

Chapter 1

Introduction

1.1 Frontier Functions and Efficiency Measurement

Neoclassical microeconomic theory specifies that a production function represents the maximum output attainable from particular quantities of inputs, given the level of technology; a cost function represents the minimum costs, given input prices and outputs; a profit function represents maximal profits, given input and output prices; and so on. However, empirical microeconomic analyses until the last three decades have been dominated by ordinary least-squares (OLS) regression and its variants, which obtain a line of best fit *through* the sample data rather than *over* the data, in the case of a production or profit function, or *under* the data, in the case of a cost function. In recent years, a body of literature has developed on the estimation of frontier functions (using either econometric or mathematical programming methods) which are more consistent with the definitions of the functions involved.

The two primary benefits of estimating frontier functions, rather than average functions, are that: (a) estimation of an average function provides a picture of the shape of the technology for an average firm, whereas the estimation of a frontier function reflects the technology of the best-performing firms and hence reflects the technology they are using, and (b) the frontier function represents a benchmark against which the efficiency of firms within the industry can be measured. It is this second use of frontiers which has provided the greatest impetus for the estimation of frontier functions in recent years.

This thesis is concerned with the estimation of frontier functions using econometric methods. This involves the estimation of econometric models known as *stochastic frontier functions*. The purpose of the thesis is to make a contribution to the specification, estimation (in particular, the computation), application and testing of stochastic frontier models. Thus this thesis may be roughly divided into four parts under these headings. Chapters 3 and 4 deal with the *specification* of two new stochastic frontiers for panel data, one dealing with time-varying inefficiency effects

and another with the incorporation of factors that may influence the values of the inefficiency effects. Chapters 5 and 6 present detailed *applications* of the second frontier model to panel data on Indian agriculture and Australian electricity generation. The *estimation* of stochastic frontiers is dealt with in Chapters 7 and 8, where the issues of computation and finite-sample properties are considered. Chapter 8 also considers the finite-sample performance of a number of *tests* of hypotheses that are regularly conducted in stochastic frontier analyses. Chapter 9 presents a summary of the major findings and suggestions for further research.

1.2 Outline of the Thesis

This thesis describes a research program which involves a collection of smaller research projects which are closely related in that they all involve the specification and/or estimation of stochastic frontier production functions. Each chapter, with the exception of Chapters 1 and 9, are closely associated with a particular research paper. At the time of submission of this thesis, papers from five chapters have been published or have been accepted for publication, while a further two papers have been submitted for publication. Furthermore, it should be noted that three of these papers were co-authored with my supervisor, Dr George Battese, while I was the sole author on the remaining four papers. I now briefly describe the contents of each chapter in turn.¹

Chapter 2 contains a review of recent developments in frontier modelling and efficiency measurement. The estimation of frontier production, cost and profit functions is discussed, along with technical, allocative, scale and overall efficiency measures relative to the estimated frontiers. The two primary methods of frontier estimation, econometric methods and mathematical programming, are discussed and compared. A survey of recent applications of frontier methods in agriculture is also provided. Much of this chapter arises from an invited paper which was presented to the 39th Annual Conference of the Australian Agricultural Economics Society. A revised version of this invited paper, Coelli (1995a), has been accepted for publication in the *Australian Journal of Agricultural Economics*.

¹ In constructing this thesis from a series of research papers, a large amount of repetitive material was removed from each of the original papers prior to inclusion in the thesis. It was not possible, however, to remove all repetitive material without seriously affecting the flow of some sections. Hence I apologise in advance for those small amounts of repetitive material that remain.

Chapter 3 describes the specification of a stochastic frontier production function for panel data in which the technical inefficiency effects are specified to be the product of a deterministic exponential function of time and time-invariant inefficiency effects associated with different firms in the panel. This model is introduced to address deficiencies of time-invariant technical inefficiency effects assumed in earlier panel data specifications. An empirical application is presented using a small data set involving Indian paddy farmers. Much of the work in Chapter 3 has been published in the *Journal of Productivity Analysis* paper, Battese and Coelli (1992).

In Chapter 4 a stochastic frontier production model is specified for panel data in which the technical inefficiency effects are permitted to be a function of firm-specific variables and time. This is an extension of the cross-sectional models, proposed by Kumbhakar, Ghosh and McGuckin (1991) and Reifschneider and Stevenson (1991), to accommodate panel data and hence time-varying inefficiency effects. The model is illustrated using the same data used in the empirical application in Chapter 3. The empirical application in Chapter 4 considers the age and education of the farmers as possible factors influencing the levels of technical inefficiency effects. The work in this chapter is closely related to the paper, Battese and Coelli (1995), which appears in *Empirical Economics*.

Chapter 5 involves an additional application of the method proposed in Chapter 4. This application involves the full set of panel data obtained from the Village Level Studies (VLS) conducted by the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) in India. This data set involves over 30 sample farmers from each of three Indian villages, observed over a ten-year period. In this study, farmer age and education, along with farm size, are considered as possible factors influencing the technical inefficiency effects of the farmers. Much of this work is also discussed in the paper, Coelli and Battese (1994), which has been revised and recently submitted for publication in the *Australian Journal of Agricultural Economics*.

Chapter 6 contains a third application of the model specified in Chapter 4. The application involves ten years of annual data on 13 coal-fired electric power plants from three Australian States: New South Wales, Victoria and Western Australia. In this application, capacity factor, unit size, plant vintage and coal quality are considered

as possible factors influencing plant inefficiency. The work in this chapter is summarised in Coelli (1995b), a report to the Australian Electricity Supply Industry Research Board (AESIRB). This report has also been submitted for publication in *Energy Economics*.

Chapter 7 involves a description of the computer program, FRONTIER Version 4.1, which has been written to estimate the models outlined in Chapters 3 and 4. This program can also be used to estimate a number of other model specifications, including cross-sectional models and cost functions. This program is presently in use in over 240 institutions around the world. The paper, Coelli (1992), published in *Economics Letters*, describes Version 2.0 of the program, which was written to estimate the model specified in Chapter 3.

The finite-sample properties of estimators for parameters of a stochastic frontier production function are investigated in Chapter 8. The relative performance of the maximum-likelihood (ML) and corrected ordinary least-squares (COLS) estimators are investigated, together with five alternative test statistics, using Monte Carlo methods. The analysis is limited to the cross-sectional stochastic production frontier, in which the technical inefficiency effects have half-normal distribution. This specification has been the most commonly assumed frontier model in empirical applications to date. Much of the work in this chapter is included in the paper, Coelli (1995c), which appears in the *Journal of Productivity Analysis*.

The final chapter provides a brief summary of the main results of the thesis, and also suggests some areas of research which are worthy of further attention.

Chapter 2

Literature Review

2.1 Introduction

This chapter surveys recent developments in the estimation of frontier functions and the measurement of efficiency. Frontier production, cost and profit functions are discussed, along with the construction of technical, allocative, scale and overall efficiency measures relative to the corresponding frontiers. The two primary approaches to the estimation of frontier functions, econometric methods and mathematical programming, are discussed and compared. A survey of recent applications of frontier methods in agriculture is also provided, along with some discussion of the potential applicability of these methods in agricultural economics.

We begin this section with clarifications of the definitions of the terms, *efficiency* and *productivity*. These terms have been used regularly in the Australian media over the last ten years by a variety of commentators. It is unfortunate that they have often been used interchangeably, although they are not precisely the same things. To illustrate the distinction between the two terms, it is useful to picture a production frontier which defines the current state of technology in an industry. Firms in that industry would presently be operating either on that frontier, if they are perfectly efficient, or below the frontier if they are not fully efficient. *Productivity* improvements can be achieved in one of two ways. One can either improve the state of the technology, for example by inventing new ploughs, pesticides, rotation plans, etc. This is commonly referred to as *technical change* and can be represented by an upward shift in the production frontier. Alternatively one can implement procedures, such as improved farmer education, to ensure farmers use the existing technology more *efficiently*. This would be represented by the firms operating more closely to the existing frontier. It is thus evident that productivity growth may be achieved through either technical progress or efficiency improvement, and that the policies required to address these two issues are likely to be quite different. This thesis concentrates upon the issue of efficiency

measurement. Issues relating to the measurement of technical change and overall productivity growth are not considered in detail.

If all that is required is a measure of efficiency, some people may ask: “why bother with fancy frontier estimators?” For example, in the case of agricultural production, what is wrong with using tonnes of wheat per hectare or litres of milk per cow as measures of farmer efficiency? Measures such as tonnes per hectare have a serious deficiency, in that they only consider the land input and ignore all other inputs, such as labour, machinery, fuel, fertiliser, pesticide, etc. The use of this measure in the formulation of management and policy advice is likely to result in excessive use of those inputs which are not included in the efficiency measure. Similar problems occur when other simple measures of efficiency, such as output per unit labour and output per unit of capital, are used.

A variety of efficiency measures have been proposed which are able to accommodate more than one factor of production. The primary purpose of Chapter 2 is to outline some of these measures and to discuss how they may be calculated relative to an efficient technology, which is generally represented by some form of frontier function. A key part of this exposition is a discussion of the two primary methods of frontier estimation, namely *stochastic frontiers* and *data envelopment analysis* (DEA), which involve econometric methods and mathematical programming, respectively. The discussion also considers multiple-output technologies and ways of accounting for a variety of behavioural objectives, such as cost minimisation and profit maximisation, through the estimation of cost and profit frontiers.

The plan of the chapter is as follows. Section 2.2 provides a brief history of modern efficiency measurement, beginning with the seminal paper by Farrell (1957). Recent developments in frontier modelling and efficiency measurement in the econometric and mathematical programming fields, are described in Sections 2.3 and 2.4, respectively. Section 2.5 provides a brief review of some frontiers applications in the agricultural economics literature, and the final section concludes and discusses some potential applications to Australian agriculture.

2.2 Early Literature

The following discussion of literature on frontier modelling and efficiency measurement is neither exhaustive nor rigorous. The purpose of this chapter is to provide an introduction to the field and a summary of the major concepts and results, which is not burdened by excessive notation and technical detail. More detailed reviews include: Førsund, Lovell and Schmidt (1980), Schmidt (1986), Bauer (1990), Seiford and Thrall (1990), Lovell (1993), Greene (1993) and Ali and Seiford (1993).

This discussion of the recent history of efficiency measurement begins with Farrell (1957) who drew upon the work of Debreu (1951) and Koopmans (1951) to define a simple measure of firm efficiency involving multiple inputs. Farrell (1957) proposed that the efficiency of a firm consists of two components: *technical efficiency*, which reflects the ability of a firm to obtain maximal output from a given set of inputs, and *allocative efficiency*, which reflects the ability of a firm to use the inputs in optimal proportions, given their respective prices. These two measures are then combined to provide a measure of total *economic efficiency*.¹

Farrell illustrated his ideas using a simple example involving firms which use two inputs (x_1 and x_2) to produce a single output (y), under an assumption of constant returns to scale.² Knowledge of the unit isoquant associated with *fully efficient firms*,³ represented by SS' in Figure 2.1, permits the measurement of technical efficiency. If a given firm uses quantities of inputs, defined by the point P, to produce a unit of output, the technical efficiency of that firm is defined to be the ratio, OQ/OP , which is the proportional reduction in all inputs that could theoretically be achieved without any reduction in output. Note that the point Q is technically efficient because it lies on the unit isoquant.

