# On the limits of creep in friction wheel drives for precision engineering applications 

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# On the Limits of Creep in Friction Wheel Drives for Precision Engineering Applications. 

# On the Limits of Creep in Friction Wheel Drives for Precision Engineering Applications. 

## PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de Rector Magnificus, prof.dr. J.H. van Lint, voor een commissie aangewezen door het College van Dekanen in het openbaar te verdedigen op vrijdag 7 oktober 1994 om 16.00 uur
door
PETRUS CAROLUS JOHANNES NICOLAAS ROSIELLE

Geboren te Eindhoven (NB)

## Dit proefschrift is goedgekeurd <br> door de promotoren <br> prof.dr.ir. P.H.J. Schellekens <br> prof <br> prof.dr.ir. M.J.W. Schouten

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## Summary

Friction Wheel Drives (FWD), of the type with ballbearing supported wheels, display fine properties for Precision Engineering applications. However, even when the tangential load is well below the limit of (macro)slip, FWD suffer from creep. Creep is an initially small, but permanently increasing, loss of motion of the output shaft during rotation. Because output motion, in time, is affected by variation in geometry, due to excentricity, out of roundness and thermal effects, creep is not easily measured.
A special measuring machine was designed, built and tested for creep measurements of very short duration (seconds), thereby avoiding the problem of non stationary temperature.
Extensive use of monolytic elastic elements provided a machine with unique repeatability. Repeatable geometric imperfections, in their effect on transmission of motion, are recorded by encoders, during wheel rotation.
One feature of the machine is that the angles between the wheel axes can be adjusted finely.
The results obtained from the measurements help quantifying the requirements for tolerances on wheel-roundness, -excentricity, -surface finish and the alignment of the axes of the FWD

Also, the amount of creep encountered in a FWD, can be predicted reliably, under practical conditions.
Software correction of creep may prove worthwhile for certain applications, compared to measuring the output motion. Correction for geometric imperfections is also envisaged for particular situations.
The thesis categorizes the routes still open for further improvements, should the creeprate be considered too high for the applications intended.

On the Limits of Creep in Friction Wheel Drives for Precision Engineering Applications.

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## Notation

a half of long axis of contact ellipse
b half of short axis of contact ellipse
c radius, distance
d distance
e argument of elliptic integral
h separation of surfaces
i transmission ratio
k torsional stiffness
1 distance rolled
p pressure
r radius
s slip
t traction, thickness
u displacement
v velocity of material particle
x coordinate axis, orthogonal set, rotating positive x towards positive y
y " gives direction of positive z by righthand corkscrew rule.
z "
A area, a friction wheel, constant
B another friction wheel, a constant, a point in a body
C a constant, a point in the contact area
E elastic Youngs modulus, Elliptic Integral
F Force, Elliptic Integral, Boussinesq Potential Function
G Shear modulus, Boussinesq Potential Function
H Boussinesq Potential Function
J Mass Moment of Inertia
K Elliptic Integral
$L$ constant
M Moment, constant
N constant
O Origin of $x, y, z$ coordinate system at initial contact point
R radii of curvature
T distant points
V velocity
greek
$\alpha$ angle
$\delta$ compression
$\mu$ friction coefficient
$\vee$ Poisson modulus
$\xi$ creeprate
$\rho$ distance between points
$\sigma$ stress component
$\tau$ shear stress component
$\varphi$ angle
$\Psi$ Boussinesq Potential Function, Spin parameter
$\omega$ angular velocity
Roman
I axis of axially translating (cylindrical) wheel
II axis of second wheel (spherical) which is tilted
indices
x,y,z in the direction of
1,2 different bodies
superscripts

- at the surface
in the principal direction
" " " "
abbreviations
FWD
d.o.f Friction Wheel Drive
IB $\quad$ Degree(s) Of Freedom
Intermediairy Body


## 1. General Introduction

### 1.1 Drive systems for positioning

Precision Engineering covers machine and systems design, the development of tools for precision manufacturing, -measurement and -control.
To meet the increasing demands, hardware is developed and software is often used to provide corrections beyond the limits of particular hardware. [Youden, 1994],
[Soons, 1993].
Positioning is probably the most difficult task in Precision Engineering.
Many different methods for positioning are under investigation at various precision engineering groups. (NIST, MIT, Stanford, Cranfield etc.).

Direct comparison of complete drive systems for positioning are made by different researchers [Hansen 1987], [Bispink 1992], [Youden, 1992]. Here, Bispink shows representative categorising, the specific properties are tested with various tests.

Although somewhat more difficult for mutual comparison, many studies, each of a specific system, are reported e.g. [Langenbeck, 1994], [Nielsen, 1987],
[Donaldson, 1984], [Otsuka, 1992], [Holmberg, 1984].
Among these tested systems Friction Wheel Drives (FWD's) have unique properties suitable for Precision Engineering. Here, electromechanical actuator movement is transferred through transmission mechanisms to position objects. (fig. 1.1)


Fig. 1.1 A mechanism adapts an actuator to an object for positioning.

Closed loop control is often used to obtain accurate movement. [e.g. Ro, 1994] (fig 1.2)


Fig. 1.2 Example of closed loop control of the driving system of a slide.

Deficiencies of transmissions such as hysteresis and imparity complicate smooth control [Bradley,1991].
For small movements, piezo devices and voice coils are practical e.g.
[Tzou,1991], [Sugihara,1985] and [Renkens,1994], for large translations linear motors or more often rotating motors with appropiate mechanisms are applied. The accuracy with which complicated components in mechanisms can be manufactured, or their limited stiffness, excludes most classical drives from Precision Engineering applications.
Positioning through rolling contact with a friction wheel drive (FWD) can be in its simplest form a two wheel stage with a rotating to rotating (R-R) transfer or a single wheel to track or rack a rotating-translating (R-T) type. (fig 1.3)


Single stage R-R drive.
 R-T drive

Wheel to fixed track drive

## R-T drive

Fig. 1.3 Elementary R-R and R-T drive examples.

This mechanism is used for movements when gears or rack and pinion are not accurate enough or when belt drives are not stiff enough.

### 1.2 Applications of various types of friction wheeldrives.

FWD's have also been used succesfully in the recent past in consumer goods of small low-power type, like audio-appliances, in medium power drives e.g. laundry centrifuges, for traction in mopeds and are still used in all wheeldriven vehicles like trains and motorcars. Variable speed drives are popular in industry for manufacturing processes (stirring and the like).
Application of a forward directed handforce to the top of a cartwheel was probably the use of FWD avant la lettre. The extra leverage to the axis was necessary to overcome difficult terrain. FWD was therefore applied long before Newtonian understanding of a force or before Coulomb's friction experiments were available. Much later, early mechanisation involved constant ratio drives to adapt motor speed to the usually much lower speed of primitive machinery. The leather belt drive was used most often, as gears were difficult to make and required correct alignment of shafts.
As shaft bearings improved, gears and FWD's became more widely used. Early FWD's were much smoother and less noisy than the gears of those days.
Larger transmission ratio's were possible with FWD's without compromising uniformity of speed. Some degree of short term overload protection was appreciated as its counterpart : gears, would loose teeth when they were severely overloaded.

### 1.3 FWD's for Precision Engineering purposes.

With increasing steel quality, hardening of special gear steels, automated grinding of gear teeth and standardisation, gears have become far more important than FWD's in machinery. This applies both to the number of drives used and their economic importance.
FWD applications in measuring equipment, all sorts of other instruments and precision manufacturing equipment may seem insignificant, compared to the more widely spread use of gears.
In terms of positioning accuracy however, FWD's, because of their use of simple basic shapes (cylindrical, conical, toroidal or spherical) which can be manufactured with small tolerances in geometry, size and roughness, will inevitably stay slightly ahead. FWD's are unique in their contribution to the advancement of Precision Engineering in experimental research. They help generate other applications, bring advances in positioning to aid in physics experiments where objects have to be positioned angularly or linearly, for instance with respect to particle- or radiation-beams.
The continuous advance of Precision Engineering requires data on which future designs can be based.

## Scope of this thesis.

The theory on rolling contact tends to be rather complex.
Actual contact between elastic bodies extends from the initial point shape to a larger area : the contact area, which can be planar theoretically but which is curved in practice. Without going into a study of the contact area for its own sake, it is necessary to describe the status quo in modelling.
The experimental verification in this thesis does not involve use of plastics for photoelasticity or elastomers for laser doppler anemometry. Measurements will be made on the actual materials being used for FWD.'s in P.E. designs. The extensive work which has been done by many scientists on modelling of contacts known as "contact mechanics", is described briefly in chapter 2 in aspects which affect the present work. Credit is due to mainly mathematicians who sought to calculate what happens in contacts. Todays' Precision Engineers need to translate their findings to products.

There are three ways of looking at contacts which will be used by Engineers. First the cartesian way, which is a stationary view as used by Heinrich Hertz. A view of space strung out by coordinate axes with the focus on the origin where contact occurs. In this case, matter is an infinitely extensive half-space loaded in the contact area, supported well away from the contact area. This stationary view is still used and successfully applied for life predictions of rollers, cams, gear teeth and other machine parts.
Second the Eulerian view. The Eulerian view allows for the contacting wheels to rotate while keeping the origin and axes stationary. All deformations around the contact, together called the deformation field, remain fixed in space, so matter is moving past the origin. Fig. 1.4 shows axes $x, y$ and $z$ and the origin $O$. In this thesis $x$ is rolling direction, y is lateral and z is normal direction. The origin is at the initial contacting point.
A point in the contact area $A$ is (even though $Z$ is not 0 ) denoted by its $x$ and $y$ coordinate only.
The third way is the Lagrangian view. It is not used in this thesis.
A surface element dxdy will support a normal load dFz and friction components dFx and dFy called surface tractions. The sum total of each component making up respectively resultant Fz , the contact preload, Fx , the desired traction and Fy, lateral traction which is to be kept to a minimum.


Fig 1.4 Eulerian view, material of two wheels moving past origin. The latter is fixed in space.

During operation of friction wheels, position information is lost in those instances when the position of the final drive member is not measured continuously. The information loss is, among other factors, related to loading of the drive. Considerable attention is given to this phenomenon in literature [Johnson, 1987], [Kalker, 1970]. Chapter 2 goes into this in some detail.

In practical design work, questions regarding the obtainable accuracy for a given FWD type are raised.

It is the intention of this thesis to verify and simplify the existing theories to the point where design rules can be drawn up. (Chapter 5). The verification is done with an instrumental setup very similar to actual high accuracy drives (Chapter 3), (except for the number of modifiable parameters). This route was chosen as it is felt that most previous research was conducted somewhat far away from actual designed drives (e.g. using plastics, photo elasticity, etc.). In those instances when existing drives were used, results of measurements were not related to the existing theory in a quantitative manner. The latter results tend to be drive specific as the influence of e.g. axis alignment is not reported or known.

To verify the behavior of FWD to the level necessary for Precision Engineering applications it was felt necessary to develop a special measuring instrument capable of quantifying the effects of sources normally influencing the behavior of FWD's. So an important part of this thesis (chapter 3) is dedicated to the design and properties of this instrument which allows to find the limits of FWD.
In this thesis FWD's are studied for PE applications. Many general FWD aspects or types which were to some extend investigated elsewhere, will not be treated here because they are considered less suitable for PE. Perpendicular or inclined axes types and planetary (often power) drives are not treated here. Proper tracking and rolling without spin is considered essential for maximum accuracy. Their influence is investigated here extensively. Chapter 4 shows the results obtained with the machine and chapter 6 provides final conclusions.

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## 2. Theoretical background

Introduction

There is a longstanding tradition of contact mechanics based on an analytical approach, started by Boussinesq and Hertz. Since then, progress was made mainly in the twenties by Carter and Fromm, and in the late sixties by Kalker. Their findings are shown in this chapter. In the past decade, progress was made with numerical techniques, e.g. using finite element analysis of contacts [Baayens,1987]. The results of the analytical methods are used for comparison with experimental work.
Each section of this chapter ends with conclusions indicating the relevance of that particular section for our purposes.

### 2.1 Theory of Hertz, normal force on stationary contact.

With the coordinate system described in 1.3 , the surface of one of two contacting bodies (perfectly smooth) can be described as $\mathrm{z}_{1}=\mathrm{A}_{1} \mathrm{x}^{2}+\mathrm{B}_{1} \mathrm{y}^{2}+\mathrm{C}_{1} \mathrm{xy}+\ldots$
(neglecting higher order terms).
By aligning the axes x and y with the plane of the two principal radii of curvature $\mathrm{R}^{\prime}$ and $R^{\prime \prime}$ at the origin (initial contact), we obtain axes $x_{1}$ and $y_{1}$ for body 1 and

$$
z_{1}=\frac{1}{2 R_{1}^{\prime}} x_{1}^{2}+\frac{1}{2 R_{1}^{\prime \prime}} y_{1}^{2}
$$

likewise for body 2

$$
z_{2}=\frac{1}{2 R_{2}^{\prime}} x_{2}{ }^{2}+\frac{1}{2 R_{2}{ }^{\prime \prime}} y_{2}{ }^{2}
$$

The separation of the surfaces $h$ is found from $h=z_{1}-z_{2}$
Using a common set of axes for body 1 and $2, \mathrm{~h}=\mathrm{Ax}^{2}+\mathrm{By}^{2}+\mathrm{Cxy}$ Cxy can be made zero by a suitable choice of axes

$$
h=A x^{2}+B y^{2}=\frac{1}{2 R^{x}} x^{2}+\frac{1}{2 R^{\prime}} y^{2}
$$

Within the contact area:

$$
\bar{u}_{z 1}+\bar{u}_{z 2}=\delta-A x^{2}-B y^{2}
$$

where $\bar{u}_{z 1}$ and $\bar{u}_{z 2}$ are the inward displacements of surface points coinciding in contact and $\delta$ is the total normal approach distance of remote points in bodies 1 and 2 , referred to as compression.

Outside the contact area:

$$
\bar{u}_{z 1}+\bar{u}_{z 2}<\delta-A x^{2}-B y^{2}
$$

Hertz assumed that:

1. local deformations can be calculated from loading an elastic halfspace in an elliptic region
2. surfaces are frictionless and perfectly smooth.

For the first assumption to be valid, the contact area must be small in dimensions compared to the size of bodies 1 and 2 (to avoid boundary influences on the contact) and also small compared to the radii of curvature on the surfaces to keep strains within the linear theory of elasticity. Actual curvature of the contact area is neglected as the halfspaces have plane surfaces.
Hertz proposed the pressure distribution

$$
p=p_{0} \sqrt{1-\left(\frac{x}{a}\right)^{2}-\left(\frac{y}{b}\right)^{2}}
$$

where a and b are respectively the long and short semi axis of the contact ellipse. The proposed distribution 2.1.6 complies with equation 2.1.4 above if

$$
\begin{align*}
& A=\frac{M}{\pi E^{*}}=\frac{p_{0}}{E^{*}} \frac{b}{e^{2} a^{2}}(K(e)-E(e)) \\
& B=\frac{N}{\pi E^{*}}=\frac{p_{0}}{E^{*}} \frac{b}{a^{2} e^{2}}\left(\left(\frac{a}{b}\right)^{2} E(e)-K(e)\right)
\end{align*}
$$

In these formulae the plane strain modulus $\mathrm{E}^{*}$ is defined by

$$
E^{*}=\frac{E}{1-v^{2}} \text { and the excentricity as } e=\sqrt{1-\frac{b^{2}}{a^{2}}}
$$

$\mathrm{E}(\mathrm{e})$ and $\mathrm{K}(\mathrm{e})$ are complete elliptic integrals of argument e .

The semi ellipsoidal pressure distribution $\mathrm{p}(\mathrm{x}, \mathrm{y})$ is integrated to the total load. $\mathrm{F}_{\mathrm{z}}$,

$$
F_{z}=\frac{2}{3} p_{0} \pi a b
$$

Since contact dimensions a and b are not initially known, for a given load $\mathrm{F}_{z}, \mathrm{p}_{0}$ can be calculated with the aid of fig 2.1.1.

$$
p_{0}=\left(\frac{6 F z E^{*}}{\pi^{3} R e^{2}}\right)^{1 / 3} F_{3}(e)
$$

$R e=\sqrt{R^{\prime} R^{\prime \prime}}$ (= equivalent relative curvature.)
a.b can then be calculated from 2.1.9,
whereas $\quad b / a \sqrt{\frac{R^{\prime}}{R^{\prime \prime}}} \quad$ is plotted in fig.2.1.1
so $a$ and $b$ can be calculated from that with known curvatures $R^{\prime}$ and $R^{\prime \prime}$. The compression $\delta$ can be calculated from the displacements:

$$
\delta=\left(\frac{9 F^{2}}{16 E^{* 2} R e}\right)^{1 / 3} F_{2}(e)
$$

$F_{2}(e)$ can be found from fig. 2.1.1.


Fig. 2.1.1 Values of $\mathrm{F}_{2}, \mathrm{~F}_{3}$ and $\mathrm{b} / \mathrm{a} \sqrt{\frac{R^{\prime}}{R^{\prime \prime}}}$ as a function of $\sqrt{\frac{R^{\prime}}{R^{\prime \prime}}}$.

To cope with fig. 2.1.1 in computation, approximate algebraic expressions were published by [Brewe \& Hamrock,1977].

In conclusion: the effect of a normal force on a stationary contraform contact, for instance two friction wheels in a P.E. application, pressed together by a force, can be calculated in terms of stiffness in the normal direction, contact size and stress distribution shape and magnitude. In practical P.E. applications the compression is not large e.g. a few $\mu \mathrm{m}$, so deformation is calculated from the undeformed condition as usual in linear elastic mechanics.

### 2.2 Normal and traction force on stationary contact [Cattaneo]

Prior to rolling, stationary contacting bodies loaded with $\mathrm{F}_{\mathrm{z}}$ and $\mathrm{F}_{\mathrm{x}}$ will show some slip. If sliding occurs, $\mathrm{F}_{\mathrm{x}}$ is the kinetic friction force.
The tangential tractions $t_{\text {xyi }}$ at the interface are equal in magnitude and opposite in direction:
$t_{x y 1}(x, y)=-t_{x y 2}(x, y)$

Therefore the normal displacements at the surface $\bar{u}_{\mathrm{z}}$ due to these tractions are proportional to the elastic constant (1-2v)/G of each body and are of opposite sign :

$$
\frac{G_{1}}{1-2 v_{1}} \bar{u}_{z l}(x, y)=-\frac{G_{2}}{1-2 v_{2}} \bar{u}_{z 2}(x, y)
$$

If the two solids have the same elastic constants, any tangential traction gives rise to equal warping of both surfaces and does not disturb the distribution of normal pressure. The shape and size of the contact area are then fixed by the profiles of the two surfaces and the normal force, and are independent of the tangential force. With solids of different elastic properties this is no longer valid and the tangential tractions do interact with the normal pressure. The influence of tangential tractions upon the normal pressure and the contact area is negligible when the coefficient of friction is appreciably less than unity. The stresses and deformation due to the normal pressure and the tangential traction are independent of each other, and can be superposed. Amontons' law of sliding friction applies to each elementary area of the interface :

$$
\frac{\left|t_{x y}(x, y)\right|}{p_{z}(x, y)}=\mu
$$

where $\mu$ is a constant coefficient of kinetic friction. Incipient sliding $\left(\mathrm{F}_{\mathrm{x}} \leq \mu \mathrm{F}_{\mathrm{z}}\right)$ is applicable in positioning. $\mathrm{F}_{x}$, fig. 2.2.1, deforms the bodies in shear. Surface points $\mathrm{A}_{1}$ and $A_{2}$ were coincident before application of $F_{x}$. Distant points $T_{1}$ and $T_{2}$ moved through rigid displacements $\delta_{x 1}, \delta_{y 1}$ and $\delta_{x 2}, \delta_{y 2}$ while $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ experience
displacements $\bar{u}_{x 1}, \bar{u}_{y 1}$ and $\bar{u}_{x 2}, \bar{u}_{y 2}$ relative to $T_{1}$ and $T_{2}$. The displacements of
$\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ relative to 0 are $\mathrm{s}_{\mathrm{x} 1}, \mathrm{~s}_{\mathrm{y} 1}$ and $\mathrm{s}_{\mathrm{x} 2}, \mathrm{~s}_{\mathrm{y} 2}$.


