A European Researchers' Night project on mechanical vibrations for high school students

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Abstract The present works were conceived to be exhibited during the 2022 European Researchers' Night (ERN 2022), at the University of Modena and Reggio Emilia. The idea is to illustrate the key concepts of mechanical vibration through the use of 3D models and virtual simulation analysis. The paper is directed to high school students planning to enroll in a mechanical engineering bachelor's degree, in order to approach or consolidate some fundamental concepts of mechanical vibration. Topics not easy to explain, such as the natural frequencies of a body, could be presented more effectively using physical models. Mathematical formalism will be kept to a minimum, as it is beyond the scope of this paper.

Keywords: Mechanical vibration, Natural frequencies, Modal analysis

1 Introduction

Any motion that repeats itself after an interval of time is called vibration or oscillation [1]. Two main types of vibrations are generally defined: *forced* and *free*. A *forced vibration* happens when the system is subjected to an external (usually periodic) force. On the other hand, if a system, after an initial disturbance, vibrates without external variable forces, one has a *free vibration*: this introduces the concept of *natural frequency*. The study of natural frequencies is extremely important because when bodies are excited at their natural frequency, a resonance occurs, which leads to high deformations of the structure [2]. Each natural frequency of a body is associated with a shape, a way in which the body vibrates, called *natural mode*. In the next sessions the natural modes and frequencies for three different structures will be identified. The first (Sec. 2) is a simple case of cantilever beams with attached point masses, whose vibrations are manually excited. The second structure (Sec. 3) is used to represent the modes of a three-storey building, excited with a harmonic force generated by a mechanical shaker, to show how these vary using a mass damper. In the last example (Sec. 4), a plate clamped at its center is adopted as object on which perform an experimental modal analysis, identifying some natural frequencies and showing mode shapes through software interface. All the examples were first explained in the simplest way, then the experiments were performed in order to visualize and convey the concepts in a more engaging and stimulating way.

2 Vertical beams with point masses

The first model developed is a small cart with three vertical beams, each carrying a concentrated mass along its length (Fig. 1). This concept was inspired by similar models [3], in which a mechanical system is designed to display several resonances at low frequencies. This way, the vibrations can be manually excited, and it is easy to distinguish the behavior at different frequencies. Our design has the following features, which we used as guidelines in this project.

1. *Configurability*: the design can be easily modified during a presentation to change the dynamic behavior of the structure. For simplicity, we consider a simplified model of a slender cantilever beam (with distributed mass M_d) vibrating with small displacements and carrying a point mass M_c at a given position a along its length L. The first natural frequency can be found analytically, but it requires solving a complex trascendental equation [4, 5]. This equation depends on L, a and M_c, on the cross-section properties of

