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Louis de Broglie's discussion on the equivalence between relativistic principles of entropy maximization and least action

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In the present work we present a didactic reconstruction of de Broglie's demonstration of the equivalence between two fundamental physical concepts: action minimization and entropy maximization. The demonstration was first developed in de Broglie's Thermodynamics of the Isolated Particle (an attempt to develop a causal interpretation of Quantum Theory in the 1960s). In this work, de Broglie argues from a particle in a hypothetical sub-quantum medium, which operates as a kind of hidden thermostat exchanging energy with the particle. In the discussion about the action minimization and entropy maximization, however, there is not any reference to the quantum nature of the system. So, it is a demonstration valid for any system in a thermal bath in some usual thermostat. Louis de Broglie concludes that the "natural" trajectory of a particle is not only the one that minimizes the action of the particle but also maximizes the entropy of the thermostat. Our objective is to reconstruct and restructure de Broglie's original demonstration in order to rescue an historical debate, that has not been addressed in the literature, and that is of interest to contemporary fields such as non-equilibrium thermodynamics.
Keywords: Hidden Thermodynamics, Louis de Broglie, relativistic entropy and action, history of science.

1. Introduction

During the whole history of Thermodynamics, its relation to Mechanics was a matter of debate. Joule (1818–1889), in the nineteenth century, was one of the few scientists to defend the idea that heat could be considered a form of energy, according to Kelvin's (1824–1907) reports [1]. Since heat and work are not completely interchangeable in a reversible cycle, Carnot (1796–1832) also assumed that they should have a different nature [2]. Fourier (1788–1830) starts his famous *Analytical Theory of Heat* [3] defending that thermal phenomena cannot be explained by mechanical laws. And, finally, Clausius' (1822–1888) studies on entropy and the recognition of irreversible processes reinforced the conception of an abyss between thermodynamics and mechanics [4].

It is well known that Statistical Physics as we know it today is a result from Boltzmann's (1844–1906) efforts to pave the way for a different direction: it relies on the fact that a macroscopic system, described by the laws of thermodynamics, is composed of particles subject to mechanical laws. Boltzmann's H-theorem and his explanation of the entropy increase and irreversibility in terms of probability is perhaps the clearest enunciation of the reduction of thermodynamics into mechanics. This conception became hegemonic in Physics in the twentieth century Physics. Callen's [5] famous textbook on Thermodynamics exemplifies this perspective when he defines thermodynamics as a theory of macroscopic systems, characterized by rough temporal and spatial measurements in relation to the atomic scale.

Scientists working in non-equilibrium thermodynamics, though, have challenged such conception [6]. In far from equilibrium systems, emergent properties are observed and their description satisfies specific relations established by non-equilibrium thermodynamics. In this context, again, the relation between mechanics and thermodynamics seems fuzzy.

Along this history, there are some resurgent questions: is there a relation between the laws of thermodynamics and the laws of mechanics? Is it possible to re-write the theorem of entropy increase as a least action principle? Which one is the most fundamental principle? Many recent works have presented reflections on this subject [7–10].

Considering the contemporary importance of such a topic, we aim to introduce a historical discussion of an episode that has not been addressed in the literature yet. In this paper, our goal is to recover a discussion proposed by Louis de Broglie (1892–1987) in 1964 about the equivalence between relativistic action minimization and entropy maximization's principles. This demonstration

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is presented in his book called Thermodynamics of the Isolated Particle, which has been rarely discussed by historians of Physics. A first paper on the subject was published by Lima and Chaib [11] in *Revista Brasileira de Ensino de Física*, where the main idea of the book was discussed. Aiming to go further on the discussion about de Broglie's idea, this paper dives into a specific section of the book with special interest to contemporary Physics.

In his presentation, Louis de Broglie shows that the path that minimizes the action of a system in a thermal bath is the one that maximizes the entropy of the thermal bath. An interesting feature of this discussion is that it allows one to connect entropy and action specifically from the standpoint of Special Theory of Relativity.

Starting from relativistic mechanics and thermodynamics, Louis de Broglie shows an interesting derivation of the interconnection between these two principles. This paper has, thus, two contributions: the first one is historical, since we recover an element of Louis de Broglie late works that has not been addressed yet; secondly, we call the attention of the Physics Education community to an important contemporary topic in the relativistic formalism, which is not usually done too. In this sense, we reconstruct de Broglie's original discussion, introducing all the important concepts necessary to understand the derivation before its presentation.