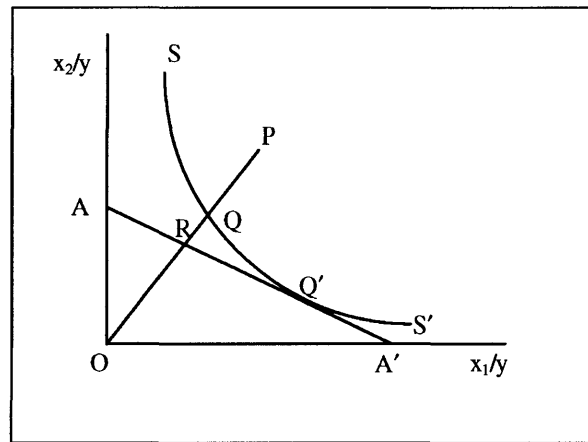
¹ Some of Farrell's terms differ from those which are used here. He used the term *price efficiency* instead of *allocative efficiency* and the term *overall efficiency* instead of *economic efficiency*. The terminology used in this thesis conforms with that which has been used most often in recent literature.

² Farrell also discussed the extension of his method to accommodate more than two inputs, multiple outputs, and non-constant returns to scale.

³ The production function of fully efficient firms is not known in practice, and thus must be estimated from observations on a sample of firms in the industry concerned. The selection of an appropriate method of estimation is the subject of considerable discussion later in this chapter.

Figure 2.1

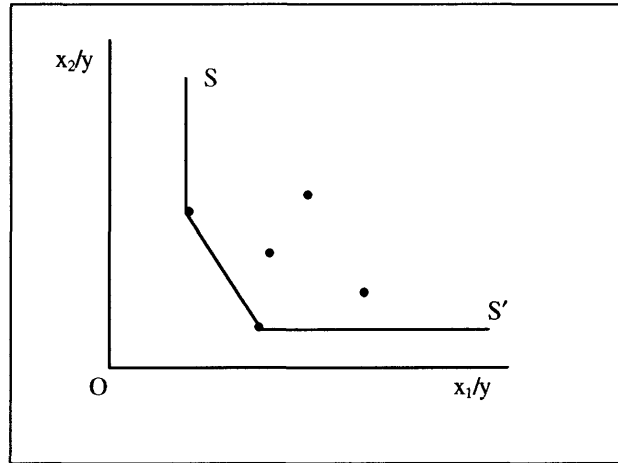
Technical and Allocative Efficiencies



If the input price ratio, represented by the line AA' in Figure 2.1, is also known, allocative efficiency may also be defined. The allocative efficiency of the firm operating at P is defined to be the ratio, OR/OQ , since the distance RQ represents the reduction in production costs that would occur if production were to occur at the allocatively (and technically) efficient point Q' , instead of at the technically efficient, but allocatively inefficient, point Q . The total *economic efficiency* is defined to be the ratio, OR/OP , where the distance RP can also be interpreted in terms of a cost reduction. Note that the product of the technical and allocative efficiencies provides the overall efficiency, $(OQ/OP)(OR/OQ)=(OR/OP)$, and all three measures are bounded by zero and one.

These efficiency measures assume that the production function of the fully efficient firms is known. In practice this is not the case and the efficient isoquant must be estimated from sample data. Farrell (1957) suggested the use of either (a) a non-parametric, piecewise-linear convex isoquant constructed such that no observed point lies to the left or below it (see Figure 2.2), or (b) a parametric function, such as the Cobb-Douglas form, fitted to the data so that no observed point lies to the left or below it. Farrell provided an illustration of his methods using agricultural data for the 48 continental States of the USA.

Figure 2.2
Piecewise-linear Convex Isoquant



The work of Farrell was subsequently adjusted and extended by several other authors. Aigner and Chu (1968) considered the estimation of a parametric frontier production function in input/output space. They specified a Cobb-Douglas production function (in log form) for a sample of N firms as

$$\ln(Y_i) = F(x_i; \beta) - U_i \quad , \quad i=1,2,\dots,N, \quad (2.1)$$

where Y_i is the output of the i -th firm; x_i is $K \times 1$ vector of input quantities used by the i -th firm; $F(\cdot)$ denotes a suitable functional form (in this case the Cobb-Douglas in logarithmic form); β is a vector of unknown parameters to be estimated; and U_i is a non-negative variable representing inefficiency in production.⁴ The β -parameters of the frontier function were estimated using linear programming, where $\sum_{i=1}^N U_i$ is minimised, subject to the constraints that $U_i \geq 0, i=1,2,\dots,N$.⁵

The ratio of observed output of the i -th firm, relative to the potential output defined by the estimated frontier, given the input vector x_i , can be used as an estimate of the technical efficiency of the i -th firm:

$$TE_i = Y_i / \exp[F(x_i; \beta)] = \exp(-U_i). \quad (2.2)$$

⁴ Note that Aigner and Chu (1968) used different notation to that used here.

⁵ Aigner and Chu (1968) also suggested the use of quadratic programming methods.

This is an output-orientated measure of technical efficiency as opposed to the input-oriented measure discussed above. It indicates the magnitude of the output of the i -th firm relative to the output that could be produced by a fully-efficient firm using the same input vector. The output- and input-orientated measures provide equivalent measures of technical efficiency when constant returns to scale exist, but are unequal when increasing or decreasing returns to scale are present (Färe and Lovell, 1978).

Afriat (1972) specified a model similar to that defined by equation (2.1), except that the U_i were assumed to have a gamma distribution and the parameters of the model were estimated using the maximum-likelihood (ML) method. Richmond (1974) noted that the parameters of Afriat's model could also be estimated using a method that has become known as corrected ordinary least-squares (COLS). The ordinary least-squares (OLS) method provides unbiased estimators of the slope parameters, but the OLS estimator for the intercept parameter has a negative bias. Hence Richmond (1974) suggested that the intercept be adjusted up using the sample moments of the error distribution, obtained from the OLS residuals. Schmidt (1976) added to the discussion on ML frontiers by observing that the linear and quadratic programming methods, proposed by Aigner and Chu (1968), yield ML estimates if the U_i were assumed to be distributed as exponential or half-normal random variables, respectively.

The two primary criticisms of the above *deterministic*⁶ frontier models are: (i) that no standard errors or tests of hypotheses based upon traditional asymptotic theory are available because the range of the dependent variable is dependent upon the parameters to be estimated (see Schmidt 1976); and (ii) no account is taken of the possible influence of measurement errors and other noise upon the shape and positioning of the frontier, since all deviations from the frontier are assumed to be the result of technical inefficiency. This latter point implies that the estimation of deterministic production frontiers may be quite sensitive to the influence of outliers. To address this issue, Timmer (1971) took up the suggestion of Aigner and Chu (1968) of permitting a percentage of observations to lie above the estimated frontier. This was done by re-estimating the frontier using a reduced sample. What Timmer (1971) called the

⁶ The term *deterministic* is used for the frontier because the (logarithm of) production is bounded above by a deterministic function of the input values. Thus the parametric form for the production frontier has a one-sided error term. The work of Aigner and Chu (1968), Afriat (1972) and Schmidt (1976) are examples of deterministic frontier models.

probabilistic frontier approach has not been widely followed because of the arbitrary nature of the selection of a percentage of observations to omit. An alternative approach to the solution of the ‘noise’ problem has, however, been widely adopted. This approach is the subject of the following section on stochastic frontiers.

2.3 Stochastic Frontiers

2.3.1 Stochastic Frontier Production Functions

Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) independently proposed the stochastic frontier production function, defined by

$$\ln(Y_i) = F(x_i; \beta) + V_i - U_i \quad , i=1,2,\dots,N, \quad (2.3)$$

where V_i is a symmetric error term (having zero mean and constant variance) which accounts for statistical noise in production. This noise is generally assumed to comprise measurement error in production and the combined influence of other variables unaccounted for in the model. Aigner, Lovell and Schmidt (1977) assumed that V_i has normal distribution and U_i had either half-normal or exponential distribution, whereas Meeusen and van den Broeck (1977) assumed that U_i had only exponential distribution. In both of these papers it was suggested that the parameters of this model be estimated by the ML method.

This model specification was referred to as a *stochastic* frontier because the (logarithm of) production is bounded from above by the stochastic term, $F(x_i; \beta) + V_i$. This stochastic frontier not only accounted for noise in production, but also permitted the estimation of standard errors and tests of hypotheses, which were not possible with the earlier deterministic models because of the violation of the above mentioned ML regularity condition.⁷

The stochastic frontier is not, however, without problems. The main criticism is that there is no *a priori* justification for the selection of any particular distributional form for the U_i . The specification of more general distributional forms, such as the truncated-normal (Stevenson 1980) and the two-parameter gamma (Greene 1990), has

⁷ Greene (1980a) observed that a particular class of distributions could be assumed for the U_i which would circumvent these regularity problems. The noise criticism, however, would still remain.

partially alleviated this problem, but the resulting efficiency measures may still be sensitive to distributional assumptions.

2.3.2 Estimation Methods

Stochastic frontier production functions can be estimated using either the ML method or using a variant of the COLS method suggested by Richmond (1974). The COLS approach could be preferred because it is not as computationally demanding as ML which requires numerical solution of the likelihood function. This distinction, however, has lessened over the past five years with the availability of software such as the LIMDEP econometrics package (Greene 1992) and the FRONTIER program (Coelli 1992, 1994), both of which automate the ML method.

The ML estimator is asymptotically more efficient than the COLS estimator, but the properties of the two estimators in finite samples cannot be analytically determined. The finite-sample properties of the half-normal frontier model are investigated using Monte Carlo methods by Olsen, Schmidt and Waldman (1980). No significant differences in the efficiencies of the two estimators are observed but it is suggested that the COLS estimator may be preferred in sample sizes smaller than 400. A more recent study by Coelli (1995c), involving substantially more replications, finds the ML estimator to significantly outperform the COLS estimator when the contribution of the variance of the inefficiency error to the sum of the two error variances is large.⁸ Given this result and the availability of automated ML routines, the ML estimator should be used in preference to the COLS estimator whenever possible.

2.3.3 Alternative Functional Forms

The Cobb-Douglas functional form has been most commonly used in the empirical estimation of frontier models. Its simplicity is a most attractive feature. A logarithmic transformation provides a model which is linear in the logarithms of the inputs and hence easily lends itself to econometric estimation. This simplicity, however, is associated with a number of restrictive properties. Most notably, that the returns-to-

⁸ Coelli (1995c) considered 11 different values of the proportion of error variance due to the inefficiency error (denoted by γ^*) ranging from 0.0 to 1.0, in steps of 0.1, for sample sizes of N=50, 100, 400, and 800. Of these 44 cases, 14 show the mean squared error of ML to be significantly smaller (at the 1% level) than that of COLS, while in only one case ($\gamma^*=0.1$, N=50) did the converse occur. This analysis is discussed in detail in Chapter 8.

scale parameter has the same value across all firms in the sample, and the elasticities of substitution are equal to one.

A variety of alternative functional forms have also been used in the frontier literature. The two most popular forms are the translog (e.g., Greene 1980b) and the Zellner and Revankar (1969) generalised production function (e.g., Førsund and Hjalmarsson 1979 and Kumbhakar, Ghosh and McGuckin 1991). The Zellner-Revankar form removes the returns-to-scale restrictions, while the translog form imposes no restrictions upon the returns-to-scale or substitution possibilities, but multicollinearity is likely to be a problem and the degrees of freedom for the errors are significantly reduced. These problems can be avoided by jointly estimating the translog production function with the first-order conditions for profit maximisation, as suggested by Greene (1980b). This systems approach, however, will increase the complexity of the estimation process.

2.3.4 Dual Forms of the Technology

The discussion above concentrates upon the direct estimation of frontier production functions using single-equation methods. The three main reasons for the consideration of alternative dual forms of the production technology, such as the cost or profit function, are to: (a) reflect alternative behavioural objectives (such as cost minimisation); (b) account for multiple outputs; and (c) simultaneously predict both technical and allocative efficiency.

