Size effect predictions by fracture models for a refractory ceramic

SIZE EFFECT PREDICTIONS BY FRACTURE MODELS FOR A REFRACTORY CERAMIC

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Abstract. Refractory ceramics used as kiln furniture are designed to operate at elevated temperatures with a high thermal shock resistance. In practice the material fails, however, due to thermal fatigue after a limited number of cycles. To predict this failure behaviour it is generally not possible to use Linear Elastic Fracture Mechanics due to the fact that the coarse grained, porous material shows a nonlinear mechanical behaviour. Therefore four different FE models widely used for concrete modelling, are investigated on their ability to describe the nonlinear failure behaviour, in particular the associated size effect phenomenon. Two of the FE models, Discrete Crack and Smeared Crack, are available in DIANA. The other two, Nonlocal Continuum Damage Mechanics and Local Continuum Damage Mechanics, have been implemented in DIANA 5.1. The results of the Nonlocal CDM model indicate that this model cannot properly describe the expected size effect. The other three models, however, give comparable results with a good description of the size effect phenomenon.

1. Introduction

When designing structures, knowledge of the failure behaviour of the materials used is of major importance in order to obtain a safe design with a satisfying service time. The aim of the present research is to model crack initiation and propagation within a ceramic component, a so-called plate saggar. This component is made of a refractory ceramic, tradename Alcorit (Sphinx Technical Ceramics), and is used as support for a plate during firing. The plate saggars fail after a number of temperature-cycles between room temperature and 1200 °C due to thermal stresses.

In order to result in a useful numerical model which describes the fracture process in the plate saggar, it is first necessary to obtain insight in the crack growth process under controlled circumstances. To accomplish this, the description of the material behaviour at room temperature subjected to mode I loading is taken as point of departure. In a later stage the effect of different loadings, temperature changes and other geometries will be taken into account.

The classical method to describe crack growth processes applies the theory of Linear Elastic Fracture Mechanics (LEFM). Disadvantage of this theory is the difficulty of incorporating the LEFM criteria in the Finite Element method (FE). The LEFM theory is still an often used and also valid model for describing crack propagation in (purely) brittle materials. LEFM models can therefore serve as a reference

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to compare with other models. All models should converge to the LEFM results for an infinite size of the structure. A deviation from the LEFM theory occurs only for finite dimensions.

For the explanation of these deviations from LEFM theory, research done in the field of concrete cracking can give an indication in the right direction. From observations of the crack tip in concrete and in ceramics, it appears that nonlinear phenomena occur in front of the crack tip, in the so-called process zone.

To describe the crack growth in a material containing cracks with a process zone properly, several FE models have been developed. The three most important models are the Discrete Crack (DC) model, the Smeared Crack (SC) model and the Nonlocal Continuum Damage Mechanics (NLCDM) model. The major difference between these models is the way in which strain localization is described. With respect to this strain localization it seems plausible that strains are localized in a gradually distributed way. Difficulties in implementing this general hypothesis have lead to more simple criteria, such as localization within a band or along a line. Among the previously mentioned FE models, the NLCDM model results in a smooth strain localization, the SC model in a localization within a band and the DC Model in a crack line localization.

In order to discriminate between the models mentioned above, an important phenomenon resulting from fracture mechanics, the so-called size effect, will be used. This size effect can be defined as the dependence of the maximum sustainable nominal stress of a structure (maximum load divided by some surface measure of the structure) on the size of this structure. To separate the size effect from other influences, one should consider structures with geometrically similar shapes (e.g., beams with the same span-to-depth ratio and the same crack length-to-depth ratio). Due to nonlinear material behaviour a deviation from the size effect as predicted by LEFM will occur. For smaller structures these deviations will become more pronounced because the size of the process zone is assumed to be a material property. For structures with a size large enough compared to the process zone, the influence of this process zone will be negligible and LEFM theory will be applicable.

In the remaining part of this paper the size effect as predicted by four FE models will be examined. A simple geometry, the three point bend beam with a notch, often used in both experimental and numerical analyses, will be evaluated with respect to the size effect. Attention will be focused on the maximum loads c.q. the load bearing capacity, because this property is believed to be the most important feature. In a later stage the softening curve will be investigated in order to obtain a complete model. The maximum loads as predicted by the different models will provide the input for the Bažant Size Effect Law which will be dicussed in section 2. In this way material properties like fracture energy and process zone size can be calculated and compared with the values which were originally used as input in the FE models.

