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An experiential program on the foundations of quantum mechanics for final-year high-school students

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Abstract

Teaching and learning quantum mechanics is one of the most demanding educational and conceptual challenges, in particular in secondary schools where students do not possess an adequate mathematical background to effectively support the description of quantum behaviour. Educational research shows that traditional approaches, generally based on historical and narrative perspectives, are only partially effective. The reason is that they do not address in depth those basic quantum concepts that radically question the fundamentals of classical physics. A research-based educational program has been proposed to two final-year classes of an Italian scientific high school. In order to build the main concepts of quantum mechanics and their formal basic representation via real and simulated experiments, the program uses the light polarization as a context. A quantum game was then integrated in the educational program, to support students' learning. Their conceptual paths, monitored by means of tutorials and questionnaires, show significant student

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learning especially on the concept of state and on appropriating the formalism meaning, whereas students more frequently referred to the geometrical vector representation instead of the algebraic-analytic formula. The quantum game has emerged to support intuition and operative experience in distinguishing the foundational concepts of superposition and entanglement.

Keywords: quantum mechanics, secondary high school, students' learning, quantum games

1. Introduction

Teaching and learning quantum mechanics (QM) is one of the most demanding educational and conceptual challenges for teachers and students [1–4]. Physics Education Research shows that QM plays a crucial role in the modern description of microscopic physics phenomena [1, 5, 6]. Its applications to modern technologies are countless, from transistors to lasers, while research actively continues in diverse fields which include quantum biology, quantum computers and cryptography [3, 7]. For these reasons, the idea has emerged in recent years that it is important to introduce the study of basic QM concepts, starting already from the secondary high-school education [3, 4]. However, studying QM involves approaching counter-intuitive concepts, which involve a reality very far from everyday experience and dealing with an abstract and sophisticated formalism. In most European national curricula, basic ideas are usually introduced through a historical perspective, starting from the black-body radiation to get to the atomic Bohr model and the de Broglie wavelength [4, 7]. In essence, this introduces the Physics of Quanta (PoQ), while neglecting, instead, fundamental QM aspects like the concept of state, superposition principle, incompatible observables. Given the cultural, scientific and technological relevance of QM in today's world, these concepts should not be left only to those students who have attended or will attend a course degree in a scientific subject, or else to professional physicists. At the level of secondary education, students do not have a sufficiently developed mathematical toolbox to support the complex description of quantum phenomena. In order to allow students to incorporate in a coherent cognitive framework the counter-intuitive QM phenomena, it is therefore necessary to conduct a careful analysis of the basic concepts, along with the teaching methods through which these concepts are to be presented [2, 8, 9]. In this sense, the, 'almost—historical' perspective through which PoQ is introduced in many European national curricula may have some disadvantages [5, 6]. The widely used approach in school practice [5, 7, 10], that proposes the quantization of different quantities as a solution to specific interpretative problems as they have historically arisen [11], does not lead to view quantum mechanics as a theory. Our approach [7, 12, 13] aims at overcoming this limitation. Similar changes of paradigm are in literature, that allow e.g. the teaching of modern Einsteinian concepts of space and time, gravity and quanta at an early student's age, by developing appropriate models and analogies [14]. Our framework offers the foundations of the QM theory explicitly addressing the following critical issues of traditional PoQ teaching. First, traditional PoQ teaching can distort students' perception of the way scientific research proceeds, providing the impression that the progress of science is characterized by linear and almost necessary developments, while the development of e.g. QM has contained non-trivial leaps. Second, elaborating an effective educational program can become a complex task when passing from PoQ, characterized by ad-hoc hypothesis for quantization, to the coherence of the QM theory. Finally, approaches where the framework of QM theory is not fully addressed, miss a valuable opportunity for analysing, even at philosophical level, the way through which modern physics describes Nature [3]. There are indications that a reflection around the concepts of measurement and non-locality (entanglement) is possible [6], contributing to develop a correct perception of the QM fundamentals [1, 4, 9, 15, 16] and play a motivational role for students. When consulting the literature on those aspects that teachers consider necessary in teaching/learning QM, one can notice a fair variability that depends on teachers' training, on observing that some topics are more easily supported by experiments in a school laboratory, and also on considering that some QM aspects are too difficult and formal for the mathematical toolbox at hand of secondary-school students [17, 18]. Researchers generally agree that the concept of state, its representation, and the superposition principle, constitute fundamental disciplinary concepts in a teaching/learning sequence about QM [2, 8–10, 12, 19].

The research-based teaching sequence on QM developed by the Research Unit in Physics Education of the University of Udine aims at implementing these objectives within an educational program proposed to two final-year classes of the public school Liceo Scientifico[4] Leonardo da Vinci in Treviso, a North-East Italian town. The program was conducted within a collaboration between the school and the Udine University in the framework of the IDIFO Project, in turn stemming from the Italian National Scientific Plan (PLS)[5].

The present work analyses some of the obtained results and is essentially organized in three parts. The first part, Section 2, presents the school context and educational program, the research questions, and the analysis tools and methods. In the second part, Section 3, the answers to selected questions of the pre/post tests are analysed qualitatively and quantitatively. For the sake of completeness, we provide an appendix where we report on the use of the quantum games. The third part, Section 4, contains a general discussion of the main outcomes, and concludes the paper by answering the research questions.

[4] In the Italian school system, Scientific Lyceums (Liceo Scientifico) correspond to about 45% of high schools, with students from 14 to 19 years old. Physics is a separate subject along all the five high-school years, taught for 2–3 h per week in the first two-last three years. Moreover they follow 5 year courses of math (4–5 h per week). Most of the students attending this type of school might enter university studies (i.e. at their will), but not necessarily in science, technology, engineering or mathematics.

[5] The interested reader can access the website of the IDIFO project: https://urdf.uniud.it/pls, accessed on 8 September 2022.

Table 1. Composition of the two final-year classes participating in the QM activity.

Class	Males	Females	Total
1	7	18	25
2	9	15	24
Total	16	33	49

2. Context and methods

2.1. The research school context

The research context includes two final-year classes of the Liceo Scientifico Leonardo da Vinci of Treviso, the activity being carried out during February and March of the school year 2020/2021. The experimentation has been conducted with all the students from two different classes, composed as shown in table 1. All students were 18–19 years old. Students had medium-high skills without observable differences, particularly in scientific disciplines. This aspect results from analysing the assessment data at the end of the first school period in physics, mathematics and science, and from the teachers opinion formulated in the two Class Councils. The students regularly attended the School's scientific laboratory. Here, they carried out experiences guided by worksheets prepared by the teachers, discussing the results and writing down reports.

