# Exchange interaction in p-type GaAs/Al_{x}Ga_{1-x}As heterostructures studied by magnetotransport

# Exchange interaction in $p$-type GaAs/Al$_x$Ga$_{1-x}$As heterostructures studied by magnetotransport

M. Kemerink, P. M. Koenraad, and J. H. Wolter
*COBRA Interuniversity Research Institute, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
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Low-temperature magnetotransport experiments have been performed on a $p$-type GaAs/Al$_x$Ga$_{1-x}$As quantum well. From activation measurements on Shubnikov–de Haas conduction minima it was found that exchange interactions can be of great importance for both odd and even filling factors and strongly influence the observed periodicity. Furthermore, it was found that the temperature dependence of Shubnikov–de Haas oscillations in the low-magnetic-field regime could not be explained within a single-particle model based on a solution of the full Luttinger Hamiltonian in a magnetic field. Numerical simulations of Shubnikov–de Haas spectra, based on a model that treats hole exchange interactions in a simplified manner, show unambiguously that exchange driven enhancement of hole ''spin'' splittings are extremely important at magnetic fields as low as 1.5 T. Also, the inclusion of a valence-band warping in the calculations is shown to be essential. Qualitatively, most experimental observations could be described within the presented model. Our results imply that, in any hole system, the effective masses obtained from temperature-dependent SdH measurements are to be treated with extreme care as they can deviate from their single-particle value by as much as a factor of 2. [S0163-1829(98)01108-4]

## I. INTRODUCTION

In the past decades there has been considerable interest in the magnetic-field dependence of the electronic $g$ factor, $g^*$. An enhancement of the electron $g$ factor owing to exchange interactions was first proposed by Janak[1] to explain experiments by Fang and Stiles.[2] Later it was shown by Ando and Uemura[3] that $g^*$ should be an oscillatory function of the magnetic field with maxima at odd filling factors, i.e., when the Fermi level is in between the spin-up and -down states of a Landau level (LL), and minima at even filling factors. The physical idea behind this periodic $g$-factor enhancement is the following: At large magnetic fields the spin-up and -down states of a Landau level near the Fermi energy have different occupations and therefore experience different exchange energies, leading to an enhanced gap between the two spin states. This enhanced splitting is usually described in terms of an enhanced $g$ factor. At odd filling factors, with $E_F$ in between, say, $N\uparrow$ and $N\downarrow$, where $N$ is the Landau number, the occupation difference between the up and down levels is at a maximum, resulting in a maximum in $g^*$. From a similar reasoning the minimum in $g^*$ at even filling factors can be understood. At even filling factors the same effect should lead to an increase of the Landau-level splitting $\hbar\omega_c$. This effect is usually neglected as for the electronic LL, in most III/V semiconductors, the energy associated with this exchange interaction is much smaller than the LL spacing.

Experimentally, the exchange enhancement of $g^*$ has been studied in various donor-doped semiconductor heterostructures and in some of them oscillatory behavior has been reported.[4–6] To our knowledge, no experimental evidence has been reported for such effects in acceptor-doped heterostructures.

In this work we report on magnetotransport experiments on an acceptor-doped GaAs/Al$_x$Ga$_{1-x}$As quantum well. We find direct evidence that, in these structures, exchange effects are important at odd and even filling factors. Furthermore, we find from numerical simulations that exchange effects are extremely important at magnetic fields as low as 1.5 T. As a result, the effective mass that is determined from temperature-dependent Shubnikov–de Haas measurements should be treated with extreme care and can often be regarded as meaningless.

The remainder of this chapter is organized as follows. In Sec. II the experimental setup and results are discussed. In Sec. III we outline the model used for the numerical simulations that are presented and discussed in Sec. IV. Our conclusions are summarized in Sec. V.

## II. EXPERIMENTS

The experiments were performed on a single 89-Å GaAs/Al$_{0.45}$Ga$_{0.55}$As quantum well (QW). The sample is $p$-modulation doped with Be and grown by standard molecular-beam epitaxy (MBE) techniques on a (001) GaAs substrate. The carrier density, as obtained from Hall and SdH measurements, is $9.55\times10^{15}$ m$^{-2}$. The sample was wet etched into a standard Hall-bar geometry and contacted with Au/Zn or Au/Sn in-diffused contacts.

