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The dynamics of working hours and wages under implicit contracts

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Abstract

In this paper, we explore the dynamics of working hours and wages in a model economy where a firm and its workforce are linked to each other by an implicit contract. Specifically, we develop a deterministic and a stochastic framework in which the firm sets its level of labor utilization by considering that workers' earnings tend to adjust in the direction of a fixed level. Without any uncertainty about firm's profitability, we show that the existence and the properties of stationary solutions rely on the factors that usually determine the enforceability of contracts and we demonstrate that wages move countercyclically towards the allocation preferred by the firm. Moreover, we show that adding uncertainty does not overturn the countercyclical pattern of wages but is helpful in explaining their dynamic behavior in response to demand shocks as well as their typical stickiness observed at the macrolevel.

KEYWORDS

consumption smoothing, implicit contract theory, out-of-equilibrium dynamics, stochastic optimal control

JEL CLASSIFICATION

D86, E24, J41

1 INTRODUCTION

The theory of implicit labor contracts—or quasi-contracts—starts from the premise that the labor market is far from being a spot market, but, on the contrary, workers and firms usually manifest the tendency to be involved in long-lasting and nonanonymous relationships characterized by a strong degree of customization (cf. Okun, 1981). Consequently, if there is some uncertainty about actual production outcomes and entrepreneurs are more risk prone than workers, then it may happen that the two parties will consensually rely on informal agreements on labor provisions and wage payments that optimally share the burden of realized labor income fluctuations (cf. Azariadis, 1975; Baily, 1974; Gordon, 1974).

The theoretical literature on implicit contracts collects a number of contributions in which labor market outcomes are determined in a time-less perspective (e.g., Azariadis & Stiglitz, 1983; Baker et al., 1997; Bull, 1983, 1987; Chiari, 1983; Geanakoplos & Ito, 1982). In more recent years, however, after the seminal work by Harris and Holmstrom (1982) in which the terms of long-run implicit contracts follow from the intertemporal maximization of workers' utility subject to the evolution of the expected profits of their employer, a number of authors spent some effort in the exploration of the dynamic consequences for hours, (un)employment, and wages arising from the existence of optimal risk sharing in labor contracts.

Within the literature on dynamic contracting, Haltiwanger and Maccini (1985) develop a framework in which the existence of implicit labor contracts may lead firms to rely on temporary layoffs and subsequent recalls. Robinson (1999) exploits the theory of repeated games to provide a dynamic model of strikes in which walkouts reduce output and are used by employed workers as punishment mechanisms to enforce implicit contracts in a context of asymmetric information. Gurtler (2006) compares repeated games of implicit contracts with infinite and finite horizons by stressing the importance of discounting for the enforcement of the agreements achieved between workers and firms (cf. Pearce & Stacchetti, 1998). Michelacci and Quadrini (2009) as well as Calmès (2007) flip the framework by Harris and Holmstrom (1982) and develop dynamic implicit-contract models in which firms maximize their profits by taking into account the evolution of the expected utility of their workers (cf. Spear & Srivastava, 1987). More recently, Pourpourides (2011), Wang (2015), and Basu and House (2016) incorporate the implicit contract hypothesis within dynamic stochastic general equilibrium (DSGE) models to exploit the amplification mechanism of macroeconomic shocks triggered by long-term employment relationships.

In this paper, we aim at contributing to the literature on dynamic implicit-contract models by deriving the smooth out-of-equilibrium dynamics of working hours and wages in a theoretical setting where workers and firms are linked to each other by an implicit contract that tends to stabilize real consumption in a long-run perspective. Specifically, we develop a self-contained theoretical framework with no information asymmetries in which a representative firm sets its optimal level of labor utilization by taking into account that workers' earnings tend to adjust in the direction of a fixed level set out in the contract that is assumed to coincide with desired long-run consumption (cf. Abowd & Card, 1987).

To the best of our knowledge, the present contribution is the first to explore the dynamic behavior of wages and working hours in an intertemporal setting with uncertainty where the optimal employment decisions of a representative firm over the intensive margin are constrained by a dynamic wage schedule that targets a fixed level of labor earnings as originally argued by Shavell (1974) in the context of risk sharing in deferred payments. Indeed, while traditional implicit-contract contributions advocate for fixed wages (cf. Azariadis, 1975; Baily, 1974; Gordon, 1974), Shavell (1976) argues that a Pareto-optimal contract between a risk-neutral payer (the firm) and risk averse payment recipients (the tenured workforce attached to that firm) both endowed with identical beliefs about future uncertainty leaves the latter ones not with a constant payment (wage), but with a constant income (labor earnings).[1]

Our theoretical exploration is split into two parts. First, we explore the disequilibrium adjustments of working hours and wages in a model economy where the representative firm is endowed with a quadratic production function, and there is no uncertainty in its profitability. Thereafter, we consider the optimal trajectories of the two mentioned variables by assuming that the effectiveness of labor is hit by random shocks that systematically alter the profitability of the firm. The former preparatory analysis allows us to discuss the conditions under which the suggested contractual agreement between the firm and its tenured workers conveys meaningful solutions. The latter provides the basis for assessing the cyclical properties of a dynamic implicit-contract economy.

Overall, our analysis provides a number of interesting findings. On the one hand, depending on selected parameter values that usually are closely linked to factors determining the self-enforceability and feasibility of contracts, the deterministic model may have one, two, or no stationary solution. Interestingly, whenever there are two steady-state allocations for hours and wages, the resting points of the economy without uncertainty can be ordered according to the preferences of each party. Moreover, in the two-solution case, the local dynamics of the model reveals that wages display the tendency to move in the opposite direction with respect to working hours by converging towards the allocation preferred by the firm. This result is consistent with the empirical tests of the implicit contract theory carried out in the United States at the microlevel by Beaudry and DiNardo (1995); indeed, in their pioneering study—controlling for labor productivity and workers' characteristics—higher wages appear associated with lower hour provision and vice versa. In addition, the deterministic economy has the property that when the initial contract wage overshoots (undershoots) its long-equilibrium value, workers' earnings remain above (below) the fixed negotiated level during the whole adjustment process.

On the other hand, simulations of the stochastic model run by targeting the observed volatility of U.S. output reveal that disturbances on firm's profitability do not overturn the countercyclical pattern of wages by mirroring the typical macroeconomic effects triggered on labor markets by aggregate demand shocks (cf. Chiarini, 1998; Fleischman, 1999; Sumner & Silver, 1989). Moreover, we show that the insurance scheme underlying the dynamic implicit contract tend to underestimate the volatility of labor earnings, but it has the potential to explain some important business cycle regularities. Specifically, whenever the conditions under which the firm finds profitable to honor a wage agreement that pegs a fixed level of labor earnings are met, we show that it is also in its best interest to comply with an hour-wage profile in which the adjustments of remitted wages are smoother than the ones of hours. Obviously, this microfounded response of working hours and wages to stochastic disturbances is consistent with the macroevidence on real wage stickiness observed in many developed countries (cf. Ravn & Simonelli, 2007; Shimer, 2005).