The direct estimation of a production function produces biased and inconsistent estimators of the parameters if the standard behavioural objectives of either profit maximisation or cost minimisation apply. This is because the input levels are not independent of the error term and hence simultaneous equation bias results. Direct estimation of the production function is justified if it is appropriate to assume either: (a) that the input levels are fixed and that the managers of the firms are attempting to maximise output given the input quantities; or (b) that the managers are selecting the levels of the inputs and the output to maximise expected (rather than actual) profits, as discussed in Zellner, Kmenta and Drèze (1966).

Given that both output prices and output quantities are rarely known with certainty when farmers make production decisions (such as to plant additional wheat or to buy

additional sheep), the assumption of expected (rather than actual) profit maximisation is the assumption which is most commonly made in production studies involving agriculture. However, in some instances the assumption of a cost minimisation objective may be more appropriate. For example, consider the case of dairy farms which are contracted to produce particular levels of output in a given year. In such cases it may be more appropriate to estimate a stochastic cost frontier of the form:

$$\ln(C_i) = C(y_i, w_i; \alpha) + V_i + U_i \quad , i=1,2,\dots,N, \quad (2.4)$$

where C_i is the observed cost for the i -th firm;

$C(\cdot)$ is a suitable functional form for the cost function;

w_i is a vector of (exogenous) input prices for the i -th firm;

α is a vector of unknown parameters to be estimated;

U_i is a non-negative random variable reflecting cost inefficiency (which is often assumed to have half-normal distribution); and

and all other variables are as defined above.

The parameters of this model can be estimated using standard econometric methods since the y_i and w_i are assumed to be exogenously determined. Schmidt and Lovell (1979) specify a Cobb-Douglas technology for steam-electric generating plants and show how the cost function can be estimated in a similar manner to the estimation of stochastic production frontiers using ML or COLS estimation. They also suggest the use of a ML systems estimator involving the cost function and K-1 factor demand equations, which provide more efficient estimators than the single-equation estimators. This systems approach also has the advantage of explicitly accounting for allocative inefficiency, which is reflected in the error terms on the factor demand equations (which represent violations of the first-order conditions for cost minimisation).

The cost-frontier approach appears to be a significant improvement in that it accounts for exogenous output and endogenous inputs, permits the measurement of technical and allocative inefficiency, and can be extended to account for multiple outputs. However, it suffers from two serious drawbacks. First, the cost-frontier approach requires data on input prices which vary among firms. In many cases, firms in an

industry either face the same prices, or, if they do not face the same prices, it is difficult to collect the relevant data on the prices.

Second, the approach of Schmidt and Lovell (1979) to systems estimation and the measurement of technical and allocative efficiencies is limited to the use of self-dual functional forms, such as the Cobb-Douglas. The specification of more flexible functional forms, which are not self-dual, such as the translog form (see Greene 1980b), results in a number of problems. The main problem is associated with selecting an appropriate way to represent the link between the allocative inefficiency errors in the input demand equations, and the allocative inefficiency error which appears in the cost function. To date, no one has solved this problem to the satisfaction of the majority, and debate continues as to how to best address these issues [see Bauer (1990) and Greene (1993) for further discussion and references]. My advice to applied economists is to avoid flexible systems estimators. If one of the existing approaches is applied [e.g., Greene (1980b) or Ferrier and Lovell (1990)] then criticism from some quarter is likely and, furthermore, estimation problems often arise when one tries to numerically maximise the rather complicated likelihood functions that are involved. The best approach to take (given that the cost minimising assumption is appropriate and suitable price data are available) is to estimate a cost function using the single equation ML method (which is automated in LIMDEP and FRONTIER) and use the method proposed by Kopp and Diewert (1982), and refined by Zeischang (1983), to decompose the cost efficiencies into their technical and allocative components. If the Cobb-Douglas functional form is considered appropriate, then the procedures involved simplify to those which are outlined in Schmidt and Lovell (1979).⁹

This section focuses upon cost functions because cost minimisation is the assumption that is most often made in the dual frontier literature. Profit maximisation has also been considered by a number of authors, and may be considered to be the more appropriate assumption in many Australian agricultural industries. Examples of frontier studies which assume profit maximisation include Ali and Flinn (1989) and

⁹ A recent paper by Kumbhakar (1996) has, for the case of a translog cost function, successfully derived an exact relationship between allocative efficiency in the share equations and the cost function. An application of this model is yet to be attempted.

Kumbhakar, Ghosh and McGuckin (1991). Ali and Flinn (1989) consider a single-equation profit frontier which is estimated using the same methods as appropriate for production frontiers, whereas Kumbhakar, Ghosh and McGuckin (1991), specify a ML systems estimator under the assumption of profit maximisation.

2.3.5 Panel Data

The previous discussion assumes that data on N firms, observed at one point in time, are available for use in the estimation of the frontier function. If data on N firms are observed in each of T different time periods, then this is what is known as *panel data*. Panel data have many potential advantages over a single cross-section of data for the estimation of frontier functions. It increases the precision of estimation of the parameters; provides consistent estimators of firm efficiencies (given sufficiently large T); removes the necessity to make specific distributional assumptions regarding the U_i ; does not require that the inefficiency effects are independent of the explanatory variables; and permits the simultaneous investigation of both technical change and technical efficiency change over time.

Pitt and Lee (1981) specified a panel data version of the Aigner, Lovell and Schmidt (1977) half-normal model:

$$\ln(Y_{it}) = F(x_{it};\beta) + V_{it} - U_{it} \quad , i=1,2,\dots,N; t=1,2,\dots,T, \quad (2.5)$$

where Y_{it} is the output of the i -th firm in the t -th time period; and the other variables are similarly defined. Pitt and Lee (1981) specify a variety of models involving different assumptions about the random errors, V_{it} , and the technical inefficiency effects, U_{it} . If the V_{it} are assumed to be i.i.d. $N(0, \sigma_v^2)$, independent of the U_{it} , which in turn are assumed to be i.i.d. half-normal, then the panel data model (2.5) is not essentially different from the cross-sectional model. Pitt and Lee (1981) also considered a model in which the inefficiency effects are constant through time:

$$\ln(Y_{it}) = F(x_{it};\beta) + V_{it} - U_i \quad , i=1,2,\dots,N; t=1,2,\dots,T. \quad (2.6)$$

Battese and Coelli (1988) extended this latter model to permit the U_i to have the more general truncated normal distribution, proposed by Stevenson (1980), and also derived panel data generalisations of the predictor for the technical inefficiency effects given by

Jondrow et al (1982). Battese, Coelli and Colby (1989) further extended the model to account for unbalanced panel data. These last three models have the advantage of providing consistent estimators of the U_i , as T becomes large. However, as T becomes large the assumption that the technical inefficiency effects, U_{it} , are time-invariant is more difficult to justify. One would expect managers to learn from previous experience.

Kumbhakar (1990) suggested a stochastic frontier for panel data in which the inefficiency effects are permitted to vary systematically with time. His model is similar to equation (2.5) with the U_{it} assumed to have structure defined by

$$U_{it} = [1 + \exp(bt + ct^2)]^{-1} U_i \quad (2.7)$$

where U_i is assumed to have half-normal distribution and b and c are parameters to be estimated. Kumbhakar (1990) suggested that the parameters of the model be estimated using the ML method but no empirical application has yet been attempted. Battese and Coelli (1992) (see Chapter 3) suggested an alternative to the Kumbhakar (1990) model, in which the U_{it} are assumed to be the product of an exponential function of time, involving only one parameter, and a time-invariant technical inefficiency effect, as follows:

$$U_{it} = \{ \exp[-\eta(t-T)] \} U_i \quad (2.8)$$

where U_i is assumed to have truncated normal distribution and η is a parameter to be estimated. The model is illustrated in an application involving data on Indian paddy farmers. The ML estimation method and efficiency calculations have been automated in the FRONTIER program (see Coelli 1994 and Chapter 7). One advantage of these latter two model specifications is that the inclusion of a time trend into the production function $F(\cdot)$ permits the estimation of both technical change and changes in the technical inefficiencies over time.¹⁰

Schmidt and Sickles (1984) noted that when panel data are available there is no need to specify an explicit distribution for the inefficiency effects. They suggested estimating a model, in which the technical inefficiency effects are time-invariant, using

¹⁰ It should be kept in mind, however, that the identification of these two effects hinges upon the distributional assumptions made regarding the U_i . If the U_i are not stochastic, then the two effects can not be individually identified.

the traditional fixed effects (dummy variables) approach or error-components estimation, depending upon what assumptions are judged appropriate regarding the independence of the inefficiencies and the explanatory variables.¹¹ The firm intercepts are then adjusted so that all firm effects are zero or negative, so that measures of the efficiencies of the firms can be obtained. One criticism that can be levelled at this approach, and all other COLS-type methods, is that the average firms are having the greatest influence upon the shape of the estimated frontier, while ML estimation of stochastic frontiers allows the more efficient firms to have a greater influence upon the shape of the estimated frontier.

The approach of Schmidt and Sickles (1984) was extended by Cornwell, Schmidt and Sickles (1990) and Lee and Schmidt (1993) to account for time-varying technical inefficiencies. Both papers suggested models for temporal variations which are more flexible than the formulations defined in equations (2.7) and (2.8). However, it should be stressed that both approaches rely upon the estimation of an average function and the use of a COLS-type intercept adjustment to identify the location of the frontier in each year.

2.3.6 Determinants of Inefficiency

A number of empirical studies (e.g., Pitt and Lee 1981 and Kalirajan 1981) have investigated the determinants of technical inefficiency variation among firms in an industry by regressing the predicted technical inefficiency effects, obtained from an estimated stochastic frontier, upon a vector of firm-specific factors, such as firm size, age and education of manager, etc. in a second-stage regression. There is, however, a significant problem with this two-stage approach. In the first stage, the inefficiency effects are assumed to be independently and identically distributed, while in the second stage they are assumed to be a function of a number of firm-specific factors which implies that they are *not* identically distributed.

Recent papers by Kumbhakar, Ghosh and McGuckin (1991) and Reifschneider and Stevenson (1991) noted this inconsistency and specify stochastic frontier models in

¹¹ The fixed-effects approach permits the explanatory variables and inefficiency effects to be correlated while the error-components method assumes independence, as does the ML estimation of the stochastic frontier models, associated with equations (2.5) and (2.6).

which the inefficiency effects are made an explicit function of the firm-specific factors, and all parameters are estimated in a single-stage ML procedure. Battese and Coelli (1995) extend this approach to accommodate panel data, which permits the simultaneous investigation of both the determinants of technical inefficiencies, along with the degree of technical efficiency change and technical change over time. This model is discussed in detail in Chapter 4.

2.4 Data Envelopment Analysis

It is evident from the discussion in Section 2.3 that stochastic frontier methods have developed a great deal over the past two decades. During this period, a separate literature on the non-parametric mathematical programming approach to frontier estimation, known as *data envelopment analysis (DEA)*, has also been developing, almost independently of the stochastic frontier literature.

Only a small percentage of agricultural frontier applications have used the DEA approach to frontier estimation. This is, in one sense, surprising, given the popularity of mathematical programming methods in other areas of agricultural economics research during the 1960s and 70s. However, DEA has a very large following in other professions, especially in management science, and in applications to service industries where there are multiple outputs, such as banking, health, telecommunications and electricity distribution. The DEA approach suffers from the same criticism as the deterministic methods discussed in Section 2.2, in that it takes no account of the possible influence of measurement error and other noise in the data. On the other hand, it has the advantage of removing the necessity to make arbitrary assumptions regarding the functional form of the frontier and the distributional form of the inefficiency effects.

