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Bit Error Evaluation of Optically Preamplified Direct Detection Receivers with Fabry–Perot Optical Filters

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Abstract— The error performance of a preamplified, direct detection receiver with an optical filter of the Lorentzian type is studied. The analysis takes into account the influence of the optical intersymbol interference (ISI). A closed-form expression of the moment generating function (MGF) of the decision variable is derived. Error probabilities are evaluated from the MGF using a saddlepoint approximation. The Gaussian approximation is also examined. The detection sensitivity in terms of a quantum limit is calculated. The results show that there exists an optimum optical bandwidth, the reason being a tradeoff between the effect of ISI and the spontaneous emission noise. It is also shown that the Gaussian approximation gives a good estimate of the error probability, allowing to find in a simple manner the optimum parameters of optically preamplified, direct detection receiver.

Index Terms— Error analysis, intersymbol interference, optical amplifiers, optical communication, optical filters, optical receivers.

I. INTRODUCTION

IN optically preamplified direct detection receiver the optical amplifier increases the power levels, but at the same time, the erbium-doped fiber amplifier (EDFA) generates spontaneous emission noise which is added to the photodetector input signal. Amplified spontaneous emission (ASE) noise is an inherent noise source of the fiber optical amplifier which impairs the receiver performance. To limit the effect of ASE, which is a wide band noise source, an optical filter is needed. Filtering, however, can distort the optical pulse and introduces intersymbol interference (ISI). Fabry–Perot filters are widely used in experimental optical transmission systems, e.g., [14]. They are well described by a Lorentzian impulse response [28].

The main question of the performance analysis is to determine the statistics of the receiver decision variable, taking into consideration ISI, and to further evaluate the bit-error probability. Most of the previous analysis of optically preamplified receivers were made under the assumption that the signal passes the optical filter unaltered, which means that the ISI is neglected or the optical filter bandwidth is assumed to be large [1]–[5]. The performance analysis for a receiver with a perfect rectangular bandpass optical filter is documented in [6], [7], and in [8] for a receiver with a traveling-wave semiconductor

optical preamplifier. Ben-Ali *et al.* [25] derived upper bounds on the bit error probability. Chernoff and modified Chernoff bounds together with an improved bound on the bit error probability are presented in [26]. Chinn [27] considered a probability density function (pdf) of the decision variable obtained by convolving individual pdf for a finite number of modes of a Karhunen–Loève expansion of the signal and noise. A Karhunen–Loève expansion approach is also used in [13] for deriving a moment generating function (MGF), but the use of the MGF is limited to finding the first and second moment of the decision variable. These works take into consideration the significance of the ISI, but a closed-form expression of the MGF, (statistics), of the decision variable that explicitly incorporate a Fabry–Perot optical filter is not given.

In this paper, a closed-form expression of the MGF of the decision variable, explicitly incorporating a Fabry–Perot optical filter, is derived. The MGF is then used to calculate bit-error probabilities by the so called saddlepoint approximation (spa). Some previous works have considered the decision variable to be Gaussian distributed [9]–[13]. In this paper the Gaussian approximation, including ISI, is also examined. The results shows that the Gaussian approximation gives a fairly accurate estimate of the error probability of optically preamplified receivers.

This paper is organized as follow: In Section II the reference scheme and the model of the receiver under analysis is presented. The general form of the MGF for the decision variable is derived with the help of a Karhunen–Loève expansion of the signal and noise. The method of deriving the MGF for the decision variable is also presented. In Section III, the expression for the error probability is presented and the saddlepoint approximation is introduced. A closed-form expression of the MGF for the decision variable is given. The performance of the Gaussian approximation is also studied. Numerical results, and comparison with previous work are presented in Section IV. Finally, in Section V, summarizing conclusions are drawn.

II. SYSTEM MODEL

The system under analysis is depicted schematically in Fig. 1. The optical preamplifier (EDFA) is characterized by an optical field amplifier with power gain G, an additive noise source $N(t)$, representing the spontaneous emission and an optical filter with complex equivalent baseband impulse
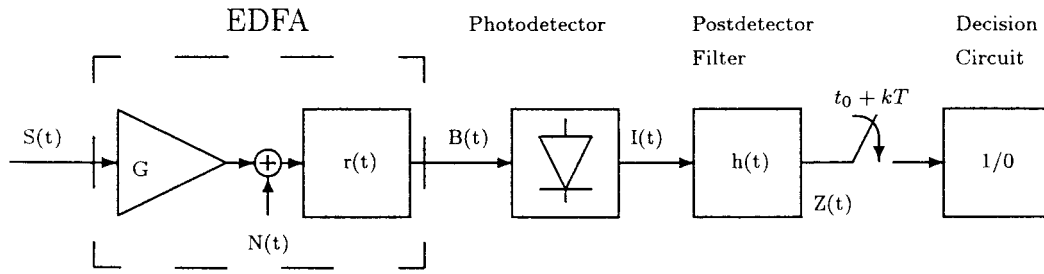
Fig. 1. Complex baseband model of preamplified direct detection receiver.