Fig 2.2.1 Shear and slip at the contact.

The component of slip between $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ :

$$
s_{x} \equiv s_{x 1}-s_{x 2}=\left(\bar{u}_{x 1}-\delta_{x 1}\right)-\left(\bar{u}_{x 2}-\delta_{x 2}\right)=\left(\bar{u}_{x 1}-\bar{u}_{x 2}\right)-\left(\delta_{x 1}-\delta_{x 2}\right) \quad \text { and similarly for } \mathrm{s}_{y}
$$

In a 'stick' region $\mathrm{s}_{\mathrm{x}}$ and $\mathrm{s}_{\mathrm{y}}$ will be zero so

$$
\begin{aligned}
& \bar{u}_{x 1}-\bar{u}_{x 2}=\left(\delta_{x 1}-\delta_{x 2}\right) \equiv \delta_{x} \\
& \bar{u}_{y 1}-\bar{u}_{y 2}=\left(\delta_{y 1}-\delta_{y 2}\right) \equiv \delta_{y}
\end{aligned}
$$

Tangential traction cannot exceed its limiting value. From Amonton's law of friction with a constant coefficient $\mu$ :

$$
\left|t_{x y}(x, y)\right| \leq \mu\left|p_{z}(x, y)\right|
$$

and where the surfaces slip :

$$
\left|t_{x y}(x, y)\right|=\mu\left|p_{z}(x, y)\right|
$$

and the direction of the frictional traction $\mathrm{t}_{\mathrm{xy}}$ must oppose the direction of slip, so:

$$
\frac{t_{x y}(x, y)}{\left|t_{x y}(x, y)\right|}=-\frac{s(x, y)}{|s(x, y)|}
$$

This is applied to the contact of cylinders under partial slip. The method is first described by [Cattaneo,1938] and [Mindlin,1949]. If the tangential force $\mathrm{F}_{\mathrm{x}}$ is increased to its limit, the tangential traction is

$$
\mathrm{t}_{\mathrm{xy}}^{\prime}(\mathrm{x})=\mu \mathrm{p}_{\mathrm{o}}\left(1-\mathrm{x}^{2} / \mathrm{a}^{2}\right)^{1 / 2} \quad\left(\mathrm{p}_{\mathrm{o}}=2 \mathrm{~F}_{\mathrm{z}} / \pi \mathrm{a}\right)
$$

By analogy with the normal displacements produced by a Hertzian distribution of normal pressure, the surface displacements are distributed parabolically. If no slip occurs at the midpoint $x=0$, then we can write

$$
\bar{u}_{x 1}^{\prime}=\delta_{x 1}^{\prime}-\left(1-v_{1}^{2}\right) \mu p_{0} x^{2} / a E_{1}
$$

and similarly for the second surface. This distribution of tangential displacement satisfy equation (2.2.5) at the origin only; elsewhere in the contact region the surfaces must slip.
An additional distribution of traction

$$
t_{x y}^{\prime \prime}(x)=-\frac{c}{a} \mu p_{0}\left(1-x^{2} / c^{2}\right)^{1 / 2}
$$

can be applied over the strip $-\mathrm{c} \leq \mathrm{x} \leq \mathrm{c}(\mathrm{c}<\mathrm{a})$. The tangential displacements follow by analogy with equation (2.2.11), :

$$
\bar{u}_{x 1}^{\prime \prime}=-\delta_{x I}^{\prime \prime}+\frac{c}{a}\left(1-v_{1}^{2}\right) \mu p_{0} x^{2} / c E_{1}
$$

Superposing the tractions $\mathrm{t}_{\mathrm{xy}}$ ' and $\mathrm{t}_{\mathrm{xy}}$ ", the resultant displacements within the central strip - $\mathrm{c} \leq \mathrm{x} \leq \mathrm{c}$ are constant :
$\bar{u}_{x 1}=\bar{u}_{x 1}^{\prime}+\bar{u}_{x 1}^{\prime \prime}=\delta_{x 1}^{\prime}+\delta_{x 1}^{\prime \prime}=\delta_{x 1}$
and for the second surface :

$$
\bar{u}_{x 2}=-\delta_{x 2}
$$

The condition of no-slip is satisfied in the strip - $\mathrm{c} \leq \mathrm{x} \leq \mathrm{c}$. Furthermore in this region the resultant traction is given by
$\tau_{x y}(x)=t_{x y}{ }^{\prime}(x)+t_{x y}^{\prime \prime}(x)=\mu p_{0}\left\{\left(a^{2}-x^{2}\right)^{1 / 2}-\left(c^{2}-x^{2}\right)^{1 / 2}\right\} / a$

In the rest of the contact $c \leq|x| \leq a$ :
$t(x)=\mu p_{z}(x)$
as required in a slip region. The size of the stick region is determined by the magnitude of the tangential force

$$
F_{x y}=\int_{-a}^{a} t_{x y}^{\prime}(x) d x+\int_{-c}^{c} t_{x y}^{\prime \prime}(x) d x
$$

$$
=\mu F_{z}-\frac{c^{2}}{a^{2}} \mu F_{z} \text { so that } \frac{c}{a}=\left(1-\frac{F_{x y}}{\mu F_{z}}\right)^{1 / 2}
$$

The physical behaviour is now clear. If, keeping $\mathrm{F}_{\mathrm{z}}$ constant, $\mathrm{F}_{\mathrm{xy}}$ is increased steadily from zero, micro-slip begins immediately at the two edges of the contact area and spreads inwards. As $\mathrm{F}_{\mathrm{xy}}$ approaches $\mu \mathrm{F}_{z}$, c approaches zero and the stick region shrinks to a line at $\mathrm{x}=0$. Increase $\mathrm{F}_{\mathrm{xy}}$ further and the contact slides.


Fig 2.2.2 Stick and slip zones and traction for cylindrical contact.

A similar path is followed for the contact of spheres under partial slip.
The 'stick' region in this case is circular and concentric with the contact circle. On the point of sliding, the distribution of traction is
$\mathrm{t}_{\mathrm{xy}}{ }^{\prime}(\mathrm{x}, \mathrm{y})=\mu \mathrm{p}_{\mathrm{z}}(\mathrm{x}, \mathrm{y})=\mu \mathrm{p}_{0}\left(1-\mathrm{r}^{2} / \mathrm{a}^{2}\right)^{1 / 2}$

The tangential displacements within the contact circle, $r \leq a$, are :

$$
\begin{align*}
& \bar{u}_{x}^{\prime}=\frac{\pi \mu p_{0}}{32 G a}\left\{4(2-v) a^{2}+(4-v) x^{2}+(4-3 v) y^{2}\right\} \\
& \bar{u}_{y}^{\prime}=\frac{\pi \mu p_{0}}{32 G a} 2 v x y
\end{align*}
$$

Consider a distribution of traction

$$
t_{x y}^{\prime \prime}(x, y)=-\frac{c}{a} p_{0}\left(1-r^{2} / c^{2}\right)^{1 / 2}, r \leq c
$$

By analogy the tangential displacements within that circle are :

$$
\begin{gathered}
\bar{u}_{x}^{\prime \prime}=-\frac{c}{a} \frac{\pi \mu p_{0}}{32 G c}\left\{4(2-v) c^{2}+(4-v) x^{2}+(4-3 v) y^{2}\right\} \\
\bar{u}_{y}^{\prime \prime}=-\frac{c}{a} \frac{\pi \mu p_{0}}{32 G c} 2 v x y
\end{gathered}
$$

The resultant displacements are found by adding equations (2.2.21) and (2.2.23) :

$$
\begin{gathered}
\bar{u}_{x}=\frac{\pi \mu p_{0}}{8 G a}(2-v)\left(a^{2}-c^{2}\right) \\
\bar{u}_{y}=0
\end{gathered}
$$

These displacements satisfy the condition for no-slip

$$
\delta_{x}=\frac{3 \mu F_{z}}{16}\left(\frac{2-v_{1}}{G_{1}}+\frac{2-v_{2}}{G_{2}}\right) \frac{a^{2}-c^{2}}{a^{3}}
$$

The stick region radius c is found from the tangential force.

$$
F_{x}=\int_{0}^{a} 2 \pi t_{x y}^{\prime} r d r-\int_{0}^{c} 2 \pi t_{x y}^{\prime \prime} r d r=\mu F_{z}\left(1-c^{3} / a^{3}\right)
$$

so:

$$
\frac{c}{a}=\left(1-F_{x} / \mu F_{z}\right)^{1 / 3}
$$

The distribution of tangential traction (2.2.20) + (2.2.22), acting everywhere parallel to the tangential force, is a good approximation to the exact solution. The ratio of $s_{y}$ to $s_{x}$ is of the order $v /(4-2 v) \approx 0.09$ (for metals) so that the inclination of the resultant slip direction to the x -axis will not be more than a few degrees.
The magnitude of the slip at a radius r within the annulus $\mathrm{c} \leq \mathrm{r} \leq \mathrm{a}$ is:

$$
\begin{gather*}
s_{x}=\left(\bar{u}_{x 1}^{\prime}+\bar{u}_{x 1}^{\prime \prime}\right)+\left(\bar{u}_{x 2}^{\prime}+\bar{u}_{x 2}^{\prime \prime}\right)-\delta x \\
s_{y}+\left(\bar{u}_{y 1}^{\prime}+\bar{u}_{y 1}^{\prime \prime}\right)+\left(\bar{u}_{y 2}^{\prime}+\bar{u}_{y 2}^{\prime \prime}\right)
\end{gather*}
$$

Neglecting terms of order $v /(4-2 v)$ :

$$
s_{x} \approx \frac{3 \mu F_{z}}{16 G a}(2-v)\left\{\left(1-\frac{2}{\pi} \sin ^{-1} \frac{c}{r}\right)\left(1-2 \frac{c^{2}}{r^{2}}\right)+\frac{2}{\pi} \frac{c}{r}\left(1-\frac{c^{2}}{r^{2}}\right)^{1 / 2}\right\}
$$

$$
s_{y} \approx 0
$$

The maximum value of micro-slip occurs at the edge of the contact. The relative tangential displacement of the two bodies is found by substituting equation (2.2.27) into (2.2.25)

$$
\delta_{x}=\frac{3 \mu F_{z}}{16 a}\left(\frac{2-v_{1}}{G_{1}}+\frac{2-v_{2}}{G_{2}}\right)\left\{1-\left(1-\frac{F_{x}}{\mu F_{z}}\right)^{2 / 3}\right\}
$$

On the point of sliding, the overall displacement $\delta_{\mathrm{x}}$ is just twice the relative slip $\mathrm{s}_{\mathrm{x}}$ at the edge of the contact circle. For bodies having the same elastic constants,
differentiating equation (2.1.11) gives a normal compliance

$$
\frac{d \delta_{z}}{d F_{z}}=\frac{2}{3}\left\{\frac{9}{4}\left(\frac{1-v^{2}}{E}\right)^{2}\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right) \frac{1}{F_{z}}\right\}^{1 / 3}=\frac{(1-v)}{2 G a}
$$

The tangential compliance for small values of $F_{x}$ is

$$
\frac{d \delta_{x}}{d F_{x}}=\frac{(2-v)}{4 G a}
$$

So that the ratio of tangential to normal compliance is independent of the normal load: (2-v)/2(1-v).

### 2.3 Creep in rolling contact

Rolling is defined as a relative angular motion between two bodies in contact about an axis parallel to their common tangent plane. The eulerian view (fig. 1.4) will be used. Material particles of each surface flow through the contact region parallel to the x -axis with a common velocity V known as the rolling speed.
If $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ are unequal, the rolling motion is accompanied by sliding and if $\omega_{z 1}$ and $\omega_{z 2}$ are unequal, it is accompanied by spin.
Tangential tractions and the resulting deformation introduce creep velocities $\delta \mathrm{V}_{1}$ and $\delta V_{2}$, each having components in both the x and y directions, which are small compared with the rolling speed V . The components of tangential elastic displacement at a surface point $(\mathrm{x}, \mathrm{y})$ are $\bar{u}_{\mathrm{x}}(\mathrm{x}, \mathrm{y}, \mathrm{t})$ and $\bar{u}_{\mathrm{y}}(\mathrm{x}, \mathrm{y}, \mathrm{t})$, whereas

$$
\frac{d \bar{u}_{x}}{d t}=V \frac{\partial \bar{u}_{x}}{\partial x}+\frac{\partial \bar{u}_{x}}{\partial t} \text { and } \frac{d \bar{u}_{y}}{d t}=V \frac{\partial \bar{u}_{y}}{\partial x}+\frac{\partial \bar{u}_{y}}{\partial t}
$$

The resultant particle velocities at a general surface point, taking into account creep, spin and deformation, are

$$
v_{x}(x, y)=V+\delta V_{x}-\omega_{z} y+V \frac{\partial \bar{u}_{x}}{\partial x}+\frac{\partial \bar{u}_{x}}{\partial t} \quad \text { and }
$$

$v_{y}(x, y)=\delta V_{y}+\omega_{z} x+V \frac{\partial \bar{u}_{y}}{\partial x}+\frac{\partial \bar{u}_{y}}{\partial t}$

If the strain field does not change with time (i.e. uniform motion under constant forces), the final terms vanish. The terms

$$
\left(\partial \bar{u}_{x} / \partial x\right) \text { and }\left(\partial \bar{u}_{y} / \partial x\right)
$$

arise from the state of strain in the surface, which can be found if the surface tractions are known. They are necessarily small compared with unity. The velocities of microslip between contacting points are then given by

$$
\dot{s}_{x}(x, y) \equiv v_{x 1}-v_{x 2}
$$

$$
=\left(\delta V_{x l}-\delta V_{x 2}\right)-\left(\omega_{z 1}-\omega_{z 2}\right) y+V\left(\frac{\partial \bar{u}_{x 1}}{\partial x}-\frac{\partial \bar{u}_{x 2}}{\partial x}\right)
$$

and

$$
\begin{align*}
& \dot{s}_{y}(x, y) \equiv v_{y 1}-v_{y 2} \\
& =\left(\delta V_{y 1}-\delta V_{y 2}\right)+\left(\omega_{z 1}-\omega_{z 2}\right) x+V\left(\frac{\partial \bar{u}_{y 1}}{\partial x}-\frac{\partial \bar{u}_{y 2}}{\partial x}\right)
\end{align*}
$$

For an elliptical contact area of semi-axes a and b, we rewrite (2.3.4 and 2.3.5) in nondimensional form to give

$$
\begin{aligned}
& \dot{s}_{x} / V=\xi_{x}-\psi y / c+\left(\frac{\partial \bar{u}_{x 1}}{\partial x}-\frac{\partial \bar{u}_{x 2}}{\partial x}\right) \\
& \dot{s}, V=\xi_{y}+\psi x / c+\left(\frac{\partial \bar{u}_{y 1}}{\partial x}-\frac{\partial \bar{u}_{y 2}}{\partial x}\right)
\end{aligned}
$$

$$
\text { where } \xi_{x} \equiv\left(\delta V_{x 1}-\delta V_{x 2}\right) / V \text { and } \xi_{y} \equiv\left(\delta V_{y 1}-\delta V_{y 2}\right) / V
$$

are the creep ratios,
$\psi$ is the non-dimensional spin parameter

$$
\psi=\left(\omega_{z 1}-\omega_{z 2}\right) c / V, \quad \text { and } \quad c=\sqrt{a b}
$$

In a stick region

$$
\dot{s}_{x}=\dot{s}_{y}=0
$$

and
$\left|t_{x y}(x, y)\right| \leq \mu p_{z}(x, y)$
where $\mu$ is the coefficient of limiting friction.

## In a slip region,

$$
\left|t_{x y}(x, y)\right|=\mu p_{z}(x, y)
$$

and the direction of $\mathrm{F}_{\mathrm{xy}}$ must oppose the slip velocity :

$$
\frac{t_{x y}(x, y)}{\left|t_{x y}(x, y)\right|}=-\frac{\dot{s}(x, y)}{|\dot{s}(x, y)|}
$$

One of the difficulties of such problems lies in finding the configurations of the stick and slip zones.

In conclusion: in this section creep of rolling contact is defined (eq. 2.3.6,2.3.7) as well as a spin parameter. In the next section creep is calculated for a plain strain situation (two perfectly aligned cylinders without end effects, consider a section in the middle)

### 2.4 Tractive rolling.

The first solution to this problem in its two-dimensional (plane strain) form was presented by [Carter,1926] and independently by [Fromm,1927]. The stick region is located adjacent to the leading edge of the contact area and slip is confined to a single zone at the trailing edge. The distribution of tangential traction comprises the superposition of two elliptical distributions $t_{x}^{\prime}(x)$ and $t^{\prime \prime}(x)$. The traction $t^{\prime}{ }_{x}(x)\left(=\mu p_{0}(1\right.$ $\left.-x^{2} / a^{2}\right)^{1 / 2}$ ) produces a tangential strain within the contact strip:

$$
\frac{\partial \bar{u}_{x}^{\prime}}{\partial x}=-\frac{2\left(1-v^{2}\right)}{a E} \mu p_{0} x
$$

Contrary to the static case the centre of $\mathrm{t}_{\mathrm{xy}}{ }^{\prime \prime}(\mathrm{x})$ is displaced by a distance $\mathrm{d}(=\mathrm{a}-\mathrm{c})$ as shown in fig. 2.4.1.

$$
t_{x y}^{\prime \prime}(x)=-\frac{c}{a} \mu p_{0}\left\{1-(x+d)^{2} / c^{2}\right\}^{1 / 2}
$$



Fig. 2.4.1 Traction for rolling cylinders.

