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Multiobjective optimization of composite plates for strength, buckling, and dynamic behavior

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Abstract: With the introduction of composite materials in the engineering field, the need for improved and cheaper designs has grown. However, multiple performance functions may be conflicting, such as the fundamental frequency, buckling load, and effective stiffness in composite plates. The individual performance functions can be optimized individually, but in doing so, it compromises the improvement of the others. Therefore, a study on multiobjective optimization tools assuming ply orientation as a design variable is implemented in this work. The angles for each ply are evaluated concerning the performance objective functions, and Pareto-Front non-dominated optimal solutions are generated. The methodology presented can be used as a reference for the best orientation for optimized designs of laminated composite plates.

1. Introduction

The use of composite materials is on a constant growth in several scientific and technological areas. There are many types of composite matrix reinforcement materials, such as carbon fiber, fiberglass, Aramid, Epoxy, just to name a few. The knowledge of the mechanical and physical behavior of these materials allows a range of structural improvements in components. Optimization processes are performed in structures built with composite materials to improve performance in terms of strength safety, dynamic behavior, buckling loads, or material costs.

This work investigates if the best fiber orientation can result in structural behavior changes without any increase in weight. The developed algorithm evaluates natural frequencies, internal stresses, and buckling load factors for composite materials based on the CLT theory. They are validated with experimental data from the literature. Later, it is expanded to multi-objective examples. Three studies are presented: the first one is a mono-objective maximization of the buckling load of a Graphite-Epoxy composite laminated with 64 layers; the second example is the maximization of the 1st natural frequency of a simply supported Graphite-Epoxy plate with 8 layers. The last example deals with the multi-objective optimization of a Graphite-Epoxy with 12 layers simply supported plate (SSSS) for (*i*) strength safety level (Tsai-Wu first ply failure criteria), (*ii*) first fundamental frequency, and (*iii*) first buckling load.

2. Theoretical basis

According to [1], optimization of a system is formulated based on a performance measure that is optimized while all other constraint requirements are satisfied. In the so-called multi-objective optimization problems, the designer may wish to optimize two or more objective functions simultaneously, stated as:

$$\begin{aligned} &Minimize \qquad f(\mathrm{X}) = \{f_1(\mathrm{X}), f_2(\mathrm{X}), \dots, f_m(\mathrm{X})\}^T \\ &Subjected\ to: \qquad g_i(\mathrm{X}) \leq 0 \quad i = 1, \dots, p \\ &h_j(\mathrm{X}) = 0 \quad j = 1, \dots, q \end{aligned} \tag{1}$$

where $\mathrm{X} = \{x_1, x_2, \dots, x_n\}^T$ is the vector of *n* design variables, and x_i are each of these variables, $g_i(\mathrm{X})$ are the *p* inequality constraints of the problem, and $h_j(\mathrm{X})$ are the *q* equality constraints of the problem. The first natural frequency equal to a defined value $\omega_1 = \omega_{lim}$, first buckling load factor less than 1, $\lambda_1 \leq 1$, safety factor, represented by the equivalent tensile strength of a composite (e.g., Tsai-Wu) less than 1, $\sigma_{Tsai-Wu} \leq 1$ are examples of such constraints. In the case of composite materials, $f(\mathrm{X})$ is the vector of *m* objective functions to be minimized simultaneously. It can represent the safety factor regarding the failure of the part, the natural frequencies, or the buckling load factor, among other objective functions. For the design variables x_i. It can comprise, depending on what you want to optimize, the angle of the fibers of the composite material, the number of layers, thickness of each layer, some elastic property of the material. Multi-objective optimization offers more freedom for design decision-making as it provides a set of equally viable solutions that meet the cost-benefit ratios of conflicting objective functions. Here the *Paretosearch* algorithm is used due to reported performance in complex multiobjective optimizations. Heuristic algorithms represent a range of algorithms that may deal with complex problems associated with non-smoothness, non-differentiability, multiple objective functions or integer design variables. In the Differential Evolution Algorithm (DEA), choose here due to reported good behavior in complex optimization, works with a population of candidate solutions. This algorithm was used in the mono-objective optimizations.