The approach suggested above will give information on the relation of the Bažant Size Effect Law and the FE models as well as information on the comparison of the FE models mutually. It must be emphasized that the analyses are focused on mode I loading and that therefore conclusions cannot be generalized to other loading conditions.

2. Size Effect Law according to Bažant

Based on observations of the size effect in concrete, Bažant has proposed the so-called Size Effect Law (SEL) to predict size- and shape-independent material properties like the true fracture energy and the size of the fracture process zone (Bažant 84; Bažant and Kazemi 90). For materials that do not behave according to the LEFM theory it is not so evident how to determine these material properties because the use of specimens of different size will result in different values for the fracture energy. These values will not only depend on the size of the structure c.q. the size of the crack, but also on the shape of the structure. Bažant states that for each material there exist two size- and shape-independent material properties. These two material properties are the fracture energy G_f and the equivalent size of the process zone c_f, occurring for infinitely large specimens. With these two parameters the Bažant Size Effect Law becomes:

$$\tau_{nom} = \sqrt{\frac{E' G_f}{c_f + \bar{d}}} \tag{1}$$

The nominal strength τ_{nom} and effective size $\bar{d}$ for two-dimensional structures are defined by:

$$\tau_{nom} = \frac{P_{max}}{A}\sqrt{g'(\alpha_0)} \tag{2}$$

$$\bar{d} = \frac{g(\alpha_0)}{g'(\alpha_0)} d \tag{3}$$

with:

$$E' = \begin{Bmatrix} E \text{ in case of plane stress} \\ \dfrac{E}{1-\nu} \text{ in case of plane strain} \end{Bmatrix}$$

$$\begin{aligned} E &= \text{Young's modulus} \\ \nu &= \text{Poisson's ratio} \\ P_{max} &= \text{maximum sustainable force} \\ A &= \text{characteristic surface measure} \\ \alpha_0 &= a_0/d \\ g(\alpha) &= \text{nondimensional function depending on } \alpha \\ &\quad \text{and resulting from LEFM analysis} \\ g'(\alpha) &= \frac{dg(\alpha)}{d\alpha}. \end{aligned}$$

For the explanation of the parameters the geometry of the single edge notched beam, as illustrated in figure 1, will be used. For the single edge notched beam with L/d=4, the function $g(\alpha)$ is defined by (Srawley 76):

$$g(\alpha) = 36\frac{\alpha\left(1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)\right)^2}{(1+2\alpha)^2(1-\alpha)^3} \tag{4}$$

and the surface measure A by $A = b \cdot d$. The result of the Bažant Size Effect Law, with $G_f = 30$ N/m, E'= 15 GPa and $c_f = 1$ mm, is illustrated in figure 2.