Before starting the program on QM presented here, the students had not yet performed neither a module on PoQ nor on QM, while they had already covered optics.

The QM program was developed online during a period of Integrated Distance Learning. Three three-hours meetings were held during curricular schedule during the regular class time devoted to physics subject, and one two-hours meeting in one afternoon, for a total of 11 h.

2.2. The research questions

The aim of this work is to investigate how to effectively teach QM in high school, as well as to understand how high-school students learn the foundational QM concepts and appropriate the physical meaning of its basic formalism.

In particular, the present study provides answers to the following research questions:

RQ1. How do students appropriate the foundational concepts of QM (RQ1a) and its basic formalism (RQ1b)?

The aim here is to investigate what kind of conception students develop on quantum state and quantum superposition. We also aim at understanding whether students distinguish between the state in which a quantum system can be found and the properties that can be attributed in the given state. The research is also focused on the concept of measurement. In particular, on whether students understand the probabilistic significance of the quantum measurement process, and whether they can distinguish between the epistemic indeterminism characterizing classical-physics phenomena with unknown initial conditions, and the non-epistemic or intrinsic indeterminism characterising QM phenomena.

RQ2. On which aspects are the students' main difficulties centred? In particular, we want to understand the main conceptual nodes, to identify overcoming strategies.

RQ3. What is the role of integrating quantum games in teaching? We focus mainly on the role for learning, while also looking at motivational aspects.

2.3. The educational program

Our educational program is in the line of research of proposals on two-states quantum mechanics [7] and more generally of approaches to quantum physics that use physical optics as a bridging phenomenological context [9, 19, 20]. Linear polarization of light is the phenomenological context faced in the educational program [12, 13, 21, 22], implementing an inquiry based learning (IBL) strategy [23–25]. By means of simple experiments, both real and ideal, even in virtual environments, this approach allows students to coherently reconstruct fundamental QM ideas such as the concept of quantum state and superposition, and the related basic mathematical formalism. An integral part of the course are exercises and tutorials [21, 22, 26] used to support the analysis of the considered experiments: they are analysed after evaluating interpretative hypotheses through peer discussion. The polarization of light is a particularly suitable context for the introduction of fundamental concepts from QM, under different points of view. First, it is easily amenable to laboratory experimentation [27], even in virtual form after use of the Java Quantum Mechanics (JQM) applet [13, 28], thereby engaging students within an IBL strategy. Second, it allows a fairly easy reflection on the concepts of quantum state and superposition, because it limits the context to two states only. Then, it introduces the active role of the measuring apparatus in QM, via the preparation of the polarization state of a beam passing through a polaroid and the measurement of the polarization state when the beam passes through other polarizers. In addition, it can be very directly treated in a quantum perspective, by imagining the polarization of beams with decreasing intensity, up to consider the state of polarization of a single photon. Finally, it introduces the epistemic role of probability in QM, by: interpreting the Malus law in a probabilistic perspective; introducing the concept of incompatible properties and the uncertainty principle by discussing the behaviour of beams that pass through calcite crystals.

Table 2 summarizes the structure of the educational program together with the timing dedicated to each module, as follows. (a) The first module contains a thorough analysis of light polarization phenomena, thought as a property of light itself, analysed and interpreted through the Malus law. While optics is among the curriculum topics in Italian high schools, we remind that the choice of treating optical polarization and Malus law is up to the teacher. In order to bridge between polarization phenomena and photon physics, experimental evidence accounting for the existence of photons themselves was briefly reconstructed. On this phenomenological basis and considering beams of light of very low intensity, rests the reinterpretation of Malus' law as describing the transmission probability of a single photon through a given polaroid. (b) Still standing on an experiential approach, but now using virtual experiments in the JQM environment [13, 27, 28], the interaction of a single photon with polarizers is analysed by exploring alternative interpretative hypotheses. This exploration constitutes a crucial conceptual link, because the identification of mutually exclusive properties leads to the definition of state and of superposition principle. The identification of incompatible properties instead, together with the identity/indistinguishability of quantum particles, leads to the uncertainty principle [12, 13]. (c) An iconographic representation helps students to distinguish between property and state, here between eigenvalue and eigenvector, and allows them to construct the vector representation of the quantum state [20–22]. (d) In order to set the stage for the formal representation of state and of transition probability, the active role of the measuring apparatus must be analysed in even greater detail. (e) The experiment of photons that cross inverse aligned calcite crystals is studied. (f) The impossibility of attributing a given trajectory to a single photon is deduced by concluding that the experiment context involves a non-local alteration of the system state. (g) The polarization phenomena are then re-analysed to introduce the basic QM formalism, which starts by casting the Malus law as a scalar product of unit vectors. (h) To operatively reinforce the learning of the faced concepts, students are engaged in a tournament based on the use of quantum games (quantum TiqTaqToe) [29, 30].

2.4. *The quantum game*

At the end of the educational program (see module VII in table 2), we have complemented the teaching/learning sequence using polarization phenomena and the JQM activities, with a playful activity: a students' tournament based on the TiqTaqToe game (QTTT) [29–31]. This is a quantum version of the classic TicTacToe, including superposition and entanglement of the occupancy states in the game grid. While the activity is described in appendix A, we refer the interested reader to [29–31] for a detailed description of the quantum game, and to our work in [32] for a pilot-study on its effectiveness for teaching/learning QM in a high-school context.

2.5. *Monitoring tools*

The whole program was supported by the use of tutorials [12, 13, 26], implementing an IBL strategy [23–25] through a sequence of stimulating questions. At the end of the program, the students filled out an online questionnaire. Table 3 provides an outline of the items and their typology. In this paper, we analyse the answers to the selected questions Q.1 and Q.8–Q.16 reported in appendix B, completed by the students at the end of the program.

Question Q.1 concerns what students believe they have learned from the course. Responses to this type of question, are not expected to provide information about what they actually learned. Rather, they are expected to provide information on which aspects the students grasped or that resonated with their learning experience. These responses support outcomes on specific learnings, which in this research were investigated with the following questions, providing useful elements to understand the role of different course parts and used tools. Questions Q.8–Q.16 focus on the concept of quantum state and superposition. These questions permit to acquire information on what concepts students have learned, what aspects they still have unclear, and a guidance on helpful teaching strategies. Questions Q.13–Q.15 provide as well specific insights into which learning students have gained on the conceptual role of the formalism and the way it is employed to make simple predictions on the analysed quantum processes. Finally, question Q.20 is useful to gain insight into the role of the QTTT game for learning.

