

Evaluation of three control concepts for the use of recipe flexibility in production planning

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**Evaluation of three control
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Evaluation of three control concepts for the use of recipe flexibility in production planning

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Abstract: Process industries often obtain their raw materials from mining or agricultural industries. These raw materials usually have variations in quality which often lead to variations in the recipes used for manufacturing a product. Another reason for varying the recipe is to minimize production costs by using the cheapest materials that still lead to a satisfactory quality in the product. A third reason for using recipe flexibility is that it may occur that at the time of production not all materials for the standard recipe are available. In earlier research we showed under what conditions the use of this type of recipe flexibility should be preferred to the use of high materials stock to avoid materials shortages. We showed that the use of recipe flexibility to account for material shortages can be justified if the material replenishment leadtime is long, the demand uncertainty is high and the required service level is high. In this paper we assume that these conditions are satisfied and we investigate three different concepts for coping with the certainty and uncertainty in demand and supply. The first concept optimizes material use over the accepted customer orders (assuming that the customer order leadtime is small compared to the material replenishment leadtime); the second concept optimizes material use over the customers orders plus expected customer orders over the material replenishment leadtime; the third concept optimizes material use of the customers orders taking into account the effect of the remaining stock positions on the future recipe costs, based on knowledge of the distribution function of demand. These three concepts are investigated via an experimental design of computer simulations of an elementary small scale model of the production planning situation. The results show that the third concept outperforms the second and first concept. Furthermore, for a realistic cost structure in feed industry under certain circumstances the use of the third concept might lead to a 4% increase in profit. However, this improvement must be weighted against the cost incurred by the operational use of this complex concept. Based on this considerations and the numerical results in this paper, we may expect that for most situations in practice the use of the first simple myopic concept, optimizing material use only over the available customer orders, will be justified from an overall cost point of view.

1. Introduction

During the last decade, various articles have been published on production control in process industries. Most of these articles focus on the typical characteristics of process industry as compared to the discrete manufacturing situations. In this body of literature two extreme types of process industry can be distinguished; the process/flow industry and the batch/mix industry (Fransoo and Rutten 1994). Process/flow is defined as: a manufacturer who produces with minimal interruptions in any one production run or between production runs of products which exhibit process characteristics such as liquids, fibers, powders, gases etc. Batch/mix is defined as a process business which primarily schedules short production runs of products (Connor 1986). In this paper we concentrate on the batch/mix process industry.

Batch/mix process industries often obtain their raw materials from mining or from agricultural industries. These raw materials have natural variations in quality. For example, crude oils from different oil fields have different sulfur contents and different proportions of naphtha, distillates and fuel oils. Oil refinery designs, production plans and operating schedules must account for this variability in crude oil qualities (Taylor et al. 1981). May (1984) observed that material variability implies that the real characteristics of the material are usually not known until the production process is started. The specific quality of a batch of raw material sometimes even determines which product will be produced out of it (Rice and Norback 1987).

Variations in raw material quality often lead to variations in bills of material (or recipes) (May 1984; Cokins 1988). For example, variation in the moisture contents, acidity, viscosity or concentration of active ingredients in different raw materials may cause variations in raw material proportions required to make a finished product according to the quality specifications (Taylor et al. 1981). This variation in raw material quality is one reason for using recipe flexibility.

A second reason for using flexible recipes is to minimize the total materials costs to produce the finished product. For each production order a recipe is determined such that the finished product quality specification is met with a combination of available raw materials which produces least costs. For example, a pet food may have specifications for the minimum amount of protein, carbohydrates and fat per pound of pet food; how-

ever, the proportions of various materials may be varied depending on their current price, quality and availability. Here recipe flexibility is used to minimize product costs.

A third reason for varying the recipe of a product is that at the time of production one or more of the raw materials which are required for the standard recipe are not available. An option could be to postpone production until all materials are available, but often this is not allowed because of customer service requirements. Generally, the finished products are commodities which can be supplied by many manufacturers and for which a short, standard leadtime must be used in the market place to maintain competitiveness (Rutten 1995). Thus postponing production often is not a realistic option. The other option is to produce the finished product with a different recipe. This leads to higher product costs since the recipe with the minimum costs will be the standard recipe. However this standard recipe can only be applied if all raw materials required are available. In an earlier paper (Rutten and Bertrand 1997) we studied the question under what conditions, or to what extent, raw materials safety stocks should be used to cope with uncertainty, and under what conditions, or to what extent, alternative more expensive recipes should be used. We found that the use of recipe flexibility will be soon profitable when a high service level is demanded and a long leadtime for raw materials exists.

Now consider the situation where the use of recipe flexibility is profitable (Rutten and Bertrand 1997). We therefore assume a situation with long materials replenishment leadtimes, high demand uncertainty and high service level requirements. The next question is whether the recipes for the successive production orders should be determined independently, using the then available materials (the single-blend concept), or whether the recipes for a number of successive production orders should be determined simultaneously, using the joint constraints on available materials (the multi-blend concept). The use of a multi-blend model, that considers several production orders simultaneously, will always result in a better use of available raw materials. However, the multi-blend concept assumes that over a sufficient horizon production orders are known with certainty. Analysis of order portfolio's in these kind of industries shows that, generally, only over a short horizon production orders are certain, as is shown in Figure 1 (we assume that a customer order is synonymous to a production order).

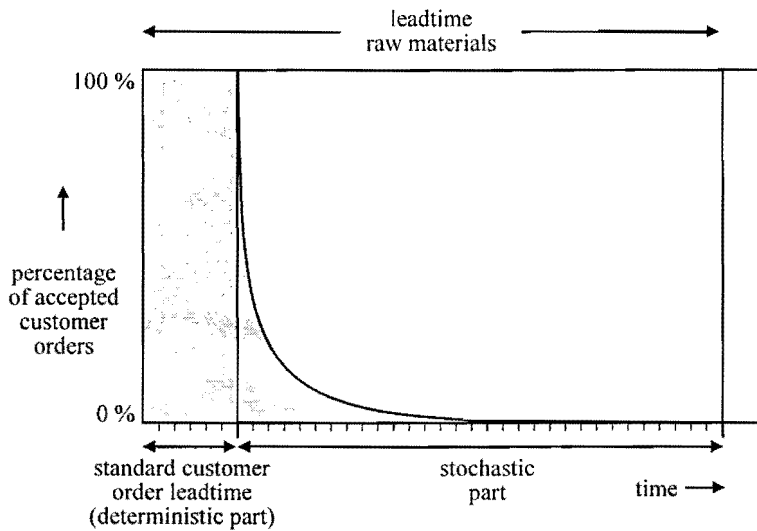


Figure 1 Accepted customer orders in time.