The review of DEA models presented here is brief, with relatively little technical detail. More detailed reviews of the methodology are presented by Seiford and Thrall (1990) and Ali and Seiford (1993).

The piecewise-linear convex hull approach to frontier estimation proposed by Farrell (1957) was considered by only a handful of papers (e.g., Sietz 1971) until Charnes, Cooper and Rhodes (1978) reformulated the approach into a mathematical

programming problem and coined the term *data envelopment analysis* (DEA). There has since been a large number of papers which have extended and applied the DEA methodology.

Charnes, Cooper and Rhodes (1978) proposed a model which had an input orientation and assumed constant returns to scale (CRS). Subsequent papers have considered alternative sets of assumptions, such as Banker, Charnes and Cooper (1984) which proposed a variable returns-to-scale (VRS) model. The following discussion of DEA begins with a description of the input-orientated CRS model in Section 2.4.1, because this model was the first DEA model to be widely used in empirical applications.

2.4.1 The Constant Returns-to-scale (CRS) Model

We begin by defining some notation. Assume there are data on K inputs and M outputs for each of N firms. For the i -th firm these are represented by the vectors, x_i and y_i , respectively. The $K \times N$ input matrix, X , and the $M \times N$ output matrix, Y , represent the data for all N firms. The purpose of DEA is to construct a non-parametric envelopment frontier over the data points such that all observed points lie on or below the production frontier. For the simple example of an industry where one output is produced using two inputs, it can be visualised as a number of intersecting planes forming a tight fitting cover over a scatter of points in three-dimensional space. Given the CRS assumption, this can also be represented by a unit isoquant in input/input space (see Figure 2.2 above).

The best way to introduce DEA is via the *ratio* form. For each firm we would like to obtain a measure of the ratio of all outputs over all inputs, such as $u'y_i/v'x_i$, where u is an $M \times 1$ vector of output weights and v is a $K \times 1$ vector of input weights. To select optimal weights we specify the mathematical programming problem:

$$\begin{aligned} & \max_{u,v} (u'y_i/v'x_i), \\ & \text{st} \quad u'y_j/v'x_j \leq 1, \quad j=1,2,\dots,N, \\ & \quad u, v \geq 0. \end{aligned} \tag{2.9}$$

This involves finding values for the vectors u and v , such that the efficiency measure of the i -th firm is maximised, subject to the constraint that all efficiency measures must be

less than or equal to one. One problem with this particular ratio formulation is that it has an infinite number of solutions.¹² To avoid this one can impose the constraint, $v'x_i = 1$, which provides:

$$\begin{aligned}
 & \max_{\mu, v} (\mu' y_i), \\
 \text{st} \quad & v' x_i = 1, \\
 & \mu' y_j - v' x_j \leq 0, j=1, 2, \dots, N, \\
 & \mu, v \geq 0,
 \end{aligned} \tag{2.10}$$

where the notational change from u and v to μ and v reflects the transformation. This form is known as the *multiplier* form of the linear programming problem.

Using the duality properties in linear programming, one can derive an equivalent *envelopment* form of this problem:

$$\begin{aligned}
 & \min_{\theta, \lambda} \theta, \\
 \text{st} \quad & -y_i + Y\lambda \geq 0, \\
 & \theta x_i - X\lambda \geq 0, \\
 & \lambda \geq 0,
 \end{aligned} \tag{2.11}$$

where θ is a scalar and λ is an $N \times 1$ vector of constants. This envelopment form involves fewer constraints than the multiplier form, and hence is generally the preferred form to solve.¹³ The value of θ obtained is the efficiency score for the i -th firm. It will satisfy, $\theta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient firm, according to the Farrell (1957) definition. Note that the linear programming problem must be solved N times, once for each firm in the sample. A value of θ is then obtained for each firm.

The piecewise-linear form of the non-parametric frontier in DEA can cause a few difficulties in efficiency measurement. The problem arises because of the sections of

¹² That is, if (u^*, v^*) is a solution, then $(\alpha u^*, \alpha v^*)$ is another solution, etc.

¹³ The forms defined by equations (2.9) and (2.10) are introduced for expository purposes. They are not used again in the remainder of this paper. These forms are, however, used in a number of empirical studies. The u and v weights may be interpreted as vectors of normalised shadow prices.

the piecewise-linear frontier which run parallel to the axes (see Figure 2.2 above) which do not occur in most parametric frontiers (see Figure 2.1 above). Figure 2.3 is useful to illustrate the problem. The firms using input combinations at points C and D, are the two efficient firms which define the frontier. Firms A and B are inefficient firms. The Farrell (1957) measure of technical efficiency gives the efficiency of firms A and B as OA'/OA and OB'/OB , respectively. However, it is questionable as to whether the point A' is an efficient point since one could reduce the amount of input x_2 used (by the amount CA') and still produce the same output. This is generally referred to as *input slack*.¹⁴ Once one considers a multiple-output situation, the diagrams are no longer as simple and the possibility of the related concept of *output slack* also occurs. Thus it could be argued that both the Farrell measure of technical efficiency (θ) and any non-zero input or output slacks should be reported to provide an accurate indication of technical efficiency of a firm in a DEA analysis.¹⁵ Note that for the i -th firm the output slacks will be equal to zero only if $Y\lambda - y_i = 0$, while the input slacks will be equal to zero only if $\theta x_i - X\lambda = 0$ (for the given optimal values of θ and λ).

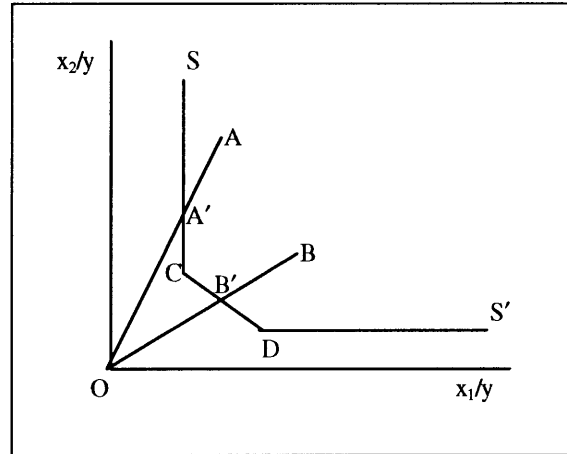
In Figure 2.3 the input slack associated with the point A' is CA' of input x_2 . In cases when there are more inputs and outputs than considered in this simple example, the identification of the nearest efficient frontier point (such as C), and hence the subsequent calculation of slacks, is not a trivial task. Some authors (see Ali and Seiford 1993) have suggested the solution of a second-stage linear programming problem to identify the nearest efficient frontier point, where 'nearest' is defined in terms of the minimum sum of slacks required to move from an inefficient frontier point (such as A' in Figure 2.3) to an efficient frontier point (such as point C). This second-stage linear programming problem may be defined by:

¹⁴ Some authors use the term *input excess*.

¹⁵ Koopman's (1951) definition of technical efficiency was stricter than that of Farrell (1957). The former is equivalent to stating that a firm is only technically efficient if it operates on the frontier and furthermore that all associated slacks are zero.

Figure 2.3

Efficiency Measurement and Input Slacks



$$\begin{aligned}
 & \min_{\lambda, OS, IS} -(M1'OS - K1'IS), \\
 \text{st} \quad & -y_i + Y\lambda - OS = 0, \\
 & \theta x_i - X\lambda - IS = 0, \\
 & \lambda \geq 0, OS \geq 0, IS \geq 0,
 \end{aligned} \tag{2.12}$$

where OS is an $M \times 1$ vector of output slacks, IS is a $K \times 1$ vector of input slacks, and $M1$ and $K1$ are $M \times 1$ and $K \times 1$ vectors of ones, respectively. Note that in this second-stage linear program, θ is not a variable because its value is taken from the first-stage results. Furthermore, note that this second-stage linear program must also be solved for each of the N firms involved.¹⁶

One major problem associated with the above second-stage approach is that it is not invariant to units of measurement. The alteration of the units of measurement, say for a fertiliser input from kilograms to tonnes (while leaving other units of measurement unchanged), could result in the identification of different “nearest” efficient boundary points and hence different slack measures.¹⁷ As a result of this problem, many studies

¹⁶ Some of the models which have appeared in the literature have used a single-stage linear program, involving the use of an infinitesimal, to solve this two-stage problem. It has been argued that the two-stage method should be preferred, since “attempts to solve these non-Archimedean models as a single linear program with an explicit numerical value for the infinitesimal frequently creates computational inaccuracies and leads to erroneous results” (Ali and Sieford 1993).

¹⁷Charnes, et al. (1987) suggest a units-invariant model where the unit worth of a slack is made inversely proportional to the quantity of that input or output used by the i -th firm. This does solve the

simply solve the first-stage linear program for the values of the Farrell technical efficiency measures (θ) for each firm and ignore the slacks completely. We believe the best approach is to use this first-stage linear program which does not explicitly account for slacks (e.g., equation 2.11), and then report both θ and the residual slacks. This approach has two advantages over the two-stage approach. It involves less programming and it also avoids the units-of-measurement problem.¹⁸

2.4.2 The Variable Returns-to-scale (VRS) Model

Given that many industries are not perfectly competitive, the CRS assumption is often not appropriate. Banker, Charnes and Cooper (1984) suggested an extension of the CRS DEA model to account for variable returns-to-scale (VRS) situations. The CRS linear programming problem can be easily modified to account for VRS by adding the convexity constraint, $N1'\lambda=1$, to equation (2.11) to provide:

$$\begin{aligned}
 & \min_{\theta, \lambda} \theta, \\
 & \text{st} \quad -y_i + Y\lambda \geq 0, \\
 & \quad \theta x_i - X\lambda \geq 0, \\
 & \quad N1'\lambda=1 \\
 & \quad \lambda \geq 0,
 \end{aligned} \tag{2.13}$$

where $N1$ is an $N \times 1$ vector of ones. This approach forms a convex hull of intersecting planes which envelope the data points more tightly than the CRS conical hull and thus provides technical efficiency scores which are less than or equal to those obtained using the CRS model.

A number of DEA applications in recent years have obtained technical efficiency estimates relative to both CRS and VRS models, with any differences between the two sets of technical efficiency estimates subsequently interpreted as scale efficiency measures. An illustration of this approach is provided in Färe, Grosskopf and Pasurka (1989).

immediate problem, but creates another, in that there is no obvious reason for the slacks to be weighted in this way.

¹⁸ Note that the technical efficiency measures obtained from this approach will be identical to those obtained from the two-stage approach. The slack measures, however, may differ slightly.

2.4.3 Output-oriented Models

In the preceding input-orientated models, discussed in Sections 2.4.1 and 2.4.2, the method sought to identify technical inefficiency as a proportional reduction in input usage. This corresponds to Farrell's input-based measure of technical inefficiency. As discussed in Section 2.3, it is also possible to measure technical inefficiency as a proportional increase in output production. The two measures provide the same value under CRS but are unequal when VRS applies. Given that linear programming cannot suffer from such statistical problems as simultaneous equation bias, the choice of an appropriate orientation is not as crucial in DEA as it is in econometric estimation. Analysts in many studies have tended to select input-orientated models because many firms have particular orders to fill and hence the input quantities appear to be the primary decision variables, although this argument may not be as strong in agriculture as it is in manufacturing and service industries.