2.6. *Methodology of the analysis*

Concerning the qualitative analysis of questions Q.1, Q.8–Q.10, Q.12–Q.16 and Q.20, we divided the answers to the proposed questions into categories based solely on what students write or say. In so doing, we aim at understanding how students build their knowledge from experience, and how far/close to the viewpoint of scientific community [33]. The analysis of the answers to the open questions Q.1, Q.8–Q.12, Q.16 and Q.20 was conducted from a qualitative point of view [34]. Here, responses are divided into categories by highlighting keywords, analysing the responses content and structure. In each table of section 3, the category indicates the class to which the reasoning or perspective of the group of students belongs. The analysis by categories/classes serves to identify students' views and is not suitable for an assessment of correctness; there is no

Table 2. Plan of activities. The table reproduces the scan of the program modules and the timing (in hours) dedicated to each module or activity.

Module	Hour	Content (see for more details [12, 13, 21, 22])
I	1	Polarization as a property of light: phenomenological approach to polarization, based on simple experiments of light interaction with polaroid and birifrangent crystals. Tutorial 1: transmission of a non-polarized light beam through three polaroids in succession
	1	Approaching quantum mechanics, crisis of classical physics: black-body radiation, atomic spectra, Bohr atom, distinction between quantum physics and quantum mechanics, role of formalism in quantum mechanics. Photoelectric effect and photon hypothesis
II	1	Probabilistic interpretation of the quantum measurement process: single-photon processes and Malus law probabilistic interpretation. Tutorial 2: experiments with JQM [13, 27, 28]
III	1	Iconographic representation of the properties of a photon (e.g. Δ, *, $\Diamond$ indicate the property associated with a vertically, horizontal, 45° polarized photon, respectively [12, 13]). Dynamic properties of a quantum system: polarization as a dynamic property of photons and quantum state, mutually exclusive properties
IV	1	Interpretative hypotheses (statistical mixture of pure states; property coexistence; quantum superposition) in the description of the polarization state at 45°. Incompatible properties
V	1	Impossibility of attributing a trajectory to a quantum system: analysis of the interaction between photons with two inverse crystals aligned
VI	2	Quantum states, (unit) state vector, formal representation of the superposition principle and transition probability by means of the Malus law, expressed by using the scalar product of the state unit vectors. Observables and linear operators: polaroids and projectors, polarization as a quantum observable, and the operator representing it [12, 18]
VII	3	Non-epistemic indeterminism. Entanglement. TiqTaqToe tournament [29–32]
VIII	2	Summary and preparation of the final questionnaire

Table 3. The final questionnaire contains 24 questions. The table shows the questions numbering and the course part which they refer to. The last column indicates the question type.

Question	Disciplinary content	Type of question
Q.2—Q.5	Experiments with polaroids and Malus law	Open
Q.6—Q.7	Application of the Malus law to single photons and probabilistic interpretation	Open
Q.8—Q.10, Q12	Mutually exclusive, incompatible properties and quantum state	Open
Q.11, Q.13—Q.15	Applications of state and transition-probability concepts	Multiple choice
Q.16—Q.18	Principle of quantum superposition and entanglement	Open
Q.19	Attributing a trajectory to a quantum particle	Open
Q.20	Role of the TiqTaqToe quantum game in learning the tackled QM concepts	Open
Q.1, Q.21—Q.24	What did I learn, what was most difficult, describe the experience	Open

correct answer for every class, precisely because students' language is often approximate and partial, rarely resembling an expert's statement. This type of analysis serves to highlight to the teacher-/researcher not only the student's perspective, but also their need to conquer the correct language, especially in terms of completeness.

The frequencies of the different categories are also evaluated.

This group of questions provides information on students ideas about the main conceptual nodes, such as mutually exclusive properties, incompatible properties, state of polarization, polarization properties, superposition principle and uncertainty.

As to questions Q.13–Q.15, a quantitative analysis was performed. Each answer was classified into three groups: Exhaustive answer

(1 point); Partially corrected answer (0.5 points); Incorrect answer (0 points) [35, 36]. The analysis was then conducted by calculating the score normalized to 100, that each student achieved in each of the examined answers. The total score recorded in each question was also evaluated.

This group of questions has a more operational character, essentially including calculations on the transition probability.

A quantitative analysis may indicate to what extent the mathematical QM formalism is effectively used as a calculus tool, as well as how its conceptual meaning is grasped. We also analysed whether there is a relationship between the way students have constructed their own representation of the quantum world and the mathematical ways in which they represent it, to solve the proposed problems.

In particular, following what emerged in previous research, the students sentences on those questions related to the formalism, were classified in the following modality: formula, when the student answers a question by just writing a formula and nothing else; declarative conceptual, when the student describes in words the formal construct or formula he/she intends to recall; conceptual-formal, when the presentation of the formula is accompanied by its physical meaning or by some conceptual element.

We highlight that not all students have answered to all questions. Thus, in what follows, for each question the ratio is reported between the number of answers of the given type, as related to the number of responding students.

3. Results

The categories of the qualitative analysis to the questions in table 3 indicate the class of reasoning by the different groups of students, identifying their different visions.

3.1. Qualitative analysis of the questions Q.1 and Q.8–Q.12

Q.1—What did I learn about Quantum Mechanics with this experience.

A common characteristic in the answers to question Q.1, is that students pointed to the complex of phenomena regarding polarization, which strictly speaking is not a quantum domain. We stress that students highlighted this as a learned aspect, even though they had already theoretically covered it with their teachers before attending the course. This indicates the importance of offering students a phenomenological exploration, performed by adopting a quantum point of view. Specifically, polarization was reconstructed operationally using real polaroids: students have been prepared to consider polarization as a property of light, setup by filtering with a specific polaroid, and analysed/measured with another polaroid emerging as an active filter, thereby anticipating their active role as measurement tools in QM.

Concerning the specific quantum aspect cited, four categories identified the students' responses. Most of them highlighted several aspects and are therefore registered in more than one category. Learning the phenomena of polarisation and behaviour of photons ('*Behaviour of light, and photons that compose light*', '*photon polarisation*') characterises the largest category (14/39). The need of overcoming classical physics (CP) characterises the second category (12/39), highlighting that '*often in physics there is a need to abandon the classical vision because many problems and many models are inexplicable through CP*'. The third category includes answers constructed as a list of contents, among which the most cited are '*superposition principle, the uncertainty principle and mutually exclusive properties*', the '*vector nature of the quantum state*'. The fourth category (8/39) highlights the relevance of probability, like in this sentence of a female student '*Thanks to the introduction of new knowledge such as the polarisation state and the vector nature of the quantum state, I was able to understand the probabilistic nature of quantum phenomena*'.