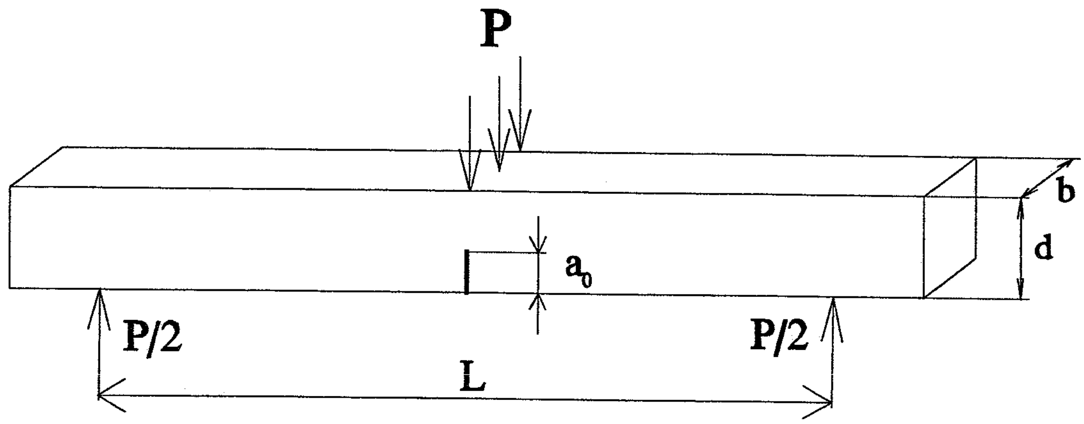

Fig. 1. Geometry of the single edge notched beam

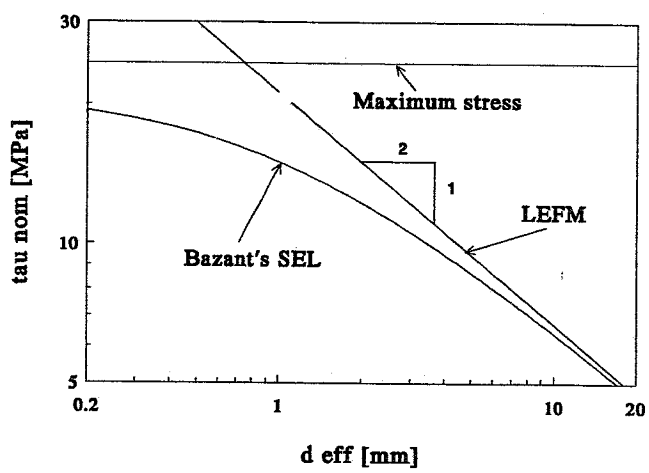

Fig. 2. Representation of the Bažant Size Effect Law

3. FE models

The Bažant SEL as introduced in the previous section will be used to compare four different FE models on their ability to predict this size effect. These models are the Discrete Crack (DC) model, the Multi Directional Smeared Crack (SC) model, the Nonlocal Continuum Damage Mechanics (NLCDM) model and an adapted Local Continuum Damage Mechanics (LCDM) model. In this section a short introduction in the fundamentals of these models will be presented.

3.1. Discrete Crack model

The Discrete Crack model, also called the Fictitious Crack model (Ngo and Scordelis 67; Hillerborg et al. 76; Rots 88), which is available in DIANA, is inspired on the classical Dugdale-Barenblatt crack concept. Interface elements are placed between the continuum elements to discretize the crack path. When the normal stress in a certain interface point exceeds the tensile strength f_t of the material, a softening constitutive behaviour is introduced in that interface point. The interface nodes connecting the continuum elements then separate, thus simulating the crack opening. The constitutive cracking behaviour is determined by the stress/crack opening relationship, where the area under this stress/crack opening curve equals the fracture energy of the material.

3.2. Smeared Crack model

In the Multi Directional Smeared Crack model (Rashid 68; Rots 88), which is also available in DIANA, a crack is simulated by introducing a softening anisotropic constitutive behaviour in element integration points in which a certain stress limit is reached. The major difference with the Discrete Crack model is the fact that the cracking behaviour is modelled in terms of stress/strain relationships instead of stress/displacement relationships. As a result, the dissipated energy depends on the volume associated with integration points. In order to predict a correct energy dissipation, the size of the elements c.q. the volume associated with the integration points should correspond to a characteristic length parameter. This length parameter is called the crack band width and is usually taken equal to three times the maximum grain size.

3.3. Nonlocal Continuum Damage Mechanics model

The Nonlocal Continuum Damage Mechanics model (Pijaudier and Bažant 87; Saouridis and Mazars 89; de Vree et al. 94) introduces an internal variable in the constitutive equations. This internal damage variable induces a softening material behaviour when a strain measure (to be defined) exceeds a certain threshold. The present model uses an isotropic damage parameter influencing the constitutive behaviour by:

$$\boldsymbol{\sigma} = (1 - D)\mathbf{C} : \boldsymbol{\varepsilon} \tag{5}$$

with:

$\boldsymbol{\sigma}$ = second order stress tensor
D = isotropic damage parameter
$\mathbf{C}$ = fourth order elasticity tensor
$\boldsymbol{\varepsilon}$ = second order strain tensor.