The output-orientated models are very similar to their input-orientated counterparts. Consider the example of the following output-orientated, VRS model:

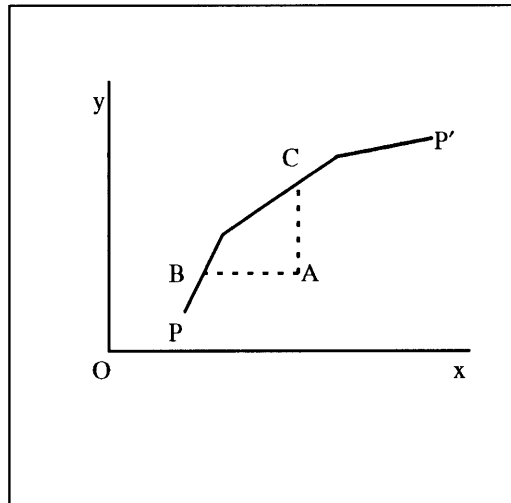
$$\begin{aligned}
 & \max_{\phi, \lambda} \phi, \\
 \text{st} \quad & -\phi y_i + Y\lambda \geq 0, \\
 & x_i - X\lambda \geq 0, \\
 & N1'\lambda = 1 \\
 & \lambda \geq 0,
 \end{aligned} \tag{2.14}$$

where ϕ is the proportional increase in outputs that could be achieved by the i -th firm with input quantities held fixed. An output-orientated CRS model is defined in a similar way, but is not presented here for brevity.

One point that should be made is that the output- and input-orientated models will yield exactly the same estimated frontier and therefore, by definition, identify the same set of firms as being efficient. The efficiency measures associated with the inefficient firms may, however, differ between the two methods. The two types of measures may be illustrated using the simple example, depicted in Figure 2.4, where output (y) is on the vertical axis and input (x) on the horizontal axis, and the production frontier is

Figure 2.4

Output- and Input-based Efficiency Measures



depicted by PP' . For the inefficient firm, operating at A , the distance AB is associated with input-based technical inefficiency, while AC is associated with output-based technical inefficiency.

2.4.4 Other Variants and Extensions

Possible extensions to the above models include: replacement of the piecewise-linear frontier with a piecewise log-linear or piecewise Cobb-Douglas frontier (Charnes, et al., 1982, 1983); incorporation of a cost minimisation behavioural objective¹⁹ (e.g., Ferrier and Lovell, 1990; and Chavas and Aliber, 1993); consideration of a stochastic element into the DEA (Sengupta, 1990); inclusion of categorical and environmental variables in the analysis (Banker and Morey, 1986a,b); and use of panel data and the Malmquist index approach to investigate technical change and technical efficiency change (Färe, et al., 1994). For a more complete list of possible extensions and variants, the reader is advised to consult Seiford and Thrall (1990) and Ali and Seiford (1993).

¹⁹ The construction of both DEA production and cost frontiers will permit the measurement of technical and economic efficiencies, and hence the calculation of allocative efficiencies as well.

2.5 Applications to Agriculture

The purpose of this section is to identify a few studies which illustrate the application of various frontier techniques to agriculture. This does not involve a comprehensive survey of frontier applications to agriculture. This has already been partially completed by the survey of applications of parametric production frontiers to agricultural industries by Battese (1992) and the survey of applications of frontier methods to developing country agriculture by Bravo-Ureta and Pinheiro (1993).

There appear to be only a few applications of frontier models to Australian agriculture. Battese and Corra (1977) estimated both deterministic and stochastic frontiers for farms involved in broadacre agriculture in Eastern Australia. Battese and Coelli (1988) applied a stochastic frontier model for panel data, which assumed time-invariant inefficiency effects, in the analysis of three years of data of sample dairy farms in NSW and Victoria.

There have been a vast number of applications of frontier methodologies to agricultural data in other countries around the world. A selection of studies from the last 12 years are listed in Table 2.1, ordered by year of publication, along with brief descriptions of the industries analysed and the methods used. This list is designed to indicate the breadth of analyses that have been conducted. It is by no means an exhaustive list.²⁰

Of the 40 papers listed in Table 2.1, only three involve DEA (non-parametric linear programming) while the remainder involve the construction of a variety of parametric frontiers.²¹ Of the latter, seven papers have estimated deterministic frontiers; 26 have estimated stochastic frontiers; and four have estimated both deterministic and stochastic frontiers. The deterministic frontiers are those discussed in Section 2.2, which specify a parametric structure and assume that all deviations from the frontier are due to inefficiency. The majority of applications of the deterministic frontiers are limited to the first half of Table 2.1. The lack of recent applications of deterministic frontier models is most likely a consequence of people becoming more aware of the

²⁰ Entries in Table 2.1 are derived from the survey papers by Battese (1992) and Bravo-Ureta and Pinheiro (1993) and a recent search conducted by the author.

²¹ The high percentage of parametric papers in Table 2.1 could be partly a consequence of the journals searched, but I believe it is an accurate depiction of the dominance of parametric methods in the agricultural economics literature.

Table 2.1

Applications of Frontier Models to Agriculture, 1985-1996

Authors	Year	Industry	Methodology
Belbase and Grabowski	1985	Nepalese farms	deterministic
Dawson	1985	Farms in North West England	deterministic
Färe, Grabowski and Grosskopf	1985	Filipino farms	DEA
Rawlins	1985	Jamaican farms	stochastic
Ray	1985	Indian farms	DEA
Bravo-Ureta	1986	New England (US) dairy farms	deterministic
Huang, Tang and Bagi	1986	Indian farms	stochastic profit frontier
Kalirajan and Shand	1986	Malay rice farms	stochastic
Phillips and Marble	1986	Guatemalan maize farms	stochastic
Taylor and Shonkwiler	1986	Brazilian farms	deterministic and stochastic
Taylor, Drummond and Gomes	1986	Brazilian farms	deterministic
Aly, et al.	1987	Illinois (US) grain farms	deterministic
Ekanayake	1987	Sri Lankan rice farms	deterministic and stochastic
Ekanayake and Jayasuriya	1987	Sri Lankan rice farms	deterministic and stochastic
Tauer and Belbase	1987	New York (US) dairy farms	deterministic
Battese and Coelli	1988	NSW and Victorian dairy farms	stochastic panel
Ali and Flinn	1989	Punjab (Pakistan) rice farms	stochastic profit frontier
Bailey, et al.	1989	Ecuadorian dairy farms	stochastic
Battese, Coelli and Colby	1989	Indian farms	stochastic panel
Dawson and Lingard	1989	Filipino rice farms	stochastic
Kalirajan	1989	Filipino rice farms	stochastic
Kalirajan and Shand	1989	Indian rice farms	stochastic panel
Kumbhakar, Biswas and Bailey	1989	Utah (US) dairy farms	stochastic system
Ali and Chaudry	1990	Punjab (Pakistan) farms	deterministic
Bravo-Ureta and Rieger	1990	North Eastern US dairy farms	deterministic and stochastic
Bravo-Ureta and Rieger	1991	North Eastern US dairy farms	stochastic
Dawson, Lingard and Woodford	1991	Filipino rice farms	stochastic
Kalirajan	1991	Filipino rice farms	stochastic panel
Kumbhakar, Ghosh and McGuckin	1991	US dairy farms	stochastic system
Squires and Tabor	1991	Indonesian farms	stochastic
Battese and Coelli	1992	Indian paddy farms	stochastic panel
Battese and Tessema	1993	Indian farms	stochastic panel
Battese, Malik and Broca	1993	Pakistani Wheat farms	stochastic panel
Chavas and Aliber	1993	Wisconsin (US) farms	DEA
Tran, Coelli and Fleming	1993	Vietnamese rubber farms	stochastic panel
Bravo-Ureta and Evenson	1994	Peasant farms in Paraguay	stochastic
Kumbhakar	1994	Farms in West Bengal (India)	stochastic system
Battese and Coelli	1995	Indian paddy farms	stochastic panel
Battese, Malik and Gill	1996	Pakistani wheat farmers	stochastic panel
Gimbol, Battese and Fleming	1996	Papua New Guinea cocoa producers	stochastic

deficiencies of deterministic frontiers and the advantages of stochastic frontiers. Lovell (1993, p.21) explained his objection to deterministic frontiers by noting that the approach “combines the bad features of the econometric and programming approaches to frontier construction: it is deterministic and parametric”. That is, deterministic frontiers may be criticised for not accounting for measurement error and other noise, and also for imposing a particular functional form upon the technology.

The stochastic frontier model has been, by far, the most popular, with 30 of the 40 papers in Table 2.1 involving applications of this model. The majority of these studies have involved the estimation of a single-equation production function using cross-sectional data. Exceptions to this are noted in the table, with ten papers considering panel data models; two estimating a profit frontier; and three estimating a system of equations involving a production function and input demand equations derived from the first-order conditions for profit maximisation. A large number of different agricultural industries are mentioned in Table 2.1. The most common frontier applications appear to be rice production, with 11 papers, and the dairy industry, with seven papers. The attention that rice has received is most likely a consequence of its vital importance to the food supplies of so many developing countries, while the attention given to dairy industries is more probably a consequence of recent debate surrounding the high degree of regulation that they attract in many developed countries. Other industries mentioned in Table 2.1 include cocoa, maize, rubber and wheat. However, the largest group of studies involve multi-product farming with 15 analyses of this type listed in Table 2.1. Applications from a total of 16 different countries are listed, ranging from developed countries, such as the USA, England and Australia, to small developing countries, such as Guatemala, Paraguay and Nepal. A researcher who wishes to conduct a frontier study of a particular agricultural enterprise, should be able to identify at least a few papers from the above list, which would be relevant to the application involved.

2.6 Conclusions

The main conclusion of this chapter is that none of the proposed methods of measuring efficiency relative to an estimated frontier is perfect. However, they all provide substantially better measures of efficiency than simple partial measures, such as output

per unit of labour or land. Given these qualifications, one frequently asked question is: "Which method of frontier estimation - stochastic frontier or DEA - should one use?" The answer to this question often depends upon the application being considered. If one is using farm-level data where measurement error, omitted variables (e.g., data on an input is not available or not suitably measured), weather, etc., are likely to play a significant role, then the assumption that all deviations from the frontier are due to inefficiency, which is made in DEA applications, may be a brave assumption. Hence the stochastic frontier method is recommended for use in most agricultural applications. This method also has the added advantage of permitting the conduct of statistical tests of hypotheses regarding the production structure and the degree of inefficiency. This advice, however, should not be viewed as an absolute rule. In some cases, where the above mentioned factors are not likely to have great influence (e.g., poultry and pig farming, abattoirs and grain silos) DEA could also be used. Furthermore, in instances where production involves more than one product, and the construction of an aggregate measure of output is difficult, DEA may be more attractive than estimating a multi-product cost or profit frontier, especially if price data are difficult to obtain.

As with all forms of empirical modelling, a frontier study can suffer from a variety of possible pitfalls. A few which warrant a mention include: the possibility that omitted or poorly measured inputs may influence technical efficiency measures; the possibility that unaccounted for environmental factors, such as soil quality or topography, may also influence technical efficiency measures; the possibility that poorly measured price variables (e.g., transport costs not properly accounted for) may influence allocative efficiency measures; and lastly, the use of data from a single season to measure efficiency may result in some farmers being labelled as inefficient, because of low stocking rates, when over a longer time frame they may be shown to be more efficient because of their more conservative approach. This last issue, points toward an interesting area of possible research. Many past analyses of farm efficiency have only involved efficiency measures derived using data for a single season. The development of a methodology to suitably account for the issues of risk aversion and multi-season efficiency would be a valuable contribution to the frontier literature.

There are a variety of agricultural policy issues which could be investigated using frontier methods. For example: identifying the influence of pollution controls upon efficiency in feedlots, abattoirs and irrigated farms; measuring the effect of salinity and soil degradation upon farm efficiency; measuring the influence of farm size upon efficiency; and investigating the effect of recent reforms upon various agricultural sectors, such as the dairy industry, using “before” and “after” data. The method could also be used to determine the extent to which the utilisation of agricultural extension advice may improve farmer efficiency.