The conceptual framework emerging from the four identified categories can be further investigated through the answers to questions Q.8—Q.10, Q.12, Q.15, Q.16.

3.1.1. Property and state in QM.

Q.8—explain to your classmate what is meant by mutually exclusive property and what is meant by incompatible properties (and in what they differ).

Table 4. Answer to question *Q.8—explain to your classmate what is meant by mutually exclusive property and what is meant by incompatible properties (and in what they differ)*. The main categories identified through the students' responses are reported. Only the 17/38 responses dealing with both properties were considered. Some examples of responses refer to the iconography used during the course, to indicate a given polarization property.

	Category			
	Exclusive	Incompatible	N	Examples of students' answers
A	One excludes the other	There is a measurement procedure whereby a system loses one and acquires the other	7/17	*'Mutually exclusive properties are those properties that if present together cancel each other out (e.g. pol. Horizontal and pol. Vertical). The incompatible properties are those for which there is a measurement operation in which the system can lose one of them and acquire the other'*.
B	May not be possessed simultaneously	If a system have one, it cannot have the other, they cannot coexist	6/17	1. '*The mutually exclusive properties are those that a photon cannot have at the same time [.]. The incompatible ones are those that the photon cannot have at the same time*'. 2. '*Incompatible if you have one, you cannot have the other. Mutually exclusive = they cannot happen together*'.
C	They cannot coexist and only up to the measurement	They cannot manifest themselves together	2/17	*'Incompatible properties are those properties that cannot be manifested at the same time, whereas mutually exclusive properties manifest themselves only at the moment of measurement'*.
D	They cannot be possessed together	Each property is not a mixture of other properties	2/17	*'Triangle vertical polarization; * pol. horizontal; rhombus polarization 45 degrees. The diamond and asterisk or triangle properties are incompatible because a rhombus cannot also have * and triangle properties. The rhombus state cannot be considered a mixture of the asterisk and triangle states. These two properties are then mutually exclusive as they cannot happen together (transmission result equal to 0).'*

Question Q.8 focuses on mutually exclusive and incompatible properties, asking for the differences between the two and highlighting the role of the measuring apparatus, as well as of the uncertainty principle. In the answers, it is interesting to observe what idea the students had about these properties, and the elements that they identified as relevant to distinguish them.

Table 4 contains the categories A–D identified in the responses of the 30 students answering the question. Only 17 of them are about both property and state.

The first three categories relate to the role of the measurement process, through different modalities. The first gives the students a more coherent vision of the distinction between the two concepts of mutually exclusivity and incompatibility. In the second, the distinction between the two properties seems rather nuanced. Students use in fact different words, but do not conceptually distinguish mutual exclusivity from incompatibility. The students were able to distinguish them when expressing the answer in terms of polarization properties. For instance: 'Properties such as rhombus and starlet or rhombus and triangle are incompatible because a photon with rhombus properties cannot also have the property of type * or triangle'. This highlights the effectiveness of the iconic representation used to represent polarisation properties (see point 4 in table 1 and [12, 13]), and the importance that learning be developed in phenomenological contexts that the students master. However, this observation also highlights the need to activate a process helping the students to gain a less contextual view of the learned concepts.

Even taking into account that only about 44% of the students respond in a complete way, we note that the most represented category is related to the measurement, that is, to the interaction of photon and polaroids. Much less perceived appears instead the relevance of the probabilistic aspect. This picture does not change even considering students who answer only part of the question.

Q.9—illustrate to a classmate what is meant by the polarization state of a photon.

Question Q.9 asks to illustrate what the polarization state of a photon is. From the analysis of the 32 answers, it emerges that the main modalities describe the state by referring to a vector, without explicitly resorting to formulas (figure 1). This mode recalls what in other studies is defined as declarative conceptual, in that the student describes in words the formal construct or formula he/she intends to recall [18], as for instance in these examples:

> *'The polarization state is a vector associated with the particle that is represented in an abstract and two-dimensional space'.*
>
> *'The polarization state of a photon is a vector associated to the particle. The latter is formed by the combination of two orthogonal unit vector representing the mutually exclusive properties'.*

The background of such an expression is in the difficulty of distinguishing the concepts of state and property, which are specifically explored in the following.

One third of the students (13/39 -category D of figure 1), responds by describing how a polarized photon behaves or is obtained rather than by defining its state. For example:

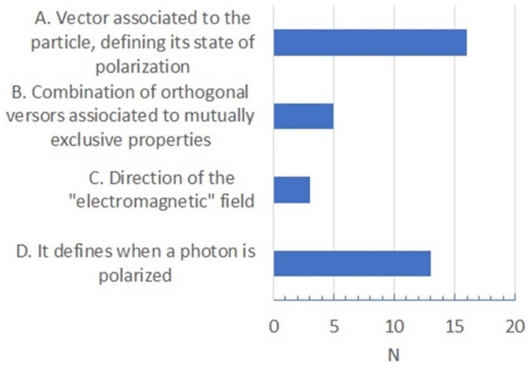

Figure 1. Main categories identified while analysing the answers to question *Q.9—illustrate to a classmate what is meant by the polarization state of a photon*. This question was answered by 32/39 students.

> *'In quantum mechanics, a beam of light is an aggregate of photons. Normally this beam of light is not polarized, as are the photons that compose it, but if they pass through a polaroid filter, which allows polarized light to pass only in the direction of its axis, a polarized photon may be obtained that is polarized in the same direction as the polaroid filter axis. In this way you can get as many photons in a polarization state'.*

The representation of the state as vector does not exhaust the characteristics of the state in QM. The reflection, conducted during the course and stimulated by question Q.8, leads to consider it as representative of the maximum information achievable on the system. This aspect is not highlighted in the students' responses.

Examples of answers that show the students difficulties are the following: *'it is a vector that's associated with the photon you are looking at. It can be identified as the direction along which light (both particle and wave) vibrates (non-polarized light vibrates in all directions'*. In this last answer, in particular, it seems that light is conceived as composed by photons that vibrate following a sinusoidal path, found in other investigations on the teaching of QM [13].

