [Gusella et al., 1991]):

$$IDI(n) = \frac{Var(X_{i+1} + \ldots + X_{i+n})}{nE^2(X)} = c_x^2 [1 + 2\sum_{j=1}^{n-1} (1 - \frac{j}{n})\rho_j],$$
(2.1)

where ρ_j is the autocorrelation of X_n at lag j. For renewal point processes, IDI(n) equals c_x^2 for all n, because $\rho_j, j \ge 1$ is 0 by definition. The IDI may have a limiting value, which is e.g. the case when the autocorrelation is 0 after a certain lag. The IDI is a generalization of the coefficient of variation from a single interval to an arbitrary number of intervals. It captures in a single number the relative variation of the arrival process during an interval of given length. If correlation is positive, the IDI increases; if correlation is negative, the IDI decreases.

The IDC at time t is defined as (see [Gusella et al., 1991]):

$$IDC(t) = \frac{Var(N_t)}{E(N_t)},$$
(2.2)

where N_t is the number of arrivals in [0, t). If the time axis is slotted into intervals of length τ and the number of arrivals in successive intervals is denoted by the presumed weakly stationary stochastic process C_n , the IDC at slot boundaries is:

$$IDC(n\tau) = \frac{Var(C)}{E(C)} [1 + 2\sum_{j=1}^{n-1} (1 - \frac{j}{n})\xi_j],$$
(2.3)

where ξ_j is the autocorrelation of C_n at lag j. The IDC is constant if the numbers of arrivals in the intervals are independent and identically distributed. As far as a limiting value is concerned, the same conditions as for the IDI apply. If they exist, the limits of the IDI and the IDC are equal.

2.2 Models for a single traffic source

Modeling a traffic source comes down to equating the characteristics that are relevant to performance between the modeled traffic and the traffic model. The heart of the matter is choosing the right characteristics to equate.

The purpose of traffic modeling is to devise a stochastic model of an ATM multiplexer that is mathematically tractable (i.e., the state space of the model should be as small as possible). The number of tractable traffic models is small. This section describes the most relevant models: a two-state Markov chain and a multiple-state Markov chain.

2.2.1 Two-state Markov chain models for on-off sources

Many ATM traffic sources show on-off behavior at the burst level. In the off-state, no cells are generated; in the on-state, cell generation is periodic. Examples of such sources are

speech sources with activity detection and computers. We describe how to model such a traffic source.

The alternation between on-periods and off-periods has given rise to the two-state Markov chain model. The discrete-time⁵ Markov chain comprises two states, an 'on-state' and an 'off-state'. State transitions occur periodically, where the period corresponds to a cell transmission slot of the multiplexer to which the traffic stream is applied. With a certain probability a state transition occurs from the current state to itself. With the complementary probability a state transition occurs from the current state to the other state. This probability depends on the state.

At a transition of the two-state Markov chain into the off-state, no cell is generated. At a transition into the on-state, one cell is generated with a certain probability, and no cell is generated with the complementary probability. So while in reality cell generation in the on-state is most often periodic, it is stochastic in the two-state Markov chain model.

This completely describes the two-state Markov chain model. The sojourn times in the states of the model are geometrically distributed. While the model is in the on-state, the cell generation process is a Bernoulli process⁶. The model is completely specified by its three parameters: mean sojourn time in the on-state, mean sojourn time in the off-state, and cell generation rate in the on-state.

The remainder of this section concerns the choice of the model parameter values:

• Model parameters according to a burst level traffic description:

The model is obviously inspired by its resemblance to on-off traffic. So, the first approach is to equate the model parameters (i.e., the mean sojourn times in the states and the cell generation rate in the on-state) to measurements of the corresponding values in the traffic stream to be modeled. The model parameters describe the traffic stream at the burst level.

• Model parameters according to a point process traffic description:

However, the model parameters may also be set by equating traffic characteristics (like burstiness and correlation) between model and source. These characteristics consider the traffic stream as a point process of cells and do not distinguish a burst level.

It will be shown that the second approach is more accurate.

Model parameters according to a burst level traffic description

The two-state Markov chain model has a certain structure, notably geometrically distributed sojourn times in the states and Bernoulli cell generation in the on-state. When

 $^{{}^{5}}$ We describe the discrete-time two-state Markov chain model. A similar model could however easily be drawn up in continuous-time.

⁶A discrete-time stochastic process is a Bernoulli process if the rv's that constitute the process take the values 0 (no cell) and 1 (1 cell) and are independent and identically distributed

the model parameters are based on the mean sojourn times in the states, it is implicitly assumed that the traffic source also has this structure or that the structure is not relevant to performance. We look into the structure of the traffic source, first into the sojourn times and then into the cell generation process.

Geometrically distributed sojourn times In two cases it is accurate to assume that the sojourn times in on- and off-state are geometrically distributed:

• In some types of sources, sojourn times are indeed geometrically distributed.

Brady [1969] shows that for conversational speech the burst length is approximately exponentially distributed. Silence periods are not, mainly because of the existence of two different kinds of silence (see 2.1.1). In traditional data networks, data packets are usually assumed to have geometrically distributed length.

• Consider statistical multiplexing of on-off sources. If too many sources are simultaneously in the on-state, the buffer starts to fill. If the buffer is small and overflows shortly after the start of overload, the probability that the buffer overflows (and the probability of cell loss) is essentially determined by the probability that a source is in the on-state. It is not determined by the distribution of the sojourn time in the on-state, so it is accurate to assume a geometric distribution (or any other distribution).

Sheng et al. [1993] verify this argument by experiments. They show that the coefficients of variation of the sojourn times have little effect on queuing performance, if the mean sojourn times are long relative to the time to transmit a full buffer load. They also show that correlation between sojourn times is irrelevant under the same condition.

Eliazov et al. [1990] simulate a multiplexer in which the mean sojourn times are small relative to the time to transmit a full buffer (mean on-time 25 slots; mean off-time 46 slots; buffer size: 100 cells). So this is exactly the opposite case, and they indeed show that in this case queue lengths are longer if the sojourn times are more variable, so that the sojourn time distributions are relevant.

Bernoulli cell generation In an ATM traffic source, the cell generation process in the on-state is most often periodic. In the two-state Markov chain model however, cell generation in the on-state is modeled by a Bernoulli process.

Qi Li et al. [1991] show that, due to this approximation, the queue length distribution in a statistical multiplexer is only slightly overestimated. In a statistical multiplexer, the effects that determine performance occur at the burst level. It is not important to accurately model the cell generation process in an on-state (see also 3.3).

Model parameters according to a point process traffic description

The second approach to establishing model parameter values is to consider source traffic as a point process of cells. The measured characteristics of the source traffic are imposed upon the traffic model.⁷

Ide [1988] approximates a voice source (with silent detection and periodic cell generation during speech) by the two-state Markov chain model. He uses three different methods of approximation, that differ with respect to the parameters that are equated between traffic model and source traffic:

- The method that we previously described: equating the mean sojourn times and the cell generation rate in the on-state.
- The three moments method: equating the first three moments of the cell interarrival interval length distribution.
- The two moments and peakedness method: equating the first two moments of the cell interarrival interval length and the exponential peakedness⁸ (with the mean service time equal to the mean interarrival time).

The motivation for choosing exactly these parameters is not given or is at least vague. The three traffic models (all of the two-state Markov chain type, but with different parameter values) are used to estimate the mean delay in a statistical multiplexer. The two moments and peakedness method provides the most accurate approximation of the mean delay.

Also Andrade et al. [1991] compare the accuracy of several sets of parameters. They finally come up with the following set:

- 1. the mean cell arrival rate,
- 2. the variance of the number of cell arrivals during a mean cell interarrival interval,
- 3. and the maximum value of the Index of Dispersion for Counts.

It is not entirely clear why this particular set provides an accurate approximation of the delay distribution in a multiplexer. The second parameter captures the burstiness of the traffic stream; the third parameter captures correlation.

In conclusion, the two-state Markov chain model is more accurate if its parameters are determined by the burstiness and correlation of the source traffic rather than by the mean sojourn times in the burst level states. The way in which burstiness and correlation could best be described is still not clear. Some solutions are presented that work well for the examples tried, but they lack a sound theoretical basis.

 $^{^7\}mathrm{This}$ approach makes it also possible to model other than on-off source traffic by a two-state Markov chain model.

⁸The exponential peakedness is the variance to mean ratio of the number of busy servers of a queue with infinitely many exponential servers to which the arrival process is hypothetically offered.

2.2.2 Multiple-state Markov chain models

The traffic model considered in this section represents the traffic stream generated by a single source, but it might also represent the aggregate traffic stream generated by a number of on-off traffic sources. It forms an appropriate transition to the next section that describes models for aggregate traffic.

Multiple-state Markov chain models are mainly used as models for video traffic. The characteristics of the traffic stream generated by a video coder depend on the type of the video sequence and on the coding algorithm. Video traffic is an isochronous sequence of frames (or images), e.g. 25 frames per second. Usually interframe coding is applied, i.e. only the difference between consecutive frames is transmitted. This makes traffic characteristics dependent on the amount of movement during scenes and on the number of scene changes.

Traffic models presented in the literature distinguish between video without and video with scene changes. Scene changes may be modeled separately; they cause a burst of cell arrivals. During a single scene, traffic of a video source can be characterized by the distribution of the number of cells per frame, $\lambda(n)$, and its autocorrelation function.⁹ In case of video-telephony and video-conferencing, the distribution of $\lambda(n)$ is bell-shaped and its autocorrelation function is roughly exponential¹⁰, see e.g. [Roberts, 1991a, sect. 2.3].

We consider a frame (burst) level model for video sources and a scene level model.

A frame level model for video

For video without scene changes, Magliaris et al. [1987] present a birth-death Markov chain model. It consists of the superposition of a number of independent and identical two-state Markov chain source models ('mini-sources'). The state of the multiple-state Markov chain denotes the number of mini-sources that is in the on-state.¹¹ The probability distribution of the state space is binomial: each source of a fixed number of sources is in the on-state with a certain probability. The state directly determines the cell generation rate. So, also the cell generation rate is binomially distributed. So, it is bell-shaped, as required. Further, it can be shown that the autocorrelation function of the cell rate is exponential, as required.

There are several extensions to this model:

• Shan Huang [1988] allows arbitrary transition probabilities in the birth-death source model, which gives a more flexible model.

 $^{{}^{9}}A$ further concern is when during the frame interval cells are transmitted: during the entire frame interval at equally spaced distance, during the entire frame interval as soon as they are generated, or at peak rate starting at the beginning of the frame interval.

¹⁰However, periodic or quasi-periodic movement will show in the autocorrelation function, see [shan Huang, 1989].

¹¹The mini-sources are not entirely independent: in a slot at most one mini-source can change state. This is an approximation that simplifies the model: in a slot the state of the multiple-state Markov chain can change by at most 1 point.

2.3. MODELS FOR AGGREGATE SOURCE TRAFFIC

- Sen et al. [1989] add a second dimension to the birth-death Markov chain. The state of the model indicates the number of mini-sources of type 1 in the on-state and the number of mini-sources of type 2 in the on-state. The second dimension describes a basic level of cell generation within a scene, and the first dimension describes the variation of the cell generation rate during the scene.
- Blondia et al. [1992] extend the model of Sen et al. by incorporating a state that is visited between scene changes. This state represents the burst of cells required to refresh the state of the receiver at scene changes.

A scene level model for video

Yasuda et al. [1989] outline a model that focuses on scene changes and neglects the frame structure of video traffic. During a scene, cells are generated according to a Poisson process with a given rate. These rates and scene durations are determined by a continuous-time Markov chain. In order to account for the burst of information transmitted at scene change, each scene change is accompanied by a batch of cell arrivals.

2.3 Models for aggregate source traffic

The aggregate traffic stream that arrives at a multiplexer consists of the traffic streams generated by individual sources. Straightforward description of the aggregate traffic stream by a Markov chain easily leads to a large state space. A large state space of the aggregate traffic model hampers analysis of the multiplexer.

In order to reduce the state space of the traffic model, the aggregate traffic stream may be approximated by a traffic model. This section describes two such approximations: approximation by a renewal process and by a two-state Markov chain. First however, we discuss two aggregate traffic models that are not simplified.

2.3.1 Markovian arrival processes

In 2.2.2 we considered the traffic model of a single video source: a multiple-state Markov chain of the birth-death type. The birth-death model represented the aggregate traffic stream generated by a number of identical two-state Markov chains ('mini-sources'). So, the birth-death Markov chain is not only a model for a single video source, but also a model for the aggregate traffic stream generated by a number of independent traffic sources, each modeled by a two-state Markov chain.

If the traffic stream of each single source is modeled by a Markov chain, the aggregate traffic stream of a number of sources can be modeled by the combination of the corresponding Markov chains. In case of the birth-death Markov chain, the combination of Markov chains is considerably simplified by:

• the assumption that the contributing Markov chains are identical and

• the approximation that in a single slot at most one of the contributing Markov chains changes state.

The birth-death Markov chain model is an instance of a very versatile class of traffic models called discrete-time batch Markovian arrival process (D-BMAP) (see [Lucantoni et al., 1990; Neuts, 1992; Blondia, 1991]). In a D-BMAP, the burst level model of an (aggregate) traffic source is an arbitrary discrete-time Markov chain. At each slot boundary, a state transition occurs in this Markov chain. (Most transitions are from a state into the same state.) The cell level is incorporated in the model by associating a batch of cell arrivals with each transition. In general, the distribution of the number of cells in the batch in general depends on the transition. (So it depends on the state before the transition and the state after the transition.)

A special case of the D-BMAP is the Markov modulated Poisson process (MMPP). We will extensively use the MMPP in the ensuing chapters. In an MMPP, at each transition in the Markov chain a Poisson distributed batch of cells is generated. Moreover, the distribution of the batch is entirely determined by the next state. (So the previous state is not relevant in this respect.)

2.3.2 A Poisson burst-arrival process

A particularly simple aggregate traffic model is obtained if many traffic sources contribute bursts of cells to an aggregate stream (see e.g. [Descloux, 1989]). During a burst cells are generated with a certain rate. The duration of a burst is typically geometrically distributed. The contribution of each individual source (in terms of number of bursts per second) should be small. If these conditions are fulfilled, the aggregate burst arrival process is approximately Poisson. ¹²

The aggregate traffic model is described by an infinite birth-death Markov chain, where the state of the Markov chain denotes the number of active bursts. In this Markov chain, the birth-rate is state independent. The death-rate is determines by the number of active bursts and the burst length distribution. (The assumption again is that at most 1 burst arrives in a slot and that at most 1 active burst ends in a slot.)

The parameters of the aggregate traffic model (i.e. burst arrival rate, cell arrival rate during bursts, and mean burst length) are set in essentially the same way as the parameters of the two-state Markov chain model, see section 2.2.1 and [Lindberger, 1991; Delbrouck, 1991].

2.3.3 Approximation by a renewal process

The models of aggregate traffic streams reviewed above have a large state space. A large state space seriously hampers analysis of the multiplexer behavior. This is the motivation

¹²This is the ATM version of the well known traffic model for packet switched networks, in which packets arrive according to a Poisson process and packet length is exponentially distributed, see e.g. [Kleinrock, 1976]. In an ATM network, a higher-layer-protocol packet is divided into a number of cells, so a packet gives rise to a burst of cells. The Poisson approximation is based on the Palm-Khintchine theorem.

to devise simple aggregate traffic models, i.e. models that are considerably less complex than the combination of the traffic models for the individual sources. We consider two simple aggregate traffic models: in this section a renewal process and in the next section a two-state Markov chain.

Sriram et al. [1986] consider the aggregate traffic stream formed by the superposition of independent and identical two-state Markov chain sources with deterministic cell generation in the on-state. They would like to approximate this aggregate traffic stream by a renewal process, but are not able to provide clear method. Note that the cell generation process of a single source is a renewal process, but the aggregate process is not.

The method of Sriram et al. is based on the mean cell arrival rate and the IDI(n) traffic characteristic. IDI(n) is a description by one number of the serial correlation in n consecutive cell interarrival intervals (see (2.1)). The nature of this description is to accumulate in some way all correlation in the interval. The IDI(n) of the aggregate traffic stream can be calculated. It is an increasing function of n. The IDI(n) of a renewal process is by definition independent of n. The IDI of the approximating renewal process is to be chosen at the appropriate value relative to IDI(n) of the superposition.

Correlation between cell interarrival intervals affects multiplexer performance only if the corresponding cells interfere with each other in the buffer of a multiplexer. (The idea is roughly: if the buffer is empty, the waiting time of subsequent cells is not affected by preceding cells.) The degree of interference depends for example on the load of the server.

Using the above argument, Sriram et al. conclude that the IDI(n) of the aggregate traffic stream is only relevant up to a maximum value of n (say, n = m). This maximum value n = m is determined by the degree to which different cell interarrival intervals of the aggregate traffic stream interfere in the multiplexer. The IDI of the approximating renewal process should be chosen equal to the IDI of the aggregate stream at n = m. Sriram et al. are not able, however, to provide an expression for m.

2.3.4 Approximation by a two-state Markov modulated process

A two-state Markov chain is the most simple traffic model that can generate a correlated traffic stream. It is therefore an obvious starting point in the quest to approximate an aggregate traffic stream.

In this section, we consider in particular the aggregate traffic stream formed by the superposition of independent and identical two-state Markov chain sources. This aggregate traffic stream is the input stream to a statistical multiplexer. The approximating two-state Markov chain is not an on-off model but a high-low model, i.e., in both states cells are generated. This Markov chain is described by four parameters: the mean sojourn time and the cell generation rate in each state. We describe a number of methods that choose the parameters of the approximating two-state Markov chain in different ways. Especially the method by Baiocchi et al. is very promising.

Heffes et al. [1986] base the approximation on the IDC. The following characteristics are matched between the aggregate and the approximating traffic stream:

- the mean arrival rate,
- $IDC(t_1)$,
- $IDC(\infty)$, and
- $E(N^3(t_2))$.

The time epochs t_1 and t_2 are chosen such that the IDC(t) functions of the aggregate traffic stream and of the approximating traffic stream coincide as much as possible. Comparison with simulation results shows that the mean and variance of the cell delay in the multiplexer are accurately estimated by this method. The tail of the delay distribution, however, is not.

Ramaswami [1988b] comments on the method of Heffes et al. He argues that the traffic characteristic IDC(∞) is not relevant (see the discussion on IDI(n) in 2.3.3). Furthermore, Ramaswami et al. [1991] observe by simulation that the method of Heffes et al. is inaccurate and does not react properly to changes of traffic source parameters.

Liao et al. [1989] comment that the method of Heffes et al. may fail to represent periods during which the instantaneous arrival rate at the multiplexer exceeds the cell transmission rate from the multiplexer, i.e. overload periods. Overload periods determine the tail of the waiting time distribution and the cell loss probability in a statistical multiplexer (see 3.3). Therefore, they propose to replace the traffic characteristic $E(N^3(t_2))$ by the mean of the instantaneous cell arrival rate at the multiplexer during overload periods. After this modification, one state of the approximating two-state Markov chain models overload of the statistical multiplexer and the other state models underload. This method more accurately estimates the mean waiting time in the multiplexer.

Delbrouck [1991] also notes the failure of the method of Heffes et al. to appropriately incorporate overload periods. A straightforward approach to incorporate overload periods is to let the two states of the approximating Markov chain correspond to underload and overload of the statistical multiplexer, respectively. He matches the following characteristics between the aggregate and the approximating traffic stream:

- mean sojourn time in underload
- mean sojourn time in overload
- mean arrival rate in underload
- mean arrival rate in overload

Then again he rejects this method, because the coefficients of variation of the sojourn times in the overload and underload states are far larger in the aggregate traffic stream than in the approximating two-state Markov chain. Further, the mean sojourn time in the underload state is much higher than the mean sojourn time in the overload state, which presents numerical problems.

2.4. CONCLUSIONS

Baiocchi et al. [1991a] [1991b] represent overload and underload of the multiplexer each by a separate state. They concentrate on long overload periods. If the buffer size is large, cell loss is almost entirely due to these long periods of overload: loss of cells while the multiplexer is underloaded is very unlikely and short periods of overload are accommodated by the buffer. Baiocchi et al. match the following characteristics between the aggregate traffic stream and the approximating traffic stream (We will go into more detail on this in section 7.2.1.):

- the mean cell arrival rate.
- the slope (at infinity and on a log scale) of the survivor function of the sojourn time in an overload period.
- the slope (at infinity and on a log scale) of the survivor function of the number of cells generated in an overload period.
- the probability of a long overload period.

This method is claimed to provide a lower bound (for large buffer size: an asymptotically exact bound) on the cell loss probability. Baiocchi et al. also provide an accurate approximation and presumed upperbound for the cell loss probability. Instead of the probability above (the fourth characteristic), they match the probability of cell loss in a bufferless multiplexer. This sets the probability of being in the overload state in a different way. Baiocchi et al. show by simulation that their method is very accurate.

2.4 Conclusions

In an ATM network, traffic is both bursty and correlated.

The two-state Markov chain is a generic building block in traffic models. Initially, the two-state Markov chain model was proposed for its obvious resemblance to on-off source traffic. It is however more accurate to set the parameters of the two-state Markov chain according to measured traffic characteristics. These characteristics should reflect the burstiness and correlation of the traffic stream. It is not clear which characteristics always provide accurate results.

The state space of the aggregate arrival stream at a multiplexer may easily become very large, if each constituent arrival stream is described by a two-state Markov chain of its own. Hence the need to model the aggregate arrival stream by a much simpler model.

We found two methods. The first method models the aggregate arrival stream by a renewal process. This method is not very well developed, and it is difficult to assess its accuracy when applied to ATM. In the second group of methods, the aggregate arrival stream is modeled by a two-state Markov chain. These methods have been developed to a large extent with application to ATM (namely, statistical multiplexing) in mind. They have been shown to achieve accurate results, especially the method by Baiocchi et al. [1991a].

Chapter 3

Multiplexing

In a communication network, efficient use of resources entails merging and splitting of traffic flows in multiplexers and switches. Further, it should be allowed that the instantaneous demand for bandwidth occasionally exceeds capacity. These observations also hold for an ATM network.

The purpose of this chapter is to study the basic operation in an ATM network: multiplexing. The multiplexer is a building block in ATM VC performance analysis. We will show that an ATM switch can be modeled as a set of multiplexers. In the next chapter, we will then interconnect ATM switches and study the network of multiplexers that is thus obtained. In this chapter, we will also study statistical multiplexing. The characteristics of a statistical multiplexer are very important in the understanding of the VC performance evaluation method for bursty traffic that we will describe in chapter 7.

In this chapter, we first model ATM switches as networks of multiplexers, then describe ATM multiplexers, and finally consider multiplexer performance. The chapter ends with conclusions. In appendix A, we review the very extensive literature on stochastic models of ATM multiplexers and their analysis.

3.1 ATM Switches

This section describes ATM switches. Like all switches, ATM switches essentially perform two operations: splitting and multiplexing of traffic streams. Splitting is distributing the switch input streams over the switch output links. Multiplexing is merging the split input streams into a single switch output stream. Multiplexing occasionally causes congestion: during a given time interval more cells arrive at a multiplexer than can be transmitted by that multiplexer. Multiplexing determines the extent of cell delay and loss in a switch.

In the remainder of this section, we describe switch types. Switch types differ in the way in which splitting and multiplexing are implemented. Many ATM switch designs have been proposed in the literature, see e.g. the survey papers [Ahmadi et al., 1989; Degan et al., 1989; Newman, 1992] and [de Prycker, 1991, Ch. 4]. After the current section we will focus on switches of a specific type. These switches show ideal behavior.



Figure 3.1: Generic functional ATM switch model

The generic functional switch model consists of a switch fabric, input modules, output modules, and a control and management block, see Fig. 3.1 and [de Vries, 1992; Newman, 1992]. The switch fabric is the main part of the switch¹. It transports cells from input port to output port. It operates synchronously: each input port is ready to accept a new cell at the same moment (unless input blocking occurs, as explained later on). Most large switch fabrics are networks themselves. They are built from smaller switch fabrics, called switch elements.

The quality of a switch design is measured by the total traffic load it can carry, at given performance requirements (loss and delay of cells). This load depends on the characteristics of the traffic and on the distribution of the traffic load over the switch input ports and output ports.

Newman [1992] provides a particularly clear classification of switch fabrics. He categorizes switch fabrics according to three dimensions: topology, contention resolution mechanism and buffering strategy. We will separately consider each dimension. For each dimension, we will indicate the optimal switch fabric (from a performance point of view). As said, after this section we will consider only ideal switches that implement the optimal solution in each dimension.

3.1.1 Switch fabric topology

Time-division topologies are distinguished from space-division topologies. In a timedivision topology, all cells cross a shared memory or a shared transmission medium. So,

¹The input and output modules essentially perform the following functions: synchronization of transmission links, translation of cell headers, addition of switching tags to cells, and control of the order of the cells on a VC. If applied, a switching tag controls routing of cells inside the switch fabric.

3.1. ATM SWITCHES

the capacity of the shared resource determines (and restricts) the throughput. This type of design is mainly used in switch elements.

In space-division topologies, not all cells pass through a single point. Two variants exist. In single-path networks, there is a single path through the switch fabric for each input-output pair. (The paths of different input-output pairs do not all pass through a single point however, like in the time-division topologies.) In multiple-path networks, several paths through the fabric exist for each input-output pair.

In general, multiple-path networks show improved performance and reliability in comparison with single-path networks. Multiple-path networks do not preserve the order of the cells on a VC, if cell routing through the switch fabric itself is connectionless.

The optimal switch fabric topology is a special case of the single-path network: each input-output pair is connected by a *dedicated* path.

3.1.2 Contention resolution

In a switch occasionally contention (i.e., blocking) occurs: cells compete for a single resource (an output port or an internal link). Distinction is made between output blocking and internal blocking. Output blocking occurs if two or more cells are simultaneously arrive at the same switch output port. Internal blocking is that a cell at an input port cannot be transported to the required output port even though it is the only cell destined to that output port.

Contention may be resolved in several ways: by buffering (either at the point of contention or upstream of the point of contention), by loss of cells and by routing the excess of cells along an alternative path. Loss of cells is obviously not the preferred solution. Re-routing is either impossible or possible only on a limited scale, depending on the architecture of the switch fabric.

In a network with dedicated paths between input-output pairs, the optimal contention resolution mechanism is buffering.

3.1.3 Buffering

Distinction is made between internal buffering and external buffering of switch fabrics. We will only describe external buffering, because internally buffered switch fabrics are networks of externally buffered switch elements. (There is one other type of internally buffered switch fabric: the time-division shared memory topology discussed previously.)

Several forms of external buffering should be distinguished, see also [Karoł et al., 1987; Hluchyj et al., 1988]. optimal performance is achieved by output buffering, in which cells are buffered only if they content for the same output port. To realize output buffering without internal blocking a dedicated path should be available for each input-output pair. If less paths are available, this will be at the expense of (some) loss. Output buffers may be shared by all output ports, they may be dedicated to output ports, and they may even be dedicated to input-output pairs. The alternative to output buffering is input buffering, in which a cell is inserted into the switch fabric only if no blocking will occur. As a consequence, in a slot all but one of the cells destined to the same output port are retained in their buffers at the input ports of the switch fabric. In case of FIFO buffering, blocked cells may in their turn block other cells in the buffers that are destined to free output ports (i.e., head of line blocking).

The optimal buffering mechanism is output buffering.

3.1.4 Model of the ideal switch

Combining the optimal solutions in the three dimensions, the ideal switch fabric has a dedicated path from each switch input port to each switch output port, so only output blocking will occur. Further, output blocking is resolved by output buffering. So, in the ideal switch splitting is performed at the input ports, multiplexing is performed at the output ports, and splitters are connected to multiplexers by dedicated paths.

In the switch models that we will use, it is assumed that buffer capacity is not shared between output ports: each output buffer is dedicated to an output port. Sharing of buffer capacity between output ports reduces the probability that a cell is lost due to buffer overflow. On the other hand, sharing makes the switch design and buffer dimensioning more complex. It requires much faster memory and more complicated memory management. Further, sharing can never be complete, because it has to be prevented that an output port is deprived of all its buffer space by other output ports.

By dedicating a buffer to a switch output port, output buffers can be studied independently of each other. The interaction between the traffic streams that pass through the switch fabric is limited to a minimum.

The ideal switch can be modeled by a set of splitters (one at each input port) and a set of multiplexers (one at each output port). The output buffer is a part of the multiplexer. There is a dedicated path from each splitter to each multiplexer. A splitter distributes the traffic stream that arrives at an input port over dedicated paths to the multiplexers at the output ports. The splitters do not cause delay or loss of cells, so they are no subject for performance analysis. A multiplexer merges the traffic streams that it receives from the splitters into a single output stream. The multiplexers determine the performance of the switch. We will examine them in more detail.

3.2 ATM multiplexer

Multiplexing is the process of collecting the cells of different traffic streams and retransmitting them on a link in the network. When the retransmission capacity is temporarily insufficient to accommodate all arriving cells, the excess of cells is buffered. As indicated in the previous section, multiplexing is performed at the output ports of the ideal ATM switch. It is however also used at the edges of the network to concentrate the traffic streams generated by individual users (see Fig. 1.1).



Figure 3.2: Multiplexer

In order to assess performance in ATM networks, we need to model multiplexers. Contention for transmission capacity in a multiplexer causes delay and (occasionally) loss of cells. Fig. 3.2 depicts the queue model of an ATM multiplexer. The service process of the queue models the transmission of cells on the output link of the multiplexer. As all ATM cells have equal length, cell transmission takes equally long for each cell, and the service time is deterministic. In an ATM network, cell transmission is slotted. So, time is divided into slots, and the transmission of a cell can only start at the beginning of a new slot. The duration of a slot equals the time to transmit a single cell.

Cells arrive at the multiplexer via a number of transmission links. The transmission rate on the input links of the multiplexer is assumed to equal the rate on the output link. The slot structures of input and output links are synchronized.

The size of the buffer is finite, i.e., the number of cells that can simultaneously be accommodated in the buffer is finite. In queuing theory parlance, *buffer* sometimes denotes storage space sufficient for only one customer/packet/cell. We have chosen to refer to the entire storage space of a multiplexer as *buffer*. So, a multiplexer has one buffer and this buffer can store cells up to a specific maximum number.

If bursty VC traffic streams are multiplexed, a multiplexer can be operated in two ways. To explain the ways of operation, consider the multiplexing of a number of statistically identical on-off VC traffic streams. The *instantaneous* cell arrival rate at the multiplexer is by definition the cell arrival rate that is determined by the number of VC traffic streams that is in the on-state at a certain point in time. The cell arrival rate on the other hand is determined only by the number of sources that is multiplexed.

Statistical multiplexing

In statistical multiplexing, the instantaneous cell arrival rate at the multiplexer occasionally exceeds the cell transmission rate from the multiplexer. During these periods of overload the multiplexer buffer fills with the excess of cells. It is especially during overload that cells are severely delayed and, eventually, get lost. Hence our special interest in periods of overload in statistical multiplexers. • Deterministic multiplexing

In the complementary case, the number of multiplexed sources is kept so low that the instantaneous cell arrival rate never exceeds the cell transmission rate. In this case, congestion is only due to variation of cell arrival epochs at given cell arrival rate.

Statistical multiplexing allows that transmission bandwidth is used more efficiently. This comes however at the expense of longer cell waiting times and (if the buffer capacity is insufficient) at the expense of lost cells.

Appendix A presents a survey of methods to calculate multiplexer performance, i.e., the probability distribution of cell delay and the probability of cell loss. In the next section, we will examine multiplexer performance itself, especially in case of statistical multiplexing.

3.3 Statistical multiplexer performance

The typical statistical multiplexer studied in the literature multiplexes independent and identical on-off sources. Each of the sources is described by a two state Markov chain (see Ch. 2). The sojourn time in the on-state and the sojourn time in the off-state are geometrically distributed. The cell generation process in the on-state is deterministic. An on-off source is described by three parameters:

- γ , the ratio of the source rate in the on-state and the output rate of the multiplexer,
- ϵ , the fraction of time that a source is in the on-state, and
- T, the mean sojourn time in the on-state.

N denotes the number of sources.

In this section, we examine the influence of buffer size and of source characteristics on cell delay and loss in a multiplexer and on bandwidth efficiency. Bandwidth efficiency is the fraction of slots of the multiplexer output link that is occupied by a cell.

Fig. 3.3 shows a typical example of the relation between cell loss probability and buffer size (see e.g. [Kroener, 1991; Roberts, 1991c; Baiocchi et al., 1991a; Liao et al., 1990; Norros et al., 1991]). A similar curve describes the survivor function of the cell waiting time, i.e., the probability that a cell has to wait longer than an indicated value. The curve can be divided into two parts:

• Cell level congestion:

The upper part describes cell loss due to cell level congestion, i.e. due to simultaneous cell arrivals while the instantaneous cell arrival rate at the multiplexer does not exceed the output rate of the multiplexer. Cell loss due to this kind of congestion can be diminished effectively by increasing the buffer size.

• Burst level congestion:

The lower part describes cell loss due to burst level congestion, i.e. due to an instantaneous cell arrival rate at the multiplexer that exceeds the multiplexer output rate.



Figure 3.3: Typical behavior of the cell loss probability as a function of the buffer size

(Note that by definition burst level congestion occasionally occurs in a *statistical* multiplexer.) Increasing buffer size is only effective against cell loss if the congestion situation is likely to end before the buffer actually starts overflowing. Otherwise, it gives only postponement of buffer overflow.

Depending on the buffer size, either of the two forms of congestion dominates cell loss and cell delay. At low buffer size, cell level congestion dominates; at high buffer size, burst level congestion dominates.

The exact form of the cell loss curve is of course a function of the source characteristics:

- The cell scale congestion part is essentially determined by the traffic load (i.e., $\gamma \cdot \epsilon \cdot N$).
- The cell loss probability due to burst level congestion is easily obtained for the special case of buffer size zero, see e.g. [Baiocchi et al., 1991a]. This cell loss probability (the intersection of the burst scale curve and the vertical axis in Fig. 3.3) provides the starting point for the burst level congestion part of the cell loss curve:

$$\frac{\sum_{i=0}^{N} \binom{N}{i} \epsilon^{i} (1-\epsilon)^{N-i} (i\gamma-1)^{+}}{N \epsilon \gamma},$$
(3.1)

This equation is based on the observations that:

- the instantaneous cell arrival rate at the multiplexer is binomially distributed,
- burst level congestion occurs when at least γ^{-1} sources are simultaneously in the on-state, and

- in case of burst level congestion, the normalized cell loss rate² is $(i\gamma 1)$. (This cell loss rate is an approximation of the actual cell loss rate. It does not take into account cell loss due to variations in the cell arrival process at given instantaneous cell arrival rate.)
- During burst level congestion, the buffer fills to a larger extent if T, the mean sojourn time in the on-state, is larger, see e.g. [Kroener, 1991; Roberts, 1991c]. At this point, the dynamic behavior of the sources is relevant.

As noted in e.g. [Kroener, 1991; Roberts, 1991c], a statistical multiplexer can be operated in either of the two congestion regions. Operation in the cell region is roughly achieved if the buffer size is chosen so low that the probability of burst level congestion is smaller than the cell loss probability. Operation in the cell level congestion region has several advantages relative to operation in the burst level region:

- Cell delay performance is better.
- Buffers are smaller, so that they are less expensive.
- Network design and traffic control are much simpler, because essentially the only relevant traffic characteristic is the mean rate.

The disadvantage of operation in the cell level congestion region is that the bandwidth efficiency is relatively low. Especially if γ is high (e.g., 0.1), the multiplexing gain³ that can be achieved in the cell level region is low. High γ means that a small number of sources can cause overload. The gain can be increased by operating the statistical multiplexer in the burst level congestion region.

If a statistical multiplexer is operated in the burst level congestion region, burst level congestion is explicitly taken into account as part of the normal operation of the multiplexer. A large buffer is installed to account for periods of overload. Dimensioning of the buffer requires detailed knowledge of the dynamic behavior of the sources (mainly the sojourn time in the on-state). This means that the dynamic source behavior should be accounted for in all traffic control rules.

Very long periods of overload cannot effectively be accommodated by a buffer, because the buffer would become unpractically large. Overload periods tend to be difficult to buffer if the source parameters T and γ are large (i.e., if the number of cells generated during the on-period of a traffic source is large on average).

In summary, statistical multiplexing allows that bandwidth efficiency is increased in comparison with deterministic multiplexing. In statistical multiplexing, there are two modes of operation. Operation in the cell level congestion region is only effective if γ

²The normalized cell loss rate is the cell loss rate divided by the multiplexer output link rate.

 $^{^{3}}$ The multiplexing gain is the ratio of the maximum instantaneous cell arrival rate at a multiplexer and the rate of the output transmission link. In a statistical multiplexer the gain exceeds 1 by definition. The gain measures the increase in bandwidth efficiency due to *statistical* multiplexing in comparison with deterministic multiplexing.

is small. Still higher bandwidth efficiency is achieved by operation in the burst level congestion region. Operation in the burst level congestion region is however only effective if both γ and T are small.

3.4 Conclusions

Multiplexing is the basic operation performed in an ATM network. At the edge of the network and in switches, multiplexers are used to increase the efficiency of cell transmission.

An ATM switch or switch element can be modeled as a network of multiplexers. Each of these multiplexers can be studied independently of the other multiplexers in the switch model. In the next chapter, we will model the entire ATM network as a network of multiplexers and splitters. A splitter distributes the output stream of a multiplexer.

If on-off traffic sources are multiplexed, the multiplexer can be operated as a statistical multiplexer: the instantaneous cell arrival rate at the multiplexer occasionally exceeds the multiplexer output rate. (The instantaneous cell arrival rate is the cell arrival rate that is indicated by the states - either on or off - of the sources.) Statistical multiplexing allows to more efficiently use transmission bandwidth. The gain in efficiency comes at the expense of worse performance (namely, longer cell waiting times and higher cell loss probability), more complex traffic control rules, and larger buffers. The gain achieved depends on the characteristics of the traffic sources. If the peak source rate is high and the mean sojourn time in the on-state is long, statistical multiplexing is not feasible.

The VC performance evaluation method for bursty VC traffic presented in chapter 7 extensively uses the characteristics of a statistical multiplexer that are summarized in Fig. 3.3: if a multiplexer is operated in the burst level congestion region, the tail of the cell loss (or waiting time) survivor function is determined by overload, but the probability of overloaded is small.

Appendix A surveys stochastic multiplexer models and their analysis.

Chapter 4

Survey of ATM VC Performance Evaluation Methods

In the previous chapter, we studied performance in individual multiplexers. In this chapter, we classify and assess methods that have been reported in the literature to determine performance on a VC in an ATM network. The relevant ATM VC end-to-end performance metrics are essentially the end-to-end cell loss probability and the end-to-end cell waiting time distribution.^{1 2} We will conclude that existing VC performance evaluation methods do not take into account the so-called *queuing network phenomena*. In the next chapter, we will study the queuing network phenomena in detail.

In order to assess ATM VC performance, a VC has to be described in terms that are amenable to mathematical analysis. So this chapter starts with modeling the ATM queuing network (see 4.1). The ATM queuing network model will be used throughout the rest of the thesis.