Within the strip ( $-\mathrm{a} \leq \mathrm{x} \leq \mathrm{c}-\mathrm{d}$ ) it produces a tangential strain :

$$
\frac{\partial \bar{u}_{x}^{\prime \prime}}{\partial x}=\frac{c}{a} \frac{2\left(1-v^{2}\right)}{c E} \mu p_{0}(x+d)
$$

The resultant tangential strain in the strip $(-\mathrm{a} \leq \mathrm{x} \leq \mathrm{c}-\mathrm{d})$ is :

$$
\frac{\partial \bar{u}_{x}}{\partial x}=\frac{2\left(1-v^{2}\right)}{a E} \mu p_{0} d=\text { constant }
$$

## The creep ratio is

$$
\xi_{x}=-4\left(1-v^{2}\right) \mu p_{0} d / a E
$$

The width of the stick region is determined by the magnitude of the tangential force:

$$
\frac{d}{a}=1-\frac{c}{a}=1-\left(1-F_{x} J \mu F_{z}\right)^{1 / 2}
$$

By using the Hertz relationship for $p_{0}$ (2.1.10), the creep ratio (2.4.5) is given by

$$
\xi_{x}=-\frac{\mu a}{R}\left\{1-\left(1-F_{x} J \mu F_{z}\right)^{1 / 2}\right\}
$$

where $1 / R=1 / R_{1}+1 / R_{2}$. The relationship between $\xi_{x}$ and $F_{x}$, plotted in fig.
2.4.2, is known as a 'creep curve'.


Fig. 2.4.2. Carters creep curve, eq. 2.4.7.
The action of a tractive force causes micro-slip at the trailing edge. The slip region spreads with increasing tangential force until it reaches the leading edge and complete sliding occurs $\mathrm{F}_{\mathrm{x}}=\mu \mathrm{F}_{\mathrm{z}}$.

When $\mathrm{F}_{\mathrm{x}} / \mu \mathrm{F}_{\mathrm{z}}$ is small, the distribution of tangential traction approaches:

$$
t_{x}(x)=\frac{P o}{2} \frac{a+x}{\left(a^{2}-x^{2}\right)^{1 / 2}} \frac{F_{x}}{F_{z}}
$$

The corresponding limit for the creep ratio is

$$
\xi_{x}=a F_{z} /\left(2 R F_{z}\right)
$$

This represents a linear creep curve whose gradient is the creep-coefficient. The results apply only when the two cylinders are elastically similar, otherwise additional tangential tractions are present. Generally the influence of the tractive force outweighs that of the difference in elastic constants whereupon the above results can be used with $\mathrm{E} /\left(1-v^{2}\right)$ replaced by $2 \mathrm{E}^{*}$. Interaction between the two effects has been analysed. [Kalker,1971]
From a tangential force $\mathrm{F}_{\mathrm{y}}$, parallel to the axis of the cylinders, tangential tractions and micro-slip arise in the axial direction.
A relationship similar to equation (2.4.1) except for the change in elastic constant from $\left(1-v^{2}\right) / E$ to $(1 / 2 \pi G)$ gives the surface strains :

$$
\frac{\partial \bar{u}_{x}}{\partial x}=0, \quad \frac{\partial \bar{u}_{y}}{\partial x}=-\frac{\mu p_{0}}{G} \frac{x}{a} \quad-a \leq x \leq a
$$

The axial traction is found by the superposition of $t_{y}^{\prime}(x)=\mu p_{0}\left(1-x^{2} / a^{2}\right)^{1 / 2}$ and $t_{y}^{\prime \prime}(=-$ $\mathrm{t}_{\mathrm{x}} \mathrm{c} / \mathrm{a}$ ) as before, where the extent of the slip region is given by (2.4.6). The axial creep ratio is given by substituting equation (2.4.10) in (2.3.7) :

$$
\xi_{y}=-\left(\frac{1}{G_{1}}+\frac{1}{G_{2}}\right) \mu p_{0}\left\{1-\left(1-F_{y}, \mu F_{z}\right)^{1 / 3}\right\}
$$

Cases of combined axial and longitudinal traction have been examined by [Heinrich \& Desoyer,1967] and [Kalker,1967].

In conclusion: Creep and the size of the stick and slip zone are quantified for a plain strain situation of steady rolling. Experimental data down to $10 \%$ of the maximum value of traction force exists to confirm this theory. In the next section the much more general case of steady rolling with traction and spin of 3D-bodies is considered.
2.5 Rolling with traction and spin. [Ollerton \& Haines,1963] and [Kalker,1967]. Three-dimensional bodies in rolling contact may have a relative angular velocity about the normal axis referred to as spin.

$$
\left(\Delta \omega_{z} \equiv \omega_{z 1}-\omega_{z 2}\right)
$$

The spin motion twists the contact interface and so causes tangential tractions and micro-slip.

1) First the situation where the coefficient of friction is sufficiently great to prevent slip is considered. No slip $(\mu \rightarrow \infty)$ : The linear creep theory.

If the coefficient of friction is sufficiently high, slip is limited to a thin zone at the trailing edge of the contact. The tangential tractions which satisfy the no-slip conditions in the contact area are sought. [Kalker,1964,1967] made use of the fact that a general traction :
$t_{x y}(x, y)=A_{m n}(x / a)^{m}(y / b)^{n}\left\{1-(x / a)^{2}-\left(y / b^{2}\right\}^{-1 / 2}\right.$
gives rise to tangential displacements $\bar{u}_{x}$ and $\bar{u}_{y}$ each of which varies throughout
the elliptical contact region as a polynomial in $x$ and $y$ of order $(m+n)$. By superposition of tractions of the form (2.5.1) the displacement gradients can be made to satisfy the condition of no-slip throughout the contact ellipse, while maintaining a zero value of tractions along the leading edge. Kalker truncated the series at $m+n=$ 5 and minimised the integrated traction round the leading edge, by making $\mathrm{F}=0$ at a finite number of points on the leading edge. The results are summarised in three creep equations :

$$
\begin{gathered}
\frac{F_{x}}{G a b}=C_{11} \xi_{x} \\
\frac{F_{y}}{G a b}=C_{22} \xi_{y}+C_{23} \psi \\
\frac{M z}{G(a b)^{3 / 2}}=C_{32} \xi_{y}+C_{33} \psi
\end{gathered}
$$

where $C_{11}, C_{22}$ etc. are non-dimensional creep coefficients.

When stick and slip zones coexist in the contact area the distribution of traction and equations for creep have been found only approximately. The contact area is divided into thin strips parallel to the rolling direction (x-axis). The theory from section 2.4 is then applied to each strip, neglecting interaction between adjacent strips. ([Haines \& Ollerton,1963] and [Kalker,1967]) An elliptical contact under the action of a tangential force $F_{x}$ is considered (fig. 2.5.1).


Fig 2.5.1 Carters cylinder theory applied to an elliptical contact.

A typical strip, distance $y$ from the $x$-axis has a length $2 a$ and supports a pressure

$$
\begin{aligned}
& p_{z}(x)=p_{0}^{*}\left(a^{2^{*}}-x^{2}\right)^{1 / 2} / a^{*} \\
& \frac{p_{0}^{*}}{p_{0}}=\frac{a^{*}}{a}=\left(1-y^{2} / b^{2}\right)^{1 / 2}
\end{aligned}
$$

where

Carter's theory for cylindrical contact can be applied to the strip. A stick region of length $2 c^{\circ}$ is located adjacent to the leading edge. The creep ratio is given by equation (2.4.5), with $\mathrm{p}_{0}, \mathrm{~d}$ and a can be replaced by $\mathrm{p}_{0}{ }^{*}, \mathrm{~d}^{*}$ and $\mathrm{a}^{*}$ respectively, so

$$
\xi_{x}=-\frac{2(1-v)}{G a^{*}} \mu p_{0}^{*} d^{*}
$$

The creep ratio $\xi_{x}$ must be a constant for the entire contact, so it follows from
equation (2.5.4) that $\mathrm{d}^{*}\left(=\mathrm{a}^{*}-\mathrm{c}^{*}\right)$ has the same value for all the strips in which a stick zone exists. Thus the mid-points of the stick zones lie on the straight line $x=-d^{*}$. The curve separating the stick and slip zones is therefore a reflexion of the leading edge in this line, giving rise to a lemon shaped stick zone as observed by [Ollerton \& Haines, 1963] on transparent photo-elastic wheels in contact. The distribution of traction on a strip is shown in fig. 2.5.1. The tangential force $\mathrm{dF}_{\mathrm{x}}$ provided by a strip is determined by equation (2.4.6)i.e.

$$
\begin{aligned}
& \mathrm{dF}_{\mathrm{x}}=\mu \mathrm{F}_{\mathrm{z}}^{*}\left(1-\mathrm{c}^{2} / \mathrm{a}^{* 2}\right) \mathrm{dy} \\
& \quad=\frac{\pi}{2} \mu p_{0} a\left(1-y^{2} / b^{2}\right)\left(1-\left(1-d^{*} / a^{*}\right)^{2}\right\} d y
\end{aligned}
$$

The total force $\mathrm{F}_{\mathrm{x}}$ is found by integration over the contact area. When $\mathrm{y}>\mathrm{b}(1-$ $\left.d^{*} z / a^{2}\right)^{1 / 2}$, the stick region vanishes so that the term $\left(1-d^{*} / a^{*}\right)$ is put equal to zero, with the result :

$$
F_{x} / \mu F_{z}=3 / 2 \zeta_{x} \cos ^{-1}\left(\zeta_{x}\right)+\left\{1-\left(1+1 / 2 \zeta_{x}^{2}\right) \sqrt{1-\zeta_{x}^{2}}\right\} \quad \text { where }
$$

## $\zeta_{x}=\xi_{x} G / \mu p_{0}$

For $\mu \rightarrow \infty$ (2.5.6) becomes

$$
F_{x}=\frac{\pi^{2}}{4} \frac{G a b}{1-v} \xi_{x}
$$

The value of the creep coefficient given by the strip theory (2.5.7) is independent of the shape of the contact ellipse. The agreement with Kalker's value for vanishingly small slip is good when the ellipse is thin in the rolling direction ( $b$ » a), but the effect of neglecting interaction between the strips becomes serious for contacts in which b < a.

To apply the strip theory to contacts transmitting a transverse force $\mathrm{F}_{\mathrm{y}}$ or rolling with spin, use is made of the two-dimensional theory of transverse traction given in [Kalker,1967]. The strip theory is not satisfactory unless b > a; it breaks down completely when the spin motion is large. The complementary energy principle of [Duvaut \& Lions,1972] and [Kalker,1979] replaces the Eulerian formulation by a Lagrangean system in which the moving contact area is followed and the traction is built up incrementally with time from some initial state until a steady state is approached. Such transient behaviour is discussed further in section 2.6.

In conclusion: By applying the theory of section 2.4 to narrow strips, parallel to the rolling direction in the contact area of 3D-bodies under steady rolling, creep was quantified. Experimental data down to $5 \%$ of the maximum value of traction force exists, though often obtained with plastic wheels. The transition from standstill to rolling is considered in the next section (2.6). It connects the traction distribution of section 2.2 (stationary contact) to that of section 2.4 (plane strain rolling).

### 2.6 Transition to rolling from standstill.

Unsteady rolling contact of cylinders in plane strain has been examined by [Kalker, 1969, 1970, 1971].
For similar elastic bodies, equation (2.3.4) for the slip velocity becomes

$$
\dot{s}(x, t)=V \xi(t)+2 V \frac{\partial \bar{u}(x, t)}{\partial x}+2 \frac{\partial \bar{u}(x, t)}{\partial t}
$$

$=V \xi(t)+2 V \frac{\partial \bar{u}(x, t)}{\partial x}=2 \frac{\partial}{\partial t}(\bar{u}(x, t)-\bar{u}(0, t))$

$$
=\frac{2 \partial \bar{u}(0, t)}{\partial t}
$$

The displacement at the centre of a rectangle $2 \mathrm{~b} \times 2 \mathrm{a}(\mathrm{b}$ » a$)$ due to a uniform traction $\mathrm{t}_{\mathrm{x}}=\mathrm{F}_{\mathrm{x}} / 4 \mathrm{a}$, is used to approximate $\bar{u}(\mathrm{o}, \mathrm{t})$ :

$$
\begin{aligned}
& \bar{u}(0, t)=\frac{2\left(1-v^{2}\right) Q(t)}{\pi E}\{1 /(1-v)+\ln (2 b / a)\} \\
& \text { and so : } \\
& \left.\frac{\partial \bar{u}(0, t)}{\partial t}=\frac{2\left(1-v^{2}\right)}{\pi E}\left\{\frac{1}{1-v}+\ln \left(\frac{2 b}{a}\right)\right\}\right\} \frac{d F_{x}(t)}{d t}
\end{aligned}
$$

where $F_{x}$ is the tangential force per unit axial length.
The distributions of traction $\mathrm{t}_{\mathrm{x}}(\mathrm{x}, \mathrm{t})$ and the value of the creep ratio $\xi(t)$ which
satisfy the conditions in the stick and slip zones must be found by following the loading history starting from given initial conditions. [Kalker,1969,1970,1971] shows the technique of solution for a constant tractive force starting from rest.
If a tractive force less than limiting friction is applied to cylinders at rest, micro-slip takes place at both edges of contact and the tangential traction is distributed according to equation (2.216), as shown in fig. 2.6.1 $(1=0)$. Initially the slip regions and tractions vary transiently with 1 , the distance rolled.

## Since inertia effects are ignored,

## $\partial / \partial t=V \partial / \partial l$

The traction distributions at various stages in the process are shown in fig. 2.6.1. for the case of partial slip in which $\mathrm{F}_{\mathrm{x}}=0.75 \mu \mathrm{~F}_{\mathrm{z}}$. As rolling proceeds, no further slip occurs at the leading edge. The original traction distribution moves through the contact with the rolling velocity until it merges with the trailing slip zone at $\mathrm{l}=0.6 \mathrm{a}$. The steady state is established by $1=1.6$ a.


In conclusion: the distance in which the traction distribution of a stationary contact develops into steady rolling is quantified. (e.g. $1=1.6 \mathrm{a}$ ).

In conclusion of chapter 2: The theory on creep, as it stands today, is now clear, it can be used to make predictions. For easy access this is done in software: geometry, loading, materials properties $\mathrm{E}, \mathrm{v}, \mu_{\text {kin. }}$ are input data, compression, contact size, shape, stick and slipzone graph, steady rolling creeprate and drive stiffness (neglecting bearing stiffness) are outputs.

What is not clear from theory are the following main questions:
1 How far down can creep rate be confirmed experimentally in setups of state-of-the-art design level, e.g. traction at $1 \%$ or $1 \%$ of normal force?
2 How serious is misalignment of wheel axes in P.E. practice?
3 What happens in practical situations under dynamic conditions?

The next chapters deal with an experimental approach to solve these three problems so that we can predict with confidence for future Precision Engineering applications involving complex, expensive hardware based on experimental interpolation rather than experimental extrapolation.

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## 3. Experimental set up

A special measuring instrument was developed to measure creep. It was based on the idea that it should be possible to measure creep in a short time span, typically a few seconds, thereby minimizing effects caused by thermal expansion or distortion. This chapter contains a discription of the apparatus used in the measurements. Fig 3.0 below gives an overview of the chapters subjects, numbers in the boxes indicate respective sections.


Fig. 3.0 Schematic diagram of apparatus description in chapter 3.

The test set up consists of a mechanical construction, described in section 3.1 and a data acquisition system, described in section 3.2. Fig. 3.1 shows an overview of those parts. Some general requirements are listed in table 3.1.

| quantity to be measured | symb. | units | range | resolution |
| :--- | :---: | :---: | :---: | :---: |
| creep rate | $\xi_{\mathrm{x}}$ | - | $\pm\left(10^{-6}-10^{-3}\right)$ | $10^{-6}$ |
| traction force | $\mathrm{F}_{\mathrm{x}}$ | N | $\pm\left(10^{-2}-10^{1}\right)$ | $10^{-2}$ |
| axial displacement | y | m | $\pm\left(0-2.10^{-3}\right)$ | $10^{-6}$ |


| Adjustable parameters | symb. | units | range | resolution |
| :--- | :---: | :---: | :---: | :---: |
| normal force | $\mathrm{F}_{\mathrm{z}}$ | N | $5.10^{2}$ | $<1$ |
| tracking angle | $\theta$ | rad | $\pm 8.10^{-4}$ | $5.10^{-6}$ |
| spin angle | $\varphi$ | rad | $\pm 8.10^{-4}$ | $5.10^{-6}$ |

Table 3.1
The various sections of chapter 3 deal with subsystems which were used to meet these requirements. The individual sections each start with a listing of a subset of requirements relevant in that section.


Fig 3.1 Overview of the experimental set-up

### 3.1 The mechanical construction

The mechanical part, as shown in fig. 3.2, consists of several sub-assemblies which will be described in their functional features here.


Fig 3.2 The mechanical part of the measuring machine

### 3.1.1 Electromechanical drive systems

The drive system was not designed first because it was considered necessary to keep it outside of the machine to allow testing of different systems later on and also to keep the heat input to the instrument to a minimum. Nevertheless it is treated here at the beginning.
Two completely different drive systems were actually used on the test machine namely:

1. A brushless motor with a two stage planetary gear reducer $i=1: 19.2$ fitted with a 500 lines incremental encoder, together with driver electronics operating from the encoder on the motorshaft. This system was initially run on both shafts with rigid shaft connection hence supported by the test machine. Later the gearbox output shaft was connected through a polythene tube to isolate drive line transients and then it was supported on an available zero stiffness elastic straight-guide when connected to the cilindrical wheel to prevent interference with the motion detection system. As the assembly has a mass of 0.376 kg , angle adjustments are required to compensate for finite stiffness after fitting or removal of motors. The soft connection of the drive does not interfere with the measurement as the measurement encoders remain rigidly attached to the wheels, thereby revealing what is happening at the contact.
2. A disc-rotor type motor with a bolted on tacho and a 1000 lines optical incremental encoder. Because of its mass ( $8 \mathrm{~kg} \mathrm{)} \mathrm{this} \mathrm{drive} \mathrm{could} \mathrm{not} \mathrm{be} \mathrm{supported} \mathrm{by} \mathrm{either} \mathrm{the}$ test machine or the existing straight-guide and was therefore put on three airbearing pads on a granite base plate.
For dynamic testing a flexible metal bellows was used between a separate flywheel and the machine axis. This flywheel was driven with an inverted Poly-V belt from the motor pulley, a ratio of 1:4 was used. The direct drive was not as smooth. Additionally a flywheel was glued to the output shaft to allow prolonged acceleration at a certain traction force without the need for revolutionary speeds so high as to create a problem with the maximum frequency the encoder board can sustain ( 60 kHz ., corresponding to 87 rpm ).
A servo system, consisting of a power amplifier, a controller, a communication card and a setpoint generator together with a security module, as shown in fig. 3.3, was used to create parabolic movement whilst rolling the wheels through an angle.

Triangular or trapezoidal velocity profiles can be programmed with max. speed and $\max$. accelleration setting. A servo observer and a digital storage oscilloscope were used to monitor and make adjustments to the servo system. A PC, running a Turbo Pascal program, allowed for communication with the system via RS232 for task and loop gain settings.


Fig 3.3 The servo drive systems.

### 3.1.2 Wheels, bearings \& shaft

Two wheels in contraform contact and a basically parallel shaft set up were used for the creep measurements. A description of the assembly, its bearings and the preload mechanism of the bearings is given below. The preloading of the wheel contact is described in section 3.1.5. The adjustment of parallellity is given in section 3.1.3. Fig. 3.4 shows a cross section of the wheelbearings.