Categories A and B show how the introduction of the formalism can help several

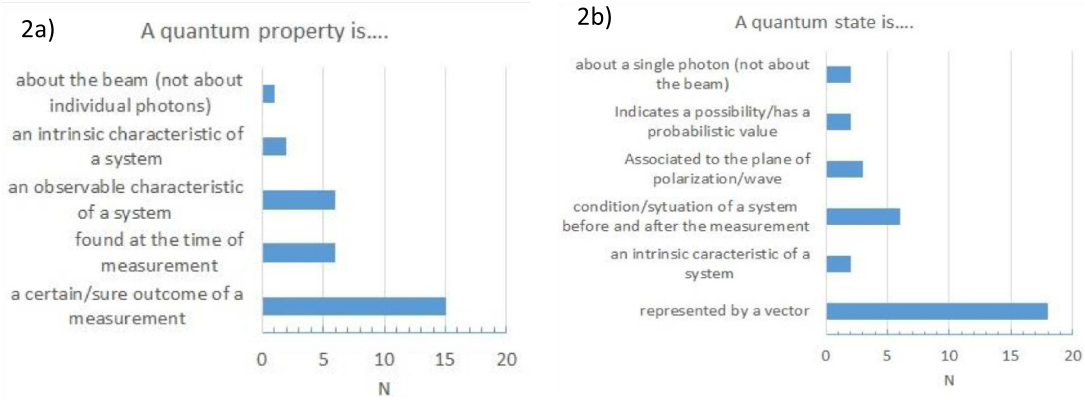

Figure 2. Main categories identified while analysing the answers to question *Q10—how would you explain to your classmate the difference between polarization properties and polarization state?* This question was answered by 30/39 students. Figure (a) shows the properties categories, figure (b) the state categories.

students (∼2/3) to identify the quantum concepts. On the other hand, the presence of categories C and D also shows that a certain number of students (∼1/3) remains in the classical frame or has difficulties in moving beyond the phenomenological framework.

Q.10—how would you explain to your classmate the difference between polarization properties and polarization state?

Question Q10 stresses as well the crucial distinction state—property. From figure 2(a) it is observed that the more represented category places the concept of property in relation to the property of a system with a certain measurement result. The state (figure 2(b)) is mainly identified with a vector, sometime specifying with mutually exclusive components. 13/30 of the responses are included in the prevailing categories, concerning both the concepts of property and of state. A typical answer along these lines is: *'The polarization property indicates a result that is certain. The polarization state represents the state of a photon with exclusive components possessed by the photon before and after polarization'*.

Within the category that associates the state to a two-dimensional vector, two students write sentences like: *'the polarization property has a certain value, the state is rather probabilistic'*, which remarks the probabilistic character of the quantum concept of state, although without elaborating on its meaning.

We can observe here that an expression like 'intrinsic characteristic' is used by some students to characterize the property of a system and that some students attribute the same expression to the concept of state. In two cases, the state is regarded as intrinsic to the system, while the property emerges upon measurement, such as in this example:

> *'the difference is that the state is what the photon really 'possesses' but the property is what I find only after the measurement. For example, if I consider an object I know that this has a certain volume (state) but I will know the value of this volume (property) only after measuring it'*. In the second part, the student reconsiders the state and property concepts in classical physics to grasp the quantum point of view.

Thirteen students identified the state with a vector in response to both Q.8 and Q.10. While consistent with each other, the responses to Q8 and Q10 are differently characterized: more clear the former and unnecessarily more complex the latter, in trying to distinguish the compared concepts meanings. In other words, it is observed that some students manifest difficulties in expressing

Table 5. *Q.12—what proves that the polarization state cannot be thought of as a statistical mixture of pre-existing properties?* The second answer in the first category recalls the iconography used during the course to specify a given polarization property.

	Category	Frequency	Examples of answers
A	Discrepancies between theoretical predictions and experimental results	10/17	*'The proof is that, since theoretically, in a statistical mixture you should get photons all equal to the polarizers, in practice, instead, you get photons with different properties'*
B	Change of state	5/17	1. '*The results appear to be compatible with photons that share the property ♢, therefore the properties are incompatible*'. 2. '*Because the placement of a 45 filter between the H and V filters allows the passage of photons along all three filters'*
C	Difference between superposition and statistical mixture	2/17	*'This shows that the union of states H and V is different from the superposition of the two'*

how two concepts differ, as highlighted in this sentence:

> '*in the polarization property, states and non-qualities of the photon are superimposed, while in the polarization state the properties of the photon itself are taken into account*'.

The student uses the concept of property to define the state, and the concept of state to define what are the properties, creating an endless conceptual loop.

The identification of the concept of state was less problematic for students, when asked independently (see figure 1). Thus, the importance emerges of including successive reflection and discussion course moments with and among students, in small and large groups, to construct shared meanings of concepts, their characterization and differentiation.

Q.12—what proves that the polarization state cannot be thought of as a statistical mixture of pre-existing properties? Here, the small number of responses is one more indicator of students' difficulty in answering questions in which they are asked to differentiate two concepts. The answers of the 17 respondents are categorized as in table 5. More frequent answers refer to the discrepancy between theoretical and experimental predictions.

This closely recalls the example discussed during the course with students: photons in a statistical mixture (e.g. equivalent to a set of photons consisting of two subsets with orthogonal polarizations) generally produces experimental outcomes that are statistically different from those produced by a set of photons in a pure state. The most represented response mode refers to this context, frequently using the iconic representation of polarization properties.

3.1.2. Superposition principle. *Q.16—explain to your classmate the superposition principle of quantum states.*

Table 6 contains the categories of the 29 answers to question Q.16. The majority (21/29) responds by citing the property of linearity, that is, the linear combination of states is still a state, a feature highlighted during the course and that can be traced back to the Conceptual-declarative modality already mentioned in the case of the answers to Q.9. A second group (5/29) writes a sentence accompanied by an analytic expression or making use of a formula, according to a Conceptual-formal modality. Finally, a few students (3/29) respond by referring to the state collapse when a system observable in a superposition state is measured.

In conclusion, from the qualitative analysis seems to emerge that the conceptual framework gained by students at the end of the program,

Table 6. *Q.16.—can you explain to a classmate the superposition principle of quantum states*? Main categories identified in the answers. The majority of the answers are in the first category. Some students, in addition to indicating the analytical expression of the superposition between two states, also recall the concept of incompatible properties using the iconography used during the course to support these concepts.

	Category	Frequency	Example of answer
A	The sum of states is one more state	21/29	*'The final quantum state is the sum of two or more distinct states'*
B	Analytic expression $\boldsymbol{U}(\boldsymbol{\theta}) = \psi_1 \boldsymbol{u_V} + \psi_2 \boldsymbol{u_H}$	5/29	1. *The superposition principle can be expressed as* $\boldsymbol{U} = \psi_1 \boldsymbol{u_V} + \psi_2 \boldsymbol{u_H}$ *and indicates the property of quantum states to add up.*
C	Connection with state measurement and collapse	3/29	1. *'The superposition principle of quantum states establishes that if I have two quantum states, at the moment of measurement one of the two decades, to allow me to calculate the other'.*

Table 7. Type of answers to question Q.12. Only 34% of the students answered. The correct answers focus on a Geometric mode, that is, by connecting the interpretation of phenomena with formalism. In this case, the angle between the allowed direction and polarization plane of the incident beam is bound with the scalar product between states.

