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Lateral dynamics vehicle model: an analysis on different approaches with increasing level of model complexity

MEMÒRIA

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Abstract

This thesis presents an analysis on different approaches with increasing level of model complexity with respect to lateral vehicle dynamics. In recent years, there has been an increasing interest in developing accurate and reliable models for lateral vehicle dynamics, in order to improve vehicle stability and control.

The analysis begins with a review of the existing literature on lateral vehicle dynamics, and the different mathematical and physical modeling approaches that have been proposed. These include linear and nonlinear models, as well as more complex models that take into account factors such as tire dynamics and suspension dynamics, among others, that are not detailed in this study.

Next, the thesis presents a comparative analysis of the different approaches based on machine learning, using both simulation and experimental data from scientific literature. The analysis focuses on the accuracy and predictive power of the different deep learning models, as well as their computational efficiency and ease of implementation.

The results of the analysis indicate that while more complex models can provide more accurate predictions of lateral vehicle dynamics, they also require significantly more computational resources and can be more difficult to implement. Therefore, the results of several publications indicate that deep learning models can provide highly accurate predictions for lateral vehicle dynamics, and can be trained with relatively small datasets.

Overall, this study provides a comprehensive analysis on the use of machine learning techniques to develop competitive and high performance controllers for lateral vehicle dynamics and state estimation, highlighting the potential contribution in autonomous driving and vehicle stability and control that these methods will allow to achieve in the next decades.

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1 Preface

1.1 Origin of the project

Since a very young age, a particular enthusiasm for the automotive industry resides into myself. There is no certain explanation of why or what guided me to this feelings about cars and motorcycles, because there is no precise answer as I feel attraction not only for driving them but also for its design, from the outer point of view yet from the inner side too.

Last year while I was taking Automatic Control as a subject in the university, I met Stefano, who was my practical class teacher. There, he showed us how he was implementing his knowledge in control for his job at Mclaren. This really kept my attention, as all the examples and illustrations were real cases executed into Mclaren vehicles. This really helped me to a better understanding of the matter and I could master it until the point I obtained the best grade in the practical exam.

This lead me to speak with Stefano about my current situation (I was looking for an internship somewhere interesting), but I didn't work for external reasons. Then Stefano proposed me to take him as the final project tutor and do something similar in order to get familiarized with this topic, as I had never been in touch before with vehicle dynamics and automotive from a technical point of view.

I was delighted for the offer and I rapidly accepted. Finally, Stefano suggested me to do an approach on different lateral vehicle dynamics models, to combine it with the knowledge I had just acquired in Automatic Control. There is where this project was born.

1.2 Previous requirements

As it is mentioned before, I had little expertise in the field of vehicle dynamics, almost absent. To mitigate this, I studied in detail several publications related to this topic, as it is shown in section 3. Some examples of it are Van Aalst dissertation [1] and Isermann literature [2]. Once a greater understanding of the subject was achieved, it was proceeded to start developing this thesis.

2 Introduction

2.1 Research objectives

The principal objectives of this research are resumed:

1. Familiarize with the most basic concepts of vehicle dynamics which imply vehicle and tire forces and moments applied, tire models, vehicle models, state estimators, time discretisation, parameters involved...

2. Introduce the existing mathematical and physical models for vehicle lateral behaviour and state estimation that will be useful to comprehend the background of all the machine learning applications into this field.

3. Master the most general aspects of artificial intelligence and machine learning, and introduce different techniques such as neural networks and its by-products deep learning and reinforcement learning.

4. Investigate the effectiveness of deep learning techniques for modeling controllers for lateral vehicle dynamics and state estimation.

5. Explore the impact of different model approaches in real life situations.

6. Develop a comprehensive understanding for training and testing different models with a wide variety of backgrounds and outcomes.

7. Evaluate the performance of every model and analyse the outcome from each approach given.

8. Investigate and learn from other professional engineers work to enhance my knowledge in this field and enrich this literature as much as possible.

2.2 State of art

This thesis is based on a literature review from several publications in the field of lateral vehicle dynamics and its approaches in deep learning for lateral control and state estimation.

3 Previous Study

In this section a previous study is made in order to get in touch with the basic concepts of vehicle dynamics, starting with a deep study into [1] and [2] to know about tire and vehicle behaviour and vehicle estimators, followed by an introduction into the vehicle dynamics with the longitudinal and lateral tire forces.

This previous research will allow to acquire the most general concepts from vehicle dynamics for the study of the lateral vehicle behaviour models and the later application of machine learning and neural networks for state estimators and vehicle dynamics controllers.
In the first section, an introduction to vehicle dynamics is made, making emphasis on tire and vehicle behaviour and its properties.
Firstly, the most basic aspects as forces and torques applied to the tire are introduced, followed by some common tire models. Later on, it is delved scantily into vehicle estimators to get acquainted with this topic.

3.1 Vehicle dynamics introduction

Vehicle dynamics is defined as the study of the vehicle motion. In the particular case of this thesis, the focus is being put on the factors like forces and torques, and the parameters like cornering stiffness or steering angle, that play a role in the lateral motion of the vehicle, in other words, the lateral vehicle dynamics.
A very important topic to take into account is the tire characteristics, as the tire is most direct implicate between the vehicle and the road surface, thus, one of the principal factors to considerate for the lateral vehicle behaviour.

To describe the tire-road interaction, it is firstly necessary to draw the coordinate frame which is directly attached to the center of the contact point between the tire and the road surface. This will allow to indicate the direction of each force, torque or angle that is applied to the tire during its motion. The tyre orientation is given by two angles: the camber angle γ and the sideslip angle α. The camber angle is the angle between the tire plane and the vertical plane measured about the x-axis. The sideslip angle is the angle between the wheel velocity vector v and the x-axis measured about the z-axis. It is given as $tan\alpha = Vy/Vx$, with V_x and V_y, respectively, the components along the x- and y-axis of the wheel velocity vector $V = [V_x V_y]^T$.

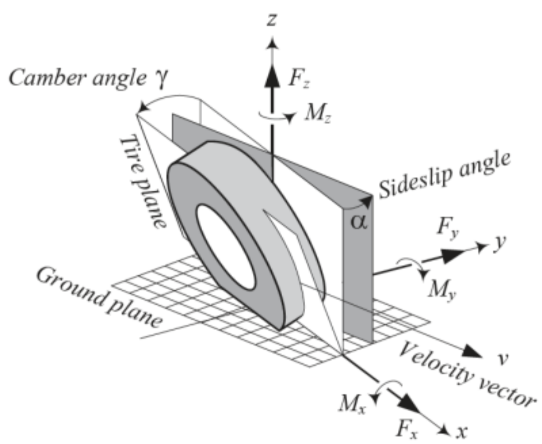

Figure 1: Tire coordinates axes example from [1]

This tire system consists in six equations system with three forces and three torques:

1. *Longitudinal Force F_x:* It is the force along the x-axis. It is positive when the car is accelerating, i.e. when a positive torque is applied and vice versa.

2. *Lateral Force F_y:* The force acting along the y-axis. It is positive when turning counterclockwise (left corner) and negative when turning clockwise (right corner).

3. *Vertical Force F_z:* The force along the vertical z-axis. This force is always positive.

4. *Overturning Moment M_x:* The moment about the x-axis. It is the moment required to tilt the tire.

5. *Rolling Resistance Moment M_y:* The moment about the y-axis. It arises from the vertical tire force that acts at a point shifted forward from the centre of the contact patch.

6. *Aligning Moment M_z:* It is the moment about the z-axis. It arises from the lateral force that acts at a point shifted backward from the centre of the contact patch.

The forces and torques that have a higher effect for the vehicle handling are the longitudinal and lateral tire forces F_x, F_y and the aligning moment M_z. This happens because those three factors are the direct outcome of the vehicle motion, in other words, when the vehicle accelerates the force is applied in the longitudinal axis, as the friction of the road surface opposes and makes resistance. A similar phenomenon occurs when the vehicle is driving through a curve, the grip between the tire and the road also applies a lateral force in the y-axis. And finally, the aligning torque is taken into consideration because is the resultant direction of the vector product between each of the latter forces mentioned and the possible distances in the 2-dimension plane that the x-axis and the y-axis describe.

As it is expressed by van Aalst in [1], the longitudinal forces F_x and F_y are developed by a shear mechanism. The shear mechanism involves a deformation of the tire tread as they deflect to develop and sustain the forces. This results in the slip velocity:

$$v_s = [v_x - v_c \; v_y]^T$$

with v_c being the circumferential velocity of the tyre:

$$v_c = \omega r_e$$

where the the angular velocity is given by ω, r_e being the rolling effective radius which is defined as the ratio v_x/ω_0 where ω_0 is the wheel angular velocity of freely rolling wheel. r_e lies between the unloaded radius r_g and the loaded radius r_l:

$$r_g > r_e > r_l$$

Tire slip is defined by normalizing the slip velocity with a reference velocity:

$$\sigma = [\sigma_x \; \sigma_y]^T = \frac{v_s}{v_c}$$
$$\kappa = [\kappa_x \; \kappa_y]^T = \frac{v_s}{v_x}$$
$$s = [s_x \; s_y]^T = \frac{v_s}{v}$$

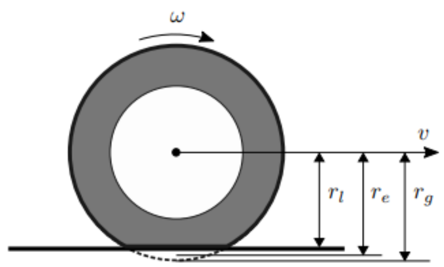

Figure 2: Radius comparison

with v being the magnitude of the wheel velocity $v = \sqrt{v_x^2 + v_y^2}$

If we take a look into the results obtained in a simulation done by van Aalst [1] in his dissertation, it is clearly observable that the vertical load (F_Z) has a direct impact on the vehicle behaviour and hence, on the tire slip.

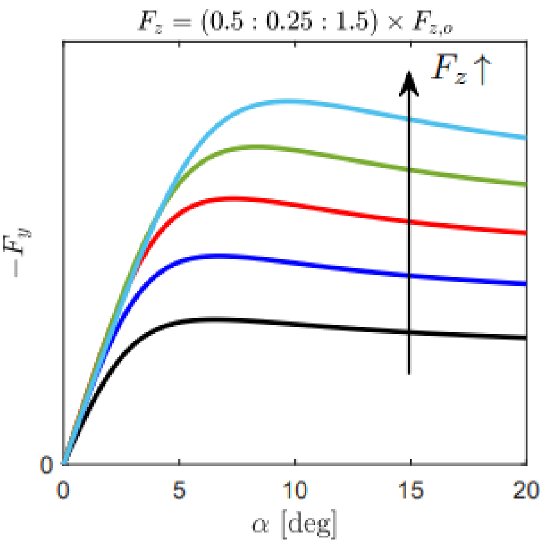

Figure 3: Lateral force versus slip angle in dependence on the vertical load

Figure 3 really shows a captivating phenomenon, where the lateral force (F_Y) decreases as the slip and the vertical load increase due to the fact that the lateral force, explained in 3.6 is commanded by the coefficient friction between the road surface and the tire which allows a maximum value before the tire blockage and the following sliding.

So as to describe the tire behaviour and define a tire model, many discussions have been held in order to develop a tire model that best fits every possible situation, thus, the main approaches fashioned are based on:

1. *Experimental data only:* Fitting full scale tire test by regression techniques, e.g the Magic Formula tire model.

2. *Using similarly methods:* Distorting, re-scaling and combining basic characteristics obtained from measurements.

3. *Simple physical methods:* Using simple mechanical representation, possibly closed form solution, e.g the "Brush" type tire models

4. *Complex physical models:* Describing tires in greater detail, computer simulation, finite element method.

3.1.1 The Linear tire model

The principal objective of the Linear tire model is to represent the tire/vehicle behaviour for small slip angles. This model approximates the tire force as a linear function with a slope equal to the tire stiffness.

$$
\begin{aligned}
F_x &= C_x \lambda \\
F_y &= -C_y \alpha
\end{aligned}
$$

Note that this model assumes pure slip conditions and does not account for the effects of vertical load. As explained in [1], pure slip is defined as the situation in which either longitudinal and lateral slip occurs in isolation.

3.1.2 The Brush tire model

The Brush tire model consists of a row of elastic bristles that touches the road plane and can deflect parallel to the road surface.

The compliance represents the elasticity of the combination of carcass, belt and actual tread elements of the real tire. The tip of the bristle will adhere to the ground as long as the available friction allows but will start sliding if the maximum friction is reached.

The Dugoff model is one of the most well-know brush tire models, it is often used to calculate longitudinal and lateral forces under pure longitudinal slip and pure side slip conditions. It assumes a uniform pressure distribution in the contact patch, a constant coefficient of friction of sliding rubber and a rigid carcass.

This model represents nonlinear tyre behaviour with minimal complexity and requires only two parameters: the longitudinal or lateral tyre stiffness, C_x or C_y, respectively, and a single tyre-road friction coefficient μ. This makes it particularly suitable for low-fidelity simulation, and for estimation and control purposes.

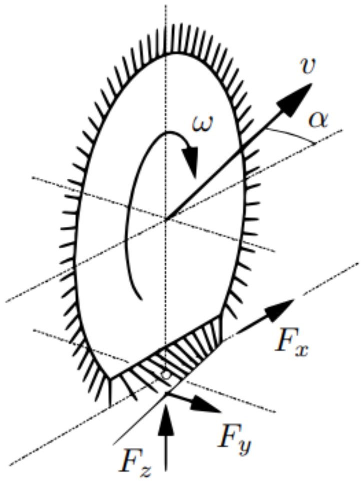

Figure 4: The Brush tire model

3.1.3 The Magic Formula model

It belongs to the first category mentioned in 3.1. The nickname "Magic" is due to the fact that Pacejka did not use any physical basis in particular to develop its formula, yet it fits a wide variety of tire options and road conditions. Tire coefficients are used to generate equations showing how much force is generated for a given vertical load F_Z on the tire, camber angle γ and slip angle α.

$$y = D\, sin(C\, arctan(B\, x - E\, [B\, x - arctan(B\, x)]))$$

with

$$Y(x) = y(x) + S_V$$
$$x = X + S_H$$

being B the stiffness factor, C the shape factor, D the peak value, E the curvature factor, S_H the horizontal shift and S_V the vertical shift.

These parameters are dependent on the physical properties of the tire, but also on the normal force, the wheel camber angle and possibly on the inflation pressure.