response $r(t)$. The equivalent baseband form of the optical field at the output of the EDFA is

$$B(t) = \left[\sqrt{G}S(t) + N(t)\right] \star r(t) \qquad (1)$$

where $\star$ stands for a convolution operation and $S(t)$ is the envelope (modulation) of the input optical signal $s(t)$, expressed as the real part of a complex field function

$$s(t) = \text{Re}\left\{S(t)\exp jwt\right\} \qquad (2)$$

where $\omega = 2\pi f$, f being the optical frequency.

The optical field $B(t)$ illuminating the photodetector produces an output shot noise current $I(t)$. The signal at the output of the postdetector filter, with impulse response $h(t)$, is

$$Z(t) = I(t) \star h(t).$$

This signal is sampled at $t = t_0 + kT$ time instants to form the decision variable. The decision device derives the estimate of a transmitted bit in a particular bit interval by comparing the decision variable with an optimal, preselected, detection threshold α. By an optimal threshold α is meant the detection threshold that yields the lowest error probability.

To continue further, we introduce some definitions and normalizations. The input signal $S(t)$ is assumed to be a rectangular pulse of duration T. The amplitude of $S(t)$ is chosen (normalized) such as m is the average number of photons contained in $S(t)$. In the sequel, it is assumed that for a transmitted "zero" bit "zero" photons are received. For equally likely symbols "one" and "zero," m is the average number of received photons per bit at the input to the EDFA.

For a given bit pattern $\mathcal{B} = (\cdots, b_{-1}, b_o, b_1, \cdots)$ the normalized information signal at the output of the optical filter is denoted by $Y(t)$

$$Y(t) = \sqrt{G}S(t) \star r(t)$$
$$= \sqrt{\frac{G2m}{T}} \sum_{k=-\infty}^{\infty} b_k g(t - kT) \star r(t) \qquad (3)$$

$$Y(t) = \sqrt{\frac{G2m}{T}} \left[b_o l(t) + \sum_{k \neq 0} b_k l(t - kT)\right] \qquad (4)$$

where

$l(t)$ $g(t) \star r(t)$;
b_k statistically independent binary symbols representing a data "zero" and a "one," respectively. $b_k \in \{0, 1\}$;
m average number of received photons per bit;
$g(t)$ input unit rectangular pulse of duration T;

The first term in (4) represents the desired information signal while the last term is the ISI. At the output of the optical amplifier, the average noise power measured in a bandwidth B is [1]

$$P_0 = n_{sp}hf(G-1)B \quad \text{W}$$

where h is the Planck's constant and n_{sp} is the spontaneous emission factor of the amplifier. For reason of compatibility with the normalization of $S(t)$ the density of $N(t)$ should be expressed in photons per second. The photon intensity corresponding to the stochastic optical field $N(t)$ is [35]

$$N_0 = n_{sp}(G-1) \quad \text{photons/s.}$$

At the output of the optical filter the real and imaginary parts of the Gaussian noise $X(t) = N(t) \star r(t)$ are independent, with mean zero and autocorrelation

$$K(\tau) = \frac{n_{sp}}{2}(G-1)R(\tau) \qquad (5)$$

where [15]

$$R(\tau) = \int_{-\infty}^{\infty} r(t)r^*(t+\tau)\,dt \qquad (6)$$

with $*$ denoting complex conjugate.

With the above notations the optical field at the output of the EDFA becomes

$$B(t) = Y(t) + X(t).$$

The photo-electron intensity is proportional to the square of the optical field (optical power) falling upon the photodetector [28]. It is assumed that the photodetector quantum efficiency η is equal to one, and the optical field is normalized so that the photo-electron intensity is

$$\lambda(t) = |Y(t) + X(t)|^2. \qquad (7)$$

The signal $Z(t)$, the postdetection filter $h(t)$ output signal, is a doubly stochastic process: it depends on the information bit pattern and on the stochastic mechanism of photodetection. The mathematical model for $Z(t)$ is the filtered compound Poisson stochastic process [16], [17] whose stochastic intensity is $\lambda(t)$. With no loss of generality we consider a time interval of duration T and denote the decision variable by $Z = Z(t = T)$. Conditioned on the value of $\lambda(t)$ the MGF for Z is [16]

$$M_Z(s)|_\lambda = \exp\left(\int_0^T \lambda(\tau)[\exp\{sh(t-\tau)\} - 1]\,d\tau\right). \qquad (8)$$