Fig 3.4 The cilindrical wheel (left) in contact with the spherical wheel (right), each mounted on preloaded precision angular contact ball bearings

Precision angular contact ball bearings were used. Wheel and shaft are a one piece part, this was done to obtain a minimal radial run-out and to make sure there could be no movement between rim and hub. The material used is a high carbon chromium steel and it was hardened to 62 Rc after machining on the lathe, before grinding. A wheel diameter of 100 mm was used on both wheels without emphasis on actual absolute size. The left side wheel is cylindrical in circumference, the right side wheel is crowned to a 50 mm radius, so its outer surface can be thought of as being cut from a sphere. This was done to allow angle adjustment. See section 3.1.3.

The actual precision bearings purchased, showed roundness measurements for the rings well within their specifications, i.e. less than $0,1 \mu \mathrm{~m}$ peak-to-valley out of roundness was observed.
Final assembly with preload was less than $1 \mu \mathrm{~m}$ peak-to-valley total radial run out on both wheel circumferences.
The hub of the wheel on the right in fig. 3.4 is mounted on a stub-axle which in turn is attached to the tilt mechanism. The latter mechanism is described in section 3.1.3. There is a rigid connection between the hub and the encoder shaft, so the encoder as a whole moves with wheel tilt.
The shaft of the cylindrical wheel on the left in fig. 3.4 is mounted in a block described in section 3.1.4. This shaft is rigidly connected to another encoder shaft, so if there is any movement, this wheel and encoder move together.
Four sets of wheels were processed together to have adequate spares in case of mishaps. Good roundness and surface finish were considered more important than actual size. The problem of actual size is addressed in section 4.1

The setting of a correct bearing preload is done by the instrument mechanics. A special bushing shown in fig. 3.5 is manufactured in such a way that it will expand axially under the desired preload for about $0,3 \mathrm{~mm}$ whilst remaining laterally stiff. The radial stiffness is high compared to the axial stiffness. The elastic mechanisms pole, part $B$ point $P$, is at the pole-point of the angular contact bearing when the part B is preloaded. The preload level is 300 N at 0.3 mm expansion, so preload is known to the order of a percent after the axial position has been assertained to approximately $3 \mu \mathrm{~m}$.

Elastic pole and ballbearing pole coincide under preload


Fig 3.5 Axially elastic radially stiff bearing preload bushing

All bearing fits are executed as fixed, so zero radial play is obtained. All diameters for fits were measured for roundness and corrected several times; especially on the elastic bushing it was not straightforward to get good roundness. It was found that even with a very well stress-relieved, believed "dead" material, the actual zero-force wire spark erosion cutting of the slots relieved enough internal stress to deform the bore several micrometers. The way to go is to cut the slots into the premachined part first, then to fill the slots by casting in resin and then the diameters are finished to their final value. The resin is subsequently dissolved in alcohol.

### 3.1.3 Spin and tracking angle adjustment

Requirements on spin and tracking angle adjustment.
There are limits to the accuracy with which shaft can be made parallel, due to manufacturing of components and assembly, and their finite stiffness. The creep measuring machine incorporates aligning mechanisms which can correct tilt angles across a range more than large enough to cover all above mentioned geometric errors.
The spherical wheel (see section 3.1.2) is adjustable by its stub-axle through two angles of tilt, the spin and the tracking angle. The part carrying the stub-axle of this wheel can be thought of as being held fixed in three translations at a single point. This point was chosen as the actual contactpoint of the wheels, so that a tilt adjustment would come down to the spherical wheel pivoting about the contactpoint. The third rotation (about an axis parallel to the axis of the cylindrical wheel) is not required and therefore suppressed by connecting a further point well away from the contact, in the mid-plane of the cylindrical wheel, to the frame. The above mentioned connections were done with elastic elements of the "folded leafspring" type, [vd Hoek, 1989]. Three identical ones were cut by wire-EDM from Dominal-H material, fig. 3.6 shows their cross section.


Fig 3.6 Crosssection of "folded leafspring", equivalent to one rod constraint, cut with wire EDM. Properties: allows two translations and three rotations.
Note that by doing this, the transition from thin leafspring to frame is now within a single piece of material, thereby eliminating a possibility for hysteresis that normally occurs when clamping a leafspring with coverplates and bolts. Also note the unique parallelity obtained this way. Furthermore, the actual "fold" which is difficult to produce from flat spring material, because its radius should be small compared to the spring thickness, is now "easily" obtained.

Three of these units, as shown in fig. 3.7, are mounted orthogonally to each other pointing with the "fold" to a point in space, later to become the contactpoint.


Fig 3.7 Elastic spherical bearing components

The folded "leafsprings" are bolted between two cones which each received three slots to locate the folded leafsprings. Fig. 3.7 shows this assembly in exploded view. This "spherical bearing" could, because of its size, not be produced as a single block of material. The wire EDM has to be done at oblique angles, so the work space of the available machine, $200 \times 250 \times 300 \mathrm{~mm}$, simply wasn't big enough. It was felt that the assembled version could be very similar in behaviour at a cost of making it to tight tolerances.

It was considered possible to produce and mount a single piece spherical bearing later. In fact a single piece version was produced early 1994 for testing, though be it only in a 150 mm diameter version.
The new type with "folded leafsprings" has the advantage of a stationary elastic pole; the pole was shown to remain within a 1 micrometer cube, at the cost of greater complexity.
Stiffness in all directions is equal, due to the orthogonal set-up. It is $100 \mathrm{~N} / \mu \mathrm{m} x, y, z$ at the contactpoint, tilt force required is $90 \mathrm{~N} / \mathrm{mm}$ at a 200 mm . arm. The inner cone is fixed at its base circle to a 22 mm thick flange, part 1 in fig. 3.7 The outer cone with the stub-axle for the spherical wheel is manipulated with respect to the flange via three 8 mm diameter rods, part 2 in fig. 3.7 that pass through holes in the flange and intersect in the midplane of a planar elastic manipulator shown in fig. 3.8.


Fig 3.8 Planar manipulator for two translations, for tilt adjustment, made with wire EDM.

Hence the rods are loaded in tension or compression only and thereby behave extremely stiff, as bending is avoided. The planar manipulator itself is supported from the thick flange on three triangular plates, part 3 in fig. 3.7.

By leaving straight connections of system points in place and removing material further away from these lines, considerable mass reduction was obtained, $50 \%$ at a moderate loss in stiffness. Hence holes were machined in the inner cone and flange and wire EDM was used on the outer cone to remove material.


Fig. 3.9 Mass reduction holes on flange, cone and outer cone.

The elastic planar manipulator provides $1: 6$ reduction of each micrometer screw actuator, which can be adjusted by hand across $\pm 1 \mathrm{~mm}$ to 0.01 mm hence to $\pm 8.5$ $10^{-6} \mathrm{rad}$ of rotation accuracy at the wheel. For further fine adjustment these commercially available components contain $30 \mu \mathrm{~m}$ stroke low voltage piezo elements of $1 \mathrm{~N} / \mu \mathrm{m}$ stiffness.

### 3.1.4 Lateral movement mechanism (zero-stiffness)

Requirements on the lateral movement mechanism.

This mechanism allows detection of axial force on the contact, whereby it is possible to adjust the tilt angles. The axial force resulting from a tracking angle changes sign with rolling direction and is thereby decernable from the axial force caused by the spin angle which does not change its sign. Below, the ideas behind the lateral movement mechanism are described, together with a compensation for its own stiffness, whereby its sensitivity to move under force is increased.
The shaft of the cylindrical wheel is mounted in angular contact ball bearings in a block mentioned in section 3.1.2 To allow free axial movement of the cylindrical wheel with its shaft and bearing, the block is provided with an integral elastic straightguide. The straight-guide is symmetrical with respect to the axis of rotation.
Fig. 3.10 shows the principle of fixing a point in a planar object (circle) in its plane by attachment of 3 tangential rods, thereby providing symmetry and a thermal center, TC.


Fig 3.10 Planar object constrained by three tangential rods, providing a thermal center TC.

In fig. 3.11 a 3D cilindrical version is shown which can be translated to a folded leafspring version, as shown in fig. 3.12 a , b.


Fig 3.11 3-D version of fig 3.10, providing a centerline in space as a thermal center, axial movement (vertical) is free.


Fig 3.12 a) Folded leafspring version of fig 3.11 . b) Same with radial springs.

Again in order to minimize hysteresis and to avoid the problem of making sharp folds in already highly deformed material, it was decided to cut folded leafsprings by wire EDM from a single piece of material. Fig. 3.13 shows the drawing of this part a leafspring thickness of 0.3 mm . Overload protection is integrated in the part.
Leafspring height is 60 mm so radial stiffness reaches $33 \mathrm{~N} / \mu \mathrm{m}$ provided loading is not near the buckling limit.


Fig 3.13 Single block, machined by wire EDM, to contain leafsprings (as depicted schematically in fig 3.12a) without hysteresis.

The buckling limit is at 225 N , whereas the axial stiffness is $80 \mathrm{~N} / \mathrm{mm}$. This axial stiffness is the only drawback of elastic straight-guides next to their limited stroke. Advantages are: there is no play or backlash, there is no friction at all, there is negligible damping in steel, movement quality is superb.

There are several methods to reduce the drawback of residual stiffness in the axial direction, one is shown in fig. 3.12 b . Here the radial part of each spring is loaded in a radial direction by another spring, in this case a helical one, to a value near the
buckling load of that particular leafspring. Axial movement thereby becomes very easy. The principle is depicted schematically in fig. 3.14.


Fig. 3.14 Two orthogonal springs of equal stiffness connected via a bar link contain constant total elastic energy in any position (= zero stiffness movement).

Two springs have their ends, which would be at the origin at zero force, connected by a bar link. Because it is difficult to attach such strong springs one by one, it was thought better to attach negative stiffness in parallel, so as to create zero resultant stiffness. The preferred principle for creating negative stiffness for large strokes is shown in fig. 3.15 a.


Fig $3.15 \mathrm{a}, \mathrm{b}$ buckled leafspring held in the center provides negative stiffness to the left and right.[vdHoek,1989]

Second order buckling yields even higher values of negative stiffness: same figure, part b.
In "direct drive", the buckled leafspring would become quite nasty e.g. $20 \times 2 \mathrm{~mm}$ in crosssection. Therefore it was decided to connect a set of normal "longstroke" negative stiffness buckled leafsprings via a cantilever reduction to the elastic straightguide. The quadratic effect of cantilever ratio on positive stiffness is shown in fig. 3.16 which is the basis of good machine design: Usually the parts are stiff by themselves, they are only loaded through excessive transmission ratios which have a quadratic stiffness reducing effect. The transmission has a similarly dramatic effect on inertia.


Fig 3.16 the effect of cantilever ratio on stiffness is quadratic.

Fortunately negative stiffness is not different from positive stiffness in the way of being "transformed" by cantilevers with the square of transmission ratio. Thereby nice thin $0,3 \mathrm{~mm}$ buckled leafsprings were found to be adequate at a moderate ratio of 7.5 : 1 .

The encoders have their shaft rigidly connected to one wheel each. The encoder bearings cannot take the weight of 35 N . Therefore the encoder housings have to be supported. A counterweight is often used for this purpose as it can easily be adjusted to the exact value required. The extra mass so added to the system is unnecessary and undesirable from the viewpoint of dynamic behaviour, as there is only a "rope and pulley" to fix it. Therefore two 1 m long, low stiffness springs with an extended length of over 3 m were lowered from the ceiling of the laboratory. These each give a constant force of $35 \mathrm{~N} \pm 35 \mathrm{mN}$.

To define the direction of each force by its attachment point on the ceiling, wires were let down on to two defined points of the machine, with respect to these points the positions were then drilled in an intermediary support plate. As the positions are thereby easily within 1 mm of true position, the horizontal component that could work on the cylindrical wheel in its elastic straight-guide is clearly less than 10 mN . After accidentally brushing these springs by hand, "slinky modes" persist for minutes and have to attenuate to allow further measurement.

### 3.1.5 Normal compression of the contact - and the traction force measurement system

Requirements on normal force and traction force measurement.

A mechanism is required which allows compression of the contact whilst constraining all remaining movements. A statically determined approach, schematically shown in Appendix 1, was used. Below is shown how the schematics were materialized to obtain maximum stiffness and minimum hysteresis for the creep measurement. The tilt mechanism that carries the stub-axle on which the spherical wheel runs, is supported by the thick flange at the bottom of the inner cone (described in section 3.1.3). From the thick flange a large diameter pipe extends towards the midplane of the cylindrical wheel, part B in fig. 3.17.


Fig 3.17 Central pipe ring frame supported by three folded leafsprings

The large diameter pipe is machined down to a 1.5 mm wall thickness except for a radially stiff flange at the other end. The mid-plane of this integral flange coincides with the mid-plane of the cylindrical wheel. Because the thin-walled pipe is kept round by flanges on either end, it is extremely stiff.

Any load on the sperical wheel will be taken out at the flanges. A gap of a few mm between the pipe and the tilting mechanism is sufficient to avoid friction. According to the principle of fig. 3.18 which can be replaced by fig. 3.19 two translations and rotation about its symmetry axis are possible for the central round body.


Fig 3.18 Table on three rods has two translations and one rotation free (all in plane).


Fig 3.19 The folded leafspring version of fig. 3.18 [vd Hoek,1989]

In the test machine the thin-walled pipe with two flanges is supported in that way. Three blocks parts A in fig. 3.17 are bolted to the flanges through small feet for minimal hysteresis. After wire EDM they each contain a "folded leafspring".

The folds are parallel to the pipes axis through machining accuracy and a fit to notches on the flanges of the thin-walled pipe. The outside part of the blocks is interconnected by a triangular frame (fig 3.17) in 10 mm plate which is assembled with long strainbolts on localized landing spots, again so as to avoid hysteresis.
Mass reduction was applied by removing material with wire EDM from the inner part of each cornerblock as this is mass that is moving with the pipe.
The flange at midplane of the cylindrical wheel is connected by a rod to a stiff piezoforce sensor $\left(5.10^{7} \mathrm{~N} / \mathrm{m}\right)$, which is installed in the top cornerblock (see fig. 3.20).


Fig 3.20 Top block containing folded leafspring, force sensor and weight support spring.

This attachment rod points exactly at the stub-axle of the spherical wheel thereby enabling measurement of the tractionforce $\mathrm{F}_{\mathrm{x}}$ transferred between the wheels. The low parasitic elastic force arising from the three "folded leafsprings" in the cornerblock is easily corrected for.

The rotation of the pipe-body is taken out at the end of the-thick-flange-that-carries-the-innercone (section 3.1.3) by means of a large coupling according to the principle of fig. 3.21.


Fig. 3.21 Original design of high quality rotationally- stiff coupling.[Koster,1990]
This coupling was made from 2 mm high strength steel sheet by CNC-milling, after having been bonded to a 25 mm aluminium backing plate. The shape, see fig. 3.22, was modified from the original coupling (fig. 3.21) which was meant to connect hubs of equal diameter to a shape that connects the (inner)pipe flange to the (outer) triangular frame.


Fig. 3.22 Adapted version of coupling of fig 3.21 to connect centerpipe to triangular frame.

The four rods A of square $2 \times 2 \mathrm{~mm}$ cross section are all at the same radial distance. The coupling plate is bolted on, with 0.5 mm spacer rings to avoid any friction leading to hysteresis.

The one remaining translation of the (inner)pipe with the tilt mechanism for the spherical wheel is used to create a well defined normal contact force $F_{z}$ between the two friction wheels. A low stiffness spring is attached to the flange in the mid-plane of the cylindrical wheel. It points exactly at the contact spot, thereby pulling the spherical wheel by its stub-axle on the tilting mechanism, see section 3.1.3, radially against the cylindrical wheel which can only move axially on its elastic straight-guide, see section 3.1.4. By extension of the spring in a tube (part 9, fig. 3.23) the force $F_{z}$ on the contact between the wheels can be modified.


Fig 3.23 Normal force on wheel contact from preloaded spring.

Again the low parasitic stiffness of the three folded leafsprings at the cornerblocks and the coupling has to be corrected for, see section 4.1.
Because the cornerblock "folded leafsprings" are 120 mm high they are extremely stiff in the direction of the "fold", order $10^{9} \mathrm{~N} / \mathrm{m}$. Because they are each at a considerable radial distance they provide extremely high stiffness against undesirable movements of the pipes relative to the triangular frame.

### 3.1.6 Frame and support structure, table.

Because small forces can disturb the delicate creep measurement, a high quality isolation of ground vibrations is required. The design information available from literature [DeBra, 1991] enabled us to come up with a low cost alternative for the commercially available tables. It nevertheless features tunable cutoff frequency and adjustable damping.

Requirements on the frame and support structure:

Ground vibrations of upto $0.1 \mathrm{~mm} / \mathrm{s}$ from the lab floor should not affect the measurements noticebly.
There should also be provisions for leveling.
The machine frame itself should have a high stiffness and negligible hysteresis. A high natural frequency is needed to created an effective filter in combination with low natural frequencies of the table.

The frame and table are described below.
The triangular plate frame mentioned in section 3.1 .5 would appear to be rigid from its shape as a triangular tube itself. However, the three lines along which the midplanes of the three 10 mm thick plates, that constitute this frame intersect in the three cornerblocks, are not coinciding with the attachment lines for the "folded leafsprings". This because of practical difficulty with locating the bolting. Therefore, the triangular frame can best be thought of as a irregular hexagonal tube itself. For maximum rigidity such a hexagonal tube requires two fixed cross sections. This would lead to the suggestion to close off either end of the frame with a steel sheet. This structurally sound idea was abbandonned for two reasons: first, the far end already carries the coupling mentioned in section 3.1.5
second, a frame is needed to connect the triangular frame to the fixed part of the elastic straight-guide for the cylindrical wheel (and this connection should be rigid and could therefore hardly be a thin sheet at the near end of the triangular frame). Instead a rigid closed box frame, which manufacture is described in section 3.5, was chosen. The closed boxes frame is depicted in fig. 3.24.


Fig 3.24 "Closed boxes" sandwich frame. See fig 3.37 for design details.

It is bolted to the three cornerblocks with two bolts each, thereby effectively taking out all internal degrees of freedom from the hexagonal tube. In Fig. 3.24 the hexagonal "hole" holds three seats for the elastic straight-guide of the cylindrical wheel. The round hole allows the connection, and the spherically crowned wheel itself, to pass through for a quick wheel change, with minimum loss of alignment. The triangular frame was erected vertically as shown in fig. 3.2 because it was considered preferable to have no influence of gravity on the delicate axial movement of the cylindrical wheel. This meant however that the central mass of the pipe and tilt mechanism had to be supported in its weight. This was done by a spring suspended in a tube on the top cornerblock of the triangle, shown in fig.3.20. The end of the spring was adjusted to its required extension at the top with a stud.
The rigid triangular frame properly locates the axes of both wheels with respect to one another. However because we want to measure small displacements and small forces without disturbances we cannot simply attach the frame to the ground. Ground vibrations typically up to $0.1 \mathrm{~mm} / \mathrm{s}$ as a velocity step occur in laboratories due to people walking, elevators holding at floors, outside traffic, air conditioning vibrations etc. All this because of the considerable mass of thick concrete floors and their finite stiffness. Passive isolation of ground vibrations is therefore required. This means that the frame is rigidly connected to a large mass which itself will be connected with "low stiffness springs" to the ground. The latter system added, provides a low pass filter function, cutting off nasty high frequency disturbances effectively.