As shown in Figure 1, two parts can be distinguished within the leadtime of raw materials; *a deterministic part*, that is equal to the standard customer order leadtime, in which all production orders are known for certain and *a stochastic part* in which none or only a few production orders are known. The stochastic part is equal to the raw materials leadtime minus the standard customer order leadtime. Generally, the deterministic part is very short (a few days). Nevertheless, previous research has shown that using the multi-blend concept, even over this short period, will give an increase in performance when compared to a single-blend model (Rutten 1995). However, the multi-blend optimization over the deterministic part does not consider the terminal conditions regarding materials availability that result at the end of the deterministic part. This may result in situations where the starting position of the materials availability for the next period is poor. For instance, a scarce material has been totally used up during the deterministic part, whereas there is a high probability that some customers orders that will likely be placed in the next period cannot be produced without it. Thus we would like to apply terminal conditions to the optimizations, or to extend the optimization beyond the deterministic part, for instance by using pseudo-orders based on forecasted demand.

An interesting question is therefore whether or not the use of raw materials can be improved by extending the horizon over which materials are allocated to production orders beyond the deterministic part. This requires the use of forecasted production orders and will result in a rolling planning approach to the materials allocation problem

(Baker 1981). The result of the optimization over the entire horizon is implemented for the first day of the deterministic part. After this day, new orders will have arrived and again an optimization is done for the allocation of the remaining available materials to the production orders over the entire horizon. As has been shown in the literature on the use of rolling schedules in production planning, the marginal benefits of an increase in the planning horizon decreases with the horizon length (Baker 1977). We may expect the same to be the case for the allocation of materials to orders. We assume that the ‘optimal’ planning horizon will be somewhere between the end of the deterministic part and the end of the stochastic part (materials leadtime).

A related question is which method should be used for forecasting the production orders in the stochastic part. Since the real production orders are not known for certain over the entire horizon, the use of a deterministic technique, such as a multi-blend optimization, seems less appropriate and more advanced techniques might be worth considering.

In this article, we compare the use of three planning concepts for materials use. In the first concept we will use a multi-blend model for the deterministic part. This gives a basic performance. In the second concept we use a simple multi-blend optimization for the complete leadtime of raw materials. The stochastic part is filled up with forecasted production orders that reflect average demand. The third concept uses a more complex stochastic model that balances the recipe costs in the deterministic part and the expected value of the recipe costs in the stochastic part that result from the terminal stock levels at the end of the deterministic part. In this third concept we use the distribution function of the forecasted demand over the stochastic part to calculate the expected value of the recipe costs. The second concept only uses information about average demand and therefore implicitly assumes that the problem is certainty equivalent (see Holt et al. 1960). Contrary to the second concept, the third concept uses information about the distribution function of demand and assumes no certainty equivalence.

We will investigate the performance obtained by these planning concepts by systematic simulations of their application to an elementary situation. This elementary situation is presented in Section 2. In Section 3 the three planning concepts are described in more detail. Next in Section 4 the experimental design of the simulations

studies is presented. The results of the simulations are discussed in Section 5 and Section 6 completes the paper with the conclusions.

2. The elementary material allocation situation

This study is based on a company that is part of a large dairy corporation. The company manufactures milk replacers for calves. A milk replacer is a powder that can replace mother's milk when it is dissolved in water. Approximately 100 different types of milk replacers are being made on order, with a total annual amount of 60,000 metric tons. A milk replacer is produced by blending a number of raw materials (powders). Every product is defined by its recipe definition, some constraints on raw materials and on their properties (e.g. fat and protein). In most cases a recipe uses six or seven raw materials. The final recipe for an individual order is computed with use of linear programming. The results of this computation (quantities of raw materials) are used by the process computer to control the production facilities in the factory. Figure 2 shows the production process.

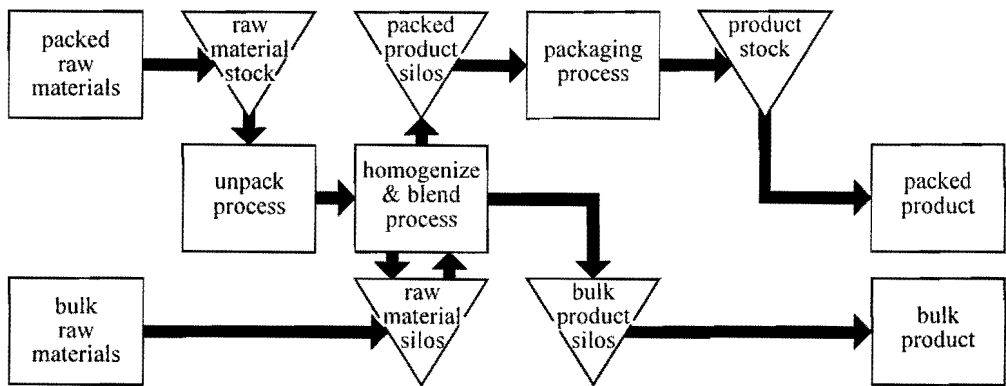


Figure 2 General flows of material in milk replacer manufacturing. Raw materials arrive in two different forms; bulk or packed. Raw materials are homogenized in a mixer before they can be used for production and then are stored in a silo. To produce a product, the computed quantities of raw materials are collected in a mixer and blended. The products can be delivered in two different forms; bulk or packed.

The main production process consists of blending raw materials in a mixer. The raw materials and final products are stored in silos. To prevent the powder from sticking, the complete factory is climate conditioned. There are 48 raw materials silos of 25

tons each, which makes the total inventory capacity approximately 1,200 metric tons. In practice, the average inventory in the silos is near 800 metric tons. The runout time of the factory (= the time the factory can produce without new deliveries of raw material) is only two days, because of the heavy usage of some raw materials. Besides the silos, a large warehouse is available for stocking materials. In this warehouse raw materials first must be packed in bags and when needed they have to be unpacked again, which gives extra handling costs.

Since quality variations occur in raw materials, each delivery of a raw material is stocked in a separate (empty) silo. Before the raw material is stocked, it is homogenized. Homogenizing is mixing a single delivery of raw material to create a homogeneous raw material. After stocking, a sample is taken and tested in the laboratory for determining the exact properties. After stocking a raw material in a silo and after the results of the laboratory tests are known, a raw material is available for production.