General surveys of the analysis of queuing networks can be found in [Sauer et al., 1981; Lavenberg, 1983; Heidelberger et al., 1984; Gelenbe et al., 1987; Kurose et al., 1988; de Souza e Silva et al., 1990; Boxma, 1990]. There is no performance evaluation method that provides a (more or less) closed form solution for the ATM queuing network model. The few queuing networks for which a closed form solution is available (essentially, product form networks) differ from the ATM queuing network model with respect to such essential features as traffic model and service time distribution. Further, the ATM queuing network model is also numerically complex. The state space size of a Markov chain description of the model easily grows out of reach of any computer when the number of queues in

 $^{^{1}}$ Next to the waiting time and loss probability for single cells, one would also like to know the joint distribution of waiting or loss for several consecutive cells. These more advanced performance metrics are, however, hard to obtain and will not be considered further.

²The end-to-end cell loss probability provides a loose upper bound on the end-to-end cell waiting times that need to be taken into consideration. Waiting times that occur with a probability that is, say, ten times lower than the cell loss probability are not important. Loss of cells that wait extremely long only marginally increases the number of lost cells. So, we may consider these cells lost and need not consider their waiting time.

the model exceeds a network of 2 or at the very most 3 queues. So, we have to resort to approximate performance evaluation methods.

In order to assess the approximate performance evaluation methods, we first describe the queuing network phenomena (QNP) in Sect. 4.2. The QNP specify the interaction between queues in a queuing network. They make a queuing network differ from a collection of independent queues and should, in principle, be accounted for by any performance evaluation method. The QNP provide us with a framework to compare VC performance evaluation methods.

Most approximate methods are based on decomposition. In decomposition, the waiting times of a cell in different queues are assumed to be independent (i.e., one of the QNP is neglected). Decomposition in general is discussed in 4.3, and two ways to apply decomposition to the ATM queuing network model in 4.4 and 4.5, respectively.

Some approximate methods try to avoid decomposition. We consider them in 4.6.

4.1 ATM network model

In this section, we model ATM networks. The model will be used in the coming chapters to develop performance evaluation methods for virtual connections in ATM networks. In this chapter, the model is used to discuss performance evaluation methods that have been found in the literature.

The ATM network model is a network of queues, each queue representing a multiplexer (see 4.1.1). The traffic stream on each VC is described by a stochastic process. A VC traffic stream follows a route through the network of queues. We consider two types of traffic: smooth traffic (see 4.1.2) and bursty traffic (see 4.1.3). Between them smooth and bursty traffic cover all traffic types that can be expected in an ATM network.

4.1.1 The queuing network model of an ATM network

We model an ATM network as a network of queues. The queuing network model consists of identical queues, each queue modeling an ATM multiplexer. The queue model of an ATM multiplexer was extensively discussed in App. A. The queue service time is fixed, and the buffer size is finite. In the following paragraphs, the queuing network model will be developed.

The queuing network model concerns the ATM layer (see 1.2.2) of the protocol reference model. Below the ATM layer, the physical layer takes care of the transmission of ATM cells. The physical layer is not reflected in the queuing network model due to assuming ideal cell transmission:

- cells are transmitted in contiguous slots,
- cells are not corrupted or lost during transmission,
- there is neither propagation delay nor transmission delay, and

4.1. ATM NETWORK MODEL

• cell transmission is synchronous: after deletion of propagation delays, cell transmission starts and ends at exactly the same moment on all links of the network.

In the queuing network model, we will almost always assume that ATM cell transmission rates are equal throughout the network. This assumption likely holds for at least parts of the network, and it facilitates model analysis. If the assumption does not hold throughout the network, the network may be split up into separate sub-networks, that are analyzed independently. The analyses of the sub-networks may them be combined to form the analysis of the original network.

The service provided by the ATM layer is the transfer of cells through the network. The model does not account for priorities of cells or adaptive traffic control protocols³ in the ATM layer. The way in which we model the ATM layer is the way it was intended to be at its conception.

Above the ATM layer, the ATM adaptation layer provides end-to-end protocols that enhance the service offered by the ATM layer. The model does not account for the relation between end-to-end protocols in the ATM adaptation layer and the traffic streams in the ATM layer. It is assumed that end-to-end protocols work in a much larger time scale than congestion in an ATM network, so that they do not have influence on congestion in the ATM network. (As a result, end-to-end protocols cannot be used to alleviate congestion in an ATM network.)

Because the model does not account for adaptive traffic control protocols in the ATM layer or end-to-end protocols in the ATM adaptation layer, there is no feedback from downstream queues to upstream queues, and the model is an open queuing network. Fig. 4.1 shows the relationship between the queuing network model of an ATM network and the ATM network model at the level of multiplexers and switches (see Fig. 1.1). The



Figure 4.1: Relation between the ATM network model at the abstraction level of multiplexers and switches and the queuing network model of an ATM network, that consists of queues and splitters

 $^{^{3}}$ An adaptive traffic control protocol influences the traffic stream that a user offers to the network in order to alleviate congestion inside the network, see 1.3.2.

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figure shows the multiplexers and switches that are on the route through the network of a designated VC, the VC under study. Cells flow through the network from left to right. The queuing network model of an ATM network represents a detailed view: the internal structure of the switches is taken into account.

The route of the VC under study through the ATM network is formed by a multiplexer, several switches and a demultiplexer. The multiplexer increases the utilization of the input ports of the first switch. It multiplexes the traffic stream on the VC under study with the traffic streams of other VCs, that originate from the same or from a nearby customer premises. After the multiplexer, the VC passes through several switches. The last element, a demultiplexer, makes possible a high utilization of the output port of the last switch.

The queuing network model takes into account the internal structure of the switches (see Sect. 3.1), multiplexers and demultiplexers. In the switch model (or in the switch element model if applicable), there is a dedicated connection from each switch input port to each switch output port. At each output port, the traffic streams that arrive from the different input ports are multiplexed on a transmission link. A switch in the ATM network model is represented in the queuing network model by the concatenation of a set of parallel splitters and a set of parallel queues. A multiplexer in the ATM network model is represented by a queue in the queuing network model. A demultiplexer in the ATM network model is represented by a splitter in the queuing network model.

The dispersion of traffic in the network will prove important later. It describes that the VC traffic streams on a transmission link tend to choose different routes through the network after that link. To explain and roughly quantify dispersion, assume that all switches in the network are identical, each having n input ports and n output ports. ⁴ Further assume that the traffic load at a switch input port is evenly spread over all output ports. Consider a route through i consecutive switches. The route starts at an input port of the first switch and ends at an output port of the *i*-th switch. The traffic load on this route is a fraction n^{-i} of the total traffic load at the designated input port of the first switch. As this example shows, the number of VCs that follows the same route through a moderate number of consecutive switches is likely very small. This conclusion should be slightly adjusted if we realize that some switch output ports may be chosen more often than others, so that traffic is not evenly spread.

4.1.2 A smooth traffic model

In order to study an ATM network, we need to model the traffic streams on VCs. The traffic models pertain to the traffic offered to the ATM layer by the ATM adaptation layer, see Fig. 1.2. The choice of a traffic model is a delicate choice. A traffic model should capture the traffic characteristics that determine performance in an ATM network, see

⁴The value of n varies according to the switch architecture. Several manufactures currently offer single chip switching elements that can be interconnected in a multi-stage switching network. The measurements of these switching elements vary between 4x4 and 32x32 (Source: Christian Paetz, TU-Chemnitz, Germany).

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Ch. 2. Further, the traffic model determines to a large extent the method to evaluate performance in the queuing network model, see the following chapters.

We will consider two types of traffic streams. In this subsection, we consider a stochastic model for a smooth traffic stream. In the next subsection, we will consider a stochastic model for a bursty traffic stream, namely, an on-off traffic stream.

The most likely type of smooth source traffic has fixed length cell interarrival intervals at the entry into the queuing network. Variable cell interarrival interval length may originate inside the network when the traffic stream is disturbed by queuing. A smooth traffic model by definition allows only small variability of the cell interarrival interval length.

Next to the obvious application (i.e., modeling smooth traffic), the smooth traffic model also has two applications in modeling on-off VC traffic streams:

1. Quasi-stationarity:

By assuming quasi-stationarity (see A.4) a queuing network with on-off VC traffic streams reduces to a set of queuing networks with fixed rate VC traffic streams. The smooth traffic model may represent the fixed rate VC traffic streams.

2. Statistical multiplexing in the cell level congestion region or deterministic multiplexing:

For some types of on-off traffic streams, a multiplexer cannot be operated as a statistical multiplexer in the burst level congestion region (see 3.3). It should instead be operated as a statistical multiplexer in the cell level congestion region or as a deterministic multiplexer.⁵ In both cases, multiplexer performance is determined by cell level congestion at the instantaneous cell arrival rate that is at most allowed.

Pascal distributed cell interarrival intervals

As an example of a smooth VC traffic stream model we consider the following stochastic process: a renewal process with Pascal distributed cell interarrival interval length. This model will be used in the next chapter to study queuing networks. It allows us to change the characteristics of the VC traffic stream model in an efficient way.

The defining characteristic of a renewal process is that the intervals between arrivals (in casu, cell arrivals) are independent and identically distributed. In the case considered here, the distribution of the cell interarrival interval length is the Pascal distribution.

The Pascal distribution is the discrete-time analogy of the continuous-time Erlang distribution. A Pascal-distributed random variable is equal to the sum of n independent and identically distributed random variables, $n \ge 1$. The distribution of these constituent variables is geometric: $\Pr(k) = (1-p)p^{k-1}, k \ge 1, 0 \le p < 1$. A Pascal distribution is completely characterized by its mean $\left(\frac{n}{1-p}\right)$ and variance $\left(\frac{np}{(1-p)^2}\right)$. If p = 0, the Pascal

⁸As noted in 3.3, buffering overload periods is ineffective if the cell generation rate during an on-period or the number of cells generated in an on-period is high. In these cases, a multiplexer should be operated either as a statistical multiplexer in the cell level congestion region or as a deterministic multiplexer. In a statistical multiplexer operated in the cell level congestion region, the probability of overload is so small that overload does not determine performance. In a deterministic multiplexer, overload is not allowed.

distribution becomes a deterministic distribution, and cell interarrival interval lengths are fixed at the value n.

In the Pascal distribution (like in the Erlang distribution), the coefficient of variation $(\sqrt{\frac{p}{n}})^{-6}$ is less than 1, which is a characteristic of smooth traffic.

4.1.3 A bursty traffic model

As concluded in chapter 2, the on-off traffic model forms a generic building block for variable bit rate ATM traffic sources. As far as bursty traffic is concerned, the traffic stream on a VC may be represented by either a single instance of the on-off traffic model or by a superposition of identical and independent instances of the on-off traffic model.

The on-off traffic model that is most often assumed is the interrupted Poisson process (IPP) or a closely resembling stochastic process. We will also use this model in the ensuing chapters. In chapter 2, the IPP model was shown to be accurate and tractable in assessing the performance in a single multiplexer.

In the IPP model, the alternation between on- and off-periods is determined by a twostate Markov chain. In case of a continuous-time model, the sojourn times in the states of this Markov chain are exponentially distributed; in case of a discrete-time model, the sojourn times in the states are geometrically distributed. In case of a continuous-time model, cell generation in the on-state is modeled by a Poisson process of cell arrivals; in case of a discrete-time model, cell generation in the on-state is represented by the arrival of a batch of cells in each slot. The number of cells in a batch is Poisson distributed (so there can be zero cells in a batch).

An IPP is described by three parameters:

- γ , the ratio of the source rate in the on-state and the output rate of the multiplexer,
- ϵ , the fraction of time that a source is in the on-state, and
- T, the mean sojourn time in the on-state expressed in slots (or, equivalently, cell transmission times).

The bursty traffic model is relevant only if the multiplexers in the queuing network are operated as statistical multiplexers in the burst level congestion region. If the multiplexers are operated as statistical multiplexers in the cell level congestion region or as deterministic multiplexers, the smooth traffic model addressed previously is the relevant model. Statistical multiplexing in the burst level congestion region is possible only if the cell generation rate in the on-state (i.e., γ) and the number of cells generated in an on-period (i.e., $T \cdot \gamma$) are both low, see Sect. 3.3.

⁶The coefficient of variation of a random variable X is the ratio of the square root of the variance of X and the mean of X: $c_X = \frac{\sqrt{\operatorname{Var}(X)}}{r_X}$.

4.2 Queuing Network Phenomena

The ATM queuing network model is not just a collection of independent queues. The queues in this model influence each other via the VC traffic streams that pass through them. This influence is described by three effects. We call them Queuing Network Phenomena, QNP⁷.

We introduce the QNP at this point, because they form the framework for the ensuing review of VC performance evaluation methods presented in the literature. In the next chapter, the QNP will be studied in detail; it will be shown that they really occur in the ATM queuing network model.

The QNP are important, because performance on a VC is influenced by them. The performance on a VC is not only determined by each of the queues that the VC passes through, but also by the relation between the queues.

1. QNP waiting time correlation:

When a cell passes through a number of queues in the ATM queuing network model, the waiting times of that cell in these queues are not independent. The waiting times are correlated.

Correlation between the waiting times of a cell is caused by the fact that VC traffic streams pass through several queues in the network. Put in another way, the queues in the network partly process the same traffic streams.

Cell waiting times are mostly positively correlated. (In the next chapter, we will also see an example of negatively correlated cell waiting times.) As a consequence, the probability of long cell waiting times is underestimated if this QNP is neglected. The performance on a VC would be worse than expected.

2. QNP VC traffic characteristics change:

When a VC traffic stream passes through a queue in the ATM queuing network model, the characteristics of this traffic stream change. This change affects queuing behavior in downstream queues.

VC traffic stream most often become smoother due to queuing. (In the next chapter, we will see examples.) As a results, congestion is less severe in any queue this VC subsequently passes through. If a model does not take this QNP into account, the model results will be pessimistic.

3. QNP VC traffic stream correlation:

When several VC traffic streams have been multiplexed on a single transmission link, they have become correlated. The traffic streams have been merged in such a way that they can be transported on a single transmission link.

At a downstream queue (that has the same service time as the queue that merged the traffic streams), these VC traffic streams do not directly interfere with each other.

⁷We will use QNP to denote both Queuing Network Phenomena and Queuing Network Phenomenon.

If they were the only traffic streams at that queue, they would pass without any congestion at all.

Due to this QNP, congestion in the downstream queue decreases.

4.3 Decomposition

The most frequently applied performance evaluation method is decomposition of the queuing network into single, supposedly independent queues. In the ATM queuing network model, decomposition does not provide exact results, and it is applied as an approximation. The approximation is that the QNP correlation between the waiting times of a cell is neglected. In this section, decomposition is outlined. In the next two sections, two specific forms of decomposition are discussed in detail.

Decomposition provides exact results in networks of the product-form type. The ATM queuing network model is not a product-form network. The exact results concern the joint distribution of queue lengths $Pr(X_1, \ldots, X_n)$ and the joint distribution of waiting times $Pr(W_1, \ldots, W_n)$, where X_i and $W_i, i \in \{1, \ldots, n\}$ are respectively the queue length and the cell waiting time in queue *i*. In networks of the product-from type, the queue lengths X_1, \ldots, X_n (all at the same epoch) and, under certain conditions, the waiting times W_1, \ldots, W_n (all for the same cell) are independent. So, the joint distribution of the queue lengths equals the product of the queue length distributions:

$$\Pr(X_1, \dots, X_n) = \Pr(X_1) \cdots \Pr(X_n). \tag{4.1}$$

A similar expression holds for the waiting times.

The product-form property holds only for some very specific types of network, and the ATM queuing network model is not one of them. The best known type of productform network is the network of continuous-time /M/1/FCFS queues, see [Jackson, 1957; Baskett et al., 1975; Kelly, 1979; Walrand, 1988].⁸ If a traffic stream arrives at a queue in this network from the outside, it is a Poisson process of cells. The cells that have received service in a queue are routed either to another queue or leave the network. The destination of such a cell is an independent random variable. The distribution of this variable depends on the queue in which the cell has been served.

This network of -/M/1/FCFS queues has the product-form property: the states of the individual queues are independent. Further, the distribution of the state of a queue can be obtained by considering the queue in isolation. The cell arrival process at the queue is then an independent Poisson process. The rate of this Poisson process equals the cell arrival rate at the queue when it is incorporated in the network. So, the queue length distribution

⁸Some other continuous-time product-form networks are described in [Baskett et al., 1975; Kelly, 1979; Walrand, 1988]; some discrete-time product-form networks are described in [Hsu et al., 1976; Bharath-Kumar, 1980; Pujolle et al., 1992]. In discrete-time queuing networks, the cell interarrival time distribution and the service time distribution are defined in terms of the same basic time unit (slot), and the service processes are slotted and synchronized. In a slotted service process, service of a cell only starts at slot boundaries even if the server is empty.

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can be determined easily. In this network, the product-form property also holds for the cell waiting time distribution if an additional property is fulfilled.⁹ In such a network, the waiting times of a cell in the queues of the network are independent.

Decomposition greatly facilitates analysis of a queuing network: techniques for single queues can be applied to the analysis of a network of queues. If the network under scrutiny is not a product-form network (like in our case), decomposition can be applied as an approximation. As we have seen, the decomposition technique to determine the end-toend queuing network performance requires three subsequent steps:

- 1. Modeling of the traffic streams inside the network.
- 2. Evaluation of performance in the individual queues, using the traffic stream models developed in step 1. Performance evaluation in single queues has been discussed at length in App. A and will not be discussed further here.
- 3. Estimation of the end-to-end performance in the queuing network assuming that the queues are independent. For the end-to-end cell waiting time distribution on a VC, this means convolution of the cell waiting time distributions in the queues that make up the route of the VC.

Decomposition correctly determines performance in each individual queue, if both traffic stream modeling (step 1) and single queue performance evaluation (step 2) are exact. However, only in (overtake-free) product-form networks the assumption in step 3 (namely, that queues are independent) is exact, so that also performance in a network is correctly determined.

Remark that dependence of queues is irrelevant for some performance measures, notably the end-to-end cell loss probability and the mean end-to-end cell waiting time. So also in non-product-form networks these measures are exactly determined by decomposition. The mean end-to-end cell waiting time is obtained as follows:

$$\mathbf{E}(W_1 + \dots + W_n) = \mathbf{E}(W_1) + \dots + \mathbf{E}(W_n).$$

This equation is a property of the 'mean value'-operator. It holds when the random variables W_i are independent and also when they are dependent. The end-to-end cell loss probability can also be considered as a mean value, so that a similar equation holds:

$$\mathbf{E}(\frac{L_{1,t}+\cdots+L_{n,t}}{N_t})=\mathbf{E}(\frac{L_{1,t}}{N_t})+\cdots+\mathbf{E}(\frac{L_{n,t}}{N_t}),$$

where N_t is the number of cells transmitted on the VC under study in an interval of length t, and $L_{i,t}$ is the number of cells in the group of N_t cells that is lost in queue i.

⁹This additional property is that the queues concerned should form an overtake free path, see [Walrand, 1988]. In an overtake-free path, a cell does not influence the waiting time of an other cell ahead of it on the path, neither directly nor indirectly. *Directly* by overtaking the cell ahead. This is for example possible if more than one route exists between two queues on the path under study. *Indirectly* when the influence of a cell overtakes the cell ahead. This influence can be transferred by other cells.

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Step 3 in the decomposition method relates to the QNP waiting time correlation. In an overtake-free product-form network, the waiting times of a cell are independent, and the QNP waiting time correlation does not occur. All performance evaluation methods that apply decomposition as an approximation neglect the QNP waiting time correlation. In the next two sections, we will discuss these decomposition methods.

In step 2, decomposition performance evaluation methods apply a single-queue-performance evaluation method to the successive queues on the route of the VC under study. In order to allow repeated use of the same single-queue-performance evaluation method, the output stream of a queue should be modeled by the same stochastic model as the input stream of a queue. Traffic stream characterization (i.e., step 1) is most often only approximate, so that additional inaccuracy is introduced in the performance evaluation method.

The performance evaluation methods that apply decomposition as an approximation differ with respect to the implementation of step 1. Step 1 relates to the other two QNP, namely QNP VC traffic characteristics change and QNP VC traffic stream correlation. We distinguish between two groups of decomposition methods. The first group focuses on the traffic streams on transmission links; the second focuses on the traffic streams on VCs:

• Modeling traffic streams on links (Sect. 4.4):

Decomposition methods in the first group focus on traffic streams on transmission links in contrast with traffic streams on VCs. They account for the QNP VC traffic stream correlation. They do not however accurately model the traffic streams on individual VCs, let alone the QNP VC traffic characteristics change.

These decomposition methods model the output stream of a queue and the cell routing process. A queue output stream is the flow of cells that leave the queue after service completion. It is the cell stream on a transmission link in the network. The cell routing process describes the distribution of the cells on a transmission link over the queues in the network model and the world outside the network model.

• Modeling traffic streams on VCs (Sect. 4.5):

Decomposition methods in the second group focus on traffic streams on VCs in contrast with traffic streams on transmission links. They account for the QNP VC traffic characteristics change. They do not however account for the QNP VC traffic stream correlation.

The method is to model the output stream from a queue that is due to a single VC. Because in this case the traffic stream on each VC is known, there is no need to model the cell routing process.

4.4 Decomposition: modeling queue output streams

The first approach to modeling traffic streams in the network is to model queue output streams. We outline this approach in three subsections. The first subsection shows how to characterize queue output streams. The second subsection shows how to model splitting of a queue output stream due to routing. The third subsection surveys performance evaluation methods that work according to the approach of this section.

4.4.1 Characterizing the queue output stream

Characterizing a queue output stream means that a description is given for the cell stream that leaves a queue. In general, this description cannot be exact. Apart from some exceptional cases, a stochastic model that exactly describes a queue output stream has a very large state space. Only the output streams of some very special queues are stochastic processes that can be described by a small number of states, see, e.g., [Daley, 1976; Hsu et al., 1976]. ATM multiplexers do not have simple output streams. So, we have to describe a queue output stream by an approximate model that shares some important characteristics with the actual queue output stream.

In Ch. 3, we saw that the predominant way to analyze multiplexers is to model them as Markov chains. The Markov chain multiplexer model can also be used to analyze the queue output traffic stream. Saito [1990] studies the output stream of the continuous-time BMAP/G/1/L queue (see also Sect. A.1) in this way. He pays special attention to the case of a deterministic service process and obtains the transform of the aggregate length of a number of cell interdeparture intervals. Takine et al. [1993] analyze the discretetime BMAP/D/1/L queue in the same way. King [1971] derives an expression for the autocorrelation function of the output process from the continuous-time M/G/1/L queue. Pack [1975] derives for the continuous-time M/D/1 queue the distribution of the aggregate length of a given number of consecutive cell inter-departure intervals.

Based on the Markov chain description, a characterization of the queue output stream can be obtained. The characterization is then used to choose the parameters of the traffic model approximating the output stream. This is part of the performance evaluation method, see 4.4.3.

A very generic, but not very efficient characterization is given by Stavrakakis [1990; 1991b]. He reduces the state space of a Markov chain describing the queue output stream by aggregating the less likely states in this Markov chain. By incorporating sufficiently many states, any desired degree of accuracy can be achieved. To achieve high accuracy, however, a considerable number of states has to be taken into account.

The output stream of an ATM multiplexer alternates between idle periods and periods during which cells depart contiguously. So, the output stream can be modeled by representing the alternation between idle and busy periods of the queue.

This approach is taken by Baiocchi et al. [1992a]. They do not, however, present a stochastic model for the queue output stream. Baiocchi et al. study the N-IDP/D/1 queue by simulation. Each of the N VC cell arrival streams is an interrupted deterministic process (IDP), i.e., a periodic cell arrival stream that is turned on and off according to the state of a two-state Markov chain. Baiocchi et al. conclude from the simulation study that the lengths of busy and idle periods in the output stream are not nearly geometrically distributed, which would facilitate modeling the alternation between idle and busy periods.
¹⁰ The distribution of busy periods is bi-modal, where the mode indicates whether the instantaneous aggregate cell arrival rate at the queue exceeds the cell service rate. Also the distribution of idle periods is bi-modal, where the mode indicates whether the instantaneous cell arrival is zero.

As a final characterization of the output stream, note that the total number of departures from an underloaded, infinite buffer queue in an interval [0, t] will (for large t and relatively to t) be very close to the number of arrivals during the same interval. So if the long term behavior is relevant, the output stream of a queue can be modeled by its input stream, see, e.g., [Daley, 1976; Whitt, 1984].

4.4.2 Modeling cell routing

The route of a cell through the queuing network model is determined by the route of the VC to which the cell belongs. So, the cells that make up a queue output stream are destined to (in general) different queues in the network model. The queue output stream is split up according to the routes of the VCs that are multiplexed in the queue output stream. This subsection describes how to model cell routing.

The problem in modeling cell routing is that multiplexer models do not account for the VCs of the cells waiting in the buffer. Incorporating these VCs in the state description of the queue would tremendously increase the size of the state space, so that analysis of the queue would be impractible if not impossible. As a consequence, cell routing can only be modeled by approximation, as cell routes are directly determined by the VCs. This means that VC traffic streams essentially loose their meaning, if the queuing network model is analyzed by the method present under study (i.e., by decomposition and modeling queue output streams).

Stavrakakis [1991a] proposes to model cell routing by an independent Markov chain. The evolution of the state of this (discrete-time) Markov chain describes the routes that are taken by the consecutive cells in the queue output stream. This approach allows modeling of the bursty character of the cell routing process even if VC identifiers are no longer represented in the traffic model. Stavrakakis observes that a complex cell routing process (modeled by the Markov chain) can in general not be replaced by a simple cell routing process (modeled by Bernoulli cell routing). He does not however indicate how to choose the parameters of the Markov chain, so that an essential part of the routing model is still missing.

The almost universally used routing model is a special case of the Markov chain model. In this routing model, cell routes are independent and identically distributed: a cells takes a certain route with a certain fixed probability independently of the routes of preceding or succeeding cells. In this model, the routing probabilities are chosen in such a way that the mean cell rates on all routes are correct. We previously called this model Bernoulli routing.

¹⁰Bonomi et al. [1992] draw similar conclusions for the N-IBP/D/1/L queue. The N VC cell arrival processes at an N-IBP/D/1/L queue are Bernoulli cell arrival processes that are turned on and off according to the state of a two-state Markov chain.

This model is only an accurate approximation, if the queue output stream multiplexes many thin VC traffic streams.

4.4.3 Decomposition methods

Finally, we review performance evaluation methods that are based on the form of decomposition presently discussed. The methods that we consider can be divided into two groups: methods that model queue output streams by renewal processes and methods that model queue output streams by on-off processes.

Renewal queue output streams

The best known decomposition method is to model the traffic streams in the network by renewal processes that are characterized by the first two moments of the interarrival interval length, see, e.g., the review papers [Kouvelis et al., 1991; Bitran et al., 1992]. The method was initiated by Reiser et al. [1974], it has developed over time, and for general applications it seems to have reached a more or less final state in the form of the Queuing Network Analyzer (QNA) developed by Whitt [1983].

The QNA decomposes a queuing network into GI/GI/1 queues that are approximately analyzed. The output stream from each queue is approximated by a renewal process. This renewal process is specified by only the first two moments of the length of the interval between cells, see [Whitt, 1984]. So the approximation does not account for correlation between intervals or for details of the interval length distribution. Cell routing is modeled by a Bernoulli routing process. The aggregate arrival process at a queue is formed by the superposition of several renewal processes (namely, output processes from other queues after they have been filtered by a routing process - and cell streams that newly arrive at the network). The method again approximates the aggregate arrival process at a queue by a renewal process. This approximation was outlined in Sect. 2.3.3.

Shroff et al. [1991] apply QNA to a VC in an ATM network. They model the ATM network by a network of GI/D/1 queues. End-to-end cell retransmission is applied to cope with cell loss due to buffer overflow and due to transmission errors. In order to estimate the cell loss probability in a queue, Shroff et al. have to assume a distribution for the length of the interval between cells. (Remember that QNA works only with the first two moments.) The cell loss probability that is obtained in this way depends however strongly on the type of distribution that is assumed. So QNA does not allow accurate analysis of the loss probability.

Abo-Taleb et al. [1985] also apply QNA to ATM (or, rather, to a network of queues with equal and deterministic service times), and they also focus on the interval between cells in the queue output stream. They explicitly account for the QNP VC traffic stream correlation. In ATM the minimum distance between two cells in the queue output stream equals the cell transmission time (i.e., the service time of the queue). As a consequence, the output stream of a queue would pass through a subsequent queue without congestion. In QNA however, there is no minimum distance between cells, because QNA has been devised

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for queues with stochastic service times. So in QNA congestion occurs that does not occur in an ATM network. Abo-Taleb et al. [1985] tailor the two-moment method of QNA to ATM by choosing a specific distribution for the interval length between cells in the queue output stream. This distribution does not allow intervals smaller than the service time. The chosen distribution gives exact results for the output stream of an M/D/1 queue and (it is claimed) 'favorable' results for other queues.

Bitran et al. [1988] propose an enhancement of the cell routing model of QNA. In QNA, cell routing is modeled by a Bernoulli process. A Bernoulli process does not allow that the burstiness of the cell routing process is taken into account. Bitran et al. model cell routing by a renewal process, that they characterize by the first two moments. They give simple expressions for the two moments of the queue output stream after application of the renewal cell routing process, so that QNA does not need any further modifications.

On-off queue output streams

Viterbi [1986], Stavrakakis [1991a], Merchant [1991], and Meliksetian et al. [1993] all model queue output streams as discrete-time on-off processes with contiguous cell generation in the on-state. They determine the parameters of the traffic model in three different ways:

- Viterbi only takes the mean cell rate into account. The two parameters that describe the on-off traffic stream are heuristically chosen on the basis of the rate.
- Merchant and Meliksetian et al. characterize traffic streams by their mean cell rate and by the probability that an empty slot is followed by another empty slot.
- Stavrakakis uses the mean rate and the probability of two consecutive occupied slots.

It holds for all methods that their accuracy is not well evaluated.

4.5 Decomposition: modeling VC traffic streams

The second way to model traffic streams inside an ATM queuing network is to focus on the traffic streams on VCs. Characterizing the traffic stream on a VC after queuing avoids the need to separately model cell routing. The main purpose of this approach is to take into account the QNP VC traffic characteristics change. The disadvantage of this approach is that the QNP VC traffic stream correlation is not accounted for.

Consider a queue in the ATM queuing network model. Let $\{A_n\}$, $\{W_n\}$, and $\{D_n\}$ denote respectively the arrival epochs at the queue of cells on the VC under study, the waiting times in the queue of these cells, and the departure epochs of these cells. $d_n = D_{n+1} - D_n$ is the departure process, and $a_n = A_{n+1} - A_n$ is the arrival process. Taking into account the cell service time of 1 slot, the following holds:

$$d_{n} = D_{n+1} - D_{n}$$

= $A_{n+1} + W_{n+1} + 1 - (A_{n} + W_{n} + 1)$
= $a_{n} + W_{n+1} - W_{n}.$ (4.2)

4.6. PARTIAL DECOMPOSITION

For a particular VC, this equation relates the cell departure process (output stream) from a queue to the cell arrival process at that queue.

If the queue is described by a Markov chain, the VC output stream is determined by the Markov chain embedded in this Markov chain at cell arrivals on the VC under study. The embedded Markov chain describes the relation between a_n , W_{n+1} , and W_n in 4.2. In general the embedded Markov chain has a large state space, so that it cannot directly serve as a description of the queue output stream. We consider three cases.

First, it is reasonable to assume that the waiting times of the cells on the VC under study are independent and identically distributed, if the server is lightly loaded and the load on the VC under study is low. See [Roberts, 1991a, Sect. 9.1] and [Diks, 1993]. Assuming that a_n, W_{n+1} , and W_n are independent, the distribution of the d_n is easily obtained on the basis of 4.2 once the distributions of the arrival process (a_n) and the cell waiting time $(W_{n+1} \text{ and } W_n)$ are known. The cell waiting time distribution follows from the embedded Markov chain. These observations are, however, only relevant if the VC traffic stream passes through many queues, so that the changes of traffic characteristics in the queues add up. If in a network of underloaded queues the load of the traffic stream on a single VC is small, the traffic characteristics of this VC will hardly change. As noted in [Whitt, 1988], the waiting times will be small relative to the cell interarrival interval length.

Second, for some specific queues the state space of the embedded Markov chain consists only of the number of cells in the queue and does not need to account for the state of the cell arrival process. An example is the discrete-time $GI+B^X/D/1$ queue. The cell stream on the VC under study is a renewal process, and the cell streams on all other VCs are collectively modeled by a B^X process (i.e., in each slot the number of cell arrivals is independent and identically distributed). Roberts [1992] studies the case in which the renewal process is a periodic process and the number of interfering cell arrivals in a slot is Poisson distributed.

Third, for most queues the state space of the embedded Markov chain consists of the number of cells in the queue and the state of the cell arrival process. Ohba et al. [1991] study the GI+N-IPP+ $B^X/D/1$ queue, where the GI process models the traffic stream on the VC under study, and the N-IPP process¹¹ and the B^X process model the traffic streams on interfering VCs. It is cumbersome to obtain the distribution of d_n by this method.

4.6 Partial decomposition

Performance evaluation methods based on decomposition of a queuing network do not account for the QNP *waiting time correlation*. Some authors have tried to adapt decomposition performance evaluation methods in such a way that correlation between cell waiting times is partly taken into account. We consider, respectively, an approach for on-off VC traffic and an approach for smooth VC traffic.

¹¹The N-IPP is a superposition of N interrupted Poisson processes (IPPs). Each IPP is described by a discrete-time two-state Markov chain. In one state of this Markov chain, no cells are generated; in the other state, a batch of cells is generated in each slot. The number of cells in the batch is Poisson distributed. See also 4.1.3

4.6.1 On-off VC traffic streams

The performance evaluation method proposed by Kroener et al. [1992] partly takes into account the QNP waiting time correlation, but it neglects the QNP VC traffic characteristics change and the QNP VC traffic stream correlation.

The method applies to an ATM queuing network model in which all VC traffic streams are identical on-off traffic streams. The purpose of the method is to assess the end-to-end cell waiting time distribution. An important concept in the method is the instantaneous cell arrival rate at a queue. The instantaneous cell arrival rate is the cell arrival rate that complies with the number of on-off VC traffic streams that is in the on-state.

To understand the method, we have to distinguish between three causes of correlation between the waiting times of a cell in different queues. (We will extensively discuss this subject in the next chapter):

• correlation between the instantaneous cell arrival rates at different queues

The waiting time distribution of a cell in a queue depends on the instantaneous cell arrival rate at that queue. If a cell passes through two queues and the instantaneous cell arrival rates at these queues are (positively) correlated, the waiting times of the cell in these queues are (positively) correlated as well.

• in general increasing cell waiting times in temporarily overloaded queues

If a queue is temporarily overloaded, the number of cells in the buffer increases steadily (until the buffer is full). So if two cells of a single VC pass through two overloaded queues, the first cell likely waits less long in both queues than the second cell. So given that both queues are overloaded, cell waiting times are positively correlated.

• correlation between the cell arrival processes at different queues at given instantaneous cell arrival rates

Consider two queues at given instantaneous cell arrival rates. Suppose that at least one VC passes through both queues. The waiting times of a single cell that passes through both queues are correlated due to variations of the cell process on the VC(s)that pass through both queues.

Kroener et al. only take the first cause of the QNP waiting time correlation into account and neglect the other two causes. They first determine the cell waiting time distribution in each individual queue conditioned on the instantaneous cell arrival rate. Then they calculate the end-to-end cell waiting time distribution essentially by determining the convolution of the cell waiting time distributions of the individual queues. In this last step, they account for the joint probability distribution of the instantaneous cell arrival rates at different queues.

So, the method of Kroener et al. applies to the case of bursty VC traffic. It partly accounts for the QNP cell waiting time correlation, and it does not account for the other two QNP (i.e., VC traffic characteristics change and VC traffic stream correlation).

4.6.2 Smooth VC traffic streams

Kruskal et al. [1988] study the end-to-end cell waiting time distribution in a network of ATM multiplexers for the case of smooth traffic streams. All multiplexers (or, queues) in the network have equal load and the output stream from each queue¹² is distributed among an equal number of other queues.

Kruskal et al. study by simulation the correlation between the waiting times of a cell in two different queues. It drops approximately geometrically as a function of the distance between the queues. (The distance indicates that the queues immediately follow upon each other, or that there is 1 queue in between, or 2 queues, etc..)

This observation allows to approximate the correlation of the waiting times in any pair of queues, once the correlation in one pair of queues is known. In this way, an approximation for the variance of the end-to-end cell waiting time distribution can be obtained (see 5.1.1).

4.7 Conclusions

In this chapter, we have surveyed ATM VC performance evaluation methods. There are no performance evaluation methods that fully account for all queuing network phenomena (QNP) in the ATM queuing network model, so we have to resort to approximate methods.

The predominant approximation is to neglect the QNP *waiting time correlation*. Performance methods based on this approximation are decomposition methods. We have categorized decomposition methods into two groups.

Methods in the first group separately model the queue output traffic stream and the cell routing process. These methods focus on the traffic streams on transmission links. They account for the QNP VC traffic stream correlation, but (in practice) neglect the QNP VC traffic characteristics change. There are several characterizations of the queue output stream. The methods still have problems to represent this stream as a stochastic process. Characterization of the routing process has hardly been considered in the literature, let alone representing it as a stochastic process. Also the relation between the queue output stream and the cell routing process has not been studied. The decomposition methods in this group can in fact only be applied if it is reasonable to assume Bernoulli routing. In case of bursty VC traffic, this assumption does not hold. If Bernoulli cell routing is applied to an on-off traffic stream, the result is a thinned on-off traffic stream. In an ATM network however, either the traffic stream passes completely or it does not pass at all.

Methods in the second group model traffic streams on VCs and the changes in these streams due to queuing. They account for QNP VC traffic characteristics change, but neglect the QNP VC traffic stream correlation. Characterization of the queue output stream due to a single VC is slightly more difficult than characterization of the aggregate queue output stream. These characterizations exist, but there are few examples of methods that represent this stream as a stochastic process.

 $^{^{12}}$ The output streams of some queues leave the network

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Some VC performance evaluation methods to some extent take the QNP *waiting time* correlation into account. They are in fact decomposition methods that do something extra to account for the QNP *waiting time correlation*. We have called them partial decomposition methods.

A partial decomposition method for on-off VC traffic streams neglects all QNP except for the QNP *waiting time correlation* as far as it is due to correlation between the instantaneous cell arrival rates at different queues.

A partial decomposition method for smooth VC traffic streams describes a heuristic way to account for the QNP waiting time correlation. The heuristic is based on simulation results.

The overall conclusion is that existing VC performance evaluation methods neglect one or more of the QNP. Especially the QNP *waiting time correlation* is almost always neglected. In the next chapter, we will show that - depending on the parameter values of the ATM queuing network model - the QNP can have a considerable effect on VC performance. So, there is a need to study the QNP more carefully and to devise performance evaluation methods that account for the QNP.