As the large mass is a flat stone table top, the rigid connection should not cause thermal distortion of our instrument frame. Ideally our rigid instrument frame should be tied down in its six degrees of freedom as a rigid body. Schematically this is depicted in fig. 3.25.


Fig 3.25 Six d.o.f. constrained on rigid body

More practical is the version with three hinged leafsprings of fig. 3.26. This in fact is "the way" to fix rigid components to each other. Differences in thermal expansion can be absorbed by the springs, without significant bending moment being introduced.


Fig 3.26 Leafspring version of fig 3.25.

As it is desired to level the instrument, some means to adjust the vertical position of two of the three leafsprings is required. To avoid further mechanisms, it was decided to go for another support design. Two vertical rods are adjustable with a simple elastic hinge, fig. 3.27


Fig 3.27 Instrument levelling rod in hinged plate adjustment.

One at a lower cornerblock, the other at a distance from the triangular frame on a tubular extension, fig. 3.28. These two allow independent leveling of the instrument with respect to the granite table top.


Fig 3.28 Tubular extension with second levelling rod in hinged plate adjustor.

With two degrees of freedom fixed in this way by fig. 3.27 and fig. 3.28, the remaining four are held in an elastic mechanism shown in Fig. 3.29, basically a cardanic pivot. Three translations of the centerpoint are held here and the rotation about the vertical axis is constrained as well.


Fig. 3.29 Cardanic pivot leaves two tilt angles free for levelling.

The granite table top is supported via foam and a frame on pistons in $0,4 \mathrm{~mm}$ thick rubber bellows under which compressed air is held in a chamber each. The chamber volume creates the extremely low stiffness. Each chamber is connected to another eight times larger chamber via a separate laminar flow restriction to provide optimum damping. On each damping chamber there is a level regulator which is connected to the table top via a $14: 1$ ratio in an elastically hinged cantilever, shown in fig 3.30.


Fig 3.30 Table support providing passive isolation and levelling.

Proper positioning of the pistons under the table controls yaw and roll frequencies. Below the table and purposefully below the center of gravity of the combined table- --and-instrument. Three well positioned thin horizontal rods connect the table top to vertical leafsprings, which cause in plane frequencies. Any motion of this kind causes yaw and roll, as the rod forces are introduced below the combined center-of-gravity, thereby effectively using the laminar damping of the vertical pistons.


### 3.2 The data acquistition system

Requirements to the data acquisition system.

To avoid thermal problems, it was considered essential to measure creep within very few revolutions (seconds). This calls for high resolution encoders, which happen to be very accurate as evidenced by their calibration records. Even if they had not been accurate enough but had the ability to reproduce, measurement would have been possible. the higher the maximum allowable frequency on the electronics, the higher the maximum rolling speed that can be measured. Recently, 100 kHz electronics came available, be it for VME-bus only. Traction force measurement should have 1 cN resolution, be fast (for the dynamic test) and have a large range ( 50 N either direction). Axial wheel position is measured to $0.3 \mu \mathrm{~m}$ full scale.

Angular motion of each wheel is measured using an ROD 800C optical incremental encoder with a grating with 36000 divisions per revolution (Heidenhain). As these two identical encoders have their shafts rigidly connected to their respective wheels and their housings coupled to the frame of the instrument, via torsionally stiff but otherwise elastic steel couplings, see fig.3.32, a mechanically sound measuring set up was created.


Fig. 3.32 Coupling for encoders

Through special shielded cables the encoder signals are sent to an IK120 card which functions as a counterboard in a $486 \mathrm{PC}(50 \mathrm{MHz}, 8 \mathrm{Mb}$ Ram, 200 MBHD$)$. The card can be set to 25 or 50 fold interpolation and one, two or fourfold evaluation of pulse flanks. Input signals up to 60 kHz can be used. During all but the initial tests 25 fold interpolation and fourfold evaluation was set. This corresponds to a measuring step of 0.0001 degree. The card is easily addressed as memory locations C800 and onwards.

Either direct writing and reading to, respectively from, memory is possible. Routines are provided in Turbo Pascal, which are supplied with their source code, so programming is relatively easy. A measurement consists of writing a large number of 16 bit long-integers to RAM and to save these to a harddisk file at the end of the measurement. A single test is done by taking a few seconds of data, which, once saved, can be evaluated in different ways (Chapter 4). Measurements can be taken at time intervals or at intervals of position of a selected encoder. The latter are used for the actual creep measurements.
The traction force measurement is done with a piezo force transducer which is connected to a piezo charge amplifier which is programmable in EPROM. Its analogue output signal is connected to a digital storage oscilloscope. The mechanical connection is described in section 3.1.5, the transfer of data to the PC is similar to what is described below for the axial movement signal.
The axial movement of the cylindrical wheel was measured with an inductive probe and an amplifier. Its analogue output signal is fed to another channel of the above digital storage oscilloscope. After the test, the data is transferred from the oscilloscope to the measuring PC, described above in this paragraph, via RS232 cable with the aid of software. A datafile is created from which the oscilloscope image is recreated on the PC screen as convenient for analysis or the datafile is analysed itself.

### 3.3 Dynamics of the mechanical construction

At the design phase calculations and simulations were used to predict the dynamic behaviour of the mechanical construction. The mechanical assembly was modelled in a lumped mass model to assess the influence of disturbance on the normal force and traction force. The stiffness of various components was calculated using classical means and occasionally verified using finite element calculations. The FEM had been used to check buckling load limits for thin plates. The stresses were only calculated for parts which show high deflections and for contact areas in assembly of components with bolting. The latter to obtain small highly loaded contacts for minimum hysteresis. Fig. 3.33 shows the model in the direction z, the direction of normal force on the contact.
Mass $\quad[\mathrm{kg}]$
Stiffness $[\mathrm{N} / \mathrm{m}]$


Fig 3.33 Lumped mass model for $z$ direction movements only.

This kind of modelling and simulation with software written in PC Matlab (The Math Works) can help identify problem areas in terms of inertia or low stiffness.
Substantial mass reductions were obtained without significantly affecting stiffness. The equivalent mass was reduced by about $50 \%$ by machining and wire-EDM on relevant 3-32
components. By modification of the stub axle on which the crowned wheel is mounted to the tilt mechanism, the equivalent stiffness was increased. Together these measures raised $\mathrm{fe}_{1}$ to 62 Hz .
Equally important is the direction of wheel rotation x . The model is shown in fig. 3.34. Loadcell stiffness is the critical component here.


Fig 3.34 Lumped mass model for movement in x-direction only.

In the $y$ direction, the natural frequency is higher due to small mass involved and high axial contact stiffness (order $10^{8} \mathrm{~N} / \mathrm{m}$ ) and wheel stiffness to an axial force at the circumference. Fig. 3.35 shows a model.


Fig 3.35 Lumped mass model for movement in the $y$ direction only.

### 3.4 Manufacturing techniques

A 4 axis programmable wire-EDM machine was used to cut critical components from bulk material. Fig. 3.36 shows some of the profiles cut in workpiece heights up to 120 mm .


Fig. 3.36 examples of wire EDM cut profiles.

This technique was available from Eindhoven University Central Workshops and allows unique parts with thin-thick transitions to be cut from high strength steels or any other conductive material. In view of the ease of programming (Delog) and automated operation of such machines, even those parts which could theoretically be produced on traditional machines, e.g. by milling, can be cut economically with minimal loss of material. For maximum effect it is essential to consider the application of this technique at the design stage of an instrument. The machine will run unattended at night and over the weekend on built in safeguards, finishing long cuts. It will also auto-thread the wire through previously drilled holes. Unlike other operations, special attention should be given to material-cut-loose, to prevent short circuiting the wire, by leaving small connections till the operator has returned.

Particularly on intersecting cuts, attention is required to avoid coolant interruption.
This is done by flooding or by using expendable cores, usually made of brass.

The crowning of the second, tilt adjustable, frictionwheel was done by copiing from a fine grained grinding wheel which had previously been dressed to the correct radius. This was done instead of using a mechanism as experience with this technique was already available. Lapping was used on both wheels to improve wheel roundness further.

The frame sandwich is of a welded construction; the technique had previously been applied. The plug welding by hand torch $\mathrm{CO}_{2}$ makes for tight assembly of milled strips and pre-drilled plates. The hole diameter is essential. If it is too small it will fill up without attaching to the strip, if it is to big, it will give to large a weld and thereby distortion of the workpiece during solidification. When producing such complicated multiple box structures, vertical seams between strips and likewise all fixtures and attachment points have to be made first, by TIG welding. This because they cannot properly be reached later everywhere and more important : their shrinkage would distort the plates if these were attached first. Next, as the plates are welded on by plug welding, the sandwich is welded on either side alternatively to minimize distortion. Some welding tests should be done before starting on the final product to allow the weldor to adjust to this technique, which requires near perfect control of heat input symmetry. After welding, several sharp blows with a plastic hammer, on ribs only, provide shakedown until a clear tone is produced. Inability to provide a clear tone, means some welding must be defective and hence hysteresis is noticeable in the part.
After stress relieve, contact spots are milled in-plane for maximum accuracy and minimal assembly stress. Fig 3.37 shows the part internally and externally.


Fig 3.37 Closed boxes frame design, in plug-welded plate construction, inside walls and fixtures and outside.

The shaft couplings fig. 3.32 were manufactured from 0.3 mm thick stainless steel sheet in an automatic microplasma weld set up. Carefull manufacture and installation of cooling jigs is essential to obtain full fusion of the sheet material as there is no filler added. Near weld clamping of the thin sheet provides good cooling. An optimally fused weld is essential if hysteresis is to be minimized. Fusion is observed through a microscope by the weldor at the controls of the programmable power source.

Other manufacturing was done using most often classical and sometimes CNC milling, turning and drilling.

## References to chapter 3

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## 4 Measurement procedures: testing and results

### 4.1 Introduction

During assembly of the measuring machine, various measurements were made to obtain a nominal position of the wheels relative to each other. Because this instrument is not one of a series of identical machines, its dimensional design is not just based upon "stacking of tolerances". During assembly, measurements indicated where reworking can enhance functionality. This localized remachining of component connecting surfaces was done on the best machines available at the University Instruments Construction Shop. The assembly is marked-up to allow reassembly to the same standards after disassembly. A complete orthogonality check with refacing of contact spots was done. One may argue that this was not absolutely necessary, but is was felt that with the elastic mechanisms involved, it would be preferable to start from zero displacement and zero force. This was successful on the cornerblocks and coupling. In the horizontal position they find their natural zero. The tilt mechanism was found at zero angle at a slight offset. This most probably has to be attributed to the way in which the manipulator is fixed : on three triangular plates. After initial tests the angle-manipulator was repositioned several tenth of a millimetre to obtain the maximum range.

### 4.2 Prerun conditions and preparations

### 4.2.1 General remarks

The following tests were done to ensure proper functioning of the instrument. With the wheels removed, stiffness of the innerpipe was measured out of plane on the three cornerblocks by measuring across the "folded leafsprings" and exerting a force on the moving part. At that time the triangular frame was in a horizontal position. Next a force lever was attached at the position of the stub-axis for the spherical wheel. With a calibrated force transducer and a calibrated displacement transducer hysteresis measurement was done in the two in plane directions $x$, traction force and $z$, normal force for calibration purposes explained in section 3.1.5. The axial stiffness of the elastic straight-guide was measured to allow matching the negative stiffness as described in section 3.1.4. On these hysteresis loop measurements 50 nm resolution was used and the variable force was introduced through a weak spring, tied with thin, souple kevlar string at both ends between the instrument and a motorized low speed spindle driven slide. A rubber belt was used to couple the motor to the spindle. The tilt mechanism was checked separately under a measuring microscope for its ability to maintain $\mathrm{x}, \mathrm{y}, \mathrm{z}$ position during tilt action and later checked for orthogonality of the two tilt axes, after completion of the assembly, just prior to the fitting of the stub-axle by glueing a mirror to the position where this axle was intended and by using an autocollimator in two directions.
Next, in the upright position, the machine assembly was completed with the wheels and encoders which were spring supported at the exact axial height and then attached. The force transducer to measure the traction force $\mathrm{F}_{x}$, was attached after the correct adjustment of the spring that supports the weight of the tilt mechanism. Next the force transducer was calibrated by adding weights to check the amplification factors of the charge amplifier and oscilloscope.
The normal force adjustment was each time checked by lifting the spring end off its seat with a low stiffness force measuring gauge.
With a low stiffness torque measuring gauge, the friction torque of the motors was measured, first separately, then with the gear reducer fixed, next with the measuring encoder, the preloaded angular contact ball bearings of each wheel. Finally the rolling resistance of the contact was added after bringing the wheels in contact.

Measurements on the assembly were repeated over several days, both directions were used, driving both wheels with the torque sensor. Repeatability was very good, within $1 \%$, even long, two weeks standing without removal of normal force, which was done to check the contacts, made no difference. Months later the same values could be found. No preference was found for either rolling direction. This is contrary to some earlier work on frictionwheels at the Eindhoven University, [Brinkgreve,1992]. It is believed this absence of difference could be due to the fact the wheels were lapped, whereby the roughness was modified from the original fine ground directional structure. Other differences with the earlier work are due to the preloading of the bearings and the wheels onto each other. This was also the first time the axial free floating was used. Further, after adjustment of the drive electronics, the uniformity of speed was checked by reading the encoders at various constant intervals of $50,20,10$ $5,2,1$ milliseconds. As the wheels' outside surface runs dry, it had been cleaned with a solvent, a journal was kept to record all tests in case there would be any shake down or wear. Throughout several months of testing there was no shakedown (subsurface plasticity) found, nor was there any wear visible. This is due to the high hardening levels of the wheels ( 62 Rc ), the low to moderate normal loading level of the contact and the fact that total running time was only of the order of 10 hours over the whole period. As indicated in Chapter 1, it was also not intended to look at "power" transmission near or even close to the maximum traction load where full slip is imminent, as this area was investigated by others [Wernitz,1958], [Johnson,1987].

### 4.2.2 Repeatability

Theoretically encoder signals may reveal:
-out of roundness of the wheels.
-excentricity of components.
-diameter difference of the wheels i.e. a non unity transmission ratio -effects like temperature.
Each time the testmachine is operated for a number of revolutions to gather data, is called a run. The initial runs were used to analyse repeatability. Fig. 4.1 through 4.4 show how this data was interpreted. All the data of later runs, (sections 4.3-4.6) was
checked in this way with similar results. Both encoders are read while the machine rotates at constant speed, measuring at fixed $1^{\circ}$ or $5^{\circ}$ intervals during several revolutions. The data file consists of three columns, encoder 1 , encoder 2 and difference 2-1. One of the encoders is used to latch the data of both simultaneously, so one column is filled with values exactly one degree apart (resolution $0.0001^{\circ}$ ). The other encoders' values differ more and more as the test continues and so does the third column of course. A copy is made of the datafile to allow visualization of the data. The difference is plotted against the angle of the fixed spaced encoder and depicted in fig. 4.1.


Fig 4.1 The difference of two encoder signals as a function of rotation.

Next the start is put at zero, so all difference data are offset by a fixed number, as depicted in fig. 4.2. This because the initial position of the encoders, and thereby their readout is random.

In fig 4.2 an ondulating signal is visible with a period of $360^{\circ}$, apart-from-that, the signal is clearly decreasing with increasing distance rolled, which is an indication of either creep or transmission ratio. All tests were repeated with the other rolling direction immediately before making any change to the adjustments. This was done to check for any directional effect.


Fig 4.2 Same as fig 4.1, but with all difference values offset so as to have zero difference at the start.

By cutting $360^{\circ}$, anywhere from this graph, full wheel revolution ensures all wheel particularities are included in the data. Shifting such a "slice" back or forth n . times $360^{\circ}$ yields results as depicted in fig. 4.3.
The striking "in place" similarity of the two graphs is confirmed by superimposing them, as shown in fig. 4.4, and finally by plotting their difference, depicted in fig. 4.5.


Fig 4.3 Two measured graphs of one revolution each, on the same basis, one of them is offset horizontally by $\mathrm{n} .360^{\circ}$.


Fig 4.4 Same as fig 4.3, but with vertical offset to get both starts at zero difference.

In fig 4.5 this "in phase" difference stays within two times the measuring step of $0.0001^{\circ}$. It is as good as can be expected from substracting two encoder signals with a resolution of $0.0001^{\circ}$ each. This indicates that changes such as temperature or residual dynamic effects occurring to the test machine during a run, do not cause the mechanical test set up to show measurable hysteresis.

The repeatable ondulating nature of the difference of two simultaneously obtained encoder signals is caused by excentricity of the wheels on their bearings.


Fig 4.5 The difference of the two graphs of fig 4.4 as a function of phase angle (throughout one revolution).

The encoders are each fixed to their respective wheels so their angular position relative to their respective wheel never changes. The bearing rings may deform slightly upon installation in their seats. As they remain fixed, they too give rise to repeatable errors. The wheel particularities, such as changing radius and surface roughness of both wheels, remain in "phase" because the level of creep coefficient is low, typically $2.10^{-5}$, i.e. $360^{\circ}$ of creep rotation occurs after 50.000 revolutions. So, because a measurement may be completed within 2 revolutions of data gathering, macro out of roundness phenomena come into the reproduceable error. The actual number of revolutions is higher, due to start up and stopping.

Asperities may come into contact with close neighbours of asperities they were in contact with during previous revolutions. As the contact area is relatively large, i.e. hundreds of micrometers, individual forces on asperities are not detectable. A statistical approach [Greenwood, 1966] based on the assumption that individual asperities deform independent of their neighbours holds when the real contact is not close to the nominal area. Contact compression is of the order of microns. Polished surfaces show roughness traces which are very smooth (approx. $0,02 \mu \mathrm{~m}$ in height) with few or no scratches inside the contact spot. The influence of surface roughness was not detectable with the test setup, which is not different from other experiments Johnson,1987]. Total peak valley of the periodic undulation in fig. 4.6 is comparable to $0.44 \mu \mathrm{~m}$ excentricity of one wheel.


Fig 4.6 Peak to valley range on the ondulation comparable to the effect of $0.44 \mu \mathrm{~m}$ excentricity of one wheel.