Answer	N	Examples of answers
Exhaustive	9/13	$\cos^2\alpha = (\boldsymbol{U} \cdot \boldsymbol{W})^2 = P_t$
Non correct	4/13	*'The event probability is* $(\psi_1\psi_1{}' - \psi_2\psi_2{}')^2$*'*

includes the main cognitive nodes proposed: the concept of state, and the superposition and uncertainty principles. The perception of these concepts appears diversified. Probably thanks to a robust phenomenological basis and to the offered experiential approach, the majority of students have clear the relation between state and measurement process. Also important, but less clear and more hardly explicit, appears the role of the state in QM as a mediator of information on the system. In some students (10%–20% depending on the question) a mixture persists between the concepts of CP and PoQ, in the interpretation of microscopic phenomena.

3.2. Quantitative analysis of the questions Q.13–Q.15

The aim of this section is to analyse the students' competences in qualitatively and quantitatively answering the simple questions Q.13, Q.14, Q.15 on state and transition probability.

Q.13—a photon prepared in the state $\boldsymbol{u}$ *makes a transition to the state* $\boldsymbol{w}$*, after interacting with a polaroid. How can I write the probability of such an event?*

The majority of students (9/13, table 7) responds exhaustively, referring to the scalar product by means of expressions like $P_t = \cos^2\theta = (\boldsymbol{U} \cdot \boldsymbol{V})^2$, or representing a figure with vectors according to the Geometric modality. A geometric aspect is used as a bridge between QM phenomena interpretation and formalism. A second group (4/13) recalls only the expression of the transition probability through the state vectors and the superposition principle, as for instance $P_t = \cos^2\theta = (\psi_1\psi_1' + \psi_2\psi_2')^2$. Given that the answer consists essentially of a formula, this can be classified as Formula modality [35]. Notice that this modality appears more problematic when the answers contain inaccuracies.

Q.14—give the example of a specific case (determine the numerical value of the transmission probability, after defining the necessary parameters).

In this case there are no wrong answers (table 8). On the other hand, there are 9/17 not-exhaustive answers, because the characteristic of the incident light (not polarized or polarized according to a given direction) is not specified. In

Table 8. Frequencies of the answers to question Q.14 and selected answers examples. The answers are considered exhaustive when including the polarization of the incident photons and the polaroid allowed direction, in addition to the average number of photons through the polaroid.

Answer	N	Examples of answers
Exhaustive	8/17	*'$\boldsymbol{U}_v$ = 10 photons → polaroid with allowed direction at 45° on average, five photons will pass through, polarized at 45° and state $\boldsymbol{U}_{45}$.'*
Partially correct	9/17	'*Starting with 10 photons with allowed direction at 45° obtaining five photons*'.

building the required example, all students use the Geometric mode.

Q.15. For each of the cases specified below (referring to the horizontal direction), write the state vector of a photon with respect to the basis $\mathbf{u}_{\mathrm{H}}$ *and* $\mathbf{u}_{\mathrm{V}}$ *(horizontal and vertical polarization states respectively) and the probability of transmission from a polaroid V both as a formal expression (formula) and as a numerical value:*

State of preparation	*State vector*	*Transmission probability from a V polaroid*	
		Formula	*Numerical value*
H (0°)			
V (90°)			
45°			
30°			
60°			

The questions Q.13, Q.14, are preparatory to Q.15, which involves the student's competences to apply and extend what is considered in the previous questions. A group of students (11/24) provides an exact answer in all parts of question Q.15 (see table 9) showing how to combine the geometric aspect with the more abstract formula. A second group (6/24) answers in a non-exhaustive way because it does not complete the column 'Vector of State' while being able to correctly calculating the required probabilities. Finally, a third group (7/29) provides a wrong answer due to a miscalculation of several transition probabilities. Also, in this group nobody completes the column 'Vector of state' that is the more formal part. In this case, however, the gap in the formal request prevents the exact calculation of the transition probability. In addition, the errors are concentrated in the last two lines (i.e. in the cases with 30° and 60°), that were less treated during the course, and involving a clear understanding of the meaning of superposition and its formal representation.

The normalized average value of the score obtained in these three answers is 49/100 (see figure 3). The observed scores range from a minimum of 17/100 to a maximum of 100/100 with a median of 40/100, slightly lower than the average, indicating a slight trend towards lower scores.

If one considers the scores of the individual answers (see figure 3(b)), the average is 67/100. It is observed that two answers, Q.13 and Q.14, have scores close or above average, while Q.15 is significantly below (50/100).

These results seem to indicate that about 50% of the answering students have developed the necessary expertise to effectively use the QM formalism within the required framework. These students connected the phenomena regarding photon polarization with its describing formalism: they anchored photon polarization to a geometric representation and, later, to an algebraic/analytic one based on the vector of state and the superposition principle. In the remaining part of the students, the photon polarization is mainly anchored to a geometric representation. The subsequent extension to a formal mode proved to be more problematic.

4. Discussion and conclusions

A research-based educational program has been proposed to two final-year classes of an Italian scientific high school, for a total of 49 students, aged 18–19. The educational program aims at the main concepts of quantum mechanics and their formalization, with a Dirac notation in the context of polarization of photons. Students explored real and simulated experiments to operatively construct concepts like quantum

Table 9. The relative frequencies of the answers to question *Q.15* for the categories contained in the first column are given. The question requires to be able to calculate different transition probabilities and to represent the state before and after the transition.