2. Context of de Broglie's Discussion

In the scenario of the development of Quantum Mechanics in the decade of 1920's, Louis de Broglie's proposal of wave-like aspects of the matter was an important achievement. In fact, de Broglie received a Nobel prize in 1929 for such a contribution. He further proposed an interpretation to quantum phenomena in terms of a particle associated with a guide wave, or pilot wave, by his double solution theory [12]. However, after the 5th Solvay meeting and the establishment of Bohr, Heisenberg and Born's interpretative[1] line, de Broglie's interpretation was, in some way, abandoned; this led de Broglie to become away from the discussions on the interpretation of quantum theory in the next few years [11].

From the early discussion on the interpretation of quantum mechanics, in the decade of 1920's, until the end of the 1950's, de Broglie was retired from the hot discussions on the topic. During this time, de Broglie taught courses in Wave Mechanics at the Poincare Institute and published some notes on the fundamentals of the theory in the French academy of science *Comptes Rendus*.

[1] Although it is very common to speak about a "Copenhagen Interpretation", it is well known that their representatives diverged about specific aspects of the interpretation of the theory. All of them, though, moved away from the materialist dualist interpretation of de Broglie's original texts.

De Broglie's interpretation was revived in a new form in the decade of 1950's by David Bohm's [13] Causal Interpretation of Quantum Mechanics. Bohm's ideas motivated the work of different research groups, especially in France with Jean Pierre Vigier. With this new movement, de Broglie retakes some of his early ideas on the double solution theory that, add to those developments, led him to propose the thermodynamics of the isolated particle [14]

In this new theory, de Broglie describes the movement of an elementary particle (an electron, for instance) as a result of its interaction with a sub-quantum medium which takes the form of a kind of fluid. That fluid permeates the whole space and exchanges energy with the usual particles, as suggested by Bohm and Vigier [15]. In this sense, de Broglie argues that it is possible to describe quantum phenomena in terms of a particle in a thermic bath. In this case, on the one hand we have the mechanical description of the particle movement. On the other hand, the possible microstates assumed by the particle are described by thermodynamics and statistical physics. This situation allows one to analyze the relation between the action of the particle and the entropy of the system.

Although the main theme of de Broglie's work is the isolated particle in a subquantum medium, his discussion of the equivalence between relativistic entropy maximization and action minimization principles do not rely in the quantum feature of the system, as we will discuss.

3. Conceptual Review

De Broglie's original demonstration involves concepts from relativistic mechanics and thermodynamics. Thus, in order to understand his proposal we present a brief review on the concepts articulated before presenting his original proposal.

3.1. Hamilton's Principle of Least Action

The Principle of Hamilton, also known as the Principle of Least Action, is a variational principle in which the dynamics of a system is described in terms of the variation of a functional based on the Lagrangian function. In this description, the system has a set of n generalized coordinates q and n generalized velocities q'. The n generalized coordinates corresponds to a particular point in the cartesian hyperspace, this space has n dimensions and we call it *configuration space*. In this sense, a system's movement corresponds to the movement of a massive point describing a path in the configuration space.

In contemporary Physics' language, Hamilton's principle is described as follows: *The motion of the system from time t_1 to time t_2 is such that the line integral (called the action or the action integral)*

$$A = \int_{t_0}^{t_1} L \, dt, \tag{1}$$

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where $L = T - V$, has a stationary value for the actual path of the motion [16]. Thus, the action variable, A, changes through the path; we can suggest any trajectory to the particle; however, the real trajectory will be the one that minimizes the value of A. That is, any other trajectory that we imagine to the particle will lead to a value of A bigger than to the real one.

The term "stationary value" means that the integral has the same value at the real path as well as all neighboring paths that differ from the real one by an infinitesimal displacement. Therefore, the variation of the integral A is zero for fixed t_1 and t_2:

$$\delta A = \delta \int_{t_1}^{t_2} L\ dt = 0. \tag{2}$$

By solving equation (2)

$$\delta \int_{t_1}^{t_2} L\ dt = \int_{t_1}^{t_2} \sum_{i=1}^{n} \left(\frac{\partial L}{\partial q_i} \delta q_i + \frac{\partial L}{\partial q_i'} \delta q_i' \right) dt \tag{3}$$

with

$$q' = \frac{d}{dt} q; \quad \delta q' = \frac{d}{dt} \delta q, \tag{4}$$

substituting the above (4) in equation (3) and integrating, one obtains

$$\delta \int_{t_1}^{t_2} L\ dt = \int_{t_1}^{t_2} \sum_{i=1}^{n} \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial L}{\partial q_i'} \right) \right] \delta q_i dt. \tag{5}$$

Since the right-hand side must be zero in order to remains stationary,

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial L}{\partial q_i'} \right) = 0, \tag{6}$$

and, thus,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial q_i'} \right) = \frac{\partial L}{\partial q_i}, \tag{7}$$

which is the Euler-Lagrange equation.