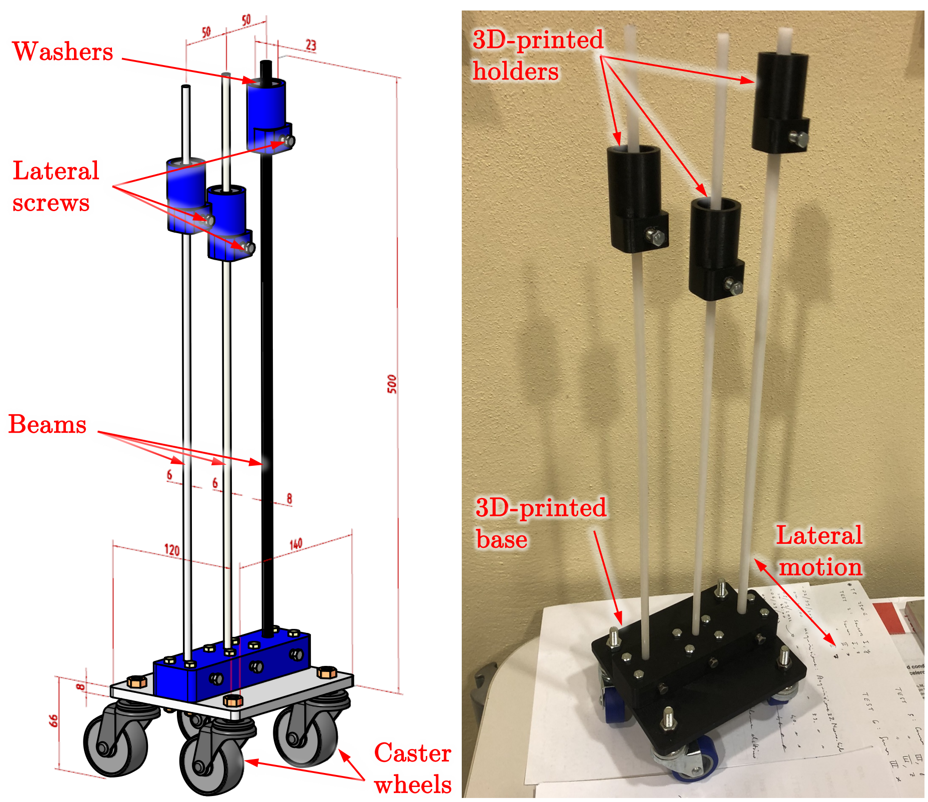

Figure 1. (a): the CAD concept we developed. (b): the final built model of the cart.

the beam (the area S and second moment of area I) and on the intrinsic properties of its material (namely, its stiffness modulus E and density ρ). Thus, the design allows to vary all these parameters independently:
   - the lengths a and L can be changed by unlocking lateral screws that fix M_c along the beam and the beam on the cart, respectively;
   - the mass M_c can be changed by adding or removing washers on the 3D-printed holders that are placed on each beam;
   - the beam properties can be changed by using beams of different materials (in our tests, we used plastic bars composed of POM, PVC and Nylon).

   The cart has three slots for placing the beams, two with a diameter of 6 mm and one with a diameter of 8 mm, so the effect of changing the cross-section can also be evaluated (by comparing the behavior of the three beams).
2. *Transportability*: the model can be disassembled and reassembled easily, for ease of use in classroom activities and at public events such as ERN.
3. *Effectiveness*: the design useas cheap elements found in hardware stores, with some 3D printed parts realized with a standard FDM printer.

The cart is manually excited, by moving it back and forth on a flat surface; the caster wheels on its base allow for quick and frictionless motion. It can then be seen how one of the beams (the one whose natural frequency is closer to the excitation frequency) vibrates with higher amplitudes: the demonstration is particularly effective if the movable masses are placed such that the beams have natural frequencies that are well apart from each other. A metronome can be used for keeping the rhythm; the experiment can also be designed as a teaching game, where the students have to find the right frequencies by trial and error.

We also developed an Excel worksheet to compute the first natural frequency of the beams: besides being useful in the design phase, this worksheet can be an interactive teaching tool, to compare the expected frequencies with the real ones.

3 Vibrating building with tuned mass damper

The second structure is a model of a three-storey building vibrating under a lateral load; the CAD concept is shown in Fig. 2. While other similar demonstration tools exist [6, 7], this model was designed as a configurable and inexpensive tool, like the one presented in Sec. 2: therefore, the same design guidelines were followed in this case, too. Thus, the floors can be moved along the length of the four columns (composed of 6 mm POM beams); their masses can also be tuned by adding or removing nuts that can be clamped on the available holes.