In this work, we restrict ourselves to a specific type of postdetector filter: the integrate-and-dump filter. The impulse response of the integrate-and-dump filter is given by

$$h(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Thus, the unconditional MGF of Z is given by

$$M_Z(s) = \int_0^\infty \exp\left\{\int_0^T \lambda(t)[\exp(s) - 1]\, dt\right\} p(\lambda)\, d\lambda \tag{10}$$

$p(\lambda)$ being the probability density function of $\lambda(t)$. In terms of the MGF for Λ

$$M_Z(s) = M_\Lambda(e^s - 1) \tag{11}$$

where

$$\Lambda = \int_0^T \lambda(t)\, dt$$
$$= \int_0^T |Y(t) + X(t)|^2\, dt. \tag{12}$$

Λ is also called the *Poisson parameter function* [16]. The expression (11) appears in an early paper by Personick [19]. The MGF $M_\Lambda(s)$ is given by

$$M_\Lambda(s) = \mathrm{E}\{e^{s\Lambda}\}. \tag{13}$$

We expand $V(t) = Y(t) + X(t)$ in a Karhunen–Loève expansion, in the time interval $[0, T]$, choosing the set of orthonormal functions $\{f_n\}$ such that

$$V(t) = \sum_{n=1}^\infty v_n f_n(t) \qquad \text{with} \quad v_n = x_n + y_n$$

and

$$y_n = \int_0^T Y(t) f_n^*(t)\, dt$$
$$x_n = \int_0^T X(t) f_n^*(t)\, dt$$

with f_n and λ_n being the eigenfunctions and eigenvalues, respectively, related to the following equation [20]:

$$\int_0^T K(t, u) f_n(u)\, du = \lambda_n f_n(t) \qquad 0 \le t \le T \tag{14}$$

where $K(t, u) = \frac{1}{2} \mathrm{E}\{X(t) X^*(u)\}$. By Parseval's theorem, the integral (12) becomes

$$\Lambda = \sum_{n=1}^\infty |y_n + x_n|^2$$
$$= \sum_{n=1}^\infty |v_n|^2. \tag{15}$$

The coefficients x_n are zero mean Gaussian independent variables whose real and imaginary part (x_{nc} and x_{ns}, respectively) have a variance $\mathrm{Var}\{x_{nc}\} = \mathrm{Var}\{x_{ns}\} = \lambda_n/2$ [20]. We observe that v_n are independent variables with mean

y_n; hence, the MGF of a particular $|v_n|^2$ is that of a stochastic variable with a noncentral chi-square distribution [15]

$$M_{|v_n|^2}(s) = \frac{1}{1 - \lambda_n s} \exp \frac{|y_n|^2 s}{1 - \lambda_n s}.$$

From (13) and (15), we have that

$$M_\Lambda(s) = \prod_{n=1}^\infty \mathrm{E}\left\{e^{s|v_n|^2}\right\}.$$

Thus, the general mathematical form for the MGF of Λ is [18], [19]

$$M_\Lambda(s) = \prod_{n=1}^\infty \frac{1}{(1 - \lambda_n s)} \exp\left(\sum_{n=1}^\infty \frac{|y_n|^2 s}{1 - \lambda_n s}\right)$$
$$\mathrm{Re}\, s < \frac{1}{\max_n \lambda_n}. \tag{16}$$

The choice of the integrate-and-dump filter simplifies the analysis, but it yields a suboptimum receiver. An MGF in the form of (16) can also be obtained for a general postdetector filter [25], [36].

The MGF (16) can be represented in terms of the resolvent kernel $h(t, u; s; \tau)$ [21], [24], [30] related to the integral equation

$$h(t, u; s; T) - s \int_0^T h(t, v; s; \tau) K(v, u)\, dv = K(t, u)$$
$$0 \le (t, u) \le T \tag{17}$$

as

$$M_\Lambda(s) = [D(s)]^{-1} \exp[F(s)] \tag{18}$$

where

$$F(s) = m_h s + s^2 \int_0^T \int_0^T Y^*(t) h(t, u; s; T) Y(u)\, dt\, du \tag{19}$$

$$m_h = \int_0^T |Y(t)|^2\, dt$$

and $D(s)$, also called the Fredholm determinant, is given by [22]

$$D(s) = \exp\left\{-\int_0^s \int_0^T h(t, t; v; T)\, dt\, dv\right\}. \tag{20}$$

The MGF given in the form of (18) is more convenient for numerical computations than the MGF expressed in terms of an infinite product [cf. (16)].