3. Results and Discussions

3.1. Example 1 - Mono-objective optimization to maximize the buckling load factor λ

This example reproduces the results of [2]. In their article, the buckling load of a composite plate is analyzed by Ansys and by Optistruct software. Experimental tests are described for different boundary conditions. They aim to find the optimal configuration of the fiber layers' orientation to maximize the buckling load and compare it to Karakaya and Soykasap, 2009 results. The LC2 plate is chosen. It contains 64 layers of graphite-epoxy composite. The material properties are $E_{11} = 127.6$ GPa, $E_{22} = 13.0$ GPa , $G_{12} = 6.4$ GPa, $\nu_{12} = 0.3$. The laminate plate is symmetrical and cross-ply, with a total thickness of 8.128 mm. It is simply supported on 4 edges, with dimensions *a* = 50.8 mm and *b* = 50.8 mm, and a biaxial load of $N_x = 1$ N/m and $N_y = 1$ N/m applied. The optimal solution found by [3] is $[\pm 45_{16}]_s$, with a buckling load factor of *λ* = 242823.1. Using Ansys Optistruct, [2], found *λ* =247942.0 for the same layup. Here, the DEA algorithm was used, assuming 16 discrete cross-ply fiber orientations as design variables $[\pm\theta_1 \ldots \pm\theta_{16}]_S$ and heuristic parameters: *n* = 10, *CR* = 0.8, *F*= 1.3, and the maximum number of iterations of 500. For the design variables, it was assumed discrete angle values $\theta_i \in \{0^0, 30^0, 45^0, 60^0, 90^0\}$, the same used in the literature. The obtained result was $[\pm 45_{16}]_s$, which results in *λ* = 242844.36 that is the same orientation found by [3]. Figure 1(a) shows the result obtained for design variables and objective function.

To evaluate the robustness of the DEA algorithm, 20 iterations of the algorithm were carried out. Results for the best value of the objective function and the respective design variables were saved, as well as the mean and standard deviation of the population. This analysis was performed for the LC2 by [2], based on the same criteria for the previous analyses. The 20 independent runs with the DEA, presented a global best objective function of *λ*=242822.18, a mean objective function of *λ*=242833.37, and a standard deviation of 2.253730×10^{-12}, which shows the robustness of the algorithm.

3.2. Example 2 - Mono-objective optimization to maximize the 1st. natural frequency ω_1

The second example is based on the example of [5], and [6]. It is about maximizing the 1st. natural frequency ω of a simply supported 8-ply graphite-epoxy composite laminate plate. The total thickness of the laminate is h =10 mm. The density is $\rho = 1450$ kg/m³. The plate has dimensions $a = 100$ mm and $b = 100$ mm. The properties used for this example are $E_{11} = 138$ GPa, $E_{22} = 8.9$ GPa, $G_{12} = 7.1$ GPa , $\nu_{12} = 0.3$. For comparison purposes, the normalized frequency will be used, as indicated in that work: $\Omega = \omega a^2 \sqrt{\rho/(E_{22}h^3/12(1-\nu_{12}\nu_{21}))}$. The problem is treated with 8 discrete design variables with lateral constraint for angles $-90^0 \le \theta_i \le 90^0$, with a variation of 5° increment. The best solution found by [5] were respectively $[-45^o/45^o/45^o/45^o]_s$ with $\Omega = 53.77$ and $[45^o/-45^o/-45^o/-45^o]_s$, with $\Omega = 56{,}32$. The small difference is attributed to the theory used for the simulation (FSDT in [5]). To solve the problem, the DEA was used assuming 8 discrete fiber orientations as design variables $[\theta_1 \ldots \theta_8]_S$, and the following DEA heuristic parameters: *n*=10, *CR*=0.8, *F*=1.3, and a maximum number of 500 iterations. This resulted in $[-45^o/45^o/45^o/45^o]_s$, with *Ω*=54.082, which is better than the result of [5]. Looking at the results of the final population of the algorithm, other orientations also resulted in a dimensionless frequency Ω equal to the best found so far, such as $[45^o/-45^o/-45^o/-45^o]_s$ or $[-45^o/45^o/-45^o/45^o]_s$, indicating that this problem has more than 1 optima. Figure 1(b) shows the result over the iterations for fiber orientation and objective function. Twenty independent runs of the algorithm were carried out to analyze the robustness of the DEA algorithm. The results for the best value, the mean and standard deviation of the population objective functions, and design variables were saved. This resulted in a global best objective function of Ω=54.082, a mean objective function of Ω=54.083, and a standard deviation of 1.530042×10^-6 and this shows that the algorithm is robust.

