Pixel-wise uncertainty quantification in electric properties tomography

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Electric properties tomography (EPT) allows estimating non-invasively the distribution of the electric properties within a human body starting from acquisitions performed with a magnetic resonance imaging (MRI) scanner [1]. Because of its possible application in detecting pathologies [2] or planning and monitoring the efficacy of therapies [3], EPT is a promising quantitative imaging technique. Nonetheless, its adoption in clinics has been slowed down by the complexity of the task, which relies on the solution of an inverse problem in the theory of the electromagnetic fields with a large sensitivity to noise in the input data [4]. Moreover, certain formulations of the EPT inverse problem are based on a number of assumptions not always verified by the input data, like the local homogeneity assumption of Helmholtz-EPT, which does not hold at the boundary between different tissues [5]. Being able to distinguish the regions where the estimation provided by EPT is trustworthy from those where the uncertainty is too large, due to the input noise or to the model assumptions, would significantly improve the effectiveness of the EPT results and accelerate its clinical adoption.

To this end, this work focuses the attention on the phase-based approximation of Helmholtz-EPT implemented in the open-source library EPTlib (https://eptlib.github.io/) [6]. This EPT approach starts from a *transceive phase* map, $\varphi^{\pm}$, acquired by the MRI scanner and estimates the electric conductivity, σ, according to $\sigma = \nabla^2\varphi^{\pm}/(2\omega\mu_0)$, where ω is the angular frequency of the radiofrequency (RF) magnetic field and μ_0 is the vacuum permeability. A classical method to compute the Laplacian of the noisy input map is the Savitzky–Golay filter [7], which consists in the local approximation of $\varphi^{\pm}$, within a kernel of given shape and size centred at the point of interest, with a second-degree polynomial, whose Laplacian can then be analytically computed as a linear combination of the polynomial coefficients.

The polynomial approximation is accomplished through a regression analysis, which allows evaluating the uncertainty that $\varphi^{\pm}$ should have to justify the fitting residual under the assumption of independent identically distributed errors in the $\varphi^{\pm}$ values. Such an uncertainty combines the contribution of the noise in the acquired map and that of the modelling error in the polynomial approximation. Finally, the pixel-wise uncertainty, $u(\sigma)$, of the estimated conductivity is obtained by propagating the uncertainty of $\varphi^{\pm}$ through the regression model and the Laplacian computation.

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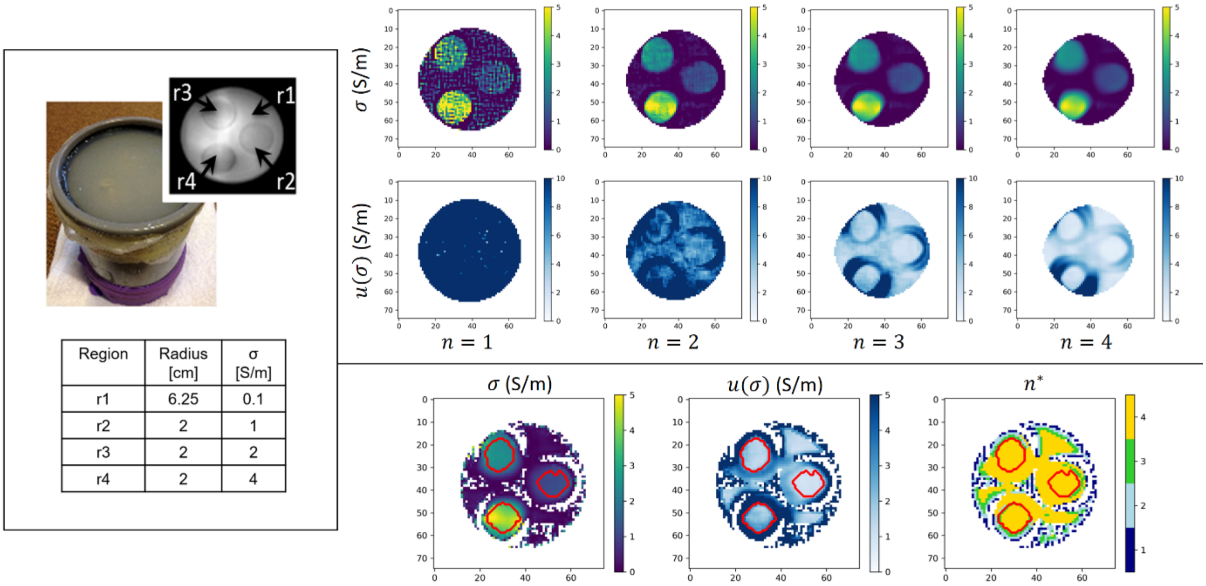
Figure 1 collects the conductivity maps and the associated uncertainties retrieved from the $\varphi^{\pm}$ maps acquired by a 3 T scanner with a body coil in transmission and a 15-channel head coil in reception, using two T1-weighted spin-echo sequences with opposite gradient polarities (TR/TE = 900/5 ms, resolution = 2 mm). Square kernels with $2n + 1$ pixels per edge are used with n from 1 to 4. The obtained images are combined to get for each pixel the optimal kernel size n^*, i.e., that one leading to a physically admissible conductivity and the minimum uncertainty $u(\sigma)$. Finally, three connected regions are identified by looking for the pixels where the relative uncertainty is less than 1. These regions coincide with the phantom compartments and their conductivities are in good agreement with the expected values. The spatial average and standard deviation of σ are (1.17 ± 0.07) S/m in r2, (2.45 ± 0.13) S/m in r3, and (4.27 ± 0.43) S/m in r4.The observed overestimations may arise from the phase-only reconstruction approach, which tends to overestimate the reconstructed conductivity values.



Region	Radius [cm]	σ [S/m]
r1	6.25	0.1
r2	2	1
r3	2	2
r4	2	4

Figure 1. Conductivity maps and associated uncertainties estimated with different kernels in a phantom.

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