Measurements in the temperature range of 60 to about 900 mK were performed with the sample mounted on the cold finger of a dilution refrigerator. For the temperature range of 1.2 to 4.2 K a pumped bath cryostat was used. Magnetic fields up to 11 T were generated by means of a superconducting coil. To exclude undesired carrier heating the measurement current was kept more then an order of magnitude below the value at which heating effects became observable. Typical values for the channel current and sheet resistance were around 25 nA and 500 Ω, respectively. Furthermore, all wiring of the dilution refrigerator was equipped with low-pass filters to prevent heating by rf noise.

A low-temperature Shubnikov–de Haas (SdH) spectrum of the QW sample is displayed in Fig. 1. The SdH spectrum is very similar to those reported in publications on similar

 

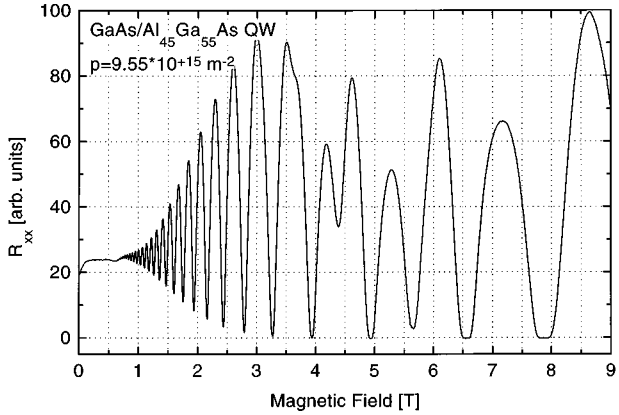


FIG. 1. Shubnikov–de Haas spectrum at 60 mK for the single quantum well.

structures by other groups.[7–9] By plotting extrema indexes vs $1/B$ (not shown) we find low- and high-field periodicities of 18.6 and 39.5 $T^{-1}$, respectively. The latter corresponds to the density that is deduced from the position of the center of the Hall plateaus. The observed periodicity doubling is obviously due to a lifting of the heavy hole spin ($m_J = \pm 3/2$) degeneracy at 4 T. The ratio of high- and low-field periodicities deviates from the expected exact value of 2, which is assigned to exchange effects, as will be discussed in Sec. IV.

In studying thermal activation of resistance minima three magnetic-field regimes can be identified. In the low-magnetic-field regime the densities of (extended) states (DOS) of many Landau levels overlap at the Fermi level. Only in this regime the Lifshitz-Kosevich[10] formula applies and effective masses can be extracted from the temperature dependence of the oscillation amplitude. The high-magnetic-field regime, on the other hand, is characterized by the existence of mobility gaps. These are regions in between well-separated Landau levels in which only localized states exist, and that, at low temperatures, give rise to plateaus in the Hall resistance and plateaus of zero conductance in the SdH spectra. As long as $k_B T$ is much smaller than this mobility gap, the temperature dependence of conduction minima will show linear activated behavior due to the thermal activation of carriers into the extended states.[11] In the intermediate-field regime the mobility gap is small or absent, and only the DOS of neighboring LL overlap. Here, activation measurements on conductance minima will only yield information about the shape of the DOS tails.

In order to analyze the thermal activation of SdH minima in the high- and intermediate-magnetic-field regimes one should, in principle, invert $R_{xx}$ to $S_{xx}$, using the well-known tensor relation $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$. In our experiments $\rho_{xx}$ is always at least one order of magnitude smaller than $\rho_{xy}$ for magnetic fields above 1.5 T, so we may safely assume proportionality between $\rho_{xx}$ and $\sigma_{xx}$. We therefore analyze $R_{xx}$, being the raw measurement data.

In Fig. 2(a) we have plotted the high-field minima at $\nu = 4$, 5, and 6 versus $1/T$, with $1.4 < T < 3$ K. Obviously there is a marked difference between the filling factors 4 and 6 on the one hand and 5 on the other. The linear activated behavior ($\nu = 4,6$) of resistance minima is what is to be expected in the absence of exchange effects. In the presence of exchange effects the same activation is at work, but then the separation between the two successive extended Landau levels, usually the spin-up and down states of one single LL, is—partially—due to the exchange and hence a function of their occupancy. As the temperature is raised the occupation difference between these states is decreased and, consequently, their separation is decreased. Since the exchange splitting is, in first order, linearly proportional to the occupation difference,[3] the total activation behavior will be quadratic in $T$. The dashed line through the data points of $\nu = 5$ is a fit to $A \exp[-(B/T)^2]$, validating the importance of exchange splitting for this conduction minimum. The behavior described