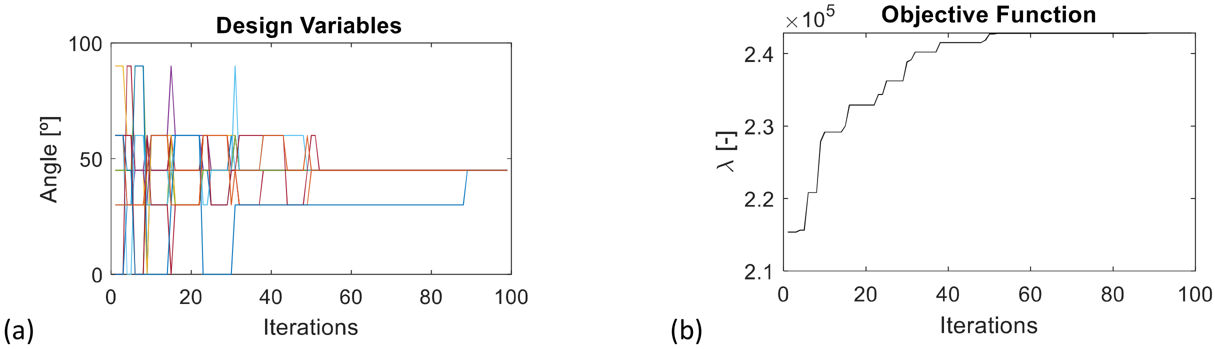


Fig. 1. (a) Convergence of Fiber angles and (b) objective function (λ) for Example 1.

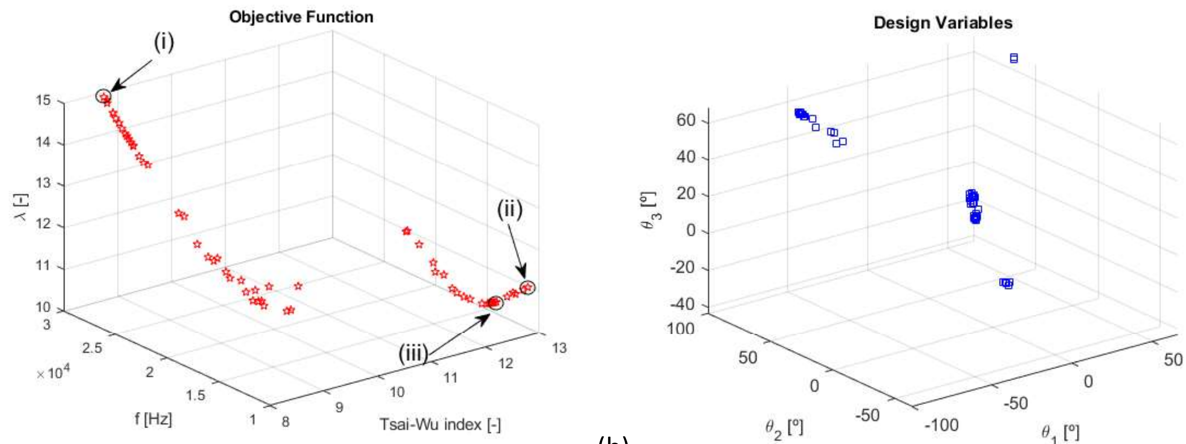
3.3. Example 3 - Multi-Objective Optimization for Buckling Load Factor, 1st. Natural Frequency and Tsai-Wu Safety Index