Australian agriculture is faced with declining world commodity prices, increased competition from both subsidised and non-subsidised overseas industries, and declining expenditure on agricultural research. A suitable rate of productivity growth is required in order to remain competitive. However, to attain this without continuing to rely upon significant advances in technology from agricultural research, the agricultural sectors must be encouraged to use the existing technology more efficiently. Frontier functions and efficiency measurement can assist in this endeavour.

Frontier applications to agriculture need not be limited to the analysis of farms. Abattoirs, livestock-selling centres, grain silos, road freight, rail, ports, etc., could also be considered. There is no necessity to avoid analysing (private or public) multi-product service industries because they do not slot easily into a traditional production model. The DEA literature abounds with applications to a variety of such service industries, including hospitals, schools, electricity distributors, etc. [see Lovell (1993) for a more complete list and references]. These analyses can provide valuable insights into how analyses of agricultural service industries could be conducted.

Chapter 3

A Stochastic Frontier Production Function with Time-Varying Inefficiency Effects

3.1 Introduction

It is noted in Chapter 2 that the use of panel data can result in a variety of benefits for the estimation of stochastic frontier models. When panel data are available, one may be able to achieve: increased degrees of freedom for efficient estimation of parameters; consistent estimators of firm efficiencies (given sufficiently large T); removal of the necessity to make specific distributional assumptions regarding the U_i ; relaxation of the assumption that the inefficiency effects are independent of the explanatory variables; and the simultaneous investigation of both technical change and technical inefficiency change over time.

Many early panel data stochastic frontier models, such as those proposed by Pitt and Lee (1981), Battese and Coelli (1988) and Battese, Coelli and Colby (1989), assume that the inefficiency effects are constant through time for each firm in the sample. This has the advantage of permitting the derivation of consistent estimators of the inefficiencies of each firm, as T becomes large.¹ However, as T increases, the assumption that technical inefficiency effects are time-invariant is more difficult to justify, because it is likely that managers learn from their previous experience.

In this chapter we attempt to address the above criticism by suggesting a stochastic frontier model for panel data in which the inefficiency effects are permitted to vary systematically with time. A review of the relevant stochastic frontier literature is provided in Section 2.3.5 of Chapter 2. In the remainder of this chapter we present our model specification in Section 3.2; illustrate the model using an empirical application to data on Indian paddy farmers in Section 3.3; and provide some concluding comments in Section 3.4. Algebraic derivations of the logarithm of the

¹ Traditional estimators of technical efficiencies for individual firms in cross-sectional analyses (see Jondrow et al., 1982) are not consistent estimators because they rely upon information from a single time-series observation for each firm.

likelihood function, the first partial derivatives of this function, and expressions for predictors for the technical inefficiencies of individual firms are collected in Appendix 1.

3.2 Model Specification

We consider a stochastic frontier production function with a simple exponential specification of time-varying technical inefficiency effects which incorporates unbalanced panel data associated with observations on a sample of N firms over T time periods. The model is defined by

$$Y_{it} = f(x_{it};\beta)\exp(V_{it} - U_{it}) \quad (3.1)$$

and

$$U_{it} = \eta_{it}U_i = \{\exp[-\eta(t-T)]\}U_i \quad ,t \in \vartheta(i); \quad i = 1,2,\dots,N; \quad (3.2)$$

where Y_{it} represents the production for the i -th firm at the t -th period of observation;

$f(x_{it};\beta)$ is a suitable function of a vector, x_{it} , of factor inputs (and firm-specific variables), associated with the production of the i -th firm in the t -th period of observation, and a vector, β , of unknown parameters;

the V_{it} s are assumed to be independent and identically distributed $N(0, \sigma_v^2)$ random errors;

the U_i s are assumed to be independent and identically distributed non-negative truncations of the $N(\mu, \sigma^2)$ distribution;

η is an unknown scalar parameter; and

$\vartheta(i)$ represents the set of T_i time periods among the T periods involved for which observations for the i -th firm are obtained.²

This model is such that the non-negative technical inefficiency effects, U_{it} , decrease, remain constant or increase as t increases, if $\eta > 0$, $\eta = 0$ or $\eta < 0$, respectively. The case in which η is positive is likely to be appropriate when firms tend to improve their level of technical efficiency over time. Further, if the T -th time period is observed for

² If the i -th firm is observed in all the T time periods involved, then $\vartheta(i) = \{1,2,\dots,T\}$. However, if the i -th firm was continuously involved in production, but observations were only obtained at discrete intervals, then $\vartheta(i)$ would consist of a subset of the integers, $1,2,\dots,T$, representing the periods of observations involved.

the i -th firm then $U_{iT} = U_i$, $i = 1, 2, \dots, N$. Thus the parameters, μ and σ^2 , define the statistical properties of the technical inefficiency effects associated with the last period of the panel. The model assumed for the technical inefficiency effects, U_i , is that which was originally proposed by Stevenson (1980) and is a generalisation of the half-normal distribution which has been frequently applied in empirical studies.

The exponential specification of the behaviour of the technical inefficiency effects over time [equation (3.2)] is a rigid parameterisation in that the technical inefficiency effects must either decrease at a decreasing rate ($\eta > 0$), increase at a decreasing rate ($\eta < 0$) or remain constant ($\eta = 0$). In order to permit greater flexibility in the nature of technical efficiency, a two-parameter specification could be required. An alternative two-parameter specification, is defined by,

$$\eta_{it} = 1 + \eta_1(t-T) + \eta_2(t-T)^2,$$

where η_1 and η_2 are unknown parameters. This model permits technical inefficiency effects to be convex or concave, but the time-invariant model is the special case in which $\eta_1 = \eta_2 = 0$.

As noted in Chapter 2, Kumbhakar (1990) was the first to specify a stochastic frontier for panel data in which the inefficiency effects are permitted to vary systematically with time. His model is similar to equation (3.1) with the U_{it} assumed to be defined by

$$U_{it} = [1 + \exp(bt + ct^2)]^{-1} U_i$$

where the U_i s are assumed to have half-normal distribution and b and c are parameters to be estimated. Kumbhakar (1990) suggested that the model be estimated using ML estimation but no empirical application has yet been attempted.

The deterministic component of the Kumbhakar (1990) specification could take values between zero and one and could be monotone decreasing (or increasing) or convex (or concave) depending on the values of the parameters, b and c . Although the more general model of Kumbhakar (1990) does permit a wider variety of temporal patterns, it would be more difficult to estimate than the simpler exponential model of equation (3.2).

Given the model, defined by equations (3.1) and (3.2), it can be shown [see Appendix 1] that the minimum-mean-squared-error predictor of the technical efficiency of the i -th firm at the t -th time period, $TE_{it} = \exp(-U_{it})$, is

$$E[\exp(-U_{it})|E_i] = \left\{ \frac{1 - \Phi[\eta_{it}\sigma_i^* - (\mu_i^* / \sigma_i^*)]}{1 - \Phi(-\mu_i^* / \sigma_i^*)} \right\} \exp[-\eta_{it}\mu_i^* + \frac{1}{2}\eta_{it}^2\sigma_i^{*2}] \quad (3.3)$$

where E_i represents the $(T_i \times 1)$ vector of E_{it} s associated with the time periods observed for the i -th firm, where $E_{it} = V_{it} - U_{it}$;

$$\mu_i^* = \frac{\mu\sigma_v^2 - \eta_i' E_i \sigma^2}{\sigma_v^2 + \eta_i' \eta_i \sigma^2} \quad (3.4)$$

$$\sigma_i^{*2} = \frac{\sigma_v^2 \sigma^2}{\sigma_v^2 + \eta_i' \eta_i \sigma^2} \quad (3.5)$$

where η_i represents the $(T_i \times 1)$ vector of η_{it} s associated with the time periods observed for the i -th firm; and $\Phi(\cdot)$ represents the distribution function for the standard normal random variable.

We also observe that if the stochastic frontier production function in equation (3.1) is of Cobb-Douglas or transcendental logarithmic type, then E_{it} is a linear function of the vector, β .

The result of equation (3.3) yields the special cases given in the literature. Although Jondrow, et al. (1982) only derived $E[U_i|V_i-U_i]$, the more appropriate result for the case when the dependent variable is the logarithm of output and the analysis involves cross-sectional data, $E[\exp(-U_i)|V_i-U_i]$, is obtained from equations (3.3)-(3.5) by substituting $\eta_{it} = 1 = \eta_i$ and $\mu = 0$. The special cases given in Battese and Coelli (1988) and Battese, Coelli and Colby (1989) are obtained by substituting $\eta_i' \eta_i = T$ and $\eta_i' \eta_i = T_i$, respectively, where $\eta_{it} = 1$ (i.e., $\eta = 0$) in both cases.

The mean technical efficiency of firms at the t -th time period,

$$TE_t = E[\exp(-\eta_t U_i)], \text{ where } \eta_t = \exp[-\eta(t-T)],$$

obtained by straightforward integration with the density function of U_i , is

$$TE_t = \left\{ \frac{1 - \Phi[\eta_t \sigma - (\mu / \sigma)]}{1 - \Phi(-\mu / \sigma)} \right\} \exp[-\eta_t \mu + \frac{1}{2} \eta_t^2 \sigma^2]. \quad (3.6)$$

If the firm effects are time invariant, then the mean technical efficiency of firms in the industry is obtained from equation (3.6) by substitution of $\eta_t = 1$. This gives the result presented in equation (8) of Battese and Coelli (1988).

Alternatively, the mean technical efficiency for period t could be estimated using the average of the predicted technical efficiencies for the firms observed in period t. These are the measures presently reported by the FRONTIER computer program.

Operational predictors for equations (3.3) and (3.6) are obtained by substituting the relevant parameters by their maximum-likelihood estimators. The maximum-likelihood estimates for the parameters of the model and the predictors for the technical efficiencies of firms can be approximated by the use of the computer program, FRONTIER.³ The likelihood function for the sample observations is presented in Appendix 1 in terms of the parameterisation of the model, suggested by Battese and Corra (1977), which involves the variance parameters $\sigma_s^2 = \sigma_v^2 + \sigma^2$ and $\gamma = \sigma^2 / \sigma_s^2$. This parameterisation has advantages in the ML estimation of the parameters. The FRONTIER computer program searches over the parameter space for a suitable starting value for γ , which is bounded by 0 and 1. For more on the FRONTIER program see Chapter 7.

3.3 Empirical Example

Battese, Coelli and Colby (1989) used a set of panel data on 38 farmers from an Indian village to estimate the parameters of a stochastic frontier production function for which the technical inefficiency effects of individual farmers were assumed to be time invariant. We consider a subset of these data for those farmers, who had access to irrigation and grew paddy, to estimate a stochastic frontier production frontier with time-varying inefficiency effects, as specified by equations (3.1) and (3.2) in Section 3.2. The data were collected by the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) from farmers in the village of Aurepalle. We consider the data for fifteen farmers who engaged in growing paddy (irrigated rice) for between

³ This model is referred to as Model 1 in the description of the FRONTIER program in Chapter 7.

four and ten years during the period, 1975-76 through 1984-85. Nine of the fifteen farmers were observed for all the ten years involved. A total of 129 observations were used and so 21 observations were missing from the panel.⁴

The stochastic frontier production function for the panel data on the paddy farmers in Aurepalle which we estimate is defined by

$$\begin{aligned} \log(Y_{it}) = & \beta_0 + \beta_1 \log(Land_{it}) + \beta_2 (IL_{it}/Land_{it}) + \beta_3 \log(Labour_{it}) \\ & + \beta_4 \log(Bullock_{it}) + \beta_5 \log(Costs_{it}) + V_{it} - U_{it} \end{aligned} \quad (3.7)$$

where the subscripts i and t refer to the i -th farmer and the t -th observation, respectively;

Y represents the total value of output (in Rupees) from paddy and any other crops which might be grown;⁵

$Land$ represents the total area (in hectares) of irrigated and unirrigated land, denoted by IL and UL , respectively;

$Labour$ represents the total number of hours of human labour (in male equivalent hours)⁶ for family members and hired labourers;

$Bullock$ represents the total number of hours of bullock labour for owned or hired bullocks (in pairs);

$Costs$ represents the total value of other input costs involved (fertiliser, manure, pesticides, machinery, etc.); and

V_{it} and U_{it} are the random variables whose distributional properties are defined in Section 3.2.