The tire stiffness is given by:

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = BCD$$

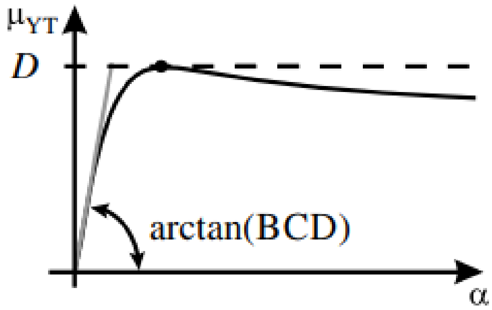

Figure 5: Pacejka Magic Formula representation

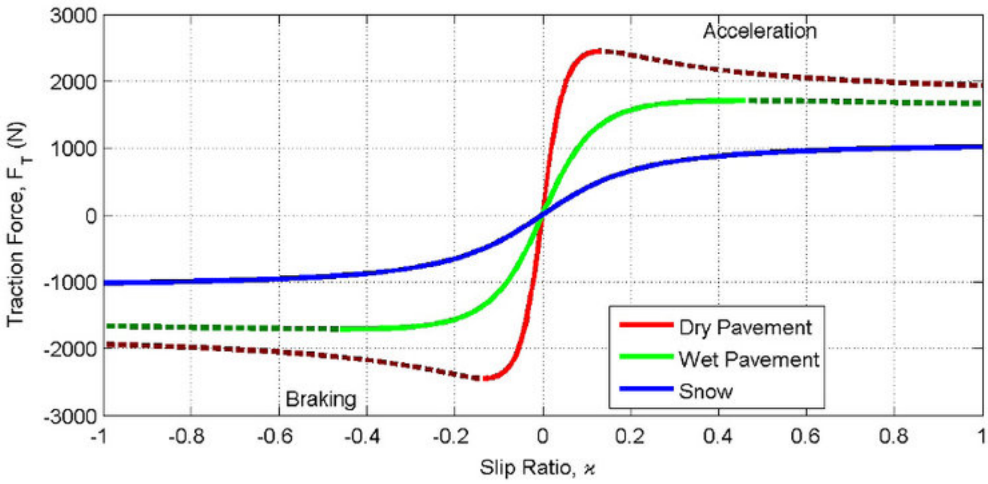

Figure 6: Example of the Magic Formula for different road surfaces

In figure 6, it is shown the difference obtained in a simulation with the same vehicle in different road conditions.
It is clearly observable that as the road conditions get rough, e.g. when the road is wet or snowy, the peak value for the traction force decreases considerably, thus, the more it decreases, the less grip the vehicle will have. Therefore, driving in these conditions is harder than in dry asphalt. This is one of the main reasons why a good set of tires has an influence in vehicle behaviour.

3.2 Model based state estimation

In control, a state estimator provides an estimate of the internal state of a given real system, from measurements of the input and output of the real system.
As van Aalst [1] says in his dissertation, it is unrealistic to assume that everything can be measured, sometimes it is impossible or too costly. That is why state estimators are used.
When using estimators there are some disturbances such as modelling errors that cause the model to not behave exactly like the real vehicle. These uncertainties have led to many different approaches (See [2] for further details).

3.2.1 Introduction to state space representation

According to multiple sources, the most general form of the state space representation is written as two functions:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x, u) \end{aligned}$$

where x is the state vector, u the input vector and y the measurement (output) vector.
The f-function is a continuous time state update function that describes the evolution of the states throughout the time while the h-function is the measurement function.
This state space representation can be written in a matrix system where the matrix A is the system matrix, B is the input matrix, C goes for the output and finally D is called the feed-through matrix:

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \\ y &= Cx(t) + Bu(t) \end{aligned}$$

This system is for continuous time invariant, hence, for continuous time variant, the matrix system would be described as:

$$\begin{aligned} \dot{x} &= A(t)x(t) + B(t)u(t) \\ y &= C(t)x(t) + B(t)u(t) \end{aligned}$$

As we have learned in Automatic Control, although time is continuous, a time discretisation is needed for the controllers implementation, thus, the state space system written in continuous time must be adapted to discrete time:

$$\begin{aligned} x_{k+1} &= f_d\left(x_k, u_k\right) = A_d x_k + B_d u_k \\ y_k &= h\left(x_k, u_k\right) = Cx_k + Du_k \end{aligned}$$

Same happens for discrete time as there can be time-invariant and time-variant systems. For the case of the time-variant system, the matrices A,B,C and D depend on the k-th state too.

To change from continuous time to discrete time an integration system is required. Van Aalst proposes a scheme for time invariant systems given as:

$$x(t) = e^{A(t-t_o)}\, x(t_o) + \int_{t_o}^{t} e^{A(t-\tau)}\, \beta\, u(\tau)\, d\tau,$$

where t_o is the initial time.
It is often assumed a zero order holder (zoh) for the input, becoming then $u = u_k$, therefore, the equation reduces to:

$$x(t_k + \Delta t) = e^{A\Delta t}\, x(t_k) + A^{-1}(e^{A\Delta t} - I)Bu_k,$$

Even though these systems are valid, the most used scheme is the Euler's scheme, as it is known for being the simplest one. It is given as:

$$x_{k+1} = f_d(x_k, u_k) = x_k + f(x_k, u_k)\Delta t,$$

To delve into state-estimation, linear filtering is the next step. This technique permits to estimate a non-measurable unknown state of a dynamic linear system. The Kalman filter is one of the most well-known examples of linear filtering used for vehicle dynamics state estimation.

3.2.2 Observability

To define an estimator, it is previously necessary to examine its observability, which will allow to determine if the information that provides is sufficient to estimate the state of the model.
Rudolf Kalman defines the observability as the capacity to measure how internal systems states of a given system can be deduced from its outputs.

Hence, observability is described as a system property and it is independent of the estimator type. Van Aalst describes the observability in both continuous and discrete time:

> *A continuous time system* is observable if for any initial state x_0 and final time $t > 0$ the initial state x_0 can be uniquely determined by knowledge of the input $u(\tau)$ and output $y(t)$ for all $\tau \in [0, t]$.
>
> *A discrete time system* is observable if for any initial state x_0 and some final time k the initial state x_0 can be uniquely determined by knowledge of the input u_i and output y_i for all $i \in [0, k]$.

"If a system is observable, then the initial state can be determined, and if the initial state can be determined, then all states between the initial and final times can be determined".

Observability is needed to develop vehicle dynamic estimators. Van Aalst [1] delves into the observability tools such as the rank criterion, the Popov-Belevitch-Hautus criterion and the observability Gramian.

3.3 Vehicle dynamics estimation

Using the knowledge of state estimation and the advantage that inventions like the Kalman filter provides us, some other parameters and characteristics of vehicle dynamics can be estimated and many models for estimation have been developed.

As it is mentioned previously, the principal forces and moments that govern the vehicle handling are:

1. The longitudinal force (F_X)
2. The lateral force (F_Y)
3. The aligning moment (M_Z)

In the next three sections the forces and torques between the tire and the road are studied in more detail for a better understanding of the vehicle dynamics basis.

3.4 Longitudinal tire forces

As it is known from Continuum Mechanics, even though it cannot be seen by the eye, every material (even the most rigid ones) experiences a certain deformation when a force is applied into it, thus, the treads of the rubber (an elastic material) do so. That is why when a force is applied to a tire, e.g when braking, the sliding phenomenon appears.
This relative slip between the tire and the vehicle is measurable, given for braking in the longitudinal direction as:

$$S_{X,b} = \frac{\Delta v_{XT}}{v_{XT}} = \frac{v_{XT} - v_{XW}}{v_{XT}} = \frac{v_{XT} - r_{dyn}\omega_w}{v_{XT}},$$

and for driving (traction):

$$S_{X,d} = \frac{\Delta v_{XT}}{v_{XW}} = \frac{v_{XW} - v_{XT}}{v_{XW}} = \frac{r_{dyn}\omega_w - v_{XT}}{r_{dyn}\omega_w},$$

being v_{XT} the longitudinal tire traction velocity and v_{XW} the rotating velocity of the wheel.

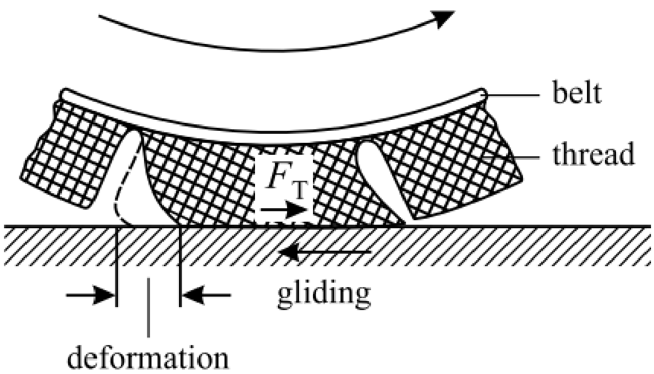

Figure 7: Rubber treads deformation

The tire slip is always positive and stable when $0 < S_x < 1$ for both S_{Xb} and S_{Xd}, and it becomes unstable when reaches a value for $S_{Xb} = S_{Xd} = 1$, then the tire loses the grip with the road. This is the equivalent for a locked wheel when braking and a spinning wheel when accelerating. Depending on the tire slip, the friction coefficient is obtained as:

$$\mu_x(S_x) = \frac{F_{XT}}{F_{ZT}},$$

where F_{XT} is the tangential tire force and F_{ZT} is the vertical load force.
Note that μ_x has a stable range within $0 \leq S_X \leq S_{Xcrit}$ and an unstable range within $S_{Xcrit} \leq S_X \leq 1$.

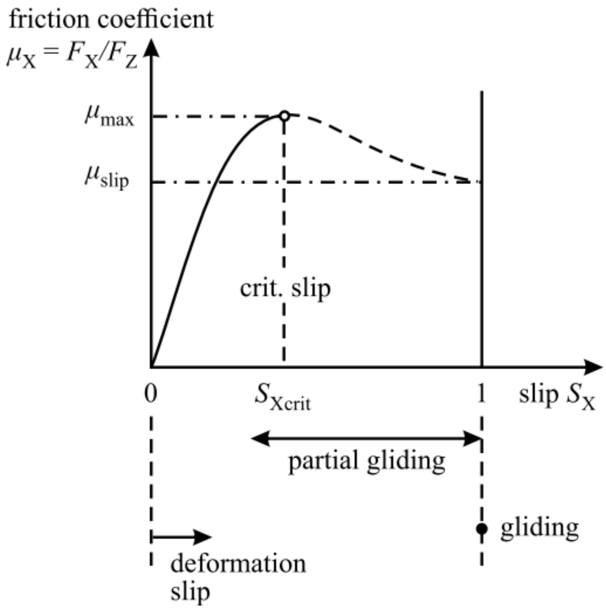

Figure 8: Friction coefficient vs slip

In figure 8 the friction coefficient in dependence on the tire slip is shown. In this graphic it can be perfectly seen the aim of the previous paragraph explanation, the maximum value for μ_x is obtained when $S_X = S_{Xcrit}$, then the system becomes unstable.
Figure 9 depicts μ_x-slip curves in different road conditions. Obviously, the highest values appear for the dry asphalt conditions, followed by other types of dry roads but with less grip, e.g cobblestones or concrete. Finally, the lowest values appear for wet roads and ice.

To determine the μ-slip curves there are different approximations, being one of the most recent and detailed approximations made by *"Pacejka"*, called *"The Magic Formula"*, as explained in 3.1.3. Its model for the longitudinal tire force is given as:

$$F_{XT} = D_X\, sin[C_X\, arctan(B_X S_X - E_X(B_X S_X - arctan(B_X S_X))]$$

with

$$D_X = \mu_{XMAX} F_Z, \qquad B_X = \frac{C_{SX}}{C_X D_X}$$

and

$$E_X = \frac{B_X S_{Xcrit} - tan(\frac{\pi}{2C_X})}{B_X S_X - arctan(B_X S_{Xcrit})}$$

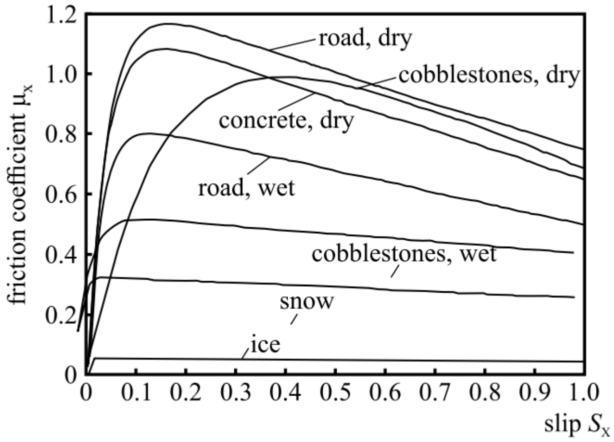

Figure 9: Friction coefficient in dependence on the road conditions

3.5 Lateral tire forces

For the case of Lateral tire forces, the same phenomenon of sliding occurs but in this case it appears during cornering, thus, the forces are applied in the lateral direction of the y-axis. That is why instead of blocking or spinning tires it is talked about the concepts of under-steering and over-steering.
The lateral tire slip is expressed by the tire side slip angle α.
As lateral tire forces affect when turning, the scenario varies from 1-dimension to 2-dimension. Then, the velocity vector must be expressed in two dimension when talking about side slip or lateral slip, given as:

$$S_Y = \frac{v_{YT}}{v_{XT}} = \frac{v_T\ sin(\alpha)}{v_T\ cos(\alpha)} = tan\ (\alpha),$$

being $S_Y = 1$ for $\alpha = 45°$.

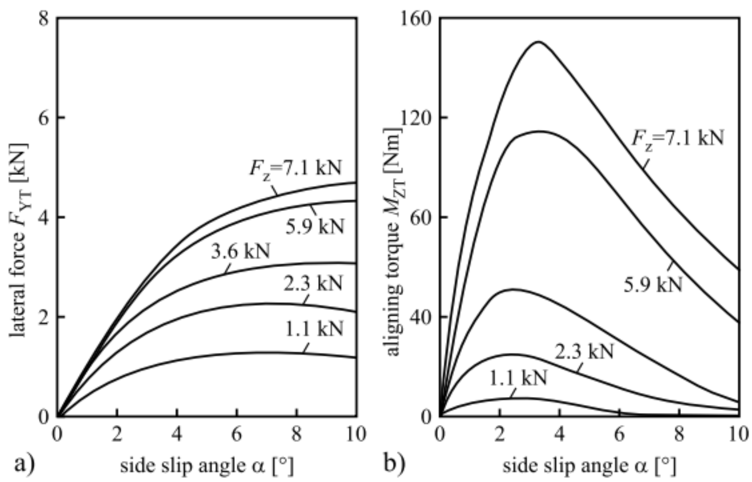

Figure 10: Lateral force and aligning torque in dependence on side slip angle and vertical load

Figure (a) from 10 depicts the lateral force F_{YT} in dependence on the side slip angle α for different vertical loads F_Z. For small slip angles $0° \leq \alpha \leq 2°$ the Lateral force is practically the same for every vertical load, but as α increases, the lateral force does so. For a certain value of α ($8° \leq \alpha \leq 10°$), even the vertical load increases, the lateral force does not increase anymore as it reaches its maximum value.