Chapter 5

Queuing Network Phenomena

The subject of this thesis is ATM Virtual Connection (VC) performance evaluation and especially evaluation of the end-to-end cell waiting time distribution. The previous chapter surveyed existing ATM VC performance evaluation methods. It was observed that these methods almost universally do not account for the interaction between the multiplexers that make up an ATM VC. This interaction is the subject of this chapter.

The interaction between multiplexers is described in terms of the three queuing network phenomena (QNP). The study of queuing network phenomena provides a basis for the ATM VC performance evaluation methods that we will develop in the next two chapters.

In order to study the queuing network phenomena, we use the ATM queuing network model introduced in the previous chapter. In this model all VC traffic streams are of the same type (either smooth or bursty). In almost all our examples the VC traffic streams will be identical.

The queuing network phenomena make performance evaluation in an ATM network difficult. If they would not exist, performance evaluation would come down to repeated performance evaluation of a single multiplexer. Performance evaluation of a single multiplexer is well understood, see chapter 3. On the basis of an understanding of queuing network phenomena, it can be decided whether the influence of a QNP should be accounted for in a performance evaluation method. If it should, the study of the QNP might provide a first indication how to incorporate it. In this chapter, we establish that the queuing network phenomena actually occur and assess their relevance to VC performance.

Performance on a VC is determined by congestion in the multiplexers that make up that VC. More precisely, performance on a VC is determined by congestion in the individual multiplexers and by the relation between the multiplexers. This is where the queuing network phenomena come into play. There are three QNP: waiting time correlation, VC traffic characteristics change, and VC traffic stream correlation. The first queuing network phenomenon directly concerns the relation between multiplexers. The other two phenomena concern the influence that other multiplexers have on congestion in a multiplexer; they indirectly concern the relation between multiplexers.

The QNP waiting time correlation (see 5.1) describes that the waiting times of a single cell in different queues of the ATM queuing network model are dependent. This dependence

makes that the end-to-end cell waiting time distribution cannot be determined on the basis of only the cell waiting time distributions of the individual queues. For an exact result, it is required to take correlation between cell waiting times into account as well.

The QNP VC traffic characteristics change (see 5.2) describes that the characteristics of the traffic stream on a VC change due to queuing. The QNP VC traffic stream correlation (see (5.3) describes that VC traffic streams become correlated if they are multiplexed on a single link.

5.1	QNP waiting time correlation
5.1.1	Smooth VC traffic
5.1.2	Bursty VC traffic
5.2	QNP VC traffic characteristics change
5.2.1	Smooth VC traffic
5.2.2	Bursty VC traffic
5.3	QNP VC traffic stream correlation
5.3.1	Smooth VC traffic
5.3.2	Bursty VC traffic

Table 5.1: Structure of chapter 5

5.1 QNP waiting time correlation

The QNP waiting time correlation describes that the waiting times of a single cell in the queues of the ATM queuing network model are correlated. The QNP is relevant when determining the end-to-end cell waiting time distribution on a VC. We will show that correlation between cell waiting times is most often positive, so that leaving the QNP out of consideration causes underestimation of the probability of long end-to-end cell waiting times. In this section, we will study the QNP in detail. We will show that it occurs, study its causes, and assess its relevance.

The cause of correlation between cell waiting times is dependence of the cell arrival processes at the queues through which the cell under study passes. This dependence is due to both the traffic stream of the VC under study (i.e., the VC to which the cell under study belongs) and traffic streams of interfering VCs, that (partly) follow the same route through the queuing network as the VC under study.

We study the QNP by means of two queues of the ATM queuing network model. The two queues are two consecutive queues on the path of the VC under study through the ATM queuing network model. Fig. 5.1 shows the possible streams of cells through two queues in tandem.

Three different cell streams through the two tandem queues are conceivable:

• stream 1 of cells that pass only through the first or upstream queue,

5.1. QNP WAITING TIME CORRELATION



Figure 5.1: Two tandem queues

- stream 2 of cells that pass only through the second or downstream queue, and
- stream 1-2 of cells that pass through both queues.

The QNP pertains to cells that follow stream 1-2. So, the VC under study is part of stream 1-2. Possibly, also other VCs are part of this stream.

In the remainder of this section, we will separately study the QNP for both the types of VC traffic stream that we previously distinguished: smooth VC traffic streams and bursty VC traffic streams.

5.1.1 QNP waiting time correlation for smooth traffic

In this section, we study the QNP for smooth VC traffic streams. We first describe the cause of the QNP, then present quantitative results, and finally draw conclusions.

The cause of the QNP

For smooth VC traffic, the cause of the QNP waiting time correlation is that the length of the interval between the cells on a VC varies. This cause is described in this section. Variation of the interval length may be a property of the traffic source, but is in addition due to the QNP VC traffic characteristics change (see 5.2).

We describe the cause of the QNP on the basis of Fig. 5.1. Each VC traffic stream is smooth, but the interval length between cells varies. The cell under study belongs to a VC that passes through both queues (i.e., a VC on stream 1-2).

Suppose that (shortly before the cell under study arrives at the first queue) the interval lengths on the VCs of stream 1-2 are so small that the first queue becomes congested and that the cell under study has to wait long. Then, the cell under study is expected to wait longer than average in the second queue as well. This is because the cells on stream 1-2 that caused long waiting times in the first queue also pass through the second queue before the cell under study. A similar mechanism increases the probability of small waiting times in both queues.

The described mechanism indicates that the waiting times in both queues of a cell on stream 1-2 are *positively* correlated: if in the first queue a cell waiting time deviates from its mean value in a given direction (longer or shorter), the cell waiting time in the second queue is more probable than average to deviate from its mean value in that direction as well. This is the QNP. If the QNP is neglected, the probabilities of long and short end-to-end cell waiting times are underestimated.

We should, however, refine the mechanism of the QNP by taking into account that the interval lengths change due to congestion in the first queue. If the first queue is congested, temporarily the cell arrival rate at the queue exceeds the cell departure rate from the queue. So, the cell waiting times tend to increase from one cell on a VC to the next. As a result, the interval length between two cells on a VC is shorter before the queue than after the queue. This effect obviously reduces the positive correlation between cell waiting times described above, because that is based on the interval lengths being more or less the same at both queues. It is difficult to determine in advance net result of both effects.

Results

In this section, we present numerical results on the QNP in the two tandem queues model of Fig. 5.1.

We use the following notation:

- The random variable W_1 denotes the waiting time of the cell under study in the first queue.
- The random variable W_2 denotes the waiting time of the cell under study in the second queue.
- The random variable $W = W_1 + W_2$.
- The random variable \tilde{W} is the approximation of W that is obtained by neglecting the QNP (i.e., by assuming that W_1 and W_2 are independent).
- w_{α} is the end-to-end waiting time that is exceeded with probability α , the α -percentile of W: $\Pr(W \ge w_{\alpha}) = \alpha$.¹

We quantify the QNP by three measures of the correlation between W_1 and W_2 :

- $\operatorname{Cor}(W_1, W_2) = \frac{\operatorname{Cov}(w_1, w_2)}{\sqrt{\operatorname{Var}(w_1)\operatorname{Var}(w_2)}}$, the traditional 'correlation'.
- $\frac{w_{\alpha}-w_{\alpha}}{w_{\alpha}}$, the (relative) error of the waiting time percentile (if the QNP is neglected).
- $\frac{\alpha \Pr(\bar{W} \ge w_{\alpha})}{\alpha}$, the (relative) error of the waiting time survivor probability (if the QNP is neglected)².

The last two measures give more detailed information on the tail distribution of the endto-end waiting time than the first measure. In case of independent waiting times W_1 and W_2 , each of the correlation measures is zero.

¹We have obtained w_{α} by linear interpolation of the function $\log(\Pr(W \ge x)), x \in \{0, 1, ...\}$.

²Remember that by definition $\alpha = \Pr(W \ge w_{\alpha})$

5.1. QNP WAITING TIME CORRELATION

Below we present results on the correlation between W_1 and W_2 . We show respectively the influence on correlation of server load and fan out and the influence on correlation of traffic characteristics. App. B describes the QNP in networks of more than two queues in tandem.

The effects of server load and fan out We first consider the effects of server load³ and fan out⁴ on the QNP.

We consider the model of Fig. 5.1, i.e., two consecutive queues of the ATM queuing network model. The two queues are equal and synchronized. In the numerical examples that follow, the loads of both queues are set to the same value (either 0.5 or 0.9), and the maximum cell waiting time in each queue is 49 slots.

Load	Fan Out	Cor	w_{lpha}	$\frac{w_{\alpha} - \tilde{w}_{\alpha}}{w_{\alpha}}$	$\frac{\alpha - \Pr(\tilde{W} \ge w_{\alpha})}{\alpha}$
			$\alpha = 10^{-3}$	$lpha = 10^{-3}$	$\alpha = 10^{-3}$
0.5	2	$1.19 \cdot 10^{-1}$	7.447	$7.75 \cdot 10^{-2}$	0.49
0.5	4	$6.30 \cdot 10^{-2}$	7.518	$4.46 \cdot 10^{-2}$	0.32
0.9	2	$1.52 \cdot 10^{-1}$	45.47	$7.41 \cdot 10^{-2}$	0.50
0.9	4	$7.58 \cdot 10^{-2}$	45.46	$3.96 \cdot 10^{-2}$	0.30
0.9	8	$3.82 \cdot 10^{-2}$	44.90	$2.03 \cdot 10^{-2}$	0.17
0.9	16	$1.92 \cdot 10^{-2}$	44.52	$1.03 \cdot 10^{-2}$	0.09
0.9	32	$9.64 \cdot 10^{-3}$	44.31	5.19 · 10 ⁻³	0.05
0.9	64	$4.83 \cdot 10^{-3}$	44.20	$2.49 \cdot 10^{-3}$	0.02

Table 5.2: Correlation between W_1 and W_2 as a function of server load and fan out.

Load	Fan Out	Cor	w_{lpha}	$\frac{w_{\alpha} - \tilde{w}_{\alpha}}{w_{\alpha}}$	$\frac{\alpha - \Pr(\tilde{W} \ge w_{\alpha})}{\alpha}$
			$lpha = 10^{-6}$	$lpha = 10^{-6}$	$\alpha = 10^{-6}$
0.5	2	$1.19 \cdot 10^{-1}$	14.05	$1.00 \cdot 10^{-1}$	0.82
0.5	4	$6.30 \cdot 10^{-2}$	13.94	$6.03 \cdot 10^{-2}$	0.63
0.9	2	$1.52 \cdot 10^{-1}$	79.29	$1.02 \cdot 10^{-1}$	0.88
0.9	4	$7.58 \cdot 10^{-2}$	77.76	$5.86 \cdot 10^{-2}$	0.69
0.9	8	$3.82 \cdot 10^{-2}$	76.01	$3.24 \cdot 10^{-2}$	0.47
0.9	. 16	$1.92 \cdot 10^{-2}$	74.91	$1.71 \cdot 10^{-2}$	0.28
0.9	32	$9.64 \cdot 10^{-3}$	74.30	$8.75 \cdot 10^{-3}$	0.15
0.9	64	$4.83 \cdot 10^{-3}$	73.98	$4.46 \cdot 10^{-3}$	0.08

In each slot a batch of cells arrives at the first queue, where the number of cells in the batch is independent and Poisson distributed. Often the batch contains no cells at all. So the aggregate cell stream on stream 1 and stream 1-2 is a discrete-time Poisson process.

³Server load is the fraction of time that the server is occupied, assuming that no cells are lost.

⁴Fan out is the number of streams into which a queue output stream is split, assuming all streams get an equal share.

The aggregate cell arrival stream on stream 2 is also a discrete-time Poisson process. The cell routing process after the first queue is a Bernoulli cell routing process, i.e., cell routes are independent. In the numerical examples, fan out varies between 2 and 64.

The Markov chain description of this model is simple. We have solved it for the joint distribution of W_1 and W_2 .⁵

Tab. 5.2 shows the calculated results for this model at different values for server load and fan out.

For all cases considered, correlation between W_1 and W_2 is positive according to each of the three measures. For the cases considered, server load has little influence on correlation. Fan out, on the contrary, is important: correlation roughly halves when fan out doubles. Correlation is more important at longer end-to-end waiting times (compare the correlation measures at $\alpha = 10^{-3}$ and at $\alpha = 10^{-6}$).

Depending on the parameters of the model, correlation may have a considerable effect on the end-to-end waiting time distribution. For example at fan out = 2 and $\alpha = 10^{-6}$, the effect on the waiting time $(\frac{w\alpha-\bar{w}\alpha}{w_{\alpha}})$ is more than 10 % and the effect on the waiting time probability $(\frac{\alpha-\Pr(\bar{W}\geq w_{\alpha})}{\alpha})$ is more than 80 %. At fan out = 4 and $\alpha = 10^{-6}$, these figures reduce to 6 % and 60 %, respectively.

The results comply with the description of the cause of the QNP given previously. If fan out increases, the load on stream 1-2 decreases relative to the server load. Traffic stream 1-2 is the cause of correlation between cell waiting times. So, it is obvious that correlation (whether positive or negative) should decrease with decreasing importance of this traffic stream.

For Poisson traffic, the QNP has a considerable effect on the end-to-end cell waiting time distribution, especially if fan out is low. Correlation is positive. The effect is larger for higher end-to-end cell waiting times.

The effect of traffic characteristics We next consider the effects of VC traffic characteristics on the QNP.

We again consider the model of Fig. 5.1, i.e., two consecutive queues of the ATM queuing network model. The two queues are equal and synchronized. In the numerical examples that follow, the loads of both queues are set to 0.9.

The traffic stream on each VC is a renewal process with Pascal distributed cell interarrival interval length (see 4.1.2). The Pascal distribution is determined by its mean value and coefficient of variation. In the numerical examples, all VC traffic streams are equal. Each of the streams 1, 2, and 1-2 comprises the same number of VCs (i.e., $N_1 = N_2 = N_{1-2}$), so fan out is 2 and the load on each stream is 0.45. In the examples, N_i varies between 1 and ∞ .

The first free parameter of the numerical examples is $N_1 = N_2 = N_{1-2}$, and the second parameter is the coefficient of variation of the Pascal distribution (c_X) . (The mean value of

⁵First, the steady-state probability distribution of the Markov chain is determined by iteration. Next, the joint distribution of W_1 and W_2 is calculated by considering all possible evolutions of the Markov chain starting from each state in the steady-state distribution.

5.1. QNP WAITING TIME CORRELATION

the Pascal distribution is implicitly determined by the load on the stream and the number of VCs in the stream).

Tab. 5.3 shows the numerical results for this model at different values of $N_1 = N_2 = N_{1-2}$ and c_X . The results have been obtained by simulation. 95 % confidence intervals (c.i.) are shown. The buffer sizes were chosen at 99 cells, and not a single cell was lost during the simulation.

The case $N_1 = N_2 = N_{1-2} = \infty$ is special: at given load on the stream, the contribution of each single VC is negligibly small. For ever increasing $N_1 = N_2 = N_{1-2}$ the aggregate cell stream on each stream is accurately approximated by a Poisson process. The Poisson model was considered in the previous section, so that we have copied the results instead of simulating this special case as well.

Table 5.3: Correlation between W_1 and W_2 as a function of VC traffic characteristics. $N_1 = N_2 = N_{12} = N$

N	c_X	Cor $(\pm 95\% c.i.)$	$w_{10^{-3}}(\pm 95\% c.i.)$	$\frac{w_{10}-3}{w_{10}-3}$	$\frac{10^{-3} - \Pr(\hat{W} \ge w_{10} - 3)}{10^{-3}}$
8	1	0.152	45.47	0.07	0.50
10	0.98	$0.150 (\pm 0.007)$	$42.63 (\pm 1.10)$	0.07	0.50
10°	0.39	$0.081 (\pm 0.003)$	$11.80 (\pm 0.08)$	0.03	0.30
10	0.07	$0.058 (\pm 0.003)$	$10.25 (\pm 0.06)$	0.02	0.23
1	0.74	$0.132 (\pm 0.006)$	$23.67 (\pm 0.55)$	0.07	0.45

For the cases considered, correlation between W_1 and W_2 is positive according to all three measures. The number of VCs on a stream has virtually no influence on correlation (compare the cases $(N_1 = \infty, c_X = 1)$, (10, 0.98), and (1, 0.74)). Remark that it has considerable influence on the waiting time percentile. The coefficient of variation however is important: smaller c_X gives considerably smaller correlation between cell waiting times (compare the cases (10, 0.98), (10, 0.39), and (10, 0.07)).

The results comply with the previously indicated cause of the QNP. The QNP is due to the variability of the cell interarrival interval length on the VCs of stream 1-2. c_X is a measure of this variability. If c_X increases, the variability increases, and the effect of the QNP increases. The influence of the variability of the VC traffic streams is clearly noticeable.

Conclusions

We have shown that for smooth traffic, the QNP *waiting time correlation* is caused by variation of the cell interarrival interval length on VCs. The correlation is predominantly positive, so that neglecting this QNP causes underestimation of long end-to-end cell waiting times.

Depending on parameter values, the effect of the QNP may be considerable, especially on the tail of the end-to-end waiting time distribution. Correlation between the waiting times of a cell increases if fan out decreases or burstiness of the VC traffic streams increases. Server load has little influence on correlation (it has of course considerable influence on the waiting times in the individual queues).

5.1.2 QNP waiting time correlation for bursty traffic

In this subsection, we study the QNP *waiting time correlation* for the case of bursty VC traffic streams. We first describe the cause of the QNP, then present quantitative results, and finally draw conclusions.

The cause of the QNP

For bursty traffic, correlation between the waiting times of a cell is mainly due to the alternation between the on- and off-state of the traffic stream on each VC. In addition, for bursty traffic also the effects observed for smooth traffic occur.

We describe the cause of the QNP on the basis of the two tandem queues network of Fig. 5.1. Unlike previously, each VC traffic stream is now a bursty traffic stream that alternates between an on-state and an off-state. To obtain numerical results, we will later model each VC traffic stream by an IPP, see also 4.1. The cell under study belongs to a VC that passes through both queues (i.e., a VC on stream 1-2).

The *instantaneous cell arrival rate* at a queue is the rate indicated by the states of the VC traffic streams.⁶ If the instantaneous cell arrival rate exceeds the service rate, the queue is said to be overloaded. During overload, the buffer of the queue fills quickly.

The queues in Fig. 5.1 are operated as statistical multiplexers in the burst level congestion region. Operation as a statistical multiplexer implies that the instantaneous cell arrival rate at a queue occasionally exceeds the service rate. Operation in the burst level congestion region implies that the buffer of the queue is designed to accommodate the excess traffic during overload periods. If an overload period persists too long, the buffer will of course in the end overflow anyway.

Next, we will further detail the cause of the QNP. The effects that we observed at smooth traffic of course also occur at bursty traffic. We have however considered them previously and will not consider them anew here. For the multiplexers that we consider (i.e., statistical multiplexers operated in the burst level congestion region), performance is determined by overload periods, so we will concentrate on the QNP during overload periods.

There are two causes for the QNP. The first cause is dependence of the instantaneous cell arrival rates at queues. The instantaneous cell arrival rate at a queue is a stochastic process. This process describes that VC traffic streams turn on and off. The processes describing the instantaneous arrival rates at the two queues in Fig. 5.1 are dependent,

 $^{^{6}}$ In the off-state, a bursty VC traffic stream does not generate cells; in the on-state, it generates cells at a predetermined rate. The instantaneous cell arrival rate due to a single VC equals zero in the off-state, and it equals the cell generation rate in the on-state. The instantaneous cell arrival rate at a queue is the sum of the instantaneous cell arrival rates due to the individual VCs.

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because the VC under study (and possibly also other VCs) passes through both queues. The cell arrival rate largely determines the waiting time distribution of a cell in a queue. So, the waiting times of a cell in the two queues are dependent and positively correlated. An important consequence of this effect is an increase of the probability that both queues in Fig. 5.1 are simultaneously overloaded.

The second cause of the QNP applies only to the case that both queues in Fig. 5.1 are overloaded. One might call this cause 'the order effect'. During overload of a queue, the number of cells in the buffer tends to increase. As a result, waiting times in an overloaded queue tend to increase from one cell on the VC under study to the next cell. When both queues are simultaneously overloaded, this effect occurs in both queues simultaneously. So if a number of consecutive cells passes through the two overloaded queues, the waiting times of a cell in both queues tend to increase from one cell to the next cell. This effect occurs, because the order of cell arrivals is the same at both queues. (Hence the 'ordereffect') Due to this effect, the waiting times of a single cell are positively correlated: when a cell waits (relatively) long in the first queue, it likely waits (relatively) long in the second queue (if both queues are overloaded).

Results

We next present simulation results on the QNP *waiting time correlation* for the case of bursty VC traffic streams and multiplexers operated as statistical multiplexers in the burst level congestion region.

The simulation model is the two tandem queues model of Fig. 5.1. The service intervals of the queues coincide. Cells that arrive at a queue in the same slot are put into the buffer in random order. All VC traffic streams are independent and identical interrupted Poisson processes (IPPs). The number of VC traffic streams on a stream is $N_i, i \in \{1, 2, 1-2\}$. Each IPP is described by the parameters:

- γ : the cell generation rate in the on-state measured in cells per slot,
- T: the mean sojourn time in the on-state measured in slots, and
- ϵ : the fraction of time that the IPP is in the on-state.

The simulation results are presented in the form of the survivor function⁷ of the endto-end cell waiting time. The end-to-end cell waiting time is the sum of the waiting time of a cell in the first queue and the waiting time of the same cell in the second queue.

Next to the actual end-to-end waiting time survivor function, we also show an approximate end-to-end waiting time survivor function. The approximate function is also based on simulation results. It is obtained by assuming that the waiting times of a cell in the two queues are independent. The difference between the actual and the approximate function is entirely due to the QNP *waiting time correlation*.

⁷The survivor function $\Pr(X > x)$ of a random variable X is the complement of the probability distribution: $\Pr(X > x) = 1 - \Pr(X \le x)$.

The simulation results concern three sets of model parameters. We compare a basic system with systems that differ from the basic system with respect to, respectively, server load and fan out. In all sets of parameters, the maximum cell waiting time in each queue separately is 149 cell transmission times. The parameters of the IPP VC traffic stream are equal to values chosen by Kroener et al. [1992] (see chapter 4): $\gamma = 0.1$ cells per slot, T = 500 slots, and $\epsilon = 0.2$.

In the basic system, $N_1 = N_2 = N_{12} = 15$, so that the server loads are 30 VCs and fan out is 2. Fig. 5.2 shows simulation results including 95 % confidence intervals.⁸ The solid lines are the actual functions (i.e., without the assumption that W_1 and W_2 are independent), and the dashed lines are the approximate function (i.e., with the assumption that W_1 and W_2 are independent). The unit of the end-to-end cell waiting time (D) is a slot.



Figure 5.2: Actual (i.e., W_1 and W_2 are not assumed to be independent) (solid) and approximate (i.e., W_1 and W_2 are assumed to be independent) (dashed) end-to-end waiting time survivor functions. 95% confidence intervals are shown. Basic system: $N_1 = N_2 = N_{12} = 15$. Server loads: 0.60

We first discuss the survivor function itself and then consider the QNP. (When considering the survivor function for two queues, it is convenient to keep the survivor function for one queue in mind, see Fig. 3.3.) The waiting time survivor function shows three distinct regions, each characterized by a different slope and corresponding to a different number of overloaded queues.

• For small waiting times (say, up to 10 slots), the situation in which both queues in the model are underloaded determines queuing behavior.

⁸Each time, three curves are shown. The middle curve describes the result that is expected on the basis of the simulation. The actual result lies with probability 0.95 in the area bounded by the two outer curves.

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- For intermediate waiting times (say, between 11 and 149 slots), the situation in which one queue is overloaded and the other queue is underloaded dominates. The probability that one of the queues is overloaded is much smaller than the probability that none of the queues is overloaded. However, if one of the queues is overloaded long waiting times are much more likely than when none of the queues is overloaded. As a result, the state of one overloaded queue dominates the state of no overloaded queue, except at small waiting times. The same behavior can be observed for a single multiplexer (see 3.3).
- For long waiting times (say, longer than 149 slots), the situation in which both queues are overloaded dominates. 149 slots is (approximately) the maximum cell waiting time in one queue. The probability that two queues are overloaded is much smaller than the probability that one queue is overloaded. However, when two queues are overloaded waiting times longer than 149 slots are more likely than when one queue is overloaded. As a result, the state of two overloaded queues dominates the state of one overloaded queue for long waiting times.

The maximum end-to-end waiting time is 298 slots.

In the basic system (Fig. 5.2), the QNP is shown to have a considerable effect for end-to-end waiting times that are determined by overload in both queues (waiting times longer than 149 slots). The effect of the QNP is that long end-to-end cell waiting times are more likely, i.e., the waiting times of a cell in the two queues are positively correlated. The QNP has only a small effect for end-to-end waiting times that are determined by overload of only one queue. If one queue is overloaded, the end-to-end cell waiting time distribution is almost entirely determined by the overloaded queue, so that the effect of correlation between the waiting times of a cell is small.

Like for the case of smooth traffic, the curves in Fig 5.2 can be compared 'vertically' (i.e., comparing probabilities) and 'horizontally' (i.e., comparing waiting times).

Not all cell waiting time probabilities are relevant. Waiting times that occur with a probability that is much smaller than the end-to-end cell loss probability are not relevant. We might as well say that these cells are lost, without really changing the end-to-end VC performance. Cell loss occurs when a cell arrives at a completely filled buffer. This is roughly equally probable as the occurrence of a maximum cell waiting time in the same queue. This means that we are interested in end-to-end cell waiting times that are not much larger than the maximum waiting time in a single queue. See also Sect. 7.1.3.

The two causes of the QNP (see 5.1.2) are clearly visible in Fig. 5.2:

- First, the probability of double overload increases due to the dependence of the instantaneous cell arrival rates at the queues. This can be observed in the figure by comparing the waiting time probabilities at 149 slots. The actual waiting time probability (solid curve) is much higher than the approximate probability (dashed curve).
- Second, during double overload the waiting times of a cell are positively correlated due to three causes simultaneously:

- dependence of the instantaneous cell arrival rates at the queues,
- the order-effect previously described, and
- the kind of correlation we observed for smooth traffic sources.

This can be observed in the figure by comparing the slopes of the survivor functions. For double overload, the actual waiting time survivor function (solid curve) decreases more slowly than the approximate function (dashed curve).

The purpose of the second system is to study the effect of the server load on the QNP. To this end, the number of multiplexed sources is decreased from 30 to 28: $N_1 = N_2 = N_{12} = 14$. Fig. 5.3 shows the results. Due to decreased server loads, the probability



Figure 5.3: Actual (solid) and approximate (dashed) end-to-end waiting time survivor functions. 95% confidence intervals are shown. Second system: $N_1 = N_2 = N_{12} = 14$. Server loads: 0.56

of overload is smaller and overload periods are more easily buffered. Fig. 5.3 shows the same QNP effects as previously described. Careful comparison of the Figs. 5.2 and 5.3 shows even that the effects are relatively larger at lower server loads. So, the QNP is more important at lower server loads. In an ATM network, the server loads will have to be smaller than in the present simulation models in order to ensure sufficiently small cell loss probabilities.

The purpose of the third system is to study the effect of fan out on the QNP. To this end, the VC traffic streams are redistributed between the streams 1, 2, and 1-2: $N_1 = N_2 = 22, N_{12} = 8$. So, fan out is $\frac{N_1+N_{12}}{N_{12}} = 3\frac{3}{4}$, instead of 2 in the basic system. Fig. 5.4 shows the results. For smooth VC traffic, we observed that fan out is an important parameter. This observation is confirmed here for bursty VC traffic as well. Comparison



Figure 5.4: Actual (solid) and approximate (dashed) end-to-end waiting time survivor functions. 95% confidence intervals are shown. Third system: $N_1 = N_2 = 22, N_{12} = 8$. Server loads: 0.60

of the Figs. 5.2 and 5.4 shows that increasing fan out considerably decreases the effect of the QNP on the end-to-end cell waiting time distribution.

Conclusions

For bursty VC traffic, the QNP waiting time correlation is mainly due to

- the alternation between an on- and an off-state of each VC traffic stream and
- (during overload of more than one queue on the route of the VC) the 'order-effect'.

In addition, for bursty VC traffic also the effects observed for smooth VC traffic occur.

Correlation between the waiting times of a single cell in different queues is positive. This means that long end-to-end cell waiting times are more likely due to the QNP. Depending on the parameter values in the model, the effect of the QNP may be considerable for long end-to-end cell waiting times. The effect is relatively more important at lower server loads. It is less important at larger fan out.

This concludes the study of the QNP waiting time correlation. In the next two sections, we will respectively study the QNP VC traffic characteristics change and the QNP VC traffic stream correlation.

5.2 QNP VC traffic characteristics change

In this section, we study the QNP VC traffic characteristics change. It describes the influence of queuing on the characteristics of the traffic stream on a VC. In a queue, the cells of a VC endure stochastic, so in general different waiting times. As a result, traffic characteristics change. A change of VC traffic characteristics influences the behavior of the queues through which the VC subsequently passes. In this way, upstream queues⁹ in the ATM queuing network model influence downstream queues.

In the ATM queuing network model, VC traffic streams in general pass through several queues. A queue may - at least in principle - not be analyzed on the basis of the VC traffic characteristics that apply at the entrance to the network, because these characteristics change when the traffic stream passes through the queues in the network. This is the QNP we are discussing. So, it is important to assess the extent of the QNP in order to determine its relevance in VC performance evaluation.

We again split the study of the QNP into two parts, one for smooth VC traffic and one for bursty VC traffic.

5.2.1 QNP VC traffic characteristics change for smooth traffic

We study the QNP on the basis of the smooth VC traffic model that was introduced in 4.1.2. In this model, the intervals between consecutive cells are independent and Pascal distributed. So, VC traffic forms a renewal process. The Pascal distribution is completely described by the mean value and the variance.

We first describe the cause of the QNP and then present numerical results.

The cause of the QNP

Queuing has the effect

- that the distribution of the interval between consecutive cells on a VC changes and
- that previously independent intervals become correlated.

We describe each effect separately.

Interval distribution The Pascal distribution is described by its first two moments (i.e., mean and variance). The mean does not change due to queuing (except for an occasional cell loss), so we further concentrate on the variance of the interval between consecutive cells.

Consider two consecutive cells of the same VC. If the waiting times of these cells in a queue were independent, the variance of the interval would increase. Independence of cell waiting times is a reasonable assumption, if the rate of the VC is very low. In general

 $^{{}^{9}}$ An upstream queue is a queue from which the traffic stream is coming. A downstream queue is a queue towards which the traffic stream is going.

however, the cell waiting times are positively correlated, i.e., they tend to be either both small or both large. This correlation obviously reduces the increase of the variance due to queuing. Nevertheless, there is a mechanism that *increases* the variance of the interval.

However, the waiting time of the second cell depends to some extent on the length of the interval itself. If the interval is short (so that the second cells arrives at the queue shortly after the first cell), an increase of the waiting time of the second cell due to congestion caused by the first cell is noticeable. So, the short interval tends to become longer. On the other hand, if the interval is long (so that the second cell arrives long after the first cell), a decrease of the waiting time of the second cell due to a relative lack of congestion due to the first cell is noticeable. The long interval tends to become shorter. So, there is a second mechanism that *reduces* the variance of the interval: short intervals become longer, and long intervals become shorter.

It is difficult to determine which mechanism prevails. It is clear that the mean interval length plays an important role: if the mean interval becomes longer, the increasemechanism is becomes important, and the decrease-mechanism becomes less important. It should, however, be noticed that a given change of variance is relatively less important at longer mean interval length.

At given mean of the cell interval, higher variance causes more congestion in a queue.

Correlation between intervals The smooth VC traffic model that we assume is a renewal process. The intervals between cells are independent at the entrance into the network. Due to queuing however, these intervals become correlated: if a cell of the VC under study is relatively much delayed, the interval that is ended by this cell tends to become longer, and the interval that is started by this cell tends to become shorter. The opposite holds if a cell is relatively little delayed. So, consecutive intervals become negatively correlated due to queuing. The effect of queuing on non-consecutive intervals is not clear.

At given mean and variance of the cell interval, a VC with negatively correlated consecutive intervals causes less congestion in a queue.

Results

In this section, we present simulation results on the QNP. The QNP is measured by comparing the cell waiting time distributions in two queues, see Fig. 5.5:

- In the left hand side system, VC traffic streams are directly fed to the queue under study. So the QNP does not occur.
- In the right hand side system, the queue under study is fed by VC traffic streams that were previously multiplexed. (Each VC traffic stream has previously passed through a *different* multiplexer.) So, the QNP influences the cell waiting time distribution in the queue under study.



Figure 5.5: System models to assess the QNP VC traffic characteristics change. Left: no change and no correlation. Right: Change, but no correlation.

The difference between the waiting time distributions of the left system and the right system is a measure of the QNP.

The cell waiting time distributions in the models of Fig. 5.5 are determined by simulation.¹⁰ At the entrance into the network, each VC traffic stream is an independent renewal process with Pascal distributed intervals. The Pascal distribution is characterized by mean m_X and coefficient of variation c_X . In each queue, an equal number of N VCs is multiplexed. Cells that arrive at a queue in the same slot are put into the buffer in random order. The buffer sizes are high enough for cell loss to have no influence on the results.

We study the influence of the traffic parameters m_X and c_X . Each time N is chosen such that the server load is high, namely 0.9. At higher server load, cell waiting times vary more so that the effect of the QNP is larger.

The influence of m_X To study the influence of m_X , we set c_X at a fixed value, namely the highest value possible for a Pascal distribution. The Pascal distribution then becomes the geometric distribution, so that the VC traffic streams are Bernoulli processes. By choosing a high value for c_X , the effect of the QNP is maximized. (See also the study of the influence of c_X that follows next.)

The Figs. 5.6 - 5.8 each compare the waiting time survivor functions for the left system (no influence of the QNP) and the right system (influence of the QNP) in Fig. 5.5. The solid line is the numerical result for the left system, and the dashed lines are the simulation result (including 95 % confidence interval) for the right system. The traffic parameters in the figures are respectively: $m_X = 2.22$, N = 2, $c_X = 0.74$; $m_X = 6.67$, N = 6, $c_X = 0.92$; and $m_X = 11.11$, N = 10, $c_X = 0.95$.

 $^{^{10}}$ An exception forms the left hand side model for the special case of Bernoulli VC traffic streams, which is solved numerically. The Bernoulli VC traffic stream is a special case of the renewal VC traffic stream with Pascal distributed intervals.



Figure 5.6: Waiting time survivor function for previously multiplexed VC traffic streams and 95 % confidence intervals (dashed lines) and waiting time survivor function for previously not multiplexed VC traffic streams (continuous lines). $N = 2, m_X = 2.22, c_X = 0.74$.



Figure 5.7: Waiting time survivor function for previously multiplexed VC traffic streams and 95 % confidence intervals (dashed lines) and waiting time survivor function for previously not multiplexed VC traffic streams (continuous lines). $N = 6, m_X = 6.67, c_X = 0.92$.



Figure 5.8: Waiting time survivor function for previously multiplexed VC traffic streams and 95 % confidence intervals (dashed lines) and waiting time survivor function for previously not multiplexed VC traffic streams (continuous lines). $N = 10, m_X = 11.11, c_X =$ 0.95.

For all sets of parameter values the same effect can be observed, although the extent of the effect is smaller at larger m_X . If VC traffic streams have been multiplexed before, waiting times tend to be shorter. The effect is relatively more important at longer waiting times.

The explanation for this observation is that VC traffic streams are less bursty after multiplexing than before due to the combined effect of the congestion reducing mechanisms described previously: reduced variance of the interval and negative correlation between consecutive intervals. The extent of the effect decreases if m_X increases. Even at $m_X =$ 6.67, the effect is small already.

The influence of c_X The second traffic parameter that we consider is the coefficient of variation c_X of the interval length. We again compare the cell waiting time survivor function for the left system and the right system of Fig. 5.5. The results are obtained by simulation.

Given m_X , c_X is chosen as low as possible for a Pascal distribution. Fig. 5.9 gives results for $m_X = 6.67$, N = 6, and $c_X = 0.13$. Solid lines again represent simulation results for the case of unchanged traffic characteristics; dashed lines again represent simulation results for the case of changed traffic characteristics. 95 % confidence intervals are shown, but hardly discernible.

It can be observed that at these traffic parameter values multiplexing has almost no



Figure 5.9: Waiting time distribution for previously multiplexed VC traffic streams and 95 % confidence intervals (dashed lines) and waiting time distribution for previously not multiplexed VC traffic streams and 95 % confidence intervals (continuous lines). $N = 6, m_X = 6.67, c_X = 0.13$.

effect on traffic characteristics. There is virtually no difference between the two sets of curves. Fig. 5.9 should be compared with Fig. 5.7, where $c_X = 0.92$. In Fig. 5.7, there is a clear effect of multiplexing. The explanation is that cell waiting times vary less if the VC traffic streams are less variable. In the extreme case of an nD/D/1 queue, traffic characteristics do not change at all, because all cells of a given VC wait equally long (see Sect. A.4).

Conclusions

For smooth VC traffic streams, we showed that the QNP VC traffic characteristics change has only a discernible effect if the rate and the burstiness of the VC traffic stream are very high. This QNP is hardly relevant to VC performance.

If these conditions are fulfilled, the VC traffic stream becomes less bursty, so that congestion in downstream queues becomes less severe.

Change of traffic characteristics for smooth traffic is hardly relevant in VC performance analysis. If it is not taken into account, performance estimates will be slightly pessimistic, which is what you would like to have.

5.2.2 QNP VC traffic characteristics change for bursty traffic

In this subsection, we study the QNP VC traffic characteristics change for the case of bursty VC traffic streams. The multiplexers in the ATM network are operated as statistical multiplexers in the burst level congestion region. Each VC traffic stream is modeled by an IPP or by a similar model, and γ is small. (γ is the maximum instantaneous cell generation rate on a VC relative to the cell transmission rate on the links in the network).¹¹

We will not present new simulation results to quantify the QNP, but we will instead refer to a paper from the literature. However, we first describe the cause of the phenomenon.

The cause of the QNP

To describe the QNP, consider the multiplexing of one on-off VC traffic stream and other VC traffic streams. The instantaneous cell arrival rate at the multiplexer is on average higher when the on-off stream is in the on-state than when it is in the off-state. So, the mean cell waiting time at the moment the stream turns off exceeds the mean cell waiting time at the moment the stream turns off an on-period of the on-off stream is stretched: the period during which the cells of an on-period leave the multiplexer is on average longer than the period during which these cells arrived at the multiplexer. The difference between the mean waiting times at the end and at the beginning of the on-period is added to the mean length of the on-period. For the same reason, the mean length of the off-period decreases. The effect is also described in e.g. [Roberts, 1991a, Sect. 9.2].

To describe the effect more precisely, recall that an IPP traffic stream is characterized by the parameters γ , T, and ϵ . The increase of the mean length of an on-period T comes with a reduction of the mean cell rate during an on-period γ and an increase of the fraction of time that the stream is in the on-state ϵ . The mean cell rate of the traffic stream is of course not affected by these changes.