In this example, multi-objective optimization of a graphite-epoxy composite laminate (12 sheets) is performed assuming (*a*)maximization of the safety coefficient for Tsai-Wu, (*b*)maximization of the first fundamental frequency, and (*c*)maximization of the buckling load factor. A plate simply supported on the 4 edges will be used, with the properties used for this example in Table 3. The plate dimensions are: a = 100 mm and b = 100 mm, and a total laminate thickness of h = 1 mm. The limiting stresses for the Graphite-Epoxy material for the Tsai-Wu failure criterion are: $(\sigma_1^T)_{ult}$ = 1.5 GPa,$(\sigma_1^C)_{ult}$ = 1.5 GPa, $(\sigma_2^T)_{ult} = 40$ MPa, $(\sigma_2^C)_{ult} = 246$ MPa $(\tau_{12})_{ult} = 68$ MPa. A biaxial compressive load of $N_x = 1000$ N/m and $N_y = -2000$ N/m. The material properties assumed are $E_{11} = 138\ GPa$, $E_{22} = 8.9\ GPa$, $G_{12} = 7.1\ GPa$, $\nu_{12} = 0.28$, $\rho = 1380\ \mathrm{kg/m^3}$. To solve the problem, the *paretosearch* algorithm is used, and 3 design variables is assumed for fiber orientations so that the 12-layer laminate is obtained as $[\pm\theta_1, \pm\theta_2 \pm\theta_3]_S$. A maximum number of 10^4 function evaluations is defined.

It is assumed ply-angles θ_i in the interval $[-90^0, 90^0]$. Figure 2(a) and (b) are the results for the Pareto frontiers and corresponding design variables. The computational time for the analysis of this case was about 50 seconds. The results of the extremes of the Pareto frontier are listed to check the extremes of the objective function. (*i*) Minimum Tsai-Wu, maximum natural frequency, and maximum buckling load factor. (*ii*) Maximum Tsai-Wu, minimum natural frequency, and intermediate buckling load factor. (*iii*) Intermediate Tsai-Wu, intermediate natural frequency, and minimum buckling load factor.

The (*i*) (*ii*) or (*iii*) index represents the same extreme points in each of the studied cases. In case A, the values found for the 3 variables θ_n and the 3 objective functions $f_n(\mathrm{x})$ are shown in Table 1, the program was executed 5 times, and an average of the results was obtained. To ease the visual interpretation of the tables, the red color represents a maximum, the green color represents a minimum, and the yellow color represents an intermediate value.

To obtain results for different situations and evaluate the obtained results, solutions with different input parameters of the problem were assessed. In case B, dimension a was replaced by 200 mm. In this and other cases, the other parameters are kept unchanged. The results obtained for case B are also found in Table 1. For case C, it is set a = 300 mm.



(a) (b)

Fig. 2. (a) Pareto frontier and (b) design variables for multi-objective optimization of the composite plate.

Table 1. Objective function results for extreme points for cases A and B.