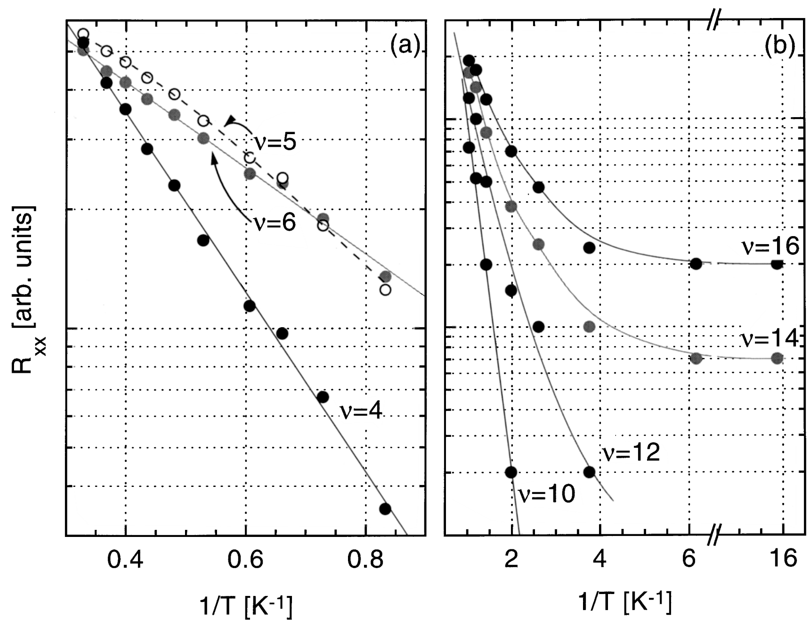


FIG. 2. (a) Temperature dependence of the resistance minima at $\nu = 4$, 5, and 6 for the quantum-well sample. The solid lines are linear least-squares fits, the dashed line is a fit with $A \exp[-(B/T)^2]$. (b) Same as panel (a), but for $\nu = 10, \ldots, 16$. Here, the solid lines guide the eye.

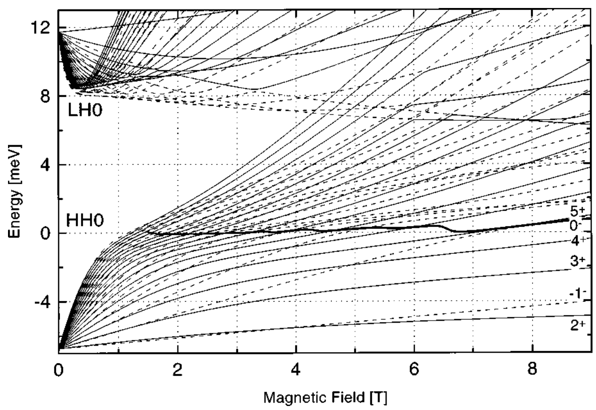


FIG. 3. Landau-level diagram for the quantum-well sample. Hole anisotropy has been included. Solid (dashed) lines indicate spin up (down) levels. For the meaning of the numbers see text. The fat lines denote the Fermi level.

for the $p$-type QW is the same as what is to be expected for $n$-type systems or any system in which the Landau splitting is much bigger than the Zeeman energy.

Interesting information can also be obtained from the thermal activation of filling factor minima in the intermediate-field regime. In Fig. 2(b) the SdH minima at $\nu = 10,...,16$ are depicted for temperatures below 1 K. Here, the solid lines are meant to guide the eye. The most remarkable feature of Fig. 2(b) is the nearly constant value of the resistance minima of $\nu = 14$ and 16 at the lowest temperatures. We find that the observed behavior cannot be explained in terms of simple, linear thermal activation nor be described in terms of the Lifshitz-Kosevich formula. The latter would predict a far stronger temperature dependence in the low-temperature regime when applied to the current minima. It is tempting to relate this nonlinear behavior to exchange effects, as for $\nu = 5$ in Fig. 2(a). In Sec. IV we will show that this indeed is the case.

## III. MODEL

The numerical calculation of SdH spectra consists of two parts. In the first part hole Landau levels are calculated that are used as input for the second part that calculates the actual $R_{xx}$ traces. Exchange effects on the spin splitting are included in the second part. Both steps will be outlined below in somewhat more detail.