Inventory is controlled via an (R,s,Q) system (cf. Silver and Peterson 1985). Periodically the inventory levels are examined. If the inventory level falls below a historically determined level, a quantity Q is ordered. A large part of the twenty main raw materials is delivered by another firm of the corporation. This firm processes raw milk into various products. Some of the residues of this process are the raw materials for the milk replacers manufacturer. It may occur that more raw material is produced than is ordered by the milk replacers manufacturer. Then the agreement is that the milk replacers manufacturer accepts all raw materials that are sent by the other firm. This sometimes can cause an overflow of certain raw materials. A small part of the raw materials is ordered externally, in which case the firm can control the quality and quantity to be delivered.

A production plan is made once a week. Orders are scheduled for production one day before delivery, in order to have sufficient time to analyze and inspect the product. The total weight of the orders per day must stay below the capacity of the available mixers. Next, the customer orders are translated to production orders, which means that the customer order is split into parts of 25 metric tons (volume of the mixer). Finally, the production plan is put into the process computer, together with the recipe definition per product. Just before the production process of an order starts, the raw materials use of the order is optimized within the limits of the currently available raw materials.

The typical production control characteristics in this case situation are:

- Production is on order.
- Customer orders are all accepted one week in advance, some are accepted two to three weeks ahead (according to Figure 1).
- Shortages are not allowed in the market, due to the strong competition. Therefore, shortage costs are very high.
- The leadtime for replenishing raw materials varies between two and three weeks.
- Raw materials arrive with variable properties, and are not homogeneous.
- There is a push of some of the raw materials from another firm of the corporation.
- The factory has a high investment level, because of the silos and the climate control system. Thus inventory is expensive, not so much because of the materials costs, but mainly because of the investments required for stock keeping facilities.
- The usable stocking capacity is limited. External stocking is possible but will imply additional (handling) costs.
- Inventories are controlled by means of an (R,s,Q) system. The reorder levels are determined historically.
- Orders are optimized just before production of an order starts. Orders are optimized one by one (single-blend).

For most companies that use recipe flexibility like the milk replacers company, determining the best recipe is very complex and usually some kind of optimization technique is used. This complexity obstructs a clear analysis of the use of recipe flexibility.

In order to gain insight in the different ways to use recipe flexibility, we will study the most simple materials allocation situation that still contains the essence of the problem. Rutten (1995) showed that the simplest situation in which recipe flexibility can be used, is a situation where four products are manufactured using three raw materials, as is depicted in Figure 3.

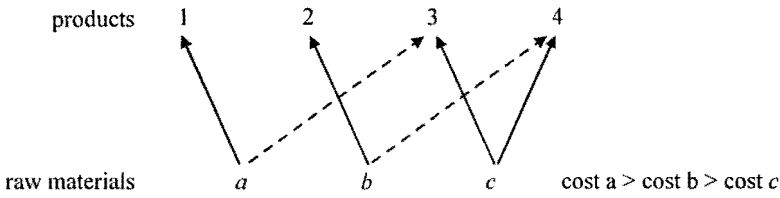


Figure 3 Four products are manufactured using three raw materials. The solid lines indicate the standard recipes and the dotted lines indicate the alternative recipes.

Behind this graphical presentation, a definition in terms of requirements of properties exists, as given in Table 1. Each product demands a minimum and/or maximum amount of three properties. The three raw materials each contain one or two of the properties. The cost per unit of raw material *a* is highest, next in cost is raw material *b* and raw material *c* is the cheapest. Due to this definition, the recipes of products 1 and 2 are fixed (raw material *a* respectively *b*) and products 3 and 4 both use raw material *c* as the standard recipe, but the alternatives recipes of these two products differ (raw material *a* respectively *b*).

Table 1 Product and raw material definitions

property	minimum amount of property in product (%)				maximum amount of property in product (%)				amount of property in raw material (%)		
	1	2	3	4	1	2	3	4	<i>a</i>	<i>b</i>	<i>c</i>
1	10	0	0	0	100	100	100	0	10	0	0
2	0	10	0	0	100	100	0	100	0	10	0
3	0	0	10	10	100	100	100	100	10	10	10

3. The three planning concepts

In this research we investigate the performance of three different material planning concepts. As a reference point we use the performance of the single-blend recipe optimization. The first concept uses a multi-blend model over the customer order lead-time (deterministic part). In the second concept we use a multi-blend optimization that

covers the replenishment leadtime of raw materials. In the third concept we balance the recipe costs in the deterministic part and the expected value of recipe costs in the stochastic part. We assume that the deterministic part is at least one day, but much smaller than the replenishment leadtime of raw materials.

THE DETERMINISTIC PART MULTI-BLEND CONCEPT (DP-MULTI)

The basic multi-blend model simultaneously considers all production orders within the deterministic part. Available raw materials are shared between these production orders. The production orders that are planned on the first day of the deterministic part are manufactured (and the used raw materials are subtracted from available inventory). This can lead to raw materials being preserved from usage on the first day, because they can be used more profitable on later days within the deterministic part.

THE TOTAL PART MULTI-BLEND CONCEPT (TOT-MULTI)

The second multi-blend model covers the complete leadtime of raw materials. Production orders during the stochastic part are based on average demand. Optimization is based on the available raw material stocks and the time-phased deliveries of raw materials during the entire horizon. The production orders that are planned on the first day are manufactured using the raw materials assigned to them (the used raw materials are subtracted from available stock and the raw materials delivered during the first day are added to the available stock).

THE BALANCE CONCEPT (BALANCE)

The more complex BALANCE concept consist of several steps:

1. First, for the deterministic part a multi-blend optimization is calculated, similar to the DP-MULTI-concept. The stock that is available at the start of the stochastic part is calculated by subtracting the used raw materials and adding the delivered raw materials during the deterministic part. This we call the deterministic part optimal terminal stock position.
2. Second we define a set of terminal stock positions in the neighborhood of the deterministic part optimal terminal stock position. With each of these terminal stock positions as a constraint, we again carry out the multi-blend optimization for the deterministic part, and we register the increase in recipe costs associated with the use of the terminal condition as a constraint.

3. In the third step for each value of the terminal stock position (including the deterministic part optimal position) the expected value of the material use in the stochastic part is calculated, based on the distribution function of demand. For the simplified situation, this can be done very accurate (see Appendix A). For each possible realization of future demand, the optimal use of raw materials can be calculated and the associated recipe costs for the stochastic part can be determined. Weighing the recipe costs associated with each value of the future demand with the probability of this demand occurring, gives the expected value of the recipe costs, given a certain terminal stock position. This step is repeated for each terminal stock position at the end of the deterministic part (determined in the second step). Thus for each possible terminal stock position, the expected value of the recipe costs in the stochastic part can be determined.
4. The total expected costs using a certain terminal stock position can be calculated by adding up the recipe costs during the deterministic and the stochastic part. The 'optimal' material use plan is the plan with the minimum total expected costs.
5. The production orders for the first day of the deterministic part of this optimal plan are carried out.