We can therefore define the generalized momentum as

$$p = \frac{\partial L}{\partial q_i'}. \tag{8}$$

Since the relativistic momentum is equal to

$$p = \frac{mv}{\sqrt{1 - \beta^2}}, \tag{9}$$

then

$$\frac{\partial L}{\partial q_i'} = \frac{mq'}{\sqrt{1 - \beta^2}}. \tag{10}$$

By integration, one obtains that the relativistic Lagrangian is

$$L = -mc^2 \sqrt{1 - \beta^2}, \tag{11}$$

and Hamilton's action integral

$$A = mc^2 \int_{t_1}^{t_2} \sqrt{1 - \beta^2}\ dt. \tag{12}$$

3.2. Entropy in statistical physics

In classical thermodynamics, the entropy of a system is defined as an extensive state function. A characteristic of the entropy function is that *The values assumed by the extensive parameters in the absence of an internal constraint are those that maximize the entropy over the manifold of constrained equilibrium states* [5]. In any isolated system, the entropy maximization principle tells us in which sense the natural process must occur; that is, in a sense that the entropy of the final equilibrium state is a maximum. If we have two systems with different temperatures interacting, for instance, there will be a heat exchange from the high temperature system to the smaller one. When both systems reach the same temperature, and thus become in equilibrium, the entropy of the composed system will be the maximum possible.

By statistical mechanics, a thermodynamic system is described in terms of a set of particles interacting with each other. Its fundamental postulate states that, in a microcanonical ensemble, all the possible microscopic states in a closed system are equally probable. The number of microscopic states, Ω, in such a system is a function of its internal energy, U, volume, V, and the amount of particles N [17]: $\Omega(U, V, N)$.

Now, in the context of the second law of thermodynamics, one could say that the entropy of a system is a function of its microstates, $S(\Omega)$. If we let the above system, with entropy $S_A(\Omega_A)$, interact with some other system, with $S_B(\Omega_b)$, the composed system will reach equilibrium when

$$S_T(\Omega_T) = S_A(\Omega_A) + S_B(\Omega_b), \tag{13}$$

in which $\Omega_T = \Omega_A \cdot \Omega_B$, then

$$S_T(\Omega_A \cdot \Omega_B) = S_A(\Omega_A) + S_B(\Omega_b). \tag{14}$$

One can suggests, by adding an additional constant, k_B, that

$$S = k_B \ln \Omega. \tag{15}$$

The above tells us that the entropy of an isolated system is equal to the Boltzmann constant times the logarithm of the number of possible microstates. On the one hand, in the cases when one has discrete values, entropy is a count of microstates. In continuous cases, on the other hand, entropy is defined as a volume in phase space.

In the relativistic domain, one can easily verify that entropy is an invariant in the discrete case. That is because a numerical count remains the same in any reference system. In continuous cases, Liouvile's theorem [18] indicates that the volume in phase space is also invariant. Thus, according to the Special Theory of Relativity, entropy is defined as an invariant physical quantity. Finally, since entropy is associated with the number of microstates, the entropy maximization principle means that the equilibrium state is the most probable state.

4. Louis de Broglie on Relativistic Thermodynamics

In Relativistic thermodynamics, usual thermodynamic concepts as heat, temperature and entropy Are not necessarily absolute values but must described accordingly to the inertial reference frame they are observed.

Since this topic is not frequently approached in undergraduate courses on Physics, we will review de Broglie's approach to it, as he presented in his book [19].

First, de Broglie argues that entropy is a fundamental invariant of thermodynamics, so as the Hamiltonian action is to mechanics. That is to say, the entropy does not vary between different reference systems. However, that is not the same with temperature and heat. According to the author, deducing the relativistic variance of temperature demands some very delicate reasoning.

Imagine a body, C, at rest in the reference system R_0, being heated by an external source. In a second reference system R', the body C is observed in a uniform movement with velocity $v = \beta c$; the amount of heat transferred to the body in this reference system is Q. These systems can be observed in the following Figure 1.