The Landau-level energies $E_N$ were calculated from the $4\times 4$ Luttinger Hamiltonian according to a method outlined elsewhere.[12,13] Valence band anisotropy[14–16] (warping) was included using standard perturbation theory in which we used the unwarped LL as basis functions for the final, warped, levels. At zero magnetic field $B$, the energy levels and the electrostatic potential were calculated self-consistently. In this zero-magnetic-field calculation, exchange-correlation potentials that account for the complications in the valence bands were included.[17] The resulting fan chart for the quantum-well structure is shown in Fig. 3. The numbers in the figures follow the Broido/Sham[18] convention for hole Landau-level indexing. From the small Landau-level spacings at the Fermi level, one may draw the conclusion that, in the absence of exchange effects, the QW sample will show a very strong temperature dependence ($\propto$larger effective mass) of the SdH oscillation amplitude. One should bear in mind that the meaning of the effective mass that can be extracted from this temperature dependence is rather limited, considering the extremely nonlinear nature of the Landau levels.

The calculation of the Shubnikov–de Haas spectra follows in principle the work of Ando and Uemura.[19] According to Ref. 19 $\sigma_{xx}$ is given by

$$\sigma_{xx} = \frac{e^2}{h} \sum_{N,i} \int_0^\infty - \frac{df}{dE} \left[ \frac{\Gamma^{\mathrm{tr}}_{N,i}}{\Gamma_{N,i}} \right]^2 \left\{ \exp\left[ -2 \left( \frac{E - E_{N,i}}{\Gamma_{N,i}} \right)^2 \right] \right\} dE, \tag{1}$$

where $f(E)$ is the Fermi-Dirac distribution and the sum runs over all occupied subbands $i$ and Landau levels $N$. From the original Ando formulas[19] the following analytical expressions were derived:[20]

$$\left[ \frac{\Gamma_{N,i}}{\Gamma_i} \right]^2 = \frac{\sum_{k=0}^{N} \binom{N}{k}^2 \alpha^{4k}}{(\alpha^2+1)^{2N=1}}, \tag{2}$$

$$\left[ \frac{\Gamma^{\mathrm{tr}}_{N,i}}{\Gamma_i} \right]^2 = \frac{2N+1+\sum_{k=1}^{N} \binom{N}{k}^2 \alpha^{4k-2} \{[2(N-k)+1]\alpha^2 - 2k\}}{2(\alpha^2+1)^{2N+2}}.$$

The quantity $\alpha = d/l$ relates the magnetic length $l$ ($l^2 = \hbar/eB$) to the range of the scatterers $d$. $\Gamma_i$ is usually given by $\Gamma_i = [(2\hbar\omega_c\hbar)/(\pi\tau_0)]^{1/2}$, which corresponds to the Born approximation for $\delta$-shaped scatterers in low magnetic fields. In this work we used the value of $\Gamma_i$ at 1 T, $\Gamma^{1\,\mathrm{T}}_{\mathrm{ext},\uparrow/\downarrow}$, as a free parameter for each subband, of which we only assumed proportionality to $B^{1/2}$. Hereby, we neglected the influence of the possible magnetic-field dependence of the hole effective mass on $\Gamma_i$. Furthermore, it was shown in Ref. 21 that, even when the effective mass is independent of $B$, $\Gamma_i$ is no longer proportional to $B^{1/2}$ in high magnetic fields. The latter effect turned out to be unimportant in the range of fields in which we used our model.

In Eq. (1) we assumed a Gaussian profile of the LL instead of the semielliptic profile that was calculated by Ando and Uemura[19] and Xu and Vasilopoulos.[21] The reason was that it turned out to be impossible to produce Shubnikov–de Haas spectra that resemble the experimental ones even slightly when semielliptic profiles were used. This is in accordance with other experimental observations[22,23] and recent calculations based on a Gaussian random potential with long-range spatial correlations.[24]

A fraction $\epsilon$ of the carriers in each Landau level was assumed to be localized due to strong localization. The localized states were also assumed to have Gaussian-shaped profiles, with a width $\Gamma^{1\,\mathrm{T}}_{\mathrm{loc},\uparrow/\downarrow}$ at $B = 1$ T, that are centered at the Landau energy $E_N$. In simulating experimental SdH spectra it turned out that the best simulations were obtained when the width of the localized states was made so large that, effectively, a constant background of localized states arose. This, again, is in agreement with earlier experimental work.[22,23]

The longitudinal conductance $\sigma_{xx}$ calculated from Eq. (1) was transformed to the longitudinal resistance $\rho_{xx}$ using the standard tensor relations and the classical approximation for the Hall conductivity $\sigma_{xy}=ne/B$, with $n$ the total 2D carrier density. Shubnikov–de Haas effective masses in the low-magnetic-field regime were determined by calculating $\rho_{xx}$ as a function of temperature and analyzing the resulting traces with standard Fourier filtering techniques and the well-known formula

$$\frac{\Delta\rho_{xx}(B)}{\rho_0}=4\sum_{s=1}^{\infty}\exp\left[\frac{-\pi s}{\mu_{q,i}B}\right]\cos\left[\frac{2\pi s(E_F-E_i)}{\hbar\omega_c}-\pi s\right]\times\frac{sX}{\sinh(sX)}, \tag{3}$$

where $X=2\pi^2k_BT/\hbar\omega_c$.