This paper is arranged as follows. Section 2 describes the theoretical setting. Section 3 analyzes the deterministic economy. Section 4 explores the stochastic economy with uncertainty in firm's profitability. Finally, section 5 concludes by discussing avenues for further developments.

[1] Similar arguments can be found also in Becker (1962), Blanchard and Fischer (1989, Chapter 9) and more recently in Beaudry and Pages (2001) and Romer (2019, Chapter 11).

2 | THEORETICAL SETTING

We consider a model economy in which time is continuous and a representative risk-neutral firm deals with a group of risk-averse identical hand-to-mouth workers that cannot purchase insurance against fluctuations in the level of their long-run labor income. Within this environment, given the different attitudes towards risk, we make the hypothesis that the firm and its tenured workers are linked to each other by an informal wage contract that seeks to stabilize the level of labor earnings. Assuming the absence of nonlabor incomes and savings on the side of workers, this means that the informal agreement between the workers and the firm will tend to stabilize real consumption in a long-run perspective (cf. Abowd & Card, 1987; Pourpourides, 2011).

On the productive side—similarly to Guerrazzi (2011, 2020, 2021) and Guerrazzi and Giribone (2021)—we assume that the representative firm is endowed with a quadratic production function so that instantaneous output $Y(t)$ is equal to

$$Y(t) = A(t)L(t) - \frac{1}{2}(L(t))^2, \tag{1}$$

where $A(t) > 0$ is a technology variable and/or a measure of the economy-wide output taken as given by the firm and its workers whereas $L(t)$ is the labor provision of the workers attached to the firm measured in hours.

A quadratic production function like the one in Equation (1) implies that the elasticity of output with respect to the labor input—say $\epsilon_L \equiv (A(t) - L(t))/(A(t) - (1/2)L(t))$—is not constant with respect to the level of factors' utilization, and this feature of the production possibilities available to the firm appears closest to the most recent attempts to estimate actual production functions (cf. Ackerberg et al., 2015).

Uncertainty enters the model economy through the variable $A(t)$ that conveys the actual realization of the state of the world observed by all the involved agents. Specifically, the higher (lower) the value of $A(t)$, the better (worse) the realized state of the world. As a matter of principle, its economic interpretation is twofold. On the supply side, $A(t)$ affects the marginal productivity of employed workers in a linear manner so that higher (lower) values of $A(t)$ imply higher (lower) values of output for each additional worked hour (cf. Gordon, 1974). On the demand side, higher (lower) values of $A(t)$ mean instead that the firm can obtain a higher (lower) relative price for each level of production (cf. Azariadis, 1975; Baily, 1974). In the remainder of the paper, we consider the implications of both perspectives, and we assume that $A(t)$ might move over time according to an Ornstein–Uhlenbeck process. Formally speaking, this means that

$$\dot{A}(t) = \kappa(\mu_A - A(t)) + \sigma_A \dot{x}(t), \tag{2}$$

where $\mu_A > 0$ is the long-run mean of the process, $\kappa > 0$ is its speed of mean reversion, $\sigma_A > 0$ is its finite instantaneous standard deviation, whereas $\dot{x}(t)$ is a standard Brownian motion with zero drift and unit variance (cf. Cox & Miller, 1967).

According to the textbook analytical treatment of the implicit contract theory offered by Blanchard and Fischer (1989, Chapter 9) and Romer (2019, Chapter 11), in a time-less contracting model where firm's profitability is stochastic, the fixed level of consumption granted to tenured workers in all the states of the world can be conveyed as a nonlinear combination of all the possible realizations of firm's profitability whose weights are affected by workers preferences, the available productive technology, and the probability distribution of the already mentioned shocks to firm's profitability (cf. Shavell, 1976). Consequently, under the assumption that agents are rational and the information on these fundamental factors is costlessly available to all the parties involved in the contract, such a critical level of consumption can be taken as exogenously given without any substantial loss of generality.[2]

Along these lines, in what follows we will not specify the form of workers' utility function and we will assume that the long-run consumption granted to employed workers who reached an agreement with the firm is fixed at the constant level $C > 0$. Thereafter, the out-of-equilibrium dynamics of the contract wage $w(t)$ aimed at equalizing the wage bill to C in a long-run perspective will be given by

$$\dot{w}(t) = \theta\left(\frac{C}{L(t)} - w(t)\right), \tag{3}$$

where $\theta > 0$ is a measure of the attrition between the actual and the long-run real wage that stabilizes consumption.

The expression in Equation (3) represents the evolution of contract wages coming from the informal agreement achieved between the firm and its workers, and it implies that in each instant $w(t)$ increases (decreases) whenever it is below (above) the level of long-run consumption per working hour. Such a differential equation can be conceived as a reduced form that binds in a dynamic way the choice of the firm regarding labor intensity by summarizing in a compact manner all the relevant terms of the implicit contract (cf. Shavell, 1976). In detail, its formal specification is not affected by the evolution of $A(t)$ to capture the idea that the wage contract is not renegotiated when the state of the world changes (cf. Ham & Reilly, 2013). Similarly, external wage and employment opportunities do not enter the differential equation for contract wages because labor mobility costs are assumed to be prohibitive (cf. Baily, 1974). Furthermore, having in mind the way in which workers' preferences usually affect the terms of implicit contracts, the parameter θ on the right-hand side of Equation (3) can be taken as a measure of the degree of aversion with respect to situations of under- or overconsumption; indeed, for any given level of L, the higher (lower) the value of θ, the faster w adjusts itself in the direction of C.[3]

The adherence of the firm and its workers to the payment trajectories generated by the reduced form that enters the model economy to represent the existence of an implicit contract by making the wage a state variable is a distinguishing ingredient of our framework and for that reason it may deserve further explanations. Following the game-theoretical arguments put forward by Bull (1987), the wage trajectory implied by Equation (3) should be thought of as the outcome of a Nash equilibrium of a posthiring trading game whose self-enforceability is supported by intrafirm reputation. In other words, given the preferences of the firm and the ones of its workers, the values of the parameters C and θ have to be selected in order to avoid the existence of any incentive to renege on the contract (cf. Michelacci & Quadrini, 2009; Pearce & Stacchetti, 1998; Thomas & Worrall, 1988). In the remainder of the paper, we will show that the factors that usually drive enforceability in intertemporal implicit contract models in our theoretical context determine the existence and dynamic properties of stationary solutions. Consequently, since reneging on the contract means

[2] A formal proof for this statement and its implications for the trajectory of remitted wages is sketched in the Online Appendix.