These changes of the VC traffic stream characteristics make it less bursty. Cells are spread more evenly in time, and as a result congestion in downstream queues is less severe.

If T is high or γ is low, the expansion of the on-period is small relative to T. The expansion occurs however anew in each queue, so that in the end it might be considerable.

Next to the change of the mean length of the on-period, the distribution of the length becomes more variable, because the first and the last cells of the on-period endure stochastic and in general different delays.

A further detail is that the lengths of successive on- and off-periods become correlated. The mechanism described indicates that the growth of an on-period is at the expense of the ensuing off-period. So if an on-period grows more than average, the following off-period is likely to shrink more than average. So, successive on- and off-periods become negatively correlated.

⁻¹¹For statistical multiplexing in the burst level congestion region to be effective, it is required that γ is small. See 4.1.3.

Results

Lau et al. [1993] extensively study this QNP by simulation. They consider the multiplexing of independent and identically distributed VC traffic streams. Each VC traffic stream is an interrupted Bernoulli process¹².

Lau et al. study the QNP by analyzing the traffic characteristics of one of the VCs in the output stream of the multiplexer. The traffic characteristics that they consider are the sojourn times in the on- and off-states. More specifically, they study the first two moments of the sojourn times and correlation between sojourn times.

The results of Lau et al. indicate that, if $\gamma < 0.05$, the relative change of the moments is smaller than 2.5 % and that correlation between sojourn times is smaller than 0.025. This holds even if other parameter values are unfavourable, e.g., if the server load is high. The above conclusions are shown to remain valid for non-identical VC traffic streams and for non-geometric sojourn time distributions in the states of the on-off process.

A special case is change of traffic characteristics due to an overloaded multiplexer. In case of overload, on-periods of VC traffic streams are spread considerably. This does not show in the simulation results of Lau et al., because in a well dimensioned multiplexer overload is very rare. If one is especially interested in overload, as we are, it may be wise to take this effect into account.

Conclusions

For bursty VC traffic streams, traffic characteristics change appreciably only if the cell generation rate in the on-state is high. For statistical multiplexing in the burst level congestion region to be effective, this rate is just required to be low. So in VC performance analysis, the QNP VC traffic characteristics change is hardly relevant for bursty traffic.

5.3 QNP VC traffic stream correlation

In this section, we consider the QNP VC traffic stream correlation. In the ATM queuing network model, VC traffic streams are multiplexed. If VC traffic streams have been multiplexed into a single stream, they no longer directly interfere with each other when they pass through another multiplexer. The reason for this effect is that all multiplexers (or, queues) have equal, deterministic servers. So, the VC traffic streams have become correlated.

A somewhat different way to look at this QNP is to consider it as change of the *collective* traffic characteristics of the VC traffic streams that pass through a multiplexer.

The QNP influences congestion in a single queue of the ATM queuing network model and thus also to end-to-end performance on a VC. Its effect is to reduce congestion.

 $^{^{12}}$ An interrupted Bernoulli process (IBP) is a discrete-time on-off process, that is very similar to the interrupted Poisson process. The alternation between on- and off-states is described by a discrete-time two-state Markov chain. In the on-state, cells are generated according to a Bernoulli process: the number of cells generated in a slot (either 0 or 1) is independent and identically distributed.



Figure 5.10: System models to assess the QNP VC traffic stream correlation. Left: Change, but no correlation. Right: Change and correlation.

To analyze the QNP, we again use simulation to compare the waiting time distributions in two queues, see Fig. 5.10. In the left system, the VC traffic streams that arrive at the queue under study (i.e., at the downstream queue) are uncorrelated. Their traffic characteristics have changed due to multiplexing in the upstream queues they have passed through¹³. In the right hand side system of Fig. 5.10, the VC traffic streams that arrive at the queue under study (i.e., at the downstream queue) are correlated due to multiplexing in upstream queues. Fan out f is smaller than N. In addition, the VC traffic characteristics have changed. Comparison of the waiting time distributions in the two systems indicates the effect of the QNP.

The parameters of Fig. 5.10 are fan out f, the number of VCs multiplexed in each queue N, and the characteristics of the VC traffic streams. Fan out f determines the degree of traffic concentration. In the right hand side system, the number of VCs that is multiplexed on each of the links between an upstream queue and the queue under study is $\frac{N}{f}$. For f = N the right system equals the left system. For f = 1, there is no congestion in the queue under study because of the effect of the QNP.

We again split the analysis of the QNP for the two traffic types, smooth and bursty traffic.

 $^{^{13}}$ We neglect this change of characteristics for bursty VC traffic streams. So for bursty traffic the left system reduces to a single queue, the left system in Fig. 5.5.

5.3.1 QNP VC traffic stream correlation for smooth traffic

The cause of the QNP

The cause of the QNP is that in a single slot at most one cell leaves a queue in the ATM queuing network model. So all cells that arrive at a queue in the same slot leave that queue in consecutive slots. In this way, clustering of cells is reduced, so that congestion in downstream queues is less severe.

For the effect to occur, it is required that the queue is congested. So the QNP is more important if the load of the queue is higher or if VC traffic streams are more bursty.

The output stream of a queue is distributed between other queues in the ATM queuing network model. (Of course, cells may also leave the network.) The parameter fan out f determines the degree to which the output stream is thinned. Obviously, thinning of the output stream diminishes the effect of the QNP. If fan out increases, less VC traffic streams pass from the same upstream queue to the queue under study, so that correlation between these VC traffic streams is smaller.

Results

The smooth VC traffic stream model is again a renewal process with Pascal distributed intervals between cells. The Pascal distribution is characterized by mean m_X and coefficient of variation c_X .

As said, we study the QNP on the basis of Fig. 5.10, and the results are obtained by simulation. More in detail, we study the effect on the QNP of fan out f and server load $\frac{N}{m_X}$.

The effect of fan out First, we consider the influence of fan out. Traffic parameters are set at $N = 6, m_X = 6.67$ and $c_X = 0.92$. So, in all queues server load is 0.9. Fig. 5.11 compares the waiting time survivor functions at fan out f = 2 (dashed lines) and at f = N (solid lines). Remember that at f = N VC traffic streams are independent. Fig. 5.12 shows results for f = 3 (dashed lines) and f = N (solid lines). All results were obtained by simulation. Confidence intervals at 95 % are shown.

Fig. 5.11 shows that the effect of the QNP is considerable at low fan out. The effect is as expected: reduced congestion or lower cell waiting times. As expected, increasing fan out from 2 to 3 greatly reduces the effect (compare Figs. 5.11 and 5.12). So, the QNP is relevant only at small fan out.

The effect of server load Second, we consider the effect of server load on correlation between VC traffic streams. Fig. 5.13 shows 4 cell waiting time survivor functions. All were obtained by simulation, and 95 % confidence intervals are shown. Like previously, $m_X = 6.67$ and $c_X = 0.92$. The server load is varied by varying N. The two right hand side curves represent the high load case: N = 6 at fan out 2 (dashed) and fan out N (solid), respectively. The two left hand side curves represent the low load case: N = 4 at fan out 2 (dashed) and fan out N (solid), respectively.



Figure 5.11: Waiting time survivor function for correlated traffic streams (f = 2) and 95 % confidence intervals (dashed lines) and waiting time survivor function for uncorrelated traffic streams and 95 % confidence intervals (continuous lines). $N = 6, m_X = 6.67, c_X = 0.92$.



Figure 5.12: Waiting time survivor function for correlated traffic streams (f = 3) and 95 % confidence intervals (dashed lines) and waiting time survivor function for uncorrelated traffic streams and 95 % confidence intervals (continuous lines). $N = 6, m_X = 6.67, c_X = 0.92$.



Figure 5.13: Waiting time survivor function for correlated traffic processes (f = 2) and 95 % confidence intervals (dashed lines) and waiting time survivor function for uncorrelated traffic processes and 95 % confidence intervals (solid lines). $m_X = 6.67, c_X = 0.92$. Left set of lines: N = 4. Right set of lines: N = 6.

Comparison of the two sets of curves in Fig. 5.13 shows that, as expected, the effect of the QNP on the waiting time percentile is smaller if the server load is lower. (More precisely: the absolute effect is smaller, but the relative effect is more or less unchanged.) If the server load decreases, dispersion of increased activity on a set of VCs is less likely to occur. This is because increased activity is less likely to cause congestion in an upstream queue.

Conclusions

The QNP has been shown to occur for smooth traffic. The effect of the QNP is to reduce congestion in downstream queues. The QNP is relevant only at very low fan out.

5.3.2 QNP VC traffic stream correlation for bursty traffic

In this section, we study the QNP VC traffic stream correlation for on-off VC traffic streams. Each VC traffic stream is modeled as an IPP. The multiplexers in the ATM queuing network model are operated as statistical multiplexers in the burst level congestion region. We concentrate on the effect of the QNP on periods of overload, because overload periods determine performance.

The cause of the QNP

During an overload period of an upstream queue in the ATM queuing network model, the instantaneous cell arrival rate from that queue to a downstream queue is throttled. The excess of cells is first buffered, and later, when the buffer of the upstream queue has become full, it is lost. There are two effects of overload in upstream queues on congestion in the downstream queue:

- If fan out is low, overload of an upstream queue is rather likely to coincide with overload of the downstream queue. If overload periods coincide, the extent of overload in the downstream queue is reduced due to the reduction of the instantaneous cell arrival rate by the upstream queue.
- The second effect concerns the probability that the downstream queue is overloaded. If the upstream queue is overloaded, it reduces the instantaneous cell arrival rate at the downstream queue. The reduction may be large enough to make the difference between overload and underload of the downstream queue. On the other hand, due to buffering in the upstream queue the cell arrival rate remains relatively high for a longer period of time. The first effect decreases the probability of overload of the downstream queue, but the second effect increases the probability of overload of the downstream queue. The net effect on the probability of overload is not clear in advance.

Results

We again study the QNP on the basis of Fig. 5.10. The cell waiting time distribution in the downstream queue of the left hand side system accounts for the case of independent VC traffic streams. The cell waiting time distribution in the downstream queue of the right system accounts for the case of correlated VC traffic streams. The cell waiting time distribution for the left system of Fig. 5.10 has been approximated by a single queue, which was numerically analyzed on the basis of its Markov chain description. The cell waiting time distribution for the right system is obtained by simulation.

All VC traffic streams are stochastically equal IPPs. The server load is equal in all queues. The parameters of the system are fan out f, number of VCs N, and the IPP traffic characteristics (i.e., T, γ , ϵ).

Fig. 5.14 compares the waiting time distributions for independent VC traffic streams (solid line) and correlated VC traffic streams (dashed lines, 95 % confidence intervals shown). The system parameters are: $N = 30, \gamma = 0.1, \epsilon = 0.2, T = 500$, fan out f = 2, and buffer size B = 100.

We observe in Fig. 5.14 that the essential effect of the QNP is to shift the overload part of the cell waiting time survivor function downward. So the probability of overload is reduced, but during overload the behavior is essentially unchanged. The effect is rather small, even at the low fan out value 2.



Figure 5.14: Waiting time survivor functions and 95 % confidence intervals. Solid: no previous multiplexing. Dashed: previous multiplexing at fan out 2. $N = 30, \gamma = 0.1, \epsilon = 0.2, T = 500, B = 100$

The decreased probability of overload is due to the second point described above (i.e., a lower probability of overload due to reduction of the instantaneous cell arrival rate at the downstream queue in case of overload of an upstream queue). The first point described above (i.e., changed overload behavior in the downstream queue due to reduction of the instantaneous cell arrival rate at the downstream queue in case of overload of an upstream queue) is not observed.

Conclusions

The QNP VC traffic stream correlation has been shown to occur for bursty traffic. Its effect is to reduce the probability that a downstream queue is overloaded. The effect is rather small, even at the low fan out value 2.

5.4 Conclusions

In this chapter, the three queuing network phenomena (QNP) have been described and analyzed.

Smooth VC traffic For smooth VC traffic, the QNP *waiting time correlation* is due to variation of the cell interarrival interval length on VCs. Depending on the parameter

values, it has a considerable effect. The effect may cause both underestimation and overestimation of the probability of long end-to-end cell waiting times. Especially at low fan out, the effect is relevant to VC performance.

The QNP VC traffic characteristics change has little effect, unless (at relatively bursty VC traffic streams) the rate of the VC traffic stream is high. The effect of the QNP is to reduce congestion in downstream queues.

The QNP VC traffic stream correlation has little effect, unless fan out is low. It reduces congestion in downstream queues.

Bursty VC traffic For bursty VC traffic, the QNP *waiting time correlation* is mainly due to the burstiness of the VC traffic streams and the 'order-effect'. The QNP causes underestimation of the probability of long end-to-end cell waiting times. Depending on the parameter values, the effect of the QNP is considerable. The effect is relevant to VC performance especially at low fan out.

The QNP VC traffic characteristics change has little effect for the kind of multiplexing that we consider, namely statistical multiplexing in the burst level congestion region. Its effect is to reduce congestion.

The QNP VC traffic stream correlation has little influence. Its effect is to reduce the probability that a downstream queue is overloaded.

In the chapters 6 and 7.1, we will present two new VC performance evaluation methods for smooth and bursty VC traffic respectively. These methods take into account the three queuing network phenomena. In this chapter we have shown that the influence of the queuing network phenomena depends especially on the fan out of the queue output streams. Of the three queuing network phenomena, the QNP waiting time correlation has a rather large and negative effect on VC performance, so that this QNP is especially relevant.

Chapter 6

ATM VC performance evaluation for non-bursty traffic

In this chapter, we present a new ATM virtual connection (VC) performance evaluation method for non-bursty traffic. An example of non-bursty traffic in an ATM network is circuit emulation, in which the cell interarrival interval is fixed. The method evaluates the end-to-end cell waiting time distribution on a VC through an ATM network. The method accounts for all three queuing network phenomena (QNP) and, of course, for congestion. Its accuracy is shown by comparison with simulation results.

The method concerns non-bursty VC traffic streams. (In the next chapter, a method is developed for the case that every VC traffic stream is bursty.) Non-bursty (or, smooth) traffic is characterized by the absence of distinct periods of high and low activity and by small variation of the cell interarrival time¹. In the examples in this chapter, we assume a periodic traffic stream on the VC under study, but the method may also be applied to other types of traffic. The traffic streams on other VCs are described by Poisson processes. In the method, the ATM network is modeled by the ATM queuing network model described previously in 4.1.

The performance evaluation method that we present differs from methods presented in the literature (see Ch. 4) by the incorporation of all three QNP (see Ch. 5). The main motivation for the method is however the incorporation of the QNP waiting time correlation. No performance evaluation method in the literature takes into account correlation between cell waiting times, except a heuristic method based on interpolation between simulation results (see Sect. 4.6). In addition, the methods in the literature either neglect the QNP VC traffic stream correlation (see Sect. 4.5) or assume a very rough cell routing model (see Sect. 4.4). Our method does neither.

The method is based on **conditional** decomposition of the tandem queuing network model of an ATM VC into single queues, where traditional methods assume straightforward decomposition. Conditional decomposition exploits that - by definition - the waiting times of a cell in different queues are conditionally independent, if the condition eliminates the

¹The coefficient of variation is typically much smaller than 1.
dependence between the waiting times. In Sect. 5.1.1, we showed that dependence between waiting times is due to dependence between the cell arrival streams at the corresponding queues. So, the method is based on conditions that describe the cell arrival processes at the queues.

The organization of this chapter is as follows. Sect. 6.1 presents the model of a VC through the ATM queuing network model. The VC model is subsequently analyzed by the conditional decomposition method, first conceptually (6.2) and then in detail (6.3). Numerical results on the accuracy of the conditional decomposition method are shown in Sect. 6.4. The chapter ends with conclusions.

6.1 A tandem queuing network model of the virtual connection under study

In this section, we model a virtual connection (VC) in the ATM queuing network model (see Sect. 4.1.1). The VC model allows to evaluate the end-to-end cell waiting time on the VC under study, without explicitly taking into account all queues in the ATM queuing network model. In the next section, the VC model will be analyzed using the conditional decomposition performance evaluation method.

The VC model represents only the queues of the ATM queuing network model through which the VC under study passes. So, it is a network of queues in tandem, a tandem queuing network. It models in detail the traffic stream on the VC under study, but it models roughly the traffic stream on every other VC. The model assumes that all VC traffic streams are smooth.

6.1.1 The traffic stream on the VC under study

At the entry point into the ATM queuing network, the most realistic stochastic model for smooth VC traffic is a periodic process, in which a new cell is generated after a fixed number of slots. The performance evaluation method that we develop in this chapter is not constrained to VC traffic of this type.

The model of the traffic stream on the VC under study is based on the lengths of cell interarrival intervals. More precisely, it is the *joint distribution of the lengths of a fixed number of consecutive cell interarrival intervals.* We will use the traffic model to describe the traffic stream on the VC under study both at the entrance into the network and inside the network.

This traffic model is very detailed. It can represent any distribution of the cell interarrival interval length. Further, it can account for correlation between the lengths of different cell interarrival intervals, if they are not further apart than the number of intervals in the model. The traffic description is an obvious extension of a description that confines itself to a single interval and assumes that the traffic process is and remains a renewal process.

The main motivation for using this traffic stream model is that it matches well with the condition on the cell waiting time distribution that we will use in the conditional decom-

6.1. VC MODEL

position method, as explained farther on. The traffic stream model captures correlation between interarrival intervals. This is however not the main motivation for this particular choice of model. Correlation between intervals is usually small. It is either a characteristic of the traffic source, or it is due to the QNP VC traffic characteristics change. This QNP was shown in the previous chapter to be of rather little importance.

A disadvantage of the traffic model is its possibly large complexity. In order to reduce the complexity, we truncate the distribution of the interval length and confine the description to a small number of consecutive intervals. As a consequence, correlation between cell interarrival intervals is only taken into account if these intervals are not far apart. This hardly affects the accuracy of the traffic model, because - as explained - this correlation is small. Note however that even when the number of intervals is small, this model is still more accurate than the renewal model.

6.1.2 The traffic streams on other VCs

The VC model represents only the queues of the ATM queuing network model through which the VC under study passes. It is a tandem queuing network. The traffic stream on the VC under study is disturbed by the traffic streams on other VCs. These VCs are *interfering* VCs. In this subsection, we model the traffic streams on interfering VCs at the moment that they enter the tandem network. (The traffic streams on interfering VCs inside the tandem network will be modeled in Sect. 6.3.)

At each queue of the VC model, interfering VCs arrive at the tandem network. The traffic streams of these VCs are collectively modeled by a single stochastic process. Assuming an aggregate traffic stream and neglecting the VC to which a cell belongs reduces the complexity of the performance evaluation method.

The stochastic model for the aggregate traffic stream is a Poisson process.² The Poisson model is not essential to the performance evaluation method. A more complex model is in principle easily incorporated in the method. The reasons for choosing the Poisson model are that it is at least fairly accurate and that it is simple.

Accuracy of the Poisson model

The Poisson model is an accurate approximation for the aggregate traffic stream if first many independent traffic streams each contribute little to the aggregate traffic stream and second a short period of time is considered, see 2.3.3. The Poisson model is, however, a less accurate approximation if a long period of time is considered. It does not account for periodicity of the individual traffic streams. Periodicity of a VC traffic stream shows in the aggregate traffic stream.

 $^{^{2}}$ A Poisson process is a continuous-time stochastic process. The ATM queuing network model is a discrete-time model, in which the unit of time is a cell transmission slot. The Poisson model is incorporated in the ATM queuing network model by registering cell arrivals of the Poisson process during a slot only at the next slot boundary.

In a realistic ATM network model the periodicity of the aggregate traffic streams is reduced by four effects:

- 1. There are likely to be non-periodic VC traffic streams among the streams that contribute to an aggregate stream, e.g. traffic streams of the on-off type.
- 2. VCs cease to exist and new VCs come into existence.
- 3. The periodicity of a periodic VC traffic stream is disturbed due to queuing in the network.
- 4. The periods on individual VCs in general differ widely. The period of an aggregate stream is the least common multiple of the periods of the individual VC traffic streams. So, formally the period of an aggregate stream is large.

Cell routing

After each queue of the VC model, the cells of the VC under study proceed to the next queue in the model. Cells of all other (i.e. interfering) VCs either proceed to the next queue in the VC model or leave the model. If all interfering cells leave after each queue, we speak of crossing interference. If no interfering cell leaves, we speak of joining interference. If only some of the interfering cells leave, we speak of partly joining interference.

In case of partly joining interference, we have to represent the cell routing process, that decides which interfering cells proceed to the next queue and which cells leave the tandem network. In compliance with the Poisson model for interfering traffic, we assume that the routing decision is independent and identically distributed for each interfering cell. The cell leaves with a fixed probability and proceeds with the complementary probability. Joining and crossing interference are two special cases of partly joining interference.

6.1.3 Summary of the tandem queuing network model

In summary, the VC model is a network of queues in tandem. All queues are identical: the service time is 1 slot and the buffer size is finite. The traffic stream on the VC under study is modeled by the joint distribution of a fixed number of cell interarrival intervals. At each queue, newly arriving interfering traffic is modeled by a Poisson process. After each queue, routing of interfering cells is independent and identically distributed for each cell. All queues and sources are synchronized at slot boundaries.

In the next section, we will present conditional decomposition. Conditional decomposition allows accurate evaluation of the end-to-end cell waiting time distribution of cells on the VC under study.

6.2 Conditional decomposition

In this section, we describe a new method to evaluate the end-to-end waiting time of a cell in the VC model. The method is called conditional decomposition (CD). It obtains

6.2. CONDITIONAL DECOMPOSITION

accurate results on the end-to-end cell waiting time distribution and accounts for the QNP waiting time correlation.

The waiting times of a cell in the queues of the VC model are correlated (see Sect. 5.1). This correlation is due to correlation between the cell arrival processes at the queues. Some of the cells that arrive at a queue before the cell under study also passed through upstream queues in the tandem network. So, these cells influence the waiting time of the cell under study in more than one queue. They are the cause of correlation between cell waiting times.

This observation forms the basis of the CD performance evaluation method. CD accounts for correlation between cell waiting times by conditioning the waiting time of the cell under study in a queue on the arrival process at this queue from the upstream queue. After conditioning, the waiting times of a cell are independent.

In the remainder of this section, we will shape these observations. First, we will describe ideal CD. Then, we will describe practical conditional decomposition (PCD), in which the cell waiting time is conditioned on only the most recent part of the cell arrival stream at the corresponding queue. In the next section, we will describe in detail the calculations that are required in PCD.

6.2.1 Ideal conditional decomposition

To describe ideal CD, we first introduce three random variables:

- W_i : the waiting time of the cell under study in queue $i, 1 \le i \le n$, where n is the number of tandem queues. (Propagation time and transmission time are not taken into account, because they are fixed and known in advance.)
- S_i : the sum of the waiting times of the cell under study in the queues 1 to *i* inclusive. $S_i = \sum_{i=1}^{i} W_j$.
- A_i : the realization of the cell arrival process at queue *i* from queue i 1 in all slots up to the arrival of the cell under study at queue *i*. A_i comprises both cells on the VC under study and interfering cells. For the moment A_i covers all cells up to the cell under study and is exact. As such a description is practically impossible, we will have to revise A_i , but this is deferred till later.

The following equation describes conditional decomposition of the distribution of S_i into the distributions of S_{i-1} and W_i (of course, $S_i = S_{i-1} + W_i$). It allows to recursively calculate the distribution of S_i , $1 < i \le n$.³

$$\Pr(S_i = s, A_{i+1} = a_{i+1}) = \sum_{a_i} \sum_{w_i=0}^{s} \Pr(S_{i-1} = s - w_i, W_i = w_i, A_i = a_i, A_{i+1} = a_{i+1})$$

³In this derivation we consecutively applied: the law of total probability (i.e. $\sum_{x} \Pr(X = x) = 1$); Bayes' rule (i.e. $\Pr(X, Y) = \Pr(X)\Pr(Y \mid X)$); and the essential step described in detail in the text.

$$= \sum_{a_i} \sum_{w_i=0}^{s} \Pr(S_{i-1} = s - w_i, A_i = a_i) \Pr(W_i = w_i, A_{i+1} = a_{i+1} \mid S_{i-1} = s - w_i, A_i = a_i)$$

$$= \sum_{a_i} \sum_{w_i=0}^{s} \Pr(S_{i-1} = s - w_i, A_i = a_i) \Pr(W_i = w_i, A_{i+1} = a_{i+1} \mid A_i = a_i)$$
(6.1)

The essential step in this derivation is to realize that $\Pr(W_i, A_{i+1} | S_{i-1}, A_i)$ equals $\Pr(W_i, A_{i+1} | A_i)$. The condition A_i is the complete description of the traffic that arrives at queue i from queue i - 1 up to the arrival of the cell under study. So by definition A_i contains all information that S_{i-1} might provide on W_i and A_{i+1} , and S_{i-1} may be deleted from the condition.

Interfering cell arrivals at queue *i* other then from queue i - 1 are not incorporated in A_i . In the VC model these cell arrivals are modeled by a Poisson process. They are not the cause of correlation. On the contrary, if the traffic load due to these arrivals at queue *i* is large, correlation between the cell waiting times in queue i - 1 and queue *i* is small.

Equation 6.1 provides a recursive procedure to calculate the distribution of S_n , the end-to-end cell waiting time. $\Pr(S_i, A_{i+1})$ is determined on the basis of $\Pr(S_{i-1}, A_i)$ and $\Pr(W_i, A_{i+1} \mid A_i), 2 \leq i \leq n$. The problem in this procedure is the traffic description A_i .

In a practical performance evaluation method, the traffic description A_i cannot comprise all cells preceding the cell under study. Also, it may be too complex to exactly describe the realization of the traffic process in the slots that are comprised by A_i . So, we have to resort to an approximate traffic description. The recursive procedure 6.1 is then no longer exact, and we consider it as an approximation.

The remainder of this section concerns PCD. The main issue is the effect on accuracy of reducing the length of the traffic description A_i . The next section will focus on the traffic description itself.

6.2.2 Practical conditional decomposition

The length of the traffic description A_i has to be rather small for CD to be a viable performance evaluation method. Because the traffic description A_i cannot comprise all cells preceding the cell under study, the CD method has to be reconsidered. The solution is that we describe only the part of the traffic stream from queue i - 1 to queue i that immediately precedes the cell under study. The traffic description of reduced length is indicated by \tilde{A}_i . The far away part of the traffic stream is not incorporated in \tilde{A}_i but is modeled by a stochastic process.

The reduction of A_i does not affect the waiting time distribution in individual queues. Correlation between the waiting times of the cell under study is however approximated. We first consider both effects and then the accuracy of PCD.

Waiting times in individual queues are unaffected

In ideal CD the cell waiting time distribution in queue i is determined by considering all possible cell arrival patterns A_i . The influence of a particular pattern on the cell waiting time distribution is determined by its probability. In PCD the length of the traffic description \tilde{A}_i is much smaller. It is however possible to retain the cell waiting time distribution even in PCD.

This is achieved by considering the state of queue i (at the moment that the traffic description \tilde{A}_i begins) as a random variable, denoted by Q_i . The distribution of Q_i is the steady-state distribution of queue i, given that \tilde{A}_i is the coming cell arrival pattern at queue i from queue i - 1. In this modified procedure, the cell waiting time distribution in queue i is again determined by considering all cell arrival patterns \tilde{A}_i and queue states Q_i . The influence of a combination of \hat{A}_i and Q_i is given by its probability. So if the distribution of Q_i is exactly known, the cell waiting time distribution is also exact.

Correlation between the waiting times of a cell is approximated

As previously remarked, reduction of the length of A_i also affects the CD method as far as correlation between the waiting times of a cell is concerned. This second effect is more important and we will discuss it at length.

The CD method is no longer exact, but is applied as an approximation. As previously indicated, the part of the traffic stream from queue i-1 to queue *i* that is not incorporated in the traffic description \hat{A}_i is accounted for by the random variable Q_i , representing the state of queue *i* at the moment that \tilde{A}_i starts to apply.

Correlation between the waiting times of a cell in different queues is due to dependence between the cell arrival streams at these queues. The waiting time of a cell is, however, only determined by the part of the arrival stream that immediately precedes the cell under study. It suffices that \tilde{A}_i allows to correctly construct the busy period⁴ to which the cell under study belongs. Previous busy periods do not contribute to the waiting time of the cell under study. Mostly a busy period comprises a rather small number of cells. So if the length of \tilde{A}_i is almost always sufficient to reconstruct the busy period of the cell under study, it is acceptable that the CD method does not account for correlation between Q_i and $Q_j, j \neq i, 1 \leq i, j \leq n$.

PCD is a combination of decomposition (to determine the Q_i 's) and considering the whole tandem network (by means of the \tilde{A}_i 's to determine the cell waiting times). Next, we elaborate this idea.

After reduction of the traffic description length, the CD method is applied as an approximation. In (6.1), the conditional probability $\Pr(W_i, \tilde{A}_{i+1} \mid \hat{A}_i)$ no longer exactly equals $\Pr(W_i, \tilde{A}_{i+1} \mid S_{i-1}, \tilde{A}_i)$, but it does so approximately. \tilde{A}_i no longer necessarily incorporates all relevant information on the cell arrival stream at queue *i*. So, S_{i-1} might add new information to the condition and may not be left out just like that.

To examine the accuracy of this approximation and to clarify the PCD procedure, we

⁴A busy period of a queue is a period in time during which the server is occupied without interruption. Immediately before and after a busy period, the server is empty.

take a closer look at the approximation by incorporating Q_i into $\Pr(W_i, \hat{A}_{i+1} \mid S_{i-1}, \hat{A}_i)^5$:

$$\begin{aligned} \Pr(W_{i} = w_{i}, A_{i+1} = \tilde{a}_{i+1} \mid S_{i-1} = s_{i-1}, A_{i} = \tilde{a}_{i}) \\ &= \sum_{q_{i}} \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1}, Q_{i} = q_{i} \mid S_{i-1} = s_{i-1}, \tilde{A}_{i} = \tilde{a}_{i}) \\ &= \sum_{q_{i}} \Pr(Q_{i} = q_{i} \mid S_{i-1} = s_{i-1}, \tilde{A}_{i} = \tilde{a}_{i}) \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1} \mid S_{i-1} = s_{i-1}, \tilde{A}_{i} = \tilde{a}_{i}, Q_{i} = \\ &= \sum_{q_{i}} \Pr(Q_{i} = q_{i} \mid S_{i-1} = s_{i-1}, \tilde{A}_{i} = \tilde{a}_{i}) \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1} \mid \tilde{A}_{i} = \tilde{a}_{i}, Q_{i} = q_{i}) \\ &\approx \sum_{q_{i}} \Pr(Q_{i} = q_{i} \mid \tilde{A}_{i} = \tilde{a}_{i}) \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1} \mid \tilde{A}_{i} = \tilde{a}_{i}, Q_{i} = q_{i}) \\ &= \sum_{q_{i}} \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1}, Q_{i} = q_{i} \mid \tilde{A}_{i} = \tilde{a}_{i}) \\ &= \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1} \mid \tilde{A}_{i} = \tilde{a}_{i}) \end{aligned}$$
(6)

The approximation is that $Pr(Q_i | S_{i-1}, \tilde{A}_i)$ is approximated by $Pr(Q_i | \tilde{A}_i)$. We will in a moment examine its accuracy, but first present the PCD method in its entirety.

The PCD procedure is obtained by substituting (6.2) for $Pr(W_i, \hat{A}_{i+1} | S_{i-1}, \hat{A}_i)$ in the ideal CD procedure:

$$\Pr(S_{i} = s, \tilde{A}_{i+1} = \tilde{a}_{i+1}) \approx \\ \sum_{\tilde{a}_{i}} \sum_{w_{i}=0}^{s} \Pr(S_{i-1} = s - w_{i}, \tilde{A}_{i} = \tilde{a}_{i}) \\ \sum_{q_{i}} \Pr(Q_{i} = q_{i} \mid \tilde{A}_{i} = \tilde{a}_{i}) \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1} \mid \tilde{A}_{i} = \tilde{a}_{i}, Q_{i} = q_{i})$$
(6.3)

Correlation between cell arrival streams is embodied in the conditional probability $Pr(W_i, \tilde{A}_{i+1} | \tilde{A}_i, Q_i)$. The calculation of the probabilities in (6.3) is deferred till the next section.

Accuracy of practical conditional decomposition

The approximation in PCD is that $Pr(Q_i | S_{i-1}, \tilde{A}_i)$ is replaced by $Pr(Q_i | \tilde{A}_i)$. It is more accurate if the cell waiting times (embodied by S_{i-1}) are to a lesser extent determined by the early part of the cell arrival stream (embodied by Q_i). If a queue empties in the interval that the traffic description \tilde{A} applies, Q does not influence the waiting time of the cell under study. When the length of the traffic description \tilde{A} increases, the probability that a queue empties while it applies increases as well, and the accuracy of the approximation increases accordingly.

We obtain numerical results on the accuracy of the approximation by comparing the length of \tilde{A} and the age of the busy period at the moment that the cell under study arrives. If the age of the busy period is smaller than the length of \tilde{A} , the queue becomes empty while \tilde{A} applies.

We simplify the analysis by considering a non-slotted M/D/1 queue. At unchanged server load, busy periods in this queue are longer than in a queue in which the arrival

⁵In this derivation, we applied consecutively: the law of total probability; Bayes' rule; the fact that, conditioned on \tilde{A}_i and Q_i , W_i and \tilde{A}_{i+1} are independent of S_{i-1} ; the approximation; Bayes' rule; and, finally, again the law of total probability.

6.2. CONDITIONAL DECOMPOSITION

#	$\rho = 0.80$	ho=0.95
0	0.80	0.95
5	0.46	0.74
10	0.32	0.63
20	0.18	0.47
30	0.11	0.35
40	0.07	0.26
50	0.05	0.19

Table 6.1: Survivor function of the age of the busy period in an M/D/1 queue at two values for the traffic load, ρ . # denotes the number of service times

process is more smooth than a Poisson process, because a more smooth arrival process decreases congestion. The busy period distribution in an M/D/1 queue provides an upperbound on the busy period distribution in the queues that we consider in this chapter. So, we are considering a worst case situation.

Kleinrock [1975] gives an expression for the distribution of the busy period length in an M/D/1 queue. He also gives the relation between the distribution of the age of the busy period at an arbitrary point in time and the distribution of the busy period length itself. Further, according to the PASTA-property (i.e. Poisson arrivals see time averages) the state of the M/D/1 queue at the moment of cell arrival is in distribution equal to the state at an arbitrary point in time. This allows calculation of the distribution of the age of the busy period at the moment of cell arrival.

Tab. 6.1 shows the survivor function of the age of the busy period (i.e. the probability that the age is longer than indicated). The results in the table show that the traffic description \tilde{A} may be confined to a small number of slots and still exceed the age of the busy period in most cases. For example, the probability that the age of the busy period exceeds 20 slots is 0.18 if the server load is 0.8. So if the traffic description length is 20 slots, correlation (between the waiting time in the queue under study and the waiting times in upstream queues) is correctly modeled in 82 % of all cases. In the other 18 % of the cases, a part of the correlation is captured and PCD is still more accurate than straightforward decomposition. Increasing the server load increases the required length of the traffic description.

In this section, CD has been described and analyzed. Later in this chapter (in Sect. 6.4), we will confirm the accuracy of the PCD method by comparison with simulation results. In the next section, we will first outline the calculations that are required to apply PCD to the VC model (see Sect. 6.1).

6.3 Application of practical conditional decomposition to the VC model

In this section, we will show how to apply the PCD method to the VC model. The description of PCD in the previous section was rather abstract and needs further clarification.

For reference purposes we repeat the PCD method (6.3) at this point:

$$\begin{aligned} &\Pr(S_{i} = s, \tilde{A}_{i+1} = \tilde{a}_{i+1}) \approx \\ &\sum_{\tilde{a}_{i}} \sum_{w_{i}=0}^{s} \Pr(S_{i-1} = s - w_{i}, \tilde{A}_{i} = \tilde{a}_{i}) \\ &\sum_{q_{i}} \Pr(Q_{i} = q_{i} \mid \tilde{A}_{i} = \tilde{a}_{i}) \Pr(W_{i} = w_{i}, \tilde{A}_{i+1} = \tilde{a}_{i+1} \mid \tilde{A}_{i} = \tilde{a}_{i}, Q_{i} = q_{i}) \end{aligned}$$

We consider the cases of crossing interference and partly joining interference. (In case of completely joining interference, the methods for partly joining interference can be applied after the cell routing probability has been chosen appropriately.)

6.3.1 Crossing interference

In case of crossing interference, the traffic stream from a queue in the VC model to the next queue consist only of cells of the VC under study.

In step *i* of the PCD method (6.3) the probability distribution $\Pr(S_i, \tilde{A}_{i+1})$ is determined, $1 \leq i \leq n$. Calculation of $\Pr(S_i, \tilde{A}_{i+1})$ requires that the following distributions are known: $\Pr(S_{i-1}, \tilde{A}_i)$, $\Pr(Q_i \mid \tilde{A}_i)$, and $\Pr(W_i, \tilde{A}_{i+1} \mid Q_i, \tilde{A}_i)$. $\Pr(S_{i-1}, \tilde{A}_i)$ is obtained in step i-1 of the PCD method. The other two distributions have to be calculated.

We first describe a Markov chain model of queue i that is required in the calculation of the two distributions. Then we indicate how the distributions are obtained from the Markov chain.

A Markov chain description of queue i

Queue i in the VC model is described by a Markov chain. The purpose of this Markov chain is twofold. As far as step i of the PCD method is concerned, it allows to calculate the two missing distributions. It should however also allow to characterize the traffic stream from queue i to queue i + 1, so that a similar Markov chain can be drawn up for queue i + 1. We first focus on the second purpose.

The traffic stream from queue i - 1 to queue i of the VC model is modeled by the joint distribution of a fixed number (say m) of cell interarrival interval lengths:

$$\Pr(L_{i,n}, L_{i,n-1}, \dots, L_{i,n-(m-1)}), \tag{6.4}$$

where n counts the cells on the VC under study, m is the number of cell interarrival intervals in the model of the traffic stream on the VC under study, and $L_{i,n}$ is the interarrival interval length on the VC under study at queue i between the cells n and n-1. The QNP VC traffic characteristics change does occur, i.e. queue i changes the characteristics of the traffic stream on the VC under study. The effect is however small and has little impact on the waiting time in a single queue, unless the rate on the VC is high (see Sect. 5.2). We do account for the effect however, because it plays a role in the QNP waiting time correlation.

The appropriate value of m is determined by correlation between cell interarrival intervals in the traffic stream. As previously indicated, the amount of correlation that is caused by queuing in the network is small. So, the number of cell interarrival intervals in the traffic description may be small as well.

The Markov chain describes queue i in the slots that a VC cell arrives from queue i-1. The state of queue i is indicated by the waiting time $W_{i,n}$ of the arriving VC cell (in this case, cell n). The waiting time completely describes the state of the queue, because we assume that the VC cell is put into the buffer of the queue before any interfering cells that arrive in the same slot.