III. ANALYSIS

A. The Error Probability

The error performance analysis is conducted by conditioning on the sent symbol b_o and considering the finite sequence $\tilde{\mathcal{B}} = (b_{-L}, \cdots, b_{-1}, b_1, \cdots, b_L)$ of symbols surrounding b_0. Assuming that the symbols $b_0 = 1$ and $b_0 = 0$ are *a priori*

equally probably, the conditional error probability given a sequence $\tilde{\mathcal{B}}$ is

$$P_e|_{\tilde{\mathcal{B}}} = \tfrac{1}{2} P_r(Z < \alpha|_{\tilde{\mathcal{B}}|b_0=1}) + \tfrac{1}{2} P_r(Z > \alpha|_{\tilde{\mathcal{B}}|b_0=0}) \quad (21)$$
$$P_e|_{\tilde{\mathcal{B}}} = \tfrac{1}{2} \{q_+(\alpha) + q_-(\alpha)\}$$

As it is shown in [31], the tail probability $q_+(\alpha)$ is approximately equal to

$$q_+(\alpha) \approx \frac{\exp[\phi(s_0)]}{\sqrt{2\pi\phi''(s_0)}} \quad (22)$$

the so-called *saddlepoint approximation*. The function $\phi(s)$ is related to the MGF for Z, $M_Z(s)$ by

$$\phi(s) = \ln[M_Z(s)] - s\alpha - \ln|s|. \quad (23)$$

The parameter s_0 is the positive root of the equation

$$\phi'(s) = 0 \quad (24)$$

and $\phi''(s_0)$ stands for the second derivative of (23) at $s = s_0$. The lower probability tail is approximated by

$$q_-(\alpha) \approx \frac{\exp[\phi(s_1)]}{\sqrt{2\pi\phi''(s_1)}} \quad (25)$$

with s_1 equal to the negative root of (24). See [31] or [35] for further details. The error probability is minimized by adjusting the detection threshold α. The optimum value of α and the parameters s_0 and s_1 may be found numerically by solving an appropriate set of equations [35]. The saddlepoint approximation has been proposed by Helstrom [31], as an efficient and numerically simple tool for analyzing communication systems. The saddlepoint approximation has shown a reasonably high degree of accuracy in the analysis of optical communication systems [32]–[34].

The average error probability, for a fixed threshold α, is obtained by averaging the conditional error probability with respect to $\tilde{\mathcal{B}}$ with a b_0 given

$$P_e = \tfrac{1}{2} \mathrm{E}_{\tilde{\mathcal{B}}}\{P_r(Z < \alpha|_{\tilde{\mathcal{B}}|b_0=1})\} + \tfrac{1}{2} \mathrm{E}_{\tilde{\mathcal{B}}}\{P_r(Z > \alpha|_{\tilde{\mathcal{B}}|b_0=0})\}. \quad (26)$$

The expression (26) is general with respect to the statistics of the transmitted binary message. In this paper, we consider the case in which the message consists of mutually independent binary symbols.

In optical communications, the (standard) *Quantum limit* is defined as the average number of photons per bit in the optical signal $S(t)$ needed to achieve a bit error probability of 10^{-9} assuming ideal detection conditions, which means that $G \gg 1$ and $n_{sp} = 1$.

B. Lorentzian Optical Filter

The normalized Lorentzian filter impulse response is specified by

$$r(t) = \sqrt{2}\mu e^{-\mu t} \qquad t \geq 0 \quad (27)$$

and consequently the covariance kernel is

$$R(\tau) = \mu e^{-\mu|\tau|} \qquad \tau \in \mathbf{R} \quad (28)$$

where $B = \mu/\pi$ is the 3-dB optical filter bandwidth.

The output signal of the EDFA (after the optical filter) is given by

$$Y(t) = \sqrt{\frac{4Gm}{T}} \left[\int_0^t b_o \mu e^{-\mu\nu} d\nu \right.$$
$$\left. + \sum_{k=1}^{\infty} b_{-k} \int_{(k-1)T+t}^{kT+t} \mu e^{-\mu\nu} d\nu \right]$$
$$= \sqrt{\frac{4Gm}{T}} \left[b_o(1 - e^{-\mu t}) + \sum_{k=1}^{\infty} b_{-k}(e^{\mu T} - 1)e^{-\mu t} \right].$$