[3] As we show in the Online Appendix, assuming that the real wage increases (decreases) when labor earnings are below (above) the long-run level of consumption specified in the contract complicates the analytical treatment of the model without any substantial modification in the conclusions achieved throughout the paper.

that one of the two parties—or both–has the desire to deviate from the achieved agreement, the enforceability of the wage contract described by Equation (3) will be assimilated to the existence of a stable stationary solution for working hours and wages.

3 | THE DETERMINISTIC ECONOMY

We begin our analysis by considering what happens in a model economy without uncertainty. In this case, the state of the world is revealed to the firm and its workers at the beginning of time and then it is assumed to remain constant thereafter. Specifically, we initially assume that

$$A(t) = A > 0 \quad \text{for all } t. \tag{4}$$

In each instant, given the values of C, θ, and A, the intertemporal problem of the representative firm in the model economy described above is to set the optimal labor input aiming at maximizing its profits by taking into account that the real wage adjusts itself over time according to the differential equation in (3). Formally speaking, considering the production function in Equation (1) and the simplifying assumption in (4), the problem of the representative firm is the following:

$$\begin{aligned} V(w_0) &= \max_{\{L(t)\}_{t=0}^{\infty}} \int_{t=0}^{\infty} \exp(-\rho t)\left(AL(t) - \frac{1}{2}(L(t))^2 - w(t)L(t)\right)dt \\ \text{s.to } \dot{w}(t) &= \theta\left(\frac{C}{L(t)} - w(t)\right) w(0) = w_0 \end{aligned}, \tag{5}$$

where $V(\cdot)$ is the value function, $\rho > 0$ is the discount rate of entrepreneurs, whereas $w_0 > 0$ is the initial level of the real wage rate specified on the implicit contract.

As it will become apparent later on, the solution of the problem in (5) defines a trajectory for working hours and a trajectory for remitted wages that may lead to the stabilization of labor income in the direction of the level of the long-run consumption established in the implicit contract.

The first-order conditions (FOCs) of the problem in (5) can be written as

$$A - L(t) - w(t) - \theta C \frac{\Lambda(t)}{(L(t))^2} = 0, \tag{6}$$

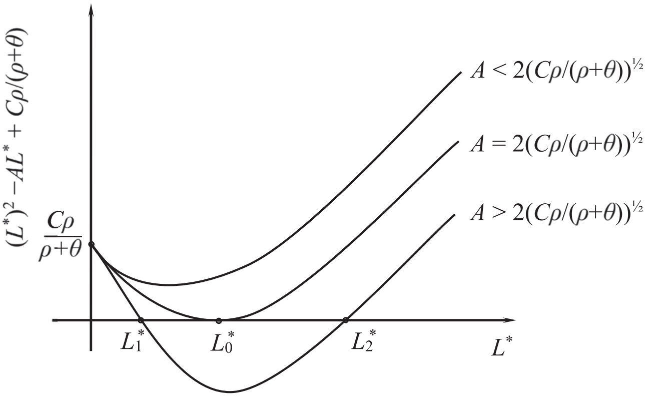
$$\dot{\Lambda}(t) = (\rho + \theta)\Lambda(t) + L(t), \tag{7}$$

$$\lim_{t\to\infty} (-\rho t)\Lambda(t)w(t) = 0, \tag{8}$$

where $\Lambda(t)$ is the costate variable associated with $w(t)$.

Equation (6) is the FOC with respect to the control variable of the firm, that is, $L(t)$. Moreover, the differential equation in (7) describes the optimal path of $\Lambda(t)$, whereas (8) is the required transversality condition.

FIGURE 1 Steady-state equilibria



After a trivial manipulation, the results in Equations (6) and (7) allow us to obtain the following differential equation for the out-of-equilibrium dynamics of working hours:

$$\dot{L}(t) = \frac{(\rho+\theta)L(t)(A - L(t) - w(t)) + \theta(2C - w(t)L(t))}{2(A - L(t) - w(t)) - L(t)}. \tag{9}$$

Starting from given initial conditions to be defined and pegging the value of A, the differential equations in (3) and (9) describe how working hours and wages move over time once an everlasting state of the world is revealed to the firm and its workers. Consequently, Equations (3) and (9) convey the dynamics of hours and wages for a given level of firm's profitability.

3.1 | Steady-state equilibria

Within the model under investigation, steady-state equilibria are defined as the set of pairs $\{L^*, w^*\}$ such that $\dot{L}(t) = \dot{w}(t) = 0$. Obviously, the elements of that set are given by allocations in which the real wage bill equals the fixed level of consumption specified on the implicit contract on which the firm and its workers reached an agreement.

From a formal point of view, the derivation of the $\{L^*, w^*\}$ pairs is straightforward. First, setting $\dot{w}(t) = 0$ in Equation (3) leads to

$$w^* = \frac{C}{L^*}. \tag{10}$$

Thereafter, setting $\dot{L}(t) = 0$ in Equation (9) and plugging the result into Equation (10) leads to the following quadratic expression:

$$(L^*)^2 - AL^* + \frac{C\rho}{\rho+\theta} = 0. \tag{11}$$

As illustrated in Figure 1, the parabola in Equation (11) allows us to state the following three propositions:

Proposition 1. *When $A = 2\sqrt{C\rho/(\rho+\theta)}$, there is only one stationary solution given by $L_0^* \equiv A/2$ and $w_0^* \equiv 2C/A$.*

Proposition 2. *When $A > 2\sqrt{C\rho/(\rho+\theta)}$, there are two distinct stationary solutions given by $L_1^* \equiv 1/2(A - \sqrt{A^2 - 4C\rho/(\rho+\theta)})$ and $w_1^* \equiv 2C/(A - \sqrt{A^2 - 4C\rho/(\rho+\theta)})$ as well as $L_2^* \equiv 1/2(A + \sqrt{A^2 - 4C\rho/(\rho+\theta)})$ and $w_2^* \equiv 2C/(A + \sqrt{A^2 - 4C\rho/(\rho+\theta)})$.*

Proposition 3. *When $A < 2\sqrt{C\rho/(\rho+\theta)}$, there are no (real) stationary solutions.*

Proposition 1 provides the parameters' combination under which there is a unique steady-state (L_0^*, w_0^*). In that allocation, equilibrium hours are an increasing function of the parameter that conveys the actual state of the world, whereas the equilibrium wage increases (decreases) with the fixed level of consumption granted by the implicit contract (the realized state of the world) virtually signed by the firm and its employees.[4] This pattern clearly points out the insurance component of the implicit contract; indeed, workers tend to work more (less) for less (more) in good (bad) states (cf. Blanchard & Fischer, 1989, Chapter 9; Romer, 2019, Chapter 11).