Fig 4.7 Ten revolutions, conditions as for fig 4.2

The general rise of the signal in fig. 4.7 can theoretically be attributed to at least three factors : the transmission ratio between the wheels, actual creep or change in wheels by temperature, i.e. the wheels each reach a different temperature. The latter was shown to be negligible with the method mentioned above during ten or fifty revolutions. The second is what we set out to measure. The first is considered a crucial problem from the start of the design of the testmachine. We asked for smooth surfaces on the wheels as a first requirement. Next on our list was roundness. Next was excentricity as low as possible. Finally, actual diameter was considered least relevant because it was anticipated that the drive arrangement should be symmetric both ways. So either shaft can be driven. Between tests, drives were swopped. Thereby one has data for two torque directions with each rolling direction.
One set of data contains the creep effect and the effect of transmission ratio, the other set contains the creep and the reciprocal of the transmission ratio. This allows elimination of the transmission ratio effect, leaving "pure creep" visible. Although it would have been no problem to accommodate a difference in diameter of several micrometers or even millimeters, it was possible to demonstrate that the real effect of nominal wheel diameter difference is negligible compared to the creep effect for the
particular wheels of our testmachine.
To find the creep ratio, the offset difference column was divided by the distance rolled, i.e. the value in the next column, and plotted as a function of distance rolled, i.e. the fixed space encoder. Theoretically, perfect wheels on perfect bearings, perfectly aligned, under constant load, would yield a horizontal line in this plot (and a constant, smooth slope in the previous one, fig. 4.7). An example of the results obtained with our machine are shown in fig. 4.8.


Fig 4.8 Time constant geometrical effects are surpassed by steadily increasing creep at constant rolling velocity. Creep rate emerges as a decernable level within one or two revolutions. Averaging from 360 values at $1^{\circ}$ interval removes geometrical effects like radius changing with angle.

During the first revolution several zero crossings occur because shape effects initially are larger than creep. After one revolution creep can be seen to emerge as the level at which the signal stabilizes, the remaining ondulation is getting much smaller than the absolute level. The final level is not dependent on the rolling direction, nor on the driving axis chosen. Fig. 4.9 and fig. 4.10 show respective raw data, note phase is random for each curve.


Fig 4.9 Comparison of rolling directions.


Fig 4.10 Comparison of driving axes.

### 4.2.3 Theory applied to the testmachine

For the particular wheels of the testmachine, the graph fig. 4.11, was prepared, based upon relations found in literature. As a function of normal force the following data are shown, scaled as indicated :
compression $\delta$ in $\mu \mathrm{m}$
creep rate $\zeta$ (lost motion devided distance rolled)

$$
\text { multiplied by } 10^{5}[-]
$$

torsional drive stiffness $\mathrm{k} \mathrm{Nm} / \mathrm{rad}$ divided by $10^{10}$
long semi axis of contact ellipse in lateral (y) direction * 10 [mm]
short semi axis of contact ellipse in rolling ( x ) direction * 10 [mm]
the average contact stress $p_{0}$ divided by $100\left[\mathrm{~N} / \mathrm{mm}^{2}\right]$.


Fig 4.11 Graphs based on theory (chapter 2) with substituted data of the wheels of the measuring machine. Fig. 4.11 is for a torque of 20 Nmm . Only the graphs for torsional stiffness and creeprate are also dependent on the torque transmitted by our F.W.D.,

Obvious conclusion from fig. 4.11 is that low levels of normal force give rise to creep rates that are more than ten times worse than the creep rate at higher values. So apparently it is wise to have a sufficiently high normal force. On the other hand, we can see that increasing the normal force excessively is not providing enormous further creep rate reduction. Fig.'s 4.12 and 4.13 show creep rate and drive stiffness at five torque levels as a parameter for each curve. From fig. 4.12 the most interesting part, in terms of normal force, is shown in the insert.

Calculated creeprate of machine


Fig 4.12 Theoretical creep ratio at five torque levels, as a function of normal force.

For the creep rate a conclusion is that in the range of traction force from $1 \%$ of normal force down to $1 \%$ creeprates have values from $3.10^{-5}$ to $3.10^{-6}$.
Lower ranges down to $0.1 \%$ were made with the testmachine by adding a system of ropes \& pulley \& weights but have little practical value. The weights help as drivers to reduce the tractionforce transmitted by the contact.

Higher values of traction force 5, 10, $15 \%$ of normal force were investigated in earlier reported studies [Johnson,1987], [Halling, 1964].


Fig 4.13 Theoretical torsional stiffness of the drive, at five torque levels, as a function of normal force.

## In conclusion:

- Drive stiffness improves with normal force, the contact size increases.
- Drive stiffness is better at low levels of traction force than at high levels, this is due to progressing slip zones, as explained in section 2.2.
4.3 Measurement of creeprate as a function of normal force.

Experiments were done to determine the creep rate $\zeta$ for a constant torque of 20

Nmm at various levels of normal force on the wheel contact. The spring end piece extension was set at normal-force levels of $20,30,40$ and 50 N . The tracking angle had to be adjusted each time the normal force was increased or decreased to compensate for bending of shaft and stub-end. At each level of normal force the creep rate was measured in two rolling directions before making any changes. Fig. 4.14 shows the results.


Fig 4.14 Creepratio during two revolutions at $\mathrm{F}_{\mathrm{z}}=20,30,40,50 \mathrm{~N}$. Note that out of roundness and excentricity blur the creep effect during the first revolution.

By cutting out a section with a length of $n$ times $360^{\circ}, n=1,2 \ldots$, the average value (see section 4.2.2) is a good representation of the creep rate, as shape problems repeating after one revolution are averaged.

We obtain an estimation of the creep rate from 360 (or $\mathrm{n}^{*} 360$ ) measurements so there is little room for doubt. Fig. 4.15 shows results of one revolution from which data the values $3.15,2.15,1.45$ respectively 1.2 all times $10^{-5}$ were derived as creep rates for the above mentioned normal force values.


Fig 4.15 Second revolution (exactly $360^{\circ}$ ) extracted from fig 4.14, contains geometric (excentricity and out of roundness) data of the wheels.

### 4.4 Measurement of creeprate as a function of tracking angle.

With the aid of visualized axial movement of the cylindrical wheel and its elastic straight-guide, it was clear to see axial force down to a few cN , once the negative stiffness was correctly adjusted. The straight guide is described in section 3.1.4., the displacement measuring instrument in section 3.2. The wheels were pulled apart till all wheel contact was lost, to zero the axial displacement reading.
Next the wheels were slowly rolled with the drive in one direction while watching the axial movement readout in a $300 \mu \mathrm{~m}$ full scale mode. Before reaching the end of the scale, the rotation direction could be changed with a switch. As the reading was changing towards the other end of the scale, the angle-manipulator micrometre screw was adjusted by hand slightly in one direction, taking care not to upset the force readout. If this resulted in the axial movement pointer moving faster towards the scale limit, the direction of tilt adjustment was changed. Each time the end of the scale was reached, the rolling direction was reversed. Adjustment of the tracking angle manipulator was typically in 0.02 mm increments, corresponding to an angle change of approx. $17 \mu \mathrm{rad}$. As the dial pointer started to move slower it was brought to the center of the scale by the rolling in the right direction. Next the full scale level of the axial displacement was adjusted to $100 \mu \mathrm{~m}$ and the adjustment continued in the same way. This proceded in the $30 \mu \mathrm{~m}, 10 \mu \mathrm{~m}$ and $3 \mu \mathrm{~m}$ full scale mode. Here finally a further adjustment increment could be shown to reverse the relation between rolling direction and axial movement direction. It was possible to adjust axial movement to a stable value where it could remain indefinitely, regardless of rolling speed, without moving visibly in the $0.3 \mu \mathrm{~m}$ full scale. This angle was named zero or "true tracking", a deviation of $5 \mu \mathrm{rad}$ to either side results in measurable axial movement of the cylindrical wheel.
The zero-angle was set in this way for all experiments and then left untouched, while merely observing the displacement readout during the experiments. After changing for instance normal force, it had to be readjusted. This was attributed to finite stiffness in the machine. At a given normal force it would remain stable for days. To quantify the effect of tracking angle on the creeprate, creeprate experiments were conducted, slowly stepwise increasing the angle between experiments from zero to one side. Next, to see the effect of zero crossing, adjustment was made in the other direction.

Fig. 4.16 shows creeprate results as a function of tracking angle at a constant torque of 20 Nmm and normal force of 30 N .

fig 4.16 Creeprate as a function of tracking angle.

## Conclusion:

From the experiment it may be concluded that the creep rate doubles at an angle of approx. $6.10^{-4}$ rad, beyond that angle the driven wheel falls even faster behind the driving wheel than would be proportional to the angle.

### 4.5 Measurement of creeprate as a function of spin angle.

From literature the effect of spin angle on creep rate is known, [Kalker, see chapter 2]. The lateral force $\mathrm{F}_{\mathrm{y}}$ called camber thrust, can be calculated, assuming its relation with the spin parameter remains linear to levels were no previous experiments were reported.
Approximately: $\mathrm{F}_{\mathrm{y}} / \mu \mathrm{F}_{\mathrm{z}}=0.3 \mathrm{X}$, were $\mathrm{X}=\omega_{\mathrm{z}} R / \mu \mathrm{V}$.
The rolling velocity V can be written as $\omega_{\mathrm{x}} \mathrm{R}$.
$\omega_{\mathrm{x}} / \omega_{\mathrm{z}}$ as approximately $\alpha_{\text {spin }}$.
For the wheels used in the test $1 / R=1 / \mathrm{R}+1 / \mathrm{R}+1 / \mathrm{R}=3 / \mathrm{R}(\mathrm{R}=50 \mathrm{~mm})$.
$\mathrm{F}_{\mathrm{y}}$ can be found by substitution, it is independent of the friction coefficient $\mu$, approx.
$0.1 . \mathrm{F}_{\mathrm{z}} / \alpha_{\text {spin }}$
This means that for the spherical bearing described in chapter 3, with a maximum tilt angle of $8.10^{-4}$ rad, axial force $\mathrm{F}_{\mathrm{y}}$ reaches a maximum of a few mN at maximum spin angle.
Inspite of efforts to get the negative stiffness adjusted equal to the positive stiffness of the elastic straight guide, it was not possible to measure in the mN range. Clearly spin angle did not cause any measurable axial movement at maximum angle. Nor was there any measurable effect on the creeprate found. Theoretically, a measurement of axial force in the mN range could be considered, its outcome would be of little practical value to Precision Engineers.

## Conclusion:

It can be concluded that spin angle is not as critical as tracking angle within the range investigated. There appears to be no difficulty to obtain small enough spin angles from manufacturing accuracy, to have a low, repeatable creep rate.

### 4.6 Measurement of creeprate as a function of rolling velocity.

Experiments were conducted at different rolling velocities. This is to exclude effects that might occur at a specific rolling velocity, due to some dynamic behaviour of our testmachine. The data gathered is relevant for application to machines for precision turning feed drives or for CMM's. It can not be extrapolated to extremely high velocities $10-100 \mathrm{~m} / \mathrm{s}$. At the high resolution used, the max. rolling speed the testmachine can measure at is $0.45 \mathrm{~m} / \mathrm{s}$. The measuring time is so short that the temperature effects will be negligible. No effect of velocity on creep rate was found in the limited range tested : from 8 to $450 \mathrm{~mm} / \mathrm{s}$ at sixteen velocities. Lower speeds are not likely to show any change from that. They were not used in the test because they were clearly not smooth enough as a consequence of the motor control/encoder used. The latter is shown in fig 4.17.


Fig 4.17 Velocity versus time, stabilizes after switch on to approx. $3 \mathrm{~mm} / \mathrm{s}$, The inserted smaller graph shows a higher time scale resolution, from which it is clear velocity is not smooth at $3 \mathrm{~mm} / \mathrm{s}$ provided 1 ms intervals are used.

As the drive system encoder has only 500 marks per revolution, speed representation is poor at low velocities and hence control gets jerky.

This is visible in the signal of the contact traction force and lead to creep rate increase in the measurement. Fig. 4.18. shows above mentioned typical effects on creeprate at high and low velocities.


Fig 4.18 Creeprate experiments at various velocities.

Conclusion:

Creep is stable provided the contact is not subjected to random high frequency forces as may occur at low speed due to poor drive characteristics, or at high speed from machine dynamics.

### 4.7 Dynamic testing

From literature it would seem, dynamic testing with emphasis on position has not been subject of extensive investigation. This is probably due to experimental difficulties. Constant speed experiments' results are apparently trusted to cover more dynamic events. In this context the truly dynamic events that occur at extremely high rolling speeds, the effect of sound waves, are not meant. One of the implications of theory is, that if effects like creep are attributed to a traction force while rolling, a certain traction force profile, should yield a certain amount of creep found by integration. In positioning i.e. moving from a given position $\mathbf{A}$ to another say $\mathbf{B}$, a nice symmetrical profile e.g. Fig. 4.19a or b is preferable for many reasons. Theoretically creep with the force direction reversed, would yield a total effect of zero after moving. It was felt that as CCM's and drives for precision turning machines and the like are used to position from A to B, some dynamic tests should be done to quantify what can be expected at the contact in terms of creep. Literature [Ro, 1994] provides some insight in maintaining a position.
$a(t)$


b)

Fig 4.19 Preferrable symmetrical profiles of acceleration and deceleration.(theory)

The drive system described under 2 in section 3.1.1 was used to generate triangular and trapezoidal velocity profiles. To make sure the repeatable geometric data was included for the entire circumference, the velocity profiles were executed during one revolution, two revolutions or half one revolution. In the latter case, two movements (and measurements) in the same direction were executed in immediate subsequence.

In the experiments, the drive was modified to contain a flywheel on the input side, connected by a rigid metal bellows to the machine. This flywheel was driven with a rubber belt (an inverted poly V belt) from a pulley on the motor shaft. A second flywheel was glued to the machines output shaft. This enabled traction force transmission at a lower accelleration rate.
This was necessary to prevent reaching the maximum speed on the measuring encoders' electronics before completion of one revolution.
Fig. 4.20 shows the oscilloscope reconstructed graph for the triangular profile, measured from the encoder on the motor shaft. Likewise for a trapezoidal profile depicted in fig. 4.21


Fig 4.20 Triangular velocity profile.
Fig 4.21 Trapezoidal velocity profile.

Fig. 4.22 shows the measurement made with the encoders on the friction wheels after removal of the offset on the difference signal.


Fig 4.22 Encodersignal difference as a function of distance rolled.


Fig. 4.23 The same data as presented in fig 4.22 shown as creeprate function.


Fig 4.24 Encoder difference data for a trapezoidal velocity profile.


Fig 4.25 The same data as shown in fig 4.24 depicted as creeprate measurement.

From these figs. $(4.22,4.23,4.24$ and 4.25$)$, the following can be observed:

## Fig.4.22:

- Four parabolic motions each of half the wheel circumference (traveled distance is 157.1 mm ) are superposed.
- Signal quality (fuzziness) is related to servodrive quality rather than a problem of the FWD contact itself.
- Inspite of the simple HEDS 500 encoder ( 500 marks) and the modest servo system, by todays standards, repeatability is good.
- Maximum tracking error half way through the motion is 4.4 or $5.3 \mu \mathrm{~m}$ depending on the wheel segment used.
- Up and down slope differ in steepness as parasitic friction torques of encoders and ballbearings are on either side of the contact. The ones after the contact help slow down the massmoment-of-inertia of the output wheel, thereby reducing the traction force during decelleration. The same friction torques increase the required traction force during the accelleration phase. The frictions before the contact are affecting the servo loop.


## Fig.4.23:

- The residual creeprate $\xi_{\text {res }}$ at the end of the motion is $10^{-5}$ which corresponds to $<1.6 \mu \mathrm{~m}$.
- Most likely, extremely low friction, e.g. airbearings could result in complete symmetry i.e. no or very little creep.


## Fig.4.24:

- The up and down slope differ for the same reasons described for fig. 4.22
- Between the up and down slope is the "constant speed" part, which looks somewhat like an inclined sinus.
- $\quad$ The travelled distance is 314 mm (one revolution) motion loss at the end of the motion is $0.004^{\circ}$. This corresponds to approx. $3,5 \mu \mathrm{~m}$.
- The maximum tracking error is $6,4 \mu \mathrm{~m}$ towards the end of the constant speed part, just prior to the decelleration.


## Fig.4.25:

- During the up slope of fig. 4.24 (the accelleration phase) the creeprate is messy due to insufficient distance rolled.
- During constant speed, the creeprate is coming down in a concave curve to a constant level which can remain stable if the constant speed were to be continued.
- During decelleration, the creeprate reduces further. The residual level could be reduced significantly by reducing friction torques.
- The reproducebility is excellent so calibration is certainly a possibility.


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## 5 Design Rules

### 5.1 Introduction

Based upon the insight and experience gained with the measuring machine, its conception, construction, commissioning and operation, the results obtained with it, their interpretability and the relation with existing theory, design rules will be drawn up in this chapter.
Calculated data (based on chapter 2) and measured data, (chapter 4, fig 4.12) are show together in fig. 5.1.


Fig. 5.1 Calculated and measured creepratio's.

The general shape of the curves in fig 5.1 is the same, but the measured data is lower than the calculated. Because most of the input values for the calculated data are known quite accurately (E.g. 100 mm wheel diameter is known to $1 \mu \mathrm{~m}$ ) and the effect of kinetic friction coefficient, which is a mere estimate at $\mu=0.1$, is very small, the difference cannot be explained easily. Fig. 5.2 shows the contact ellipse at various levels of traction force $\mathrm{F}_{\mathrm{x}}$. The slip area is relatively small compared to the stick area of the contact. In the slip area, the transferred tangential force component is the product of the local normal pressure and the kinetic friction coefficient.


Fig. 5.2 The contact area, with stick and slip zone.

Even if the friction coefficient is not known to a factor 10, the effect on creep ratio is negligible at these traction loads, due to the small size of the slipzone relative to the stick zone.
Clearly the effect of Youngs modulus on creep is large enough to notice it when changing for very high modulus materials e.g. Tungstencarbide-6\%Cobalt: $\mathrm{E}=6^{*} 10^{11}$ $\mathrm{N} / \mathrm{m}^{2}, v=0.22$. But even while creep is small, typically $2^{*} 10^{-5}$ or less at moderate to low loading, changing to a high modulus material such as Tungstencarbide-Cobalt still leaves considerable creep, enough to warrant compensation requirements.

It is considered possible to model the behavior of the contact in more realistic detail. This because in the actual contacts many of the assumtions made in the mathematical models of chapter 2 do not hold. There are no perfect mathematical surfaces, friction coefficient is not a single number but varies from rest to sliding situations as occur across the transition of the slip zone, there is some lateral sliding besides the sliding in the rolling direction, true contacts can be considered etc. These modelling aspects are not included in this thesis work, as it focussed on an experimental method.

### 5.2 Alignment of axes.

From fig 4.16 it is clear that misalignment causes increased creep. Accurate description of creep requires a very small angle of misalignment, $1^{*} 10^{-4} \mathrm{rad}$. and that angle should not vary noticebly e.g. $1^{*} 10^{-5}$ or $2^{*} 10^{-6} \mathrm{rad}$ variations.
There are basically at least four ways in dealing with the misalignment problem in ballbearing supported FWD. Schematically they are depicted in fig. 5.3 through 5.6, in order of ascending cost.


Fig 5.3 Alignment based on tolerances.

Method a) relies on the ability to bore parallel holes i.e. relying on the ability to copy an existing high quality machine's guideways. Preloading is enabled by cutting an elastic hinge into the block. This can be done prior to final cutting of the bore for the bearing, to minimize distortion. (Alignment based on tolerances).