This design uses lateral wires for bracing the structure: this way, the shear motion (along the x axis of the coordinate frame, which is aligned with the axis of a shaker) corresponds to a relatively low stiffness, while other motions (such as bending or torsion of the whole structure, axial displacements along z or shear motion along y) have much higher stiffness. Since the shaker operates at low frequencies in this case, only the natural modes corresponding to shear along the x axis are of interest. We also use stiffening rods, to reinforce the floors (composed of 3D printed plastic and thus relatively flexible); this way, they can

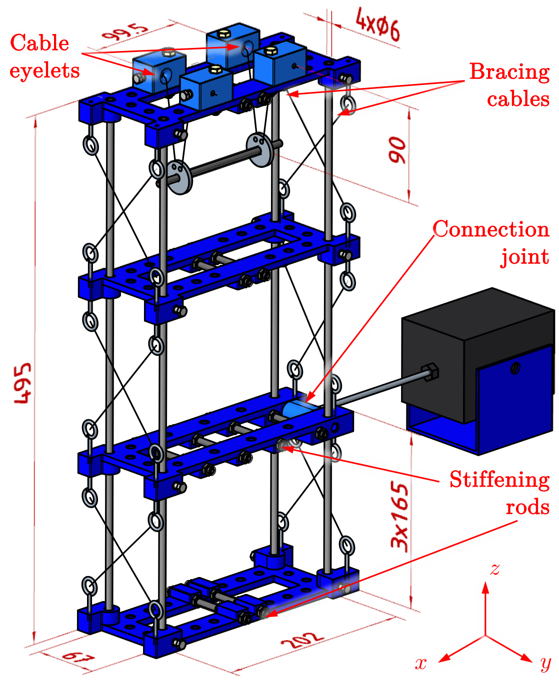

Figure 2. CAD model of the building; dimensions in mm. The shaker is also shown.

be considered as rigid masses. In any case, it was also verified that the model can be used without these wires and rods, to tune its stiffness as desired.

The model is connected to the shaker through a rotary joint on the first floor: this was required to avoid misalignment of the shaker connection. In this way, one can also lower the shaker with respect to the model and verify the effect of having both horizontal (shear) and axial excitation forces. For simplicity, the shaker axis was kept horizontal in the tests, by raising the shaker base.

The final built model is in Fig. 3. The frequencies of this building may be found through analytical methods [8, 9], by considering the structural compliance to be concentrated in the columns. This, however, proved to be complex due to the uncertainties in the structural parameters, such as the properties of the material used for the columns. Instead, an experimental modal analysis was performed, by exciting the structure with an impulse (which is produced by a function generator) and measuring the vibrational response. Two piezoelectric accelerometers (with sensitivities of 10 mV/g) were placed on the model: one close to the shaker connection point, measuring the input excitation, and the other one on the top floor, which is expected to have the largest oscillations. Three-axial accelerometers were used, in which the channels of interest are the axes of measurement aligned with the x axis; the accelerometers were connected to a NI cDAQ Ethernet chassis sampling at 51200 Hz. By computing the FRF between