By contrast, Proposition 2 reveals the condition under which—similarly to what happens in the dynamic-search model with multiple equilibria developed by Diamond (1982)—there are two different steady states, that is, (L_1^*, w_1^*) and (L_2^*, w_2^*).[5] Assuming separability between leisure and consumption in the utility function of workers, the two stationary solutions pointed out in Proposition 2 can be unambiguously ordered according to the preferences of the two parties involved in the contract (cf. Azariadis, 1975). Specifically, since the implied level of consumption—or the implied labor earnings—is the same in both allocations, (L_1^*, w_1^*), that is, the stationary solution with low equilibrium hours and high equilibrium wage is the most preferred by workers because it implies more leisure, whereas (L_2^*, w_2^*), that is, the stationary solution with high equilibrium hours and low equilibrium wage, is the most preferred by the firm because—everything else being equal—it implies higher profits.

Furthermore, Proposition 3 shows the condition under which a steady state does not exist. For a given value of the state of the world conveyed by A, the impossibility of retrieving a stationary solution for the dynamics of working hours and wages appears alternatively related to an excessive degree of impatience on the side of the firms mirrored in the value taken by ρ, to an excessive fixed level of long-run consumption granted to workers embodied in the actual level of C and/or to a mild rate of mean reversion of contract wages conveyed by the value of θ. Overall, this proposition suggests that in our dynamic implicit-contract model the existence of a stationary solution requires appropriate levels of firm's profitability and workers' risk-aversion combined with not exorbitant discount rates on the side of entrepreneurs and sober long-run levels of insured labor earnings (cf. Shavell, 1976).

The requirements for the existence of a steady state summarized by Proposition 3 replicate the usual combination of factors that according to the literature reviewed in the introduction should determine the existence and the enforceability of implicit contracts. In detail, a certain degree of risk aversion is the main reason why a group of workers may decide to engage in a long-run relationship with a risk-neutral firm (cf. Azariadis, 1975; Baily, 1974; Gordon, 1974). Moreover, the

[4] It is worth noticing that the unique stationary solution falls in the concave part of the production function in Equation (1).

[5] Obviously, for (L_1^*, w_1^*) to be feasible it must hold that $A > \sqrt{A^2 - 4C\rho/(\rho+\theta)}$. In the remainder of the paper, we will assume that when the condition pointed out by Proposition 2 is met such an inequality is always fulfilled.

result on discounting recalls the one achieved by Gurtler (2006) in a repeated-game setting where higher values of the discount rate yield a decrease in the future value of firm's profits. Consequently, it becomes less worthwhile for the firm to honor the implicit agreement achieved with its workers since the punishment for reneging on the contract decreases and in that case the firm may find profitable to withdraw from the agreement (cf. Pearce & Stacchetti, 1998). Furthermore, similar arguments hold for the measure of firm's profitability; indeed, a reduction of output can make it difficult for the firm to honor the terms of the wage contract (cf. Harris & Holmstrom, 1982).

3.2 | Local dynamics

Given the stationary solution $\{L^*, w^*\}$, the local dynamics of working hours and wages is described by the following 2×2 linear system:

$$\begin{pmatrix} \dot{L}(t) \\ \dot{w}(t) \end{pmatrix} = \begin{bmatrix} j_{1,1} & j_{1,2} \\ -\frac{\theta C}{(L^*)^2} & -\theta \end{bmatrix} \begin{pmatrix} L(t) - L^* \\ w(t) - w^* \end{pmatrix}, \tag{12}$$

where $j_{1,1} \equiv \partial \dot{L}(t)/\partial L(t)|_{L(t)=L^*, w(t)=w^*}$ and $j_{1,2} \equiv \partial \dot{L}(t)/\partial w(t)|_{L(t)=L^*, w(t)=w^*}$.

In general, the two unspecified elements on the first row of the Jacobian matrix in (12) can be written as

$$j_{1,1} = \frac{\left((\rho+\theta)\Phi(L^*) - \frac{\theta C}{L^*}\right)(2\Gamma(L^*) - L^*) + 3\left((\rho+\theta)\left(AL^* - (L^*)^2 - C\right) + \theta C\right)}{(2\Gamma(L^*) - L^*)^2}, \tag{13}$$

$$j_{1,2} = \frac{2\left((\rho+\theta)\left(AL^* - (L^*)^2 - C\right) + \theta C\right) - (\rho + 2\theta)L^*(2\Gamma(L^*) - L^*)}{(2\Gamma(L^*) - L^*)^2}, \tag{14}$$

where $\Phi(L^*) \equiv (AL^* - 2(L^*)^2 - C)/L^*$ and $\Gamma(L^*) \equiv (AL^* - (L^*)^2 - C)/L^*$.

Under the condition pointed out in Proposition 1, that is, when there is only one stationary solution given by (L_0^*, w_0^*), the Jacobian matrix of the system in (12) merely reduces to

$$\begin{bmatrix} \rho + \theta & \rho \\ -\frac{\theta(\theta+\rho)}{\rho} & -\theta \end{bmatrix}. \tag{15}$$

The trace of the matrix in (15) is equal to ρ, whereas its determinant is equal to zero. This means that one eigenvalue of the system is zero, whereas the other is equal to ρ. Consequently, when the parameters of the deterministic model deliver a unique stationary solution, the out-of-equilibrium dynamics of working hours, and wages cannot be assessed; indeed, this characterization represents a degenerate case in which convergence towards the steady state denoted by the pair (L_0^*, w_0^*) is possible only if time flows in reverse (cf. Lesovik et al., 2019). From an economic point of view, this result can be rationalized by arguing that when the condition indicated by Proposition 1 is met, the agreement achieved between the firm and the workers—described by the problem in (5)—is

TABLE 1 Numerical solutions for different values of A ($\rho = 0.05$, $\theta = 0.10$, $C = 1$)