Type of answer	N	Indicators
Exhaustive	11/24	All columns in the table are completed with the correct answer
Partially correct	6/24	Missing analytic expression of the state vector
Erroneous	7/24	Calculations of different transition probabilities are wrong

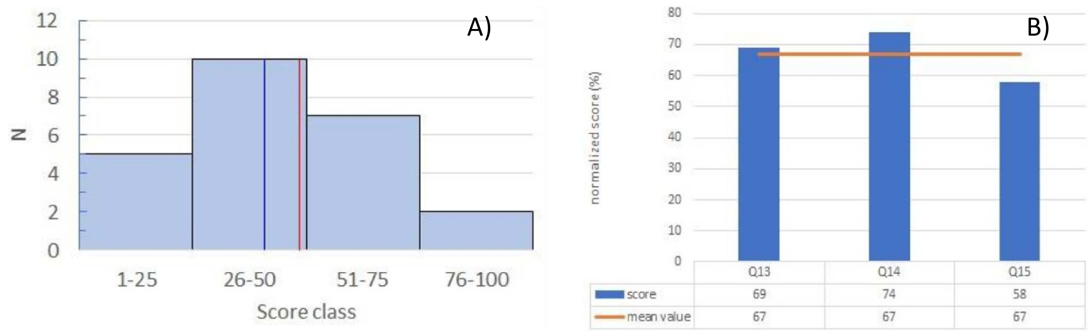

Figure 3. (A) Distribution of scores obtained by students in the answers to questions Q.12, Q.14, Q.15 (normalized to 100). The distribution average is 49/100 (red vertical line) and the median 40/100 (blue). Notice that the distribution is asymmetric with a tendency towards lower scores. About 54% of students rank below average. (B) Average scores obtained by the students in questions Q.13, Q.14, Q.15 (normalized to 100). The orange line represents the average of the three scores (67/100).

state and property (and their distinction), superposition, uncertainty and measurement, and the impossibility of attributing a trajectory to a quantum system. Non-locality and entanglement were also discussed by analysing the behaviour of photon beams passing through a birefringent crystal. These concepts, and in particular the concepts of superposition and entanglement, were discussed also after introducing a quantum game, engaging students in a tournament at the end of the educational program. Relevant issues appear to emerge from the examined data, as follows.

First, the students have formed a final picture of QM that prioritizes two aspects. On the one hand, the phenomenological aspects of photon polarization with the limiting behaviour of individual photons. On the other hand, the role of overcoming the QM as compared to CP, due to the relevance played by probability in quantum phenomena, in a non-epistemic sense.

Regarding what concepts were learned by students (RQ1) and what open nodes remained (RQ2), several outcomes are in order.

About half of students, focussing on the measurement process, developed a coherent view of the distinction between properties of a quantum system and the state in which the system is. In particular, these students showed an adequate clear distinction between mutually exclusive and incompatible properties embedded with the superposition state, the former recognized as the outcomes of measurements made on system observables. Relevant is the importance perceived by students of the active role played by the measuring apparatus in QM, a distinctive aspect with respect to photon polarization. The in-depth phenomenological analysis of Malus law as interpreted in a probabilistic perspective, contributed to form this conception (RQ3).

It should be emphasized the evident role played by the phenomenological exploration of polarization for the learning of the other students as well. However, a portion of them (identifiable between 10% and 20%) either did not go beyond the classical view, or did not go beyond the phenomenological aspect, albeit referring to the behaviour of photons (RQ2).

However, the use of classical concepts, or of expressions in which concepts are confused, is clearly contextual. In particular, responses to questions in which two concepts were asked to be distinguished (e.g. state and property, mutual exclusivity-incompatibility) are those in which learning difficulties most often emerged. For this reason, it is important to include a suited amount of time in an instruction course, where quantum concepts can be shared and constructed through the discussion of different viewpoints (RQ3).

The concept of incompatibility plays a central role in quantum theory, being the basis of all the counterintuitive and peculiarly nonclassical aspects of QM. It is no coincidence, therefore, that it is also the most complex for students to learn (RQ2). It has been seen, however, that the operational approach offered e.g. by the gameplay, at least provides students with reference contexts in which experiencing what incompatibility entails (e.g. potentially occupying two distinct positions while not having a specific position). This is an important outcome of the present work: the quantum game allows the reintroduction of intuition and direct experience of otherwise inaccessible phenomena (RQ3).

One more relevant outcome of the present research is that for many students (about 2/3 of our sample), the introduction of the formalism can help to identify quantum concepts in a coherent manner. The identification of the state with a vector that can be expressed in components corresponding to the mutual exclusive results of a measurement, is the most robust construct gained by these students (RQ1). All of this, obviously, emerged in a whole spectrum of a variety of slightly different views. The main way through which students tend to represent the quantum state, is by specifying an formal characteristic such as its vector nature (RQ1b). The geometric representation of the state was privileged with respect to the analytic-algebraic form.

Although the representation as a vector does not exhaust the concept of state in QM, it shows that several students have begun to build a connection between the phenomenological and the formal parts, unrelated to the geometric representation.

In addition, some students prefer to translate the mathematical constructs in words, adopting what we call Declarative modality. However, the majority of them adopts a Formal Declarative modality, including both mathematical expressions and explicative sentences (RQ1).

The significant role played by the iconographic representation introduced into the educational program to bridge the interpretation of phenomena and mathematical formalism, emerged again as noted in previous research [13, 21, 22] (RQ2).

The representation by means of an algebraic/analytic expression of the concepts of quantum state and superposition appears for some students to be more problematic (RQ2).

Only part of the students explains the role of quantum state in QM as a mediator of the maximum information available from the system. This lack is probably due to the distinction between mutually exclusive and incompatible properties, which is not entirely clear for about half of students. In any event, despite some difficulties in the formal representation of the quantum state, several students (about 2/3) manage to correctly apply the QM formalism and calculate the transition probability in different contexts.

We highlight the significant engagement showed by students during the QTTT tournament played at the end of the QM program. As in the words of students, the game-based activity helps developing the ability of building intuition on quantum behaviour, comprehending concepts like superposition and entanglement, and what distinguishes an entangled from an ordinary superposition state. This confirms the encouraging results that we have obtained in a dedicated pilot study [32]. We believe that the combined use of quantum games and interactive tools with experimented educational programmes like the one described in the present work, opens up an exciting perspective to be explored, which future research efforts should be dedicated to.

Data availability statement

The data presented in this study are available on request from the corresponding authors.

The data that support the findings of this study are available upon reasonable request from the authors.

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Conflict of interest

The authors declare no conflict of interest.

Ethics statement

The involved students have voluntarily participated to the educational activity and the questionnaires. This study complies with the IOPScience ethical policy.

Appendix A. The quantum game

For the sake of completeness, in this appendix we provide the main concepts and results related to the use of a playful activity realized at the end of the educational program, as indicated in module VII of table 2 of the main text: a tournament among students based on the game TiqTaqToe (QTTT) [29–32], complementing the teaching/learning sequence based on photon polarization and the JQM activities.