4. Experimental design of simulations

We use systematic simulation to investigate the performance of the three planning concepts. In the simulations some parameters of the problem will be kept constant and other parameters will be varied. In a previous article (Rutten and Bertrand 1997), we found that a long leadtime of raw materials and a high target service level are two significant factors that favor the use of recipe flexibility. Therefore we will only investigate problems with a long raw material leadtime and a high target service level.

PARAMETERS VARIED IN THE SIMULATIONS

The length of the deterministic part will be varied, since we expect that the differences between the concepts become smaller for larger values of the length of the deterministic part. This means that when the deterministic part is larger than one day, scheduled receipts due to material replenishment orders will be available for allocation after the first day. If variation in raw materials quality occurs, then scheduled receipts due to

the replenishment orders as used in the production planning, may deviate from the actual future deliveries. This increases the uncertainty in the planning situation. Therefore the factor ‘variation in raw materials quality’ also should be included in the simulations.

Furthermore, the two concepts that optimize over the total leadtime of raw materials (the latter two concepts) differ in the way demand is forecasted; the TOT-MULTI concept takes the expected value of demand and treats this as certain, while the BALANCE concept uses the complete probability density function of demand. Including the factor ‘coefficient of variation of demand’ for products 3 and 4 should enable us to identify a difference in performance between using these two concepts.

Summarizing, we have three parameters that will be varied in the simulations. The experimental design is depicted in Table 2. For each parameter we use at least two values. For the situation in which we expect the largest differences between concepts, simulations are also performed for a length of the deterministic part of 3 and 4 days. The other parameters in the simulation model will be fixed during the simulations; the leadtime of raw materials is 16 days, mean demand for each product is 400 units per day, and the coefficient of variation in demand for products 1 and 2 is 1.00.

Table 2 Design matrix for comparing the performance of the concepts.

Run	Variation in raw materials quality (= probability of right quality)	Coefficient of variation in demand for products 3 and 4	Length of the deterministic part (=customer order leadtime)
1	0.90	1.00	1
2	0.90	1.00	2
3	0.90	1.00	3
4	0.90	1.00	4
5	1.00	1.00	1
6	1.00	1.00	2
7	0.90	0.50	1
8	0.90	0.50	2
9	1.00	0.50	1
10	1.00	0.50	2

RAW MATERIAL COSTS

For the use of recipe flexibility to be justified, the costs of raw materials are chosen within the boundaries as given in Rutten and Bertrand (1997). However, since the balancing concept uses the actual costs in the calculations for the stochastic part, the use of alternative raw materials will depend on the specific material cost structure. To account for this factor, we use three different raw material cost structures. In Table 3, the set of three raw material costs is given that corresponds to each setting. For the BALANCE concept, the 10 runs of the design are simulated in every setting (high, medium and low).

Table 3 Settings of raw material costs (per unit) as used in the simulations. In the high, medium respectively low setting, the relation $(c_a - c_c)/(c_b - c_c)$ equals 100, 10 respectively 2.

Setting	High	Medium	Low
cost of raw material a	1.2206	1.1791	1.0861
cost of raw material b	1.0022	1.0178	1.0426
cost of raw material c	1.0000	1.0000	1.0000

THE PERFORMANCE MEASURE

In order to be able to uniquely order the performance of using the concepts, we need one single performance measure. The simulation model will give several measures; the α service level per product, the usage of alternative raw materials and the mean inventory levels. To order the concepts, we have to aggregate these measures into one measure.

The use of alternative recipes and the inventory levels can be expressed in annual costs. However, it is difficult (or even impossible) to translate the α service level into costs. To avoid this problem we decided to increase the target service level to 100%. After all, a high target performance level is a necessary condition for justifying the use of recipe flexibility (Rutten and Bertrand 1997). Furthermore, we are interested in the effects of differences in alternative recipe use per concept. Therefore, we made the available inventory of raw materials a and b infinite. This results in an α service level of 100% for each product, because there will be always inventory available (standard or alternative) for each product. In that case, the α service level is no longer an interesting output value. The inventory of raw material c will be limited (the order-up-to level is calculated to achieve a service level of 90% under the fixed recipe regime). In this way, recipe flexibility will still be used and the outputs of the simulation can be translated into one output for each concept; the annual costs needed to realize the target service.

T-TEST VALUE

To determine differences between two concepts, each concept is simulated for 250,000 days, divided in 25 subruns of 10,000 days each. These subruns can be considered to be independent (Von Neumann statistic; Kleijnen and Van Groenendaal 1992) and hence we have 25 replications of each run. We use common random numbers for the simulations of the concepts, hence each concept deals with the same sequence of customer orders. We can test whether a difference in performance between two concepts exists by creating a new sample that exists of the differences between the 25 observations and next we can use the Student's t statistic to test whether this new sample is significantly different from zero. We wish to test at a significance level of 90%, 95% and 99%; the experimentwise error rate equals 10%, 5% and 1%. Since we compare 3 concepts, we make 3 comparisons (multiple comparison, see Kleijnen 1987) and therefore have to use a per comparison error rate which is the experimentwise error rate divided by 3 (Bonferroni inequality). The resulting per comparison error rates can not be found in the available tables. We therefore used linear interpolation to determine the t -test value per comparison; they were determined at respectively 2.31, 2.60 and 3.30. The calculated t -value must be larger than these values to be significant.

5. Simulation results

We simulated each of the ten runs of the design for each concept. The α service level was 100% in every run, due to the unlimited availability of raw materials a and b . The order-up-to level of raw material c was the same in all concepts for each run, thus the inventory costs are the same for all concepts. The use of alternative recipes for products 3 and 4, however, differs per concept and is given in Table 4 for the highest cost setting (the complete simulation results are given in Appendix B).

Table 4 Fraction alternative recipe use, as measured in the simulations. The results of the BALANCE concept are presented for the highest cost setting (see Table 3).