A	L_1^*	w_1^*	$r_1(L_1^*, w_1^*)$	$r_2(L_1^*, w_1^*)$	L_2^*	w_2^*	$r_1(L_2^*, w_2^*)$	$r_2(L_2^*, w_2^*)$
1.3	0.351	2.845	$0.025 + 0.039i$	$0.025 - 0.039i$	0.948	1.054	0.091	−0.041
1.4	0.304	3.287	$0.025 + 0.043i$	$0.025 - 0.043i$	1.095	0.912	0.100	−0.050
1.5	0.271	3.686	$0.025 + 0.046i$	$0.025 - 0.046i$	1.228	0.813	0.107	−0.057
1.6	0.246	4.061	$0.025 + 0.048i$	$0.025 - 0.048i$	1.353	0.738	0.112	−0.062
1.7	0.226	4.421	$0.025 + 0.049i$	$0.025 - 0.049i$	1.473	0.678	0.117	−0.067

TABLE 2 Numerical solutions for different values of θ ($\rho = 0.05$, $A = 1.5$, $C = 1$)

θ	L_1^*	w_1^*	$r_1(L_1^*, w_1^*)$	$r_2(L_1^*, w_1^*)$	L_2^*	w_2^*	$r_1(L_2^*, w_2^*)$	$r_2(L_2^*, w_2^*)$
0.08	0.328	3.046	$0.025 + 0.039i$	$0.025 - 0.039i$	1.171	0.853	0.092	−0.042
0.09	0.296	3.368	$0.025 + 0.042i$	$0.025 - 0.042i$	1.203	0.831	0.100	−0.050
0.10	0.271	3.686	$0.025 + 0.046i$	$0.025 - 0.046i$	1.228	0.813	0.107	−0.057
0.11	0.250	4	$0.025 + 0.049i$	$0.025 - 0.049i$	1.250	0.800	0.115	−0.065
0.12	0.231	4.311	$0.025 + 0.052i$	$0.025 - 0.052i$	1.268	0.788	0.122	−0.072

not self-enforcing. In fact, when there is only one resting point in the system of Equations (3) and (9), the insurance mechanism provided by the implicit contract becomes pointless. In a forward-looking environment, despite the constancy of labor effectiveness, the actual implementation of an agreement on hours provision and wage payments between the firms and its workers requires at least the existence of multiple equilibria. Therefore, when the condition for the uniqueness of the stationary equilibrium actually holds, the solution of the firm problem is not able to pin down meaningful out-of-equilibrium dynamics for contract hours and wages.[6]

Under the condition pointed out by Proposition 2, that is, when there are two distinct stationary solutions given by (L_1^*, w_1^*) and (L_2^*, w_2^*), analytical results are difficult to be derived. Fixing the value of ρ and relying on a computational software, however, it becomes possible to assess—for different values of A, θ, and C—the magnitude of the eigenvalues associated with the Jacobian matrix in (12)—say r_1 and r_2—for each implied stationary solution.[7] Specifically, setting the value of the discount rate according to the figure suggested for entrepreneurs by Itskhoki and Moll (2019) and considering values of A in the order of magnitude of total factor productivity (TFP) indexes provided by Solow (1957) for the postwar period, some sets of numerical solutions are collected in Tables 1–3.[8]

The numerical results in Tables 1–3 can be summarized in the following proposition:

Proposition 4. *When $A > 2\sqrt{C\rho/(\rho+\theta)}$, the stationary solution (L_1^*, w_1^*) defined in Proposition 2 is an unstable source with complex dynamics whereas (L_2^*, w_2^*) is a saddle point.*

[6] A unique stationary solution characterized by saddle-path dynamics could be obtained by assuming that the representative firm is endowed with a Cobb–Douglas production function instead of the quadratic specification in Equation (1). Given the dynamic of wages conveyed by Equation (3), however, this assumption would deliver an unrealistic acyclic equilibrium output (cf. Baily, 1974). Formal details are available from the authors upon request.

[7] All the MATLAB codes used throughout the paper are available from the authors upon request.

[8] The same value for ρ is taken by Alvarez and Shimer (2011).

TABLE 3 Numerical solutions for different values of C ($\rho = 0.05$, $A = 1.5$, $\theta = 0.10$)

C	L_1^*	w_1^*	$r_1(L_1^*, w_1^*)$	$r_2(L_1^*, w_1^*)$	L_2^*	w_2^*	$r_1(L_2^*, w_2^*)$	$r_2(L_2^*, w_2^*)$
0.8	0.206	3.881	$0.025 + 0.049i$	$0.025 - 0.049i$	1.293	0.618	0.116	−0.066
0.9	0.237	3.787	$0.025 + 0.047i$	$0.025 - 0.047i$	1.262	0.713	0.112	−0.062
1.0	0.271	3.686	$0.025 + 0.046i$	$0.025 - 0.046i$	1.228	0.813	0.107	−0.057
1.1	0.307	3.577	$0.025 + 0.044i$	$0.025 - 0.044i$	1.192	0.922	0.103	−0.053
1.2	0.346	3.459	$0.025 + 0.042i$	$0.025 - 0.042i$	1.153	1.040	0.098	−0.048

Proposition 4 reveals that when the condition for multiple stationary solutions is met, the steady state with low equilibrium hours and high equilibrium wage is unstable, whereas the steady state with high equilibrium hours and low equilibrium wage is characterized by saddle-path dynamics. This means that given an initial value for the contract wage—say $w(0) = \overline{w}_0 > 0$—there is only one trajectory that satisfies the dynamic system in (12) which converges to (L_2^*, w_2^*) while all the others diverge. In other words, the equilibrium path towards the steady state with high equilibrium hours and low equilibrium wage is locally determinate, that is, taking the contract value of $\overline{w}_0$ there is only a unique value of the initial hours—$L(0)$—in the neighborhood of L_2^* that generates a trajectory converging to (L_2^*, w_2^*), whereas all the others diverge. Strictly speaking, the value of $L(0)$ should be selected in order to verify the transversality condition in (8) by placing the system in (12) exactly on the stable branch of the saddle point (L_2^*, w_2^*). For the arguments put forward above, the fact that there is a unique optimal converging trajectory means that the dynamic wage contract described by Equation (3) is self-enforceable; indeed, all the diverging trajectories imply implosive or explosive profits for the firm and do not allow workers to achieve the insured level of consumption.