A more concise expression for $Y(t)$ is presented in [25]

$$Y(t) = \sqrt{\frac{4Gm}{T}} [b_o + \rho e^{-\mu t}] \qquad t \in [0, T] \quad (29)$$

in which

$$\rho = (e^{\mu T} - 1) \sum_{k=-\infty}^{-1} b_k e^{\mu kT} - b_o. \quad (30)$$

In order to obtain an expression for the MGF of the type in (18) the resolvent kernel $h(t, u, s; T)$ should be known. For the case of the Lorentzian filter the resolvent kernel is given in the literature, e.g., [20], [30]

$$h(t, u, s: T) = \frac{[h_1 e^{\beta t} + h_2 e^{-\beta t}][h_1 e^{\beta(T-u)} + h_2 e^{-\beta(T-u)}]}{v[h_1^2 e^{\beta T} - h_2^2 e^{-\beta T}]} \quad (31)$$

for $0 \leq t \leq u \leq T$. For $u < t$ the roles of u and t just interchange in (31), in which

$$h_1 = v + 1$$
$$h_2 = v - 1$$

with

$$v = \sqrt{1 - 2\sigma^2 s}$$
$$\beta = v\mu$$

and

$$\sigma^2 = n_{sp}(G - 1).$$

The Fredholm determinant is given by (see the Appendix)

$$D(s) = \frac{(v + 1)^2 e^{\beta T} - (v - 1)^2 e^{-\beta T}}{4v e^{\mu T}}. \quad (32)$$

The expression for

$$F(s) = m_h s + s^2 \mathcal{F}(s)$$

in which

$$\mathcal{F}(s) = \int_0^T \int_0^T Y^*(t)h(t, u; s; T)Y(u) \, dt \, du$$

turns out to be

$$F(s) = \frac{4mG}{\mu T} [b_o^2 F_1(s) + b_o \rho F_2(s) + \rho^2 F_3(s)]. \quad (33)$$

The expressions for $F_1(s)$, $F_2(s)$, and $F_3(s)$ are shown in (33a), (33b), and (33c) at the bottom of the next page. The derivation of the above expressions is straightforward but

tedious. In the Appendix a more detailed presentation is given. According to (11) the MGF for the decision variable, Z, is

$$M_Z(s) = M_\Lambda(e^s - 1). \tag{34}$$

The validity of the derived MGF can be tested by considering the following cases: 1) Only noise being present. The MGF for the decision variable is then given only in terms of the Fredholm determinant $D(s)$. We obtain the same result as for the well studied case of detecting purely incoherent light with a Lorentz spectral density, e.g., [21]. 2) If both signal and noise are present, then the mean and variance of the decision variable derived from (36) and those obtained from (37) and (38) are identical, as expected from the properties of the MGF.

C. The Gaussian Approximation

The Gaussian approximation to the error probability $P_e|_{\tilde{\mathcal{B}}}$[cf. (21)] is given by

$$P_e|_{\tilde{\mathcal{B}}} = \frac{1}{2} Q\left(\frac{E_1 - \alpha}{\sigma_1}\right) + \frac{1}{2} Q\left(\frac{\alpha - E_0}{\sigma_0}\right) \tag{35}$$

with $E_{0,1}$ and $\sigma_{0,1}^2$ being the mean and the variance of the decision variable for a transmitted binary symbol "zero" and "one," respectively. The function $Q(x)$ is the normalized Gaussian tail probability

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-s^2/2} \, ds.$$

If the MGF for Λ is known, the mean E_Λ and variance σ_Λ^2 are given by [15]

$$E_\Lambda = \frac{d[\ln M_\Lambda(s)]}{ds}\bigg|_{s=0}$$

$$\sigma_\Lambda^2 = \frac{d^2[\ln M_\Lambda(s)]}{ds^2}\bigg|_{s=0} \tag{36}$$

respectively. Alternatively, E_Λ and σ_Λ^2 are also given by the following relations:

$$E_\Lambda = \int_0^T |Y(t)|^2 \, dt + \int_0^T K(t,t) \, dt \tag{37}$$

and

$$\sigma_\Lambda^2 = 2 \iint_0^T Y(t)Y(u)K(t,u) \, dt \, du$$
$$+ \iint_0^T K^2(t,u) \, dt \, du \tag{38}$$

respectively. The mean and the variance of the decision variable Z are given in term of the mean and the variance of Λ according to the *Poisson transform* by [29]

$$E_Z = E_\Lambda$$
$$\sigma_Z^2 = E_\Lambda + \sigma_\Lambda^2.$$

For the case of the Lorentzian optical filter the covariance of the noise $X(t)$ is from (28)