The evolution of the damage parameter is coupled to the development of strains. At local level the Modified von Mises equivalent strain (de Vree et al. 94) is used:

$$\varepsilon_{eq} = \frac{k-1}{2k(1-2\nu)} J_1 + \frac{1}{2k}\sqrt{\left(\frac{k-1}{1-2\nu} J_1\right)^2 - \frac{12k}{(1+\nu)^2} J_2'} \tag{6}$$

with the strain invariants:

$$J_1 = tr(\varepsilon) \tag{7}$$

$$J_2' = -\frac{1}{6}\left\{3tr(\varepsilon\cdot\varepsilon) - (tr(\varepsilon))^2\right\} \tag{8}$$

and with k the ratio of the compressive strength f_c and the tensile strength f_t:

$$k = \frac{f_c}{f_t}. \tag{9}$$

To avoid mesh-sensitivity, strain localization is regularized by the introduction of a nonlocal equivalent strain defined by:

$$\bar{\varepsilon}_{eq}(\vec{x}) = \frac{1}{\int\limits_{|\vec{r}|<\lambda} w(|\vec{r}|)dV} \int\limits_{|\vec{r}|<\lambda} w(\vec{r})\varepsilon_{eq}(\vec{x}+\vec{r})dV \tag{10}$$

with the introduction of the nonlocal weighing parameter λ, also called the characteristic length, and a Gaussian weight function:

$$w(|\vec{r}|) = e^{-\left(\frac{2\cdot|\vec{r}|}{\lambda}\right)^2}. \tag{11}$$

The local actual damage is a function of the maximum exceed of the threshold by the equivalent strain $\bar{\varepsilon}_{eq}$. This function determines the character of the softening as a result of equation (5).

3.4. Adapted Local Continuum Damage model

The adapted Local CDM model (Brekelmans and de Vree 94) is a refinement of the original CDM model in order to reduce the mesh-dependence. The present LCDM model shows an adequate energy dissipation by adjusting the material properties to the element size. The dependence of the dissipated energy on the element integration scheme is reduced by forcing the damage to be constant within an element. The critical strain value ε_c, satisfying $D(\varepsilon_c) = 1$, will depend on a certain length L_e which is a representative value for the size of the element. For a linear softening constitutive relation this critical strain equals:

$$\varepsilon_c = \frac{2\cdot G_f}{f_t\cdot L_e}. \tag{12}$$

The characteristic size L_e for two-dimensional elements is defined by:

$$L_e = \sqrt{A_E} \tag{13}$$

with A_E the area of the element.

4. Results

For the comparison of the different models the load-displacement curves of differently sized single edge notched beams are calculated. The beam sizes (bxdxL) with $\alpha_0 = 0.25$, see figure 1, chosen are:

2.5	x	2.5	x	10	mm^3	(beam 1)
10	x	10	x	40	mm^3	(beam 2)
25	x	25	x	100	mm^3	(beam 3)
50	x	50	x	200	mm^3	(beam 4)

The material parameters are chosen so that they represent the refractory ceramic Alcorit most closely. The actual values of these material properties should be derived from experiments, but the parameters used for the calculations are reasonable estimations of the true values. For the softening curve, $\sigma(u)$ or $\sigma(\varepsilon)$, a constant slope is assumed with the following material parameters:

$$f_t = 9 \text{ MPa} \qquad G_f = 30 \text{ N/m}$$
$$E = 15 \text{ GPa} \qquad \lambda = 2.5 \text{ mm}$$
$$\nu = 0.25 \qquad k = 8$$

Second order plane stress elements with four integration points were used in the analyses. Riks' arc length method was used for the largest beam to capture the snap-back behaviour in the force/displacement curve. Figures 3 and 4 display the force/associated displacement curves for the smallest and the largest beam using the different FE models.