In order to validate our model we calculated SdH masses by the procedure outlined above, but using linear LL as input, i.e., assuming parabolic bands. We found that the resulting masses were, at least up to two decimal places, the same as those used for the calculation of the LL. This agreement was totally independent of the used parameter set, as long as the minima in $\rho_{xx}$ did not reach zero. Using semielliptic broadening profiles instead of Gaussian profiles did not alter this agreement.

The effects of exchange on the splitting between spin-up and -down LL's were included *a posteriori*. We used a simplified version of the model proposed by Ando and Uemura,[3] which was derived for, and successfully applied to, electron systems.[4–6] The exchange energy of a Landau level $N$ is written as

$$E_{\mathrm{Ex}}^{N}=E_{\mathrm{Ex}}^{0}(n_{\uparrow}^{N}-n_{\downarrow}^{N}), \tag{4}$$

where $E_{\mathrm{Ex}}^{0}$ is only dependent on the magnetic field,[5,6] $E_{\mathrm{Ex}}^{0}=E_{\mathrm{Ex}}^{1\,\mathrm{T}}B^{1/2}$. $E_{\mathrm{Ex}}^{1\,\mathrm{T}}$ is used as a free parameter and $n_{\uparrow}^{N}$ and $n_{\downarrow}^{N}$ are the relative occupations of the two spin states of the $N$th Landau level. For hole LL's the question remains as to which $N$'s should be assigned to the various hole Landau levels. The only consistent way to do this is by assigning the lowest spin-up and -down Landau levels ($2^{+}$ and $-1^{-}$ in Fig. 3) to the up and down states of the $N=0$ Landau level, and by assigning subsequent up and down hole LL's to $N=1,2,\ldots$.

Several objections can be made against the application of Eq. (4) to a 2D hole gas. Bobbert *et al.*[17] have shown that the change from a spin doublet ($s=\pm 1/2$) for electrons to a spin quadruplet ($m_J=\pm 3/2, \pm 1/2$) for holes strongly affects the exchange interaction. Furthermore, in the derivation of Eq. (4) pure spin states were assumed. Due to the strong band mixing, spin is a poorly defined quantity in hole systems. Since there seems to be no theory on exchange in the Landau-level regime that includes these features, we will use Eq. (4) as an educated guess. In the next paragraph we will show that most experimental features can be explained qualitatively within the model outlined above.

Finally, it should be pointed out that the Fermi energy and the $E_{\mathrm{Ex}}^{N}$ are mutually dependent, and therefore form a self-consistency problem that is solved by iteration.

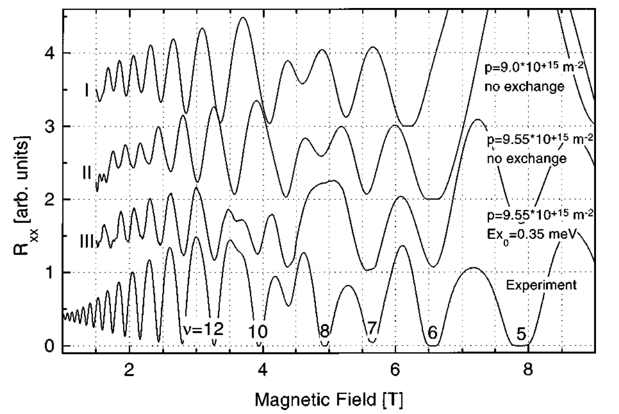


FIG. 4. Experimental and simulated Shubnikov–de Haas spectra of the single quantum well. The parameters used in the simulation are given in Table I.