$$K(\tau) = K(t - u)$$
$$= \sigma^2 \mu e^{-\mu|\tau|}. \tag{39}$$

The mean of Λ results to be

$$E_\Lambda = 4mGb_0^2 + \frac{8mG}{\mu T} b_0\rho(1 - e^{-\mu T})$$
$$+ \frac{2mG}{\mu T} \rho^2(1 - e^{-2\mu T}) + \sigma^2\mu T \tag{40}$$

and the variance

$$\sigma_\Lambda^2 = \frac{\sigma^4}{2} (2\mu T + e^{-2\mu T} - 1)$$
$$+ \frac{16mG\sigma^2}{\mu T} b_0^2(e^{-\mu T} + \mu T - 1)$$
$$+ \frac{8mG\sigma^2}{\mu T} \rho b_0(e^{-2\mu T} - 4e^{-\mu T} - 2\mu T e^{-\mu T} - 3)$$
$$+ \frac{4mG\sigma^2}{\mu T} \rho^2(1 - 2\mu T e^{-2\mu T} - e^{-2\mu T}). \tag{41}$$

IV. RESULTS

The Lorentzian filter is a causal filter [see (27)] and the ISI is caused by the bits preceding the information bit. We are going to examine the situation for two past information bits. Averaging over a larger sequence of past bits does not substantially changes the result for the average error probability [25], [36]. The computations are performed for the On–Off keying (OOK) modulation format with a value $G = 100$ and $n_{sp} = 1$. The observation time is the interval $[0, T]$. The value of ρ was calculated for all possible sequences $\mathcal{B} = \{b_{-2}, b_{-1}, b_0\}$ and the average error probability was evaluated by (26) using a saddlepoint approximation for each term. The receiver optimum threshold α, yielding the lowest error probability, is determined numerically.

The quantum limit for different values of the bandwidth bit-time product BT, yielded both by the saddlepoint and the Gaussian approximation, is displayed in Fig. 2. The quantum limit, with optimized BT $= 7$ and optimum decision threshold, is 49.9 [photons/bit] compared to the 38 [photons/bit] for a receiver with a matched optical filter [1]. The bounds on the error probability derived in [25] yielded a quantum limit of

$$F_1(s) = \mu Ts + \frac{2\mu Ts^2\sigma^2}{v^2} + \frac{4s^2\sigma^2\{2 - [(v+1)e^{\beta T} - (v-1)e^{-\beta T}]\}}{v^3[(v+1)^2e^{\beta T} - (v-1)^2e^{-\beta T}]} \tag{33a}$$

$$F_2(s) = 2s\left\{1 - \frac{4 - 2s\sigma^2[(v+2)e^{\beta T} - (v-2)e^{-\beta T}]}{v[(v+1)^2e^{\beta T} - (v-1)^2e^{-\beta T}]}\right\} \tag{33b}$$

$$F_3(s) = s\left\{\frac{1}{2} + \frac{\sigma^2 s(e^{\beta T} - e^{-\beta T}) - [(v-1)e^{\beta T} + (v+1)e^{-\beta T}]}{[(v+1)^2e^{\beta T} - (v-1)^2e^{-\beta T}]}\right\}. \tag{33c}$$
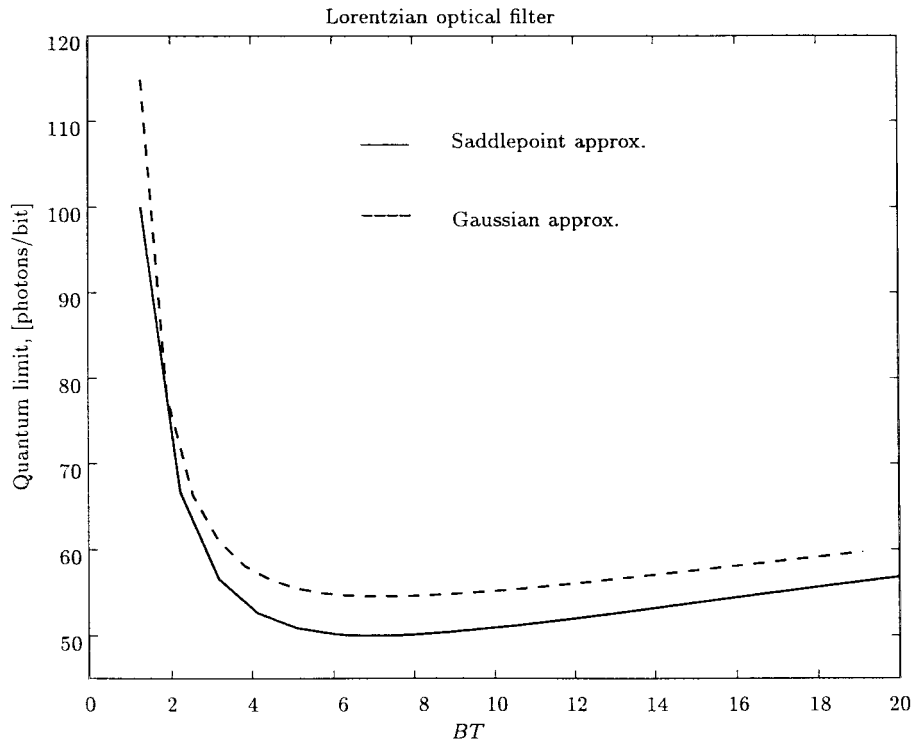
Fig. 2. The Quantum limit as a function of the bandwidth bit-time product BT. The solid line shows the results of the exact analysis (spa). The dotted line illustrates the results by the Gaussian approximation. $G = 100$, $n_{sp} = 1$.