A summary of the data on the different variables in the stochastic frontier production function is given in Table 3.1. It is noted that about 30 per cent of the total land operated by the paddy farmers in Aurepalle was irrigated. Thus the farmers involved

⁴ All model specifications in this thesis permit unbalanced panel data. This is believed to be very important in frontier analysis, since the enforced exclusion of some observations because they are not observed in all time periods may result in the introduction of serious sample-selection biases.

⁵ Because the output variable in the stochastic frontier production function is *value of total output*, the measures of technical efficiencies obtained below will, in fact, be measures of the total economic efficiencies of the farmers. Hence the U_{it} s are hereafter referred to as “inefficiency effects” rather than technical inefficiency effects.

⁶ Labor hours were converted to male equivalent units according to the rule that female and child hours were considered equivalent to 0.75 and 0.50 male hours, respectively. These ratios were obtained from ICRISAT.

Table 3.1
Summary Statistics for Variables in the Stochastic Frontier Production Function
for Paddy Farmers in Aurepalle

Variable	Sample Mean	Standard Deviation	Minimum Value	Maximum Value
Value of Output (Rupees)	6939	4802	36	18094
Total Land (hectares)	6.70	4.24	0.30	20.97
Irrigated Land (hectares)	1.99	1.47	0.00	7.09
Human Labour (hours)	4126	2947	92	6205
Bullock Labour (hours)	900.4	678.2	56.0	4316.0
Other Input Costs (Rupees)	1273	1131	0.7	6205

were generally also engaged in dryland farming. The minimum value of irrigated land was zero because not all the farmers involved grew paddy in all the years involved.

The production function, defined by equation (3.7), is related to the function which was estimated in Battese, Coelli and Colby (1989, p.333), but family and hired labour are aggregated (i.e., added).⁷ The justification for the functional form considered in Battese, Coelli and Colby (1989) is based on the work of Bardhan (1973) and Deolalikar and Vijverberg (1983) with Indian data on hired and family labour and irrigated and unirrigated land. The production function of equation (3.7) is a linearised version of that which was directly estimated in Battese, Coelli and Colby (1989)⁸ [cf. the model in Defourny, Lovell and N'gbo (1990)].

⁷ The hypothesis that family and hired labour were equally productive was tested and accepted in Battese, Coelli and Colby (1989). Hence only total labour hours is considered in this empirical study.

⁸ The deterministic component of the stochastic frontier production function estimated in Battese, Coelli and Colby (1989), considering only the land variable (consisting of a weighted average of unirrigated land and irrigated land), is defined by,

$$Y = a_0[a_1UL + (1-a_1)IL]^\beta.$$

This model is expressed in terms of Land = UL + IL and IL/Land, as follows

$$Y = a_0 \times a_1^\beta (\text{Land})^\beta [1 + (b_1 - 1)(\text{IL}/\text{Land})]^\beta, \text{ where } b_1 = (1 - a_1)/a_1.$$

By taking logarithms of both sides and considering only the first term of the infinite-series expansion of the function we obtain

$$\log Y = \text{constant} + \beta \log(\text{Land}) + \alpha (\text{IL}/\text{Land}), \text{ where } \alpha = \beta(b_1 - 1).$$

The data, consisting of 129 observations for each variable, collected from 15 paddy farmers in Aurepalle over the ten-year period, 1975-76 to 1984-85, were collected by the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) as part of its Village Level Studies, see Binswanger and Jodha (1978).

The original values of output and input costs used in Battese, Coelli and Colby (1989) are deflated by a price index for the analyses in this paper. The price index used was constructed using data, supplied by ICRISAT, on prices and quantities of crops grown in Aurepalle.

The stochastic frontier model, defined by equation (3.7), contains six β -parameters and the four additional parameters associated with the distributions of the V_{it} - and U_{it} -random variables. Maximum-likelihood estimates for these parameters were obtained by using the computer program, FRONTIER. The frontier function (3.7) is estimated for five basic models:

Model 1.0 involves all parameters being estimated;

Model 1.1 assumes that $\mu = 0$;

Model 1.2 assumes that $\eta = 0$;

Model 1.3 assumes that $\mu = \eta = 0$; and

Model 1.4 assumes that $\gamma = \mu = \eta = 0$.⁹

Model 1.0 is the stochastic frontier production function (3.7) in which the inefficiency effects, U_{it} , have the time-varying structure defined in Section 3.2 (i.e., η is an unknown parameter and the U_{it} s of equation (3.2) are non-negative truncations of the $N(\mu, \sigma^2)$ distribution). Model 1.1 is the special case of Model 1.0 in which the U_{it} s have half-normal distribution (i.e., μ is assumed to be zero). Model 1.2 is the time-invariant model considered by Battese, Coelli and Colby (1989). Model 1.3 is the time-invariant model in which the inefficiency effects, U_{it} , have half-normal distribution. Finally, Model 1.4 is the traditional average response function in which the paddy

⁹ We note that if the restriction, $\gamma=0$, is true then the U_{it} are not stochastic and so μ and η are not identifiable. We will list all three parameters in the restriction, however, to remind the reader that when the inefficiency effects are not present, then the model involves three less parameters.

farmers are assumed to be fully technically efficient (i.e., the inefficiency effects, U_{it} , are absent from the model).

Maximum-likelihood estimates for the parameters of these five models are presented in Table 3.2. Tests of hypotheses involving the parameters of the distributions of the inefficiency effects, U_{it} , are obtained by using the generalised likelihood-ratio statistic. Several hypotheses are considered for different distributional assumptions and the relevant statistics are presented in Table 3.3.

The generalised likelihood-ratio test statistic is calculated as

$$\lambda = -2\{\log[L(H_0)] - \log[L(H_1)]\}$$

where $L(H_0)$ and $L(H_1)$ represent the likelihood function under the null hypothesis, H_0 , and the alternative hypothesis, H_1 , respectively. In most situations this statistic has asymptotic chi-square distribution with degrees of freedom equal to the difference in the number of parameters in H_1 and H_0 , if H_0 is true.

This statistic does not, however, have a chi-square distribution when one or more of the restrictions involves a one-sided alternative. Thus when the null hypothesis involves $\gamma=0$, the alternative hypothesis can only involve positive values of γ (recalling that it is the ratio of two variances). As discussed in Chapter 8, Coelli (1995c) notes that the distribution of any likelihood-ratio statistic involving the γ -parameter has distribution which is a mixture of chi-square distributions. The 5% critical value for the null hypothesis of $\gamma=0$, when U_i is assumed to have a half-normal distribution, is shown to be 2.71.

In cases involving more than one parameter restriction the calculation of the appropriate critical value for the mixed chi-square distribution is a very complicated exercise. For example, a test of the null hypothesis, $H_0: \gamma=\mu=\eta=0$, versus the alternative hypothesis, $H_1: \gamma>0$ and (μ,η) free, involves a mixture of equality and inequality restrictions. To avoid the difficulty of deriving the appropriate distribution in such cases, one can utilise Table 1 in Kodde and Palm (1986) which lists lower and upper bounds for the appropriate critical value, when a mixture of equality and

Table 3.2

Maximum-likelihood Estimates for Parameters of Stochastic Frontier Production Functions for Aurepalle Paddy Farmers

Variable	Parameter	Model	Model	Model	Model	Model
		1.0	1.1	1.2	1.3	1.4
Constant	β_0	3.74	3.86	3.90	3.87	3.71
		(0.96)*	(0.94)	(0.73)	(0.68)	(0.66)
log(Land)	β_1	0.61	0.63	0.63	0.63	0.62
		(0.23)	(0.20)	(0.15)	(0.15)	(0.15)
IL/Land	β_2	0.81	1.05	0.90	0.89	0.80
		(0.43)	(0.33)	(0.30)	(0.29)	(0.27)
log(Labour)	β_3	0.76	0.74	0.74	0.74	0.74
		(0.21)	(0.18)	(0.15)	(0.14)	(0.14)
log(Bullocks)	β_4	-0.45	-0.43	-0.44	-0.44	-0.43
		(0.16)	(0.11)	(0.11)	(0.11)	(0.12)
log(Costs)	β_5	0.079	0.058	0.052	0.052	0.053
		(0.048)	(0.038)	(0.042)	(0.042)	(0.043)
Variance Parameters:						
	σ_s^2	0.129	0.104	0.136	0.142	0.135
		(0.048)	(0.010)	(0.040)	(0.028)	(0.019)
	γ	0.22	0.056	0.11	0.14	0
		(0.21)	(0.012)	(0.26)	(0.17)	
	μ	-0.8	0	-0.07	0	-
		(1.8)		(0.43)		
	η	0.27	0.138	0	0	-
		(0.97)	(0.047)			
Log-likelihood Function		-40.79	-40.80	-50.41	-50.42	-50.81

* The estimated standard errors for the parameter estimators are presented below the corresponding estimates to two significant digits. The parameter estimates are given correct to the number of digits behind the decimal place in the estimated standard errors.

Table 3.3

Tests of Hypotheses for Parameters of the Distribution of the Inefficiency Effects

Assumptions	Null Hypothesis H_0	Test Statistic λ	Critical Value	Decision
Model 1.0	$\gamma = \mu = \eta = 0$	20.04	5.14-7.05	Reject H_0
Model 1.0	$\mu = \eta = 0$	19.26	5.99	Reject H_0
Model 1.0	$\mu = 0$	0.02	3.84	Accept H_0
Model 1.0	$\eta = 0$	19.24	3.84	Reject H_0
Model 1.1	$\gamma = \eta = 0$	20.02	5.14	Reject H_0
Model 1.1	$\eta = 0$	19.24	3.84	Reject H_0

inequality restrictions are involved. This is the reason a range of values are reported in the critical value column in Table 3.3 for the test of the null hypothesis, $H_0: \gamma=\mu=\eta=0$.

Given the specifications of the stochastic frontier with time-varying inefficiency effects (Model 1.0), it is evident that the traditional average production function is not an adequate representation of the data (i.e., the null hypothesis, $H_0: \gamma = \mu = \eta = 0$, is rejected).¹⁰ Further, the hypotheses that time-invariant models for inefficiency effects apply are also rejected (i.e., both $H_0: \mu = \eta = 0$ and $H_0: \eta = 0$ would be rejected). However, the hypothesis that the half-normal distribution is an adequate representation for the distribution of the inefficiency effects in the last period of the panel is not rejected using these data. Given the specifications of the time-varying inefficiency effects, for which the half-normal distribution is appropriate to define the distribution of the inefficiency effects in the last period of the panel, the hypothesis that the yearly inefficiency effects are, in fact, time invariant is also rejected by the data.