The aligning torque M_Z has a completely different behaviour for the variation of α. M_Z reaches its maximum value when $\alpha \approx 3°$.

For small side slip angles the lateral force can be described linearly by $F_{YT} = c_\alpha \, F_{ZT} \, \alpha$, where c_α is the cornering stiffness. Cornering stiffness is linearly approximated for small side slip angles and quadratic for larger angles, and it is given as:

$$c_\alpha = \left.\frac{dF_{YT}(\alpha)}{d\alpha}\right|_{\alpha=0°}$$

The friction coefficient in the lateral direction can be defined as:

$$\mu_Y(S_Y) = \frac{F_{YT}}{F_{ZT}} \;\rightarrow\; c_\alpha = \left.\frac{d\mu_Y(\alpha)}{d\alpha}\right|_{\alpha=0°} F_{ZT}$$

being $\mu_Y = c_{\alpha_1}\alpha$ for small side slip angles and small vertical forces.

As Bauer highlighted in his study and van Aalst [1] mentions, there are some mathematical models for lateral tire friction coefficients, e.g the linear model (for small α), HSRI Dugoff (1969), Burckhardt (1993), Pacejka (2012), Ammon (1997) and Bauer (2015) being the Pacejka model the reference for this previous study and for 4:

$$\mu_{YT}(\alpha) = D\ sin(arctan(D\alpha - E(B\alpha - arctan(B\alpha))))$$

where **B** is the stiffness coefficient, **C** a free parameter, **D** the maximum value and **E** a form coefficient.

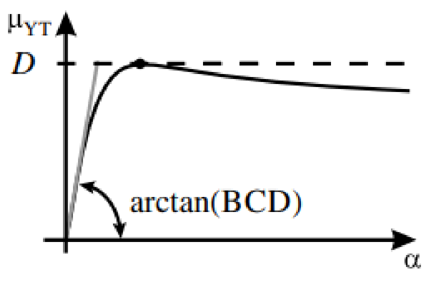

Figure 11: Pacejka friction coefficient model

3.6 Combined longitudinal and lateral tire forces

The aim of this section is to combine both longitudinal and lateral tire forces to understand some of the vehicle behaviour when accelerating or braking in corners.
Now a new concept appears, the resultant slip S_{res}, obtained by applying Pythagoras between the longitudinal slip S_X and the lateral slip S_Y studied previously, hence, the resultant slip is:

$$S_{res} = \sqrt{S_x^2 + S_y^2}$$

Due to the combination of both longitudinal and lateral tire forces, now instead of α, S_Y is taken into account when analysing the side slip. Hence, the generated forces are then:

$$F_{XT} = \mu_{XT}(S_X, S_Y)F_{ZT}$$
$$F_{YT} = \mu_{YT}(S_X, S_Y)F_{ZT}$$

being then the resultant force and friction coefficient:

$$F_{res} = \sqrt{F_{XT}^2 + F_{YT}^2} \qquad \mu_{res} = \sqrt{\mu_{XT}^2 + \mu_{YT}^2} \qquad F_{res} = \mu_{res}F_{ZT}$$

Note that for anisotropic tires with tread profiles the maximal tire friction coefficients are different and they follow $\mu_{Y_{max}} < \mu_{X_{max}}$.
For the combined forces, the Pacejka Magic Formula, can be brought in its most general expression but changing the input *"x"* by the resultant slip S_{res}:

$$y(x) = D\ sin(C\ arctan(BS_{res} - E(BS_{res} - arctan(BS_{res}))))$$

being, for this section, **y(X)** the longitudinal or lateral tire force or the aligning torque, while **B** stands for the slope at zero slip, **C** for the form of the characteristic, **D** for the maximal force or torque and **E** for the relaxation of the characteristic.

3.7 Lateral tire dynamics

As van Aalst [1] mentions in his dissertation, the lateral tire force $F_{YT} = c_\alpha \alpha$ is valid for stationary behaviour. After a change on the side slip angle, the tire builds up a deformation (Δy) due to the lateral stiffness c_y of the tire, thus, we can talk about a lateral force variation $\Delta F_{YT} = c_y \Delta y_T$.
The lateral deformation Δy_T initially builds up with time:

$$\Delta y_T = v_{YT} \Delta t \;\rightarrow\; \Delta y_T = v_T \alpha \Delta t$$

and using the stiffness relation, the expression of the time variation is obtained:

$$\Delta t = \frac{c_\alpha}{c_y v_t} = T_{YT}$$

The time behaviour fot the lateral force ΔF_{YT} after a change on the side slip angle $(\Delta\alpha)$ can be approximated by a dynamic first order differential equation:

$${}_{YT}\Delta \dot{F}_{YT}(t) + \Delta F_{YT}(t) = c_\alpha \alpha \;\rightarrow\; G(s) = \frac{\Delta F_{YT}(s)}{\Delta\alpha(s)} = \frac{c_\alpha}{1 + T_{YT} s}$$

being G(s) its transfer function.

4 Lateral vehicle behaviour models

Up to now, the principles of lateral vehicle dynamics have been introduced for a better understanding of the upcoming content.

The aim of this thesis is to comprise the different lateral vehicle behaviour models, starting from the most simple ones as the kinematic model, which does not take into consideration any of the forces and torques participants in the vehicle motion. Then the dynamic models are presented, making emphasis on one-track model for the main reason that are the most used in scientific experiments and simulations, but not avoiding the principal aspects of general one-track and two-track models for a deeper knowledge in this field. The intention of this section is to introduce the models behind all the simulations and scientific articles brought in this study.

The comprehension of this models will allow to understand much better the background behind all of the Machine Learning applications and their outcomes.

4.1 Kinematic models for lateral dynamics

The kinematic model describes the motion of the vehicle in terms of kinematics. In other words, it only takes into consideration the position, velocity, and the acceleration of the vehicle and disregards the forces acting on the it.

For this part, one of the most simple one-track vehicle-model is used, concretely, the approximation known as the Bicycle model.
The Bicycle model takes a 4-wheel representation and merges the front and rear wheels respectively to form a 2-wheeled model. The target of this approach is to simplify the usually used 4-wheel models with two steering angles into a model with only one steering angle and two wheels, which eases the understanding and the calculations.

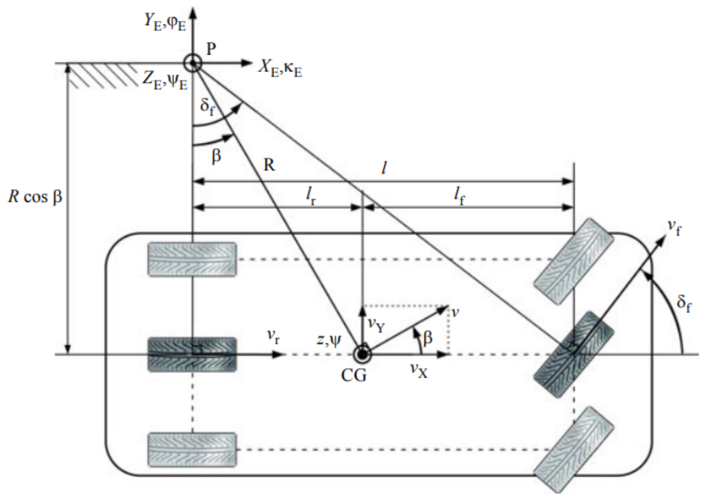

Figure 12: Bicycle model scheme from [2]

In figure 12 the Bicycle model scheme is depicted. As we can see, δ_f is the front wheel steering angle, relative to the x-axis while β is the relation between the velocities in each axis, expressed as:

$$\beta = arctan\left(\frac{v_y}{v_x}\right) \rightarrow \beta \approx \frac{v_y}{v_x}$$

being the approximation only valid for small values of β.
A new concept appears in this section, the yaw rate ($\dot{\psi}$), which is the angular velocity for the vehicle rotation and it is expressed as:

$$\dot{\psi} = \frac{v}{R}$$

being v the module of the velocity vector $[v_x, v_y]$ and R the turning radius from the center of gravity, henceforth CG.
The centripetal acceleration a_c for the CG is given by the next expression, which leads into the vehicle acceleration in the y-axis:

$$a_c = \frac{v^2}{R_p} = v\dot{\psi} \rightarrow a_Y = a_c\, cos\beta$$

Note that for small β the yaw rate can be approximated to $\dot{\psi} = \frac{a_Y}{v}$

Knowing all the above, a motion equation system for this model that can be summarized by:

$$\dot{x} = vcos(\psi + \beta)$$
$$\dot{y} = vsin(\psi + \beta)$$

A more accurate representation of the vehicle requires modeling its dynamics, which considers behaviors like side-slipping, oversteering and understeering and friction. The dynamic models are more accurate than the kinematic models in the sense that they include the applied forces and torques on the vehicle, especially on the tires. Newton's second law of motion and Euler Lagrange methods are applied on the vehicle system to obtain dynamic models. The complete dynamic model is very complex and nonlinear accounting for translation and rotation motions in the 3D space and considering a full vehicle with four wheels. These models are used mainly for validation purposes and are too complicated, thus, simplified two-wheel dynamic models are used instead.
In the next sections, different dynamic models are explained in detail, Starting from the most simple one, the bicycle model, to the basis of dynamic two-track models, which are much more complex and require heavier calculations and a wider range of equations, hence, huge matrix systems will develop when applying two-track models with a large amount of degrees of freedom.

4.2 Dynamic one-track models

The dynamic representation is far more accurate than the kinematic by the fact of including the forces and torques applied to the tire.
Dynamic vehicle behaviour models are obtained for dynamic cornering. In this situation, slow cornering behaviour must be differentiated from speed cornering behaviour.
For slow cornering (v→0), β angle can be neglected for very small values of the steering angle δ_f, as it holds:

$$tan(\delta_f) = \frac{l}{R} cos\beta \approx \frac{l}{R} \quad for \ R <<< l$$

On the other hand, for speed cornering, centrifugal force must be compensated by lateral tire forces. this requires the both front and rear side slip angles α_f, α_r.

In this section a new parameter is presented, the heading angle or course angle (ν) which stands for the direction of the vehicle velocity vector $\vec{v_h}$, thus the course angle is given as $\nu = \psi + \beta$, where ψ is the yaw angle and β the slip angle.

4.2.1 Non-Linear One-Track models

Two representations for one-track model vehicle cornering are given, firstly the yaw rate and velocity representation, then the yaw rate and slip angle representation. These two models show the equations of motion, which are made out of two force balances and one torque.

For the **yaw rate and velocity representation**, the motion equations are:

$$\begin{aligned} \dot{v}_x &= \frac{1}{m} \left[F_{xf} + F_{xr} - F_{xA} - F_{xR} \right] \\ \dot{v}_y &= \frac{1}{m} \left[F_{yf} + F_{yr} - F_{yA} \right] \\ \ddot{\psi} &= \frac{1}{J_Z} \left[F_{y_f l_f} - F_{y_r l_r} \right] \end{aligned}$$

where **A** stands for air drag and **R** stands for Rolling Resistance and being x the longitudinal direction, y the lateral direction and ψ the vertical axis.
As shown in 3.5, the lateral forces depend on the side slip angle α and the vertical load F_Z. For small α, the lateral tire force is proportional to the side slip angle α.
The side slip angle α depends on the steering angle δ_f, the vehicle slip angle β and the yaw rate $\dot{\psi}$. A relation between the tire side slip angles and the vehicle slip angles of all wheels can be established by considering the longitudinal and lateral velocity components of the center of gravity and the tires. See *Isermann's* [2] for further details.
This can lead to obtain the front and rear side slip angles values, given by the next expressions:

$$\begin{aligned} \alpha_r &= arctan \left(\frac{l_r \dot{\psi} - vsin\beta}{vcos\beta} \right) \\ \alpha_l &= \delta_f - arctan \left(\frac{l_f \dot{\psi} - vsin\beta}{vcos\beta} \right) \end{aligned}$$

Now, for the **yaw rate and side slip angle** representation, the motion equations for the longitudinal direction, lateral direction and vertical axis respectively are respectively:

$$-m\dot{v}cos\beta + m\frac{v^2}{R_p}sin\beta + F_{xr} + F_{xf}cos\delta - F_{yf}sin\delta_f - F_{xA} - F_{xR} = 0$$
$$-m\dot{v}sin\beta - m\frac{v^2}{R_p}cos\beta + F_{yr} + F_{xf}sin\delta_f + F_{yf}cos\delta_f + F_{yA} = 0$$
$$-J_Z\ddot{\psi} + (F_{yf}cos\delta_f + F_{xf}sin\delta_f)l_f - F_{y_rl_r} - F_{yA} = 0$$

4.2.2 Linearized One-Track models

In order to simplify the motion equations seen in the previous section, small steering and side slip angles are assumed so some approximations can be made:

$$sin\delta \approx 0 \qquad cos\delta \approx 1$$
$$sin\beta \approx \beta \qquad cos\beta \approx 1$$

Hence, the expressions for the front and rear side slip angle become:

$$\alpha_r = arctan\left(\frac{l_r\dot{\psi} - vsin\beta}{vcos\beta}\right) \approx -\beta + \frac{l_f\dot{\psi}}{v}$$
$$\alpha_l = \delta_f - arctan\left(\frac{l_f\dot{\psi} - vsin\beta}{vcos\beta}\right) \approx \delta_f - \beta - \frac{l_f\dot{\psi}}{v}$$

As *Isermann* says in his literature [2], these approximations can only be made for $\beta \leq 5°$

The same two representations, the yaw rate versus longitudinal velocity and the yaw rate versus side slip angle for stationary cornering, seen previously, can also be simplified by linearizing some of the parameters as it was did before.