The discrete-time Markov chain describing queue *i* is:

$$\{(W_{i,n}, W_{i,n-1}, \dots, W_{i,n-(m-1)}, L_{i,n}, L_{i,n-1}, \dots, L_{i,n-(m-2)}), n \in \{1, 2, \dots\}\},$$
(6.5)

where n counts the cells on the VC under study, m is the number of cell interarrival intervals in the model of the traffic stream on the VC under study, $W_{i,n}$ is the waiting time of cell n in queue i, $L_{i,n}$ is the interarrival interval length on the VC under study at queue i between the cells n and n-1. Note that the number of cell interarrival intervals in the Markov chain is m-1 instead of m.

This Markov chain describes a possibly very large state space, which might be an impediment for the practical application of the method. The size of the state space is essentially determined by m. In the examples that follow, we will set m to 2, and we will show that this provides an accurate traffic description.

The transition probabilities in the Markov chain are easily obtained:

$$L_{i,(n+1)-\{m-2\}} = L_{i,n-(m-3)}$$

:
 $L_{i,(n+1)-1} = L_{i,n}$

The distribution of $L_{i,(n+1)}$ conditioned on $(L_{i,(n+1)-1},\ldots,L_{i,(n+1)-(m-2)})$ follows from the description (6.4) of the VC traffic stream from queue i - 1 to i.

$$W_{i,(n+1)-(m-1)} = W_{i,n-(m-2)}$$

:
 $W_{i,(n+1)-1} = W_{i,n}$

The distribution of $W_{i,(n+1)}$ is obtained by analyzing the evolution of queue *i* between the arrivals of VC cell *n* and cell n + 1. $W_{i,n}$ is the waiting time of cell *n*. In the slot that cell *n* arrives, also interfering cells arrive at queue *i*. In the following $L_{i,(n+1)} - 1$ slots only

interfering cells arrive. $W_{i,(n+1)}$ is the waiting time of cell n + 1 that arrives at queue $i L_{i,(n+1)}$ slots after cell n.

The fully specified Markov chain (6.5) can be solved for its steady-state distribution.

The first application of the Markov chain is to establish the traffic description of the VC traffic stream from queue i to queue i + 1. The traffic description can be derived from the random variable:

 $(W_{i,n}, W_{i,n-1}, \ldots, W_{i,n-(m-1)}, W_{i,n-m}, L_{i,n}, L_{i,n-1}, \ldots, L_{i,n-(m-2)}, L_{i,n-(m-1)})$

This random variable is the state of the Markov chain extended with 1 cell interarrival interval. Its distribution is obtained by considering the Markov chain at two consecutive epochs. The relation between the random variable and the random variable describing the traffic stream on the VC under study is:

$$(L_{i+1,n},\ldots,L_{i+1,n-(m-1)}) = (L_{i,n} - W_{i,n-1} + W_{i,n},\ldots,L_{i,n-(m-1)} - W_{i,n-m} + W_{i,n-(m-1)})$$

We next indicate how the two probability distributions required by PCD can be derived from the Markov chain (6.5).

Calculation of $Pr(Q_i \mid \tilde{A}_i)$

The traffic description \hat{A}_i describes the traffic stream from queue i - 1 to queue i during the interval that immediately precedes the cell under study. The traffic description is again a fixed number k of cell interarrival interval lengths:

$$\Pr(\tilde{A}_i) = \\ lim_{n \to \infty} \Pr(L_{i,n}, \dots, L_{i,n-(k-1)})$$

The appropriate number k is determined by considerations on the QNP waiting time correlation. If k is larger, the QNP is taken into account more accurately. Normally, we would choose $k \ge m$. The number of slots that the traffic description \tilde{A}_i comprises varies.

 Q_i is the waiting time in queue *i* of the first cell in A_i : $Q_i = W_{i,n-k}$.

$$\Pr(Q_i \mid \tilde{A}_i) = \lim_{n \to \infty} \Pr(W_{i,n-k} \mid L_{i,n}, L_{i,n-1}, \dots, L_{i,n-(k-1)})$$

 $\Pr(Q_i \mid \tilde{A}_i)$ is derived from the steady-state distribution of:

$$(W_{i,n}, W_{i,n-1}, \dots, W_{i,n-k}, L_{i,n}, L_{i,n-1}, \dots, L_{i,n-(k-1)}),$$
 (6.6)

which is obtained by considering m - k + 1 consecutive epochs of the Markov chain (6.5).

Calculation of $Pr(W_i, \tilde{A}_{i+1} | Q_i, \tilde{A}_i)$

$$\Pr(W_i, \tilde{A}_{i+1} \mid Q_i, \tilde{A}_i) = \lim_{n \to \infty} \Pr(W_{i,n}, L_{i+1,n}, \dots, L_{i+1,n-(k-1)} \mid W_{i,n-k}, L_{i,n}, \dots, L_{i,n-(k-1)})$$

The probability distribution follows again by considering m - k + 1 consecutive epochs in the Markov chain (6.5).

6.3.2 Partly joining interference

In case of partly joining interference, the traffic stream from queue i to queue i + 1 in the VC model consists of cells of the VC under study and interfering cells.

To apply the PCD method to the VC model at partly joining interference, we have to modify the description of the traffic stream between queues. To this end, we extend the model for the traffic stream on the VC under study to include a description of interfering cells.

We first describe the model and its accuracy. Then we present the Markov chain model of a queue in the VC model at partly joining interference. The Markov chain forms the basis for the calculations in the PCD method. Finally, a simplified model for the internal traffic stream is given.

Internal traffic stream model

The ideas that have formed the internal traffic stream model are first that it extends the model of the traffic stream on the VC under study and second that it is as simple as possible and yet captures the QNP *waiting time correlation*.

We modeled the traffic stream on the VC under study by the joint distribution of a number of cell interarrival interval lengths. We add interfering traffic to this model by indicating the rate of interfering cells during each cell interarrival interval on the VC under study. So, the internal traffic stream is described by the joint distribution of a number of paired random variables. Each pair of random variables describes respectively the length of a cell interarrival interval on the VC under study and the rate of interfering cells during that interval. It is essential that the rate of interfering cells is allowed to change after each cell on the VC under study. This change captures the QNP *waiting time correlation*.

By accounting only for the rate of interfering cells, we neglect any details and confine ourselves to a minimum traffic description. We do not indicate the cell positions in the interval or even the exact number of cells. The internal traffic stream model assumes that interfering cells form a Bernoulli process⁶. The rate of the Bernoulli process is the rate of interfering cells indicated by the traffic description. So, it is in general different in each cell interarrival interval on the VC under study.

⁶In the present context, a stochastic process is a Bernoulli process if the probability that a cell appears in a slot is independent and identically distributed. Either one or no cell appears. The probability that a cell appears equals the rate of the process.

The internal traffic stream model approximately accounts for the three QNP. The model accounts for the QNP waiting time correlation (due to interfering cells) and the QNP (interfering) traffic characteristics change by changing the rate of interfering cells after each cell on the VC under study. The model incorporates the QNP VC traffic stream correlation by representing the VCs that form the internal traffic stream in a single model. The model allows at most one cell in each slot and simultaneously changes the traffic characteristics of all VCs.

Accuracy of the internal traffic stream model

The output stream of a queue is an on-off process: during a busy period of the server cells are transmitted in contiguous slots, during an idle period no cells are transmitted. All cells of the VC under study in the output stream are routed to the next queue in the VC model. Interfering cells are routed to the next queue at random. The essential point concerning the accuracy of the internal traffic model is that only the rate of interfering cells is indicated. The number of interfering cells and the slots in which they occur are not indicated.

In the next section, we will show that the internal traffic model is sufficiently accurate for the PCD method. With respect to the PCD method, it is important that the QNP *waiting time correlation* is accurately captured. Here, we argue that the internal traffic model allows accurate estimation of the cell waiting time in a single queue of the VC model. We show that the traffic model is exact at three extremes, so that it is reasonable to assume that the model is accurate near those extremes.

Heavy and light traffic The internal traffic model is exact at two extremes: heavy and light interfering traffic. In heavy traffic, no idle periods occur in the queue output stream, so the traffic model is exact. In the light traffic limit, never two cells simultaneously arrive at the queue. So, the queue output stream is the superposition of the input streams (namely, the internal traffic stream and newly arriving interfering cells). In the light traffic limit, the Poisson process model for interfering cells equals a Bernoulli process. So, the internal traffic model is again exact.

Few interfering cells in the internal traffic stream The probability that an interfering cell in the queue output stream proceeds to the next queue in the VC model tends to be rather small. (A part of the internal traffic stream has to be attributed to the VC under study.) If the probability approaches 0, the internal traffic model is exact. The numerical results that follow show that the internal traffic model is accurate at small routing probability.

For the sake of the argument, we consider an extreme case of the VC model at partly joining interference: 2 queues and the rate on the VC under study is 0. The internal traffic stream model is a Bernoulli process, without rate changes. Obviously, the accuracy of the internal traffic model is better if the rate on the VC under study is high and the rate of interfering cells changes.



Figure 6.1: Comparison between the waiting time in the second queue of two tandem queues (solid) and in an B+M/D/1 queue (dashed). Server loads are 0.5. Lower pair of lines: fan out is 2 and load Bernoulli process is $\frac{0.5}{2}$, respectively; upper pair of lines: fan out is 4 and load Bernoulli process is $\frac{0.5}{2}$, respectively.

The complexity of this extreme case is small. So small that the Markov chain describing it can be solved numerically (see also Ch. 4). In this way, we determine the cell waiting time distribution in the second queue for cells that pass through both queues.

The internal traffic model for the traffic stream from the first to the second queue is a Bernoulli process. So, the approximation of the second queue is an B+M/D/1 queue, where the Poisson process (M) represents newly arriving interfering cells. The Markov chain describing the B+M/D/1 queue is also easily solved numerically. In this way, we approximate the cell waiting time distribution in the second queue for cells that pass through both queues.

Comparison of the two cell waiting time distributions gives an indication of the accuracy of the internal traffic model. In the examples that follow, the 2 tandem queues are equally highly loaded. Cells that simultaneously arrive at a buffer are put into the buffer in random order. Buffer sizes are finite: 50 cells.

Figs. 6.1 and 6.2 show waiting time survivor functions for server loads 0.5 and 0.9, respectively. In each figure, two different values for the fan out from the first to the second queue are considered. Comparison of the solid (exact) and dashed (internal traffic model) curves shows that the internal traffic model provides a lower bound on the actual cell waiting time distribution. The explanation is that the output stream from the first queue in the two tandem queues is more bursty than a Bernoulli process. The figures show that the internal traffic model becomes more accurate if fan out increases. (Remark that if



Figure 6.2: Comparison between the waiting time in the second queue of two tandem queues (solid) and in an B+M/D/1 queue (dashed). Server loads are 0.9. Lower pair of lines: fan out is 2 and load Bernoulli process is $\frac{0.9}{2}$, respectively; upper pair of lines: fan out is 4 and load Bernoulli process is $\frac{0.9}{4}$, respectively.

fan out increases both distributions approach the waiting time distribution in an M/D/1 queue.)

A Markov chain description of queue i

In case of partly joining interference the calculations in the PCD method closely resemble the calculations in case of crossing interference. The main adjustment to be made is an extension of the Markov chain describing a queue in the VC model.

The traffic stream from queue i - 1 to queue i of the VC model is modeled by the joint distribution of m cell interarrival interval lengths and m rates of interfering cells:

$$\Pr(L_{i,n},\ldots,L_{i,n-(m-1)},R_{i,n},\ldots,R_{i,n-(m-1)}),$$

where $R_{i,n}$ is the rate of interfering cells in the internal traffic stream from queue i - 1 to queue i in the interval between the VC cells n - 1 and n.

The extended Markov chain describing queue i is:

 $\{(W_{i,n}, W_{i,n-1}, \dots, W_{i,n-(m-1)}, \\ I_{i,n}, I_{i,n-1}, \dots, I_{i,n-(m-2)}, \\ L_{i,n}, L_{i,n-1}, \dots, L_{i,n-(m-2)}, \\ R_{i,n}, R_{i,n-1}, \dots, R_{i,n-(m-2)}, n \in \{1, 2, \dots\}\},$

where $I_{i,n}$ is the number of arrivals of interfering cells at queue *i* in the interval between the arrivals of VC cells n - 1 and n. $I_{i,n}$ includes cells that arrive from queue i - 1 and newly arriving interfering cells. It is required to determine $R_{(i+1),n}$.

The transition probabilities in this Markov chain are again easily obtained. The distribution of $(L_{i,(n+1)}, R_{i,(n+1)})$ follows from the description of the internal traffic stream. The distribution of $(W_{i,(n+1)}, I_{i,(n+1)})$ follows from the evolution of queue *i* between the arrivals of the cells *n* and *n* + 1 on the VC under study.

The model of the internal traffic stream from queue i to queue i + 1 is obtained by considering that:

$$L_{i+1,n} = L_{i,n} - W_{i,n-1} + W_{i,n}$$

$$R_{i+1,n} = \frac{1}{f} \cdot \frac{I_{i,n}}{L_{i+1,n} - 1},$$

where f is the fan out from queue i to queue i + 1. $\frac{1}{f}$ accounts for cell routing.

A simplified model of the internal traffic stream

The internal traffic model developed in this section is still too complex in case of partly joining interference. The model is the joint distribution of a fixed number of VC cell interarrival interval lengths and the rates of interfering cells during these intervals. This traffic description easily becomes too complex for numerical evaluation of (6.3), regarding both storage space and calculation time in a computer. In order to circumvent this problem, we introduce a rougher and less costly traffic model. In the next section, we will show that it is accurate.

The idea behind the simplified traffic model is that it is not required to indicate the exact value of the rate of interfering traffic during a VC cell interarrival interval, but that – to capture correlation between cell waiting times – it suffices to indicate essentially the direction into which the rate deviates from its mean value. Moreover, as the description of interfering traffic is approximate anyway, it is not useful to maintain a false idea of accuracy by allowing the rate to vary virtually continuously.

In the simplified traffic model, the range of possible values of the rate R of interfering traffic during a VC cell interarrival interval is divided into non-overlapping areas, e.g. $0 \leq R < r_{l(ow)}, r_{l(ow)} \leq R < r_{h(igh)}, r_{h(igh)} \leq R$. The simplified traffic model indicates the area in which R lies, instead of R itself.

The probability that a particular area is selected is the probability that the actual rate lies in this area, e.g. $\Pr(r_l \leq R < r_h) = \sum_{r_l \leq r < r_h} \Pr(R = r)$. When calculating cell waiting times, we have to choose a rate for each area. We have chosen the mean rate in the area, so that the rate of interfering traffic is not changed by the simplification of the traffic model. E.g. the rate corresponding to the area $r_l \leq R < r_h$ is: $\sum_{r_l \leq r < r_h} r \cdot \Pr(R = r)/\Pr(r_l \leq R < r_h)$.

6.4 Accuracy of conditional decomposition

In this section, we consider the end-to-end cell waiting time distribution in the ATM VC model. We compare numerical results obtained by the PCD performance evaluation method with simulation results. The purpose of this comparison is to asses the accuracy of the PCD method.

The tail of the end-to-end cell waiting time distribution cannot be estimated accurately by simulation at the extremely low cell loss probabilities that are required in an ATM network. So, we validate the PCD method at a relatively high loss probability. A lower cell loss probability can be achieved in two ways: lower server loads or longer buffers. If server loads are lower, the PCD method is more accurate, so that the validation at high loss probability is sufficient. If buffers are longer, the PCD method might be less accurate, so that in that case a somewhat longer traffic description may be required than expected on the basis of the validation that follows.

The ATM VC model is a tandem network of queues. The service time in each queue is 1 slot. The servers are synchronized and service operation is slotted. Buffer sizes are finite. The cell interarrival time on the VC under study is fixed. At each queue, interfering cells that newly arrive at the tandem network form a Poisson process. Their routes are independent and identically distributed.

Next, we consider crossing and partly joining interference, respectively.

6.4.1 Crossing interference

In this subsection, we consider the case of crossing interference. We compare with simulation results the assessment by the PCD performance evaluation method of the end-to-end cell waiting time in the VC model.

In the present example, the VC model consists of 5 queues in tandem. The server load (i.e., the fraction of slots in which the server would be occupied if there were no cell loss) is 0.8 for all queues. The traffic stream on the VC under study is initially (i.e., at the entrance into the tandem network) periodic with period 10. So, the rate of the Poisson interfering traffic is 0.7 at each queue. We consider two buffer sizes: 10 and 20, respectively.

The Figs. 6.3 and 6.4 show the survivor function of the end-to-end waiting time in the VC model at queue buffer size 10. Fig. 6.3 shows results obtained by simulation. In this figure, two end-to-end waiting time distributions are shown together with the corresponding 95 % confidence intervals. The solid, lower set of lines represents the actual end-to-end waiting time distribution. The dashed, upper set of lines represents the approximation of the end-to-end waiting time distribution that is obtained by neglecting correlation between the waiting times of a cell in different queues. The end-to-end distribution is the convolution of the distributions for the individual queues. Note that the confidence interval is so small that it is hardly discernible.

Comparison of the two sets of curves in Fig. 6.3 shows that the waiting times of the cell under study are negatively correlated. Long end-to-end waiting times are less likely in reality (solid lines) than predicted by assuming independent waiting times (dashed lines).



Figure 6.3: End-to-end cell waiting time distribution in 5 tandem queues. Crossing interference only. Period traffic under study: 10. Rates interfering Poisson traffic: 0.7. Buffer size: 10. Simulation results with 95 % c.i. (solid lines: no decomposition; dashed lines: decomposition).

Long waiting times in one queue are likely to be followed by short waiting times in a subsequent queue and vice versa. Negative correlation is entirely due to change of traffic characteristics on the VC under study. So, change of VC traffic characteristics is relevant to the end-to-end waiting time. Change of VC traffic characteristics does, however, not significantly influence the waiting time in a single queue (see Sect. 5.2).

Fig. 6.4 repeats the simulation results of Fig. 6.3 (without confidence intervals) and adds results obtained by the PCD performance evaluation method and by straightforward decomposition. The results obtained by calculation are indicated by crosses (lower set of crosses: conditional decomposition, upper set of crosses: decomposition). The traffic descriptions used in the PCD method comprise 2 VC cell interarrival intervals. This holds for both the traffic description to determine Q and for A in (6.3).

The calculated results give a perfect approximation of the simulation results. For the case of decomposition, this implies that the waiting time distribution in an individual queue can accurately be obtained on the basis of a VC traffic description that is the joint distribution of 2 VC cell interarrival intervals. For the case of the PCD performance evaluation method, this implies that it is sufficient if the condition comprises 2 VC cell interarrival intervals.

In the preceding example we assumed a buffer size of 10 cells. The numerical complexity of the PCD method increases considerably with increasing buffer size. Buffer size 10 is rather small, so the probability that a cell is lost is high, namely approximately 10^{-3} in



Figure 6.4: End-to-end cell waiting time distribution in 5 tandem queues. Crossing interference only. Period traffic under study: 10. Rates interfering Poisson traffic: 0.7. Buffer size: 10. Simulation results (solid line: no decomposition; dashed line: decomposition). Calculated results (lower set of crosses: conditional decomposition; upper set of crosses: decomposition).



Figure 6.5: End-to-end cell waiting time distribution in 5 tandem queues. Crossing interference only. Period traffic under study: 10. Rates interfering Poisson traffic: 0.7. Buffer size: 20. Simulation results (solid line). Results calculated by PCD (dashed: 1 interval in condition; point-dash: 2 intervals in condition).

each queue. Loss of cells reduces the length of a busy period. A busy period of which the length is reduced by loss is likely to be a long busy period. Reduction of the length of busy periods increases the accuracy of the PCD method. Next, we consider the same VC model, however, with buffer sizes doubled to 20 cells. The probability of cell loss is then much lower, namely approximately 10^{-5} in each queue. As a result, the results of the performance evaluation method are less accurate.

Fig. 6.5 shows the survivor function of the end-to-end cell waiting time in the VC model at buffer size 20. It compares simulation results (solid line) with results calculated according to the PCD method. In the PCD method, the traffic description used to determine Qcomprises 2 VC cell interarrival intervals. The dashed line is the result if the condition Ais a traffic description of 1 VC cell interarrival interval. The point-dash line is the result if the condition A is a 2 VC cell interarrival interval description.

Comparison of the point-dash and dashed curves with the solid curve shows that the PCD method is clearly less accurate at buffer size 20 than at buffer size 10. Accuracy can be improved by increasing the length of the traffic description in the condition, as follows by comparing the curves at a 1 interval description (dashed) and at a 2 interval description (point-dash).



Figure 6.6: End-to-end cell waiting time distribution in 3 tandem queues. Partly joining interference. Fan out: 2. Period traffic under study: 10. Rates interfering traffic: 0.7. Buffer size: 10. Simulation results with 95 % c.i. (solid lines: no decomposition; dashed lines: decomposition).

6.4.2 Partly joining interference

In this subsection, we consider the case of partly joining interference. We compare with simulation results the assessment of the end-to-end waiting time survivor function obtained by the PCD method.

In the present example, the VC model consists of 3 queues in tandem. The server load is 0.8 in all queues. Fan out is 2. The traffic stream on the VC under study is initially periodic. We consider two periods: 10 and 15, respectively. Buffer size is 10 in all queues. The main issue is the accuracy of the description of interfering cells in the internal traffic stream.

The Figs. 6.6 and 6.7 show the survivor function of the end-to-end cell waiting time in the VC model for period 10 on the VC under study. Fig. 6.6 shows simulation results. The solid lines give the end-to-end waiting time survivor function and 95 % confidence intervals. The dashed lines give the approximation for the end-to-end waiting time survivor function (and 95 % confidence intervals) that is obtained by neglecting correlation between the waiting times of a cell in different queues.

Correlation between cell waiting times is observed to be positive, i.e. high waiting times are likely followed by high waiting times in subsequent queues. In Fig. 6.6, positive correlation shows by the decrease of waiting times if independence is assumed. Note that positive correlation is due to the interfering cells. The traffic on the VC under study



Figure 6.7: End-to-end cell waiting time distribution in 3 tandem queues. Partly joining interference. Fan out: 2. Period traffic under study: 10. Rates interfering traffic: 0.7. Buffer size: 10. Simulation results (solid line: no decomposition; dashed line: decomposition). Calculated results (upper point-dash line: conditional decomposition; lower point-dash line: decomposition).

causes negative correlation, as observed in the previous subsection. In case of partly joining interference, both effects are simultaneously active. In this example, positive correlation prevails.

Fig. 6.7 repeats the simulation results of Fig. 6.6 (without confidence intervals) and adds results obtained by the PCD method and by straightforward decomposition. The upper point-dash line is the result of the PCD method. The lower point-dash line is the result obtained by assuming independent queues. The traffic description used in the performance evaluation method is of the simplified type. It comprises 2 VC cell interarrival intervals. This holds for the traffic description to calculate Q and for A in (6.3). The rate of interfering cells during a VC cell interarrival interval is indicated by one of three areas. Before cell routing, the mean rate of interfering cells during an interval is $0.7 \cdot \frac{10}{9} = \frac{7}{9}$ cells/slot.⁷ The 3 areas are rather arbitrarily chosen: smaller than $\frac{7}{9} - 0.2$, larger than $\frac{7}{9} + 0.2$, and the center area. ⁸

⁷The rate of interfering traffic is 0.7. The rate of the VC understudy is 0.1. So if we only consider slots that are not occupied by the VC under study, the rate of interfering traffic in the queue output stream is $\frac{7}{9}$.

⁸We tried several other divisions into areas: a less wide center area, a wider center area, and 5 areas. The result of the method is hardly sensitive to the choice of the areas, except that accuracy is much worse for the wide center area case that we considered: smaller than $\frac{2}{3} - 0.3$, larger than $\frac{2}{3} + 0.3$, and the center



Figure 6.8: End-to-end cell waiting time distribution in 3 tandem queues. Partly joining interference. Fan out: 2. Period traffic under study: 15. Rates interfering traffic: 0.733. Buffer size: 10. Simulation results and 95 % confidence intervals (solid lines: no decomposition; dashed lines: decomposition). Calculated results (upper point-dash line: conditional decomposition; lower point-dash line: decomposition).

For the case of decomposition, the results in Fig. 6.7 are very close to each other. So, the traffic model is sufficiently accurate to capture the cell waiting time distribution in a single queue. The result of the PCD method closely resembles simulation results. So, also correlation between cell waiting times is accurately captured.

Fig. 6.8 repeats Fig. 6.7, however the period on the VC has been increased from 10 to 15. The results in the figure show that the accuracy of the method is virtually unaffected, if the rate on the VC under study is decreased. Decrease of the rate on the VC under study makes the description of internal traffic more rough for interfering cells.

Note that fan out 2, as assumed in this example, is low. At higher fan out, results are more accurate due to the filtering effect of cell routing.

6.5 Conclusions

We have shown that the PCD method provides a powerful tool in the analysis of end-toend cell waiting times in the ATM VC model. Most importantly, it captures the QNP *waiting time correlation*. In case of crossing interference, this correlation is negative. In

area. If the center area is too narrow or too wide, the effect of correlation between cell waiting times is not captured.

6.5. CONCLUSIONS

case of partly joining interference, it is positive. Moreover the method incorporates the QNP change of VC traffic characteristics and correlation between VC traffic steams.

The method allows a trade-off between accuracy of the results and complexity of the description of traffic inside the VC model. The traffic description is used to condition the waiting times of the cell under study in the queues of the VC model. It is the joint distribution of a number of cell interarrival interval lengths on the VC under study and the rates of interfering cells during these intervals.

As far as the QNP VC traffic characteristics change is concerned, the accuracy of the PCD method can be increased by incorporating more cell interarrival intervals in the traffic description (i.e., larger m) or by describing interfering traffic in more detail.

As far as the QNP waiting time correlation is concerned, the accuracy of the PCD method can be increased by incorporating more cell interarrival intervals in the condition on the waiting time (i.e., larger k) or by describing interfering traffic in more detail. The numerical complexity of the method hardly depends on k, and is essentially determined by m. So, the method is capable of taking the QNP waiting time correlation into account.

Further, the accuracy of the method is higher if the busy periods of queues are shorter, e.g. due to lower server load or due to decreased buffer size. We have examined the accuracy of the PCD method in several examples. Conditioning the waiting time on a traffic description of 2 cell interarrival intervals is sufficient at small buffer size, say smaller than 20 cells. (This holds at parameter values that are typical for ATM: server load not larger than 0.8, fan out not smaller than 2, rate on VC under study not larger than 0.1.) For larger buffer size, more interarrival intervals should be incorporated in the condition on the waiting time (i.e., k should be larger).

While we demonstrated the application of the method to the case of periodic traffic on the VC under study and Poisson interfering traffic, the basic idea of the method can be applied to many more types of traffic processes. Extension of the method to other VC traffic models is straightforward. In case of partly joining interference, application of the method to other cell routing methods than independent and identically distributed cell routes is however more difficult. It is difficult to devise other realistic cell routing models.

Chapter 7

VC performance evaluation for bursty traffic

In this chapter, we present a new ATM VC performance evaluation method for bursty traffic. An early version of this work can be found in [van Rijnsoever, 1993]. In bursty traffic, distinct periods of high and low activity can be distinguished. Examples are many forms of computer data and video.

The performance evaluation method takes into account the queuing network phenomena (QNP) as far as they are due to overload of the multiplexers on the VC under study. The main motivation for the method is the QNP waiting time correlation. The restriction to overload is accurate, because during underload the QNP are much less relevant than during overload, and performance is mainly determined by overload. The accuracy of the performance evaluation method is shown by comparison with simulation results.

The method that we present differs from methods presented in the literature (see Ch. 4) by the incorporation of all three QNP. Most performance evaluation methods presented in the literature neglect correlation between cell waiting times. In addition, they either neglect correlation between VC traffic streams (see 4.5) or assume a very rough cell routing model (see 4.4). One method partly takes into account correlation between cell waiting times (see [Kroener et al., 1992] and 4.6). The method presented here fully accounts for this QNP.

We model the traffic streams on the VCs in the network by independent and identical on-off processes of the interrupted Poisson process (IPP) type. The performance evaluation method requires that the multiplexers that form the ATM VC model are operated as *statistical* multiplexers in the burst level congestion region.¹ All multiplexers are identical, and they have equal loads.²

 $^{^{1}}$ A multiplexer is a statistical multiplexer if the demand for transmission capacity is allowed to occasionally exceed the transmission capacity of the multiplexer during a relatively long period of time. Such a period is called an overload period. Operation in the burst level congestion region means that an attempt is made to buffer all excess traffic during an overload period.

²If the multiplexers are not identical, the performance evaluation method can independently be applied to the parts of the VC that do have identical multiplexers. It is not possible to take into account the QNP

The basic operation in the performance evaluation method is to reduce the VC model of, say, n queues in tandem to only two queues in tandem. The traffic streams through these two queues are chosen such that the overload behavior of the two queues models the relevant part of the overload behavior of the n queues that form the ATM VC model. Modeling of the traffic streams is based on a traffic aggregation technique. The two tandem queues model is solved numerically.

This chapter is outlined in such a way that the ATM VC performance evaluation method is gradually developed. First it presents the VC model. Then it extends traffic aggregation and performance analysis first from a single statistical multiplexer to two queues in tandem and then to more than two queues in tandem. The chapter ends with conclusions.

7.1 A tandem queuing network model of the VC under study

In this section, we model a VC through the ATM queuing network model for the case of bursty VC traffic streams (see 4.1.1). (Smooth VC traffic streams have been addressed in the previous chapter.) The VC model allows to evaluate the end-to-end cell waiting time distribution on the VC under study, without explicitly taking into account all queues in the ATM queuing network model. It represents only the queues of the ATM queuing network model through which the VC under study passes. So, it is a network of queues in tandem, a tandem queuing network.

The ATM VC model is a tandem network of, say, n identical queues. In each queue the service time is fixed and the buffer size is finite. To simplify the model, we assume that the queues are synchronized. The VC under study passes through all n queues. Other (i.e. interfering) VCs join the tandem queuing network at one of the queues and depart from the tandem network after this queue or after a subsequent queue. So, we consider the case of partly joining interference or of crossing interference.

We assume that the same number of VCs passes through each queue in the VC mod. In addition we assume that the same number of VCs passes through each pair of consecutive queus. This is less restrictive than it seems, because network design will be based on a maximum load of the network. Further, fan out³ should not be very small (e.g., not smaller than 2).

In the remainder of this section, we model traffic streams on VCs, describe the mode in which the multiplexers in the network operate, and indicate the performance measure of interest.

waiting time correlation between queues that are in different parts.

 $^{^{3}}$ Fan out is the ratio of the number of VCs multiplexed by a queue and the number of VCs that remains in the VC model after that queue.



Figure 7.1: Interrupted Poisson Process

7.1.1 IPP model of a VC traffic stream

This chapter concerns bursty VC traffic. The type of bursty traffic that we consider is on-off traffic. On-off traffic is a generic building block in traffic modeling (see Ch. 2). Several on-off traffic sources may together represent a single, more complex traffic source. The specific on-off VC traffic model that we use is the interrupted Poisson process (IPP). (See also 4.1 for a motivation of the IPP model.)

The traffic stream on each VC is modeled by an independent and identical IPP. In an IPP, the alternation between on- and off-periods is described by a two-state discrete-time Markov chain. In the on-state cells are generated; in the off-state no cells are generated. The lengths of on- and off-periods are geometrically distributed. In the on-state, cells are generated according to a Poisson process⁴.

The IPP model is described by three parameters (see Fig. 7.1): T the mean sojourn time in the on-state (unit: cell transmission time or, equivalently, service time), γ the cell generation rate in the on-state (unit: cell transmission rate), and ϵ the fraction of time that the IPP is in the on-state.

VC traffic streams that newly arrive at the VC model may previously have passed through queues of the ATM queuing network model that do not belong to the VC model. These traffic streams have been influenced by the QNP traffic characteristics change and the QNP VC traffic stream correlation (see Ch. 5). The performance evaluation method neglects both effects if they occur in queues that do not belong to the VC model. As a result, all VC traffic streams entering the VC model can be described by the same, independent IPP.

Neglecting QNP that are due to queues outside the VC model is accurate:

⁴The VC model is a discrete-time model in which all events occur at slot boundaries. We describe cell generation in the on-state of the IPP model by a Poisson process. The Poisson process is a continuous-time process. In order to make the IPP traffic model compatible with the VC model, we register cell arrivals not until the next slot boundary. This is equivalent to assuming that in each slot of an on-period a batch of cells is generated, where the number of cells in a batch is independent and Poisson distributed.

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- In 5.2, we concluded that the QNP VC traffic characteristics change is only relevant if the rate of the VC traffic stream in the on-state is high (relative to the cell transmission rate). This does not apply to the VC model that we consider⁵. So, this QNP is not relevant and is justifiably neglected.
- In 5.3, we showed that the QNP VC traffic stream correlation is relevant only if a very small number of upstream queues contributes to the traffic load in the queue under study. The effect of the phenomenon is essentially to reduce the probability of overload of the queue under study. In the performance evaluation method presented in this chapter, the QNP is neglected if it is due to queues that do not belong to the VC model. It is however incorporated if it is caused by a queue in the VC model. We are especially interested in correlation between the waiting times of a cell in the queues of the VC model. Simultaneous overload of two queues of the VC model is important in this respect.

7.1.2 Statistical multiplexing

Each queue in the VC model represents a multiplexer. The multiplexers operate as statistical multiplexers in the burst level congestion region (see 3.3). This implies that cell loss is determined by burst level congestion. Congestion at the burst level (or, equivalently, overload) occurs when the *instantaneous* cell arrival rate at a multiplexer exceeds the cell transmission rate of that multiplexer. The instantaneous cell arrival rate is determined by the number of VC traffic streams in the on-state. (In contrast, the mean cell arrival rate is determined by the *mean* number of VC traffic streams in the on-state.)

The performance evaluation method presented in this chapter assumes that the probability of m + 1 simultaneously overloaded queues in the VC model is much smaller than the probability of m simultaneously overloaded queues, $1 \le m \le n - 1$. As we will show later, this is ensured if first the probability of overload of a queue is small (e.g., smaller than 10^{-3}) and second fan out is not extremely small (e.g., not smaller than 2). These requirements are usually fulfilled in an ATM network.

7.1.3 Performance measures

The performance evaluation method presented in this chapter concerns the end-to-end cell waiting time distribution in the VC model. Large end-to-end cell waiting times are not relevant if they occur with a probability much smaller than the probability of cell loss. (One might consider a cell as lost if it has to wait extremely long.) We examine the relation between waiting time and cell loss, first in a single queue and then in the VC model.

 $^{{}^{5}}$ The performance evaluation method developed in this chapter applies to a tandem network of statistical multiplexers that operate in the burst level congestion region, see the next subsection. In 3.3, we showed that effective application of statistical multiplexing requires the source rate in the on-state to be low relative to the cell transmission rate.

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The probability that a cell is lost in a specific queue of the VC model is of the same order of magnitude as the probability that the cell waiting time in that queue is maximum. When a queue is overloaded, the excess of cells is first accommodated in the buffer. If the overload state persists and the buffer has become completely full, the excess of cells is lost. Maximum cell waiting time occurs if one place is left in the buffer of the queue at the moment of cell arrival. Loss occurs if no place is left in the buffer. ⁶ The probability of maximum cell waiting time in a queue and the cell loss probability in that queue are of the same order of magnitude.

The probability that a cell is lost in the VC model approximately equals the sum of the cell loss probabilities of the individual queues. The end-to-end cell loss probability is of the same order of magnitude as the probability that a cell waits for the maximum time possible in one of the queues of the VC model. So, we are not interested in end-to-end cell waiting times that are much less likely to occur than the waiting time that is caused by a single full buffer.

7.1.4 Summary of the ATM VC model

In summary, the VC model is a network of queues in tandem, each queue representing a multiplexer operating as statistical multiplexer in the burst level congestion region. All queues are identical: the service time is 1 slot and the buffer size is finite. All VC traffic streams are modeled by independent and identical IPPs. All queues and traffic streams are synchronized at slot boundaries. Through each queue the same number of VCs passes, and also through each pair of queues the same number of VCs passes.

In the next sections, we will gradually develop the performance evaluation method. The method allows accurate evaluation of the end-to-end cell waiting time distribution in the VC model. It is based on aggregation of the VC traffic streams that are multiplexed by a queue. The Sects. 7.2, 7.3 and 7.4 concern respectively a single queue in the VC model, two consecutive queues in the VC model and all queues in the VC model.

7.2 Single queue

In this section, we consider an arbitrary queue in the VC model. The issue is not analysis of the queue itself, but the way in which traffic is modeled. The VC traffic streams that arrive at the queue are collectively modeled by an aggregate traffic model. The aggregate traffic model is a simple stochastic process. It allows to describe the queue by a Markov chain that has a small state space size. Reduction of the state space size is especially important when considering several queues in tandem (see the next sections).

The VC traffic streams that arrive at a queue are aggregated by a slight variant of the aggregation procedure for IPPs developed by Baiocchi et al. (see 2.3.4). Baiocchi's aggre-

⁶If the instantaneous cell arrival rate during an overload period is ρ (cells / service time), on average ρ cells arrive at the queue in each slot. Roughly 1 of ρ cells is put into the buffer, and $\rho - 1$ of ρ cells are lost. So the ratio of cell loss probability and probability of maximum cell waiting time is roughly $\rho - 1$.

gation method approximates the superposition of, say, m independent and identical IPPs by a two-state Markov modulated Poisson process $(MMPP(2))^7$. The aggregation method was developed to assess cell loss in a statistical multiplexer. One state of the MMPP(2) corresponds to queue overload; the other state to queue underload. The parameters of the MMPP(2) are chosen such that the behavior that is most relevant, namely overload behavior, is accurately captured. In this section, we first summarize Baiocchi's method and then propose a slight modification to better approximate the cell waiting time distribution.

7.2.1 Summary of Baiocchi's method

In contrast with our VC model, Baiocchi's aggregation method applies to continuous-time VC traffic stream models. In order to apply Baiocchi's method, we first interpret the VC traffic streams as continuous-time processes, then apply the method, and finally interpret the aggregate traffic stream model as a discrete-time process. This hardly influences the accuracy of the method.

Baiocchi's method approximates an m-IPP/D/1/K queue by an MMPP(2)/D/1/K queue. It focuses on overload periods, i.e. on periods during which the instantaneous cell arrival rate exceeds the service rate. The overload periods of the MMPP(2) represent the overload periods of the m-IPP that do most harm and determine performance. Harmful overload periods last long and comprise many cells. The frequency of overload periods in the MMPP(2) is however chosen smaller than in the m-IPP, in order not to exaggerate the probability that a cell is lost. The frequency reduction is such that the cell loss probability in a queue without buffer is the same for the MMPP(2) and for the m-IPP.⁸. The aggregation method gives a tight upperbound on the cell loss probability in a queue with buffer.

In the *m*-IPP, the number of VC traffic streams in the on-state is described by the continuous-time Markov chain $\{M_i, t \ge 0, M_i \in \{0, 1, \ldots, m\}\}$. The overload states are: $\{i \mid i\gamma > 1\}$. Overload of the queue starts at the transition of $\{M_i\}$ from $i = \lfloor \frac{1}{\gamma} \rfloor$ to $i = \lceil \frac{1}{\gamma} \rceil$. It ends at the reverse transition. The sojourn of the queue in overload is described by the sojourn of $\{M_i\}$ in the states $i > \lfloor \frac{1}{\gamma} \rfloor$. This is a phase-type distribution.