On the basis of these results it is evident that the hypothesis of time-invariant inefficiency effects for paddy farmers in Aurepalle would be rejected. Given the specifications of Model 1.1 (involving the half-normal distribution), the efficiencies of the individual paddy farmers are calculated using the predictor, defined by equation (3.3). The values obtained, are presented in Table 3.4.

¹⁰ Unless otherwise stated, all tests of hypotheses conducted in this thesis assume a 5% level of significance.

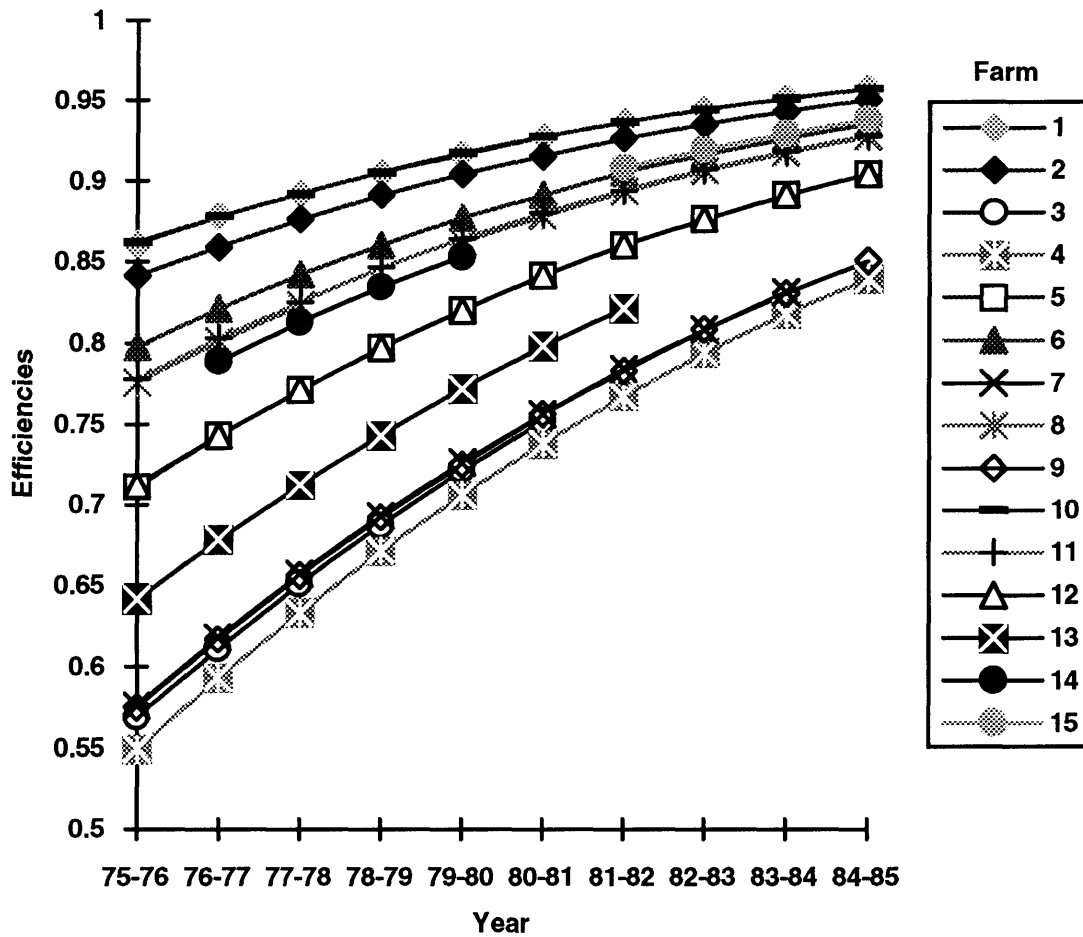
Table 3.4
Predicted Efficiencies of Paddy Farmers in Aurepalle
for the years, 1975-76 through 1984-85

Farmer Number	75-76	76-77	77-78	78-79	79-80	80-81	81-82	82-83	83-84	84-85
1	.861	.878	.892	.905	.916	.927	.936	.944	.951	.957
2	.841	.859	.876	.891	.904	.915	.926	.935	.943	.950
3	.569	.611	.651	.687	.721	.752	.*	-	-	-
4	.549	.593	.633	.671	.706	.738	.767	.794	.818	.839
5	.711	.743	.771	.797	.820	.841	.860	.876	.891	.904
6	.798	.821	.842	.860	.877	.891	.905	.916	.926	.935
7	.576	.618	.657	.693	.726	.756	.784	.808	.831	-
8	.776	.801	.823	-	.862	.878	.893	.906	.917	.927
9	.575	.617	.656	.692	.725	.756	.783	.808	.830	.850
10	.862	.878	.892	.905	.917	.927	.936	.944	.951	.957
11	.778	.803	.825	.846	.864	.880	.894	.907	.918	.928
12	.712	.743	.771	.797	.820	.841	.860	.876	.891	.904
13	.641	.678	.712	.743	.772	.798	.821	-	-	-
14	-	.789	.813	.834	.853	-	-	-	-	-
15	-	-	-	-	-	-	.908	.919	.929	.938
mean	.711	.745	.772	.793	.820	.838	.867	.886	.899	.917

* In years when particular farmers were not observed, no values of technical efficiencies are calculated.

The efficiencies range between 0.549 and 0.862 in 1975-76 and, between 0.839 and 0.957 in 1984-85. Because the estimate for the parameter, η , is positive ($\eta = 0.138$) the inefficiency effects decrease over time, according to the assumed exponential model, defined by equation (3.2). As a consequence, the efficiencies of the paddy farmers increase over time, as indicated in Table 3.4. These predicted efficiencies of the 15 paddy farmers are graphed against year of observation in Figure 3.1. These data indicate that there exist considerable variation in the efficiencies of the paddy farmers, particularly at the beginning of the sample period. Given the assumption that the inefficiency effects change exponentially over time, it is expected that the predicted efficiencies converge over a period of generally decreasing levels of inefficiency.

Figure 3.1
Predicted Technical Efficiencies of Aurepalle Paddy Farmers



The above results are, however, based on the stochastic frontier production function (3.7), which assumes that the parameters are time invariant. In particular, the presence of technical progress is not accounted for in the model. Given that year of observation is included as an additional explanatory variable, then the maximum-likelihood estimates for the parameters of the stochastic frontier production function are presented in Table 3.5 in the column labelled, Model 2.0. Also presented in Table 3.5 are the parameter estimates for the corresponding traditional response function in which the inefficiency effects, U_{it} , are absent from the model.

Table 3.5
Maximum-likelihood Estimates for Parameters of Production Functions Which
Account for Technical Change

Variable	Parameter	Model 2.0	Model 2.1
Constant	β_0	2.80 (1.75)*	2.73 (0.63)
log(Land)	β_1	0.50 (0.37)	0.51 (0.13)
IL/Land	β_2	0.53 (0.43)	0.50 (0.26)
log(Labour)	β_3	0.91 (0.32)	0.91 (0.14)
log(Bullocks)	β_4	-0.489 (0.098)	-0.48 (0.11)
log(Costs)	β_5	0.051 (0.040)	0.048 (0.040)
Year	β_6	0.050 (0.019)	0.054 (0.011)
Variance Parameters			
	σ_s^2	0.130 (0.084)	0.113 (0.029)
	γ	0.21 (0.44)	0
	μ	-0.69 (0.98)	-
	η	0.11 (0.65)	-
Log-likelihood Function		-38.504	-38.719

* The estimated standard errors for the parameter estimators are presented below the corresponding estimates to two significant digits.

The generalised likelihood-ratio test of the null hypotheses that the inefficiency effects are absent from the stochastic frontier model (i.e., $H_0: \gamma=\mu=\eta=0$) yields a test statistic, $\lambda=0.43$, which is highly insignificant. Thus the inclusion of the year of observation in the model (i.e., Hicksian neutral technical change), leads to the conclusion that the stochastic frontier production function is not significantly different from the traditional average response model.¹¹

This estimated response function, Model 2.1 of Table 3.5, is such that the returns-to-scale parameter is estimated by 0.990 which is not significantly different from one, because the estimated standard error of the estimator is 0.065. Thus the null hypothesis of constant returns to scale for the paddy farmers would not be rejected using these data.

The coefficient of the ratio of irrigated land to total land operated, $IL/Land$, is significantly different from zero. Using the estimates for the elasticity of land and the coefficient of the land ratio, one hectare of irrigated land is estimated to be equivalent to about 1.98 hectares of unirrigated land for Aurepalle farmers who grow paddy and other crops. The calculations involved here are: $\hat{\beta}_1 = 0.512$, $\hat{\beta}_{21} = \hat{\beta}_1(\hat{b}_1 - 1) = 0.501$ implies $\hat{b}_1 = 1.98$, where \hat{b}_1 is the value of one hectare of irrigated land in terms of unirrigated land for farmers who grow paddy and other crops. This estimate compares with the estimate of 3.50 hectares obtained by Battese, Coelli and Colby (1989) using data on all 38 farmers in Aurepalle. The smaller value obtained using only data on paddy farmers is probably due to the smaller number of unirrigated hectares in this study than in the earlier study involving all farmers in the village.

The estimated elasticity for bullock labour on paddy farms is negative. This result was also observed in Saini (1979) and Battese, Coelli and Colby (1989). A plausible argument for this result is that paddy farmers may use bullocks more in years of poor

¹¹ Some authors have recently questioned whether it is possible to identify both technical change and technical inefficiency change in the stochastic frontier. Although we do not have a formal theoretical proof of the identification of the parameters associated with these two trends, we make the following two observations. First, if these parameters are not identified then the μ parameter in Stevenson's (1980) truncated normal model is also not identifiable (i.e., discernible from β_0). Second, in estimating the above models, involving both time trends, we considered a number of different sets of starting values ranging from the technical change parameter (β_6) having all weight to the technical inefficiency change parameter (η) having all weight. In all cases the computer program converged to the same set of parameter estimates.

production (associated with low rainfall) for the purpose of weed control, levy bank maintenance, etc., which are difficult to conduct in years of higher rainfall and higher output. Hence, the bullock-labour variable may be acting as an inverse proxy for rainfall.

The coefficient, 0.054, of the variable, year of observation, in the estimated response function, given by Model 2.1 in Table 3.5, is statistically significant and it implies that the value of output (in real terms) is estimated to have increased by about 5.4% over the ten-year period for the paddy farmers in Aurepalle.

3.4 Conclusions

The empirical application of the stochastic frontier production function model with time-varying technical inefficiency effects, defined by equations (3.1) and (3.2), in the analysis of data from paddy farmers in an Indian village, reveals that the efficiencies of the farmers were not time invariant when year of observation was excluded from the stochastic frontier. However, the inclusion of year of observation in the frontier model led to the finding that the corresponding technical inefficiency effects were time invariant. In addition, the stochastic frontier was not significantly different from the traditional average response function. This implies that, given the state of technology among paddy farmers in the Indian village involved, technical inefficiency is not an issue of significance provided technical change is accounted for in the empirical analysis. However, in other empirical applications of the time-varying model which have been conducted [see Battese and Tessema (1992)], the inclusion of time-varying parameters in the stochastic frontier has not necessarily resulted in time-invariant inefficiency effects or the conclusion that the inefficiency effects do not exist.

The stochastic frontier production function estimated in Section 3.3 does not involve farmer-specific variables. To the extent that farmer- (and farm-) specific variables influence the level of the technical inefficiency effects, the empirical analysis presented in Section 3.3 does not appropriately predict the overall efficiencies of the paddy farmers. More detailed modelling of the variables influencing production and the statistical distribution of the random variables involved will lead to improved analysis

of production and better policy decisions concerning productive activity. The following chapter introduces a model which attempts to address this issue.