Run	Fraction alternative in product 3 (%)				Fraction alternative in product 4 (%)			
	single-blend	DP-MULTI	TOT-MULTI	BALANCE	single-blend	DP-MULTI	TOT-MULTI	BALANCE
1	3.3218	3.3218	3.0050	2.3407	7.1125	7.1125	7.5317	10.6922
2	3.3218	2.3593	2.2493	1.7181	7.1125	8.2766	8.4072	10.3870
3	3.3218	1.9086	1.7850	1.5500	7.1125	8.6297	8.6026	9.9956
4	3.3218	1.6352	1.6143	1.3540	7.1125	8.8519	8.8768	9.5905
5	1.4209	1.4209	1.2815	0.8305	3.7863	3.7863	3.9440	7.1569
6	1.4209	0.7386	0.6944	0.4815	3.7863	4.4691	4.5146	6.5377
7	2.4319	2.4319	2.2293	1.4905	7.7708	7.7708	8.0324	10.7809
8	2.4319	1.9379	1.9017	1.3968	7.7708	8.3929	8.4849	9.6679
9	0.2064	0.2064	0.1980	0.0674	2.4362	2.4362	2.4450	4.8084
10	0.2064	0.0425	0.0411	0.0195	2.4362	2.6004	2.6017	3.3156

As can be seen in Table 4, in runs 1 to 4 (increasing length of the deterministic part) the multi-blend concepts decrease the use of alternative in product 3 (the most expensive alternative) meanwhile increasing the use of alternative in product 4. The resulting use of alternative recipes was aggregated into one output ‘the change in additional recipe costs’, as follows:

- First we calculate the *base cost* of additional recipe costs, which is the cost due to the use of alternative recipes when using the single-blend optimization (the reference point). Equation (1) shows the calculation of the additional recipe costs (c_r denotes the cost of raw material r , D_i denotes annual demand for product i ($D_i = 400 \times 250$ days) and fa_i denotes the fraction alternative (%) used in product i).

$$(c_a - c_c)D_3 \frac{fa_3}{100} + (c_b - c_c)D_4 \frac{fa_4}{100} \quad (1)$$

- Next, we calculate the *cost of concept_i* which is the cost due to the use of alternative recipe use when using concept *i* (*i* = DP-MULTI, TOT-MULTI, BALANCE). Again Equation (1) is used with the specific values of *fa_i* for the current concept *i*.
- Finally, the decrease of additional recipe costs due to the use of concept *i* is then calculated as in Equation (2):

$$\text{Decrease of additional recipe costs}_i = \frac{\text{BaseCost} - \text{Cost of Concept}_i}{\text{BaseCost}} \quad (2)$$

The decrease of additional recipe costs for each concept is given in Table 5.

Table 5 Decrease of additional recipe costs according to Equation (2) for the highest cost setting.

Run	DP-MULTI	TOT-MULTI	BALANCE
1	0.00%	9.22%	27.87%
2	28.03%	31.23%	46.31%
3	41.21%	44.86%	51.38%
4	49.20%	49.81%	57.27%
5	0.00%	9.45%	38.17%
6	46.31%	49.31%	62.52%
7	0.00%	7.97%	36.32%
8	19.44%	20.84%	40.49%
9	0.00%	3.60%	50.02%
10	70.35%	70.94%	77.21%

6. Discussion of simulation results

The following observations can be made from the simulation results. In general, a multi-blend concept performs better over a larger horizon, as may be expected (length deterministic part; compare runs 1 to 4). However, the marginal increase in performance (= decrease of additional recipe costs) of the multi-blend concepts becomes smaller as

the length of the deterministic part increases, as is depicted in Figure 4. Nevertheless, the change in costs per concept is statistically significant for all runs given, but decreases every time the deterministic part becomes larger. Baker (1977) already observed that the value of extra information at the end of the planning horizon decreases as a function of the planning horizon. This means that for a situation in which many customer orders are accepted in advance (large deterministic part), it may not always be necessary to include all these orders in the optimization.

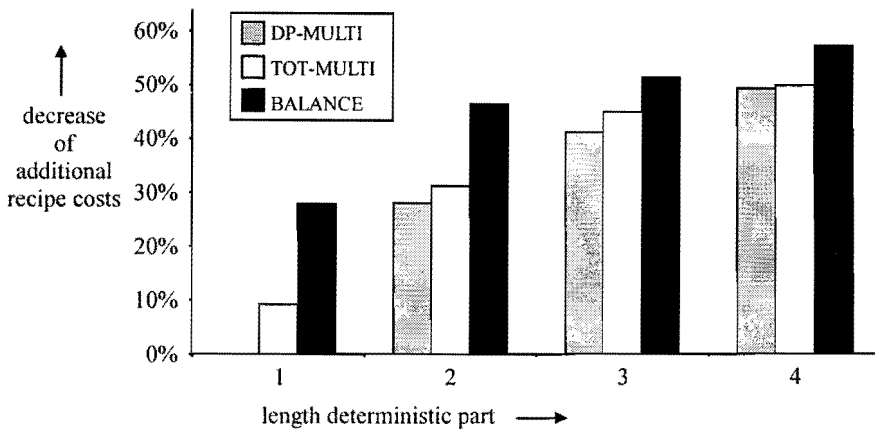


Figure 4 Decrease of additional recipe costs of the concepts for several lengths of the deterministic part (runs 1 to 4), for the highest cost setting.

If the uncertainty in replenishment orders is low (variation in raw materials; compare runs 1 & 5 and 2 & 6), the performance of all concepts is higher than for the situation with a high uncertainty. Furthermore, a lower coefficient of variation in demand for products with alternative recipes (compare runs 1 & 7 and 2 & 8) also increases the performance of all concepts. Combining the latter two factors (compare runs 1 & 9 and 2 & 10) results in a situation in which recipe flexibility is hardly used. For the medium and low cost settings (see Appendix B) the same conclusions hold.

By combining the additional recipe cost values of two concepts, we can calculate a t-test value for measuring the performance difference between the concepts as described. In Table 6, the t-test values are given per run for each combination of two concepts.

Table 6 Calculated t-test values for each concept combination. Concepts are abbreviated by their first character (D= DP-MULTI, T= TOT-MULTI, B= BALANCE). The columns in which the BALANCE concept is included are given for all three cost settings. A t-test value larger than 2.31, 2.60 resp. 3.30 can be considered significant at a 90%, 95% resp. 99%-level.

Run	D-T	high cost		medium cost		low cost	
		D-B	T-B	D-B	T-B	D-B	T-B
1	5.45	15.62	12.18	11.07	5.35	1.45	0.20
2	2.52	15.07	10.93	5.59	3.17	2.47	1.48
3	2.91	7.75	6.11	5.04	1.14	1.30	1.11
4	0.53	5.85	6.75	2.26	2.31	0.82	0.99
5	19.02	29.75	23.67	22.79	14.57	14.07	5.48
6	18.16	16.43	13.25	16.00	10.88	15.25	3.27
7	5.28	27.45	17.34	14.97	7.70	1.63	0.24
8	1.01	15.84	14.19	5.89	4.56	3.02	3.37
9	9.48	20.19	18.01	11.02	8.37	13.65	11.51
10	3.73	7.18	6.66	5.13	4.66	7.70	6.96

The DP-MULTI and TOT-MULTI concepts (first column) do not show substantial differences in their performance for every run (especially when the length of the deterministic part is large, the difference is small). Since the DP-MULTI concept is a multi-blend of all accepted orders (deterministic part), this means that adding pseudo-orders (TOT-MULTI) has no significant effect on the performance for these situations.