An interesting implication of Proposition 4 is that—unless the system rests in (L_1^*, w_1^*)—working hours and wages tend to converge towards (L_2^*, w_2^*), that is, the allocation that leads to higher profits with respect to (L_1^*, w_1^*). To some extent, the difference in the levels of profits achieved in these two allocations, that equals $A(L_2^* - L_1^*) - 1/2((L_2^*)^2 - (L_1^*)^2)$, can be taken as a proxy of the equilibrium reward that the firm receives for its insurance service.[9] Moreover, everything else being equal, the absolute value of the convergent root (r_2) is an increasing function (decreasing) of A and $\theta(C)$. Obviously, this means that high levels of firm's profitability as well as a strong risk-aversion for under- or overconsumption imply a quick convergence towards (L_2^*, w_2^*). By contrast, high values of the constant level of consumption granted by the underlying implicit contract delay the process of convergence.[10]

Using the baseline calibration indicated in the fourth row of Tables 1–3 and assuming that $w(0)$ is 1% below or above w_2^*, the saddle path dynamics of hours, wage and their product—which is assumed to coincide with workers' consumption stated by the implicit contract—is illustrated in the two panels of Figure 2.

The two plots in Figure 2 show that when the starting level of the wage undershoots (overshoots) its stationary reference by 1%, hours overshoot (undershoot) their long-run equilibrium value by 0.35%, whereas earnings undershoot (overshoot) their fixed contractual value by 0.65%. Thereafter, consistently with the microeconometric tests of the implicit contract theory, wages move countercyclically until (L_2^*, w_2^*) is reached. Moreover, given the absence of savings, the whole

[9] In a similar manner, if $U(C) - V(L)$ is the separable utility function of workers, then the equilibrium cost of the insurance service measured in utils amounts to $V(L_2^*) - V(L_1^*)$.

[10] In addition, it would be possible to show that firm's impatience works against convergence; indeed, the modulus of r_2 results in being a decreasing function of the value of ρ.

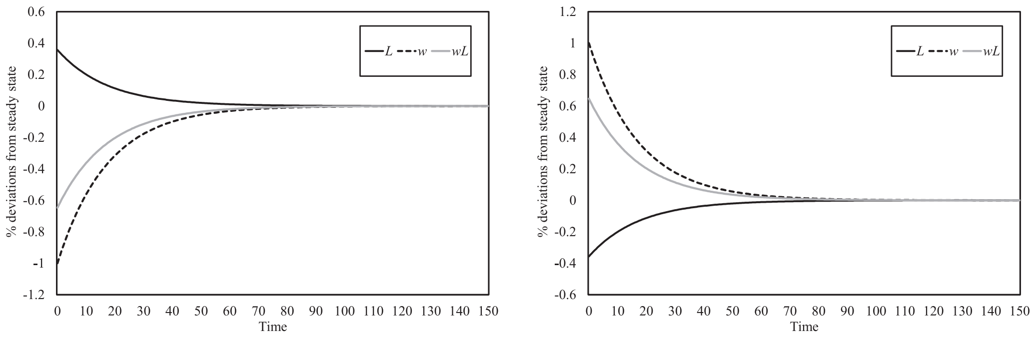


FIGURE 2 Saddle path adjustments of hours, wages, and earnings ($A = 1.5$, $\rho = 0.05$, $\theta = 0.10$, $C = 1$)

adjustment process of hours and wages is characterized by a pattern of under- or overconsumption depending on the initial value of the contract wage.

The dynamic behavior of hours and wages described above follows in a straightforward manner from the role played by the wage rate in the model economy under investigation; indeed, taking into account the insurance scheme provided to workers by the self-enforcing implicit contract, the wage does not play any allocative function, but it can be thought of as a sort of indemnity that the firm corresponds to its workers with the aim of stabilizing their consumption (cf. Barro, 1977; Hall, 1980). On the firm's side, large (small) indemnities are profitable only when its profitability is high (low) and this happens when the amount of working hours of its employees is low (high). On the workers' side, given the targeted stability of consumption, higher (lower) indemnities will be used to buy additional (sell some) leisure—which is assumed to be a normal good—by leading the insured employees to work for a lower (higher) amount of hours. In other words, consistently with wage equations run in the United States at the microlevel by controlling for labor productivity and other observable job characteristics, higher (lower) wages have only a positive (negative) income—or endowment—effect that leads workers to work less (more) (cf. Beaudry & DiNardo, 1995).

Finally, very different arguments hold when the condition indicated by Proposition 3 is met, that is, when the stationarity loci for hours and wages do not intersect with each other. In this case, the dynamics of L and w is still described by Equations (3) and (9). A stationary solution does not exist, however, and hours (wages) tend to implode (explode). Obviously, this pattern cannot be optimal since it violates the transversality condition in (8).

4 | THE STOCHASTIC ECONOMY

Now we deal with the more realistic case in which the variable that conveys the realized state of the world and the firm's profitability is not constant, but it follows instead the stochastic process enclosed in Equation (2). In this case, the firm problem becomes the following:

$$\begin{aligned} V(A_0, w_0) = \max_{\{L(t)\}_{t=0}^{\infty}} E_0 \left[\int_0^{\infty} \exp(-\rho t) \left(A(t)L(t) - \tfrac{1}{2}(L(t))^2 - w(t)L(t) \right) dt \right] \\ \text{s.to } \dot{w}(t) = \theta \left(\frac{C}{L(t)} - w(t) \right) \\ \dot{A}(t) = \kappa(\mu_A - A(t)) + \sigma_A \dot{x}(t) \\ w(0) = w_0, \quad A(0) = A_0 \end{aligned}, \tag{16}$$

where $E[\cdot]$ is the expectation operator whereas $A_0 > 0$ is the initial value of the state of the world.

Denoting by Q and S, respectively, the set in which are defined all the eligible functions for the control variable L and the set in which are defined all the eligible functions for the state variables A and w, the Hamilton–Jacobi–Bellman equation for the firm problem can be written as

$$\begin{aligned}\rho V(A_0, w_0) = \max_{L \in Q} \Big\{ & AL - \tfrac{1}{2}L^2 - wL + \theta\left(\tfrac{C}{L} - w\right)\tfrac{\partial V(A_0,w_0)}{\partial w} + \kappa(\mu_A - A)\tfrac{\partial V(A_0,w_0)}{\partial A} \\ & + \tfrac{1}{2}\sigma_A^2 \tfrac{\partial^2 V(A_0,w_0)}{\partial A^2} \Big\},\end{aligned} \tag{17}$$

where $Q \subseteq \mathbb{R}_+$ whereas $(A, w) \in S \subseteq \mathbb{R}_+^2$.

Obviously, $AL - \frac{1}{2}L^2 - wL$ will be a function defined in $S \times Q$ which returns nonnegative values.