## IV. NUMERICAL RESULTS AND DISCUSSION

The model outlined in the previous paragraph has been used to simulate the Shubnikov–de Haas spectrum of Fig. 1. The results for the quantum well are plotted in Fig. 4. The Landau levels that are displayed in Fig. 3 are used as input for the simulations; other parameters used are given in Table I. It turned out to be impossible to simulate both the high- and low-field regions of the experiment without including exchange interactions, see the upper two curves of Fig. 4. When we use the density from the high-field SdH periodicity, $9.55\times10^{15}$ m$^{-2}$, the low-field oscillations are exactly out of phase with the experiment. Using the low-field density, $9.0\times10^{15}$ m$^{-2}$, solves this deviation but introduces large deviations in the high-field regime. Only when we include exchange interactions in our model and use the high-field density, satisfying agreement with the experiment can be obtained in the whole magnetic-field range. It should be pointed out that when the low-field density is used in the simulations with exchange, the agreement with the experimental curve is very poor. From a comparison of the curves with $p=9.55\times10^{15}$ m$^{-2}$ with and without exchange (middle two curves of Fig. 4) we can conclude that the low-field resistance minima occur at even instead of odd filling factors due to an exchange-driven rearrangement of Landau levels. This implies that exchange interactions are important at magnetic fields as low as 2 T. Furthermore, we observe that the exchange indeed enhances the spin splitting at $\nu=5$, in agreement with our interpretation of the activation measurements in Fig. 2(a). At $\nu=8$ our simplified model fails in

TABLE I. Input parameters used in the calculations of Figs. 4 and 5. The roman numbers and the symbols refer to the curves in Fig. 4 and the symbols in Fig. 5, respectively. For all calculations we took $\epsilon=0.2$, $d=250$ Å and $\Gamma_{\mathrm{loc},\uparrow/\downarrow}^{1\,T}=1.5/1.5$ $meV$.

| | Calculation | | | |
|---|---|---|---|---|
| Parameter | I | II,× | III,* | + |
| $p$ [$10^{+15}$ m$^{-2}$] | 9.0 | 9.55 | 9.55 | 9.55 |
| $\Gamma_{\mathrm{Ex},\uparrow/\downarrow}^{1\,\mathrm{T}}$ [meV] | 0.28/0.28 | 0.25/0.25 | 0.55/0.55 | 0.28/0.28 |
| $E_{\mathrm{Ex}}^{1\,\mathrm{T}}$ [meV] | 0 | 0 | 0.35 | 0 |
| warping | yes | yes | yes | no |

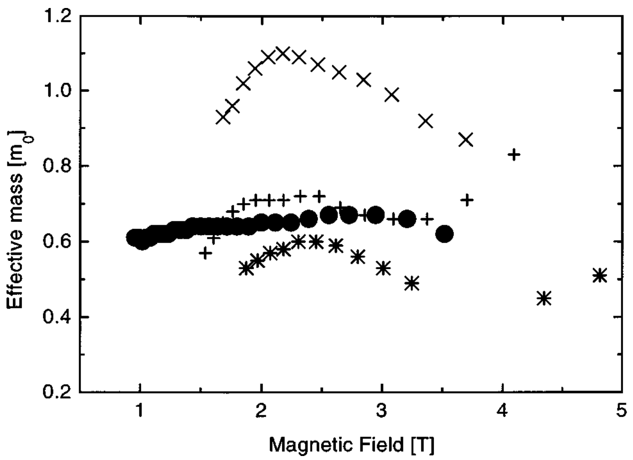


FIG. 5. Experimental (solid circles) and calculated effective masses vs magnetic field. The parameters used are the same as for the simulations of Fig. 4 (where appropriate) and are given in Table I, in which also the meaning of the other symbols is given. The effective masses plotted in this figure should only be regarded as a measure of the temperature dependence of the SdH oscillation amplitude; see text.

giving a proper description of the experiments. The reason for this is unclear at present.

If, as stated above, exchange is important in the low-magnetic-field regime, this should also be observable in the temperature dependence of the Shubnikov–de Haas oscillations. In Fig. 5 we plotted experimental and calculated effective masses versus magnetic field. Because of the nonlinear nature of the hole Landau levels and the presence of exchange interactions, the masses plotted in the figure should only be regarded as a measure of the temperature dependence of SdH oscillation amplitudes, i.e., as an indication of the splitting between the highest occupied and lowest unoccupied Landau level. As from the simulation of the raw low-temperature SdH spectra, we can conclude from the effective-mass calculations that the single-particle model, which neglects exchange effects, cannot explain the experimental observations. Once again it should be stated that the calculated effective masses do not depend significantly on any model parameter except $E_{\mathrm{Ex}}^{1\,\mathrm{T}}$. The good agreement between the experimental masses and those of the model calculations without warping and exchange $(+)$ is due to a cancellation of errors: When the band warping is included in the calculation of the hole Landau levels, an extra, repulsive interaction between the highest occupied (HO) and lowest occupied (LO) states is included.[14,16] This results in a ''compression'' of the HO Landau fan, which can be expressed in terms of an increased effective mass $(\times)$. This demonstrates that the inclusion of band warping in hole Landau-level calculations is not only essential for reproducing resistance minima in SdH experiments,[15] but also for the determination of single-particle masses. When exchange interactions are included in the calculations, spin-up and -down splittings at the Fermi level are increased. The thereby enhanced splitting between the highest occupied and lowest unoccupied Landau level is observed as a decrease of the effective mass $(*)$. For this reason we believe that the effective mass that is extracted from Shubnikov–de Haas measurements is rather meaningless. It should be mentioned that the calculated SdH mass is usually very different from the calculated cyclotron mass, $m_{\mathrm{cycl}}^* = \hbar eB/(E_{N\uparrow,\downarrow} - E_{N-1\uparrow,\downarrow})$, even when the exchange interaction is neglected in the former calculation.