The BALANCE concept outperforms the other two concepts in all runs (last six columns). However, as mentioned before, when raw material costs change, the performance of the BALANCE concept will also change. The last six columns of Table 6 give the t-test values of the BALANCE concept with the two other multi-blend concepts for all three cost settings. As can be seen, the magnitude of t decreases as the differences in raw material costs become smaller. This is in line with our expectation, because the importance of preserving raw materials becomes less as the costs incurred by not preserving raw materials decrease. For the lowest cost setting a significant difference be-

tween the BALANCE concept and the two other concepts can only be found in runs 5, 6, 9 and 10 (at 99%-level). From this we can conclude that a simple myopic concept like the DP-MULTI concept will be adequate in most situations.

We observed an increase in performance, when adding pseudo-orders after a short deterministic part or by balancing the terminal stock position. However, not for all parameter values the results were statistically significant. The differences between the concepts were largest for a high cost difference between raw materials.

Apart from this statistical relevance of the differences in performances, we also are interested in the practical relevance. We are aware that the model we used is too simple to be realistic and that larger, more realistic models have to be investigated. However, given the existing technical and timing constraints, simulation with larger models was not possible.

Suppose that raw material costs constitute 80% of the product cost and that the company demands a profit margin of 2% on the product cost (this is not unusual in feed industries). The standard recipe costs per year equal 200,000 (800 (mean demand) \times 250 (days) \times 1.000 (cost per unit)) in each run. After adding the additional recipe costs of the concept, we can calculate the annual profit for a specific run as follows:

$$profit = \left(Total\ recipe\ costs \times \frac{100}{80} \right) \times \frac{2}{100} \quad (3)$$

Table 7 gives the increase in profit when compared to the single-blend, for each concept in each run of the design. As the length of the deterministic part increases (runs 1 to 4), the differences between the multi-blend concepts become very small. The differences between the multi-blend concepts are largest for a short deterministic part. For the highest cost setting these differences are significant. Note that for runs with the length of the deterministic part equal to one, there is no increase in profit for the DP-MULTI concept. This is caused by the elementary material allocation situation as used in this paper.

If the uncertainty in replenishment orders is high (runs 1 and 2), the increase in profit is higher than for the situation with a low uncertainty in replenishment orders (runs 5 and 6). A more stochastic demand (runs 1 and 2) also increases the possible

increase in profit (runs 7 and 8). Especially in these runs, the differences between the BALANCE concept and the two other concepts are larger.

Nevertheless, the effort that is needed for implementing the sophisticated and complex BALANCE concept with the highest performance must be weighted against the effort that is needed for implementing the much simpler TOT-MULTI or even the DP-MULTI concept with a lower performance. For instance, in the first run the difference in profit increase between the DP-MULTI and BALANCE concept is 4.16% (for the highest cost setting; for the lowest cost setting the difference is not significant). If the length of the deterministic part increases by one day (run 2), this difference decreases to 2.62% ($= (6.91 - 4.18) / 4.18$). One day more (run 3), and the difference becomes 1.42%. Since in practice most companies will have a customer leadtime that is at least two days, the differences between the multi-blend concepts will be relatively small in practice.

Table 7 Increase in profit compared to the single-blend concept.

Run	DP-MULTI	TOT-MULTI	BALANCE
1	0.00%	1.37%	4.16%
2	4.18%	4.66%	6.91%
3	6.15%	6.69%	7.66%
4	7.34%	7.43%	8.54%
5	0.00%	0.61%	2.45%
6	2.98%	3.17%	4.02%
7	0.00%	0.88%	4.01%
8	2.15%	2.30%	4.47%
9	0.00%	0.04%	0.51%
10	0.72%	0.72%	0.79%

7. Conclusions

In this paper we have investigated the performance of three planning concepts for the use of flexible material recipes to cope with occasional shortages in raw materials.

We have studied the situation with long material replenishment leadtimes, uncertainty in demand and a high required service level. The three concepts studied differed in the horizon over which information about the future production orders is taken into account, and in the way in which this information is used. The first concept optimizes the use of available materials over the customer order leadtime (the DP-MULTI concept). The second concept optimizes material use over the entire material replenishment leadtime, using actual customer orders and pseudo orders based on the expected value of demand (the TOT-MULTI concept). The third concept optimizes the material use over the customer order leadtime, taking into account the effect that the terminal material stock position has on future recipe costs given the available knowledge about the distribution function of demand (the BALANCE concept).

We have investigated the performance of the three concepts for an elementary small scale model of the production situation and we have identified the parameters of the production situation that may influence differences in performance. These parameters are: the variation in raw materials quality, the variation of demand and the length of the customer order leadtime. The performance of the BALANCE concept might be sensitive to the material costs structure. Therefore the performance is investigated for three different settings of material costs. Finally we have developed a costs performance measure to evaluate the differences in performance between the three concepts. A thorough experimental design based on multiple comparisons has been developed to investigate the performance differences. Student's t-statistics have been used to establish significance in performance difference.

Inspection of the simulation results reveals that the three concepts produce significantly different results. In particular, the TOT-MULTI concept performs better than the DP-MULTI concept, and the BALANCE concept in turn performs better than the TOT-MULTI concept. However, the performance improvement of the TOT-MULTI concept over the DP-MULTI concept is quite small as compared to the improvement of the BALANCE concept over the DP-MULTI concept. This indicates that the certainty equivalence assumption underlying the TOT-MULTI concept is not justified.

Furthermore, the differences in performance diminish if the customer order leadtime increases. The performance differences are the highest for a short customer order leadtime, a high variation in material quality, a high variation in demand and for a large

difference in material costs. To evaluate the practical relevance of the differences in performance we have calculated the relative increase in profit due to the use of each of the concepts, for a realistic cost structure for the feed industries. From the simulation we may conclude that the increase in annual profit over the use of the DP-MULTI concept, for the BALANCE concept can lead to 4.16% whereas the TOT-MULTI concept in that case leads to only a 1.37% increase in profit. From these figures we might conclude that especially the BALANCE concept under certain circumstances promises an interesting profit improvement.