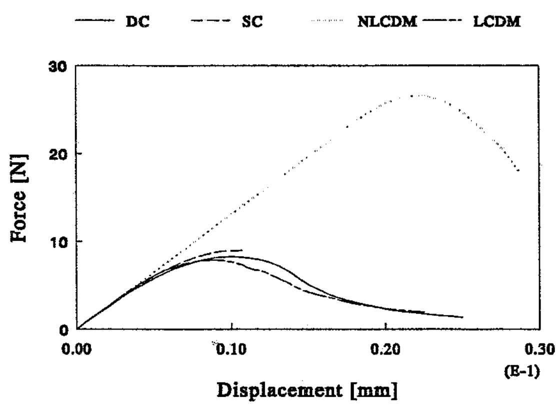

Fig. 3. Force/displ. curves for beam 1

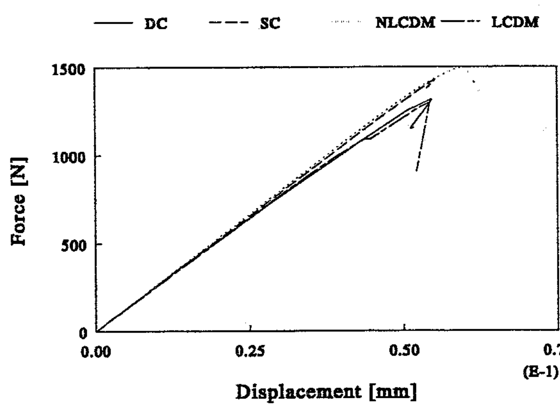

Fig. 4. Force/displ. curves for beam 4

From these figures it is clear that the larger beam shows a behaviour more brittle than the smaller beam. For larger beams the deviation from the linear behaviour until maximum load decreases because the relative size of the process zone decreases. From figure 3 and to a lesser extend figure 4, a striking deviation of the NLCDM model from the other models can be observed. This deviation decreases for larger beams. The three other models result in almost identical curves with slight differences in the maximum loads. It is clear that the nonlocal formulation causes a strongly deviating prediction of the maximum loads and the dissipated energy when the nonlocal parameter λ becomes relatively large compared to the size of the structure. This effect can also be found in the size effect curves based on the maximum loads which are shown in figures 5-8. For the NLCDM model several values of the characteristic length λ were evaluated (figure 7). It appears that an increase of this characteristic length results in a steeper size effect curve. The fitted parameters of the Bažant SEL for the results of the different models are given in table 1.

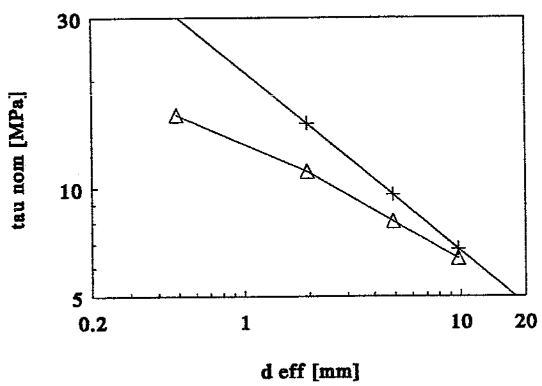

Fig. 5. Size effect DC model
△ DC + LEFM

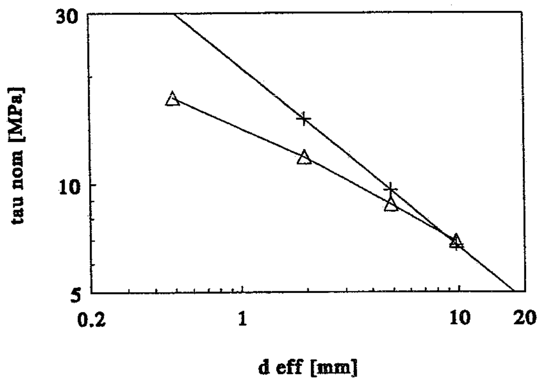

Fig. 6. Size effect SC model
△ SC + LEFM

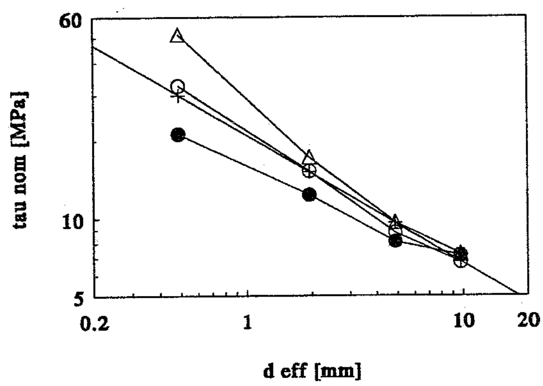

Fig. 7. Size effect NLCDM model
△ $\lambda = 25mm$ ○ $\lambda = 12.5mm$
• $\lambda = 6.25mm$ + LEFM