The aggregation method uses the sojourn time in overload and the number of cell arrivals during overload. To determine the distribution of the number of arrivals it assumes fluid-flow given the state of $\{M_t\}$. The distribution of the number of arrivals is obtained by modeling the number of cell arrivals in a state of $\{M_t\}$ instead of the sojourn time.

The aggregation method equates the following parameters between the m-IPP and the MMPP(2):

• the mean cell arrival rate.

 $^{^{7}}$ The MMPP(2) type of stochastic process closely resembles the IPP type of stochastic process. The alternation between the two states of the MMPP(2) is again described by a two-state Markov chain. In contrast to the IPP, in the MMPP(2) cells are generated in both states, where the cell generation rate depends on the state.

 $^{^{8}}$ In this step of the procedure a fluid-flow traffic model (see A.3) is assumed, which considerably simplifies the calculations.

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- the slope (at infinity and on a log scale) of the survivor function of the sojourn time in an overload period.
- the slope (at infinity and on a log scale) of the survivor function of the number of cells generated in an overload period. Fluid-flow is assumed.
- the probability of cell loss in a bufferless queue. Fluid-flow is assumed.

Assuming that it exists, the slope (at infinity and on a log scale) of the survivor function of random variable X is:

$$\lim_{x\to\infty}\frac{d}{dx}(\ln(\Pr(X\geq x))).$$

If X is exponentially distributed (i.e., $\Pr(X \ge x) = e^{-\lambda x}$, this evaluates to $-\lambda$. The MMPP(2) approximation represents the most harmful overload behavior of the *m*-IPP process by copying the asymptotic distribution of overload periods.

As said, the distributions of sojourn time and number of cell arrivals in overload are phase-type distributions. The slope (at infinity and on a log scale) of the survivor function of a phase-type distribution is relatively easily obtained. (It is the smallest eigenvalue of the matrix that describes the distribution see [Neuts, 1989]).

7.2.2 Modification of Baiocchi's method

Baiocchi's method was developed to estimate the cell loss probability. The performance evaluation method developed in this chapter is about cell waiting times. We found that a slight variant of Baiocchi's method provides more accurate results for the cell waiting time distribution. The variant is that instead of the cell loss probability in a queue without buffer, we equate the probability of cell arrival at an overloaded queue. In calculating this probability, we also assume fluid flow.

The probability of cell arrival at an overloaded queue is important, because it determines the transition from cell scale congestion to burst scale congestion in a queue (see 3.3). This transition is marked by the 'knee' in the cell waiting time distribution. The modified aggregation method ensures that the knee is accurately modeled.

Figs. 7.2 and 7.3 compare the cell waiting time distributions in an *m*-IPP/D/1/K queue and an MMPP(2)D/1/K queue at different buffer sizes. The MMPP(2) approximates the *m*-IPP according to the modified method described above. The results shown in the figures are obtained by solving the Markov chain descriptions of the queues. The Markov chains are solved by iteration. The parameters of each IPP are set to values chosen by Kroener et al. [1992] (see Ch. 4): $T = 500, \gamma = 0.1, \epsilon = 0.2$.

Comparison of the solid and dashed curves in the Figs. 7.2 and 7.3 shows that the modified aggregation method is accurate for the relevant part of the distribution (i.e., the tails) and that it slightly overestimates the probability of long cell waiting times.



Figure 7.2: Waiting time distribution for an m-IPP/D/1/K queue (solid) and the approximating MMPP(2)/D/1/K queue (dashed). Parameters: m = 30 (top) or 20 (bottom), $T = 500, \gamma = 0.1, \epsilon = 0.2, K = 100$ (top) or 90 (bottom).



Figure 7.3: Waiting time distribution for an m-IPP/D/1/K queue (solid) and the approximating MMPP(2)/D/1/K queue (dashed). Parameters: m = 30 (top) or 20 (bottom), $T = 500, \gamma = 0.1, \epsilon = 0.2, K = 200$.

7.3. TWO TANDEM QUEUES



Figure 7.4: Two tandem queues

7.3 Two tandem queues

In this section, we take the next step in the development of the VC performance evaluation method. We extend the method from a single queue in the VC model (see 7.2) to two consecutive queues in the VC model. The purpose of simultaneously analyzing two queues is to incorporate the QNP *waiting time correlation*. In the next section, we will analyze the entire VC model.

Fig. 7.4 shows the two tandem queues model. The queues are two consecutive queues in the VC model (see 7.1). The VCs that pass through the two queues are classified into three streams: VCs that pass only through the first, upstream queue; VCs that pass only through the second, downstream queue; and VCs that pass through both the first and the second queue. The streams are indicated by, respectively, stream 1, stream 2, and stream 1-2.

The main point in this section is the extension of the traffic aggregation method from a single queue to two queues in tandem. The purpose of traffic aggregation is to describe all VC traffic streams in the two tandem queues model by a simple stochastic model. Traffic aggregation should considerably facilitate (numerical) analysis of the Markov chain describing the two tandem queues model. Traffic aggregation is even more important in the next section when we consider the entire VC model.

This section is organized as follows. The first subsection describes the approach to traffic aggregation. The traffic streams through the two tandem queues are described by a four-state Markov modulated Poisson process (MMPP(4)). The following two subsections describe how to choose the parameters values of the MMPP(4). The fourth subsection summarizes the choice of parameters. The final subsection shows the accuracy of the traffic aggregation method by comparison with simulation results.

7.3.1 Approach to traffic aggregation

The VC traffic streams through the two tandem queues are described by the Markov chain $\{(M_{1,n}, M_{2,n}, M_{12,n}), M_{1,n} \in \{0, \ldots, m_1\}, M_{2,n} \in \{0, \ldots, m_2\}, M_{12,n} \in \{0, \ldots, m_{12}\}, n \in \{0, 1, \ldots\}\}$, where $M_{i,n}, i \in \{1, 2, 12\}$ is the number of VC traffic streams on stream *i* that is in the on-state in slot *n*. The state space of this Markov chain is in general too large to allow numerical analysis of a Markov chain description of the two tandem queues model. The purpose of traffic aggregation is to reduce the state space size.

The traffic aggregation method for single queues can be extended to two tandem queues



Figure 7.5: MMPP(4) model for the aggregate arrival traffic and cell routing in a two tandem queues network.

in two 'obvious' ways. Each of them comes however with problems:

- represent the VC traffic streams on each stream by an independent MMPP(2): Application of the single queue traffic aggregation method is now difficult at both queues for the same reason.
- represent the VC traffic streams on stream 1 and stream 1-2 by an MMPP(2) and the VC traffic streams on stream 2 by an other MMPP(2): Application of the single queue traffic aggregation method to stream 2 is not straightforward at all, because there is no fixed arrival rate on stream 2 that marks the overload state of the second queue.

Therefore, we have chosen a less obvious way to model the traffic streams 1, 2 and 1-2. The cell arrival rates on the streams are described by a single, four-state Markov modulated Poisson process (MMPP(4)), see Fig. 7.5. In te remainder of this section, the MMPP(4) model is elaborated. In the following sections we will describe how to choose the parameters of the MMPP(4).

States of the MMPP(4)

The traffic aggregation method is again based on the distinction between overloaded and underloaded queues, because overload essentially determines the cell waiting time distribution in the two queues. Each state of the MMPP(4) corresponds to a specific overload situation of the two queues:

- (u, u): both queues underloaded,
- (u, o): queue 1 underloaded and queue 2 overloaded,
- (o, u): queue 1 overloaded and queue 2 underloaded, and

7.3. TWO TANDEM QUEUES

• (o, o): both queues overloaded.

A queue is overloaded if the aggregate instantaneous cell arrival rate at the queue exceeds the service rate. This definition of overload cannot be applied directly when considering queues in tandem. The problem is the span of time that passes between the arrival of a cell at queue 1 and its arrival at queue 2. While a cell is waiting in the first queue, the number of active sources on stream 2 may change and overload of queue 2 may turn into underload.

The traffic aggregation method requires unambiguous mapping of $\{(M_{1,n}, M_{2,n}, M_{12,n})\}$ onto the states of the MMPP(4). This is not possible with the above definition of overload. We resolve this issue by defining overload of queue 2 at the moment that the cell under study arrives at queue 1 (instead of at arrival at queue 2). Taking into account that the traffic on stream 1-2 is throttled by an overloaded queue 1, the mapping of $\{(M_1, M_2, M_{12})\}$ onto MMPP(4) is:

$$\begin{array}{rcl} (u,u) &=& \{(i_1,i_2,i_{12}) \mid (i_1+i_{12})\gamma \leq 1 \land (i_2+i_{12})\gamma \leq 1\} \\ (u,o) &=& \{(i_1,i_2,i_{12}) \mid (i_1+i_{12})\gamma \leq 1 \land (i_2+i_{12})\gamma > 1\} \\ (o,u) &=& \{(i_1,i_2,i_{12}) \mid (i_1+i_{12})\gamma > 1 \land (i_2\gamma + \frac{i_{12}}{i_1+i_{12}}) \leq 1\} \\ (o,o) &=& \{(i_1,i_2,i_{12}) \mid (i_1+i_{12})\gamma > 1 \land (i_2\gamma + \frac{i_{12}}{i_1+i_{12}}) > 1\} \end{array}$$

Not all transitions between states in the MMPP(4) model are taken into account. Transitions between the single overload states (o, u) and (u, o) are so unlikely that their effect on performance is negligible. These transitions comply with two simultaneous transitions in $\{(M_{1,n}, M_{2,n}, M_{12,n})\}$, namely of $\{M_{1,n}\}$ and $\{M_{2,n}\}$. Two simultaneous transitions are improbable, and we neglect them.

Cell routing

Next to traffic modeling, modeling cell routing is also an impediment in the numerical analysis of the two tandem queues model. Cells that have received service in the first queue are either routed to the second queue or they leave the model. The appropriate cell route is determined by the route of the VC to which the cell belongs. The VC-identity of cells buffered by the first queue can, however, not be incorporated in a Markov chain description of the two tandem queues, because the state space size would become much too large. The solution to this problem is to device a stochastic model for cell routing.

If the VC traffic streams in the streams 1 and 1-2 were Poisson processes and cells that arrive at queue 1 in a single slot were put into the buffer in random order, cell routing would be modeled exactly by a Bernoulli process. In Bernoulli routing, the routing decision is independent and identically distributed for each cell. In the model under study however, VC traffic streams are *modulated* Poisson processes, so Bernoulli routing is not exact.

The cell routing model that we will use is based on and used in conjunction with the MMPP(4) aggregate traffic model. We let the MMPP(4) determine the cell routing
probability. Given the state of the MMPP(4) the routing process is a Bernoulli process, but when the MMPP(4) changes state the routing probability changes accordingly.

We expect this routing model to be accurate and validate it together with the traffic aggregation method by comparison with simulation results. The routing model only falters at state transitions of the MMPP(4). The cell routing probability is determined by the state of the MMPP(4) at the moment that a cell is served. The state of the MMPP(4) may however have changed in the interval between cell arrival at queue 1 and cell departure from queue 1. The cells waiting in queue 1 at the moment the MMPP(4) changes state are routed according to a Bernoulli process with incorrect routing probability.

Calculation of the cell waiting time distribution

Traffic aggregation and the cell routing model considerably reduce the complexity of the two tandem queues model. The state space of the corresponding Markov chain description is now small enough to allow calculation of the steady-state distribution.

The end-to-end cell waiting time distribution is calculated on the basis of the steadystate distribution of the two tandem queues model at arrival of a cell on stream 1-2. The cell waiting time distribution is derived from the steady-state distribution by considering all possible evolutions of the two tandem queues after the cell under study has arrived. In this part of the procedure, a trick is applied that ensures that at least the cell under study is routed according to the correct cell routing probability. When considering the evolutions of the two tandem queues between arrival of the cell under study and service of this cell in queue 1, the routing probability remains fixed at its initial value, so that the cell under study and immediately preceding cells are routed according to the correct probability distribution. (So in the last part of the calculation, the cell routing probability no longer changes according to the state of the MMPP(4).)

The traffic aggregation method has now been described in principle. What remains is to choose the parameters of the MMPP(4) in such a way that the performance evaluation method accurately assesses the end-to-end cell waiting time distribution.

In the remainder of this section, we first separately consider the parameters of the double overload and single overload states of the MMPP(4). We then list all parameters that determine the MMPP(4). The section ends with results on the accuracy of the traffic aggregation method.

7.3.2 Double overload

In this subsection, we model double overload in two tandem queues. Double overload was defined as a state in which the instantaneous arrival rates on the streams 1, 2, and 1-2 indicate that both queues are overloaded. Throttling of the traffic on stream 1-2 due to overload of queue 1 is to be taken into account. In the MMPP(4) aggregate traffic model, double overload is represented by a single state, state (o, o).

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We model double overload in essentially the same way as overload in a single queue (see 7.2). So the (o, o)-state of the MMPP(4) represents the asymptotic behavior of double overload in $\{(M_{1,n}, M_{2,n}, M_{12,n})\}$. In order not to overestimate the effect of double overload, we reduce the frequency of double-overload periods, so that the probability that a cell arrives in double overload is retained.

The following parameters are equated between the MMPP(4) and the unaggregated traffic:

- the slope (at infinity and on a log scale) of the survivor function of the sojourn time in double overload.
- the slope (at infinity and on a log scale) of the survivor function of the number of cells generated during double overload on, assuming fluid flow⁹:
 - stream 1 plus stream 1-2
 - stream 2 plus stream 1-2
 - stream 1 plus stream 2 plus stream 1-2
- the probability that a cell arrives during double overload.

The slopes are obtained in the same way as for a single queue (see 7.2). The distributions of the sojourn time and of the number of generated cells in double overload are again of the phase type. They are three-dimensional, each dimension describing one of the traffic streams.

7.3.3 Single overload

Single overload refers to the state that one of the two queues in the two tandem queues model is overloaded and the other queue is underloaded. Single overload is relevant by itself as a form of congestion, but it also occurs in combination with double overload.

The MMPP(4) model describes single overload by two states, namely (o, u) and (u, o). In this subsection, we consider how to choose the parameters of the MMPP(4) that concern single overload. We first focus on the most likely form of single overload: single overload preceded and succeeded by double underload (double underload = no overload). We then cater for single overload in combination with double overload.

⁹We need to know the number of cells generated during double overload on each stream. We do not calculate these number directly but derive them from the parameters indicated. The reason is that even during double overload, it is possible that no cells are generated on a stream in some states of the unaggregated cell arrival process. By considering combinations of streams this is avoided, which considerably facilitates the calculation of the slopes

Single overload preceded and succeeded by double underload

To model single overload preceded and succeeded by double underload, we again use the technique that we used in modeling overload of a single queue: the asymptotic behavior of overload periods is represented and the frequency of overload periods is reduced in order not to overestimate the effect of overload.

Single overload corresponds to a set of states in the Markov chain $\{(M_{1,n}, M_{2,n}, M_{12,n})\}$. The sojourn time and the number of cells generated in a single-overload period depend on the state (either a double underload state or a double overload state) that precedes the period and the state that succeeds the period. The *asymptotic* behavior depends however only on the succeeding state.

In order to simplify the traffic aggregation method, we do not distinguish between single overload followed by double underload and single overload followed by double overload when calculating the asymptotic behavior. Instead the asymptotic behavior is approximated by the asymptotic overload behavior in a *single* queue. The single queue is either the first or the second queue of the two tandem queues model, while the other queue is neglected. This approach is equivalent to considering the Markov chain $\{(M_{1,n}, M_{12,n})\}$ (or $\{(M_{2,n}, M_{12,n})\}$) instead of $\{(M_{1,n}, M_{2,n}, M_{12,n})\}$.

The approximation provides an upperbound, because the state of overload in the single queue includes the state of double overload in the two tandem queues. It is however accurate because succession of single overload by double overload is improbable in a well designed ATM network.

The frequency of single-overload periods is determined by the probability that a cell arrives at the two tandem queues during a single-overload period. (Note that not the state probabilities are used, but the probabilities at the moment of cell arrival.) This probability is easily calculated on the basis of the unaggregated traffic description. So, double overload is not neglected in this part of the aggregation method.

In summary, the following parameters determine single overload in the MMPP(4):

- the slope (at infinity and on a log scale) of the survivor function of the sojourn time in overload of queue 1.
- the slope (at infinity and on a log scale) of the survivor function of the number of cells generated on stream 1 and stream 1-2 during overload of queue 1, assuming fluid-flow.
- the probability that a cell on stream 1-2 arrives at overload of queue 1 and underload of queue 2.
- the slope (at infinity and on a log scale) of the survivor function of the sojourn time in overload of queue 2.
- the slope (at infinity and on a log scale) of the survivor function of the number of cells generated on stream 2 and stream 1-2 during overload of queue 2, assuming fluid-flow.

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• the probability that a cell on stream 1-2 arrives at underload of queue 1 and overload of queue 2.

In addition, the aggregate arrival rate at an overloaded queue is attributed to the two streams involved in proportion to the number of VCs on each stream.

Single overload in combination with double overload

The parameters of the single overload states in the MMPP(4) are almost completely determined by the case of single overload preceded and succeeded by double underload. The sojourn time distribution, the cell arrival rate at the overloaded queue, and the probability that a cell arrives at an overloaded queue have already been established. The parameters not yet established are the probabilities that single overload is preceded by double underload or by double overload and that single overload is succeeded by double underload or by double overload. These probabilities correspond to the transitions to and from the states (o, u) and (u, o) in Fig. 7.5. They can be used to represent in the MMPP(4) single overload periods preceded or succeeded by double overload.

In the aggregate traffic model, we have modeled double overload periods by their asymptotic behavior. The contribution to congestion of a single overload period (in the unaggregated traffic model) that precedes or succeeds such a double overload period is most often very small in comparison with the contribution of the asymptotic period itself.

In the aggregate traffic model we have modeled both single overload periods and double overload periods by their asymptotic behavior. So if in the aggregate traffic model a single and a double overload period occur one after the other, they are both very long. Unlike previously however, this effect can only partly be compensated for by reducing the frequency with which these combinations occur.

The frequency of consecutive overload periods can be reduced so that the probability is retained that a cell belongs to a (single or double) overload period that is preceded by another (double or single) overload period. This is achieved by properly choosing the transition probabilities in the MMPP(4) from single to double overload and vice versa. What cannot be compensated for in this way however is that the preceding (single or double) overload period has the asymptotic distribution. As a result, the aggregate traffic model provides a rough upperbound.

Especially in case of double overload succeeded by single overload the bound is not tight, as we will show in the results section. An asymptotic double overload period needs to be compensated for by frequency reduction. As this is not possible in the model, we choose not to incorporate in the MMPP(4) the possibility of transitions from double to single overload. The MMPP(4) can then no longer be guaranteed to provide an upperbound. It still gives a good approximation (and most likely an upperbound) because the case of single overload after double overload contributes relatively little to congestion, as explained previously.

7.3.4 Parameters

In the previous subsections, we described how to obtain the parameter values of the MMPP(4) traffic model. The treatment was directed towards arguing which metrics of the unaggregated traffic model should determine the parameters of the MMPP(4) traffic model. In this subsection, we list all these metrics in order to provide an overview of the traffic aggregation method for two tandem queues. We add some metrics not mentioned previously: the mean arrival rates on the three streams through the two tandem queues; the mean arrival rate on stream 1 in (u, o); the mean arrival rate on stream 2 in (o, u).

An MMPP(4) is fully described by the transition rates between the states and the cell generation rates on each stream in each state. There are four states ((u, u), (u, o), (o, u), (o, o)) and three streams (1, 2, 1-2). We do not allow transitions from (o, o) to (o, u), from (o, o) to (u, o), and transitions between (o, u) and (u, o). So, the MMPP(4) has 20 free parameters.

The 20 parameters of the MMPP(4) are determined by matching the 18 metrics of the MMPP(4) to the corresponding 18 metrics of the unaggregated traffic. The remaining 2 parameters of the MMPP(4) are determined by the assumption that in state (o, u) the arrival rate at queue 1 is distributed over stream 1 and stream 1-2 in proportion to the numbers of traffic sources in these streams. A similar assumption holds for the state (u, o). The metrics of the unaggregated traffic are listed below:

- 1. Double overload:
 - (a) slope (at infinity and on a log scale) of the survivor function of the sojourn time in (o, o).
 - (b) slope (at infinity and on a log scale) of the survivor function of the number of cell arrivals to queue 1 in (o, o). Fluid-flow is assumed.
 - (c) slope (at infinity and on a log scale) of the survivor function of the number of cell arrivals to queue 2 in (o, o), not taking into account cell throttling by the first queue. Fluid-flow is assumed.
 - (d) slope (at infinity and on a log scale) of the survivor function of the number of cell arrivals to the network formed by queue 1 and queue 2 in (o, o). Fluid-flow is assumed.
 - (e) Probability that a stream 1-2 cell arrives during (o, o).
 - (f) Probability that (o, o) was preceded by (u, o).
 - (g) Probability that (o, o) was preceded by (o, u).
- 2. Single overload:
 - (a) slope (at infinity and on a log scale) of the survivor function of the sojourn time in overload of queue 1.

- (b) slope (at infinity and on a log scale) of the survivor function of the number of cell arrivals to queue 1 during overload. Fluid-flow is assumed.
- (c) slope (at infinity and on a log scale) of the survivor function of the sojourn time in overload of queue 2 (neglecting any influence of queue 1).
- (d) slope (at infinity and on a log scale) of the survivor function of the number of cell arrivals to queue 2 during overload (neglecting any influence of queue 1). Fluid-flow is assumed.
- (e) Probability that a stream 1-2 cell arrives during (o, u).
- (f) Probability that a stream 1-2 cell arrives during (u, o).
- (g) Mean cell arrival rate on stream 2 during (o, u).
- (h) Mean cell arrival rate on stream 1 during (u, o).
- 3. Mean rates:
 - (a) Mean rate on stream 1.
 - (b) Mean rate on stream 2.
 - (c) Mean rate on stream 1-2.

7.3.5 Results

In order to validate the traffic aggregation method for two tandem queues, we compare model results with results obtained by simulation. The model results concern the MMPP(4) aggregate traffic model: the two tandem queues are described by a Markov chain that is solved by iteration. The simulation results concern unaggregated VC traffic streams. Model and simulation results are compared for two examples that differ with respect to the number of VC traffic streams. By comparing model and simulation results at different traffic loads, we can assess whether the model is accurate at the different cell loss probabilities that correspond to these traffic loads.

The two tandem queues are consecutive queues in the VC model (see 7.1). Service time is 1 slot, and buffer size is 150 cells. The VC traffic streams are described by the same parameter values as previously (see 7.2): $T = 500, \gamma = 0.1, \epsilon = 0.2$. The number of VC traffic streams, *m*, that feeds each queue is 30 and 28 in, respectively, the first and the second example. Fan out is 2, i.e. half the VC traffic streams multiplexed by queue 1 proceeds to queue 2

Fig. 7.6 shows the cell waiting time distribution for m = 30. The solid lines show the end-to-end waiting time survivor function obtained by simulation. 95 % confidence intervals are also shown. The dashed lines also show the end-to-end waiting time survivor function obtained by simulation, however assuming independent waiting times of a cell in the two queues. Comparison of the two sets of curves clearly shows the influence of the QNP waiting time correlation (see 5.1).



Figure 7.6: Waiting time survivor function in two tandem queues. $m = 30, T = 500, \gamma = 0.1, \epsilon = 0.2, B = 150$, fan out = 2. Simulation results with 95 % confidence intervals (solid: no decomposition; dashed: decomposition) and results for the traffic aggregation method (upper point-dash and lower point-dash: see text). Server loads: 0.6

The point-dash curves in Fig. 7.6 show the end-to-end waiting time survivor function in case the aggregate traffic model is applied. The upper point-dash curve concerns the case that transitions from double overload to single overload are allowed. The lower point-dash curve concerns the case that after double overload always double underload follows. As noted before, allowing transitions from double to single overload considerably overestimates the probability of long waiting times. We will not allow these transitions in the aggregate traffic model. Fig. 7.6 shows that the simulation results are then closely approximated.

As previously remarked (see 7.1.3), we are only interested in end-to-end waiting times that are not much more improbable than the probability that a cell is lost in any of the two queues. As cell loss is determined by overflow of a buffer, the probability of cell loss is related to and of the same order of magnitude as the probability that a cell is delayed by a buffer-worth of cells (in the present example, 150 cells).

At very small waiting times, the aggregation method is not accurate. However, small waiting times are hardly interesting. The aggregation method provides a very rough upperbound on the cell waiting time if single overload after double overload is allowed. If such transitions are banned, the method provides an accurate estimate of the end-to-end waiting time survivor function in the area that is relevant. It can however not be guaranteed to provide an upperbound on the tail of the cell waiting time distribution, due to neglecting direct transitions from double to single overload. The latter version of the method should obviously be preferred over the former, and we will hold on to it in the remainder of this



Figure 7.7: Waiting time survivor function for two tandem queues. $m = 28, T = 500, \gamma = 0.1, \epsilon = 0.2, B = 150$, fan out = 2. Simulation results with 95 % confidence intervals (solid: no decomposition; dashed: decomposition) and results for the traffic aggregation method (point-dash). Server loads: 0.56

chapter.

In order to show that the traffic aggregation method retains its accuracy if the probability of cell loss decreases, Fig. 7.7 shows results for m = 28. All other parameters values are unchanged. The simulation time required to achieve accurate results increases steeply if m decreases. So, we confine ourselves to m = 28. Comparison between the figures shows already clear differences, however. Fig. 7.7 complies with Fig. 7.6. The figure shows that the accuracy of the method is maintained if the server load (and the cell loss probability) is decreased.

7.4 n tandem queues

In the previous section, we described the performance evaluation method for two consecutive queues in the VC model. In this section, we extend the performance evaluation method to the entire VC model, a network of n queues in tandem (see 7.1).

The main point made in this section is a reduction of the VC model (a network of n tandem queues) to a network of two queues in tandem. The queues in the two tandem queues model are of the same type as the queues in the VC model. The traffic streams in the reduced model are described by an MMPP(4), like in the previous section. The MMPP(4) is chosen such that the reduced model represents the relevant part of the overload behavior

in the VC model. (End-to-end cell waiting times in the VC model are not relevant if they occur with a probability much smaller than the cell loss probability.)

The reduction of the VC model to two queues is accurate if the probability that a multiplexer is overloaded is small and fan out is not very small. These requirements are fulfilled in almost any ATM network in which (as in the VC model) multiplexers operate as statistical multiplexers in the burst level congestion region.

All three QNP are partly taken into account when reducing the VC model to two tandem queues. The incorporation of the QNP is restricted to the effects that are due to overload of a queue of the VC model. The QNP are however most relevant during overload, and we are interested in performance during overload. So, the performance evaluation method incorporates the essential elements of the QNP.

This section first describes why it is possible to reduce the VC model to two tandem queues. The second subsection indicates how to choose the parameters of the MMPP(4) traffic model. In a final subsection, the performance evaluation method is validated by comparison with simulation results.

7.4.1 Reduction of the number of queues

The VC performance evaluation method reduces a tandem queuing network of n queues to a tandem network of only 2 queues. This reduction maintains the relevant overload behavior of the n tandem queues. It is based on two propositions that we make plausible in this section:

- simultaneous congestion of more than two queues in the VC model is not relevant to the performance measures of interest.
- two queues are sufficient to represent the VC model up to double overload.

For these propositions to hold, it is required that the probability that a multiplexer in the VC model is overloaded is small and that fan out in the VC model is not very small.

Simultaneous congestion of more than two queues is not relevant to the performance measures of interest

The first proposition is that simultaneous congestion of more than 2 of the *n* tandem queues in the ATM VC model is not relevant to the end-to-end cell waiting time distribution. For this proposition to hold, it is required that fan out is not very small (e.g., at least 2). A second requirement for the proposition to hold is that the probability that a queue is overloaded is small (e.g., at most 10^{-3}). Both requirements are very likely fulfilled in an ATM network.

In support of the proposition, we will elaborate the following argument. Let K denote the buffer size of the queues in the VC model. The maximum cell waiting time in a single queue is then (K-1). For waiting times up to $m \cdot (K-1)$, simultaneous overload of at most m queues, $1 \le m \le n$, dominates simultaneous overload of (m+1) queues.

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To see how the argument supports the proposition, recall that in 7.1.3 it was observed that the cell loss probability in the VC model is of the same order of magnitude as the probability that a cell is delayed by an almost full buffer of cells in a queue of the VC model. End-to-end cell waiting times much less probable than cell loss are not relevant. So, we need only consider single and double overload (m = 1 and m = 2).

The argument is elaborated in two steps. In the first step, we will show that the probability of (m+1)-fold overload is much smaller than the probability of m-fold overload. In the second step, we will show that the increase of the end-to-end cell waiting time is approximately equal during m-fold overload and during (m+1)-fold overload. Both parts add up to the conclusion that, unless the accumulation of cells during m-fold overload is blocked by full buffers, m-fold overload predominates (m + 1)-fold overload as far as the cell waiting time distribution is concerned.

Pr((m+1)-fold overload) $\ll Pr(m$ -fold overload) In a well designed and controlled ATM network, the probability that a queue in the VC model is overloaded is small. At first, we will neglect correlation between the instantaneous cell arrival rates at the queues (see Sect. 5.1); then we will take this correlation into account. Correlation between instantaneous cell arrival rates at the queues increases the probability of multiple overload.

At first, we neglect correlation between instantaneous cell arrival rates at the queues in the VC model, i.e. we assume that the states of the queues (overload or underload) are independent. Note that queues far apart in the VC model are approximately independent at the moment that a cell under study arrives. Denote by p the probability that a queue is overloaded. The mean number of overloaded queues in the VC model is much smaller than 1: $np \ll 1$. Assuming independence, the number of overloaded queues is binomially distributed with parameters n and p:

$$\begin{array}{rcl} \Pr(i \text{ queues overloaded}) &=& \binom{n}{i} p^i (1-p)^{n-i} \\ \frac{\Pr(i+1 \text{ queues overloaded})}{\Pr(i \text{ queues overloaded})} &=& \frac{n-i}{i+1} \frac{p}{1-p} \\ &<& \frac{np}{1-p} \\ \ll & 1 \end{array}$$

So the probability of (i + 1)-fold overload is much smaller than the probability of *i*-fold overload.

Next, we present an example that takes into account correlation between the instantaneous cell arrival rates at the queues. We consider the joint distribution of the instantaneous cell arrival rates at 5 queues in tandem: n = 5. Through each queue, 16 VCs pass. The traffic stream on each VC is modeled by an independent IPP. When 10 VCs are simultaneously in the on-state, the server load is 1 (i.e., $\gamma = 0.1$). The probability that an IPP is in the on-state is 0.2 (i.e., $\epsilon = 0.2$). Fan out is 2.¹⁰ We assume bufferless

¹⁰Fan out 2 means that the number of VCs that entered the tandem network at a specific queue and

Table 7.1: Probability of multiple overload for five tandem queues, $N = 16, \epsilon = 0.2, \gamma = 0.1$, fan out = 2

i	dependent queues	independent queues
0	$9.99 \cdot 10^{-1}$	$9.99 \cdot 10^{-1}$
1	$1.16 \cdot 10^{-3}$	$1.24 \cdot 10^{-3}$
2	$3.55 \cdot 10^{-5}$	$6.13 \cdot 10^{-7}$
3	$1.02\cdot 10^{-6}$	$1.52\cdot10^{-10}$
4	$2.47 \cdot 10^{-8}$	$1.89 \cdot 10^{-14}$
5	$4.12 \cdot 10^{-10}$	$9.31 \cdot 10^{-19}$

queues. This allows us to neglect lengthening of on-periods due to queuing, so that the joint distribution of instantaneous cell arrival rates can be determined numerically. Tab. 7.1 shows the probability of multiple overload among the 5 tandem queues, both for dependent queues and for independent queues. The results in the table show that, even for this low fan out case, the probability of multiple overload is small and decreases sharply if the number of overloaded queues increases.

The increase of end-to-end waiting time during overload is almost independent of the number of overloaded queues The sum of the queue lengths in the VC model may serve as an indication of the end-to-end cell waiting time. The increase of this sum is approximately equal during *m*-fold overload and during (m + 1)-fold overload.

We first assume that the queues are independent and that their overload periods are identically distributed. We further assume that the cell arrival process at an overload queue is a fixed rate (r) fluid-flow and the distribution of the sojourn time in overload is exponential (α) . Assuming this model, the increase of the sum of the queue lengths during an overload period is independent of the multiplicity of the overload period. It does not matter whether m or (m + 1) queues are overloaded during the multiple overload period. While overload lasts, the sum of queue lengths grows faster in case of (m+1)-fold overload. However because in (m+1)-fold overload more queues are involved, the sojourn in this type of overload is shorter than in m-fold overload. $(mr \cdot \frac{1}{m\alpha} = (m+1)r \cdot \frac{1}{(m+1)\alpha})$ Both effects cancel each other, so that the net result is independent of the multiplicity of overload.

In ATM VC model, simultaneous overload periods in different queues are not independent. The increase of the sum of queue lengths is not proportional to the number of overloaded queues, because the cell arrival rate at downstream queues is throttled by overloaded upstream queues. This decreases the growth of the sum of queue lengths. Due to correlation between the instantaneous cell arrival rates at the queues, the sojourn time in

remains inside the tandem network is halved after this queue and after each subsequent queue. For example: at the first queue, 16 VCs enter the network; of these 16 VCs, $\frac{16}{24} = 1$ VC (i.e., the VC under study) passes through the fifth queue. Through each queue 16 VCs pass. In this example, the total number of VCs is 48. So, the joint probability distribution comprises 2^{48} states.

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a multiple overload state is not inversely proportional to the number of overloaded queues. It is longer than when assuming independent queues. This effect increases the growth of the sum of queue lengths during a multiple overload period. Both effects described work against each other, so that it may be expected that at least the order of the increase of the sum of queue lengths is independent of the number of simultaneously overloaded queues. The validity of this conclusion increases if fan out increases.

Two queues are sufficient to represent the VC model up to double overload

The second proposition is that two queues are sufficient to model single and double overload in the VC model. For this proposition to hold, it is required that the probabilities of single and double overload are small. This requirement will be fulfilled in a well designed and controlled ATM network that operates on the basis of statistical multiplexing in the burst level congestion region.

After a period of single or double overload in the tandem queuing network, the overloaded queues will likely empty (almost) completely before a next overload period starts in the same or other queue(s) of the tandem. For this to hold, it is required that the probability of single or double overload is small, so that the interval between overload periods is long. (Baiocchi's technique of modeling overload periods even further reduces the frequency of overload periods.) As a result, there is no need to distinguish between the overload periods of different queues in the performance evaluation method. It is not relevant that an overloaded queue through which a cell passes is the first or the last or any other queue in the tandem network. It is only relevant that it passes through an overloaded queue. Single overload periods of different queues in the tandem queuing network may be modeled by a single queue in the performance evaluation method; double overload periods of different pairs of queues in the tandem queuing network may be modeled by a single pair of queues in the performance evaluation method.

If the buffers have not emptied completely at the start of a next overload period, the performance evaluation method gives an upper bound. In the VC model, the not yet fully emptied queue continuous to empty, while the newly overloaded queue starts to fill. In the reduced system of two queues, emptying of a buffer stops as soon as a new overload period starts.

Outline of the performance evaluation method

The performance evaluation method comprises two steps. In the first step essentially the relevant overload behavior of the ATM VC model is addressed. In the second step, the overload behavior is expanded with the underload behavior.

The first step of the performance evaluation method is to represent single and double overload in the VC model by the overload behavior in two tandem queues. The two tandem queues model is the model described in 7.3. The parameters of the MMPP(4) traffic model are chosen such that matching of the overload behavior is achieved. This is discussed at length in the next section. In this procedure, the three QNP are accounted for as far as they relate to overload.

In the second step of the performance evaluation method, the end-to-end waiting time in the two tandem queues obtained in the first step is added to the waiting times in n-2independent M/D/1/K queues. The two tandem queues represent the overload behavior in the *n* tandem queues and the underload behavior in 2 of the *n* tandem queues. In order to account for the underload behavior in the remaining n-2 queues, we add the waiting times in n-2 M/D/1/K queues. We do not account for the QNP in underload, so we may assume that the queues are independent. As performance is determined by the overload behavior, neglecting QNP in underload is accurate. Further, the aggregate arrival process at a queue in underload is modeled by a Poisson process, like in the traffic aggregation method for a single queue. Note that, like in the first step, there is no one-to-one correspondence between the queues in the VC model and the n-2 M/D/1/K queues.

7.4.2 Parameters

The performance evaluation method reduces the n queues of the ATM VC model to two tandem queues, while maintaining single and double overload behavior. The two tandem queues model and its analysis were discussed in 7.3. In this subsection, we show how to choose the parameters of the MMPP(4) traffic model in the two tandem queues in such a way that the overload behavior in the n tandem queues is accurately captured. We consider the following aspects of the VC model: overload behavior, probabilities of cell arrival in a specific state, probability that double overload is preceded by a specific state, and mean rates on the streams through the two tandem queues.

Overload behavior

Like for a single queue and for two tandem queues, the asymptotic behavior during single and double overload in n tandem queues is represented. For single overload, the asymptotic behavior is equal in each of the n queues, so we choose this behavior also in the two tandem queues model. For double overload, asymptotic behavior of two overloaded queues depends on which two of the n queues are overloaded. We study double overload in more detail.

The pairs of queues in the ATM VC model differ with respect to the number of VCs that passes through both queues, i.e., the number of VCs on stream 1-2 (see Fig. 7.4). If queues are further apart, the number of VCs on stream 1-2 is lower, and the double overload behavior will be different.

As an example, we consider the asymptotic behavior of double overload in two tandem queues at different numbers of VCs on stream 1-2. Through each of the two queues m = 16VCs pass. The traffic process on each VC is an IPP, described by $\epsilon = 0.2, \gamma = 0.2$. (*T* has no influence in this example.) We reduce the asymptotic double overload behavior to two figures, namely the mean increase of the length of each queue during an asymptotic double overload period. Double overload is defined as in Sect. 7.3, i.e., two queues are in double overload if the instantaneous arrival rates on the streams through these queues indicate that the two queues are overloaded. Traffic throttling because of the overloaded

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Table 7.2: Asymptotic mean increase of the queue length during a double overload period. Traffic throttling by the first queue is taken into account. $m = 16, \epsilon = 0.2, \gamma = 0.2$

fan out	mean increase queue 1	mean increase queue 2	sum
2	200.4	183.5	383.9
4	189.0	179.5	368.5
16	177.6	172.1	349.7

first queue is taken into account. Tab. 7.2 shows numerical results as a function of fan out. The number of VCs on stream 1-2 is $\frac{16}{\text{fan out}}$.