	Case A				Case B			
Point	Orientation [°] $\theta_1; \theta_2;\ \theta_3$	$f_1(\mathrm{x})$ [-]	$f_2(\mathrm{x})$ [Hz]	$f_3(\mathrm{x})$ [-]	Orientation [°] $\theta_1; \theta_2;\ \theta_3$	$f_1(\mathrm{x})$ [-]	$f_2(\mathrm{x})$ [Hz]	$f_3(\mathrm{x})$ [-]
(*i*)	$68.62; -69.80; -69.16$	8.68	31729.84	15.60	$-69.59;\ -71.06; 70.77$	8.68	34082.37	16.40
(*ii*)	$37.25; -35.27; 34.87$	12.96	10268.51	11.37	$36.73;\ 34.95; -35.31$	13.07	10286.31	11.35
(*iii*)	$39.66; -37.98; 37.16$	12.24	10565.75	10.94	$-39.84;\ 37.92; -37.20$	12.20	10587.18	10.93

Finally, in the case D, the 3 design variables are replaced by a symmetric layup $[\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]_S$. The algorithm now assumes 6 design variables to perform the Multiobjective Optimization. The results for case D can be found in Table 2.

Table 2. Objective function results for extreme Pareto Frontier points for cases C and D.

	Case C				Case D			
Point	Orientation [°] $\theta_1; \theta_2; \theta_3$	$f_1(\mathrm{x})$ [-]	$f_2(\mathrm{x})$ [Hz]	$f_3(\mathrm{x})$ [-]	Orientation [°] $\theta_1; \theta_2;\ \theta_3; \theta_4; \theta_5; \theta_6$	$f_1(\mathrm{x})$ [-]	$f_2(\mathrm{x})$ [Hz]	$f_3(\mathrm{x})$ [-]
(*i*)	$67.96\ ; -69.02; -68.99$	8.69	30579.56	15.26	$67.46;\ 69.01; 68.34; 77.81; 99.37; -25.31$	8.69	29723.80	14.96
(*ii*)	$36.69\ ; 34.60; 34.32$	13.15	10242.91	11.34	$37.91; -35.91; 35.48\ ; -18.22; -75.09; -32.87$	12.76	10287.41	11.09
(*iii*)	$39.62; 37.79; 37.16$	12.24	10542.80	10.93	$-39.78; 37.87; 37.19; -61.45; 89.29; 89.89$	12.21	10573.80	10.93

Note that in all the data presented in the previous table, points (*i*), (*ii*), and (*iii*) are representative of the extremes found in the Pareto Frontier. It can also be seen from cases C to D (Table 2) that, when more freedom was allowed for the ply-angles (e.g., no cross-ply or symmetry), the gains of the objective functions were minimal, presenting practically the same results for one case (case D), more complex in terms of optimization (6 design variables). The results for each objective function can be better compared if the maximum values for each of the objective functions in the four cases are reported. Starting from the variation of each one regarding case A, the differences for each of the points in the Pareto Frontier $f_n(\mathrm{x})$ is found in Table 3.

Table 3. Comparison of cases A, B, C, and D.

	$f_1(\mathrm{x})$ [-]	Differences	$f_2(\mathrm{x})$ [Hz]	Differences	$f_3(\mathrm{x})$ [-]	Differences
A	12.96	-	31729.84	-	15.60	-
B	13.07	-0.82%	34082.37	-7.41%	16.40	-5.10%
C	13.15	-1.46%	30579.56	3.63%	15.26	2.22%
D	12.76	1.57%	29723.80	6.32%	14.96	4.12%

*Difference=$100(f_1 - f_n)/f_1$

4. Conclusions

It is concluded that choosing the fiber orientation directly affects the values of objective functions. The designed composite withstand application-specific strength, buckling conditions, and dynamic requirements. The study also predicts that the number of design variables had little impact on the results, all of which have variations within an absolute value of less than 7.4% concerning the original objectives of case A.

Declaration of Competing Interest

The authors declare no conflict of interest.

CRediT author statement

Sager, G.; Hofstadler, F.: Methodology, Software, Writing-original draft. **Grotti, E.; Santana, P.B.; Casas, W.J.P.; Gomes, H. M.**: Conceptualization, Formal Analysis, Supervision, Writing-review.

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