By using these simplifications, i.e using the sinus and cosinus simplifications from 4.2.2, the motion equations evolve into:

$$m\dot{v} + m\frac{v^2}{R_p}\beta + F_{xr} + F_{xf} - Fy_f\delta_f - F_{xA} - F_{xR}$$
$$-m\frac{v^2}{R_p} - m\dot{v}\beta + F_{yr} + F_{x_f\delta_f} + F_{yf} + F_{yA}$$
$$-J_Z\ddot{\psi} + (F_{yf} + F_{x_f\delta_f})lf - F_{y_rl_r} - F_{yA}dA$$

From the **yaw rate versus side slip angle representation**, a state-space form can be described for continuous time-variant which leads into a matrix system. This matrix system will be the input for the signal flow system that will allow the simulations to be done:

$$\dot{x}(t) = A(t)x(t) + b(t)u(t)$$
$$y(t) = C(t)x(t)$$

being y(t) = x(t) the matrices of dimension 2x1 that contain the vehicle slip β and the yaw rate $\dot{\psi}$. These lead to $\dot{x}(t)$ when derived. **A** and **b** are the matrices containing the rest of parameters isolated from the motion equations to obtain the previous system. Finally, **C** being the identity matrix and u(t) the variation of the steering angle δ_f throughout the time, $\delta_f(t)$.

The state-space matrix system then becomes:

$$\underbrace{\begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \\ a_{\mathrm{Y}} \end{bmatrix}}_{\mathbf{y}'} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{c_{\alpha\mathrm{f}} + c_{\alpha\mathrm{r}}}{m} & \frac{c_{\alpha\mathrm{r}} l_{\mathrm{r}} - c_{\alpha\mathrm{f}} l_{\mathrm{f}}}{mv} \end{bmatrix}}_{\mathbf{C}'} \underbrace{\begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{c_{\alpha\mathrm{f}}}{m} \end{bmatrix}}_{\mathbf{d}'} \delta_{\mathrm{f}}$$

This model contains parameters that can vary throughout the time. If this happens, then the model will acquire a time variant behaviour, although being linear. These parameters depend on the velocity v and both front and rear cornering stiffness c_{α_f} and c_{α_r}.
To solve this and go for the simulation, as stationary cornering is supposed, then $\dot{v} = 0$ has to be assumed. This allows to neglect the motion equation in the longitudinal direction shown in 4.2.2. Air drag and rolling resistance are neglected too for the simulations. These simplifications lead to a reduced system to develop the signal flow diagram shown in Isermann's simulations [2].

$$\dot{\beta} = \frac{1}{mv}(F_{yf} + F_{yr}) - \dot{\psi}$$
$$\ddot{\psi} = \frac{1}{J_Z}(F_{y_f} l_f - F_{y_r} l_r)$$

For the simulation, the input signal is the steering angle δ_f and the outputs are the yaw rate $\dot{\psi}$ and the slip angle β. As a conclusion for this simulation, the author ends underlining the importance of the rear cornering stiffness c_{α_r} for the vehicle stabilisation.

For the **yaw rate versus velocity** representation $\dot{v} = 0$ is also assumed, and F_{yA} is neglected too, while $\beta \approx \frac{v_y}{v}$. Hence, the motion equations reduce from 3 to 2 equations, as the motion equation in the longitudinal direction is neglected because of the assumption made for $\dot{v}$.

$$\dot{v}_y = \frac{1}{m}\left[c_{\alpha_f}\left(\delta - \frac{v_y}{v} - \frac{l_f\dot{\psi}}{v}\right) + c_{\alpha_r}\left(-\frac{v_y}{v} + \frac{l_r\dot{\psi}}{v}\right)\right]$$
$$\ddot{\psi} = \frac{1}{J_Z}\left[c_{\alpha_f l_f}\left((\delta - \frac{v_y}{v} - \frac{l_f\dot{\psi}}{v}\right) - c_{\alpha_f l_f}\left(-\frac{v_y}{v} + \frac{l_r\dot{\psi}}{v}\right)\right]$$

Dynamic one-track models are the most used in simulations because of its simplicity. There are studies and simulations where models with higher complexity are used to enhance precision, e.g. a model the case of Devineau et al. [4] where a 9-degree-of-freedom (DOF) model is used, or in Setiawan et al. (2009), where a 14 DOF vehicle model was constructed for vehicle dynamics modeling taking into consideration the vehicle ride, handling and tire characteristics among others. Another example is Parra et al. (2019), where a 16-DOF full vehicle model was developed. In those cases then matrices and calculations are much more heavy and the process is more difficult and too costly than they are when bicycle model is used.

4.2.3 Stability and driving conditions

A very important parameter for the lateral behaviour of a given vehicle is the characteristic velocity, which is expressed as:

$$v_{ch}^2 = \frac{c_{\alpha r} c_{\alpha f} l^2}{m(c_{\alpha r} l_r - c_{\alpha f} l_f)}$$

The smaller the characteristic velocity the higher the tendency to understeer. A normal daily car's characteristic velocity oscillates between 60 kph and 120 kph, having a normal passenger car a tendency to 60 and a sports car to 120.

In terms of stability, for single track-models, the characteristic velocity determine the stability conditions throughout driving.

Stability condition is given by the following expressions:

$1 + \frac{v^2}{v_{ch}^2} > 0 \rightarrow$ stable condition

$1 + \frac{v^2}{v_{ch}^2} = 0 \rightarrow$ neutral condition

$1 + \frac{v^2}{v_{ch}^2} < 0 \rightarrow$ unstable condition

Stability is ensured while $v^2 \leq v_{ch}^2$.

A very interesting aspect in lateral dynamics, and probably one of the most iconic topics in automotive is the vehicle behaviour throughout a curve, i.e how the car responds while cornering, which can be described as four different situations:

1. **Neutral-steering:** Is defined as the situation when a vehicle turns at a rate exactly proportional to the rate at which the steering wheel is turned, it is said to have neutral steering.

2. **Under-steering:** Occurs when a the vehicle turns less than desired when cornering, i.e when turning the steering wheel to the left relatively sharply, but the car only turns left slightly. Understeer is usually accompanied by the tires screeching as they lose grip.

3. **Over-steering:** This condition is more usual in rear wheel vehicles. It often happens when too much power is delivered the vehicle lose traction. In extreme circumstances oversteer can cause the rear of the car to spin around the front. This condition is used by professional drivers and delivered in a technique known as drifting.

4. **Counter-steering:** This condition is used to redirect the vehicle when it has completely lost the grip and it is sliding, i.e to take control of the vehicle while drifting.

Figure 13 depicts the different driving conditions cited.

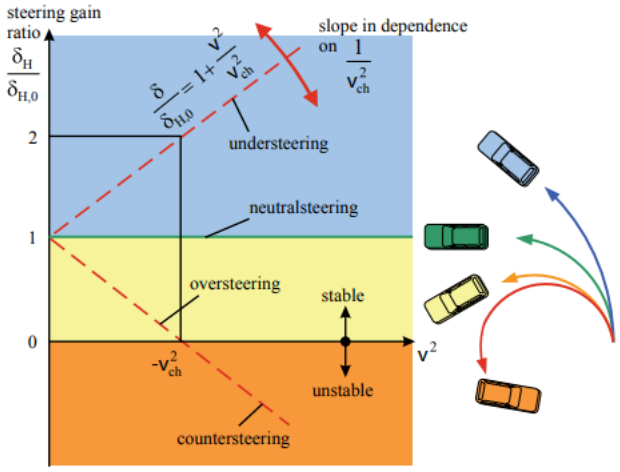

Figure 13: Vehicle behaviour while cornering

4.3 Dynamic two-track models

Unlike one-track models, dynamic two-track models take into consideration the four wheels of the vehicle, which increases the complexity.

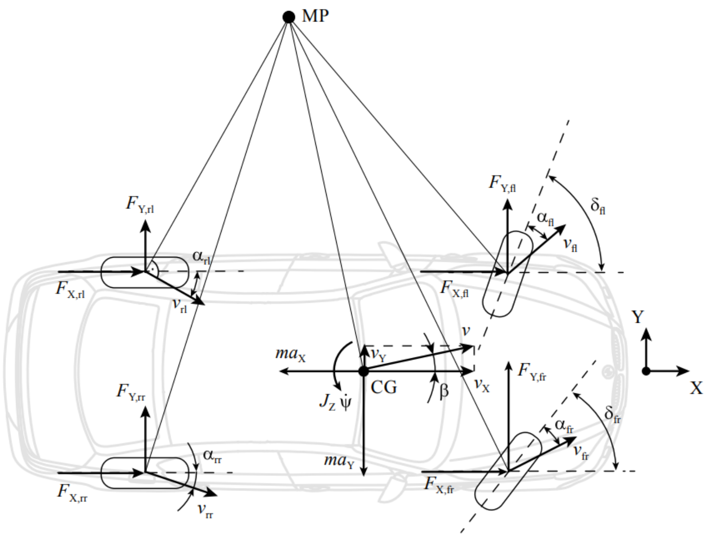

Figure 14: Two-Track model example from [2]

For these models, there are the same two representations than in the one-track models, but now considering four wheels instead of two. Apart from this, as the wheel axes are not centered to the CG of the vehicle, some new momentum arise, so all the calculus and equations become tricky and extensive than they appear in one-track models. Hence, that is one of the main reasons why the bicycle model is usually used in the vast majority of simulations and studies to fashion the vehicle model. Although this condition, a two-track model must be used in order to minimize the error as it ensures accuracy. Some studies show that using machine learning can be useful to ease the parameter estimations, and it (referred to neural networks) also can handle with high complex problems.

4.4 Effects of parameter variations

In his literature, Isermann [2] depicts a very interesting simulation using a two-track model based to show how parameter variations on the vehicle can have a considerable effect. For these simulations, a very similar to Neural Networks algorithm is used, conceretely, a local linear neuronal net model (LOLIMOT).
A steering angle δ_f of 60 degrees is imposed. Then the simulation is done for different ranges of velocity, figure 15, to obtain the results for the lateral acceleration a_Y and for the yaw rate $\dot{\psi}$. As in this study it is not delved into two-track models, the roll angle ϕ which is not taken into account for the simpler models will be omitted. The most interesting part of this simulation is to observe a very usual effect on our daily life, the changing load force, as when driving a vehicle the number of passengers and the trunk load can vary in dependence on the situation.

For this simulation, 4 different situations are experimented:

- Only the driver
- The driver and the copilot
- The driver and three other person
- The driver, three other person and a loaded trunk without exceeding the maximum allowable load

Figure 15 and 16 depict the results obtained in the simulations illustrated by Isermann [2] mentioned in the previous paragraph

These simulations show that as velocity and vertical load increase, they both tend to destabilize the vehicle motion throughout the time. That is why normal vehicles have a maximum tare acceptable to ensure safety driving. Same happens for velocity, depending on how the car is built, and the tires it uses, the driving condition while cornering will be more stable or unstable. For example, a sports car tends to be stable in curves while speeding because of the way it is shaped, but also because of the tires it mounts, which may have more influence in vehicle dynamics than it is thought.