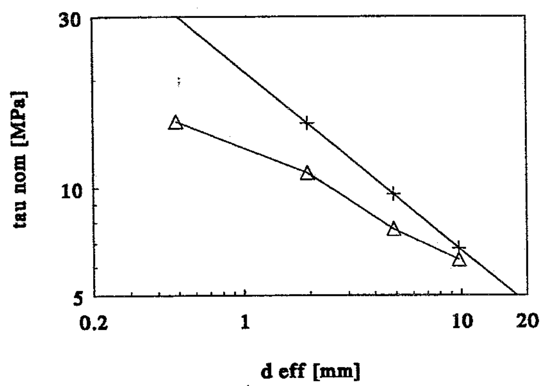

Fig. 8. Size effect LCDM model
△ LCDM + LEFM

From the size effect curves it becomes clear that the present NLCDM model cannot describe the size effect as expected to occur in experiments. As a result of the weighing process the peak stresses at the crack tip decrease with increasing value of λ. This phenomenon has two results:
1) Increasing τ_{nom} with increasing λ. This is in contrast to the SEL where an increasing c_f results in a *decreasing* τ_{nom}.

2) $\dfrac{\partial^2 ln \tau_{nom}}{\partial ln^2 \bar{d}} > 0$. This is true for all λ in a certain range of $\bar{d}$. According to the SEL,

TABLE I
Size Effect Law parameters

DC	SC	NLCDM λ=2.5 cm	LCDM
G_f=26.06 N/m	G_f=30.13 N/m	G_f=31.2 N/m	G_f=25.3 N/m
c_f=1.05 mm	c_f=1.03 mm	c_f= -0.31 mm	c_f=1.134 mm

however, this curvature is always *negative* and according to LEFM it is equal to zero.

The three other models show comparable results with regard to the size effect and the fitted parameters of the Bažant SEL show a relatively good agreement with the original values. It must be mentioned, however, that these models all show a slight deviation from the shape of the Bažant SEL (figure 2).

5. Conclusions

The Discrete Crack model has the disadvantage, especially in more complex geometries and loading conditions, that it is necessary to know a priori the crack path in order to model it with interface elements. Another disadvantage is the fact that the crack can only develop in mode I. Crack initiation due to shearing or high compressive loading is not possible. Nevertheless the DC model is useful, especially to describe the mode I behaviour. It can be considered as reference for other models and for determining (mode I) material parameters.

The Smeared Crack model suffers from mesh-dependence as a result of the absence of strain localizers in the model. The present method of manual adaptation of the input parameters can only be effective for those problems where the size of the crack band is known. For complicated geometries with different element sizes this method will not yield consistent data. Incorporating a sort of strain localization limiter or an automatic adaptation of the material parameters (as in the LCDM model) could, however, result in an attractive model. The advantage of such a model is the fact that an anisotropic damage model based on stress/strain relations can simulate the actual cracking process without the need for complicated meshes.

The Nonlocal CDM model gives results that converge with mesh-refinement and is probably the model that resembles the strain field in the localization zone most closely. It suffers, however, from several problems due to the nonlocal formulation. The averaging process at the crack tip smears out the peak stresses, resulting in lower damage values and higher maximum loads. This effect causes a size effect which is incomparable with that of the Bažant SEL. Large values of the characterictic length λ result in an increasing size effect in comparison with the LEFM model.

The alternative Local CDM model doesn't have the nonlocal difficulties of the NLCDM model but can still give mesh-independent results with a proper description of the size effect. An advantage of this model in comparison to the SC model is the automatic adjustment of the softening parameters to the element size. A disadvantage of this model is the fact that the size of the damage zone is meshdependent and not representative for the actual size of the process zone. This fact doesn't, however, restrict the applicability of the model because the global response is hardly influenced.

The DC, SC and LCDM models are principally all suitable to describe the expected size effect. An increasing deviation from the LEFM results can be seen for the nominal strength values in case of smaller structures. The shape of the resulting size effect curve is somewhat different from the prediction of the Bažant SEL. Especially for smaller structures the Bažant SEL nominal strength values are smaller than the FE results. Based on the Bažant SEL preliminary estimations of the material properties can be determined. The correct shape of the size effect curve should follow from experiments.

An untouched issue is the fact that the present evaluation is restricted to mode I loading. The influence of mode II, mode III and mixed mode loading on the mechanical behaviour of the material and on the performance of the different damage models is unknown. Both experimental and numerical evaluation of test geometries displaying other stress fields will have to be carried out to completely evaluate the damage models for static loading at room temperature.

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