Since in R the body remains with constant velocity, an amount of work, A, must be done over the body while it receives heat (since its mass is increasing). We can write the energy of the body, in the system R, as[2],

$$E = \frac{M_0 c^2}{\sqrt{1-\beta^2}}. \tag{16}$$

The internal energy of the body can increase, as a result of the heat, Q, and work, A, received: from M_0 to $M_0 + \Delta M_0$.

> In other words, the heat and work that is absorbed by the body C in motion will have increased its internal energy, which must make its proper mass increase, from the principle of inertia for energy [19].

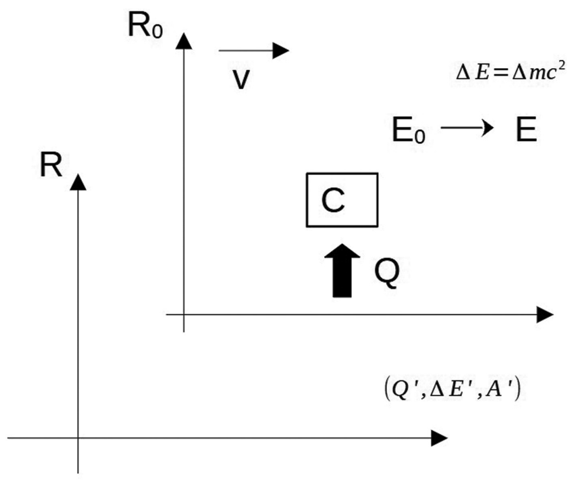

Figure 1: Representation of a body receiving heat. The phenomenon is observed in two inertial reference systems.

[2] Although, nowadays, rest, or proper, mass is not used, [21], we will follow de Broglie's original notation.

From the first law of thermodynamics, or the principle of conservation of energy, we can write

$$\frac{\Delta M_0 c^2}{\sqrt{1-\beta^2}} = Q + W, \tag{17}$$

that is, the variation in the internal energy of the system results from the received heat and work – in accordance with the conservation of energy. It is important to notice that to change its mass, the body C must have some kind of internal structure, so the energy is distributed into internal bonds or vibration modes [20].

Now, an amount of work A done over the body C means that a force F was performed so that this work was communicated to the body in the reference system R (with velocity $v = \beta c$ in relation to R_0). By integrating the force in respect to time we obtain, for the changes in momentum,

$$\int dp = \frac{(M_0 + \Delta M_0)v}{\sqrt{1-\beta^2}} - \frac{M_0 v}{\sqrt{1-\beta^2}} = \int F dt. \tag{18}$$

Since the velocity v is constant,

$$\frac{(M_0 + \Delta M_0)v}{\sqrt{1-\beta^2}} - \frac{M_0 v}{\sqrt{1-\beta^2}} = \frac{1}{v}\int F v dt, \tag{19}$$

and taking $W = \int F dt$ we have

$$\frac{1}{v}\int F v dt = \frac{W}{v}. \tag{20}$$

Thus, one obtains

$$W = v^2 \frac{\Delta M_0}{\sqrt{1-\beta^2}}. \tag{21}$$

That is, the work done on the body leads to an increase in its pseudo-kinetic[3] energy. This increase, the right hand of eq. (16), is equal to twice of the pseudo-kinetic energy. By combining (17) and (21),

$$W = Q\frac{\beta^2}{1-\beta^2}, \tag{22}$$

and then combine (21) in (22), we obtain

$$Q = \Delta M_0 c^2 \sqrt{1-\beta^2}. \tag{23}$$

That is the expression for the transformation of the heat received by the body when passing from the

[3] The expression is called pseudo vis-viva because it resembles the classical kinetic energy $mv^2/2$, while the relativistic kinetic energy is $mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$.

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reference frame R_0 to R. Comparting (11) and (23), we obtain

$$Q = -\Delta L. \tag{24}$$

That is, the relativistic heat is the negative of the variation in the Lagrangian when the body's proper mass varies in some reference system. In other words, when the body's proper mass varies in some reference system, with $v = \beta c$, its Lagrangian varies equals to the negative of the heat ceded. This is an important relation in de Broglie's arguments on the equivalence between entropy and action.

Also, the mass variation is equal to

$$\Delta M_0 = \frac{\sqrt{1-\beta^2}}{c^2}(W + Q) = \frac{Q}{c^2\sqrt{1-\beta^2}}. \tag{25}$$

From the above, according to de Broglie:

> *One sees that all of these considerations finally describe the principle of the energy of inertia, which permits one to envision variations of the proper mass of a body that result from the variation of its internal energy* [19].