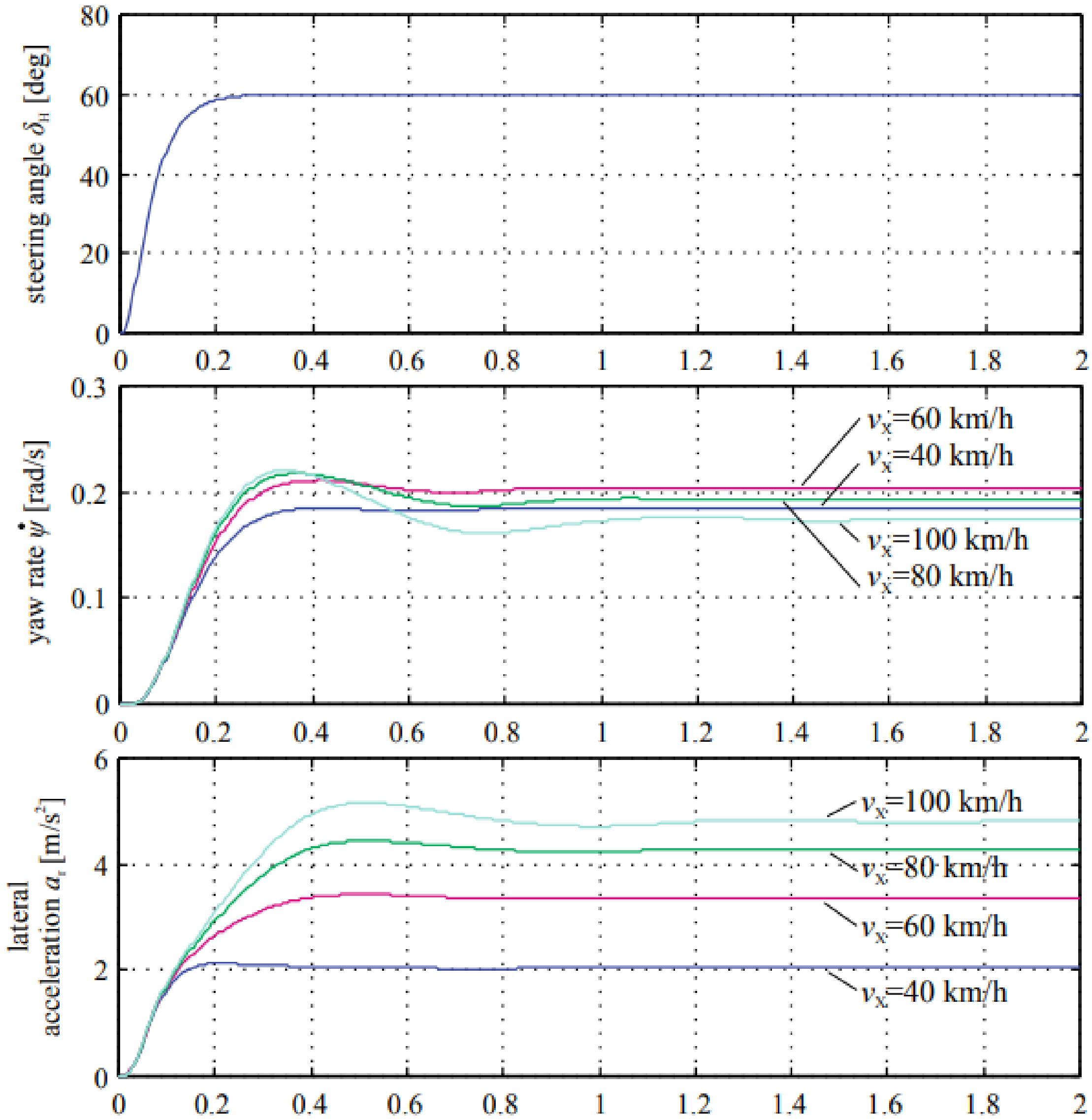

Figure 15: Yaw rate and lateral acceleration for different velocities

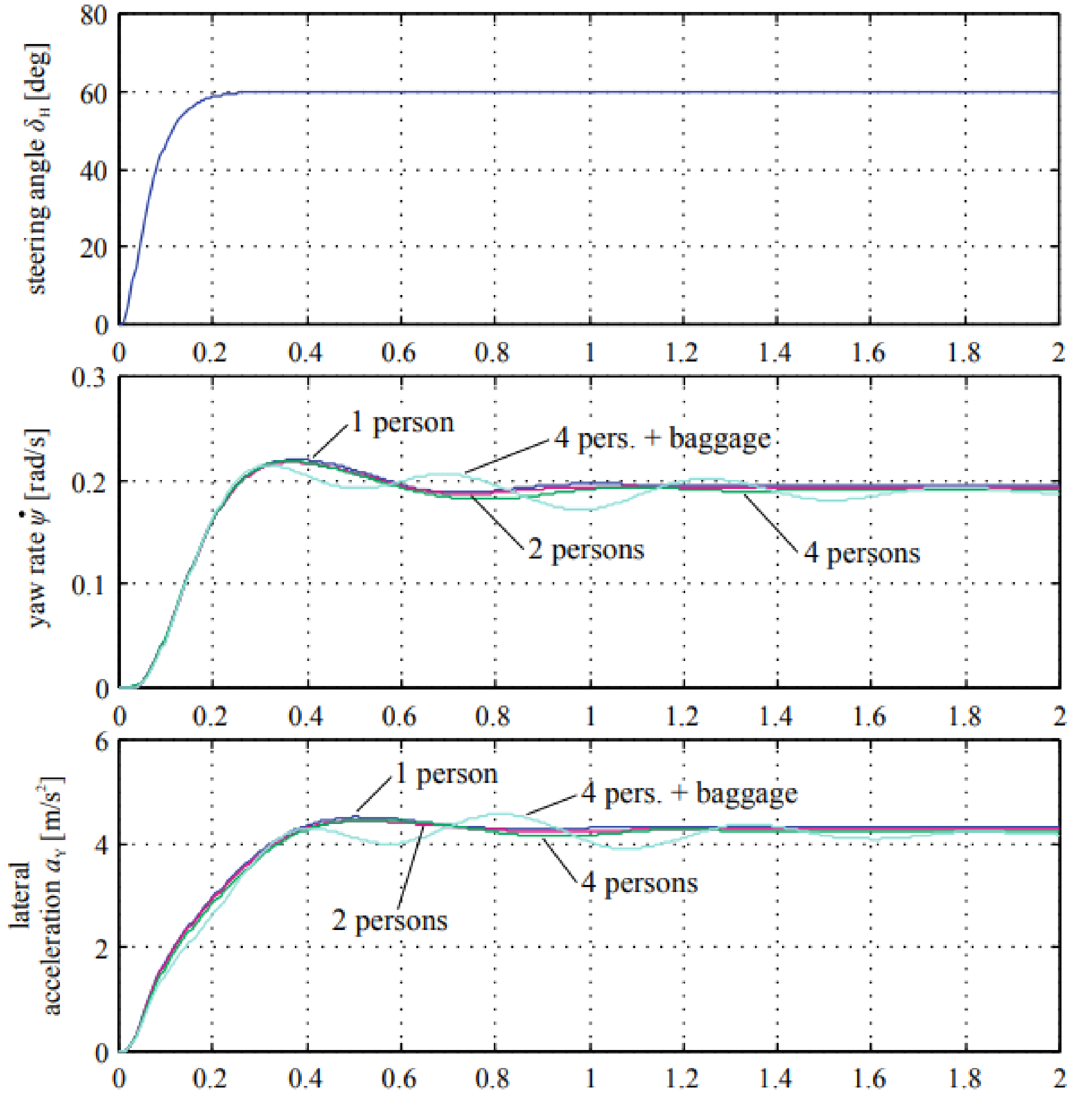

Figure 16: Yaw rate and lateral acceleration for different load distribution

5 Neural network applied to lateral vehicle dynamics

The aim of this first section 5.1 is to make a quick look into the fundamentals of machine learning, this will help to understand the different papers and thesis brought into this study, as they all bring together a joint between lateral dynamics and machine learning and its derivatives.

5.1 Machine Learning introduction

Machine learning is a branch of Artificial Intelligence (AI henceforth) and computer science which focuses on the use of data and algorithms to imitate the way humans learn, gradually improving its accuracy, e.g. Chat GPT, one of the most well-known AI applications.

Three phases constitute the learning process:

- **A decision process** where algorithms are used to make predictions and classifications. Based on the input data, the algorithm will produce an estimation about a pattern in the data.

- **An error function** that evaluates the model prediction.

- **A model optimization process** where the algorithm accuracy will be improved progressively.

There are also three (plus one) primary machine learning methods:

1. **Supervised machine learning** is defined by its use of datasets to train algorithms to classify data or predict outcomes accurately. Input data is fed to the model in order to adjust its weights until it is well-fitted. Neural networks is one of the methods used in supervised learning, among others.

2. **Unsupervised machine learning** uses machine learning to analyze and cluster unlabeled datasets. The principal goal of these algorithms is to find patterns or data groupings without the need of human intervention.

3. **Semi-supervised machine learning** is on its way between both methods cited previously. It uses a little labeled dataste for its training so it can subsequently feature extraction from larger unlabeled datasets.

4. **Reinforcement learning** is similar to supervised learning but in this case, the algorithm learns from trial and error. A sequence of successful outcomes will be reinforced to develop the best recommendation for incoming problems.

There are several machine learning common algorithms, being neural networks one of the most used, among others. This study focuses in some cases where neural networks are applied to take advantage in the investigation and development of vehicle dynamic controllers (and state estimators) using the models seen in 4.

5.1.1 How does a neural network learn?

A usual neural network consists of multiple layers which are the input layer, the hidden layer or hidden layers, depending on the depth of the network, and the output layer. In each layer every node, called neuron, is connected to all nodes in the next layer with parameters called weights.

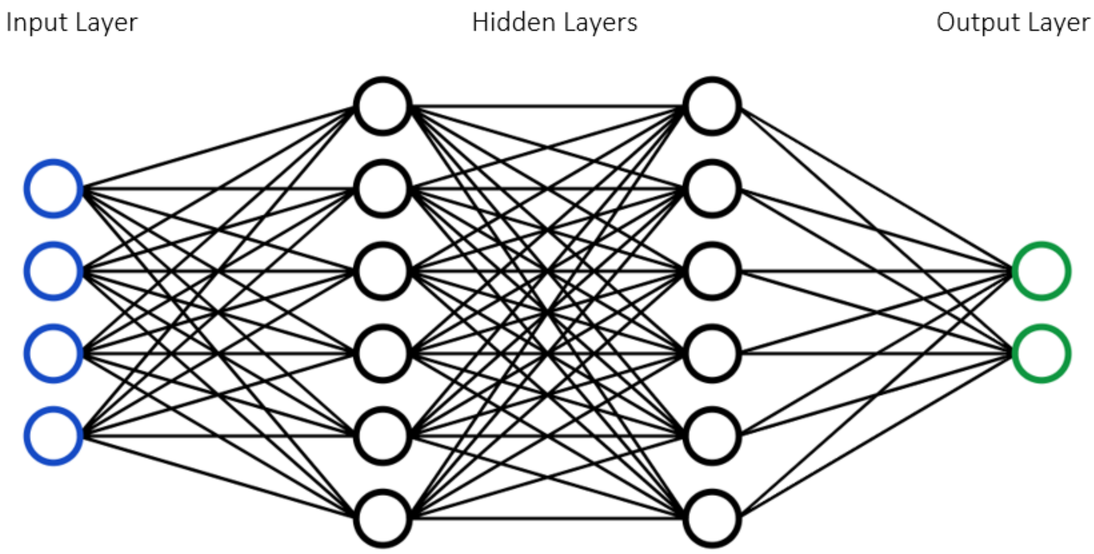

Figure 17: Neural Network example

Weights determine the importance of the node. As much as larger is the weight, as much as important will be the decision or the outcome. It is also important to distinguish the different types of neurons, which are the MP-neurons, the perceptrons and the sigmoid neurons.
Every neuron works as an individual regression model, composed of input data, a bias or threshold, and output. Both input and output data can be Real or Boolean (i.e binary) depending on which type of the above cited neurons it is. Finally, every neural network needs an activation function to determine the output.
Throughout its training, the accuracy is constantly evaluated by means of a loss function. The loss or cost function its often calculated taking into consideration the predicted outputs($\widehat{y}$) and the obtained outputs(y). There are several ways of calculating the loss function, i.e the Mean Squared Error (MSE), the Mean Absolute Error (MAE) or the Hubber loss, among others:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \widehat{y}_i|$$

Deep learning and Neural Networks are usually interchanged and brought together in scientific papers, articles, conversations..., and both could be misunderstood. The word "deep" from Deep learning is referring to the depth of the hidden layers, so Deep learning is just delving deeper into Neural Networks.

5.2 Machine Learning for vehicle lateral dynamic controllers

There are a great amount of examples of how AI and machine learning have helped the automotive industry, e.g. to develop autonomous cars, which may be the most impressive consequence of the machine learning implementation on vehicles. The shift towards autonomous driving is encouraged by the recent developments in artificial intelligence, big data, and information processing techniques. Autonomous driving will considerably reduce road accidents and traffic congestion, air pollution will be mitigated, and energy consumption will be optimized.

Autonomous driving applications are based on three types of neural networks, according to Kebbati et al.[3]:

- Fully Connected Neural Networks
- Convolutional Neural Networks (CNN)
- Recurrent Neural Networks (RNN)

A good example for Supervised learning is Sharma et al.[9], where they tried to implement Deep learning to design a controller for both lateral and longitudinal autonomous vehicle behaviour, concretely, a Convolutional Neural Network (henceforth CNN). They used two CNN-based controllers, one for the steering control and the other one for the speed and a 2-DOF vehicle model, being the the degrees of freedom both cited speed and steering.

One of the keys of this simulation is image processing. The controllers and the models used in the literature use image filtering to perceived the environment where the vehicle is moving, in other words, to detect lanes, path, other vehicles, obstacles... and apply control logic to improve speed and steering autonomy.

Although at first it was tried to fashion one single CNN controller for both velocity and steering control, it did not show the expected success as it was not able to extract a profitable relation between the processed images and the desired output, Sharma et al. say. Hence, two different controllers were used, showing acceptable results and obtaining full autonomy for the vehicle in two different circuits. It did not reach total autonomy in other tracks due to the fact that the model was trained with a database from a concrete circuit, shown in figure 18, and sometimes, the road conditions were very different from the training ones.

A very captivating aspect to point out is that the speed model showed a very good behaviour, which may be crucial in a real life situation, e.g if the car in front suddenly stops.
Combining longitudinal and lateral controllers is useful for safety improving in street cars. The most well-known example is the combination that it is always present in our cars, the Electronic Stability Programme, also known as ESP, which combines traction control (TCS) and Anti-Blockage System (ABS). This controller prevents the car from sliding when accelerating and the tires to block when braking hard. Thus, unifying combined longitudinal and lateral control with the development of a neural network model can evolve into a powerful and robust autonomous vehicle model.

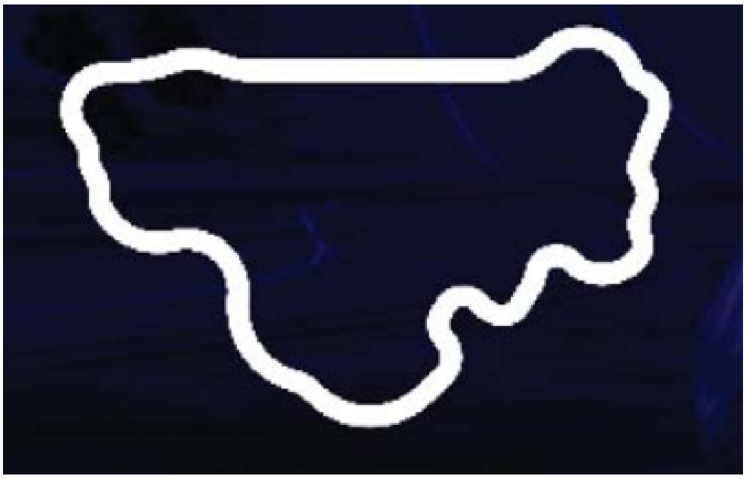

Figure 18: Track used for data acquisition in [9]

Figure 19: Track used in [9]

Devineau et al. [4], as they say that their combined longitudinal and lateral models could be used, with restraint, for "highly" dynamic situations, as obstacle avoidance can be, because of the combination of both phenomena.

Kebbati et al. [3] introduces in section 3.2.3 what is called *The Fuzzy Logic*, that is basically a seeking to model human expertise by using linguistic variables which conceptualize fuzziness in human decision making. Elsayed et al. [10] uses the Fuzzy logic for obstacle avoidance which could be a complementary solution to what is described previously.
A strong point to take into consideration is that Fuzzy logic does not require any mathematical models, thus, high computational costs are not considered.

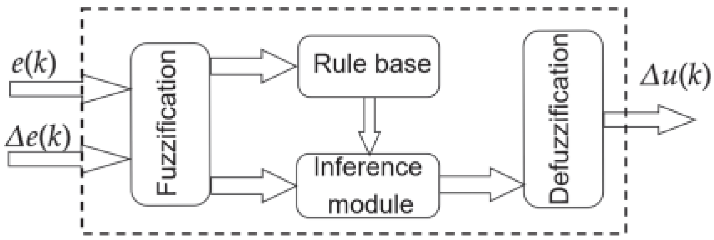

Figure 20: Fuzzy Logic Controller example from [3]

The fuzzy logic process is depicted in figure 20, which consists in four main components apart from the inputs and the output:

1. **The fuzzification process:** It is used to convert the inputs, also known as crisp numbers, into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing.

2. **The rule base:** It contains the collection of rules and the IF/THEN conditions that are set to govern the decision making system.

3. **The inference module:** It determines the matching degree between fuzzy inputs and rules given and decides which rules should be taken out.

4. **The defuzzification process:** It is used to convert the fuzzy sets obtained by the inference module into a crisp value.

Some advantages provided by the usage of the fuzzy logic are the facility to adapt to any type of input, the easiness to design and deploy and little memory storage needed as it can be described with a little range of data. It also provides efficient solutions for complex problems.
Even though all the advantages that kits out, it has some inconveniences. Many researchers have found that using fuzzy logic may lead to some ambiguity and there is no systematic approach to solve a given problem through fuzzy logic.