Research in different fields highlighted the role of games in favouring the connection between abstraction and formalization. Games designed with a purpose (GWAPs) can be powerful tools to boost research and science education [29–32, 37–43], in particular favouring the use of intuition to quickly solve very complex computational problems [44], and reinforcing motivations and voluntary participation, ensuring that the experience can remain enjoyable with no fears of failures [45]. Recently, GWAPs are significantly flourishing for quantum physics education [29–32, 46–49], along with the use of interactive tools, i.e. applications designed to visualize and simulate quantum-mechanical phenomena, thus relevant components in QM teaching/learning [50–52]. These tools are used to help students understanding complex and largely non-intuitive behaviours such as those analysed so far.

In the case of QTTT, the foundational concepts of quantum state, properties, superposition, entanglement and measurement can be experienced in an engaging and direct way, and stimulate in students a kind of 'quantum intuition'. In the game, the conventional X and O of the classical TicTacToe have been formalized as quantum particles, which can be in a position state $|n\rangle_X$ or $|m\rangle_O$ in Dirac's notation, with n, m numbers from 1 to 9 in the TicTacToe grid. A quantum superposition state of particle, e.g. X can then be formalized as $|Q\rangle_X = \frac{1}{\sqrt{2}}(|n\rangle_X + |m\rangle_X)$, while the entangled state between particle X and particle O as e.g. $|Q\rangle_{XO} = \frac{1}{\sqrt{2}}(|n\rangle_X|m\rangle_O + |m\rangle_X|n\rangle_O)$. The two players add their X and O in the grid, each time choosing whether to add a classical particle, or a superposition state of one own's particle, or else an entangled state of one own's particle with that of the other player. On completion of the grid, the system automatically performs a measurement, which projects the quantum states in either one of their possibilities, with a 50% probability (other options can become available). Using the quantum states can lead to advantages (and also to parity matches!), which is what the students were challenged to find out. In the educational program, the QTTT tournament has been preceded by a very compact module where the game rules have been formalized in Dirac's notation as shown above, and the foundational concepts have been introduced via creative animations, the so-called Quantum pills, available on QPlayLearn [29, 30].

As in a previous pilot study [32], students showed a significant interest in the QTTT tournament, suggesting the role of the game in motivating them to tackle a topic otherwise hostile to deal with.

Students gave a feedback on the role of the QTTT game, answering to question

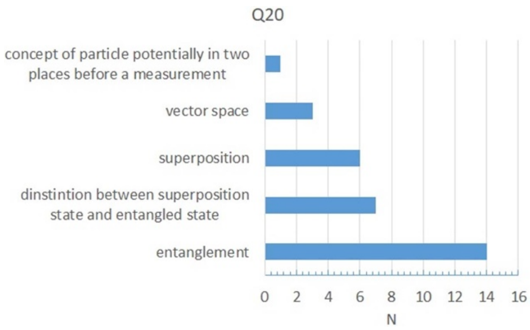

Figure 4. Concepts that 25/27 students claim to have better understood thanks to the TiqTaqToe quantum game.

Q.20—illustrate whether and how the TiqTaqToe game helped you to understand some QM concepts/aspects. Explain which concept, if any.

Among the 27 respondents, 25 indicated specific concepts learned, as summarized in figure 4. Prominent is the role of game-playing in understanding the concept of entanglement and how an entangled state differs from an ordinary superposition state.

A few students sentences provide a better understanding of the role that integrated game-playing can have in a QM learning path. '*It helped me, though without I could realize it, to distinguish between superposition and entangled states'*. Here, the operative context setup by the QTTT game emerges, in which students become immersed. Almost without realizing it, this leads to functional understanding [23] of the concepts, i.e. they understand what the different quantum concepts imply more than they understand the concepts themselves. *'The TiqTaqToe game allows one to understand the concepts of quantum and entangled superposition states in a visual and very intuitive way, and then see the result through measurement'*. In this sentence, representing almost other 1/3 of answers, the role of the QTTT game emerges in visualizing the concepts, by bringing them intuitively closer to students: *'It helped me understanding the concept of a particle potentially being present in two different places before the measurement occurred'*. This sentence highlights how the gameplay is able to give a representation, other than a mathematical one, of a concept otherwise inaccessible via classical tools.

Among the 2/27 students who did not indicate specific concepts learned, one explained that the game allowed to look from a different perspective at the set of concepts learned during the educational journey, and better understand them: [The QTTT] '*game did not help me understanding single concepts, but it helped me to see them in a different light and understand them more'*. Here the role emerges that the game can play as a context in which students can get a synthetic view of quantum concepts. Finally, one students pointed out that the game was fun, but did not feel that it enabled a better understanding of specific concepts ('*funny, but it did not help me understand*').

The data reported here are consistent with findings from a previous pilot study [49]. There, a thoroughly study on the role of QTTT in QM learning could be performed with a smaller group of students. The main outcome emerged from the study is that students could grasp the concepts of superposition and, with special awareness, entanglement, the game proving to be effective in helping students to experience their implications in quantum behaviour.

The present data reinforce the results on the role played by QTTT in a teaching/learning environment, by enhancing awareness about non-intuitive concepts, offering students a context, though virtual, where to visualize quantum concepts, have experience of what they imply, and build up intuition for learning.

In addition, it emerges the complementary role that QTTT may play with respect to the experimental and mathematical literacies involved in the scientific process, boosting student engagement, and overall improving the efficiency and effectiveness of the program.

Appendix B. Questions posed to students and analyzed in the paper

Q.1— What did I learn about Quantum Mechanics with this experience (Open question)

Q.8—explain to your classmate what is meant by mutually exclusive property and what is meant by incompatible properties (and what they differ) (Open question)

Q.9—illustrate to a classmate what is meant by the polarization state of a photon (Open question).

Q.10—how would you explain to your classmate the difference between polarization properties and polarization state? (Open question)

Q.12—what proves that the polarization state cannot be thought of as a statistical mixture of pre-existing properties? (Open question)

Q.13—a photon prepared in the state ***u*** *make a transition in the state* **w***, after interacting with a polaroid. How can I write the probability of such an event?*

Q.14—give the example of a specific case (determine the numerical value of the transmission probability, after defining the necessary parameters).

*Q.*15. *For each of the cases specified below (referring to the horizontal direction), write the state vector of a photon with respect to the basis* $\mathbf{u}_H$ *and* $\mathbf{u}_V$ *(horizontal and vertical polarization states respectively) and the probability of transmission from a polaroid V both as a formal expression (formula) and as a numerical value.*

State of preparation	*State vector*	*Transmission probability from a V polaroid*	
		Formula	*Numerical value*
H (0°)			
V (90°)			
45°			
30°			
60°			

Q.16.—can you explain to a classmate the principle of superposition of quantum states?

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