56.5 [photons/bit] for an optimum BT $=$ 8. The quantum limit derived in [27] is 44.5 [photons/bit] for an optimum BT $=$ 3.7 and optimized observation time. Experimental results for a receiver with a value BT $=$ 7 reported a quantum limit of 76 [photons/bit] [14]. The present work predicts for this case a quantum limit of 49.9 [photons/bit], which is in good agreement with the experimental result, considering that penalties may be incurred in the postdetection signal treatment.

The Gaussian approximation, the dotted line in Fig. 2, gives a good estimate of the error probability. The resultant quantum limit is 54.5 compared to 49.9 [photons/bit] yielded by the exact analysis (spa). The Gaussian approximation also predicts the optimum bandwidth bit-time product with high degree of accuracy.

V. CONCLUSIONS

In this paper, the impact of ISI on the performance of optically preamplified, direct detection OOK receivers with a Lorentzian optical filter has been studied. A closed-form expression for the MGF of the decision variable has been derived. Bit-error probabilities have been calculated by the spa (exact analysis) and the Gaussian approximation. The optimum filter bandwidth, minimizing the bit-error probability, and the penalty incurred by using a nonmatched filter, Lorentzian, is found.

The Gaussian approximation predicts the performance of the optically preamplified receiver with good accuracy; see Fig. 2. The parameters required by the Gaussian approximation, the variance and the mean of the decision variable, may be found without the knowledge of the MGF. Different type of optical filters [covariance kernels $K(t, u)$] may be considered with no need of solving integral equations of the Fredholm type. Thus, optimum parameters of optically preamplified, OOK direct detection receiver may be determined by the simple method of the Gaussian approximation.

Although this paper deals only with OOK modulation format, the technique employed here can be used for receivers with other types of modulation. Independent additive noise contributions at the receiver can be incorporated in the exact analysis by just multiplying their MGF. The Gaussian approximation is expected to work well for modulation schemes with nonzero decision threshold [37].

APPENDIX

In this Appendix is presented the derivation of the MGF for the direct detection, optically preamplified receiver with an optical filter of the Lorentzian type.

Introducing the following auxiliary functions:

$$f_1(t;s) = (\beta + \mu)e^{\beta t} + (\beta - \mu)e^{-\beta t}$$
$$f_1(u;s;T) = (\beta + \mu)e^{\beta(T-u)} + (\beta - \mu)e^{-\beta(T-u)}$$
$$f_2(t;s;T) = (\beta + \mu)e^{\beta(T-t)} + (\beta - \mu)e^{-\beta(T-t)}$$
$$f_2(u;s) = (\beta + \mu)e^{\beta u} + (\beta - \mu)e^{-\beta u}$$

and

$$C(s;T) = \frac{\mu^2}{\beta[(\beta + \mu)^2 e^{\beta T} - (\beta - \mu)^2 e^{-\beta T}]}$$

$h(t, u; s; T)$ in (31) can be expressed as

$$h(t, u; s; T) = C(s;T)\{f_1(u;s;T)f_1(t;s)\theta(u - t) + [1 - \theta(u - t)]f_2(u;s)f_2(t;s;T)\}$$

with

$$\theta(t-u) = \begin{cases} 0 & t < u \\ 1 & t > u. \end{cases} \tag{42}$$

The moment generating function is expressed as [see (18)]

$$M_\Lambda(s) = [D(s)]^{-1} \exp[F(s)]$$

where

$$F(s) = m_h s + s^2 \int_0^T \int_0^T Y^*(t) h(t, u; s; T) Y(u) \, dt \, du \tag{43}$$

$$m_h = \int_0^T |Y(t)|^2 \, dt$$

and $D(s)$, the Fredholm determinant, is given by [22]

$$D(s) = \exp\left\{ -\int_0^s \int_0^T h(t, t; v; T) \, dt \, dv \right\}. \tag{44}$$