The FOC for L requires that along an optimal path it must hold that

$$\frac{\partial V(A_0, w_0)}{\partial w} = \frac{L^2(A - L - w)}{C\theta}. \tag{18}$$

It is worth noting that the expression for $\partial V/\partial w$ in Equation (18) is equal to the expression for Λ implied by Equation (6). Moreover, the envelope condition for w is given by

$$\begin{aligned}(\rho + \theta)\tfrac{\partial V(A_0,w_0)}{\partial w} &= \theta\left(\tfrac{C}{L} - w\right)\tfrac{\partial^2 V(A_0,w_0)}{\partial w^2} \\ & + \kappa(\mu_A - A)\tfrac{\partial^2 V(A_0,w_0)}{\partial A \partial w} + \tfrac{1}{2}\sigma_A^2 \tfrac{\partial^3 V(A_0,w_0)}{\partial A^2 \partial w} + \sigma_A^2 \tfrac{\partial^2 V(A_0,w_0)}{\partial A^2}.\end{aligned} \tag{19}$$

Intuitively, since $V(\cdot)$ is concave in w, Equation (19) states that the firm optimally allocates workers' wages such that they are smoothed across states and time (cf. Wang, 2015).

Despite the simplicity of the stochastic process used to describe the evolution of firm's profitability, analytical results for the dynamics of working hours and wages may be difficult to derive. Nevertheless, the solution of the stochastic model can be retrieved by using numerical techniques aimed at approximating the value function over a given grid (cf. Kushner & Dupuis, 1992). In what follows, after the calibration of the model, we provide the output of some simulations grounded on a Markov decision chain approximation.[11]

4.1 | Calibration

The stochastic model is discretized and simulated to match the volatility of the log-deviations of U.S. GDP from its long-run level as reported by Ravn and Simonelli (2007). In other words, we calibrate the model with the aim of replicating the volatility of the observed output fluctuations. To this end, the baseline calibration indicated in the fourth row of Tables 1–3 is integrated by calibrating the stochastic process in Equation (2) in the following manner. First, the long-run mean of the stochastic process that conveys the profitability of the firms (μ_A) is set at the same value exploited for the deterministic simulations whose outcome is illustrated in Figure 2. Second, the speed of mean reversion of the profitability of the firm (κ) is fixed at the value of the convergent

[11] An extensive review of the implemented computational tool is given in the Online Appendix.

TABLE 4 Calibration

Parameter	Description	Value
C	Long-run consumption	1.000000
ρ	Discount rate	0.050000
θ	Attrition of the contract wage	0.100000
μ_A	Long-run profitability	1.500000
κ	Attrition of profitability	0.057000
σ_A	Standard deviation of profitability	0.004225

TABLE 5 Simulation results

Variable		$\overline{Y}$	$\overline{wL}$	$\overline{L}$	$\overline{w}$
Standard deviation (%)		1.56	0.57	0.92	0.69
		(1.56)	(1.01)	(0.51)	(0.86)
	$\overline{Y}$	1	0.63	0.94	−0.71
				(0.67)	(0.18)
Correlation matrix	$\overline{wL}$	–	1	0.66	−0.04
	$\overline{L}$	–	–	1	−0.76
					(0.01)
	$\overline{w}$	–	–	–	1

root implied by the baseline calibration of the deterministic model. Moreover, the volatility of the profitability of the firm (σ_A) is tuned to achieve the targeted value of the standard deviation of output.[12] The whole set of parameters, their description, and the respective values are collected in Table 4.

4.2 | Simulation results

Given the parameters' value in Table 4, the theoretical values implied by the model economy are obtained by replicating the typical steps followed in business cycles contributions (cf. Shimer, 2005). Specifically, we first generate 1200 theoretical observations. Throwing away the first 1000 in order to mitigate the possible butterfly effect, we remain with 200 observations that represent the corresponding quarterly figures of the typical 50-year horizon covered by business cycle analyses. For each variable of interest, we take the standard deviation and the correlation matrix of the log deviations from the corresponding deterministic long-run reference. Thereafter, such a procedure is repeated for 10,000 times, and theoretical values are obtained by averaging the outcomes of each replication. Defining $\overline{z}$ as $\ln z - \ln z^*$, where z^* is the stable stationary solution for the variable z, the simulation results for a set of selected variables are collected in Table 5 (observed values in parenthesis).[13]

[12] The calibration is completed by fixing $w_0 = 0.81$, $A_0 = 1.51$ and setting the simulation time-step to 0.004.

[13] Ravn and Simonelli (2007) explore U.S. quarterly data over the period 1959–2003. Within that sample period, they measure output by taking the figure of GDP in constant chained prices retrieved by the Bureau of Economic Analysis, working hours by using the average hours worked per worker in the private nonfarm sector retrieved by the Bureau of Labor Statis-

The figures in Table 5 suggest the following broad conclusions. On the one hand, the stochastic model understates the volatility of labor earnings and wages but it overstates the one of working hours. According to simulated values, earnings should be the variable with the smaller volatility while in real data the lowest dispersion around the mean is observed instead for hours. Interpreting earnings as a measure of consumption, the figure of volatility is still understated though to a lower extent; indeed, the observed standard deviation of consumption amounts to 0.86% which is definitely higher than 0.57%. An explanation for this pattern is that our theoretical framework does not account for the consumption of unemployed workers which is usually more volatile than the consumption of the employed ones (cf. Pissarides, 2004).

On the other hand, as opposed to what is shown by the deterministic model, the stochastic model displays a sound degree of real-wage stickiness; indeed, the standard deviation of simulated output is more than double with respect to the one of wages (cf. Shimer, 2005). In comparison with actual data, however, our theoretical model tends to exacerbate the cyclical correlation of working hours with respect to output. Moreover, the stochastic model replicates in a strong manner the countercyclicality of wages that also characterizes the saddle-path trajectories of the deterministic model. Operatively, the arrangement that leads to this kind of dynamics could be implemented in different ways. For instance, consider an industry characterized by some seasonality in output demand that is known by the firms and their workers. Within such an industry, in good states, that is, in periods in which there are production peaks because output demand is strong, tenured workers could be asked for some overtime work whose actual payment occurs, however, in depressed periods, that is, in bad states in which output demand is feeble.

The dynamic patterns described above straightforwardly reveals that the insurance scheme implied the dynamics of hours and wages is prima facie unable to explain the mild procyclicality of wages observed at the macrolevel even when uncertainty on firm's profitability is taken into consideration (cf. Calmès, 2007; Harris & Holmstrom, 1982). In an implicit contract economy, however, the pattern of real wages documented in Table 5 can be explained by the occurrence of composition effects driven by the flows of firms and workers that usually characterize periods of expansions and recessions (cf. Elsby et al., 2015). In detail, the new productive units that often enter the market in good states are likely to sign more expensive wage agreements because they need to attract and motivate applicants from the existing firms (cf. Fiorillo et al., 2000). By contrast, in bad states some productive units exit the market by firing their employees, and the availability of additional dismissed workers looking for a new position may allow the remaining ones to renegotiate less expensive wage agreements. More generally, the alternation of good and bad states may affect the bargaining power of workers and this is likely to have an influence on implicit (and/or explicit) wage contracts (cf. Gottfries & Sjostrom, 2000).