A further confirmation of our interpretation of the present experiments can be obtained from the calculated thermal activation of resistance minima in the high- and intermediate-field regimes. In the discussion of Fig. 2(a) the nonlinear activation of the $\nu=5$ minimum was taken as an indication for an exchange-driven enhancement of a spin splitting. Our simulations reproduce the observed behavior qualitatively: The $\nu=4$ and 6 minima exhibit linear activated behavior, whereas the $\nu=5$ minimum indeed shows a pronounced quadratic activation, see Fig. 6(a). To produce a good local fit around $\nu=5$, including the broad plateau of zero resistance at 60 mK, we used $\Gamma_{\mathrm{ext},\uparrow}^{1\,\mathrm{T}}, \Gamma_{\mathrm{ext},\downarrow}^{1\,\mathrm{T}} = 0.25$ meV instead of 0.55 meV that gives the best overall simulation, and that therefore is used in the simulations of Figs. 4 and 5. It should be pointed out that also an increase of the exchange parameter $E_{\mathrm{Ex}}^{1\,\mathrm{T}}$ can be used to reproduce the zero-resistance plateau, but also in this case a quadratic temperature dependence is found. Since neither $E_{\mathrm{Ex}}^{1\,\mathrm{T}}$ nor $\Gamma_{\mathrm{ext},\uparrow}^{1\,\mathrm{T}}$ and $\Gamma_{\mathrm{ext},\downarrow}^{1\,\mathrm{T}}$ have a quantitative meaning in our model, and only the qualitative results of our model are important, this freedom in parameter choice is not troublesome.

In Sec. II the activation of resistance minima in the intermediate-field regime of the quantum-well SdH spectrum are discussed, and the claim was made that the extremely nonlinear activation of the $\nu=14$ and 16 minima [see Fig. 2(b)] is related to effects of exchange-enhanced spin splitting. In Fig. 6(b) the activation behavior of various minima in the same magnetic-field regime of calculated SdH spectra are shown. The same parameters have been used as in the calculation of curves II and III in Fig. 4. Note that for the calculation without exchange resistance minima occur at odd filling factors, in contrast to the calculation with exchange and the experiment. Apart from this, additional support for our claim can be extracted from a comparison of Fig. 6(b) with Fig. 2(b). It is clear that the saturation at low temperatures of the experimental SdH resistance minima at $\nu=14$ and 16 is qualitatively far better reproduced by the model calculations that include exchange. As in the intermediate-field regime no mobility gaps are present; the observed activation behavior is far more complicated than the simple $1/T$ or $1/T^2$ that is found in the high-field regime. We found that at the lowest temperatures, the observed (lack of) temperature dependence predominantly reflects the shape and width of the tails of the LL density of states. The essential parameter determining this $T$ dependence is therefore $\Gamma_{\mathrm{Ext}}^{1\,\mathrm{T}}$. In the calculations with and without exchange $\Gamma_{\mathrm{Ext}}^{1\,\mathrm{T}}$ equals 0.55 and 0.25 meV, respectively, giving rise to the lesser temperature dependence at low temperatures in the former case. The importance of exchange interaction lies in the fact that it enhances the Landau-level splitting at the Fermi level so that resistance oscillations are still observable, notwithstanding the fact that the Landau-level broadening is larger than the splitting in the single-particle calculation; see Fig. 3.