If fan out increases, two relevant effects occur:

- The duration of double overload periods becomes shorter. This is because more sources are involved in double overload if fan out is larger.
- The number of VCs on stream 1-2 decreases, while the numbers of VCs on the streams 1 and 2 increase. As a result the effect of traffic throttling decreases, because traffic throttling applies to stream 1-2.

The first effect reduces the mean increase of the first queue; the second effect does not have influence on the first queue. Both effects work in opposite directions for the second queue. Both effects are confirmed by the results in the table. that the net

In the two tandem queues model, a single state is available to model the double overload behavior of all pairs of queues in the ATM VC model. We choose to represent in the two tandem queues model the asymptotic double overload behavior that corresponds to the lowest fan out in the n tandem queues network. There are two reasons for this choice:

- Double overload behavior is worse if fan out is lower. So, the behavior at low fan out provides an upper bound on behavior at higher fan out. This bound is not too bad.
- The probability that a pair of queues is doubly overloaded decreases if fan out increases. In the *n* tandem queues network, fan out between two queues increases, if the number of queues between the two queues under study increases. So, the most likely double overload behavior is the behavior at the smallest fan out.

Probability of cell arrival in a specific state

As the second set of parameters of the n tandem queues model, we consider the probabilities that a cell under study arrives at the n tandem queues while they are in a specific state. These states correspond to the states of the MMPP(4) traffic model: universal underload, single overload, and double overload. Arrival at the n tandem queues is negligible for the case that more than 2 queues are overloaded, because it has no discernible influence on the performance measure of interest. Exact evaluation of these probabilities in the n tandem queuing network requires consideration of a prohibitively large state space. Therefore, we provide an approximate procedure that is based on the observation that overload of more than two queues is improbable enough to be neglected. The procedure may be characterized as a decomposition procedure, because it considers all possible pairs of queues in the n tandem queues network. We consider the probability of cell arrival at double and at single overload, respectively.

First, we consider the probability of cell arrival at two overloaded queues:

Pr(A cell arrives at two overloaded queues)

- = $\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Pr(A \text{ cell arrives at overloaded queues i and j and further at underloaded queues})$
- $\leq \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Pr(A \text{ cell arrives at overloaded queues i and j})$
- $\leq \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Pr(A \text{ cell arrives at double overload in a two tandem queues system})$

modeled after the queues i and j)

We have found an upper bound on the probability that a cell arrives at two overloaded queues in the n tandem queues model. This upperbound is accurate. We started by applying the law of total probability. The first inequality is also based on the law of total probability. The upper bound is tight, because the probability of more than 2 overloaded queues is small relative to the probability of 2 overloaded queues. The second inequality is approximate. What it does is that it takes the queues i and j out of the network of n tandem queues and considers them in isolation. Together they form a two tandem queues network, like we previously analyzed. When you consider overload of the queues i and j in isolation, you neglect the possibility of traffic throttling caused by overload of any of the other queues. Hence the upperbound. Also this upperbound is tight, again because the probability of more than double overload is small relative to the probability of double overload.

Second, we consider the probability of cell arrival at one overloaded queue:

Pr(A cell arrives at one overloaded queue)

$$= \sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue i and further at underloaded queues})$$

$$= \sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue i}) - \sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue i and at at least one other overloaded queue})$$

$$\leq \sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue i}) - \sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue i}) - \sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue i})$$

 $\sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue i and at one other overloaded queue})$

 $= \sum_{i=1}^{n} \Pr(A \text{ cell arrives at overloaded queue } i) -$

2 Pr(A cell arrives at two overloaded queues)

 \leq Pr(A cell arrives at overloaded queue I) +

 $\sum_{i=2}^{n} \Pr(\stackrel{\text{Cell arrives at overloaded queue 2 in two tandem queues}}{\text{modeled after the queues i-1 and i}}) = 2 \Pr(\text{A cell arrives at two overloaded queues})$

We have found an approximation for the probability that a cell arrives at one overloaded queue in the n tandem queues model. This approximation is good. We started by applying the law of total probability twice. Then, we neglected the occurrence of more than double overload. Subsequently, we substituted the probability that a cell arrives at two overloaded queues. Note that we previously assessed this probability. Finally, we neglected traffic throttling in other than the immediately upstream queue. It may be expected that traffic throttling in the immediately upstream queue is far more important than traffic throttling in other upstream queues, because of dispersion of traffic due to routing. Note that this last step indicates how we incorporate correlation between VC traffic streams. If we would neglect this correlation, we would have approximated the probability of cell arrival at one overloaded queue by $n \Pr(A \text{ cell arrives at two overloaded queue 1}) - 2 \Pr(A \text{ cell arrives at two overloaded queues}).$

We have distributed the probability of cell arrival at a single overloaded queue equally among the first and the second queue in the two tandem queues model. The difference between single overload of the first and of the second queue is only relevant if single overload is succeeded by double overload. The probability of these events is determined by the probabilities discussed next.

Probability that double overload is preceded by single overload

The probability that a double overload period is preceded by single overload (i.e., (u, o) or (o, u)) are again determined by decomposing the VC model into pairs of queues. For each pair, the probabilities are determined. Then, they are weighted with the probability that a cell arrives at such a pair, given that it arrives at two overloaded queues.

Mean rates on the streams

The queues in the n tandem queues network are all equally highly loaded, but the distribution of the loads among the streams 1, 2, and 1-2 differs depending on the pair of queues. The mean rate on stream 1-2 is higher and, correspondingly, the mean rates on stream 1 and stream 2 are lower if there are less queues between the two queues under consideration.

In the traffic aggregation method, the mean rates on the streams are used to establish the arrival rates on the streams in the state (u, u). Further, the mean rate on stream 1-2

is also used to establish the state probabilities in the MMPP(4). The state (u, u) has no important effect on the performance measures of interest. Nor have the state probabilities of the MMPP(4). (The probabilities of cell arrival in a specific state are very important, but they have been dealt with previously.) So, the choice of the mean rates on the streams is not very relevant.

The mean rates on the streams are set to values that correspond to a pair of adjacent queues. This has the advantage that the probability of sojourn in state (u, u) of the MMPP(4) model is relatively small. This probability tends to be very high, which impedes fast numerical solution of the two tandem queues model.

7.4.3 Results

In the previous subsections, the performance evaluation method was described completely. We are now in the position to actually use the method to determine the end-to-end waiting time distribution in a tandem queuing network. In this subsection, we compare numerical results obtained by the performance evaluation method with simulation results. The purpose of this comparison is to show the accuracy of the performance evaluation method.

We consider a tandem network comprising three tandem queues: n = 3. The traffic stream on each VC is, at its entry into the network, an IPP described by the following parameters (see also 7.2): $T = 500, \gamma = 0.1, \epsilon = 0.2$. Buffer sizes of all queues equal 150 cells. The number of VCs that passes through each queue is 32: N = 32. Fan out after each queue is $2.^{11}$

A network of 3 tandem queues is the smallest network we can use to validate our method. The reason for restricting us to 3 queues is that it is difficult to obtain accurate simulation results for larger networks. When the number of queues in the tandem is increased, the time required for the simulation increases for two reasons:

- The added queues have to be simulated as well.
- The rate of the cell stream that passes through all queues in the tandem decreases because of fan out. To maintain the number of events in the simulation (i.e., the number of cells that passes through all queues in the tandem), the duration of the simulation has to be increased by a factor that corresponds to the fan out.

So the simulation time grows exponentially as a function of the number of queues, if (as usual) fan out thins the rate of the traffic stream that passes through all queues. The cell loss probability in the simulation is higher than in an ATM network. This is also to keep simulation time manageable.

Fig. 7.8 shows the end-to-end waiting time survivor function obtained by the performance evaluation method and by simulation. The solid lines represent simulation results

¹¹The distribution of VC streams in the network is given by the following parameters: N = 32, fan out = 2, and n = 3. The complete enumeration of VC streams is as follows: 8 VCs pass through queue 1, 2, and 3; 8 VCs pass through queue 1 and 2; 16 VCs pass through queue 1; 8 VCs pass through queue 2 and 3; 8 VCs pass through 2; 16 VCs pass through queue 3



Figure 7.8: Waiting time distribution for 3 tandem queues. Simulation results with 95 % confidence interval (solid lines: no decomposition; dashed lines: decomposition). Method results (point-dash line). Parameters: $N = 32, T = 500, \gamma = 0.1, \epsilon = 0.2, B = 150$, fan out = 2. Server loads: 0.64

and 95 % confidence intervals. The dashed lines also represent simulation results and 95 % confidence intervals, however, with the additional assumption that waiting times in different queues are independent. The positive correlation between cell waiting times shows clearly for waiting times exceeding one buffer size. See Sect. 5.1 for a discussion of this queuing network phenomenon. The point-dash result was obtained by the performance evaluation method.

The method is shown to be accurate in the area that is relevant for our purpose, i.e., for waiting times around the buffer size of a single queue (150 cells).¹² For waiting times little lower than twice the buffer size in a single queue, the method waiting time survivor function sharply declines relative to the simulation results. This is a consequence of the reduction of the tandem queuing network from n to 2 queues by the performance evaluation method.

 $^{^{12}}$ We are not interested in cell waiting times that are much less probable than cell loss. As previously observed, the cell loss probability is roughly equal to the probability that a cell waits for the maximum time in one of the queues.

7.5 Conclusions

We have presented a new ATM VC performance evaluation method. The multiplexers that form the VC operate on the basis of statistical multiplexing in the burst level congestion region. VC traffic is bursty and is described by an interrupted Poisson process (IPP). The performance evaluation method takes into account the QNP (i.e. *waiting time correlation*, *traffic characteristics change*, and VC traffic stream correlation) as far as they are due to overload of a multiplexer on the VC under study. So, the QNP are taken into account when they have appreciable influence on performance.

The method is based on two propositions. For these propositions to hold, it is required that the probability of overload of a multiplexer is low and that there is dispersion of traffic in the network (i.e. fan out from one multiplexer to the next may not be very small, say, not smaller than 2). In an ATM network operated on the basis of statistical multiplexing in the burst level congestion region, these conditions are virtually always fulfilled. The first proposition is that simultaneous overload of more than two of the nqueues in the VC model is not relevant to performance. The second proposition is that single and double overload in the n tandem queues of the VC model may be represented by a single pair of queues, even though in the n tandem queues network different queues become overloaded. The method reduces the VC model of n tandem queues to essentially two queues in tandem. The aggregate cell arrival streams at these two queues are described a four-state Markov-modulated Poisson process (MMPP(4)). The states of the modulating Markov chain correspond to the overload state of the two queues.

The performance evaluation method represents the QNP waiting time correlation by analyzing two queues at once.

The method was shown to accurately assess the end-to-end waiting time survivor function for waiting times that are relevant. When correlation between cell waiting times is relevant, it is much more accurate than assuming independent cell waiting times.

Chapter 8

Application of VC waiting time evaluation methods

In chapter 1, the need for ATM VC performance evaluation methods was identified. VC performance evaluation, was said, is required in network design, traffic control, and design of end-to-end protocols and of end-terminal functions. In the chapters 2 to 7, two VC waiting time evaluation methods have been developed, one for smooth VC traffic and one for bursty VC traffic. In this chapter, the methods will be applied to three design problems. The purpose of this chapter is first to show that the methods are applicable to ATM design problems and second to show that the methods are more accurate than conventional methods when the Queuing Network Phenomena (see Ch. 5) play a role.

The waiting time evaluation methods apply to VCs, virtual connections between two end-terminals. They apply however equally well to a part of a VC, and the design problems in this chapter show several examples. The design problems that we study are¹:

• Usage Parameter Control:

Checking that the traffic stream on a VC complies with the characteristics agreed upon between user and network at connection set-up. The traffic characteristic that we focus on is the peak cell rate.

• Smoothing Buffer:

Restoring the interval between the cells in a VC traffic stream. Sometimes it is required that the network does not disturb the lengths of the intervals between the cells of a VC. At the receiving side, the intervals are restored by a smoothing buffer.

• Switch Design:

Designing a multi-stage switch. In switch design, there is a limited budget for the end-to-end delay through the switch. The methods accurately assess this delay, so that the delay budget can be used entirely.

¹The problems apply to both smooth and bursty VC traffic streams.

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Before addressing the three design problems, we first present numerical results obtained by the two methods. These results will be used to illustrate the application of the methods to the design problems.

8.1 Results

We use the two methods to determine the end-to-end cell waiting time distribution in a network of queues in tandem in case of, respectively, smooth and bursty VC traffic streams. In the next sections, the results of this section will be applied to the three design problems.

8.1.1 Smooth VC traffic

The results in this subsection concern the end-to-end cell waiting time distribution in 20 tandem queues for the case of smooth VC traffic streams. The waiting time distribution is obtained by the method presented in Ch. 6.

The VC model consists of 20 queues in tandem. The buffer size of each queue is 20 cells, and the maximum cell waiting time in a queue is 19 cell transmission intervals. At the entrance into the network, the traffic stream on the VC under study is periodic: the interval between two consecutive cells is 15 slots long. Interfering VCs only cross the VC under study: they leave the tandem queuing network immediately after the queue at which they entered the network. The aggregate load of the interfering VCs at a queue is 0.63, so the total load of a queue is $0.63 + \frac{1}{15} = 0.7$, and fan out is $0.7/\frac{1}{15} = 10.5$. For a description of the interfering traffic, see Ch. 6.

Fig. 8.1 shows the end-to-end cell waiting time distribution in the VC model after 5 queues and after 20 queues. The solid curves are the result of the conditional decomposition method (see Ch. 6)². Where appropriate, this method takes into account the QNP and especially correlation between the waiting times of a cell in the queues of the VC model. The dashed curves are obtained by applying decomposition instead of conditional decomposition to the VC model. The dashed curves differ from the solid curves only because the QNP waiting time correlation is neglected.

As expected on the basis of the conclusions of Ch. 5 and 6, comparison between the solid and dashed curves in Fig. 8.1 shows that in this example correlation between the waiting times of a cell is negative: end-to-end cell waiting times tend to be smaller when correlation between cell waiting times is taken into account (or, equivalently, the dashed curves are above the solid curves). The effect is larger after 20 queues than after 5 queues. Other VC performance evaluation methods do not take this effect into account.

The cell loss probability in the tandem queuing network is $3.3 \cdot 10^{-7}$ after 5 queues and $1.4 \cdot 10^{-6}$ after 20 queues.

²In the present instance of the conditional decomposition method, the waiting time of the cell under study in a queue is conditioned on the lengths of the 2 preceding cell interarrival intervals on the VC under study. Also the joint probability distribution of adjacent cell interarrival intervals on the VC under study is incorporated.



Figure 8.1: End-to-end cell waiting time distribution in the VC model for smooth traffic. Deterministic traffic on the VC under study (period: 15). Crossing interference. Load of each queues: 0.7. Solid curves: dependent cell waiting times; dashed curves: independent cell waiting times. Upper set of curves: 20 tandem queues; lower set of curves: 5 tandem queues.

8.1.2 Bursty VC traffic

The results in this subsection concern the end-to-end cell waiting time distribution in 5 tandem queues for the case of bursty VC traffic streams. The waiting time distribution is obtained by the method presented in Ch. 7.

The VC model consists of 5 queues in tandem. The buffer size of each queue is 150 cells, so the maximum cell waiting time in a queue is 149 cell transmission intervals. Each VC traffic stream is an interrupted Poisson process (IPP) with the parameters (see Ch. 4): $T = 500, \gamma = 0.1, \epsilon = 0.2$. m = 20 VC traffic streams pass through each of the queues, so the total load of each queue is $\gamma \cdot \epsilon \cdot m = 0.4$. The case of partly joining interference is considered: interfering VCs may pass through more than one queue of the VC model.

To describe the organization of interfering VC traffic streams, we number the queues in the VC model consecutively 1 through 5. If $|i - j| \ge 3$, the queues *i* and *j* share 1 VC (namely, the VC under study) (fan out $\frac{20}{1} = 20$). If |i - j| = 2 they share 2 VCs (fan out $\frac{20}{2} = 10$), and if |i - j| = 1 they share 5 VCs (fan out $\frac{20}{5} = 4$).

Fig. 8.2 shows the end-to-end cell waiting time distribution in the VC model. The solid curve is the result of the waiting time evaluation method for bursty traffic VC streams (see Ch. 7). This method takes into account the queuing network phenomena (QNP), especially correlation between the waiting times of a cell in the queues of the VC model. The dashed curve is obtained by neglecting all QNP: the cell waiting time distribution in the first queue of the VC model (i.e., in an m-IPP/D/1/K queue) is determined, and subsequently the waiting times in the queues 1 through 5 are assumed to be independent and identically distributed.

As expected on the basis of the conclusions of Ch. 5 and 7, comparison between the solid and dashed curves in Fig. 8.2 shows that correlation between the waiting times of a cell is positive. The effect is clearly visible in the double overload part of the cell waiting time distribution (say for waiting times exceeding 150).

The cell loss probability in the tandem queuing network is $5.3 \cdot 10^{-7}$.

8.2 Usage Parameter Control and Smoothing Buffer

The purpose of Usage Parameter Control (UPC) is: verification and enforcement of the traffic characteristics on a VC. The traffic characteristics on a VC are mutually agreed upon between subscriber and network provider at the establishment of a VC. For the network provider, they are the basis for Connection Admission Control (i.e., the decision if the VC is admitted to the network) and for charging the subscriber. So the subscriber has an interest to offer more cells to the network than the traffic characteristics. The problem that the provider should verify and enforce the traffic characteristics. The problem that the provider encounters is that the traffic characteristics on a VC may have changed between terminal (where the subscriber measures them) and the entry into the network on the user premises.



Figure 8.2: Cell waiting time distribution in the VC model (5 tandem queues). IPP VC traffic streams: $T = 500, \gamma = 0.1, \epsilon = 0.2$. Partly joining interference. m = 20. Load of the queues: 0.4. Solid curves: dependent waiting times; dashed curves: independent waiting times.

We focus on a simple traffic characteristic, namely, the peak cell rate, and on a specific UPC algorithm, namely, the Leaky Bucket algorithm.

The purpose of a Smoothing Buffer is: removal of the variable delay that the cells on a VC have encountered while they were passing through the network. A Smoothing Buffer can fulfill this task if (in case of smooth traffic) the traffic stream on a VC is initially periodic or if (in case of on-off traffic) the traffic stream on a VC is periodic during onperiods. The Smoothing Buffer may be part of the network, so that the network provider can offer a VC of which the cells all take equally long to pass through the network. (This is a so called circuit emulation service, in which circuit switching - that has constant delay - is emulated.) The Smoothing Buffer may on the other hand also be part of the receiving terminal. For example for telephony, it is important that the cell stream that enters the speech decoder (in the receiving terminal) is equal to the cell stream that left the speech encoder (in the sending terminal). If this condition is not fulfilled by a Smoothing Buffer), the continuity of speech is disturbed.

We will address each of the design problems UPC and Smoothing Buffer in a separate subsection. Both problems are however based on the same queue, and we will first address that queue. The queue consist of a buffer that can contain K cells. The traffic stream on the VC under study is fed into this buffer, and periodically (with period T) one cell is removed from the buffer. The queuing discipline is first-in first-out.

For UPC by means of the Leaky Bucket algorithm, T corresponds to the enforced peak

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cell rate on the VC under study (i.e., the peak cell rate is frac1T cells per slot). A cell is assumed not to comply with the designated peak rate (frac1T), if the buffer is completely full when it arrives at the queue. The issue is the choice of K:

- If K is too large, the peak rate on the VC may (temporarily) exceed frac1T.
- If K is too small, cells may unjustly be considered non-conforming.

We will go into more detail later.

For the Smoothing Buffer, T is the interval between cells on the VC under study (or, in case of on-off traffic, T is the interval between cells during on-periods). The issue is the choice of K and of R. R is the artificial delay between arrival of the first cell in the buffer and reading of the first cell from the buffer. After the first cell, cells are read out of the buffer periodically (period: T). R ensures that the buffer is filled to some extent at the moment that the playing out of cells starts.

- If K is too small, the buffer may overflow and cells may get lost.
- Large K means that the storage space of the Smoothing Buffer is large, which is more expensive.
- If R is too small, the buffer may underflow³, so that dummy cells have to be inserted into the cell stream on the VC.
- Large R means that all cells on the VC are artificially delayed by R (slots).

We will go into more detail later.

8.2.1 Inequalities for buffer overflow and underflow

We follow the development in [Gravey and Blaabjerg, 1994]. A periodic cell stream (period: T) is offered to the ATM network. In the network, the cells encounter stochastic delays $\{W_n, n \ge 0\}$, where n counts the cells. After the network, the cell stream arrives at a queue. The arrival epochs are denoted by $\{a_n, n \ge 0\}$. The traffic stream, the network, and the queue are all synchronized. The relation between a_n and W_n is: $a_n = a_0 - W_0 + n \cdot T + W_n$.

The queue has a buffer of size K. Cells are read out of the buffer every T slots. The first cell however is read out R slots after its arrival. $\{d_n, n \ge 0\}$ denotes the cell departure epoch from the buffer. Assuming that the buffer has not overflown or underflown, the following holds: $d_n = d_0 + n \cdot T = a_0 + R + n \cdot T$.

The buffer does not underflow due to cell n if it is not empty at d_n (see [Gravey and Blaabjerg, 1994]):

$$a_n \leq d_n$$

$$a_0 - W_0 + n \cdot T + W_n \leq a_0 + R + n \cdot T$$

$$W_n \leq R + W_0$$
(8.1)

³'Underflow' means that the buffer is empty at the moment that a cell should be read out of the buffer.

The buffer does not overflow due to cell n if it is not full at d_n (see [Gravey and Blaabjerg, 1994]):

$$a_n \geq d_{n-K}$$

$$a_0 - W_0 + n \cdot T + W_n \geq a_0 + R + (n-K) \cdot T$$

$$W_n + K \cdot T \geq R + W_0$$
(8.2)

R and K can be chosen on the basis of requirements for the probabilities of overflow and underflow. For UPC, underflow is not an issue, so that R = 0 suffices. For the Smoothing Buffer, R can be set on the basis of (8.1) by requiring that the probability of buffer underflow is sufficiently small:

$$\Pr(W_n \le R + W_0) > 1 - \eta_R \Pr(W_n - W_0 > R) < \eta_R,$$
(8.3)

where W_n and W_0 may be considered as independent random variables if n is sufficiently large. W_0 is the waiting time distribution of the first cell generated on the VC under study. The waiting time of this cell is on average lower than of ensuing cells, because the first cell of a VC does not meet congestion that is due to the VC itself. A worst case is obtained by assuming that the distributions of W_0 and W_n are equal.

K can be set on the basis of (8.2) and the previously calculated value of R:

$$\Pr(W_n + K \cdot T \ge R + W_0) > 1 - \eta_K$$

$$\Pr(W_n - W_0 < R - K \cdot T) < \eta_K$$

$$\Pr(W_0 - W_n > K \cdot T - R) < \eta_K.$$
(8.4)

If R and K are chosen as low as possible and $\eta_K = \eta_R$, it follows from (8.3) and (8.4) that $K = \lceil 2\frac{R}{T} \rceil$.

8.2.2 Usage Parameter Control

The design problem that we consider is UPC of the peak cell rate on a VC by means of the Leaky Bucket algorithm. The Leaky Bucket algorithm is based on the queue that we have described. The main problem in verifying the peak cell rate is that the length of the interval between cells is disturbed due to queuing in the user premises network. The Leaky Bucket algorithm solves this problem by allowing smaller intervals between cells up to the point that the buffer overflows.

Inequality 8.4 (where R should be set to 0), describes the probability that a cell is designated as non-conforming to the cell peak rate, even though the traffic source strictly adheres to the peak rate. This event may occur only rarely (i.e., η_K is a very small probability). We will use the methods developed in this thesis to determine the distribution of $W_0 - W_n$. On the basis of the required value of η_K , the appropriate value of $K \cdot T$ can then be established.

It is important that K is set to the right value, and the accurate methods developed in this thesis are important tools in this respect:

- If K is too small, too many conforming cells are unjustly marked as non-conforming. The performance of UPC is a part of the Quality of Service (QOS) offered by the network.⁴ If the probability of unjust marking is too large, the QOS is degraded.
- If K is too large, a malicious subscriber can offer more cells than agreed at connection set-up. A network operator needs to guarantee the QOS. Such a guarantee is only possible, if a worst case scenario is assumed with respect to the traffic streams on VCs: the operator should assume that traffic streams behave as badly as they possibly can, and the traffic streams are only restricted by UPC.

At given mean rate, the worst type of traffic stream is an on-off traffic stream in which cells are sent back-to-back in the on-period. If the on-period is longer, the traffic stream is worse and the efficiency of the network is lower. If a traffic source offers such a traffic stream to a Leaky Bucket, the buffer size K directly determines the length of the on-period (and thus how bad the traffic stream is).

This means that the value of K plays a crucial role in the efficiency of the network and that relatively small changes in this value can have considerable effect.

The numerical results presented in 8.1 allow calculation of the distribution of $(W_0 - W_n)$ in (8.4). As an example, we set the value of η_K at 10^{-7} and 10^{-8} for smooth and bursty traffic respectively. These values are well below the respective cell loss probabilities. The quantiles of $(W_0 - W_n)$ are shown in Tab. 8.1⁵.

Table 8.1: Quantiles of $(W_0 - W_n)$ in the VC model. indep. is independent waiting times of a cell in different queues. dep. is dependent waiting times of a cell in different queues.

smooth; 10^{-7} quantile	indep.	dep.
5 queues	29	28
20 queues	46	43
bursty; 10 ⁻⁸ quantile	indep.	dep.
5 queues	168	180

For the case of smooth traffic, conventional methods overestimate the required value of $K \cdot T$: 29 in stead of 28 after 5 queues and 46 in stead of 43 after 20 queues. In the examples we used, T was 15 slots. Conventional methods, obtain the correct value of K after 5 queues (namely, $\left\lceil \frac{29}{15} \right\rceil = \left\lceil \frac{28}{15} \right\rceil = 2$), but a too high value of K after 20 queues: $\left\lceil \frac{46}{15} \right\rceil = 3$ in stead of $\left\lceil \frac{43}{15} \right\rceil = 2$.

The methods presented in this thesis allow a more efficient use of the network by providing more accurate and lower values for the buffer size in the Leaky Bucket algorithm.

⁴So, the probability of unjust marking should be of the order of the cell loss probability (if nonconforming cells are deleted).

⁵The α -quantile of a random variable X is the smallest value x such that $\Pr(X \ge x) \le \alpha$.

In the example given, the buffer size (and as a consequence the burst length of the worst case traffic) is reduced from 3 to 2 cells.

For the case of bursty traffic, conventional methods underestimate the required value of $K \cdot T$: 168 in stead of 180. At $K \cdot T = 168$, conventional methods, underestimate the probability of unjustified cell marking: 10^{-8} in stead of $2.1 \cdot 10^{-8}$.

The methods in this thesis ensure that the Quality of Service is maintained. In the example given, conventional methods allow that the probability of unjustified cell marking is increased by a factor 2.

8.2.3 Smoothing Buffer

We consider the design problem of compensating for variable delays in the network by means of a Smoothing Buffer. The delay of a cell in the smoothing buffer should exactly complement the delay of that cell in the network, so that a fixed delay results for all the cells of a VC. The smoothing buffer is described by the queue that we previously introduced.

Inequality 8.3 describes the probability that the smoothing buffer is empty at a moment that a cell should have been played out (i.e., underflow). Inequality 8.4 describes the probability that the smoothing buffer overflows. The probabilities concern the distribution of $W_0 - W_n (W_n - W_0)$, and we will use the methods developed in this thesis to determine this distribution. On the basis of the required values of η_K and η_R , the appropriate values of $K \cdot T$ and R can then be established.

The values of K and R should be chosen carefully, because they determine when the mechanism of the smoothing buffer fails:

- Cell delay in the network may become so large that the buffer is not filled in time: at the moment that a next cell should have been played out, this cell has not arrived yet (see 8.3). Instead, a dummy cell is played out, which will have an effect on the quality of the service similar to the loss of a cell. Larger R reduces this effect, at the expense of additional delay.
- Cell delay in the network may become so small that many cells arrive at the buffer shortly after each other. In this way, the buffer may overflow: an arriving cell cannot be accommodated in the buffer and is lost (see 8.4). Larger K reduces this effect, at the expense of additional buffer size.

For the probability distribution of the waiting time, we again use the numerical results of section 8.1 that were repeated in Tab. 8.1. We set $\eta_K = \eta_R$, so that we rate buffer overflow equally heavy as buffer underflow. As a result $K = \left[2\frac{\pi}{R}\right]$, as observed previously.

For the case of smooth traffic, conventional methods overestimate the value of R: 29 and in stead of 28 after 5 queues and 46 in stead of 43 after 20 queues.

The methods developed in this thesis provide a lower value of R. The consequences of a slightly overestimated value of R are however small: a slightly increased cell waiting time in the smoothing buffer.

For the case of bursty traffic, conventional methods underestimate the required value of R and K: 168 and 34 in stead of 180 and 36. At R = 168 and K = 34, conventional methods underestimate the probabilities of buffer underflow and overflow: 10^{-8} in stead of $2.1 \cdot 10^{-8}$.

The methods developed in this thesis ensure the QOS. In the example given, conventional methods make that the probabilities of buffer underflow and buffer overflow are both increased by a factor 2.

8.3 Switch design

In 3.1, we reviewed ATM switch architecture. A well-known switch architecture is a multistage network with internal buffering. In such a switch architecture, the switch input ports are connected to the output ports by a network of multiplexers. The multiplexers are grouped into stages. A cell passes from a multiplexer in one stage to a multiplexer in the next stage, until it has passed through all stages. The multiplexers of a stage are connected to the multiplexers of the next stage by a network that has some regular pattern. The route through the switch of a VC forms a tandem network of multiplexers.

A multi-stage switch is typically built from identical building blocks, where a building block comprises a fixed number of multiplexers out of the same stage.

The design of a multi-stage switch comprises three steps:

- 1. Establish the switch architecture: the number of stages, the number of multiplexers in each stage, and the network that interconnects the multiplexers in the stages.
- 2. Determine the routes of cells through the network. Depending on the network, there may be only a single route from a switch input port to a switch output port. In that case, this step is trivial.
- 3. Choose the cell transmission rate inside in the switch and the buffer size of the multiplexers.

After the design has been completed, the end-to-end performance in the switch should be determined and compared with the requirements. The end-to-end performance of a switch is the cell loss probability and the cell waiting time distribution as a function of the traffic applied to the switch. If the end-to-end performance is not satisfactory, the design should be adjusted. If the end-to-end performance is satisfactory, the design may be optimized.

After the switch itself has been designed, rules for Connection Admission Control have to be established. This is closely related to switch design.

If all cells of a single VC follow the same route through the switch, this route forms a tandem network of queues. If in addition the cell transmission rate inside the switch equals the cell transmission rate outside the switch⁶, the end-to-end waiting time distribution in the tandem network can be determined by the two methods provided in Ch. 6 and 7.1).

⁶If the transmission rate inside the switch differs from the transmission rate outside the switch, there are two different service times in the queues of the tandem network. The service time in the last queue

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The end-to-end cell waiting time in a switch should be small. The most important measure of the cell waiting time is the waiting time that is exceeded with a certain probability, i.e. the quantile of the waiting time. The probability is comparable with the cell loss probability. If a cell waits longer than the quantile of the waiting time this is as bad as loss of a cell. Typically the quantile of the waiting time in a switch should be of the order of 100 μ s (see [de Prycker, 1993]). At a transmission rate of 150 Mbit/s and a cell size of 53 \cdot 8 bits, 100 μ s is equivalent to approximately 35 cell transmission slots. At 600 Mbit/s, it is equivalent to approximately 142 slots.

Table 8.2 shows the quantiles of the end-to-end cell waiting times in the models of Sect. 8.1. Consider the case that a switch achieves the required waiting time performance

Table 8.2: Quantiles of W_n in the VC model. indep. is independent waiting times of a cell in different queues. dep. is dependent waiting times of a cell in different queues.

smooth; 10 ⁻⁷ quantile	indep.	dep.
5 queues	31	30
20 queues	54	51
bursty; 10 ⁻⁸ quantile	indep.	dep.
5 queues	170	182

according to the conventional methods, that assume independent waiting times.

 For smooth traffic, this means that a 10⁻⁷-quantile of 31 (54) slots is acceptable. The method developed in this thesis however shows that the actual end-to-end waiting time is lower: 30 in stead of 31 slots (51 in stead of 54). This means that the switch load can be increased.

The new method allows the design of a more efficient switch.

• For bursty traffic, this means that a 10^{-8} -quantile of 170 slots is acceptable. The new method shows however that the actual 10^{-8} -quantile is higher, namely 182 slots. So that the conventional methods accept a switch design that in reality exceeds the waiting time budget by 12 slots or 7%.

The new method avoids that the delay budget is exceeded.

The performance requirements on a switch are tight. To design an efficient switch that meets the performance requirements, it is important that delay performance can be examined carefully as a function of traffic load. The two waiting time evaluation methods serve this purpose.

of the tandem represents transmission of cells between switches; the service times in the other queues of the tandem represent transmission of cells inside the switch. In a multi-stage switch without internal buffering, the internal cell transmission rate is often higher than the external rate, in order to reduce the probability that a cell is blocked inside the switch. In a multi-stage switch with internal buffering, internal cell blocking is alleviated by buffering, so it is not required to increase the internal cell transmission rate.

8.4 Conclusions

The methods developed in this thesis have been shown to be applicable to rather realistic design problems. The results obtained show that the methods make possible the design of more accurate and more efficient ATM systems.

In Usage Parameter Control (UPC) design, the methods allow tighter control over the traffic characteristics on a VC. This may in turn lead to a more efficient use of network resources. Further, they ensure that UPC respects the Quality of Service and does not unjustly mark cells that are compliant with the agreed traffic characteristics.

In Smoothing Buffer design, the methods allow shorter delay in the smoothing buffer and guard that the Smoothing Buffer maintains the Quality of Service.

In Switch design, the methods allow more efficient switches and switches that meet their delay budget.

The methods have been developed to incorporate the influence of the three Queuing Network Phenomena (QNP) on the end-to-end cell waiting time distribution. This means that, in comparison with conventional methods, the methods provide significantly more accurate results only when the QNP have a noticeable effect on the end-to-end cell waiting time. (Otherwise they are neither more accurate nor less accurate.)

Already in chapter 5, we observed that fan out (i.e., the degree to which the output stream of a queue is distributed over different destinations) is decisive for the relevance of all three QNP. Fan out should by the way be interpreted in terms of the loads of the actually realized traffic streams and not in terms of the rates of the transmission links.

The numerical results in this chapter clearly show that the methods of this thesis perform better than conventional methods. The improvement is rather large in some cases and small in other cases. This confirms that the methods should predominantly be applied to queuing network models in which fan out from one queue to the next is small.

Chapter 9

Conclusions

The subject of this thesis is performance analysis of a Virtual Connection (VC) in an ATM network. VC performance is measured in terms of cell loss and delay (of which delay by far receives most attention in this thesis). VC performance is part of the contract between the provider and the user of a VC. The provider needs performance estimates in the design and control of the network. The user needs performance estimates in the design and control of his private network and terminal equipment.

The problem addressed in this thesis is that performance analysis of ATM VCs is not well understood and that existing performance evaluation methods are not accurate in specific circumstances. An ATM network can be modeled as a network of multiplexers through which VC traffic streams pass. VC traffic models and multiplexer models are well understood in the literature. ATM VC performance evaluation itself has however received little attention. In the literature it is almost always assumed that the multiplexers that make up the ATM network model are independent. The performance evaluation methods developed in this thesis are not based on this assumption.

The approach taken in this thesis towards ATM VC performance analysis is stochastic modeling. The behavior of an ATM VC is represented in a stochastic model (namely, a Markov chain), and this model is subsequently analyzed with respect to the waiting time of a cell on the VC under study. The distribution of the end-to-end cell waiting time allows to give a statistical guarantee on the performance on a VC.

This chapter first lists the claims that follow from the work in this thesis. Then, it discusses the limitations of the work. Finally, it describes related research issues that have not been considered.

9.1 Claims

This thesis presents two new methods to determine the end-to-end cell waiting time distribution on an ATM VC. One method applies to smooth VC traffic ; the other method to bursty VC traffic. The methods are more accurate than existing methods, because they take the three Queuing Network Phenomena into account. For our purposes an ATM network may be modeled as a network of queues, where each queue represents a multiplexer. We assume that the queues are identical and synchronized. The VC under study passes through a given subset of the queues. This subset forms a tandem network of queues. This tandem network forms the VC model, together with a description of the traffic streams on the VCs that pass through it. In the thesis, the VC model is analyzed step by step:

• The chapters 2, 3 and 4 provide a survey of the literature on respectively traffic modeling, multiplexer modeling and VC performance evaluation. These chapters provide the basis for the ensuing chapters. Their contribution to the development of the field is that they provide a framework to categorize and assess the work by others on the subject. Especially, VC performance evaluation in the context of ATM has received little attention in the literature.

The conclusion from chapters 2 and 3 is that traffic models and multiplexer models are well developed. Two multiplexer models should be distinguished: deterministic multiplexing of smooth traffic and statistical multiplexing of bursty traffic¹. In case of deterministic multiplexing, the smooth VC traffic streams that arrive at the multiplexer may collectively be modeled by a Poisson process. In case of statistical multiplexing, the bursty VC traffic streams that arrive at the multiplexer may collectively be modeled by a two-state Markov-modulated Poisson process.

The conclusion from chapter 4 is that existing VC performance evaluation methods essentially neglect the queuing network phenomena (QNP) and that the influence of the QNP on VC performance should be studied. Three QNP have been distinguished:

- 1. QNP *waiting time correlation*: correlation between the waiting times of a single cell in different queues of the network.
- 2. QNP traffic characteristics change: change of the characteristics of a VC traffic stream due to queuing.
- 3. QNP traffic stream correlation: correlation between VC traffic streams that have been multiplexed on a single transmission link.
- Chapter 5 describes and assesses the three QNP for both smooth traffic and bursty traffic (i.e., deterministic multiplexing and statistical multiplexing, respectively). The description and analysis of the QNP in the context of ATM is original (except for the QNP *traffic characteristics change* at bursty traffic). It provides the motivation for VC performance evaluation methods that account for the QNP.

The main conclusion of this chapter is that the effect of the QNP on VC performance is relevant in certain cases. In general, the QNP *waiting time correlation* and the QNP *traffic stream correlation* are more relevant if fan out is smaller (i.e. if the

 $^{^{1}}$ In deterministic multiplexing, the instantaneous cell arrival rate at a multiplexer never exceeds the cell transmission rate from that multiplexer. In statistical multiplexing, the instantaneous cell arrival rate occasionally exceeds the cell transmission rate.

9.2. LIMITATIONS

output stream of a multiplexer is divided over a smaller number of downstream multiplexers); the QNP *traffic characteristics change* is more relevant if the rate of the traffic stream is higher. For both smooth and bursty traffic, the most important QNP is *waiting time correlation*. It should be accounted for by VC performance evaluation methods.