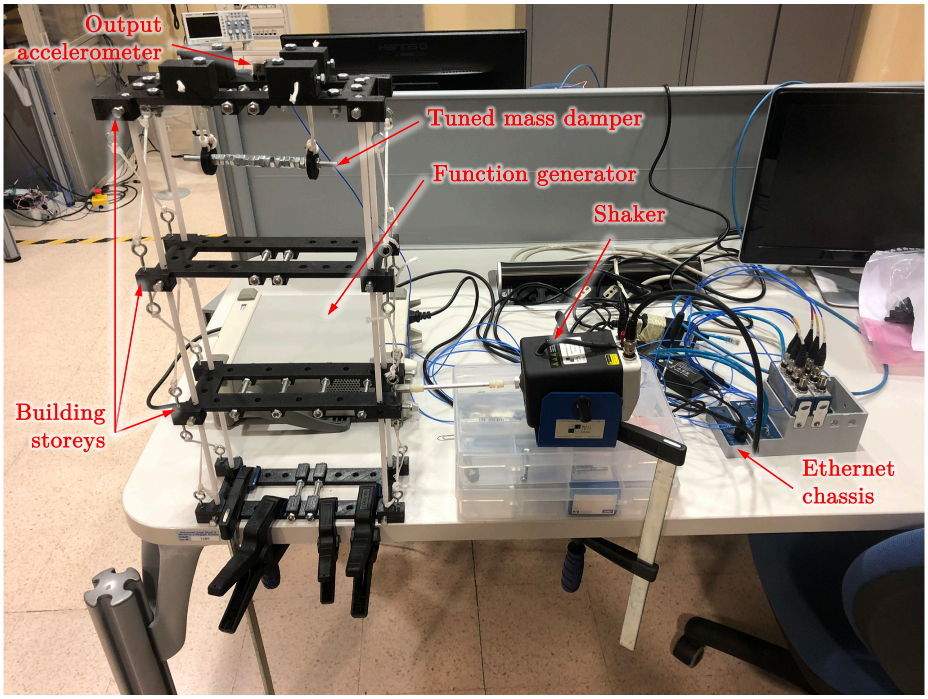

Figure 3. Photo of the vibrating building clamped on the test table for modal analysis.

the input and the output channels, the natural frequencies of the structure were found at 1, 6.5 and 24 Hz. By exciting the system at these frequencies with sinusoidal forces (again by controlling the function generator), the vibration modes can be clearly shown, since the displacement of each floor visibly changes.

This model was also designed to be used with a tuned mass damper, which can be rapidly connected and disconnected to show the different behavior in each case. This is composed of a pendulum suspended on the third floor: while a simple point mass attached to a rope could be applied [10], it would be difficult to constrain its motion along the x axis. Therefore, in our design, we used a threaded bar suspended at its ends by four cables (which pass through cable eyelets on supports fixed to the floor): since the cables are of the same length, the bar always remains horizontal and no inertia torque is introduced. Thus, its behavior is dynamically equivalent to that of a simple pendulum: if an excitation is applied at the frequency of the pendulum (which depends only on the cables' lengths), the energy introduced is in large part absorbed by the pendulum, which swings with large amplitudes, while the vibration on the rest of the structure is reduced. This was verified experimentally, by measuring the output vibration both with and without the damper: the RMS amplitude of the acceleration is reduced by > 10 %. This effect can be varied by changing the mass of the pendulum (on which nuts can be screwed), without changing its natural frequency.

4 Plate with center point clamped

4.1 Setup

This section presents the setup used to carry out the experimental modal analysis of a square plate with the central point clamped. As shown in Fig. 4, the plate was constrained in its central point by interlocking to a massive vertical support bar. Being interested in out of plane modes, a uniaxial accelerometer with sensitivity of 10 mV/g was fixed on the lower face of the plate with vertical acquisition direction, while the impulses were supplied by an instrumented hammer with a sensitivity of 2.25 mV/N with a metal tip. Both devices were connected to the acquisition system case through BNC connectors cables. Siemens equipment, LMS SCADAS Mobile (hardware) and LMS Test.LAB (software) were used for acquisition and analysis respectively.