We start by integrating with respect to u in (19)

$$\mathcal{I}_1 = \int_0^T h(t, u; s; T) Y(u) \, du. \tag{45}$$

After substitution of (29) $\mathcal{I}_1$ can be expressed as

$$\mathcal{I}_1 = \sqrt{\frac{4mG}{T}} C(s; T) \{ f_1(t; s) \int_0^T \underbrace{f_1(u; s; T)(b_o + \rho e^{-\mu u})}_{g_1(u)}$$

$$\times \theta(u - t) \, du + f_2(t; s; T) \int_0^T [1 - \theta(u - t)]$$

$$\times \underbrace{f_2(u; s)(b_o + \rho e^{-\mu u})}_{g_2(u)} \, du. \tag{46}$$

We recall that

$$\int_0^T g(u) \theta(u - t) \, du = [G(u) - G(t)] \theta(u - t) \big|_0^T$$

$$\text{in our case } 0 \le t \le T$$

$$= G(T) - G(t) \tag{47}$$

where $G(t)$ is the primitive function of $g(t)$. Then

$$\mathcal{I}_1 = \sqrt{\frac{4mG}{T}} C(s; T) [f_1(t; s) G_1(T) - f_1(t; s) G_1(t)$$

$$- f_2(t; s; T) G_2(0) + f_2(t; s; T) G_2(u)]$$

$$\equiv \sqrt{\frac{4mG}{T}} C(s; T) [I_1^1 - I_2^1 - I_3^1 + I_4^1]. \tag{48}$$

The integration operation leading to I_n^1, $n = 1 \cdots 4.$, is straightforward but tedious. The resulting expressions contain many terms. In the derivation that follows we do not reproduce the long intermediate expressions, but focus on the main steps toward the final result for the desired MGF.

Continuing with the derivation, we now perform integration with respect to t

$$\int_0^T \int_0^T Y^*(t) h(t, u; s; T) Y(u) \, dt \, du$$

$$= \int_0^T \mathcal{I}_1 Y^*(t) \, dt$$

$$= \frac{4mG}{T} C(s; T) \int_0^T \mathcal{I}_1 (b_o + \rho e^{-\mu t}) \, dt.$$

The expression for the variable m_h turns out to be

$$m_h = \int_0^T |Y(t)|^2 \, dt$$

$$= \frac{4mG}{T} \int_0^T (b_o + \rho e^{-\mu t})^2 \, dt$$

$$= \frac{4mG}{\mu T} \left[b_o^2 \mu T + 2 b_o \rho (1 - e^{-\mu T}) + \frac{\rho^2}{2} (1 - e^{-2\mu T}) \right]. \tag{49}$$

Finally, rearranging common terms in b_o^2, $b_o \rho$, and ρ^2 we get

$$F(s) = \frac{4mG}{\mu T} [b_o^2 F_1(s) + b_o \rho F_2(s) + \rho^2 F_3(s)]$$

where $F_1(s)$, $F_2(s)$, and $F_3(s)$ are shown at the bottom of the page and the Fredholm determinant takes the form

$$D(s) = \frac{(v+1)^2 e^{\beta T} - (v-1)^2 e^{-\beta T}}{4 v e^{\mu T}}$$

with

$$v = \sqrt{1 - 2\sigma^2 s}$$

and

$$\beta = v\mu$$

The same result for $D(s)$ (considering the difference in notation) is given in an early paper by Helstrom [21].

$$F_1(s) = \mu T s + \frac{2\mu T s^2 \sigma^2}{v^2} + \frac{4 s^2 \sigma^2 \{ 2 - [(v+1) e^{\beta T} - (v-1) e^{-\beta T}] \}}{v^3 [(v+1)^2 e^{\beta T} - (v-1)^2 e^{-\beta T}]}$$

$$F_2(s) = 2s \left[1 - \frac{4 - 2s\sigma^2 [(v+2) e^{\beta T} - (v-2) e^{-\beta T}]}{v[(v+1)^2 e^{\beta T} - (v-1)^2 e^{-\beta T}]} \right]$$

$$F_3(s) = s \left\{ \frac{1}{2} + \frac{\sigma^2 s (e^{\beta T} - e^{-\beta T}) - [(v-1) e^{\beta T} + (v+1) e^{-\beta T}]}{[(v+1)^2 e^{\beta T} - (v-1)^2 e^{-\beta T}]} \right\}$$

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