Now, in relation to the reference system R_0, the transformation of heat follows

$$Q = Q_0\sqrt{1-\beta^2}. \tag{26}$$

Since the entropy is invariant, and $dS = \delta Q/T$, the absolute temperature transforms according to,

$$T = T_0\sqrt{1-\beta^2}, \tag{27}$$

and that is the temperature when one pass from the system R_0 to R.

5. De Broglie's Description of the Movement of a Particle in the Presence of a Thermostat

Suppose a very small particle in a thermal bath. The particle is very small in relation to the macroscopic system, but it still has some sort of internal structure, so its mass can vary when it receives heat. If the particle's mass change due to heat, its Hamiltonian will also change and, thus, we may find what will be its new trajectory.

The main idea of de Broglie's demonstration is the following: classically g, a particle moves in a trajectory that minimizes the action; if it receives a quantity of heat, its mass changes. From the variation in mass, the particle has a new trajectory. However, by the heat exchange, the entropy of the thermostat is reduced. Thus, the most probable trajectory must be the one that not only minimizes the action of the particle but also maximizes the entropy of the thermostat. Now, we show how de Broglie describes mathematically the situation.

In a thermostat-particle system, the total entropy of the thermostat, S, is equal to an amount of entropy coming from the thermostat S_0 which is independent of the variations in mass, plus an amount of entropy of the thermostat that is subject to the particle mass variation $S(M_0)$.

$$S = S_0 + S(M_0), \tag{28}$$

using the thermodynamic definition of entropy and the relativistic heat we have that the change in entropy is

$$\delta_{M_0} S = -\frac{\delta Q}{T} = \frac{\delta M_0 L}{T}. \tag{29}$$

The minus sign that appears in (29) is due to the fact that de Broglie defines entropy for the thermostat and not for the particle, while the heat Q is the heat that the particle receives. Thus, the entropy of the thermostat is reduced when the particle's mass increase.

Now, suppose that the particle describes a minimum action, or 'natural', trajectory, C, between A, at t_0, and B, at t_1. One can then imagine a 'varied' fictitious trajectory, C', that is close to the 'natural' one, in a sense that the points A and B, and the time t_0 and t_1 remains equal to the one of the 'natural' motion. In the graph below one can identify the initial 'natural' movement of the particle and its varied movement.

Considering, at first, the trajectory C, in which the particle does not change its mass (that is, where M_0 is equivalent to m_0, the regular value of the particle), and that the changes in the lagrangian are $[\delta L]_{M_0}$, by Hamilton's principle one can then write

$$\int_{t_0}^{t_1} [\delta L]_{M_0} dt = 0. \tag{30}$$

Recalling the conditions of global minimums of integral differential calculus, a condition for guaranteeing that the minimum of the functional is global is

$$\int_{t_0}^{t_1} [\delta^2 L]_{M_0} dt > 0, \tag{31}$$

because the second variation of L has to be greater than zero in order to guarantee the minimization of the trajectory.

Following, an important step is given by de Broglie. If one assumes that the proper mass of a particle is subject to fluctuations in its value, then one can consider a varied motion, as is C' in which both space and time intervals are equal to the 'natural' one, that is no longer imaginary, or fictitious, but with a real physical meaning. In this sense, one can assume, by instantaneous fluctuations in the proper mass during the interval of time $t_0 \to t_1$, that the fictitious, or varied, motions are physically real. By such hypothesis, we can write the motion AC'B (Figure 2), according to Hamilton's Principle, as

$$\int_{t_0}^{t_1} \delta(L + \delta L)dt = \int_{t_0}^{t_1} (\delta L + \delta^2 L)dt = 0. \tag{32}$$

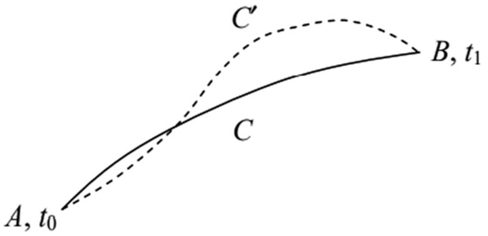

Figure 2: The body executing two different trajectories.