Resuming the case of Elsayed et al. [10]. Two fuzzy controllers were used for collision decision making. Two inputs are given, obstacle distance and obstacle angle, and two outputs are obtained, safe speed and steering maneuvering.

Figure 21 shows the structure of the fuzzy controller designed by Elsayed et al. in their literature. Weight controller is the responsible for deciding the following state of the vehicle, i.e it decides whether the TGC or the CAC controller is used in dependence on the given situation, while TGC and CAC decisions are based on the weights associated to the risk of the circumstances.ñ In other words, if there is or not an obstacle to avoid.
To determine the fuzzy rules, three linguistic variables for the obstacle distance are defined, followed by six obstacle angle linguistic rules too. This makes a list of eighteen rules that will allow to decide, by considering the weights, the actions that the vehicle has to do in every situation. Figures 22 and 23 show the variables and the weights used in this literature respectively.

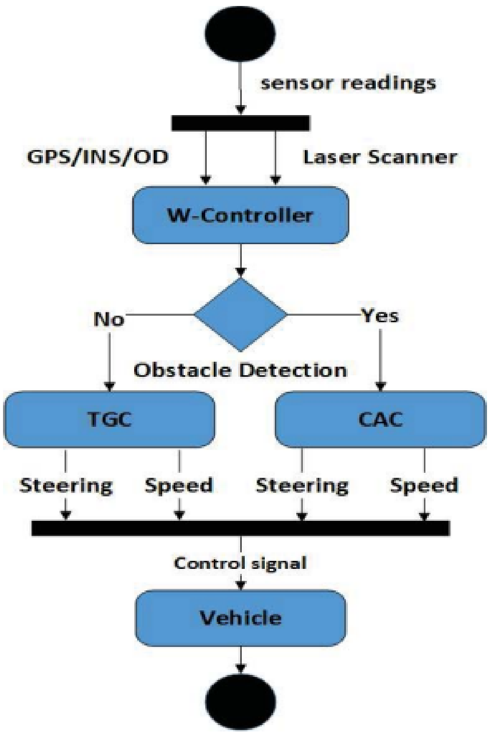

Figure 21: Fuzzy controller scheme from [10]

Note that these variables generate very logical outputs, which is truly captivating.

Rule	Inputs		Output	
	OD	*OA*	*SS*	*SM*
1	Far	Far Left	Med	Right
2	Far	Med Left	Med	Big Right
3	Far	Close Left	Med	VB Right
4	Far	Close Right	Med	VB Left
5	Far	Med Right	Med	B Left
6	Far	Far Right	Med	Left
7	Med	Far Left	Slow	Right
8	Med	Med Left	Slow	B Right
9	Med	Close Left	Slow	VB Right
10	Med	Close Right	Slow	VB Left
11	Med	Med Right	Slow	B Left
12	Med	Far Right	Slow	Left
13	Close	Far Left	V Slow	Right
14	Close	Med Left	V Slow	B Right
15	Close	Close Left	V Slow	VB Right
16	Close	Close Right	V Slow	VB Left
17	Close	Med Right	V Slow	B Left
18	Close	Far Right	V Slow	Left

Figure 22: Fuzzy linguistic variables

Rule	Inputs		Output
	LD	***OD***	***W***
1	Far	Out	VBig
2	Far	Far	Big
3	Far	Med	Med
4	Far	Close	Small
5	Med	Out	VBig
6	Med	Far	Big
7	Med	Med	Med
8	Med	Close	Small
9	Near	Out	VBig
10	Near	Far	Big
11	Near	Med	Med
12	Near	Close	Small

Figure 23: Fuzzy weights

The results obtained are quite good. The obstacle avoiding process is simple. Firstly, once the obstacle is detected, the impulse is sent to the weight controller, who determines that the controller responsible to act when an obstacle is detected, in this particular case, the CAC controller. Then, the CAC adapts the vehicle velocity and the steering to safely elude the obstacle.
These applications can have such a beneficial impact in the traffic conditions as some implementation like this in a normal car could avoid lots of accidents and traffic jams. Although its smooth performance, the specified controller designed and tested in this literature is only available for straights, so as the authors emphasise, there is still lot of room for improvements, starting by adapting this knowledge to develop a similar controller that can be efficient in every driving situation. This is a good starting point though, at least the response from a fuzzy logic controller is pretty much faster and error minimizing than the humans are.

Another very good example of neural networks application for autonomous vehicles is Devineau et al. [4]. A comparison between Multi-Layer Perceptron (MLP) and Convolutional Neural Networks (CNN) is made, using what they call a "challenging track" that combines long straights and tight curves and a dataset with high-fidelity simulations of vehicle dynamics. In this case, a 9-DOF dynamic two-track vehicle model is used instead of using the typical 2-DOF bicycle model (See figures 12 and 14 for further details about both vehicle models), with 5 inputs being the steering angle δ_f and the torque applied to each wheel $(\tau_{\omega_{fr}}, \tau_{\omega_{fl}}, \tau_{\omega_{rr}}, \tau_{\omega_{rl}})$.
The target of this approach is to use two neural networks instead of classic controllers to drive a vehicle around a closed track which combines long straights with sharpen curves, to compare whether or not the implementation of neural networks can be beneficial and more effective than conventional controllers.

Before analysing the results obtained and consequently bringing in the knowledge gained after deepen into their study, it might be interesting to go into detail on how both MLP and CNN networks are based, from the examination of the equations systems to the scheme of each network, in order to get some more insight about neural networks and how they work.

For the Multi Layer Perceptron (MLP), its equations are:

$$h^{(0)} = x$$
$$h^{(k)} = \sigma^{(k)}(W^{(k)T}h^{(k-1)} + b^{(k)})$$

for k=1...L and L being the number of layers.

x $\rightarrow$ the input vector

$h^{(k)} \rightarrow$ the output layer

$\sigma^{(k)} \rightarrow$ the k-th activation function

$h^{(L)} \rightarrow$ the output vector of the neural network

In this case there are 5 layers with 32,32,128,32,128 neurons each respectively and for the activation function, a Rectified Linear Unit (ReLU) is used, $\sigma(x) = max(0, X)$. The loss function is the same for both CNN and MLP, a mean square error (MSE), see equation 56. between the predictions made by the neural network and the controls applied for the simulation.
The Rectified Linear Units transform the values given, cancelling the negative ones and only taking into consideration the positives. This phenomenon is called *sparse activation*.

$$ReLU \rightarrow f(x) = max(0, x)$$
$$f(x) = 0 \; for \; x < 0$$
$$f(x) = x \; for \; x \geq 0$$

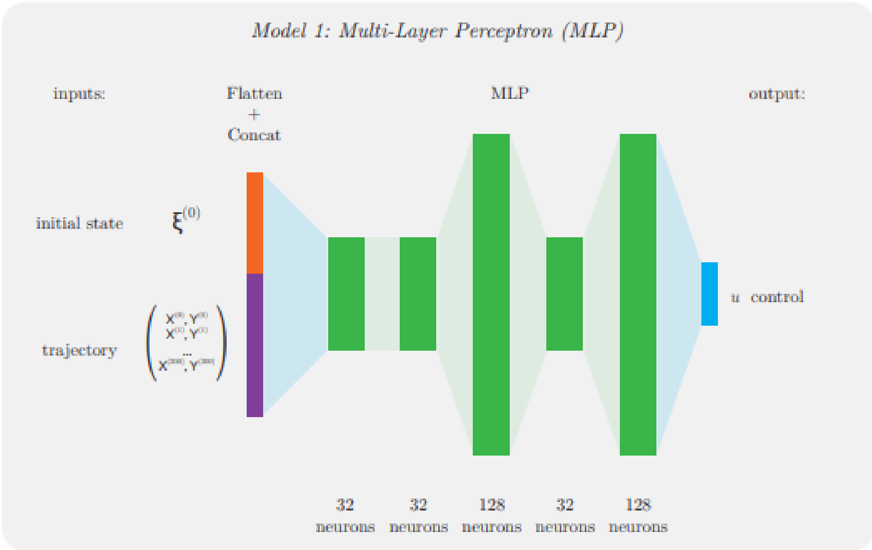

Figure 24: Multi Layer Perceptron from [4]

Now, for the Convolutional Neural Network, the equations are given as:

$$h^{(0)} = x$$
$$h^{(k)} = \sigma^{(k)}(\pi^{(k)}(W^{(k)T}h^{(k-1)} + b^{(k)}))$$

for k=1...L′ and L′ being the number of layers.

$h^{(L')}$ → the output of the CNN

$\sigma^{(k)}$ → the k-th activation function

$\pi^{(k)}$ → the k-th pooling function

In figure 26 it is clearly depicted the module of the Convolutional network. The pooling layer adjusts the number of parameters the network needs to process by reducing the sample size of a feature map, creating a pooled feature map.

It is preferred to delve into their conclusions about the MLP and CNN controllers in sections 2 and 6 (also sections 4 and 7 apply) as they are narrowest curves of the circuit, i.e. where lateral dynamics arise. The results of Devineau et al. show that CNN controller is much better than the MLP one for the main reason that CNN values are smoother and less erratic.
The simulation shows that the neural network brakes on purpose, even if the speed is below the reference and this phenomenon appears because the loss function used during its training, penalizes more the steering angle error than the torque error, thus, it can be assumed that those models prioritise the lateral over longitudinal dynamics.

A PP pure-pursuit controller (green), as well as the Stanley (grey), are compared with the MLP (blue) and CNN (red) for lateral control.

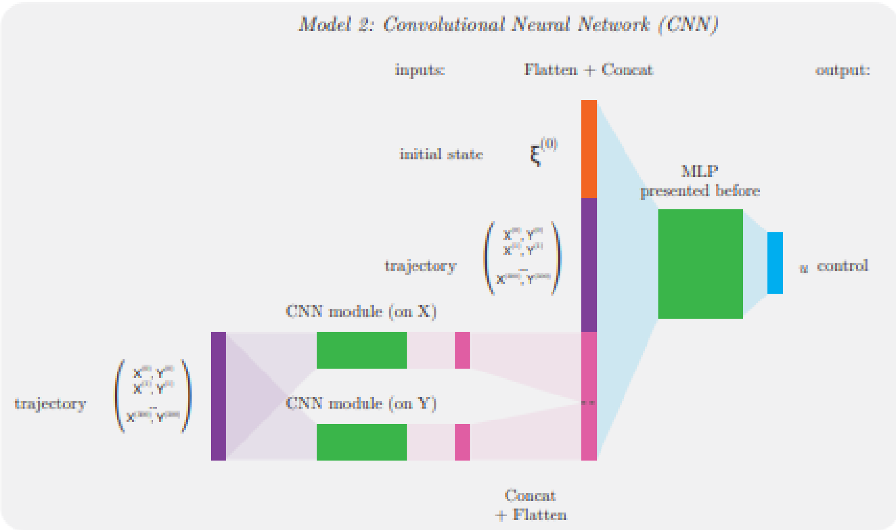

Figure 25: Convolutional Neural Network from [4]

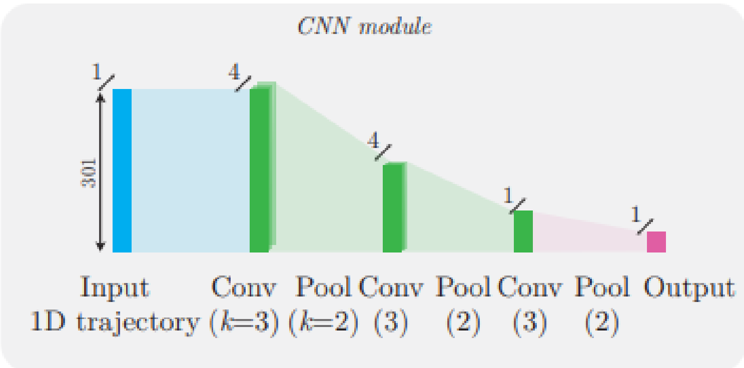

Figure 26: Convolutional Neural Network module from [4]

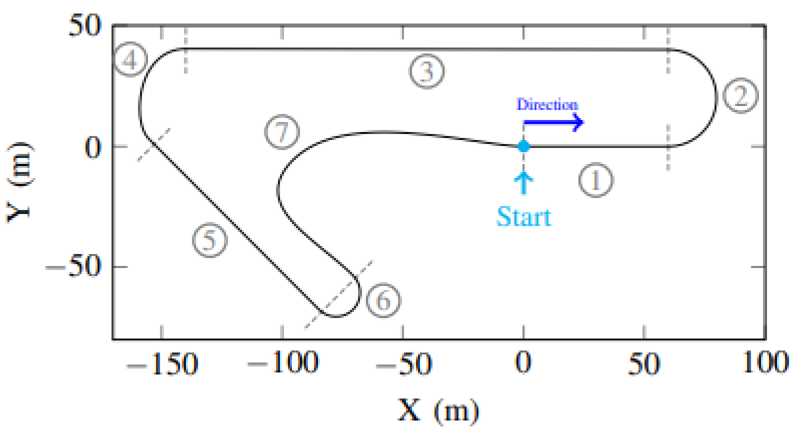

Figure 27: Circuit used to test the models

If we take a look into the results obtained in the simulation for the steering angle shown in figure

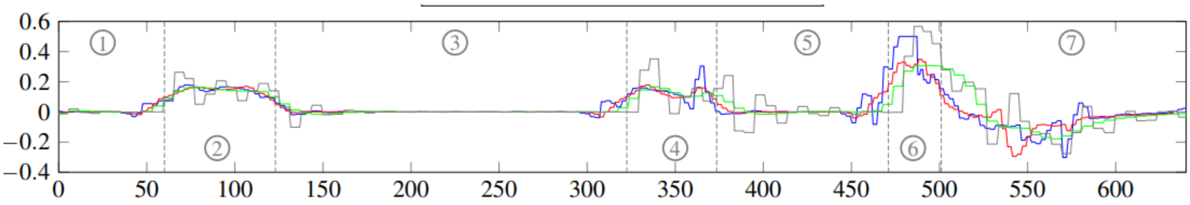

Figure 28: Simulation results for the steering angle

28, it is clearly observable that the areas where the lateral controllers are obviously sections 2, 4, 6, and 7 as they are curved sections, reaching steering angle peak value probably in the apex of the section 6 curve. Apart from the evidenciary, a really captivating phenomenon is the huge difference of smoothness between the conventional controllers and the neural networks. If we put the focus on the the MLP and the CNN trajectories (see figure 29, some corrections of the steering wheel are noticeable, just before entering the curve and at the exit, while the PP has certainly quite more uniform behaviour in terms of steering.