The actual behavior of real wages is a strongly debated issue among business cycle scholars (cf. Basu & House, 2016). Remaining on a macroeconomic ground and considering the disturbance A as a measure of the economy-wide output, the countercyclical pattern of real wages displayed by our dynamic implicit-contract model can also be used as a theoretical underpinning for a more refined empirical evidence that shows a negative response of U.S. real wages to aggregate demand shocks. Indeed, Sumner and Silver (1989) find that during periods dominated by shifts in aggregate demand, that is, years characterized by procyclical inflation rates, real wages are highly

tics and real wages by computing the ratio of nominal wages to the price deflator retrieved by the Federal Reserve Bank of Saint Louis. Very similar figures are reported also by Pourpourides (2011).

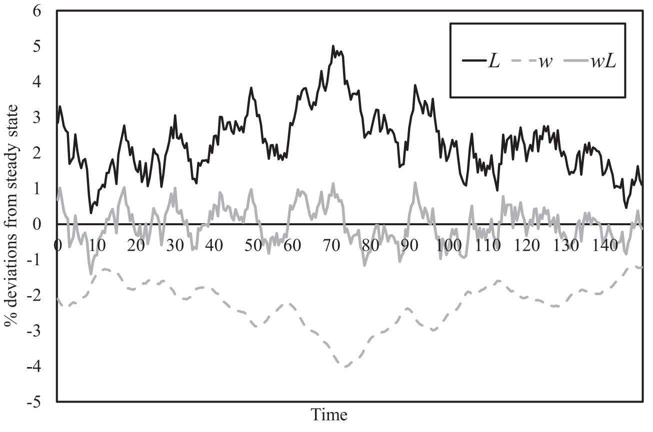


FIGURE 3 Stochastic adjustments of hours, wages, and earnings

countercyclical as predicted by a number of non-Walrasian business cycles models (cf. Neftici, 1978; Sargent, 1978). Similarly, Fleischman (1999) estimates that the correlation of real wages and U.S. output in response to aggregate demand shocks amounts to −0.49. An example of a typical trajectory of hours, wages, and labor earnings is illustrated in Figure 3.

The plot in Figure 3 clearly shows the distinct consumption smoothing operated by the dynamic implicit contract via the dynamics of labor earnings as well as the countercyclical behavior of wages; indeed, working hours (wages) are always above (below) their stable long-run references. Such a pattern reveals the existence of a strong amplification mechanism of the shocks to firm's profitability inside the stochastic model coming from the rigidity of wages.[14] Although the negative correlation between hours and wage appear at odds with the available macroevidence in which there is no specification of the kind of disturbances that hit the economy, that dynamic behavior is a direct consequence of the insurance scheme described above and is also consistent with the empirical tests on the implicit contract theory carried out with microdata on hours and wages as well as with macroeconometric assessments of wage cyclicality in response to demand shocks performed even outside the United States (cf. Bellou & Kaymak, 2012; Chiarini, 1998).

5 | CONCLUDING REMARKS

In this paper, we developed a dynamic implicit-contract model grounded on optimal control. Specifically, we explored the out-of-equilibrium dynamics of working hours and wages in a model economy where a risk-neutral representative firm endowed with a quadratic production function and its risk-averse tenured workers are linked to each other by an implicit contract that is assumed to smooth labor earnings and consumption in a long-run perspective. In detail, we built a theoretical framework in which the firm intertemporally sets its optimal level of labor utilization by taking into account that the implied wage bill tends to adjust in the direction of a fixed level that seeks to stabilizing workers' equilibrium consumption (cf. Becker, 1962; Beaudry & Pages, 2001; Blanchard & Fischer, 1989, Chapter 9; Shavell, 1976; Romer, 2019, Chapter 11).

[14] On a quantitative ground, a similar finding is obtained by Pourpourides (2011) by assuming that workers are only slightly more risk averse than entrepreneurs.

On the one hand, ignoring uncertainty in firm's profitability revealed that our theoretical setting may have one, two, or no stationary solution depending on factors traditionally related to the enforceability of contracts. The out-of-equilibrium dynamics of the deterministic economy, however, can be assessed only in the two-solution case, and it reveals that wages tend to moving in the opposite direction with respect to working hours by converging towards the allocation in which firm's profit is relatively higher than the corresponding workers' utility. This result corroborated the microeconometric evidence on the implicit contract theory obtained by regressing wages on hours by controlling for productivity (cf. Beaudry & DiNardo, 1995). Moreover, we showed that when the initial value of the contract wage falls above (below) its long-run equilibrium value, the pattern of workers' consumption is characterized by overconsumption (underconsumption).

On the other hand, adding uncertainty in firm's profitability with the aim of replicating the magnitude of observed output fluctuations revealed the potential of the model to mimick the real wage stickiness conveyed by macrodata (cf. Ravn & Simonelli, 2007; Shimer, 2005). The insurance mechanism provided by our dynamic implicit contract, however, understates the volatility of labor earnings and confirms the countercyclicality of wages observed in microdata as well as in the macroeconomic response to aggregate demand shocks detected in a number of developed countries (cf. Bellou & Kaymak, 2012; Chiarini, 1998; Fleischman, 1999; Sumner & Silver, 1989).

In the absence of any substitution effect on workers' labor provision and omitting to consider the possibility of composition effects, the failure of our model to predict a procyclical pattern of wages in response to supply shocks may also be due to the lack of adjustments on the extensive margin of the labor input. If labor mobility costs are not so high as we assumed in Section 2 and positive shocks to the effectiveness of labor lead the firm to hire additional workers, then—for a given path of contract and external wages—the marginal productivity of working hours does not necessarily move in the same direction of the effectiveness of labor because not only its vertical intercept but also its slope will be affected by the level of employment. Obviously, this may open the door to a positive comovement of hours, employment, and wages as observed in real macrodata where there is no distinction between supply and demand shocks. Furthermore, there might be too much symmetry in our model economy. For instance, it is quite likely that the firm would be willing to lower wages when earnings above long-run consumption, but it may be much more reluctant to raise them when it holds the opposite, especially in a context of incomplete information (cf. Barro, 1977). This kind of asymmetric behavior may have important cyclical consequences both on hours and wages. The implied extensions of the model, however, are left to further developments.

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SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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