As the proper mass is no longer constant, and each term in (32) is a sum of two terms, one then obtains

$$\delta L = [\delta L]_{M_0} + \delta_{M_0} L. \tag{33}$$

And

$$\delta^2 L = [\delta^2 L]_{M_0} + \delta^2_{M_0} L, \tag{34}$$

where $\delta^2{}_{M_0} L$ represents the terms in $\delta^2 L$ that depends on the changes in mass and, as in the previous case, $[\delta^2 L]_{M_0}$ represent the terms that do not depend on the changes. Therefore, by substituting the above relations in eq. (32) one gets

$$\int_{t_0}^{t_1} \{[\delta L]_{M_0} + \delta_{M_0} L + [\delta^2 L]_{M_0} + \delta^2{}_{M_0} L\} dt = 0. \tag{35}$$

De Broglie assumes that the term related to the second-order variation in mass, $\delta^2{}_{M_0} L$, can be neglected and thus equation (35) becomes

$$\int_{t_0}^{t_1} \{[\delta L]_{M_0} + \delta_{M_0} L + [\delta^2 L]_{M_0}\} dt = 0, \tag{36}$$

and, according to Hamilton's principle, the first integral is zero

$$\int_{t_0}^{t_1} \delta_{M_0} L \ dt + \int_{t_0}^{t_1} [\delta^2 L]_{M_0} dt = 0, \tag{37}$$

and thus,

$$-\int_{t_0}^{t_1} \delta_{M_0} L \ dt = \int_{t_0}^{t_1} [\delta^2 L]_{M_0} dt. \tag{38}$$

As the right side must be > 0, according to (31), we know that the left side must also be > 0. The absolute value of the left side can be written as

$$\int_{t_0}^{t_1} \delta_{M_0} L \ dt < 0. \tag{39}$$

We can also write the above equation as

$$-\int_{t_0}^{t_1} \delta_{M_0} L \ dt = -(t_1 - t_0)\underline{\delta_{M_0} L} = \int_{t_0}^{t_1} [\delta^2 L]_{M_0} dt > 0. \tag{40}$$

$t_1 - t_0$ is always positive, then

$$-\delta_{M_0} L > 0. \tag{41}$$

And, according to (29), one has

$$\delta_{M_0} \mathrm{S} < 0, \tag{42}$$

which means that the entropy of the thermostat has reduced. Thus, "the natural trajectory", without receiving any energy fluctuation, is associated to the maximum entropy. As we have discussed, entropy is associated to the probability of a determinate state. So, the natural path, without being heated by the thermostat, is the most probable path. In this sense, de Broglie shows that the classical path is not obtained from an absolute law but it is simply the most probable path.

According to de Broglie,

> It then results that the entropy S is reduced in mean when one passes from the motion ACB to the motion AC′B. The entropy is therefore maximal on the natural trajectory with respect to the fluctuations, subject to the conditions of the Hamiltonian variation, and one can say that the natural motion is more probable than the varied motion. A very remarkable relation between the principle of least action and the second law of thermodynamics can thus appear [19].

Also, the author tries to provide a comparison between Action minimization and entropy maximization by referring to the concept of "negentropy": *In figurative terms, one can say that the natural trajectory follows a curve along the bottom of a valley of negentropy* [19].

An interesting historical question is how important this result was to Louis de Broglie. It is important to notice that his contribution in the early 1920s with the wave-particle duality was a result of his unification of Maupertuis and Fermat's Principles ("unifying particle mechanics and Wave Theory). Now, Louis de Broglie is trying to build Quantum Theory upon a new unification, now between Mechanics and Thermodynamics:

> In its beginnings, wave mechanics had to establish a relationship between the action of a corpuscle and the phase of its associated wave that would permit one to identify the principle of Maupertuis with Fermat's principle. Pursuing the same type of identification, the preceding theory attaches the principle of least action to the second law of thermodynamics and the increase in entropy [19].

6. Final Remarks

In this paper, we have reconstructed de Broglie's demonstration of the equivalence between the relativistic principle of least action and maximum entropy. The demonstration is presented in the book *The Thermodynamics*

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of the Isolated Particle as an essential point in his new theory.

This discussion is especially interesting nowadays, when the discussion of the relation between mechanics and thermodynamics is still a matter of debate. The didactic reconstruction that we proposed allows the theme to be discussed in undergraduate courses, for students that have already studied thermodynamics and special theory of relativity.

Moreover, this paper has an important historical contribution, since we recall an interesting result obtained by de Broglie in the context of the development of a causal interpretation for Quantum Theory in the 1960s – which has been rarely explored in the literature. We hope that the paper contributes to the motivation of news studies on this important work.

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