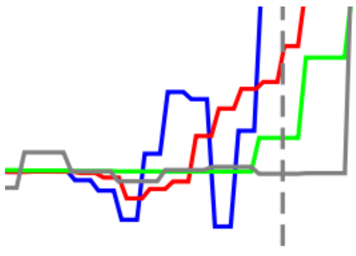

Figure 29: Difference of smoothness between controllers

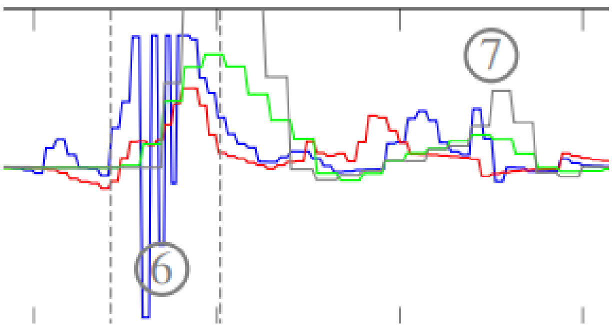

Figure 30: MLP erratic torque response

Figure 30 shows a peculiar response in section 6 in terms of torque, front torque particularly, where the MLP controller seems to not being able to display the torque uniformly while cornering a sharpen curve, while CNN controller really does. Same happens for the rear torque, the MLP is incapable to perform regularly in turn 6.

To summarize the outcome, note that CNN is the best performer with regard to lateral error, as it outperforms every other controller. The PP is the most erratic one.

Authors conclude that CNN is smoother and more accurate compared to other controllers and it can have such interesting outcome in the automotive industry, for autonomous driving and to reduce road accidents and collisions, by the reason that it shows good response to situations with strongly coupled longitudinal and lateral dynamics in a short time, which is pretty similar to the situation described previously when analysing the fuzzy logic controllers.

5.3 Machine Learning for state estimators

With the usual lateral vehicle dynamic models existing, high accuracy and precision are not guaranteed due to the simplifications applied when modelling. These are essential when talking about stability and state estimation.

To solve this issue, Zhou et al. [5] propose to develop a hybrid lateral dynamics model combining physical models with Data-driven, i.e using the existing and simplified 2-DOF bicycle model combined with a fully connected neural network, which they call *Neural Network-based bicycle model*.
This hybrid model is fascinating by the fact that not only combines a traditional one-track model for lateral state estimation and control and a neural network but it also uses a wide range of items seen throughout this study. For instance, to describe the curves of tire lateral forces the Magic Formula, detailed in 3.1.3, is adopted.

After modelling the bicycle model, a wide range of errors appeared by the simple reason of assuming a single-track model and for the fact that some external parameters like road conditions or time varying vertical load are not taken into consideration because of its complexity. Hence, a combined neural network with the existing one-track dynamic model is proposed, using the longitudinal velocity v_x, the steering angle δ_f and the yaw rate $\dot{\psi}$ as inputs and being both front and rear cornering stiffness C_f and C_r the outputs of the neural network. Lateral velocity v_y is not considered for the inputs for the reason that it cannot be accurately measured without an specific sensor, which implies spending a huge amount of money.

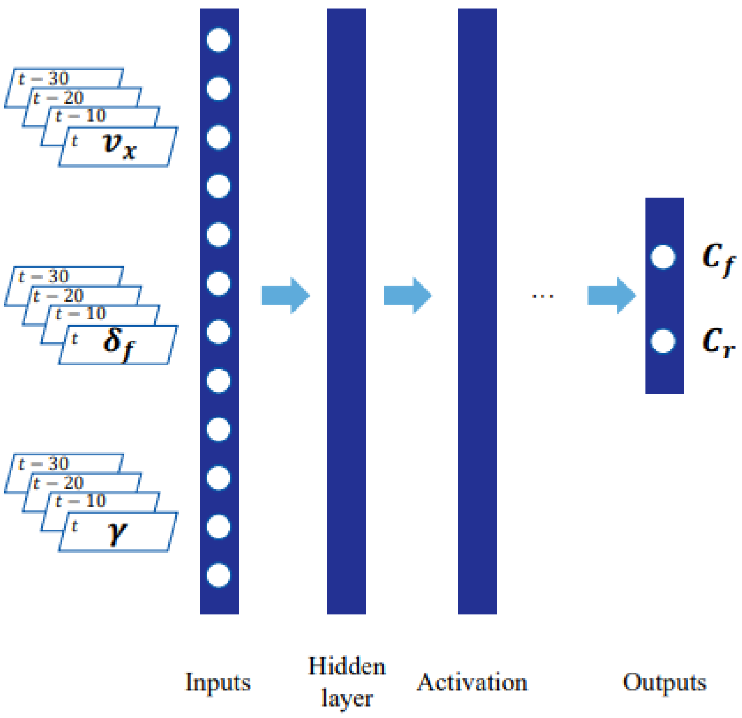

Figure 31: Neural Network used by [5] for parameter estimation

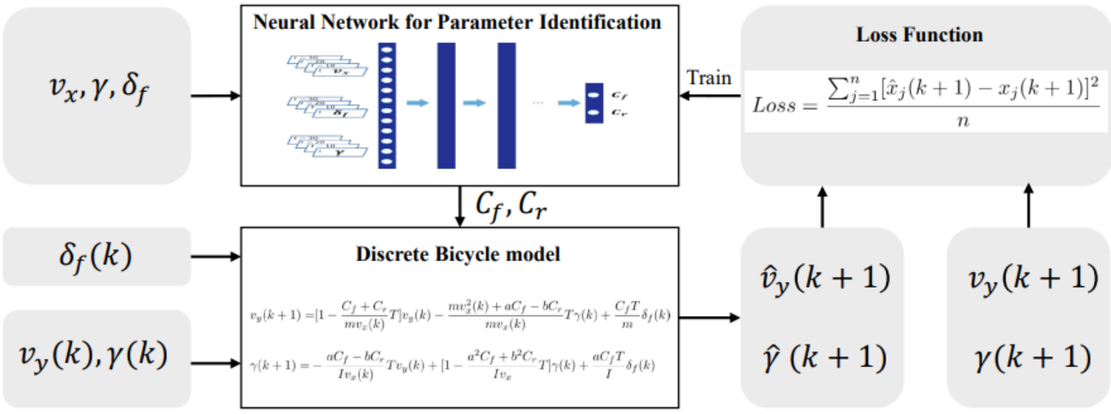

Figure 32: Full hybrid model used for the simulation in [5]

To train the Neural Network, Zhou et al. [5] try to cover the most possible scenarios in order to reflect the vehicle lateral dynamic characteristics with maximum detail. The results obtained in the simulation clearly reflect that the hybrid neural network-based bicycle model is able to predict the velocities and the parameters and it can precisely describe the vehicle lateral dynamics with far more effectiveness than the traditional single-track bicycle model does

Following what the former literature brings in, concretely the establishment of a hybrid model that combines conventional physical models with machine learning derivatives, it is interesting to take a deep look into Sieberg et al. [13] publication as it introduces some aspects seen before in the previous study, as the Kalman filter, which is the base of van Aalst's [1] vehicle state estimation section.

In this particular case, the hybrid state estimation model is given for the roll angle combining and Unscented Kalman Filter with an Artificial Neural Network (UKF and ANN henceforth), so that is considerably complex state to measure but at the same time is needed as an input for safety-critical stabilisation.

The UKF is used instead of the Extended Kalman Filter by the fact that does not require any linearization and as some publications have mentioned, the UKF tends to outperform the EKF. Van Aalst [1] details both extensions.

Another thrilling detail of this literature that has nothing to do with state estimation neither with artificial intelligence, yet it does with pure lateral vehicle dynamics, is the particularity that instead of the x-y plane, the y-z plane is taken for the physical model sketch, which is not very usual to see. In any case, it is useful for the roll angle determination.

Now, delving into the technical aspects of this literature, it is of a significant importance to introduce the hybrid model, which combines the UKF with an Artificial Neural Network (ANN henceforth).
The ANN is used as software sensor to estimate the roll angle ϕ based on a sensor data for lateral acceleration a_Y, longitudinal velocity v_X and the yaw rate $\dot{\psi}$. The UFK is modeled as loop predictor-corrector with the measurement and the system input to estimate states. Figure 33 shows exactly how

this loop works, describing the discrete-time linear system and the measurement respectively:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$$
$$z_k = Hx_k + v_k$$

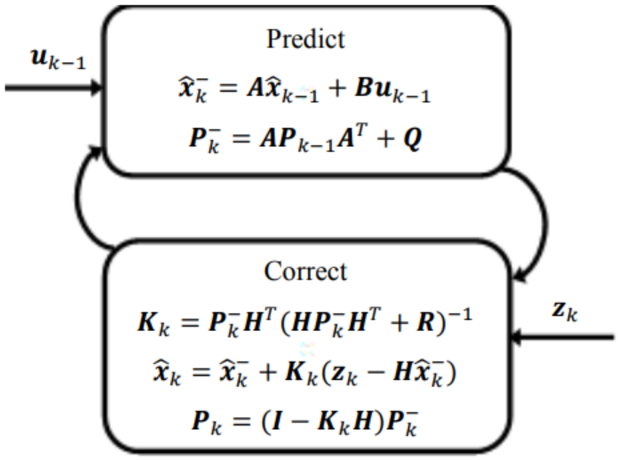

Figure 33: Kalman Filter from [13]

Finally, the ANN and the UKF are unified with the physical model (Non Linear Roll estimator, NLRE henceforth) to become the hybrid state estimator.

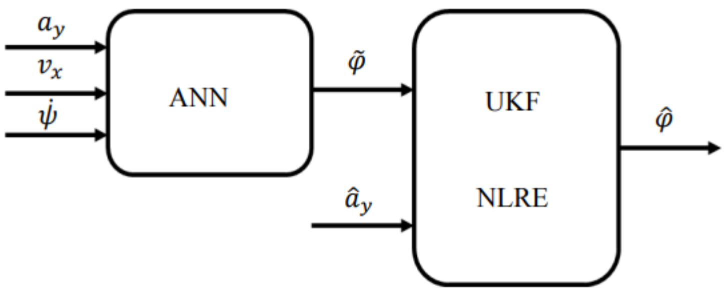

Figure 34: Hybrid State Estimator from [13]

Figure 34 shows the Hybrid state estimator scheme, the UKF filter combines the prediction done by the ANN and the NLRE prediction to achieve the best fit and accomplish higher accuracy.

If we take a look into the estimations done by the ANN and the MLRE a very outstanding phenomenon can be observed. See that for the positive values of the roll angle ϕ, the physical model estimations are very close to the measures, while the ANN tends to be more overfitted. Contrary, when the roll angle is negative the ANN is far more accurate and closer to the measure than MLRE. Separately, both estimators make a poor estimation in terms of accuracy, so here is where the importance of the UKF and the hybrid state estimator arises. As it is previously mentioned, the UKF picks the best fits from each estimator by filtering all the values given and it draws a very much closer estimation to the real measures.

This outcome answers the question of how high accuracy and precision can be guaranteed although model simplifications. Furthermore, ensuring a roll angle stability is beneficial for nowadays vehicles, as the market tendency points to a constant increase in the SUV sales. SUV's have the center of gravity at a considerable height, which makes them eligible to overturn in case of high dynamic situation as an obstacle avoiding can be.

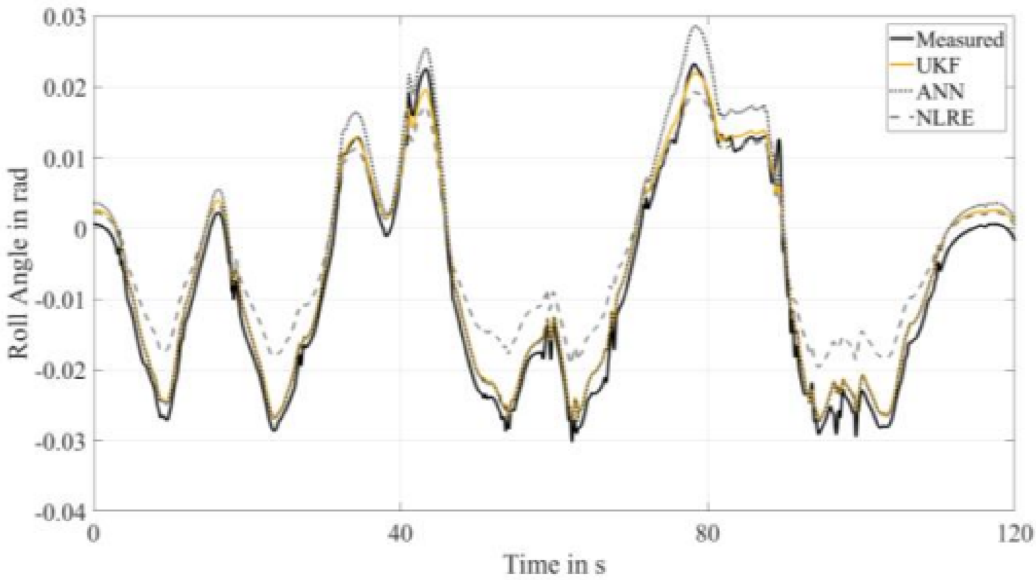

Figure 35: State estimation from [13]

5.4 Model-Based versus Machine Learning comparison

Computer simulation is now gaining ground because the traditional methods of testing are costly as well as risky, and take a long time to prepare. Furthermore, with simulation it is easy to change and modify the sample system and study all design possibilities, operation conditions, and the system response so that the designer can take the appropriate decision on the performance of the system components. Thus mathematical models are realized which are both reliable and trustworthy. The mathematical models are developed by finding out the relationship between all state variables, taking into consideration all changes in the related parameters when operation conditions change affecting the system dynamic response.

In his study, Dr. Abdullah [6] combines the use of a lateral vehicle dynamics model, concretely the bicycle model (as it is the simplest one) with the use of machine learning.
When using these type of models, i.e bicycle model, linear or simplified non-linear one-track models, if a parameter is changed in the vehicle, then they are not reliable anymore. See how parameter variations affect to the lateral dynamics in section 4.4.
The one-track model that Dr. Abdullah [6] uses is considered to be turning. A multi-layer perceptron with two hidden layers is used to predict the lateral force which is trained through learning the relations between the lateral force. the tire air pressure variations, the friction coefficient with the road surface, the side slip angle and the load in the vertical axis obtained previously.