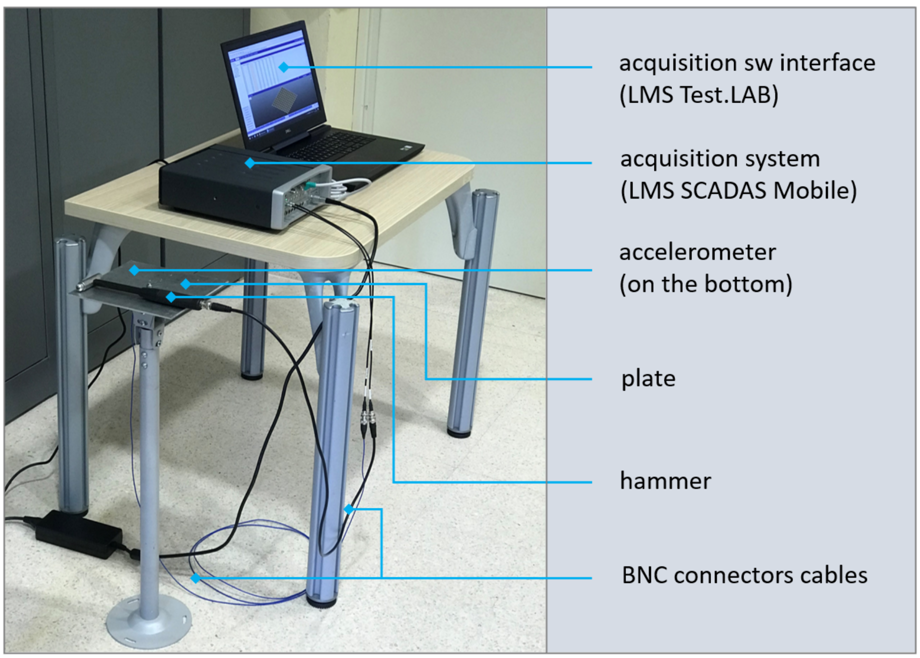

Figure 4. Setup

4.2 Measurements

The plate (200x200x3 mm) was discretized giving a regular grid of 121 points to be excited. The measurements were carried out by exciting the structure with 5 impacts per point, averaged in *coherence* and *frequency response function.*

4.3 Results

Starting from the complete set of acquisitions, in the post-processing phase a global frequency response function was calculated in order to analyze all the data simultaneously, and thanks to the Least Square Complex Exponential method, estimate values of natural frequencies and related damping percentage has been obtained (see Tab. 1). The range covered is up to 1000 Hz, due to limits related to impact energy supplied manually, and a good measurement accuracy is also required. The corresponding modal shapes has been finally obtained, Fig. 5 shows 2 of the 8 identified modes: for each of them, the maximum amplitude deformations in opposite phases are shown, nodal lines are marked in red.

Table 1. Natural frequencies detected [0-1000 Hz]

mode	frequency [Hz]	damping [%]
1	63.259	1.42
2	190.424	2.05
3	230.459	0.65
4	339.916	0.49
5	542.651	4.50
6	601.901	0.75
7	750.183	2.24
8	997.938	1.10

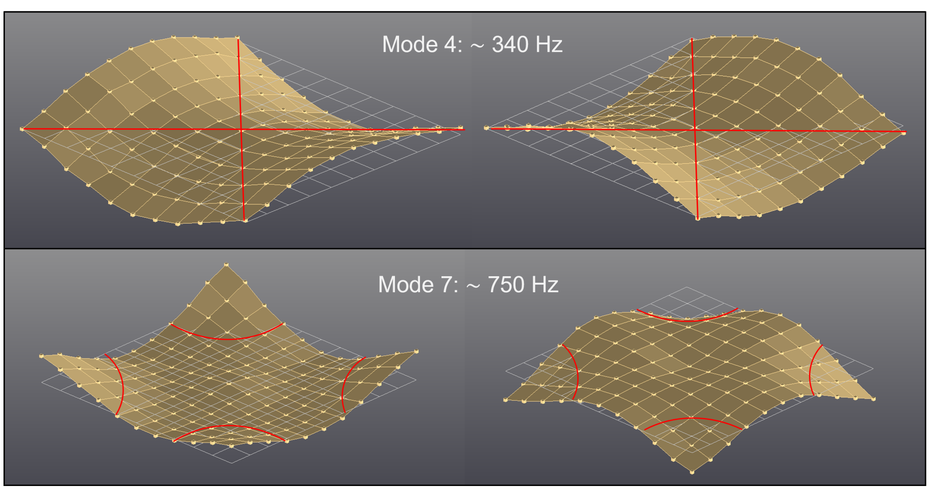

Figure 5. Modal shapes

5 Conclusions

The different projects define a course in mechanical vibration analysis, starting from systems with a single degree of freedom (Sec. 2) and moving to models with many degrees of freedom that better approximate a continuous body (e.g. the third project). Positive and full of admiration feedback are been received from high school students, who, driven by a strong interest in mechanics, have expressed their desire to enroll in a degree course. In particular, they appreciated how the use of physical models is fully effective in understanding complex phenomena.

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