Fig. 5.4 Self alignment prior to fixation.

Method b) is based on the idea that the contact can indicate how it would like to run, once known this position is fixed. Alignment based on in plane contact forces causing axial movement.


Fig. 5.5 External manipulator, alignment based on measured creep ratio followed by fixation.

Method c) involves searching for minimal creep while adjusting angles and measuring creep. Subsequently, the angle is fixed and the measuring and or adjustment gear is removed. Alignment based on creep measurement while using an external manipulator.
d)


Fig 5.6 Manipulator as part of the product, (re)adjustment based upon measurement of creepratio.

Method d) Involves a complicated design which contains manipulators for adjustment and possible later readjustment. Using an integral manipulator and possibly axial movement as an indicator.


Fig 5.6 Using flat and square with airbearings equivalent of method a)

### 5.3 Design rule precursors.

For the area of tangential load to normal preload investigated:

Creep is considerable: $\xi=2^{*} 10^{-5}$ means $20 \mu \mathrm{~m}$ is lost on 1 m of distance travelled for well aligned wheels.

Creep is not significantly dependent on friction coefficient, which is essential in avoiding the problem of predicting, or maintaining a friction coefficient.

Creep is very well repeatable i.e. it is due to material elasticity $\mathrm{E}, \mathrm{v}$. Geometric errors of the wheel (roundness and excentricity) are similarly repeatable. A design with little hysteresis and good alignment allows calibration of creep and repeatable geometrical errors and from that prediction of over $90 \%$ of these errors. Provided that velocities are comparable to the range used in this thesis and the velocity is smooth. (no spikes)

To get good repeatability consider:

Good roundness: i.e. $0.1 \mu \mathrm{~m}$ peak to valley or less, preferably wheels from a hydrostatic machine, grinder or lathe. For wheel much smaller than the 100 mm used in the test, roundness should be even better, if possible.

Good surface: Roughness $\mathrm{R}_{\mathrm{a}}$ less than $5 \%$ of hertzian compression or better e.g. use lapped wheels, optical quality surface.

Little excentricity: Design the wheel for the grinder and clamping method selected, and machine in one go with the bearing surfaces.

Provide long elliptical contact: the short axis in the rolling direction, yields the least creep. Provide crowning on at least one of the wheels. The amount of crowning needed can be less than used in the measurement machine, provide just a little more than would be required to allow alignment.

Make reductions with a single contact rather than multiple stages to allow simpler fingerprinting (characterisation of repeatable errors) and computing.

Use precision angular contact ballbearings with an elastic bushing (section 3.1.2 or membrane preload, or do better, use hydrostatic or aerostatic bearings.

Use constant normal force (fig4.12) preload with a low stiffness spring with sound attachments. High normal contact stiffness should be obtained together with a "low mass" design.
5.4 Design Rule Flow Chart for positioning through FWD.



## 6 Conclusions

A dedicated machine is designed, built, commissioned and operated for the purpose of measuring creep in rolling contact, both at low tangential loading level at constant speed and under dynamic conditions. It features extensive use of monolytic elastic components made with wire EDM from high strength steels.

Experiments down to extremely low creep rate $\left(2.10^{-6}\right)$ were conducted to assertain whether existing creep theory can be extrapolated to such low tangential loading levels for which up till now no experimental verification was available.

The creep rates found in the machine while rolling friction wheels at a constant speed with carefully aligned shafts are consistently lower ( $25 \%$ ) than the predicted creeprates from theory. However, on long distance motions, build up of lost motion remains unacceptably high i.e. $20 \mu \mathrm{~m}$ per m distance rolled at a tangential load of 1 $\%$ of normal force or still $2 \mu \mathrm{~m}$ at $1 \%$ of normal force.

A relatively short motion can be "zeroed" frequently. Under those circumstances, geometric errors due to excentricity and out of roundness are likely to prevail, so component and bearing quality is vital

At the above presented tangential loading levels, creep rates would improve to $15 \mu \mathrm{~m} / \mathrm{m}$ respectively $1.5 \mu \mathrm{~m} / \mathrm{m}$ for extremely high modulus material like Tungstencarbide-6 \% Cobalt with three times the Young's modulus of steel

A larger improvement could come from a compound wheel, consisting of a high modulus base with a coating of lower Youngs modulus. It is anticipated that such a compound wheel, if it can be designed and made properly, could display much less creep. This should/will be investigated further.

The influence of friction coefficient on motion transfer is negligible at loads well below those required for complete slip. This is fortunate as it means friction wheel drives are stable performers, provided they are kept clean from surface degrading
substances like corrosives or abrasives

Ball bearing friction on the output shaft, though constant under constant preload, sets limits to the minimum creep. Bearings with negligible friction, e.g. aerostatic bearings help to get good symmetry in dynamic loading, i.e. acceleration followed by deceleration and hence creep reduced towards zero in the second half of the motion. The extent to which some portion of this effect can be obtained with ballbearings is shown in figs. 4.23 and 4.25

For longer motions, correction will be required in the form of measured output displacement i.e. closed loop operation or as a beforehand calculated correction. The former is frequently used but cannot always be applied. Where it does work, it also compensates for many other errors by its nature of measuring the output.

The latter method of calculating creep, either beforehand or online, and providing just so much more input motion as required seems a good alternative for some situations as repeatability proved excellent (chapter 4). The motion should be known, to estimate the load. Additional correction for geometric errors, due to excentricity and out of roundness, should be included.

Alignment of the wheel axes is critical, if the amount of creep is to be repeatable and low. Five methods are mentioned to deal with the alignment problem, four of which are applicable to ballbearing supported wheels, all of them are applicable to aerostatic bearings. Selection of any of these principles is based on the requirements of the specific application.

Design of state-of-the-art instruments, requires knowledge of the limitations of hard(and soft)ware of the highest quality. Deficiencies in transmission of motion or force are hard to detect here. Certainly, errors can be compensated, but only those which are repeatable can be predicted. Statically and kinematically determined design (Appendix 1), thermal analysis, low hysteresis design, the use of monolytic thin-thick transitions on elastic connections are essential ingredients for good repeatability. To get good dynamic behavior, a low inertia/high stiffness design is needed, as well as vibration isolation

Because of their simple shape, which can be generated with high precision, friction wheels are highly useful mechanisms.

## Appendix I: Degrees of freedom

In the design phase of machinery and instruments it is worthwhile to simplify things first. E.g. by beginning to assume components are rigid bodies [Maxwell, 1890], each initially having potentially six degrees of freedom, like in fig. AI.1. for the cube.

fig Al. 1 Rigid body having six degrees of freedom (three translations $\mathrm{x}, \mathrm{y}$ and $z$ and three rotations $\varphi, \psi$ and $\theta$ ).

The next step is then to determine which movements of our rigid components are desirable to enhance functions and which movements are undesirable and have to be suppressed. Schematically, suppressing a translation means : attaching a thin rod, rigid in the suppressed direction and ideally flexible in all other directions fig. AI.2. The rod is thus capable of sustaining axial tension and compression. The compressive load is intended never to cause buckling of the rod.

fig AI. 2 A rod rigidly constrains a body in one translation (x).

In a friction wheel set-up, two wheels are pressed together, as fig. AI. 3 shows. So, two wheels each originally having 6 degrees of freedom, touch and in the normal direction have one d.o.f. less, whilst contact is maintained. Because of friction, two further translations are lost. If in fig. AI. 3 wheel A rotates about an axis fixed in space whilst being axialy fixed, then 5 d.o.f. are fixed for that wheel. Wheel B approaching radially, will, upon contact in a point, at first loose three degrees of freedom (translations at the contact) hence three rotations remain. The one in the direction of normal contact is called spin (drilling).

fig. AI. 3 Two friction wheels pressed together, three d.o.f. are shared.

In general, the axes of wheel A and B are crossing lines in space, their distance is measured through the contact point. If these axes happen to be coplanar (in general they intersect), the situation is referred to as proper tracking.
The rotation parallel to A's axis is called rolling and the normal component is called spin.
In still more general terms : two bodies touch in a point thought of as stationary in space. The bodies may each have a velocity at the contact point, a vector. Both vectors are considered to be in the contact plane (contact is maintained).

If within the plane the vectors coincide in direction, than we have proper tracking. If they are unequal in size we have (macro)sliding, which is not very useful for precision FWD's and is hence to be avoided. Furthermore the two bodies have a possibility for rotation about axes, normal to the contact plane. If there is a difference in such a rotational velocity, this is referred to as spin. Poor tracking is shown to be detrimental towards transferring positional information from wheel A to B at the contact point. Ideally an R-R set up would look like fig. AI. 4

fig. AI. 4 Ideal schematic of R to R transfer if I and II are parallel axes.
Properly constrained d.o.f. on two contacting friction wheels.

Here rods 1-4 are defining rotational axis I. Rod 5 is preventing axial movement of wheel A. On wheel B, rods 6 and 7 prevent translation in the rolling direction and rotation about the vertical (normal axis).

Rods 8 through 11 prevent, with the aid of an intermediary body IB, a rotation perpendicular to the plane of IB, in which lay rods 8 through 11 also. Note that axial movement of wheel B is prevented by the contact, rather than through an additional rod.
Also note that both axial and radial runout of either wheel is absorbed without significantly affecting rolling and motion transfer.
Likewise for the R to T situation (fig. AI.5)

fig. AI. 5 Properly constrained R - T drive.

fig. AI. 6 Properly constrained R-T drive.

## Reference to Appendix I

[Maxwell, 1890]
Maxwell J.C. : "The pieces of our instrument are solid but not rigid. If a solid piece is constrained in more than six ways it will be subject to internal stress, and will become strained or distorted, and this in a manner which, without the most exaxt micrometrical measurements, it would be impossible to specify".

## Appendix 2 Theory of Boussinesq and Cerruti

In general terms fig. A2.1 shows the contact area A , a point in the contact $C(\xi, \eta, z=0)$ and a point in the body, $B(x, y, z)$.

The normal pressure $p_{z}(x, y)$ and the tractions $t_{x}(x, y)$ and $t_{y}(x, y)$ may vary in both $x$ and $y$, these distributions are considered specified for $x$ and $y$ coordinates within A , and zero outside A .

The stress system consists of six components: $\sigma_{x}, \sigma_{y}, \sigma_{z}, \tau_{x y}, \tau_{y z}, \tau_{z x}$
These components fall to zero for points xyz far away from A.
The distance $C B \equiv \rho=\sqrt{(\xi-x)^{2}+(\eta-y)^{2}+z^{2}}$

## Distributions of traction

$$
t_{x}(\xi, \eta), t_{y}(\xi, \eta) \text { and } p_{z}(\xi, \eta) \text { act on the area } \mathrm{A} \text {. }
$$



Fig A2.1 The contact area.

The following potential functions are defined (A2. 2,3,4):

$$
\begin{aligned}
& F_{1}=\int_{A} \int t_{x}(\xi, \eta) \Omega d \xi d \eta \\
& G_{1}=\int_{A} \int t_{y}(\xi, \eta) \Omega d \xi d \eta \\
& H_{1}=\int_{A} \int p_{z}(\xi, \eta) \Omega d \xi d \eta \text { with } \Omega=z \ln (\rho+z)-\rho \\
& F=\frac{\partial F_{1}}{\partial z}=\int_{A} \int t_{x}(\xi, \eta) \ln (\rho+z) d \xi d \eta \\
& G=\frac{\partial G_{1}}{\partial z}=\int_{A} \int t_{y}(\xi, \eta) \ln (\rho+z) d \xi d \eta \\
& H=\frac{\partial H_{1}}{\partial z}=\int_{A} \int p_{z}(\xi, \eta) \ln (\rho+z) d \xi d \eta
\end{aligned}
$$

$\Psi_{1}=\frac{\partial F_{1}}{\partial x}+\frac{\partial G_{1}}{\partial y}+\frac{\partial H_{1}}{\partial z}$ and
$\Psi=\frac{\partial \Psi_{1}}{\partial z}=\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}+\frac{\partial H}{\partial z}$

The displacement vector $\underline{u}=\left(u_{x}, u_{y}, u_{z}\right)^{T}$ for a point in the body relative to far away points, which are fixed, can be shown to be

$$
\begin{align*}
& u_{x}=\frac{1}{4 \pi G}\left\{2 \frac{\partial F}{\partial z}-\frac{\partial H}{\partial x}+2 v \frac{\partial \Psi_{1}}{\partial x}-z \frac{\partial \psi}{\partial x}\right\} \\
& u_{y}=\frac{1}{4 \pi G}\left\{2 \frac{\partial G}{\partial z}-\frac{\partial H}{\partial y}+2 v \frac{\partial \Psi_{1}}{\partial y}-z \frac{\partial \psi}{\partial y}\right\}  \tag{A2. 5}\\
& u_{z}=\frac{1}{4 \pi G}\left\{\frac{\partial H}{\partial z}+(1-2 v) \Psi-z \frac{\partial \psi}{\partial z}\right\}
\end{align*}
$$

The displacements $\underline{\underline{u}}$ can be used to calculate the stress components.

$$
\begin{aligned}
& \sigma_{x}=\frac{2 v G}{1-2 v}\left(\frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}+\frac{\partial u_{z}}{\partial x}\right)+2 G \frac{\partial u_{x}}{\partial x} \\
& \sigma_{y}=\frac{2 v G}{1-2 v}\left(\frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}+\frac{\partial u_{z}}{\partial z}\right)+2 G \frac{\partial u_{y}}{\partial y} \\
& \sigma_{z}=\frac{2 v G}{1-2 v}\left(\frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}+\frac{\partial u_{z}}{\partial z}\right)+2 G \frac{\partial u_{z}}{\partial z}
\end{aligned}
$$

In a frictionless situation $F=F_{1}=G=G_{1}=0$, so

$$
\Psi_{1}=\frac{\partial H_{1}}{\partial z}=H=\int_{A} \int p_{z}(\xi, \eta) \ln (\rho+z) d \xi d \eta
$$

$\psi=\frac{\partial H}{\partial z}=\frac{\partial \psi_{1}}{\partial z}=\int_{A} \int p_{z}(\xi, \eta) \frac{1}{\rho} d \xi d \eta$

$$
\begin{aligned}
& \tau_{x y}=G\left(\frac{\partial u_{x}}{\partial y}+\frac{\partial u_{y}}{\partial x}\right) \\
& \tau_{y z}=G\left(\frac{\partial u_{y}}{\partial z}+\frac{\partial u_{z}}{\partial y}\right) \\
& \tau_{z x}=G\left(\frac{\partial u_{z}}{\partial x}+\frac{\partial u_{x}}{\partial z}\right)
\end{aligned}
$$

$u_{x}=-\frac{1}{4 \pi G}\left\{(1-2 v) \frac{\partial \Psi_{1}}{\partial x}+z \frac{\partial \Psi}{\partial x}\right\}$
$u_{y}=-\frac{1}{4 \pi G}\left\{(1-2 v) \frac{\partial \Psi_{1}}{\partial y}+z \frac{\partial \psi}{\partial y}\right\}$
$u_{z}=\frac{1}{4 \pi G}\left\{2(1-v) \Psi-z \frac{\partial \Psi}{\partial z}\right\}$

$$
\begin{aligned}
& \sigma_{x}=\frac{1}{2 \pi}\left\{2 v \frac{\partial \psi}{\partial z}-z \frac{\partial^{2} \psi}{\partial x^{2}}-(1-2 v) \frac{\partial^{2} \psi_{1}}{\partial x^{2}}\right\} \\
& \sigma_{y}=\frac{1}{2 \pi}\left\{2 v \frac{\partial \psi}{\partial z}-z \frac{\partial^{2} \psi}{\partial y^{2}}-(1-2 v) \frac{\partial^{2} \psi^{1}}{\partial y^{2}}\right\} \\
& \sigma_{z}=\frac{1}{2 \pi}\left\{\frac{\partial \psi}{\partial z}-z \frac{\partial^{2} \psi}{\partial z^{2}}\right\}
\end{aligned}
$$

$$
\tau_{x y}=-\frac{1}{2 \pi}\left\{(1-2 v) \frac{\partial^{2} \Psi_{1}}{\partial x \partial y}+2 \frac{\partial^{2} \psi}{\partial x \partial y}\right\}
$$

$$
\tau_{y z}=-\frac{1}{2 \pi} z \frac{\partial^{2} \psi}{\partial y \partial z}
$$

$$
\tau_{z x}=-\frac{1}{2 \pi} z \frac{\partial^{2} \psi}{\partial x \partial z}
$$

At the surface of the solid, the normal stress

$$
\bar{\sigma}_{z}=\frac{1}{2 \pi}\left(\frac{\partial \Psi}{\partial z}\right)_{z=0}=\left\{\begin{array}{cc}
-\rho(\xi, \eta) & \text { inside } A \\
0 & \text { outside } A
\end{array}\right\}
$$

and the surface displacements are

$$
\begin{align*}
& \bar{u}_{x}=-\frac{1-2 v}{4 \pi G}\left(\frac{\partial \Psi_{1}}{\partial x}\right)_{z=0} \\
& \bar{u}_{y}=-\frac{1-2 v}{4 \pi G}\left(\frac{\partial \Psi_{1}}{\partial y}\right)_{z=0} \\
& \bar{u}_{z}=\frac{1-v}{2 \pi G}\left(\frac{\partial \Psi_{1}}{\partial z}\right)_{z=0}=\frac{1-v}{2 \pi G}(\Psi)_{z=0}
\end{align*}
$$

In practice, obtaining expressions for the stresses in any but the simplest problems, presents difficulties. The approach is to start from stresses and displacements produced by concentrated normal and tangential forces. The stress distribution and deformation resulting from a distributed loading, can then be found by superposition.

## Concentrated normal force

The stresses and displacements produced by a concentrated point force $F_{z}$ acting normally to the surface at the origin (fig. A2.2) can be found by making the area A over which the normal pressure acts approach zero.


Fig A2.2 Concentrated normal force.