A very interesting comparison between the linear model and the neural network calculations can be made when observing the results that Dr.Abdullah obtained throughout the simulations done. Due to the fact that the bicycle model is a very simplified linear approximation model of a real vehicle its limitations are visible when parameters vary or changes arise. The neural network can face and take into account a larger amount of inconveniences and unexpected changes while the bicycle model is not capable. So, that is why neural networks can be helpful, this type of artificial intelligence ease calculations and can hold high complex models, making the simulations much more closer to the reality than a simplified model does.

Figures 36 and 37 depict the results obtained by Dr.Abdullah in [6] which show the difference explained previously.

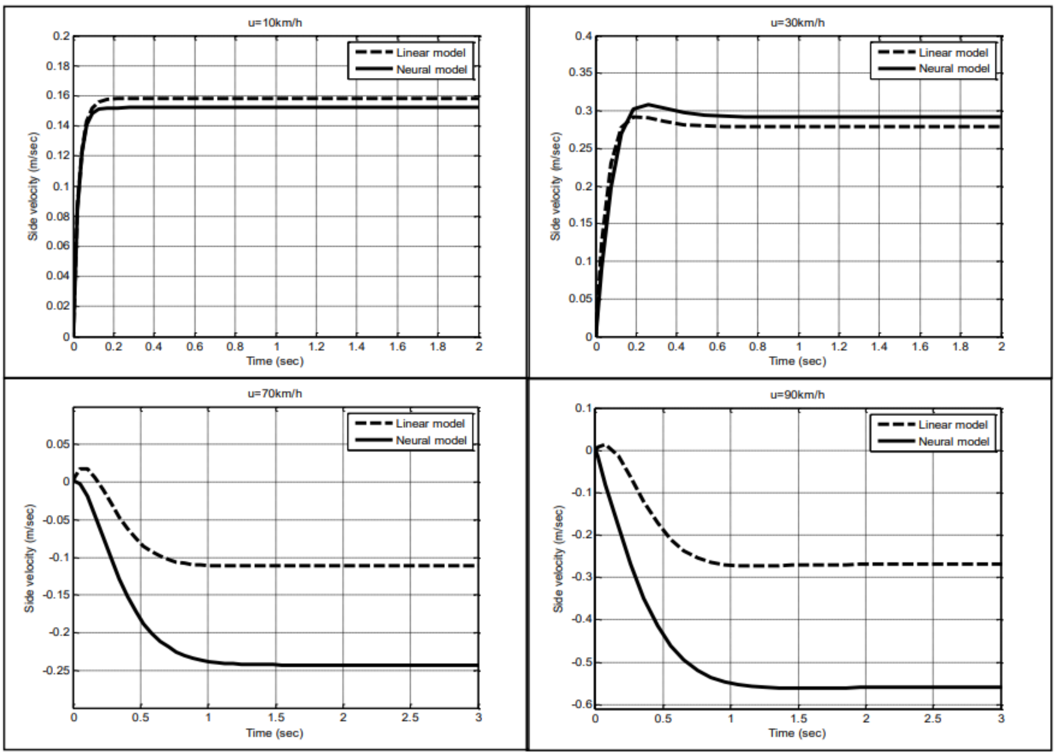

Figure 36: Side velocity response comparison

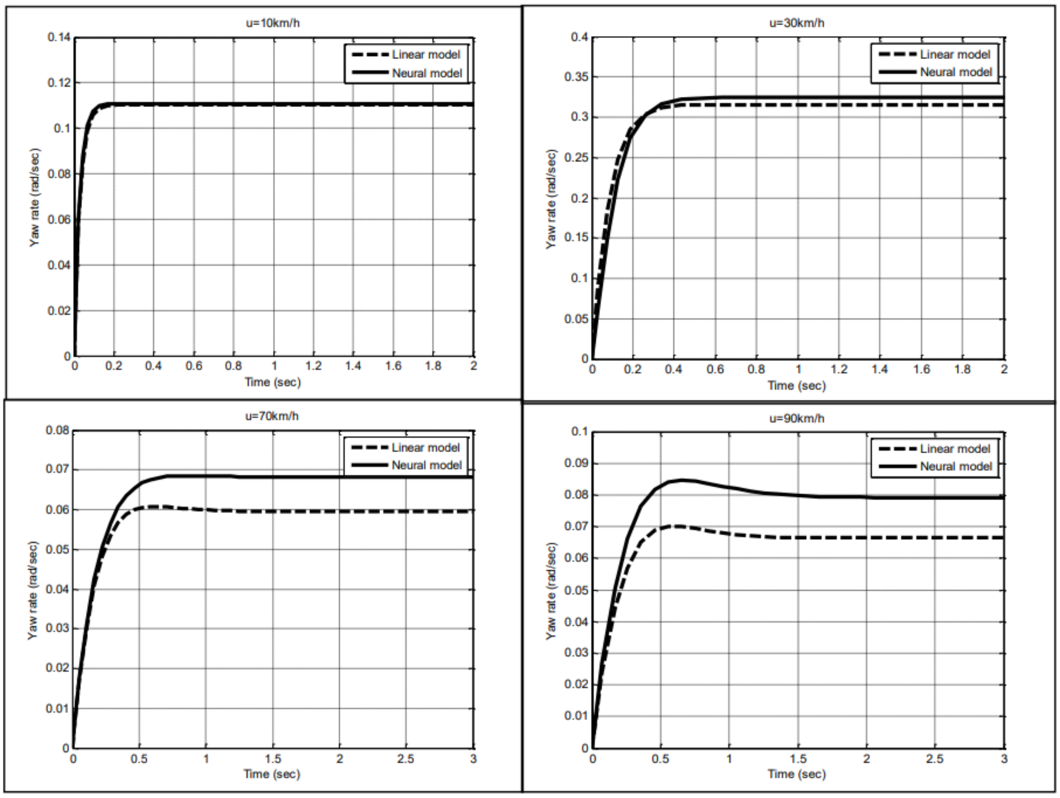

Figure 37: Yaw rate response comparison

Up to now, multiple approaches have been brought into this literature. By far, neural networks based models have demonstrated a clear level of effectiveness, accuracy and precision when compared to conventional physical models. Several publications are based on developing lateral dynamic models for autonomous driving, as it seems this is the fate of the automotive industry for the next decades. Hence, it is vital to adequate our knowledge to improve our vehicles by implementing, whenever is possible, a larger amount of self-taught neural network-based controllers and state estimators which can learn by training them with a wide variety of driving situations and conditions. This can lead to a significant decrease in the number of traffic accidents as well as an increase in safety, comfort and ergonomics for drivers and passengers.

From a technical point of view, it is clear that the evolution in Artificial Intelligence models, such as Neural Networks composed by deep learning, have excelled in terms of performance, as several publications reflect, to estimate parameters as steering angle, slip angle, roll angle, yaw rate, torques, acceleration, velocity and so on, which play a crucial role on vehicle dynamics. This is such an important achievement from the point of view of cost-effectiveness. The use of Neural Networks and computational simulations and predictions allow to engineers and vehicle designers to test as much times as it is needed without sparing a huge amount of money.

Over the last decades, vehicles have been equipped with active systems such as the prior mentioned ABS or ESC controllers. Now, it is possible to enhance these systems by combining it with the knowledge acquired in machine learning and developing superb controllers or state estimators like the ones designed by some of the publications brought into, that combine physical models with deep learning algorithms and state estimation filters which lead into far more accurate predictions than only the physical model would do. Other factors that make deep learning implementation a significant upgrade are, for example, the higher complexity in calculations that deep learning can achieve with a smaller time interval. Model-based methods are less effective in coupling with non-linearities and uncertainties.

This can lead to a notable change in urban and road mobility that we are used to. Conducting the automotive industry to the autonomous driving can have a great impact, by reducing accidents, traffic jams and pollution due to the adaptability and the efficiency that deep learning provides, as it has the capability to mimic human thinking but at the same time it shows the precision and the accuracy of a classic machine. Control is essential for achieving the goals set in terms of autonomy, as it is the responsible of managing the power deployment as speed and the steering in terms of traceability.

Thus, there is no doubt that the combination of the existing physical models with the implementation of machine learning for lateral control and state estimation will be the principal actor in the present and future development of autonomous vehicles and the automotive industry in general.

6 Others

6.1 Schedule planning

The first step in planning the thesis was to choose a topic that interested me. As it is explained in section 1.1, the topic was proposed by Stefano, and it really kept my attention, so I did not have to put too much effort into this field.

During the first months, I invested my time in reading and studying a few vehicle dynamics literature, emphasising in lateral dynamics, as I had very little expertise in this discipline.

Due to a lack of time, it was not possible to do our own simulations to determine the effectiveness and performance of each different lateral vehicle dynamics model presented, which was the first goal.

Later, it was decided to deep into the study of different scientific articles and papers related to machine learning applications, which lead this work to its final objective, a technical review on different approaches with increasing level of model complexity with respect to lateral vehicle dynamics. At this moment, a thorough research was conducted to gather information and data related to machine learning applications for lateral vehicle dynamics controllers and state estimators. This involved reading academic journals, books, and other relevant sources.

Finally, some of the most relevant publications found were brought into the study to develop a technical review and the corresponding conclusions for the entire thesis.

Figure 38 shows the planning schedule followed during this months of work.

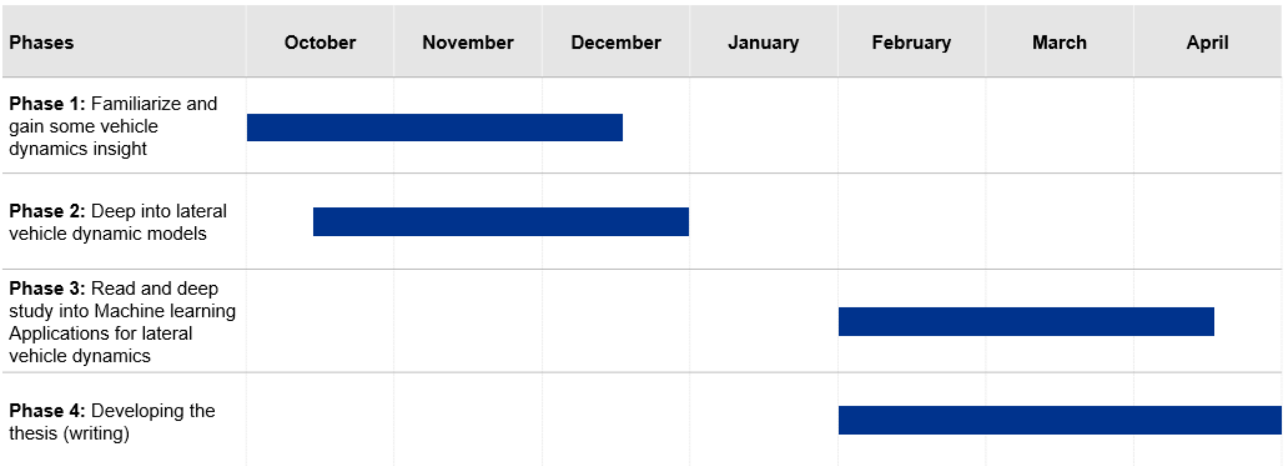

Figure 38: Gant diagram for planning

6.2 Environmental impact

Considering that the actual work is based on a technical review this is not applicable. Though, as it is mentioned before, the environmental impact that several models outcomes can have on our ecosystem is significantly positive, as the implementation of deep learning for autonomous driving can guide the automotive industry to a considerable decrease in pollution levels, due to effectiveness in fuel consumption management, among others.

6.3 Economical viability

Considering that the actual work is based on a technical review this is not applicable.

To take budget cap into account, my own salary for the hours dedicated could be considered, putting my wage in 8€/hour, the same I receive for my internship agreement that I'm enrolled on actually.

6.4 Social and gender equity study

Considering that the actual work is based on a technical review this is not applicable.

Although not being applicable to this work outcome, it is remarkable the difference between authors genders. Statistically, the majority of the articles, publications, journals, books..., in this field are written by men. Maybe it should be a must to encourage women to participate in more dissertations in this discipline in order to enrich diversity and widen the range of opinions as each person has it own point of view.

Conclusions

Lateral vehicle dynamics, which involve the vehicle's ability to move sideways or turn, are crucial for vehicle safety and stability. Historically, engineers have relied on mathematical and physical models to predict and control lateral dynamics, but these models can be complex and limited in accuracy as it is evidenced in section 4. Recently, deep learning techniques have emerged as a promising approach for improving vehicle dynamics, offering the potential for more accurate predictions and better control as the analysis done in this literature confirms.

Deep learning is a subset of machine learning that involves training neural networks with large amounts of data to make predictions and decisions. In the context of lateral vehicle dynamics, deep learning techniques have been used to develop more accurate models for predicting vehicle behavior and to improve control systems for better stability and safety.

One approach to using deep learning for lateral vehicle dynamics is to train neural networks on large datasets of real-world driving data. By analyzing this data, the neural network can learn to predict the vehicle's behavior in various situations, such as lane changes, turns, or emergency maneuvers like the obstacle avoiding controller developed by [10]. These predictions can be used to improve control systems, such as Electronic Stability Control (ESC) or Anti Blockage System (ABS), to prevent the vehicle from sliding or spinning out of control or even wheel blocking when cornering or braking hardly.

Another approach is to use deep learning to directly control the vehicle's lateral dynamics in real-time. This has a direct impact in autonomous driving. As it is shown in several publications, using this technique has lead to engineers developing autonomous driving models capable of reaching full autonomy and drive through a given real track showing a good level of performance when compared to the real measurements. Although autonomous driving is such a captivating achievement, it has not to be forgotten that the use of deep learning can really enhance the actual way of driving, by improving the existing systems and implementing new upgrades to make non-autonomous driving more safety and comfortable. This approach can be used to improve the performance of advanced driver assistance systems (ADAS), such as lane-keeping assist, automated emergency braking or adaptive cruise control.

The use of machine learning and its derivatives for lateral vehicle dynamics has shown several potential benefits over traditional approaches. These include improved accuracy in terms of prediction and control and more flexibility as artificial intelligence can couple with a wide range of driving situations and road conditions. It is more adaptable and can be constantly updated and refined with the data that is being collected periodically by one himself or other engineers. Finally, something to really take into consideration is the real level of improving in terms of safety, by enhancing the accuracy of lateral vehicle dynamics predictions and control, deep learning can contribute to safer driving experiences and reduce the risk of accidents.

Finally, I want to thank Stefano for his involvement, sharing with me some of his insight about vehicle dynamics and machine learning and giving me some useful tools, yet some examples so I could really improve this literature the most.

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