However, the value of this performance improvements must be weighted against the effort that is needed for implementing and operating the complex BALANCE concept. The same goes of course for the TOT-MULTI concept, although here the procedure is much less complex. Therefore, in many practical situations it may turn out that the best solution is to use the simple DP-MULTI concept which optimizes the use of raw materials over the accepted customer orders. The relative loss of performance is then accepted in order to avoid the costs that would be incurred by using a more complex planning concept.

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Appendix A

For determining the expected costs in the stochastic part as a function of the terminal stock position, we have to determine the expected usage of the different raw materials. Let I_c denote the inventory position (raw material c), $E_r(I_c)$ denote the expected usage of raw material r given inventory I_c and c_r denote the cost of raw material r , then the expected value of recipe costs is:

$$ECost(I_c) = E_a(I_c)c_a + E_b(I_c)c_b + E_c(I_c)c_c \quad (4)$$

The expected usage of the different raw materials follows from the demand function. From an 'optimization'-view, the demand for products 3 and 4 can be graphically displayed, see Figure 5.

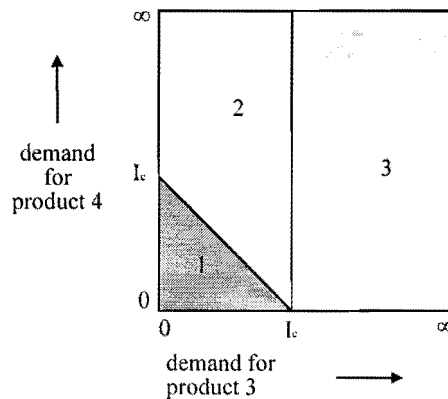


Figure 5 Graphical presentation of demand for products 3 and 4.

The complete area displayed in Figure 5, can be divided into three areas that have a different optimal solution:

1. Total demand ($x_3 + x_4$; demand is denoted by x_i) is lower than or equal to the inventory position I_c ; only raw material c will be used. The amount used of c equals the demand for products 3 and 4.
2. Demand for product 3 is lower than or equal to the inventory position and total demand is larger than the inventory position; product 3 will use raw material c and product 4 will first use the remaining inventory of c and some raw material b . Raw

material c will be used completely (I_c) and the amount used of raw material b equals $x_4 - (I_c - x_3)$.

3. Demand for product 3 is larger than the inventory position I_c ; product 3 will use all available raw material c and some raw material a to fulfill demand. The order for product 4 will consist completely of raw material b . Again raw material c is used completely, the amount used of raw material b equals demand for product 4 (x_4) and the amount used of raw material a equals $x_3 - I_c$ units.

We now are able to evaluate Equation (4) (where $f_i(x_i)$ denotes the demand function of product i):

$$E_c(I_c) = \int_0^{I_c} \int_0^{I_c - x_3} (x_3 + x_4) f_3(x_3) f_4(x_4) dx_4 dx_3 + \int_0^{I_c} \int_{I_c - x_3}^{\infty} I_c f_3(x_3) f_4(x_4) dx_4 dx_3 + \int_{I_c}^{\infty} \int_0^{\infty} I_c f_3(x_3) f_4(x_4) dx_4 dx_3 \quad (5)$$

$$E_b(I_c) = \int_0^{I_c} \int_{I_c - x_3}^{\infty} (x_4 - (I_c - x_3)) f_3(x_3) f_4(x_4) dx_4 dx_3 + \int_{I_c}^{\infty} \int_0^{\infty} x_4 f_3(x_3) f_4(x_4) dx_4 dx_3 \quad (6)$$

$$E_a(I_c) = \int_{I_c}^{\infty} \int_0^{\infty} (x_3 - I_c) f_3(x_3) f_4(x_4) dx_4 dx_3 \quad (7)$$

The three 'usage'-functions can be plotted for every value of I_c which gives Figure 6. As can be seen from Figure 6, when the inventory of raw material c increases (in other words, raw material c becomes less scarce), the expected usage of the alternatives decreases, whereby the usage of the most expensive alternative a decreases fastest. If the costs per unit are known, the expected costs can be calculated by Equation (4).

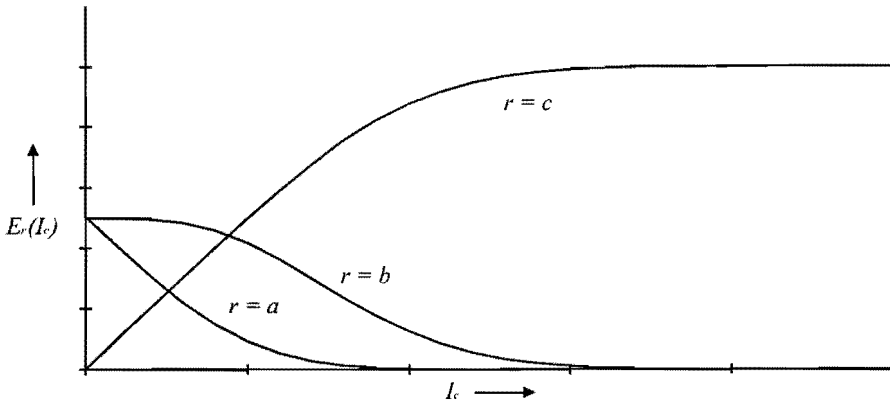


Figure 6 Expected usage of raw materials a , b and c .

Appendix B

In this appendix the tables of the complete simulation results are given. Table 8 gives the fraction of alternative raw materials used in the recipes for products 3 and 4, as measured in the simulations. The additional recipe costs that can be calculated from the use of alternatives is given in Table 9. The last two tables give the calculated t-test values.

Table 8 Mean fraction alternative measured

Run	single-blend	fraction alternative in product 3 (%)				
		DP-MULTI	TOT-MULTI	cost setting		
				high	medium	low
				BALANCE	BALANCE	BALANCE
1	3.3218	3.3218	3.0050	2.3407	2.5785	3.1005
2	3.3218	2.3593	2.2493	1.7181	2.0298	2.3048
3	3.3218	1.9086	1.7850	1.5500	1.7061	1.8478
4	3.3218	1.6352	1.6143	1.3540	1.5044	1.6104
5	1.4209	1.4209	1.2815	0.8305	0.9556	1.2956
6	1.4209	0.7386	0.6944	0.4815	0.5733	0.7017
7	2.4319	2.4319	2.2293	1.4905	1.7052	2.1290
8	2.4319	1.9379	1.9017	1.3968	1.7086	1.8015
9	0.2064	0.2064	0.1980	0.0674	0.0834	0.1406
10	0.2064	0.0425	0.0411	0.0195	0.0240	0.0346