The Boussinesq potential functions $\Psi_{1}$ and $\psi$ (A2.4), reduce to

$$
\begin{align*}
\Psi_{1} & =\frac{\partial H_{1}}{\partial z}=H=F \ln (\rho+z)  \tag{A2. 13}\\
\Psi & =\frac{\partial H}{\partial z}=F / \rho
\end{align*}
$$

The elastic displacements at any point in the solid

$$
\begin{align*}
& u_{x}=\frac{F}{4 \pi G}\left\{\frac{x z}{\rho^{3}}-(1-2 v) \frac{x}{\rho(\rho+z)}\right\} \\
& u_{y}=\frac{F}{4 \pi G}\left\{\frac{y z}{\rho^{3}}-(1-2 v) \frac{y}{\rho(\rho+z)}\right\} \\
& u_{z}=\frac{F}{4 \pi G}\left\{\frac{z^{2}}{\rho^{3}}+\frac{2(1-v)}{\rho}\right\}  \tag{A2. 15}\\
& \text { for } z=0 \quad u_{z}=\frac{1-v}{4 \pi G} \cdot \frac{F}{r}
\end{align*}
$$

distance and with an infinite deflexion at O . The stress components at any point are :

$$
\begin{aligned}
& \sigma_{x}=\frac{F}{2 \pi}\left[\frac{(1-2 v)}{r^{2}}\left\{\left(1-\frac{z}{\rho}\right) \frac{x^{2}-y^{2}}{r^{2}}+\frac{z y^{2}}{\rho^{2}}\right\}-\frac{3 z x^{2}}{\rho^{5}}\right] \\
& \sigma_{y}=\frac{F}{2 \pi}\left[\frac{(1-2 v)}{r^{2}}\left\{\left(1-\frac{z}{\rho}\right) \frac{y^{2}-x^{2}}{r^{2}}+\frac{z x^{2}}{\rho^{3}}\right\}-\frac{3 z y^{2}}{\rho^{5}}\right] \\
& \sigma_{z}=-\frac{3 F}{2 \pi} \frac{z^{3}}{\rho^{5}} \\
& \tau_{x y}=\frac{F}{2 \pi}\left[\frac{(1-2 v)}{r^{2}}\left\{\left(1-\frac{z}{\rho}\right) \frac{x y}{r^{2}}-\frac{x y z}{\rho^{3}}\right\}-\frac{3 x y z}{\rho^{5}}\right] \\
& \tau_{x z}=-\frac{3 F}{2 \pi} \frac{x z^{2}}{\rho^{5}} \\
& \tau_{y z}=-\frac{3 F}{2 \pi} \frac{y z^{2}}{\rho^{5}} \\
& \text { where } \mathrm{r}^{2}=\mathrm{x}^{2}+\mathrm{y}^{2} .
\end{aligned}
$$

## Concentrated tangential force.

The effect of a force $F_{x}$ is calculated. $\left(F_{y}, F_{z}=0\right)_{x y}$. Then the Boussinesq potential functions $\mathrm{G}_{1}=\mathrm{H}_{1}=\mathrm{G}=\mathrm{H}=0$

$$
\begin{array}{ll}
\text { so } \Psi_{1}=\frac{\partial F_{1}}{\partial x}, \Psi=\frac{\partial^{2} F_{1}}{\partial x \partial z} & \text { and } \\
u_{x}=\frac{1}{4 \pi G}\left\{2 \frac{\partial^{2} F_{1}}{\partial z^{2}}+2 v \frac{\partial^{2} F_{1}}{\partial x^{2}}-z \frac{\partial^{3} F_{1}}{\partial x^{2} \partial z}\right\} & \text { A2.18 } \\
u_{y}=\frac{1}{4 \pi G}\left\{2 v \frac{\partial^{2} F_{1}}{\partial x \partial y}-z \frac{\partial^{3} F_{1}}{\partial x \partial y \partial x}\right\} & \text { A2.19 } \\
u_{z}=\frac{1}{4 \pi G}\left\{(1-2 v) \frac{\partial^{2} F_{1}}{\partial x \partial z}-z \frac{\partial^{3} F_{1}}{\partial x \partial z^{2}}\right\} & \\
F_{1}=\int_{A} \int t_{x}(\xi, \eta)\{z \ln (\rho+z)-\rho\} d \xi d \eta & \text { where } \\
\text { and from }(\mathrm{A} 2.1) & \text { A2.20 } \\
\rho^{2}=(\xi-x)^{2}+(\eta-y)^{2}-z^{2}
\end{array}
$$

When the appropriate derivatives are substituted in equations (A2.19), we get

$$
\begin{aligned}
& u_{x}=\frac{1}{4 \pi G} \int_{A} \int t_{x}(\xi, \eta)\left\{\frac{1}{\rho}+\frac{1-2 v}{\rho+z}+\frac{(\xi-x)^{2}}{\rho^{3}}-\frac{(1-2 v)(\xi-x)^{2}}{\rho(\rho+z)^{2}}\right\} d \xi d \eta \\
& u_{y}=\frac{1}{4 \pi G} \int_{A} \int_{t}(\xi, \eta)\left\{\frac{(\xi-x)(\eta-y)}{\rho^{3}}-(1-2 v) \frac{(\xi-x)(\eta-y)}{\rho(\rho+z)^{2}}\right\} d \xi d \eta \\
& u_{z}=-\frac{1}{4 \pi G} \int_{A} \int t_{x}(\xi, \eta)\left\{\frac{(\xi-x) z}{\rho^{3}}+(1-2 v) \frac{(\xi-x)}{\rho(\rho-z)}\right\} d \xi d \eta \quad A 2.22
\end{aligned}
$$

The tangential traction is now taken to be concentrated on a vanishingly small area at the origin, so that

$$
\int_{A} \int t_{x}(\xi, \eta) d \xi d \eta
$$

reduces to a concentrated force $\mathrm{F}_{\mathrm{x}}$ acting at the origin $\quad(\xi=\eta=0)$ in a direction parallel to the x -axis. Equations (A2.22) for the displacements throughout the solid reduce to

$$
\begin{align*}
& u_{x}=\frac{F_{x}}{4 \pi G}\left[\frac{1}{\rho}+\frac{x^{2}}{\rho^{3}}+(1-2 v)\left\{\frac{1}{\rho+z}-\frac{x^{2}}{\rho(\rho+z)^{2}}\right\}\right] \\
& u_{y}=\frac{F_{x}}{4 \pi G}\left[\frac{x y}{\rho^{3}}-(1-2 v) \frac{x y}{\rho(\rho+z)^{2}}\right] \\
& u_{z}=\frac{F_{x}}{4 \pi G}\left[\frac{x z}{\rho^{3}}+(1-2 v) \frac{x}{\rho(\rho+z)^{2}}\right] \tag{A2. 23}
\end{align*}
$$

where now $\rho^{2}=x^{2}+y^{2}+z^{2}$.

$$
\text { for } z=0, \rho=r=\sqrt{x^{2}+y^{2}}
$$

$$
u_{x}=\frac{F_{x}}{4 \pi G}\left[\frac{1}{\rho}+\frac{x^{2}}{\rho^{3}}+(1-2 v)\left\{\frac{1}{\rho}-\frac{x^{2}}{\rho^{3}}\right\}\right]=\left[\frac{(1-v)}{\rho}+\frac{v x^{2}}{\rho^{3}}\right] \frac{F_{x}}{2 \pi G}
$$

$$
u_{y}=\frac{F_{x}}{4 \pi G}\left[\frac{x y}{\rho^{3}}-(1-2 v) \frac{x y}{\rho^{3}}\right]=\frac{F_{x} v}{2 \pi G} \frac{x y}{\left(x^{2}+y^{2}\right)^{3 / 2}}
$$

$$
\begin{equation*}
u_{z}=\frac{F_{x}}{4 \pi G}\left[(1-2 v) \cdot \frac{x}{\rho^{3}}\right]=\frac{F_{x}(1-2 v)}{4 \pi G} \frac{x}{\left(x^{2}+y^{2}\right)^{3 / 2}} \tag{A2. 24}
\end{equation*}
$$

## Appendix 3 Design principles

Design of precision machinery and instruments has special emphasis on specific problems.
How to design for minimal sensitivity to changing temperatures or how to design for minimal sensitivity to spatial thermal gradients. How to design for minimum hysteresis of an instrument, the keys to reproducebility and accuracy. How to predict the dynamics of a design and how to design in a way that the designs are highly predictable. The designs of instruments should be tolerant to assembly and yet only the finest instrument mechanics are allowed critical operations. A good design calls for minimal use of "shiny surfaces" produced at great cost, yet that is what many expect to see in an instrument. Since the early sixties Prof. Ir. W. van der Hoek, who worked at Philips Centrum voor Eabrikage Technieken and taught at Eindhoven University, collected and generated the essentials to good machine and instrument design. It is amazing to see how much of his work (he retired in 1985 from the university) is still appliable in todays Precision Engineering. Mass produced products and the tools to make them called for unconventional thinking. The ideas from that "school" together with todays manufacturing techniques are the basis for new designs. Some of the classics and the modifications made to them will be presented here.

One of the main rules of predicting a designs' performance is that one should know about stress. At low stiffness that is visualized by deflection, at high stiffness internal loads are invisible and may well be enormous, unless one avoids them altogether by considering rigid objects first. Internal stress comes from assembly -mainly bolting \& welding- or, if one manages a near stress free assembly, from internal expansion differences. By correctly connecting only certain d.o.f. of the individual rigid bodies, one can control movement directions. If a construction is statically determined, one knows the loads and can calculate limits for other effects to prevent failure.
Without knowledge of internal loads, one has to accept small undefined movements at component connections which occur at the most awkward moments on external influences like heat input: standing near an instrument, or touching it or operating an actuator or vibration: a truck in the street, an elevator in the building, a slamming door, colleagues that walk or even move their chair. Sensitive instruments easily pick up these effects.

The use of elastic elements with a high ratio of stiffness in one direction when compared to another direction is the key to good instrument design. The constraint of a single degree of freedom can be done by a rod, Fig. A3.1, in its simplest form; a ballbearing ball or airbearing pad or any hydrostatic or magnetic or electrodynamic bearing pad can do the same function.


Fig A3.1 constraining one d.o.f. with a rod, a preloaded ball or an airbearing pad.

A leafspring constrains three degrees of freedom, Fig. A3.2, by cutting a hinge into it, we can constrain two d.o.f., Fig. A3.3.


Fig. A3.2 leafspring constrains three d.o.f. $\mathrm{x}, \mathrm{y}, \mathrm{\theta}$


Fig. A3.3 hinged leafspring constraining two d.o.f.

By creating a folded leafspring we can constrain one d.o.f., Fig. A3.4.


Fig. A3.4 folded leafspring, constraining one d.o.f.

To constrain one angle only, we can use four rods and an intermediary body IB Fig. A3.5 or use two leafsprings coupled by a torsionally rigid tube.


Fig. A3.5 Constraining one angle only.

With combinations of these basics, or by using more similar elements, we can create all other connections between rigid bodies, constraining up to 6 d.o.f.s. Fig. A3.6 shows only a few classical examples. For a complete list the reader is referred to a mechanical designers case book.
This modular thinking resulted in the design of section 3.1. Many other combinations of elastic elements were considered along the route of design. Practical aspects guided the selection of the final set.
Equally important besides above lay-out principles, are aspects of manufacturing details. How to design light-weight rigid bodies, how to attach rods and leafsprings, how to find a suitable frame. In general it can be said that a rod creates a high ratio of axial over lateral stiffness.

$$
\frac{E A}{1} \text { over } \frac{12 E I}{l^{3}} \text { equals } \frac{A}{12 I l^{2}}
$$

or for a circular cross section $\frac{4}{3 d^{2} l^{2}}$


Fig. A3.6 A few classical examples of proper constraint.

However, the concentrated force from a rod localizes body and frame loading and that lowers stiffness of the surroundings.

A kinematic drawback of the rod is the axial shortening $\Delta z$ under lateral deflection

$$
\Delta z=0,5 \mathrm{x}^{2} / 1
$$

Rods should not be attached by clamping but preferably by preloading onto an axial flat.

Hinged leafsprings have a nasty habit of concentrating the load. In out-of-plane bending, the hinge easily fails. A triangle bolted at its angles is a highly practical version e.g. part 3, Fig. 3.7.

The folded leafspring should have a really tight fold to function properly, the radius of bending should be small, not only compared to the planar dimensions but also compared to the plate thickness. The fold can be avoided by the use of an intermediairy body, Fig. A3.7, but such bodies are not desirable from a dynamic point of view.


Fig. A3.7 One d.o.f. constrained with two leafspring and an intermediairy body IB.

The intermediary body can rotate about the intersection line of the two attached planes, at random. The "folded leafspring" cut from a massive block of material is hence the best choice. The transitions from the thin leafspring to the frame and body are difficult to make by clamping without introducing hysteresis. As the thin part strains a lot and its thicker clamp much less, there occurs microslip inevitably under load. There are a number of repair techniques for this, but the subject can be avoided altogether if the thin to thick transition is cut in the same massive block. The thick parts have low stress and hence low strain and can therefore be attached without much hysteresis, or they can be the frame or the body. For these advantages this element was used thirteen times in this instrument alone. Fig. A3.8 shows two versions which were used succesfully.


Fig. A3.8 a) dimensions of the folded leafspring constraint
b) a version with a changing cross section.

## Samenvatting

WrijvingsWielOverbrengingen (WWO), veelal kogelgelagerd, zijn geschikte bouwstenen voor Precision Engineering. Echter, zelfs bij tangentiale wielbelastingen ver onder de (macro)slipgrens vertonen WWO kruip. Kruip veroorzaakt tijdens rollen een aanvankelijk gering maar gestadig toenemend positie-informatieverlies aan de uitgaande as. Omdat de geometrie in de tijd verandert ten gevolge van excentriciteit, onrondheid en thermische expansie, is kruip niet gemakkelijk te meten.

Er is een meetmachine gerealiseerd, die speciaal voor korte-duur kruipexperimenten is ontworpen. Hiermee wordt het probleem van niet stationaire temperatuur vermeden. Monolytische elastische elementen (verkregen door draadvonkerosie van hoog waardig staal) vormen de basis van deze machine die een uitzonderlijke herhalingsnauwkeurigheid bezit. De invloed van systematische geometrische afwijkingen op de overdracht van positie-informatie tijdens rollen, wordt vastgelegd door middel van encoders. De machine heeft nauwkeurige verstelmogelijkheden van de onderlinge wielashoeken van een paar voorgespannen wrijvingswielen.

De meetresultaten maken het mogelijk de eisen ten aanzien van toleranties op wielonrondheid, excentriciteit, oppervlakte gesteldheid en asparallelliteit bij WWO te kwantificeren. Bovendien is het mogelijk de onder praktijkcondities optredende kruip te voorspellen met hoge betrouwbaarheid. Softwarecorrectie van kruip via de input zal voor bepaalde toepassingen de moeite waard blijken, vergeleken met systemen waarbij wordt gemeten aan de uitgang. Tevens kan correctie voor systematische geometriegebonden afwijkingen worden meegenomen. Dit proefschrift toont de weg naar toekomstige verbeteringen voor toepassingen van WWO waarin de thans optredende kruip onaanvaardbaar hoog lijkt.

Nick Rosielle born May 31st 1956

| 1974 | Diploma Gymnasium $\beta$ Eindhoven <br> 1980 <br> Masters degree Mechanical Engineering, Materials <br> group. |
| :--- | :--- |
| $1980-1982$ | For Shell Intl. in Kon. Shell lab. Amsterdam, Materials <br> Research dept. |
| $1982-1984$ | For Shell Intl. at SCNV Curaçao Refinery: Corrosion <br> Engineering dept. |
| $1984-1986$ | Shell Intl. the Hague, Manufacturing Oil \& Gas, <br> Corrosion Engineering. |
| 1987 | PDVSA Curazao Refinery, Corrosion Engineering, <br> Head Inspection \& Corrosion. <br> Eindhoven University of Technology, Faculty of |
| 1994 | Mechanical Engineering. <br> Phd. Mechanical Engineering. |

Bij Wrijvingswiel Overbrengingen is as-uitlijning dermate essentieel, dat onderzoeksresultaten gepubliceerd door Bispink, Donaldson, Maddux en anderen, waarbij gebruik gemaakt werd van WO zonder instelbare uitlijning en met slecht gedefinieerde lagervoorspanning, ter discussie staan.
\ll dit proefschrift \gg
In tegenstelling tot de publicatie van Ro, 1994, kunnen bewegingen van 100 nm van uit stilstand, nauwelijks beschouwd worden als het testen van WO, aangezien rollen dan nog niet is opgetreden en dien ten gevolge de beweging niet van transleren te onderscheiden is.

$$
\ll \text { dit proefschrift } \gg
$$

Aangezien de wrijvingscoefficient vrijwel geen invloed heeft op het kruipproces is er weinig noodzaak om onderzoek met verschillende smeerolien te doen zoals gedaan door Holmberg, 1984. Zijn ontdekking dat voor acht sterkverschillende olien nauwelijks kruipverschillen optreden is daarom niet verwonderlijk.

$$
\ll \text { dit proefschrift } \gg
$$

In tegenstelling tot hetgeen is voorgesteld door van der Hoek, 1989, is een lichaam, dat op een vlakke kant wordt ondersteund op een vastgemonteerd lichaam, dat ter plaatse van het kontakt voorzien is van een grote radius ("brilleglas"), in drie vrijheidsgraden (alle drie ongeveer even stijf) vastgelegd en niet in éen.

$$
\ll \text { dit proefschrift } \gg
$$

Een bladveer als verbindingselement, tussen de aangedreven stang van een WO en een rechtgeleide slede, dient centraal, in plaats van zijdelings zoals bij Ro, 1994 en Bispink 1992, te worden bevestigd.

$$
\ll \text { D. de Bra, N. Rosielle>> }
$$

Gedetailleerde standaardisatie van inspectiebevindingen in olieraffinaderijen kan er toe bijdragen dat in het onderhoudsbudget de post "voorziene uitgaven" in grootte toeneemt in verhouding tot de post "onvoorziene uitgaven". Hierbij wordt dan óók nog een groter beroep gedaan op het verantwoordelijkheidsgevoel van individuele inspecteurs.
\ll N. Rosielle, 1987 \gg

Computersimulatie van het ontgassen van de reactorwand van reactoren in waterstof service kan helpen de afkoelfase aanmerkelijk te bekorten zonder toename van het risico voor schade.

$$
\ll \text { N. Rosielle, } 1985 \gg
$$

$\mathrm{K}_{\text {Iscc }}$ experimenten bij hoge temperatuur ( $300{ }^{\circ} \mathrm{C}$ ) en hoge druk ( 10 MPa ) aan voorvermoeide proefstukken van het "Compact Tension" type, welke op vóórspanning worden gebracht en waarbij door middel van de wisselspanningsproefstukweerstandsmeetmethode (bekend als ACPD) de scheuruitbreiding wordt bewaakt, kunnen in een autoklaaf zonder trekinrichting worden gedaan. Dit kan door gebruik van een voorspanbout, mits die uit het proefmateriaal vervaardigd is, in combinatie met kalibratie van de scheuropening (met clip gauges). Dupliceren van de test is dan ook mogelijk met slechts geringe bijkomende kosten.
\ll N. Rosielle, 1982 >>

Het lijkt welhaast onmogelijk, dat de suggestie: "de kosten van het verwijderen van asbesthoudend materiaal uit gebouwen blijven aanvaardbaar, mits onderaannemers met minder stringente normen ten aanzien van personeelbeschermende maatregelen voorrang krijgen in het offerte stadium" afkomstig is van nederlands wetenschappelijk onderzoek.

$$
\ll \text { sept. } 93 \text { E.D >> }
$$

Ondanks het nog altijd toenemende begrotingstekort (> 370 Gfl ), hebben nederlanders bedenkingen omtrent het functioneren van Justitie (budget 3.9 Gfl) of politie (budget van BiZa: 6.4 Gfl). Dit terwijl de rentelast thans 28.4 Gfl bedraagt en nog slechts wordt overtroffen door 33.6 Gfl voor Onderwijs en Wetenschap, hoelang nog?

$$
\ll \text { Begroting 1993/1994 >> }
$$

Een serieus voorstel om het wiel opnieuw uit te vinden?
<< dit proefschrift >>