There are two last questions that we want to address with respect to these simulations. The first concerns the magnetic field in which exchange effects become important. Our observations indicate that rearrangement of Landau levels due to exchange interactions is essential for a proper description

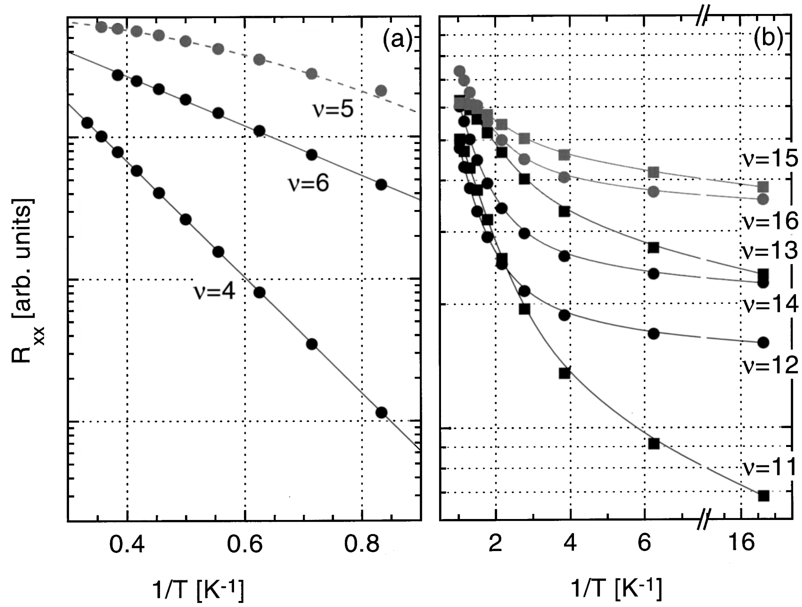


FIG. 6. (a) Calculated thermal activation of the conduction minima $\nu=4$, 5, and 6 of the quantum well, including exchange. As in Fig. 2(a), the $\nu=5$ minimum shows a quadratic activated behavior, indicative of exchange enhanced spin splitting. (b) Calculated activation of resistance minima in the intermediate-magnetic-field regime. The density in the quantum well is taken to be $9.55\times 10^{15}\ \mathrm{m}^{-2}$. The solid squares (circles) correspond to calculations without (with) exchange. The parameters used are the same as for the middle two curves in Fig. 4.

of the transport experiments on $p$-type heterostructures at *all* magnetic fields. This is in marked contrast with the situation for $n$-type heterostructures, for which exchange effects become only significant at much higher magnetic fields.[5] The reason for this contrast is that the hole spin splitting is not only due to the Zeeman effect, but also due to the different interactions with other bands for up and down hole levels. As a result, the total spin splitting at the Fermi level is already much larger than the Landau-level splitting for magnetic fields as low as 1 T, as can be seen in Fig. 3.

The second question concerns the universality of our results. In the above we have shown that for one particular GaAs/$Al_xGa_{1-x}$As QW exchange dominates the SdH resistance and the effective masses derived from its temperature dependence. It is, however, well known that, qualitatively, the dispersion relations and Landau levels of most GaAs/$Al_xGa_{1-x}$As heterostructures are the same. This also holds for the valence bands of most III/V-based heterostructures, in the sense that the hole $g$ factor and the interactions with other hole subbands lead to a much stronger spin splitting than in the corresponding electron bands. Consequently, it is very likely that the exchange interaction is important in magnetotransport experiments on these structures. We feel that one should therefore be extremely careful in using temperature-dependent SdH measurements as a tool to obtain the single-particle hole mass.

## V. CONCLUSIONS

We have performed magnetotransport measurements on a $p$-type quantum well. From the thermal activation behavior of Shubnikov–de Haas conduction minima we concluded that exchange enhancement of spin splittings can be important at both odd and *even* filling factors. Numerical simulations of Shubnikov–de Haas spectra, based on realistic Landau levels and a simplified model for the exchange interaction confirmed these observations. Furthermore, simulations showed that exchange interactions lead to a drastic rearrangement of hole Landau levels around the Fermi level at *all* magnetic fields, which is reflected in the observed Shubnikov–de Haas spectra. This conclusion is strengthened by the observed temperature dependence of the Shubnikov–de Haas oscillation amplitude in the low-magnetic-field regime. We found that the observed behavior cannot be described within the single-particle model in which the Landau levels are calculated. Inclusion of exchange effects in the model did greatly improve the agreement of the calculations with the experimental data. These findings, in combination with the extremely nonlinear nature of hole Landau levels, lead us to the conclusion that hole effective masses, deduced from temperature-dependent SdH measurements, should be treated with extreme care and often can be regarded as totally meaningless.

Although the simplified model that we applied to take the effects of exchange on the hole Landau levels into account seems to give a qualitatively correct description of our experimental findings, a more extensive theoretical model could greatly improve on the understanding of many-body effects in $p$-type heterostructures.

## ACKNOWLEDGMENTS

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