• Each of the chapters 6 and 7.1 describes a new method to assess the end-to-end cell waiting time distribution on a VC. The method in chapter 6 applies to smooth VC traffic (i.e., deterministic multiplexing); the method in chapter 7.1 to bursty VC traffic (i.e., statistical multiplexing). The methods approximately solve a stochastic model of a VC. The methods are an original contribution. They are more accurate than existing numerical methods, because they take all relevant QNP into account.

The method for smooth VC traffic (Ch. 6) is an enhancement of the traditional decomposition method, that assumes independence of the waiting times of a cell in the queues of the VC model. Correlation between cell waiting times is caused by correlation between the arrival processes at the corresponding queues. The enhanced method takes correlation between arrival processes into account by conditioning the cell waiting time in a queue on the arrival process at that queue.

The method for bursty traffic (Ch. 7.1) is based on the observation that the relevant part of the end-to-end waiting time distribution on a VC is determined by the occurrence of simultaneous overload of 1 or 2 queues in the VC model. The method exploits this observation and reduces the VC model to a network of 2 queues in tandem while maintaining the relevant overload behavior of the VC model. The method accounts for the QNP waiting time correlation and traffic stream correlation during overload of a queue.

• Chapter 8 shows the application of the methods developed in the two preceding chapters to the design of ATM networks.

9.2 Limitations

The two methods are useful tools in the design of ATM networks. Their development has however required limitations in three areas: VC model, performance measures and computation time.

9.2.1 Limitations regarding the VC model

Smooth VC traffic

• The method for smooth VC traffic applies to a VC model of identical queues. The method can easily be adapted to a VC model of non-identical queues, after the synchronization of these queues has been given some thought.

• In the VC model, interfering traffic² is collectively modeled by a Poisson process, and the routes of interfering cells are supposedly independent and identically distributed.

The Poisson model is generally accepted as an accurate model for aggregate traffic stremas that are composed of many small contributions. Only if a high-rate non-Poisson traffic source contributes to the interfering traffic, the Poisson model may be less accurate.

Other processes may be chosen for both the cell arrival process and the cell routing process, but a useful choice would require new research. The method can model cell routing only by a stochastic process. It is not possible that the route of a cell is determined at the moment that it arrives at a queue. The route of a cell after a queue is determined at departure from that queue by a stochastic process.

Bursty VC traffic

- The method for bursty VC traffic applies to a VC model of identical queues and cannot account for the occurrence of different cell transmission rates in an ATM network. The method represents the n queues in the VC model by only 2 queues, so it is essential that the queues are identical.
- The method applies to a VC model in which all VC traffic streams (on the VC under study and on interfering VCs) are identical and the loads of all queues are equally high. The method cannot account for different VC traffic streams. Again, this is a result of the method that reduces the number of queues in the VC model.

In the VC model, VC traffic streams are modeled as interrupted Poisson processes (IPP). The choice of the IPP type of traffic model is essential to the traffic aggregation technique.

• The multiplexers in the VC model should be operated as statistical multiplexers in the burst level congestion region. The probability that a multiplexer is overloaded should be small, for the reduction of the number of queues to be accurate. This requirement hardly limits the applicability of the method.

9.2.2 Limitations regarding performance measures

The two performance evaluation methods assess the end-to-end waiting time distribution of a single cell. They do not consider the *joint* end-to-end waiting time distribution of several cells on the VC under study. They also do not consider cell loss.

The joint distribution of cell waiting times is not a very important performance measure. It determines the probability that a number of consecutive cells are lost in a smoothing buffer or in UPC/NPC. The *joint* cell loss probability is important for some source coding

 $^{^{2}}$ The description of the traffic stream on the VC under study is already very versatile, so it does not provide a limitation.

techniques. It also determines the probability of packet loss, where a packet is a data unit that is divided into a number of cells that are transported via an ATM network.

The cell loss probability is very important. The end-to-end cell loss probability is obtained by adding the cell loss probabilities in the queues that make up the VC model. In the end-to-end cell loss probability (unlike in the end-to-end cell waiting time distribution), correlation between the queues of the VC model does not play a role. So, decomposition of the queuing network is an appropriate approach in determining the cell loss probability. Correlation between queues does play a role in the joint probability of cell loss.

9.2.3 Limitations regarding computation time

Both methods require a considerable amount of computation, so that they cannot be used in the control of an ATM network. In control, real-time decisions have to be taken on e.g. the admission to the network of a requested VC. The methods should be applied to the design of ATM networks (including traffic control rules), where the computation time is relatively unimportant.

9.3 Future research

Future research may be directed towards alleviation of limitations of the methods and, more importantly, towards application of the methods to the design of ATM networks.

9.3.1 Alleviation of the limitations

- The main limitation of the method for smooth traffic resides in the trade off between accuracy of the results and computation time. Possibly, an improvement of this trade off is achieved by simplification of the traffic description.
- The main limitations of the method for bursty traffic are (1) that the queues in the VC model should be identical and (2) that the VC traffic streams should be identical IPPs. Removal of these limitations would allow application of the method to configurations that more closely resemble real life ATM networks.

9.3.2 Application of the methods

Application of the methods to the design of ATM networks is an interesting topic for future research. The role of the methods is to provide accurate results on the end-to-end cell waiting time distribution. Future research should concern the implications of these results to the design of ATM networks and of traffic control rules.
Appendix A Multiplexer Models

In Sect. 3.2, an ATM multiplexer was described as a queue. In this appendix, several methods to analyze ATM multiplexers are discussed. Earlier versions of this text are [van Rijnsoever, 1991a; van Rijnsoever, 1991b]. Focus is on statistical multiplexers.

Mostly, it is straightforward to obtain the cell delay distribution and the cell loss probability once you know the joint distribution of queue length and source state. We will therefore concentrate on the queue length distribution.

Let the number of cells in the multiplexer, i.e. buffer and server, be denoted by the stochastic process X_n , and denote by the random variable L_{n+1} the number of cell arrivals to the multiplexer in slot n + 1. K is the maximum number of cells in the multiplexer. To describe the operation of the multiplexer, we have to determine the relative order of three events that are repeated endlessly in that order: arrival of a batch of new cells, departure of a cell from the multiplexer after service, and loss of cells due to buffer overflow. (Of course not all three events will actually occur in each slot.) There are basically two different orders. (Other orders differ from the two basic orders only w.r.t. the position of the observation moment relative to the events.) We call them the arrivals-first and departures-first orders of events. They, respectively, give rise to the following stochastic equations for X_{n+1} :

$$X_{n+1} = min[max(X_n + L_{n+1} - 1, 0), K]$$
(A.1)

$$X_{n+1} = min[max(X_n - 1, 0) + L_{n+1}, K].$$
(A.2)

Both orders describe essentially the same system.

The queue model of an ATM multiplexer can be solved in several ways. The evolution of the queue length in a multiplexer is described by either a Markov chain (like in A.1 and A.2) or by a set of differential equations. Both can, in principle, be solved numerically. (See [Krieger et al., 1990] for a survey on the numerical solution of Markov chains.) We will however not consider purely numerical techniques. They do not provide insight into the problem at hand and – although they are getting more sophisticated, and computing power and storage capacity are increasing steadily – the model state space may easily exceed the problem size they can handle. The techniques that we do consider try to exploit the structure of the problem under scrutiny. The ideal of a fully explicit solution is, however, almost never attained. Sects. A.1 and A.2 solve the Markov chain multiplexer description. Sect. A.1 provides an algorithmic solution method; the methods in sect. A.2 are based on probability generating functions. Sect. A.3 solves the fluid-flow model of an ATM multiplexer. This model describes a multiplexer in terms of differential equations. The basic problem that all these methods encounter is how to determine the probability distribution at the edge of the state space, i.e., at an empty queue (most method assume infinite buffer size). Finally, Sect. A.4 solves a multiplexer that multiplexes periodic traffic streams.

A.1 Algorithmic solution of the D-BMAP/D/1 queue

In this section, we consider an algorithmic technique to solve the D-BMAP/D/1 queue for its steady-state probability distribution. A more general version of this technique is extensively described by Neuts [1989]. The treatment here is very short and confined to the context of ATM.

We consider respectively the D-BMAP/D/1 queue, its solution, and solution of the D-BMAP/D/1/K queue.

A.1.1 The D-BMAP/D/1 queue

The Discrete-time Batch Markovian Arrival Process (D-BMAP) was introduced in 2.3.1. It is a discrete-time Markov renewal process in which an event is the arrival of a batch of cells. The events of a Markov renewal process coincide with transitions in a Markov chain, see [Cinlar, 1975, Chap. 10, Def. 1.1]. In a D-BMAP, the transition that induces an event also determines the distribution of the number of cells in the batch associated with that event. The distribution depends on the states that precede and succeed the transition.

A D-BMAP can be described as follows, see [Blondia, 1991]. The Markov chain that induces the D-BMAP has transition probabilities matrix D. D is an irreducible¹, finite stochastic matrix². The state of this Markov chain is called the phase of the D-BMAP. To describe the batch size distribution, the transition probabilities matrix D is split up into a set of substochastic matrices $D_k, k \ge 0$, such that $D = \sum_{k=0}^{\infty} D_k$, and $I - D_0$ is non-singular. Entry (i, j) in matrix D_k is the probability that, given that the phase in the current slot is i:

- the phase in the next slot is j and
- a batch of k cells is generated during the transition from phase i to phase j.

The D-BMAP describes the aggregate cell arrival process at an ATM multiplexer. In the context of ATM, the traffic stream on a single VC is often an on-off process with periodic

 $^{^{1}}$ A Markov chain is said to be irreducible if its only closed set is the set of all states. A set of states is closed if no state outside it can be reached from any state in it. See [Cinlar, 1975, p. 127].

 $^{^{2}}$ A square matrix is stochastic if its elements are non-negative and each row sums up to 1. The transition probabilities matrix of a discrete-time Markov chain is stochastic.

cell generation in the on-state. The alternation between on- and off-states is described by a two-state Markov chain. In case of identical VC traffic streams, the burst level state of the aggregate arrival process is then a birth-death process, which is easily represented in a D-BMAP. Periodic cell generation however cannot easily be represented in a D-BMAP. So for the sake of tractability, it is mostly assumed that cell generation in the on-state is Bernoulli instead of periodic. As noted in [qi Li et al., 1991; Ramaswami et al., 1991], this hardly affects the accuracy of the model.

The D-BMAP is synchronized to the service process of the multiplexer: both processes have a slot structure and these structures coincide. The multiplexer is described by the Markov chain $\{(X_n, J_n)\}$, where X_n denotes the number of cells in the system in slot n, and J_n denotes the phase of the D-BMAP in slot n.

If the states of $\{(X_n, J_n)\}$ are ordered lexicographically, the transition probabilities matrix has a block matrix structure: it can be written down in terms of the smaller matrices D_k . It is given below for, respectively, the arrivals-first and departures-first orders of events:

$$\begin{pmatrix} D_0 + D_1 & D_2 & D_3 & D_4 & \cdots \\ D_0 & D_1 & D_2 & D_3 & \cdots \\ 0 & D_0 & D_1 & D_2 & \cdots \\ 0 & 0 & D_0 & D_1 & \cdots \\ \vdots & & \ddots & \ddots & \ddots \end{pmatrix}$$

$$\begin{pmatrix} D_0 & D_1 & D_2 & D_3 & \cdots \\ D_0 & D_1 & D_2 & D_3 & \cdots \\ 0 & D_0 & D_1 & D_2 & \cdots \\ 0 & 0 & D_0 & D_1 & \cdots \\ \vdots & & \ddots & \ddots & \end{pmatrix}$$
(A.3)
$$(A.4)$$

A.1.2 Algorithmic solution

Assuming steady-state exists, denote the steady-state probabilities of $\{(X_n, J_n)\}$ by $\pi_{ij} = \lim_{n\to\infty} \Pr(X_n = i, J_n = j)$, and form the vectors $\pi_i = (\pi_{i1}, \pi_{i2}, \ldots, \pi_{im})$, where m is the number of states of the phase process. Obviously, $\pi = \sum_{i\geq 0} \pi_i$ is the steady-state probability distribution of the phase process, i.e. $\pi = \pi D$.

For the arrivals-first order of events, the vectors π_i are related by (see also equation A.1):

$$\boldsymbol{\pi}_{o} = \boldsymbol{\pi}_{o}(D_{0} + D_{1}) + \boldsymbol{\pi}_{i}D_{0} \tag{A.5}$$

$$\pi_i = \sum_{j=0}^{i+1} \pi_j D_{i+1-j}, \qquad i \ge 1$$
(A.6)

For the departures-first order of events, the vectors $\boldsymbol{\pi}_i$ are related by (see also equation

A.2):

$$\pi_{i} = \pi_{o} D_{i} + \sum_{j=1}^{i+1} \pi_{j} D_{i+1-j}, \qquad i \ge 0$$
(A.7)

The Markov processes described by the matrices A.3 and A.4 are of the M/G/1-type³, see [Neuts, 1989]. The essential characteristics of these processes are that the behavior is independent of the number of cells in the system (the *level*), except when the level is 0, and that the level may decrease only in single steps. The transition of the Markov chain from a level to the next lower level is called the fundamental period; *fundamental* because it is independent of the level.

The algorithmic solution determines π_{\circ} . π_i , i > 0, may then be obtained in an iterative algorithm based on A.1 or A.2, see [Ramaswami, 1988a].

The algorithm examines the fundamental period of the process. The distribution of the fundamental period is the joint distribution of:

- the number of slots required to reach the next lower level for the first time and
- the phase in which this level is reached.

The distribution of the fundamental period is described by a recursive matrix-equation based on the observations that the level may decrease only in single steps and that system behavior is independent of the level from level 1 onwards.

To calculate π_{o} , it is not required to determine the distribution of the fundamental period. It suffices to determine less complex measures: the distribution of the phase transition in the fundamental period and the mean number of slots in the fundamental period, given the initial phase. The computationally most intensive part of the algorithm is the recursive solution of the polynomial matrix equation that describes the distribution of the phase transition.

A.1.3 Algorithmic solution of the D-BMAP/D/1/K queue

The transition probabilities matrix of the D-BMAP/D/1/K queue is easily obtained from the transition probabilities matrix of the D-BMAP/D/1 queue. In comparison with the D-BMAP/D/1 queue, in the D-BMAP/D/1/K queue a transition into a state is impossible if the number of cells in this state exceeds K. A transition into a state with more than K cells in the infinite queue is 'translated' to a transition into a state with K cells in the finite queue. For the departures-first order of events, the following transition probabilities

 $^{^{3}}$ The Markov chain embedded immediately after departure moments in the M/G/1 queue is also of this type, hence the name.

matrix is obtained:

$$\begin{pmatrix} D_0 & D_1 & D_2 & \cdots & D_{K-1} & D_K \\ D_0 & D_1 & D_2 & \cdots & D_{K-1} & \tilde{D}_K \\ 0 & D_0 & D_1 & \cdots & D_{K-2} & \tilde{D}_{K-1} \\ 0 & 0 & D_0 & \cdots & D_{K-3} & \tilde{D}_{K-2} \\ \vdots & & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & D_0 & \tilde{D}_1 \end{pmatrix},$$

where $\tilde{D}_i = \sum_{j=i}^{\infty} D_i$.

This queuing model can be solved numerically. One may, however, exploit the structure of the transition matrix in an algorithmic solution procedure to obtain a more efficient solution method. Such procedures are outlined in [Blondia, 1991; Baiocchi et al., 1993]. The notation is most clear in [Baiocchi et al., 1993], and this procedure will be outlined. Equation A.7 still holds for π_o to π_{K-1} . This allows expressing all probability vectors in $\pi_o: \pi_i = \pi_o C_i, i \in \{0, \ldots, K\}$, where

$$C_0 = I \tag{A.8}$$

$$C_1 = [I - D_0]D_0^{-1} \tag{A.9}$$

$$C_{i} = [C_{i-1} - D_{i-1} - \sum_{j=1}^{i-1} C_{j} D_{i-j}] D_{0}^{-1}$$
(A.10)

 π_{\circ} follows from the normalization equation:

$$\boldsymbol{\pi} = \sum_{i=0}^{K} \boldsymbol{\pi}_{i} = \boldsymbol{\pi}_{o} \sum_{i=0}^{K} C_{i}, \qquad (A.11)$$

where π is the steady-state distribution of the phase process. If several buffer sizes are to be considered, the matrices C_i can be used repeatedly.

A.2 Transform solutions

This section surveys application of the well-known probability generating function (PGF) technique to the determination of the steady-state probability distribution of ATM multiplexers. Focus is on the D-BMAP/D/1 queue.

A general overview of PGFs is given in, e.g., [Kleinrock, 1975, App. I]. Only in a small number of cases an explicit expression for the PGF is known. The PGF of an integer, non-negative random variable X is defined as:

$$E(z^X) = \sum_{i=0}^{\infty} \Pr(X=i) z^i.$$
 (A.12)

The simple PGF $E(z^X)$ is not well suited to analyze a queue like the D-BMAP/D/1 queue, that is described by the two-dimensional Markov chain $\{(X, J)\}$. There are two more advanced PGFs:

• The vector PGF (E($z^X | J = 0$), E($z^X | J = 1$),..., E($z^X | J = m$)).

This PGF is used to study D-BMAP/D/1-like queues in e.g. [Daigle et al., 1990]. We will subsequently address it in two separate subsections.

An expression for the PGF is obtained that contains constants that are the roots of a set of equations.

• The multi-dimensional PGF $E(z^X \cdot y^J)$.

The series of papers [Xiong et al., 1993; Bruneel, 1993; Bruneel, 1988; Xiong et al., 1992] uses a multi-dimensional PGF to study queuing behavior when multiplexing onoff traffic streams. In the on-state, cells arrive in contiguous slots at the multiplexer.

A functional equation for $E(z^X \cdot y^J)$ is found, from which explicit expressions for the moments of X can be derived. However, the algebra involved is very complex except for the first moment.

A.2.1 Vector probability generating functions

In section A.1, the D-BMAP/D/1 queue was solved according to an algorithmic method. This section addresses its solution by transform methods, see e.g. [Daigle et al., 1990].

Consider the departures-first D-BMAP/D/1 queue. Its transition probabilities matrix is given in A.4. Define for each state of the phase process a generating function $G_j(z)$ and form the vector of generating functions $\mathbf{G}(z) = (G_1(z), \ldots, G_m(z))$.

$$G_j(z) = \sum_{i \ge 0} \pi_{ij} z^i \tag{A.13}$$

$$\mathbf{G}(z) = \sum_{i \ge 0} \boldsymbol{\pi}_i z^i \tag{A.14}$$

The functions $G_j(z)$ are not called *probability* generating functions, because they are not constructed from a complete probability distribution. Multiplying equation A.7 by z^i and summing over all *i*, readily yields⁴:

$$\mathbf{G}(z)[zI - D(z)] = (z - 1)\pi_{o}D(z), \tag{A.15}$$

where $D(z) = \sum_{i\geq 0} D_i z^i$. $\mathbf{G}(z)$ is now completely determined up to the vector $\boldsymbol{\pi}_{\circ}$, which denotes the joint probability that the system is empty and in a given phase. (Note, however, that even when $\boldsymbol{\pi}_{\circ}$ is known determination of $\mathbf{G}(z)$ and of the probability distribution is not straightforward at all.)

Postmultiplying both sides of equation A.15 by $\mathbf{e} = (1, \dots, 1)^T$, taking derivatives with respect to z, and substituting z = 1 yields:

$$1 - \boldsymbol{\pi} D'(1) \mathbf{e} = \boldsymbol{\pi}_{\circ} \mathbf{e}, \tag{A.16}$$

⁴For the arrivals-first order of events, the equation is: $\mathbf{G}(z)[zI - D(z)] = (z - 1)\pi_0 D_0$.

where π the steady-state probability distribution of the phase process. $\pi = \mathbf{G}(1)$. This equation states that 1 minus the mean number of cell arrivals per slot equals the probability of an empty system.

In some systems, the D-BMAP is such that there is only one phase that allows the system to be empty (i.e. there is only one phase that can be entered without generating a cell). The unknown vector π_o then follows immediately from A.16.

A more general way to obtain π_{\circ} in A.15 is the following. The matrix [zI - D(z)] is singular for some complex values of z.⁵ For values of z that are inside or on the unit circle in the complex plane ($\{|z| \leq 1, z \in C\}$), $|\mathbf{G}(z)| \leq |\mathbf{G}(1)| = |\pi|$, because in A.13 $\pi_{ij} > 0$. The essential point is that $|\mathbf{G}(z)|$ is bounded inside and on the unit circle of the complex plane. If [zI - D(z)] is singular for a value of z that is inside or on the unit circle in the complex plane, the right hand side of equation A.15 has to equal 0, because only then $\mathbf{G}(z)$ is bounded. Values of z for which [zI - D(z)] is singular are the roots of $\mathrm{Det}(zI - D(z)) = 0$. The set of linear equations that is obtained in this way, together with the normalization equation A.16, should allow determination of π_{\circ} .

Qi Li et al. [1991] show how to obtain π_o and solve A.15 by an alternative procedure that uses eigenvalues and eigenvectors. In their procedure, one may take advantage of the structure of the matrix D(z) if the D-BMAP is formed by superposition of independent traffic processes.

A.2.2 Other probability generating functions

We consider three types of PGF that resemble vector PGFs.

Hashida et al. [1991] apply transform methods to the D-BMAP(2)/G/1 queue, and Liao et al. [1989] to the D-BMAP(2)/D/1 queue. The D-BMAP(2) is a D-BMAP in which the phase process has 2 states. They take a somewhat different approach and determine the two PGFs of the queue length at state changes of the phase process.

Murata et al. [1990] study the discrete-time $G+GEO^A/D/1$ queue by transform methods. The G-stream represents traffic of which the performance is required; interfering traffic is represented by a GEO^A process. In a GEO^A process, a batch of cells arrives in a slot with a certain probability and with the complementary probability no batch arrives. If a batch arrives, the number of cells in the batch is distributed according to distribution A. In steady-state, the PGF of the queue length at an arrival moment on the G-stream, $G_k(z)$, depends on the number of slots k since the last arrival on the G-stream. The PGF of the queue length at arrival moments on the G-stream equals the weighted sum of the $G_k(z)$'s, where the weighting factor is given by the interarrival time probability distribution of the G-stream. This expression entails the infinite set of unknowns $\{G_k(0), k \ge 1\}$. After considerable manipulation, this expression is converted into an expression with finitely many unknowns that can be solved for by requiring that $G_0(z)$ is analytic in the unit circle of the complex plane.

 $^{{}^{5}[}zI - D(z)]$ is singular for z = 1, because $\pi[I - D(1)] = \pi - \pi D = 0$. This value of z is of no use here, however, because the right hand side of A.15 has a factor (z - 1).

Brown et al. [1988] consider a special case of the previous model: the D+GEO^A/D/1 queue. The set $\{G_k(z), 0 \le k \le d-1\}$ is to be determined, where d is the period of the D-stream. The unknowns $\{G_k(0), 1 \le k \le d-1\}$ can now be determined directly from the expression for $G_0(z)$.

A.3 Fluid flow models

If congestion mainly occurs when the instantaneous aggregate arrival rate at the multiplexer exceeds the multiplexer output rate, the arrivals of individual cells need not be taken into account to accurately estimate the multiplexer performance. Fluid-flow models represent the arrival and service processes by continuous-streams of information (see also Sect. 2.1.1). As a result, buffer content and waiting time are no longer integer values. Fluid flow models account for the net flow of information into a buffer. They only consider burst level congestion and do not model cell level congestion, see Sect. 3.3. If burst level congestion prevails, however, they are remarkably accurate and underestimate the cell loss probability only slightly. In this section, we will focus on fluid-flow models in which the arrival rate is modulated by a continuous-time Markov chain. A survey of fluid-flow models is given in [Roberts, 1991a, sect. 7.2].

Most results pertain to the case in which the modulating Markov chain is a birthdeath process. Kosten [1974] studies the case of a Poisson process of burst arrivals (rate λ). The burst length is exponentially distributed (rate β), and the arrival rate during a burst (r) is fixed. See also section 2.3.2. Anick, Mitra and Sondhi (AMS) [1982] model the arrival process as the traffic generated by the superposition of N independent and identically distributed on-off sources, that have exponentially distributed on- and off-times (rate off-times: α) and a fixed generation rate in the on-state. The model is to be solved for $F_i(x) = \lim_{t\to\infty} \Pr(J_t = i, X_t \leq x)$. A set of differential equations is drawn up by allowing in an infinitesimal interval only one of the following events: a step upwards or downwards in the birth-death chain or a change in the buffer filling level:

$$\begin{aligned} &\Pr(J_{t+dt} = i, X_{t+dt} \le x) = \\ & \mu_{i+1} \, dt \, \Pr(J_t = i+1, X_t \le x) \\ &+ \lambda_{i-1} \, dt \, \Pr(J_t = i-1, X_t \le x) \\ &+ (1 - (\lambda_i + \mu_{i+1}) \, dt) \Pr(J_t = i, X_t + (r_i - 1) \, dt \le x) \\ &+ \mathsf{o}(dt), \qquad 0 \le i \le N, \end{aligned}$$

where λ_i is the birth-rate in state i (λ in the Kosten model; $(N-i) \cdot \alpha$ in the AMS model); μ_i is the death-rate $(i \cdot \beta)$; and r_i is the arrival rate $(i \cdot r)$. The service rate is 1. Parameters with an index value out of range are zero. Subtraction of $\Pr(J_t = i, X_t \leq x)$ at both sides of the equation, division by dt and rearranging yields:

$$\frac{\Pr(J_{t+dt}=i, X_{t+dt} \le x) - \Pr(J_t=i, X_t \le x)}{dt} =$$

$$\begin{split} & \mu_{i+1} \Pr(J_t = i+1, X_t \le x) \\ &+ \lambda_{i-1} \Pr(J_t = i-1, X_t \le x) \\ &- (\lambda_i + \mu_{i+1}) \Pr(J_t = i, X_t + (r_i - 1) dt \le x) \\ &+ \frac{\Pr(J_t = i, X_t + (r_i - 1) dt \le x) - \Pr(J_t = i, X_t \le x)}{dt} \\ &+ \frac{\mathrm{o}(dt)}{dt}, \quad 0 \le i \le N. \end{split}$$

Next, let t approach ∞ and dt approach 0. In the steady-state, the derivative of the joint probability with respect to time (i.e. left hand side of equation) is 0. Substitution of $dx = (r_i - 1)dt$ and rearrangement gives:

$$(r_i - 1)\frac{dF_i(x)}{dx} = \mu_{i+1}F_{i+1}(x) + \lambda_{i-1}F_{i-1}(x) - (\mu_i + \lambda_i)F_i(x), \qquad 0 \le i \le N.$$
(A.17)

The set of differential equations can be cast into matrix format:

$$D\frac{d\mathbf{F}(x)}{dx} = M\mathbf{F},\tag{A.18}$$

where $\mathbf{F}(x) = (F_0(x), F_1(x), \dots, F_N(x))^T$, $D = \operatorname{diag}(r_0 - 1, r_1 - 1, \dots, r_N - 1)$, and M is the infinitesimal generator matrix of the birth-death process. In a stable system, the generic solution of this model is:

$$\mathbf{F}(x) = \mathbf{F}(\infty) + \sum_{\{i|Re(z_i)<0\}} a_i \phi_i e^{z_i x},\tag{A.19}$$

where z_i is a (complex valued) eigenvalue of $D^{-1}M$, ϕ_i the corresponding right eigenvector, and the parameters a_i are set to comply with the boundary conditions. Eigenvalues with positive real part cannot be part of the solution, because they would give unbounded probabilities for large enough x. Eigenvalues zero give the steady-state distribution of the phase process. The asymptotic behavior of $\mathbf{F}(x)$ in x is determined by the largest negative eigenvalue.

For the Kosten model only an algorithmic solution is known. For the AMS model, however, an explicit solution is given in [Anick et al., 1982]. There are one eigenvalue zero and $N - \lfloor \frac{r_{mux}}{r} \rfloor$ negative eigenvalues, where r_{mux} denotes the multiplexer rate, that is assumed not to be a multiple of r.

Tucker [1988] study the AMS model in case of a finite buffer. For a finite buffer, the positive eigenvalues cannot be discarded. The set of unknown constants $a_i, 0 \le i \le N$ is solved numerically from a set of linear equations that ensures that the boundary conditions are fulfilled. These boundary conditions are that the buffer can never be empty if the phase process is in an overload state and that the buffer can never be full if the phase process is in an underload state.

Kosten [1984] studies the AMS model with heterogeneous traffic sources, i.e. traffic sources are divided into groups of independent and identically distributed sources. An explicit solution is not found, but equations for and properties of the eigenvalues are given. In [Baiocchi et al., 1992b] this model is further analyzed for the finite buffer case. It is shown that the calculation of the eigenvectors can be decomposed according to the groups of sources. The set of equations that determines the constants in the solution according to the boundary conditions readily becomes too large to solve. An approximation for the loss probability is proposed that (1) leaves out the contributions from the positive eigenvalues and (2) sets the constants corresponding to the negative eigenvalues to their values at buffer size zero. The latter values can be determined relatively easy. Comparison with exact results shows that this approach yield an upper bound on the cell loss probability. The relative error is less than 10% if the server load does not exceed 0.7.

Suruagy Monteiro et al. [1991] show that in the AMS model the loss probability depends on the buffer size and on the mean burst length only through their ratio. So, a model with small buffer size may replace a model with large buffer size in performance studies if the burst length is proportionally decreased.

A.4 Multiplexing periodic traffic sources

If the arrival process to a queue with a deterministic server is periodic and the queue is not overloaded or the buffer size is finite, the queue length distribution will, after a transition period, be periodic as well (see Sect. 2.3.3). This phenomenon may lead to widely different performance for otherwise statistically indistinguishable sources. The period equals the smallest common multiple of the periods of the individual sources. It is commonly assumed that the phase of the sources is chosen randomly.

Especially the nD/D/1 queue has received much attention. Roberts et al. [1991b] and Humblet et al. [1993] give explicit expressions for the queue length distribution of the nD/D/1 queue. The expressions differ with respect to their complexity. Dron et al. [1991] give an approximate expression for the queue length distribution that is shown to be accurate for high load and large n. It is shown in e.g. [Roberts et al., 1991b; Dron et al., 1991] that the M/D/1 queue is a good approximation (slight overestimation of the queue length) for the nD/D/1 queue, if the system load is small and n is large, see also section 2.3.3.

In the nD/D/1/K queue, a buffer size exceeding *n* will not contribute to a lower cell loss probability, independently of whether the queue is overloaded or not. Cidon et al. (see [Roberts, 1991a, sect. 6.5.2]) and Huebner et al. [1991] solve the nD/D/1/K queue by considering all possible cell arrival patterns. The queue length regenerates once each period. In case of an underloaded queue, the queue is empty in the regeneration point; in case of an overloaded queue, the queue is full in the regeneration point. The queue length distribution is determined by following the queue length evolution starting in a regeneration point for all possible arrival patterns in a single period.

Roberts et al. [1991b] give upper and lower bounds on the queue length distribution for the $\sum_{i=1}^{m} n_i D_i / D/1$ queue.

A.4.1 Quasi-stationary approximation

The quasi-stationary approximation can be applied to multiplexing on-off sources with periodic cell generation in the on-state, see [Fuhrmann et al., 1991; Huebner et al., 1991; Norros et al., 1991; Chen, 1993; Kamitake et al., 1989]. The approximation applies only to the case of underload: the instantaneous cell arrival rate at the multiplexer should be smaller than the output rate of the multiplexer.

If the source states change slowly, the queue length distribution reaches a limit distribution long before the next change, and it is reasonable to assume that this limit distribution holds during the entire sojourn time in the corresponding states. The 'overall' queue length distribution can be assessed by first determining the steady-state distribution corresponding to each set of source states, and subsequently averaging these distributions according to the probability of each set.

An alternative to the quasi-stationary approximation is approximation by a M/D/1/K queue. It is rather accurate, see e.g. [Kroener, 1991; Baiocchi et al., 1991a].

APPENDIX A. MULTIPLEXER MODELS

Appendix B

Waiting time correlation for more than two queues

This appendix describes the QNP waiting time correlation if more than two queues are involved for the case of smooth VC traffic.

The model that we study is a network of n queues in tandem. These are n queues of the ATM queuing network model that are passed (one after the other) by the VC under study (and possibly also by other VCs). All queues are identical and synchronized.

In the numerical examples that we consider, the loads of all queues are equal. After each queue in the tandem except after the last queue, cells are routed according to a Bernoulli model. A fraction $\frac{1}{f}$ of the cells passes on to the next queue in the tandem, and a fraction $1 - \frac{1}{f}$ of the cells leaves the tandem network. So f is the fan out. At each queue new cells join the tandem network. The traffic stream that they comprise is modeled by a discrete-time Poisson process¹.

For two reasons, the QNP is in general less important if it refers to two non-consecutive queues in the tandem network:

• Decreased traffic load:

The traffic load on the route that passes through two queues in general decreases if there are other queues in between. In 5.1.1 it is shown that a lower load on this route decreases correlation between waiting times.

• Disturbance of traffic stream:

Previously, it was argued that the QNP is due to the similarity of the traffic arrival streams at two queues. The traffic stream on the route that passes through the two queues is however disturbed by any queues in between.

We argued that the effect of the QNP is smaller if the queues concerned are further apart. This does not mean that the QNP may be neglected for far apart queues. If the number of tandem queues n in the model grows, the number of pairs that can be formed

¹In each slot an independent and Poisson distributed batch of cells arrives

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 $\binom{n}{2}$ rapidly increases. Each pair of queues contributes its own, possibly small share to the effect of the QNP on the end-to-end cell waiting time distribution. The number of pairs of adjacent queues is only a linear function of n, namely, n - 1.

We break the numerical study down into two parts, according to the two points above. At first we neglect the second point above (i.e., the disturbance of the traffic stream) and take into account only the first point. This allows us to study arbitrarily long networks. Then we take into account both points.

B.1 The effect of decreased traffic load

If we neglect the disturbance of the traffic stream, the QNP is the same for consecutive queues and non-consecutive queues, as long as the load of the traffic stream through the queues concerned is adjusted. We will use this observation to approximate the variance of the end-to-end waiting time in an arbitrary number of tandem queues. In 5.1.1, we analyzed the QNP for two tandem queues and Poisson traffic. We will use exactly these results here again.

Let W_i , $i \in \{1, ..., n\}$, be a random variable denoting the waiting time of the cell under study in the *i*-th queue of the *n* tandem queues in the model. The variance of the end-to-end waiting time can be approximated as follows² ³:

$$Var(\sum_{i=1}^{n} W_{i}) = \sum_{i=1}^{n} Var(W_{i}) + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Cov(W_{i}, W_{j})$$

$$\approx n \cdot Var(W_{1}) + 2\sum_{i=1}^{n-1} \sum_{j=1}^{n-i} Cov(W_{1}, W_{1+j}).$$
(B.1)

In order to obtain a compact expression, we make a further approximation by inserting into B.1 approximate values for $Cov(W_1, W_{1+j}) = Cor(W_1, W_{1+j}) \cdot Var(W_1)$.

Tab. B.1 shows correlation between the waiting times of a cell in two tandem queues, for server loads of 0.9. We used the two tandem queues model with Poisson traffic of 5.1.1 and copied the results from Tab. 5.2. The column labeled *ratio* shows the ratio between correlation at fan out 2^{i-1} and at fan out 2^i , $2 \le i \le 6$. It is observed that an increase of fan out by a factor 2 corresponds to a decrease of correlation by a factor $\frac{1}{2}$. So, correlation is inversely proportional to fan out. This observation was also made by others (see 4.6).

In the tandem queuing network model, we have chosen a special fan out rule: the load of the traffic stream through two queues is proportional to f^d , where f is fan out from one queue to the next queue and d is the distance between the two queues. (For consecutive queues, d = 1). In combination with the observation that correlation is inversely proportional to fan out, we obtain the following approximation: $Cov(W_1, W_{1+d}) = Cor(W_1, W_{1+d}) \approx Cor(W_1, W_2) \cdot f^{-d+1} \cdot Var(W_1), d \geq 1$.

 $^{{}^{2}}Cov(W_{i}, W_{i}) = E(W_{i} \cdot W_{i}) - E(W_{i}) \cdot E(W_{i})$

³The approximation is due to neglecting disturbance of the traffic stream: $Var(W_i) = Var(W_1), 1 < i \le n$ and $Cov(W_i, W_i + j) = Cov(W_1, W_{1+j}), 1 \le i \le n-1, 1 \le j \le n-i$.

Table B.1: Correlation between cell waiting times in a two tandem queues network as a function of fan out. Traffic processes are Poisson. Cell routing is Bernoulli.

i	Fan out 2 ⁱ	$Cor(W_1, W_2)$	ratio
1	2	$1.52 \cdot 10^{-1}$	
2	4	$7.58 \cdot 10^{-2}$	2.01
3	8	$3.82 \cdot 10^{-2}$	1.98
4	16	$1.92 \cdot 10^{-2}$	1.99
5	32	$9.64 \cdot 10^{-3}$	1.99
6	64	$4.83\cdot10^{-3}$	2.00

Insertion of this approximation into B.1 gives:

$$Var(\sum_{i=1}^{n} W_i) \approx n \cdot Var(W_1) \left(1 + \frac{2}{1 - f^{-1}} \left(1 - \frac{1 - f^{-n}}{n \cdot 1 - f^{-1}} \right) Cor(W_1, W_2) \right)$$
(B.2)

Equation B.2 is the result we were looking for. It allows us to obtain numerical results on the correlation between cell waiting times.

The variance of the end-to-end cell waiting time in the tandem queuing network is $Var(\sum W_i)$. If the waiting times were independent (i.e., if the QNP would not exist), this would equal $Var(\sum W_i) = \sum Var(W_i) = nVar(W_1)$. Tab. B.2 shows the (relative) underestimation of the variance of the end-to-end waiting time if the QNP is neglected, i.e., $\frac{Var(\sum W_i) - nVar(W_i)}{Var(\sum W_i)}$. This is a measure of the QNP. The results in B.2 are based on B.2 and Tab. B.1.

Table B.2: Relative underestimation of end-to-end waiting time variance as afunction of the number of queues n and fan out f.

n	f = 2	f = 4	f = 16
2	0.13	0.07	0.02
3	0.20	0.10	0.03
10	0.32	0.15	0.04
∞	0.38	0.17	0.04

The results in Tab. B.2 show that correlation between cell waiting times is positive and may have a considerable effect on the end-to-end waiting time, depending on the system parameters. The effect increases with decreasing fan out f or increasing number of queues n. The increase is, however, less than proportional to n. If fan out is high, the effect increases slowly as a function of n.

B.2 The effect of decreased traffic load and disturbance of the traffic stream

We simulated a network of three queues in tandem, in order to take into account the effect of disturbance of the traffic stream on the QNP. The model is the model we previously used. Fan out is 2, and server loads are 0.9.

The simulation shows that the actual value of the previously introduced measure of correlation $\frac{Var(\sum W_i)-nVar(W_1)}{Var(\sum W_i)}$ is 0.18. Assuming the simpler model, we found the approximate value 0.20 (entry for n = 3, f = 2 in Tab. B.2). So, disturbance of the traffic stream occurs and has the influence that was expected: a decrease of correlation. The influence is, however, rather small.

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