Run	single-blend	fraction alternative in product 4 (%)				
		DP-MULTI	TOT-MULTI	cost setting		
				high	medium	low
				BALANCE	BALANCE	BALANCE
1	7.1125	7.1125	7.5317	10.6922	8.6495	7.3667
2	7.1125	8.2766	8.4072	10.3870	8.7509	8.0816
3	7.1125	8.6297	8.6026	9.9956	8.8487	8.6180
4	7.1125	8.8519	8.8768	9.5905	8.9963	9.0125
5	3.7863	3.7863	3.9440	7.1569	4.9615	3.9356
6	3.7863	4.4691	4.5146	6.5377	4.8531	4.5075
7	7.7708	7.7708	8.0324	10.7809	9.2829	8.2014
8	7.7708	8.3929	8.4849	9.6679	8.6541	8.3226
9	2.4362	2.4362	2.4450	4.8084	3.2330	2.5338
10	2.4362	2.6004	2.6017	3.3156	2.6686	2.6083

Table 9 Mean and standard deviation of the additional recipe costs

highest cost setting; $(c_a - c_c)/(c_b - c_c) = 100$

Run	single-blend		DP-MULTI		TOT-MULTI		BALANCE	
	mean	std.dev.	mean	std.dev.	mean	std.dev.	mean	std.dev.
1	748.44	70.89	748.44	70.89	679.47	47.67	539.88	63.95
2	748.44	70.89	538.68	59.01	514.69	51.77	401.87	51.41
3	748.44	70.89	440.02	55.69	412.70	41.91	363.92	49.35
4	748.44	70.89	380.19	47.59	375.64	53.87	319.78	35.42
5	321.77	49.98	321.77	49.98	291.37	46.36	198.96	42.30
6	321.77	49.98	172.76	31.53	163.11	30.54	120.60	28.61
7	553.58	35.66	553.58	35.66	509.46	39.29	352.53	35.41
8	553.58	35.66	445.97	39.78	438.19	44.69	329.41	24.82
9	50.90	8.47	50.90	8.47	49.07	8.47	25.44	4.85
10	50.90	8.47	15.09	4.59	14.79	4.44	11.60	2.79

medium cost setting; $(c_a - c_c)/(c_b - c_c) = 10$

Run	single-blend		DP-MULTI		TOT-MULTI		BALANCE	
	mean	std.dev.	mean	std.dev.	mean	std.dev.	mean	std.dev.
1	721.54	61.09	721.54	61.09	672.26	43.04	615.77	61.02
2	721.54	61.09	569.88	53.21	552.50	45.67	519.30	51.37
3	721.54	61.09	495.44	50.19	472.82	36.48	463.07	51.52
4	721.54	61.09	450.42	42.42	447.12	50.77	429.57	43.96
5	321.87	46.08	321.87	46.08	299.71	43.33	259.46	37.69
6	321.87	46.08	211.83	31.50	204.72	30.68	189.06	29.53
7	573.88	33.50	573.88	33.50	542.25	35.68	470.64	29.41
8	573.88	33.50	496.48	37.01	491.63	41.38	460.06	34.00
9	80.34	8.24	80.34	8.24	78.99	8.24	72.48	6.37
10	80.34	8.24	53.89	5.03	53.67	4.97	51.80	3.70

lowest cost setting; $(c_a - c_c)/(c_b - c_c) = 2$

Run	single-blend		DP-MULTI		TOT-MULTI		BALANCE	
	mean	std.dev.	mean	std.dev.	mean	std.dev.	mean	std.dev.
1	589.00	39.64	589.00	39.64	579.58	35.49	580.77	35.72
2	589.00	39.64	555.72	40.13	551.81	36.32	542.72	42.08
3	589.00	39.64	531.96	39.27	520.16	31.86	526.22	37.08
4	589.00	39.64	517.88	36.25	517.14	43.66	522.59	42.28
5	283.63	35.47	283.63	35.47	278.35	34.78	279.21	34.69
6	283.63	35.47	253.97	31.45	252.11	31.22	252.44	31.28
7	540.43	27.29	540.43	27.29	534.12	27.83	532.69	34.14
8	540.43	27.29	524.39	29.91	525.19	32.48	509.65	24.62
9	121.56	9.36	121.56	9.36	121.21	9.35	120.04	9.15
10	121.56	9.36	114.43	8.65	114.37	8.66	114.09	8.67

Table 10 The t-test value for the increase in performance of a concept when the length of the deterministic part increases. A t-test value larger than 2.31, 2.60 resp. 3.30 can be considered significant at a 90%, 95% resp. 99%-level.

Concept	cost setting	Runs compared (runno. equals the length of the deterministic part)		
		Runs 1-2	Runs 2-3	Runs 3-4
DP-MULTI	high	22.01	9.71	6.84
TOT-MULTI	high	19.07	13.36	3.85
BALANCE	high	9.24	4.32	5.29
DP-MULTI	medium	18.25	8.70	6.41
TOT-MULTI	medium	16.95	12.23	3.10
BALANCE	medium	8.79	6.02	4.32
DP-MULTI	low	5.61	4.37	3.15
TOT-MULTI	low	6.29	6.39	0.51
BALANCE	low	7.33	3.29	0.61

Table 11 The t-test values for comparing the concepts per run. Concepts are abbreviated by their first character (D= DP-MULTI, T= TOT-MULTI, B= BALANCE). A t-test value larger than 2.31, 2.60 resp. 3.30 can be considered significant at a 90%, 95% resp. 99%-level.

Run	high cost			medium cost			low cost		
	D-T	D-B	T-B	D-T	D-B	T-B	D-T	D-B	T-B
1	5.45	15.62	12.18	4.76	11.07	5.35	1.57	1.45	0.2
2	2.52	15.07	10.93	2.16	5.59	3.17	0.72	2.47	1.48
3	2.91	7.75	6.11	2.80	5.04	1.14	2.15	1.30	1.11
4	0.53	5.85	6.75	0.47	2.26	2.31	0.17	0.82	0.99
5	19.02	29.75	23.67	18.74	22.79	14.57	15.99	14.07	5.48
6	18.16	16.43	13.25	18.04	16.00	10.88	16.64	15.25	3.27
7	5.28	27.45	17.34	4.33	14.97	7.70	1.22	1.63	0.24
8	1.01	15.84	14.19	0.71	5.89	4.56	0.16	3.02	3.37
9	9.48	20.19	18.01	9.47	11.02	8.37	9.13	13.65	11.51
10	3.73	7.18	6.66	3.73	5.13	4.66	3.79	7.70	6.96